

**FINAL REPORT**

# Meshfree Modeling of Munitions Penetration in Soils

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**September 2022**

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| <b>REPORT DOCUMENTATION PAGE</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                               |                                             |                                                  |                                         | Form Approved<br>OMB No. 0704-0188                                 |  |
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| <b>1. REPORT DATE (DD-MM-YYYY)</b><br>30/09/2022                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                               | <b>2. REPORT TYPE</b><br>SERDP Final Report |                                                  |                                         | <b>3. DATES COVERED (From - To)</b><br>9/28/2018 - 9/27/2022       |  |
| <b>4. TITLE AND SUBTITLE</b><br><br>Meshfree Modeling of Munitions Penetration in Soils                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                               |                                             |                                                  | <b>5a. CONTRACT NUMBER</b><br>18-C-0099 |                                                                    |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                               |                                             |                                                  | <b>5b. GRANT NUMBER</b>                 |                                                                    |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                               |                                             |                                                  | <b>5c. PROGRAM ELEMENT NUMBER</b>       |                                                                    |  |
| <b>6. AUTHOR(S)</b><br>Sheng-Wei Chi                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                               |                                             |                                                  | <b>5d. PROJECT NUMBER</b><br>MR-2628    |                                                                    |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                               |                                             |                                                  | <b>5e. TASK NUMBER</b>                  |                                                                    |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                               |                                             |                                                  | <b>5f. WORK UNIT NUMBER</b>             |                                                                    |  |
| <b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b><br>University of Illinois at Chicago<br>842 W. Taylor Street<br>3087 Engineering Research Facility<br>Chicago, IL 60607-7023                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                               |                                             |                                                  |                                         | <b>8. PERFORMING ORGANIZATION REPORT NUMBER</b><br><br>MR-2628     |  |
| <b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b><br>Strategic Environmental Research and Development Program<br>4800 Mark Center Drive, Suite 16F16<br>Alexandria, VA 22350-3605                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                               |                                             |                                                  |                                         | <b>10. SPONSOR/MONITOR'S ACRONYM(S)</b><br>SERDP                   |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                               |                                             |                                                  |                                         | <b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b><br>MR-2628           |  |
| <b>12. DISTRIBUTION/AVAILABILITY STATEMENT</b><br>DISTRIBUTION STATEMENT A. Approved for public release: distribution unlimited.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                               |                                             |                                                  |                                         |                                                                    |  |
| <b>13. SUPPLEMENTARY NOTES</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                               |                                             |                                                  |                                         |                                                                    |  |
| <b>14. ABSTRACT</b><br><p>The objective of this project is to develop physics-based models and predict the penetration depth of common military munitions in various soil conditions. Ultimately, the models will be used to determine probable depths of munitions in the soil of formerly used defense sites in support of planning for remediation. The simulation results can be used to aid sensor detection and removal of these munitions.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                               |                                             |                                                  |                                         |                                                                    |  |
| <b>15. SUBJECT TERMS</b><br><p>Meshfree Modeling, Munitions Penetration, Soils</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                               |                                             |                                                  |                                         |                                                                    |  |
| <b>16. SECURITY CLASSIFICATION OF:</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                               |                                             | <b>17. LIMITATION OF ABSTRACT</b><br><br>UNCLASS | <b>18. NUMBER OF PAGES</b><br><br>126   | <b>19a. NAME OF RESPONSIBLE PERSON</b><br>Sheng-Wei Chi            |  |
| <b>a. REPORT</b><br><br>UNCLASS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | <b>b. ABSTRACT</b><br>UNCLASS | <b>c. THIS PAGE</b><br>UNCLASS              |                                                  |                                         | <b>19b. TELEPHONE NUMBER (Include area code)</b><br>(312) 996-5024 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                               |                                             |                                                  |                                         |                                                                    |  |

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## **List of Acronyms**

|       |                                                      |
|-------|------------------------------------------------------|
| APCBC | Armor-Piercing Capped Ballistic Cap                  |
| APDS  | Armor-Piercing, Discarding Sabot                     |
| ASTM  | American Society for Testing and Materials           |
| CD    | Consolidated Drained                                 |
| CFL   | Courant-Friedrichs-Lewy                              |
| CPU   | Central Processing Unit                              |
| DNI   | Direct Nodal Integration                             |
| DOP   | Depth of Penetration                                 |
| FEM   | Finite Element Method                                |
| HPC   | High Performance Computing                           |
| MPIM  | Midpoint Integration Method                          |
| MSCNI | Modified Stabilized Conforming Nodal Integration     |
| MSNNI | Modified Stabilized Non-conforming Nodal Integration |
| NSNI  | Naturally Stabilized Nodal Integration               |
| RK    | Reproducing Kernel                                   |
| SCNI  | Stabilized Conforming Nodal Integration              |
| SNNI  | Stabilized Non-conforming Nodal Integration          |
| UIC   | University of Illinois at Chicago                    |
| VCI   | Variationally Consistent Integration                 |

## List of Symbols

|            |                       |
|------------|-----------------------|
| $E$        | Young's Modulus       |
| $\nu$      | Poisson's ratio       |
| $\beta$    | friction parameter    |
| $\rho$     | Density               |
| $\alpha$   | cohesion              |
| $\sigma$   | Cauchy stress         |
| $^{\circ}$ | angle degree          |
| $m$        | Meter                 |
| $cm$       | Centimeter            |
| $mm$       | Millimeter            |
| $m/s$      | meter per second      |
| $s$        | Second                |
| $ms$       | Millisecond           |
| $N$        | Newton                |
| $kg$       | Kilogram              |
| $\%$       | Percent               |
| $mm/min$   | millimeter per minute |
| $kPa$      | kilo Pascal           |
| $psi$      | pound per square inch |

# **Meshfree Modeling of Munitions Penetration in Soils**

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## **1. Project Objective**

The objective of this project is to develop physics-based models and predict the penetration depth of common military munitions in various soil conditions. Ultimately, the models will be used to determine probable depths of munitions in the soil of formerly used defense sites in support of planning for remediation. The simulation results can be used to aid sensor detection and removal of these munitions.

## **2. Summary of Outcomes**

- A robust meshfree code based on reproducing kernel (RK) approximation has been developed for penetration simulation. Several improvements have been developed and implemented to ensure solution accuracy and stability, including
  - Developing the midpoint integration method (MPIM), which offers better robustness for penetration simulation compared to existing methods.
  - Developing and implementing the absorbing boundary condition in the RK formulation to remove the reflected wave due to truncated boundary.
  - Developing the consistent semi-Lagrangian (SL) formulation to address the algorithmic issue due to the convective term.

- Implementing the quasi-linear reproducing kernel formulation, which ensures the consistency of RK approximation under material separation.
- Implementing a gradient-based stabilization technique to improve the stability when a nodal integration is used.
- Implementing a pressure projection method for the u-p formulation, which resolves the instability due to the inf-sup condition.
- Soil constitutive models have been developed and implemented in the meshfree framework to simulate soil behaviors under different saturation conditions during the penetration process. Developments and improvements include:
  - Reformulating the cap surface to better reproduce experimental data.
  - Modifying the plastic potential function so that plastic compaction of the soil may occur even as the shear strength increases.
  - Implementing some algorithmic improvements to enhance the robustness and efficiency of the soil models.
  - Developing strategies to calibrate material parameters for soil models.
  - Performing the sensitivity analysis to identify critical parameters in the soil models.
- Soil properties tests and penetration tests under different saturation conditions have been conducted in the laboratory. The data have been used to verify and validate the developed soil models and meshfree framework.
- The laboratory-tested data and several penetration experiments from the literature have been used to validate the developed meshfree framework for predicting the penetration metrics under different saturation conditions.
- Journal paper publications:
  - Mahdavi, A., Chi, S.W., and Zhu, H., “A gradient reproducing kernel collocation method for high order differential equations,” *Computational Mechanics*, 64: 1421–1454, 2019.
  - Mahdavi, A., Chi, S.W., and Atif, M.M., “A two-field semi-Lagrangian reproducing kernel model for impact and penetration simulation into geo-materials,” *Computational Particle Mechanics*, 7: 351–364, 2020.
  - Parvaneh, S.M., Foster, C.D, and Chi, S.W., “A hardening/softening viscoplastic model for large deformation of soil,” *International Journal for Numerical and Analytical Methods in Geomechanics*, 2022. <https://doi.org/10.1002/nag.3373>

### 3. Task 1 – Enhancement of Multi-Field Reproducing Kernel Computation Framework

#### 3.1. Review of semi-Lagrangian Reproducing Kernel Formulation

This section recapitulates the reproduction kernel (RK) formulation, the main framework for the meshfree simulations in this project. It is to provide the necessary background to present the enhancements we developed and implemented during the performance period.

The RK approximation of a function  $u(\mathbf{x})$ , constructed based on a set of points/nodes  $\{\mathbf{x}_I\}_{I=1}^{NP}$ , is expressed as:

$$u(\mathbf{x}) \approx \sum_{I=1}^{NP} \Psi_I(\mathbf{x}) d_I = \sum_{I=1}^{NP} C(\mathbf{x}; \mathbf{x} - \mathbf{x}_I) \phi_a(\mathbf{x} - \mathbf{x}_I) d_I \quad (1)$$

where  $\mathbf{x}$  is the spatial evaluation point,  $\mathbf{x}_I$  is the position of node  $I$ ,  $\Psi_I$  is the RK shape function associated with node  $I$ ,  $d_I$  is the corresponding coefficient, and  $NP$  is the number of nodes.  $\phi_a(\mathbf{x} - \mathbf{x}_I)$  is the kernel function with a support of radius  $a$  and controls the smoothness and domain of influence of the RK shape function.  $C(\mathbf{x}; \mathbf{x} - \mathbf{x}_I)$  is the correction function and is introduced to ensure solution accuracy up to the desired order. The correction function is commonly expressed in terms of the linear combination of a set of  $m$ -th order complete monomials, the vector form of which is denoted as  $\mathbf{H}(\mathbf{x} - \mathbf{x}_I)$  in the following:

$$C(\mathbf{x}; \mathbf{x} - \mathbf{x}_I) \equiv \mathbf{H}(\mathbf{x} - \mathbf{x}_I) \mathbf{b}(\mathbf{x}) \quad (2)$$

The coefficient  $\mathbf{b}(\mathbf{x})$  can be determined by the reproducing condition [1] [2]. Subsequently, the RK shape function is given as follows.

$$\Psi_I(\mathbf{x}) = \mathbf{H}^T(\mathbf{0}) \mathbf{M}^{-1}(\mathbf{x}) \mathbf{H}(\mathbf{x} - \mathbf{x}_I) \phi_a(\mathbf{x} - \mathbf{x}_I) \quad (3)$$

where  $\mathbf{M}(\mathbf{x})$ , called the moment matrix, is expressed as:

$$\mathbf{M}(\mathbf{x}) = \sum_{I=1}^{NP} \mathbf{H}(\mathbf{x} - \mathbf{x}_I) \mathbf{H}^T(\mathbf{x} - \mathbf{x}_I) \phi_a(\mathbf{x} - \mathbf{x}_I) \quad (4)$$

It is noted that for  $\mathbf{M}(\mathbf{x})$  to be invertible, any spatial point  $\mathbf{x}$  in the domain must be covered by sufficient kernels, the number of which depends on the order of accuracy and the spatial dimension [2].

### 3.2. Quasi-Linear Reproducing Kernel Approximation

To ensure that the RK approximation is constructible and achieves desired order of accuracy, the necessary condition is to have sufficient kernel functions to cover any evaluation point in the problem domain. In other words, the kernel size of RK approximation needs to be large enough to cover any points in the problem domain. However, it may be difficult to achieve under extreme deformations involving fragmentation and material separation. To ensure that the moment matrix (4) is always invertible and the approximation achieves at least linear-order accuracy, the quasi-linear RK approximation [3] is introduced in the current formulation and implemented in the code.

An additional set of  $NQ$  sampling points is introduced in the derivation, as shown in Figure 1.

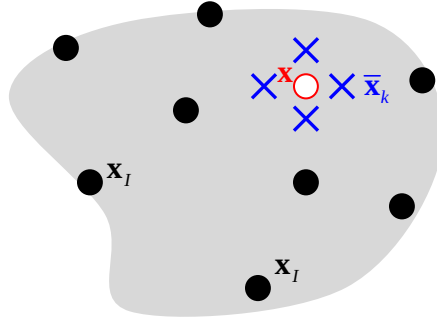


Figure 1. Schematic of additional sampling points for quasi-linear reproducing kernel approximation.

After imposing the reproducing kernel, the quasi-linear RK approximation takes the following form:

$$\tilde{\Psi}_I(\mathbf{x}) = \mathbf{H}^T(\mathbf{0}) \tilde{\mathbf{M}}^{-1}(\mathbf{x}) \tilde{\mathbf{H}}(\mathbf{x}, \mathbf{x}_I) \phi_a(\mathbf{x} - \mathbf{x}_I) \quad (5)$$

where

$$\tilde{\mathbf{M}}(\mathbf{x}) = \mathbf{M}(\mathbf{x}) + \varpi \bar{\mathbf{M}}(\mathbf{x}) \quad (6)$$

$$\bar{\mathbf{M}}(\mathbf{x}) = \sum_{I=1}^{NP} \sum_{k=1}^{NQ} \mathbf{H}(\mathbf{x} - \bar{\mathbf{x}}_k(\mathbf{x})) \mathbf{H}^T(\mathbf{x} - \bar{\mathbf{x}}_k(\mathbf{x})) \phi_a(\mathbf{x} - \mathbf{x}_I) \quad (7)$$

In Equation (6),  $\varpi$  is a tunable parameter, which is chosen as 0.1 in the numerical results unless otherwise mentioned [3].

### 3.3. $u$ - $p$ Formulation for Modeling Soil Materials

A  $u$ - $p$  mixed RK formulation based on the Biot mixture theory (Biot, 1956; Zienkiewicz et al., 1999) has been used to account for the coupling effect between solid and fluid in geomaterials. The governing equations are the balance of momentum equation and continuity equations for fluid phases, respectively:

$$\sigma_{ij,j} + \rho g_i = \rho \ddot{u}_i \quad (8)$$

$$\alpha \dot{u}_{i,i} + \frac{\dot{P}}{M} + q_{i,i} = 0 \quad (9)$$

where  $\sigma_{ij}$  is the total stress,  $g_i$  is the gravity,  $u_i$  is the displacement of the solid skeleton,  $\alpha$  is the Biot's coefficient,  $P$  is the water pressure,  $M$  is the Biot compressibility modulus, and  $q_i$  is the superficial velocity.  $\rho$  is the average mass density of the porous medium defined as:

$$\rho = (1 - n^F) \rho^S + n^F s^W \rho^W \quad (10)$$

where  $n^F$  is the porosity,  $s^W$  is the degree of water saturation, and  $\rho^S$  and  $\rho^W$  are the densities of the solid and water phases, respectively. The total stress can be decomposed into partial stresses from solid and fluid, i.e.  $\sigma_{ij} = \sigma_{ij}^S + \sigma^F \delta_{ij}$ , where  $\sigma_{ij}^S$  is the solid partial stress,  $\sigma^F$  is the fluid partial stress, and  $\delta_{ij}$  is the Kronecker delta. By introducing the Biot coefficient  $\alpha$ , the total stress can be alternatively represented as

$$\sigma_{ij} = \bar{\sigma}_{ij} - \alpha s^W P \delta_{ij} \quad (11)$$

where  $\bar{\sigma}_{ij}$  is the effective stress of the solid phase. The soil constitutive law developed in the following section can be introduced for determining the effective stress  $\bar{\sigma}_{ij}$ . The Biot coefficient,  $\alpha$ , and Biot compressibility modulus,  $M$ , can be defined as

$$\alpha = 1 - \frac{K}{K^S} \quad (12)$$

$$\frac{1}{M} = C^S + \frac{n^F s^W}{K^W} + \frac{s^W (\alpha - n^F) (s^W + C^S P / n^F)}{K^S} \quad (13)$$

and

$$C^S = n^F \frac{\partial s^W}{\partial P} \quad (14)$$

where  $K$  is the bulk modulus of the porous medium, and  $K^S$  and  $K^W$  are the bulk moduli of the solid grains and water, respectively. For isotropic porous media, the relationship between the flux or superficial velocity  $q_i$  and the pore pressure  $P$  can be described by using Darcy's law as

$$q_i = -k_{ij}^W (P_{,j} - s^W \rho^W g_j) \quad (15)$$

where  $k_{ij}^W = (k^{rw} / \mu) k_{ij}^{in}$  is the permeability tensor,  $k_{ij}^{in}$  is the intrinsic permeability,  $k^{rw}$  is the relative permeability to water, and  $\mu$  is the water dynamic viscosity.

The general boundary conditions for Equations (8) and (9) are:

$$\begin{aligned} u_i &= \bar{u}_i & \text{on } \Gamma_u \\ n_i^t \sigma_{ij} &= \bar{t}_j & \text{on } \Gamma_t \\ P &= \bar{P} & \text{on } \Gamma_p \\ n_i^q q_i &= \bar{s} & \text{on } \Gamma_q \end{aligned} \quad (16)$$

where  $\bar{u}_i$  is the prescribed displacement on  $\Gamma_u$  and  $\bar{t}_j$  is the prescribed effective traction on  $\Gamma_t$  with the outward unit normal  $n_i^t$ ;  $\bar{P}$  is the prescribed water pressure on  $\Gamma_p$  and  $\bar{s}$  is the prescribed water influx on  $\Gamma_q$  with the outward unit normal  $n_i^q$ .

The variational forms, and subsequently the Galerkin formulation of Equations (8) and (9) are given as follows:

$$\int_{\Omega} \delta u_{i,j}^h \bar{\sigma}_{ij}^h d\Omega - \int_{\Omega} \delta u_{i,j}^h \alpha s^w P^h \delta_{ij} d\Omega + \int_{\Omega} \delta u_i^h \rho \dot{u}_i^h d\Omega = \int_{\Gamma_t} \delta u_i^h \bar{t}_i d\Gamma + \int_{\Omega} \delta u_i^h \rho g_i d\Omega \quad (17)$$

$$\int_{\Omega} \delta P^h \alpha s^w \dot{u}_{i,i}^h d\Omega + \int_{\Omega} \delta P^h \frac{\dot{P}^h}{M} d\Omega + \int_{\Omega} \delta P_{,i}^h \frac{k}{\mu} P_{,i}^h d\Omega + L2 = \int_{\Gamma_q} \delta P^h \bar{s} d\Gamma + \int_{\Omega} \delta P_{,i}^h \frac{k}{\mu} s^w \rho^w g_i d\Omega \quad (18)$$

Above, the approximation functions of displacements  $u_i^h$ , and water pressure  $P^h$ , as well as their variations,  $\delta u_i^h$  and  $\delta P^h$ , are constructed using the semi-Lagrangian RK shape function given in Equation (1). The term  $L2$  in Equation (18) is an additional term introduced for stability due to the *inf-sup* condition. Many different schemes have been proposed, for example, White and Borja, 2008; Zienkiewicz et al., 2010; Sun et al., 2013. We have employed a least-squares projection method (Wei et al. 2020) when using implicit time integration and  $L2$  in (18) reads,

$$L2 = \int_{\Omega} w (\delta P^h - \mathbf{L} \delta P^h) (P^h - \mathbf{L} P^h) d\Omega \quad (19)$$

where  $\mathbf{L}$  is the projection operator, and  $w$  is a stabilization parameter (Sun et al. 2013; Wei et al. 2020). One of the critical computational issues under the meshfree formulation is the domain integration involved in Equations (17) and (18) [1]. The Stabilized Nonconforming Nodal Integration [2] [3] has been implemented to achieve a stable and sufficiently accurate scheme for penetration modeling. The stability and accuracy have been further enhanced using a midpoint integration. This method is described in the following section.

### 3.4. Midpoint Integration Method

In the meshfree formulation used in this project, the semi-Lagrangian Reproducing Kernel formulation [4], the approximate solution is obtained based on the Galerkin formulation, where the integration over the entire domain is required. Despite efforts made over the past two decades, domain integration remains challenging in Galerkin-based meshfree methods due to instability and sub-optimal convergence. Although background cell integration with a high order quadrature rule

can achieve convergence and stability, it requires the integration mesh for quadrature points and becomes computationally inefficient in the semi-Lagrangian formulation.

While a node-based integration scheme, such as direct nodal integration (DNI), is desired in the meshfree methods since it can naturally cope with the point-based approximation and offer simplicity and computational efficiency, it suffers from instability and low convergence rates. Several nodal integrations were developed to alleviate the abovementioned drawbacks. Chen et al. [5] introduced the stabilized conforming nodal integration (SCNI) to achieve optimal convergence and remove spurious zero-energy modes due to nodal integration. Since constructing conforming integration cells is almost infeasible in the semi-Lagrangian formulation, a non-conforming version of SCNI, termed stabilized non-conforming nodal integration (SNNI), was later proposed [6]. Nonetheless, spurious, oscillatory low-energy modes still exist in both SCNI and SNNI and cause instability. Additional stabilization terms were introduced to further remove the low-energy spurious modes, resulting in modified SCNI (MSCNI) and SNNI (MSNNI) [7]. Chen et al. [8] introduced a variationally consistent integration (VCI) to restore the desired order of exactness for any numerical integration schemes in the Galerkin formulation and achieve optimal convergence. Hillman and Chen [9] introduced the naturally stabilized nodal integration (NSNI) to eliminate the spurious low energy modes in direct nodal integration with a tunable parameter. However, a combination of NSNI with VCI maintains optimal convergence and stability, and the requirement of contour integration in the VCI posts a major challenge in meshfree simulations involving contact and materials separation. A novel domain integration method, termed the midpoint integration method (MPIM), is developed to achieve accuracy and stability for extremely large deformation analyses. The MPIM is derived based on the modified Simpson's rule. It needs neither conforming integration cells nor contour integrals to ensure accuracy and stability, while the resultant discrete system is symmetric, and thus the momentum conservation is preserved.

The schematic of the integration subdomain associated with a meshfree node in the MPIM is illustrated in Figure 2, where a one-dimensional domain is discretized non-uniformly, and the midpoint, node, and evaluation points are depicted. The modified Simpson's rule for the integration of a function  $f$  within  $\Delta x = \Delta x_1 + \Delta x_2$ , which is sixth-order accurate, can be expressed as follows.

$$\int_{x-\Delta x_1}^{x-\Delta x_2} f \, dx = \left( \frac{7f(x-\Delta x_1) + 16f(x_m) + 7f(x-\Delta x_2)}{30} \right) \Delta x - \left( \frac{f'(x-\Delta x_2) - f'(x-\Delta x_1)}{60} \right) \Delta x^2 + O(\Delta x^6) \quad (20)$$

where  $x_m$  is the middle point of the subdomain, and  $\Delta x$  is the length of the subdomain.

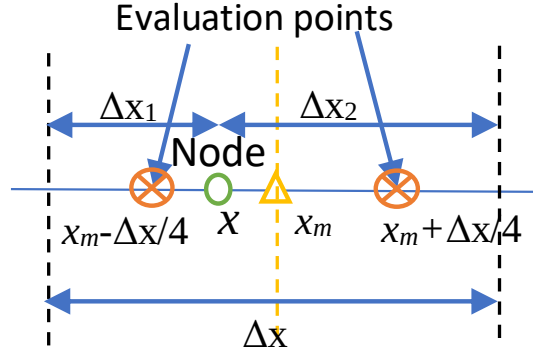


Figure 2. Schematic of the node subdomain.

The modified Simpson's rule is derived based on a conforming mesh, which becomes computationally very expensive in the penetration simulations. To overcome the above issue, a midpoint integration method (MPIM) is developed without conforming integration subdomains. It is designed to achieve optimal integration accuracy using only evaluation points that are at an equal distance from the midpoint of the subdomain. The MPIM in one dimension is given as follows and can be shown to achieve the accuracy of 4th order with a non-uniform mesh.

$$\int_{x-\Delta x_1}^{x-\Delta x_2} f \, dx = \left( \frac{f(x_m - \Delta x/4) + f(x_m + \Delta x/4)}{2} \right) \Delta x + \left( \frac{f'(x_m + \Delta x/4) - f'(x_m - \Delta x/4)}{48} \right) \Delta x^2 + O(\Delta x^4) \quad (21)$$

The MPIM can be extended for two and three dimensions using the tensor product. Figure 3 illustrates the integration subdomains, a rectangle for 2D and a cuboid for 3D, around the midpoint. The corner points are selected as the evaluation points. For 2D, the midpoint integration rule is calculated as:

$$\iint f dx dy \approx \left( \sum_{I=1}^4 f(x_I) / 4 \right) \iint dx dy + g_x ml_x + g_y ml_y \quad (22)$$

where  $ml_x$ ,  $ml_y$  are derived to achieve desired accuracy [10]. In this study,  $ml_x = \Delta x A / 96$  and  $ml_y = \Delta y A / 96$  with  $A = \iint dx dy$  are derived and  $g_x = \sum_{I=1}^4 \nabla_x f(x_I) n_x$ ,  $g_y = \sum_{I=1}^4 \nabla_y f(x_I) n_y$  with  $n_x$  and  $n_y$  the x- and y- components of the outward unit normal, respectively. Similarly, for 3D, the midpoint integration rule is calculated as:

$$\iiint f dx dy dz \approx \left( \sum_{i=1}^8 f(x_i) / 8 \right) \iiint dx dy dz + g_x ml_x + g_y ml_y + g_z ml_z \quad (23)$$

Above  $ml_x = \Delta x V / 192$ ,  $ml_y = \Delta y V / 192$ , and  $ml_z = \Delta z V / 192$  with  $V = \iiint dx dy dz$  are derived and  $g_x = \sum_{I=1}^8 \nabla_x f(x_I) n_x$ ,  $g_y = \sum_{I=1}^8 \nabla_y f(x_I) n_y$ , and  $g_z = \sum_{I=1}^8 \nabla_z f(x_I) n_z$ . As a result, the MPIM approximates the domain integration of a function around the midpoint of the subdomain created at an initial configuration. One of the advantages of the method is that it does not require VCI correction, which is not straightforward to obtain when dealing with continuous evolving surfaces, to attain the desired convergence.

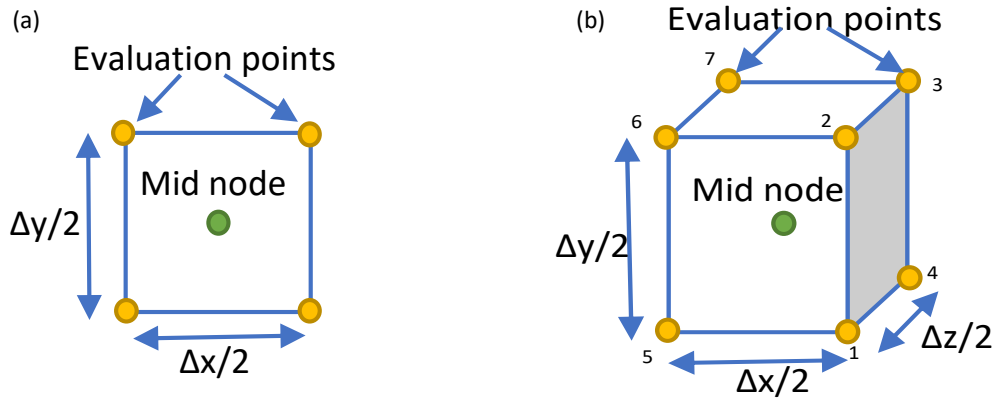


Figure 3. Schematic of integration subdomain and evaluation points in MPIM for (a) 2D and (b) 3D.

### 3.5. Verification of Midpoint Integration Method

The performance of the newly derived MPIM is examined using several benchmarks and compared with existing integration techniques before being applied to the penetration simulations. An eigenvalue analysis can be employed to examine the stability of the domain integration scheme in Galerkin formulation, and it is presented in Appendix A.1. The accuracy of the MPIM is also examined in a convergence study described in Appendix A.2. Several benchmarks related to impact and penetration are presented in the following sections.

#### 3.5.1. Elastic Wave Propagation

The 1D wave propagation in an elastic bar of length  $L=5.0$  m with an initial velocity of 30 m/s is analyzed to examine the performance of the MPIM in the semi-Lagrangian formulation, as illustrated in Figure 4. The material properties of the elastic bar are Young's modulus = 40 MPa and density = 2000 kg/m<sup>3</sup>. The semi-Lagrangian RK with a normalized support size of 2.0 is considered for the problem. The domain is discretized uniformly with 21 nodes, and an explicit temporal integration is employed for the numerical solution. The numerical solutions are compared with domain integration schemes from SNNI, NSNI, and NSNI+VCI. The reference solution is obtained by using 9-point Gauss integration in the Lagrangian formulation.

From Figure 5, it is observed that the SNNI solution becomes unstable, whereas the NSNI solution yields a significant error in the amplitude and phase when the wave propagates. On the other hand, NSNI+VCI and MPIM both provide accurate solutions at first glance. On a closer inspection in Figure 5c, it can be observed that the MPIM yields a lower error when compared with the NSNI+VCI. The MPIM achieves stable and accurate solutions without additional integration correction compared to other methods.

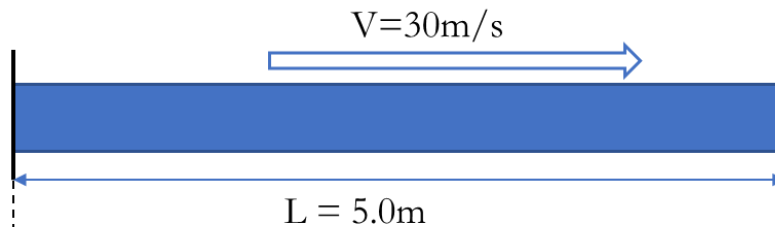


Figure 4. Schematic of a linear elastic bar with an initial velocity.

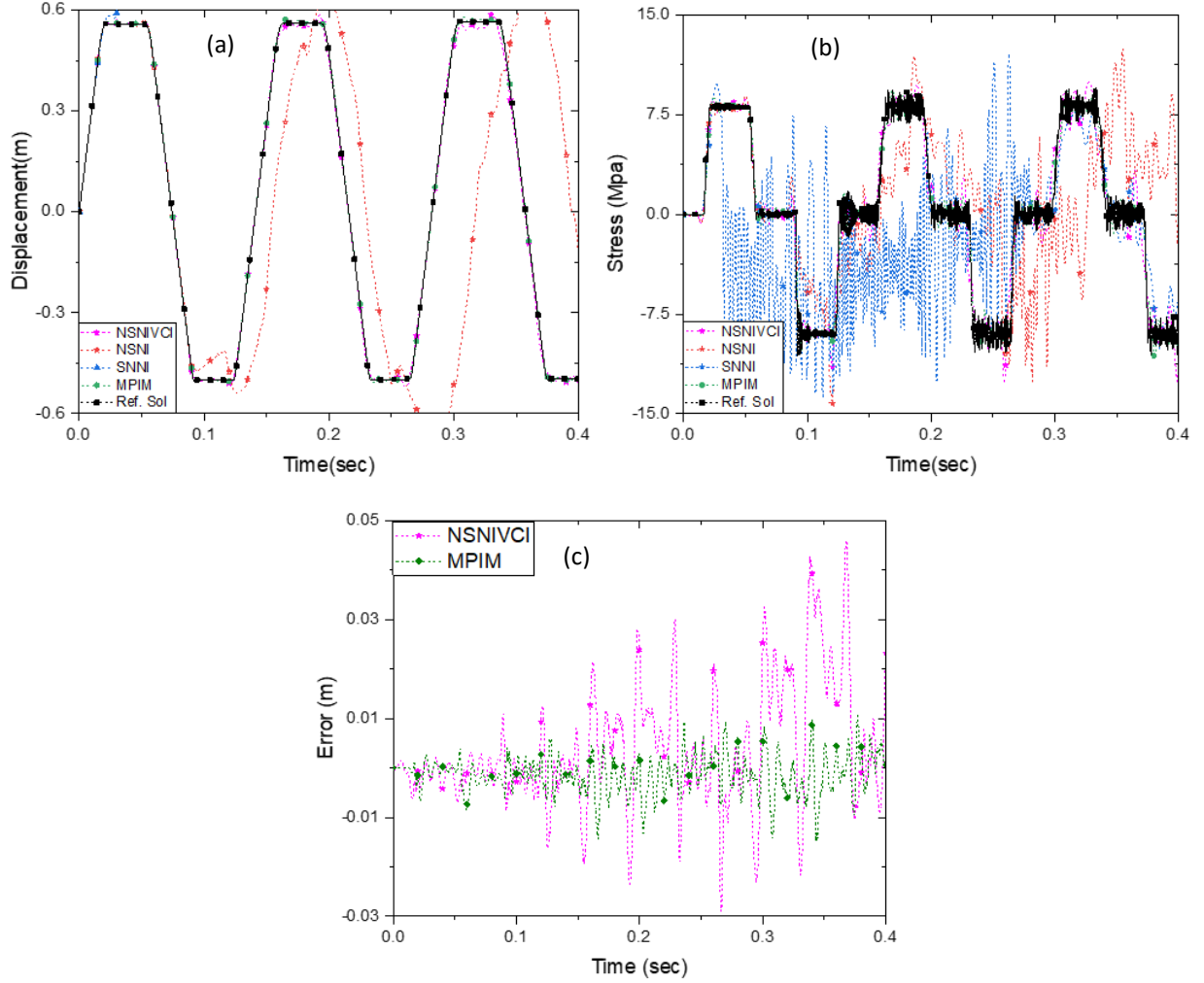


Figure 5. Time history of (a) displacement, (b) stress, and (c) displacement error at the middle node of the bar.

### 3.5.2. Taylor Bar Impact Problem

A classical Taylor bar impact problem [11] is employed to investigate the performance of the MPIM in the 3D dynamic impact problem. An aluminum bar impacts a rigid frictionless wall with an initial velocity of 373 m/s. Its material properties of the aluminum bar are given as follows: Young's modulus ( $E$ ) = 78.2 GPa, Poisson's ratio = 0.3, density = 2700 kg/m<sup>3</sup>, and yield stress ( $\sigma_y$ ) = 290 MPa. The initial height and diameter of the aluminum cylinder are 23.46 cm and 0.782 cm, respectively. The updated Lagrangian framework is adopted, and the cylinder is discretized by 25000 nodes with a normalized support size of 1.65. Linear basis and cubic B-spline kernel

functions are used in the RK approximation. A J2 plasticity model with isotropic hardening is considered for the aluminum, and the yield function is given as

$$f(s, \bar{e}_p) = \|s\| - \sqrt{\frac{2}{3}} \sigma_y (1 + 125 \bar{e}_p)^{0.1} \quad (24)$$

where  $s$  is the deviatoric portion of the Cauchy stress,  $\bar{e}_p$  is the equivalent plastic strain, and  $\sigma_y$  is the yield stress. The numerical solutions of the MPIM alongside MSNNI and NSNI+VCI are shown in Figure 6, where the color map indicates the plastic strain. The overall deformation of all three integration schemes has good agreement with the experimental measurements. Moreover, the smooth contour and the nodal connectivity (gridlines) demonstrate that the solutions are stable without spurious oscillation modes.

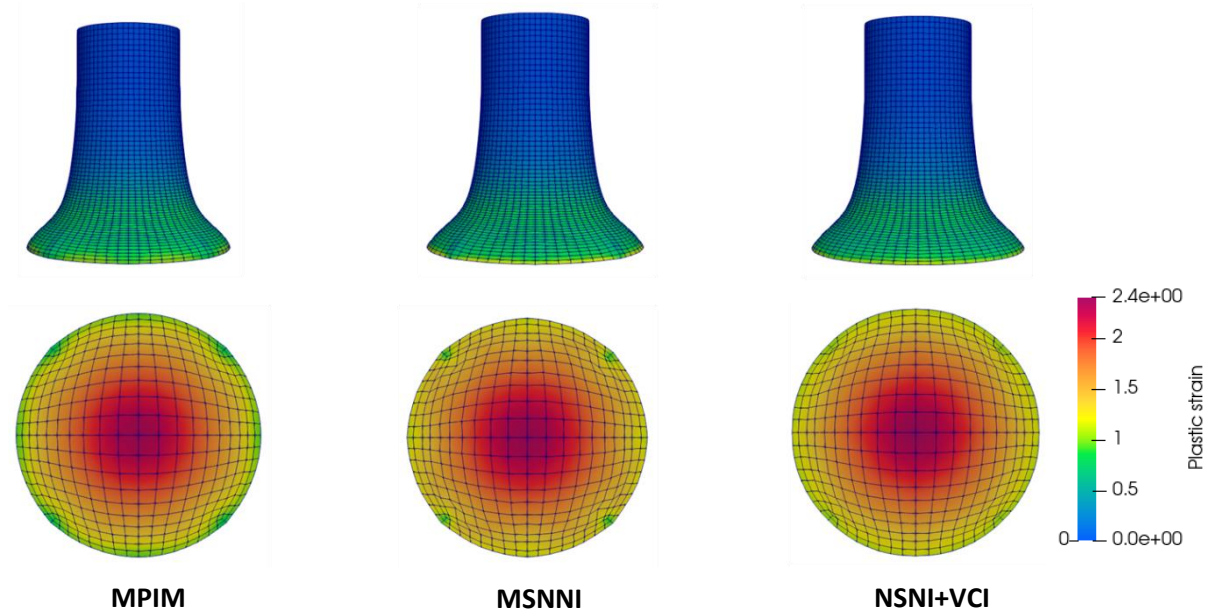


Figure 6. Plastic strain contour of the Taylor impact problem.

### 3.5.3. Elastic Ball Collision with Cuboid Target

To assess the performance of the MPIM in the elastic impact problem, a spherical ball with a velocity of 80 m/s (equivalent to 60% of the elastic wave speed) colliding with the stationary target is studied. The radius of the ball is 27.25 mm, and the length of the cuboid in each direction is 200

mm. Both objects are elastic with the same material properties: Young's Modulus ( $E$ ) = 40 MPa, Poisson's ratio = 0.3, and density = 2000 kg/m<sup>3</sup>. The semi-Lagrangian framework with the kernel contact algorithm [4] is employed, and a normalized support size of 1.50 is considered. The discretization of the problem is shown in Figure 7.

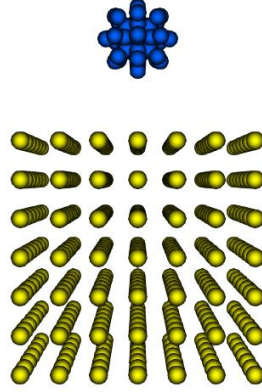


*Figure 7. Discretization of the spherical ball and cuboid target.*

The simulation solutions in Figure 8 show that the deformation of the spherical ball and cuboid target in the MPIM solution remains symmetric and stable throughout the simulation, whereas the solutions of MSNNI and DNI are stable initially but become unsymmetrical and oscillatory at a later time. To further investigate, the linear momentum and total energy of the system are examined in Figure 9 (a) and (b). It is observed that the linear momentum and total energy are conserved in the MPIM and DNI solutions, whereas they are steady at initial instances but diverge at a later time in the MSNNI and NSNI solutions. The histories of the mean velocities of the spherical ball cuboid target are shown in Figure 9 (c) and (d) using several different integration schemes. It can be seen that the numerical solutions obtained by the MPIM offer better accuracy and stability in comparison with those obtained by commonly used nodal integration methods.

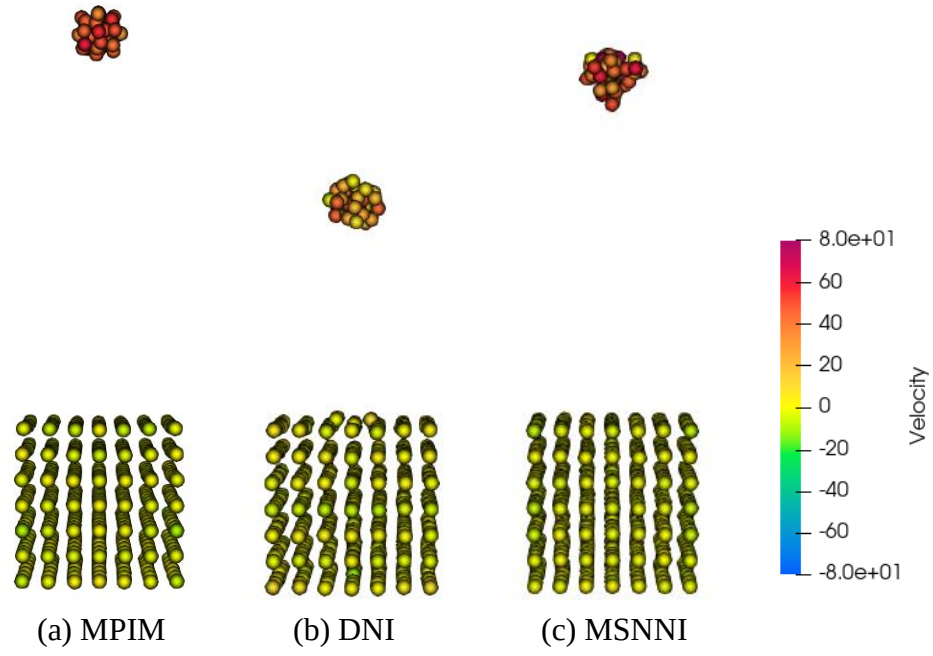


Figure 8. Deformation of the spherical ball and cuboid target at 0.012 second after collision.

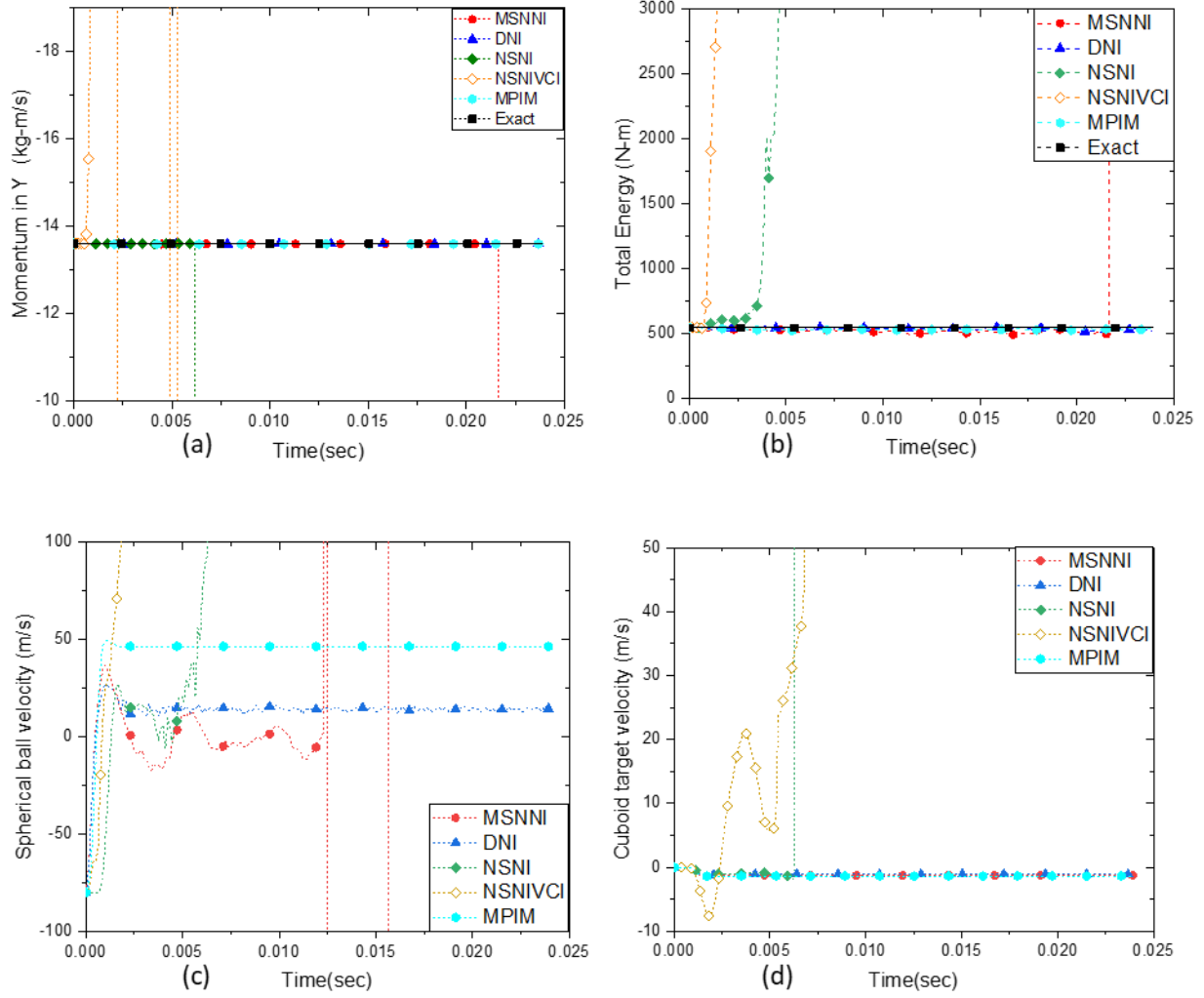


Figure 9. Time history of (a) linear momentum, (b) total energy, (c) spherical ball mean velocity, and (d) cuboid target mean velocity.

### 3.6. Consistent Semi-Lagrangian Formulation

The shape functions in the semi-Lagrangian framework are developed in the current configuration [12], and as a result, the shape functions are dependent on time. The time-dependent variables such as velocity and acceleration can be expressed as the material derivatives of the RK approximated displacement, which is described below,

$$\begin{aligned}
u^h &= \boldsymbol{\psi} \mathbf{d}, \\
v^h &= \frac{Du^h}{Dt} = \boldsymbol{\psi} \dot{\mathbf{d}} + \bar{\boldsymbol{\psi}} \mathbf{d}, \\
a^h &= \frac{D^2 u^h}{Dt^2} = \boldsymbol{\psi} \ddot{\mathbf{d}} + 2\bar{\boldsymbol{\psi}} \dot{\mathbf{d}} + \bar{\bar{\boldsymbol{\psi}}} \mathbf{d}
\end{aligned} \tag{25}$$

where  $u^h$ ,  $v^h$ , and  $a^h$  represent numerical displacement, velocity, and acceleration, respectively.

$\boldsymbol{\psi}$ ,  $\bar{\boldsymbol{\psi}}$ ,  $\bar{\bar{\boldsymbol{\psi}}}$  represent RK shape functions, first time derivative, and second time derivative of the RK shape functions, respectively.  $d$ ,  $\dot{d}$ , and  $\ddot{d}$  represent the generalized nodal displacement, velocity, and acceleration, respectively.

Due to acceleration (25) in the equation, the linear momentum equation produces a convective component. Applying the Galerkin weak form and introducing the approximation functions (25) in the linear momentum equation leads to

$$\begin{aligned}
\int_{\Omega} \rho \boldsymbol{\psi} \boldsymbol{\psi} d\Omega \ddot{\mathbf{d}} + \int_{\Omega} \rho \boldsymbol{\psi} \bar{\boldsymbol{\psi}} d\Omega \dot{\mathbf{d}} + \int_{\Omega} \rho \boldsymbol{\psi} \bar{\bar{\boldsymbol{\psi}}} d\Omega \mathbf{d} + \int_{\Omega} \nabla \boldsymbol{\psi} C_e \nabla \boldsymbol{\psi} d\Omega \mathbf{d} \\
= \int_{\Omega} \boldsymbol{\psi} \mathbf{f}_b d\Omega + \int_{\Gamma_h} \boldsymbol{\psi} \mathbf{h} d\Gamma
\end{aligned} \tag{26}$$

$$\mathbf{M} \ddot{\mathbf{d}}_{n+1} + \mathbf{C} \dot{\mathbf{d}}_{n+1} + \mathbf{K}_x \mathbf{d}_{n+1} = \mathbf{F} \tag{27}$$

where  $\mathbf{M} = \int_{\Omega} \rho \boldsymbol{\psi} \boldsymbol{\psi} d\Omega$ ,  $\mathbf{C} = \int_{\Omega} \rho \boldsymbol{\psi} \bar{\boldsymbol{\psi}} d\Omega$ ,  $\mathbf{K}_x = \int_{\Omega} \rho \boldsymbol{\psi} \bar{\bar{\boldsymbol{\psi}}} d\Omega + \int_{\Omega} \nabla \boldsymbol{\psi} C_e \nabla \boldsymbol{\psi} d\Omega$  and

$\mathbf{F} = \int_{\Omega} \boldsymbol{\psi} \mathbf{f}_b d\Omega + \int_{\Gamma_h} \boldsymbol{\psi} \mathbf{h} d\Gamma$ ,  $C_e$  is the elastic modulus.

An inconsistent time integration can be used directly for dynamic analysis since the convective component ( $\bar{\boldsymbol{\psi}}$ ,  $\bar{\bar{\boldsymbol{\psi}}}$ ) under small deformation has a negligible impact on the solution. However, the convective component contributes more when there is a significant deformation, making the inconsistent time integration ineffective. Therefore, a consistent time integration should be utilized in the semi-Lagrangian framework to get an accurate solution.

Newmark  $\beta$  method (Inconsistent Time Integration)

$$\begin{aligned} d_{n+1} &= d_n + \dot{d}_n dt + (0.5 - \beta) \ddot{d}_n dt^2 + \beta \ddot{d}_{n+1} dt^2 \\ \dot{d}_{n+1} &= \dot{d}_n + (1 - \gamma) \ddot{d}_n dt + \gamma \ddot{d}_{n+1} dt \end{aligned} \quad (28)$$

Newmark  $\beta$  method (Consistent Time Integration)

$$\begin{aligned} u_{n+1}^h &= u_n^h + v_n^h dt + (0.5 - \beta) a_n^h dt^2 + \beta a_{n+1}^h dt^2 \\ v_{n+1}^h &= v_n^h + (1 - \gamma) a_n^h dt + \gamma a_{n+1}^h dt \end{aligned} \quad (29)$$

where  $u_n^h$ ,  $v_n^h$  and  $a_n^h$  represent displacement, velocity and acceleration approximation evaluated at  $t = t_n$ .

To the above equation, an approximation for displacement, velocity and acceleration from Equation (25) can be introduced as:

$$\begin{aligned} \psi d_{n+1} &= u_n^h + v_n^h \Delta t + (0.5 - \beta) a_n^h \Delta t^2 + \beta \psi \ddot{d}_{n+1} + 2\bar{\psi} \dot{d}_{n+1} + \bar{\bar{\psi}} d_{n+1} \Delta t^2 \\ \psi \dot{d}_{n+1} + \bar{\psi} d_{n+1} &= v_n^h + (1 - \gamma) a_n^h \Delta t + \gamma \psi \ddot{d}_{n+1} + 2\bar{\psi} \dot{d}_{n+1} + \bar{\bar{\psi}} d_{n+1} \Delta t \end{aligned} \quad (30)$$

The semi-discrete equation (27) and the aforementioned consistent time integration technique (30) (Standard semi-Lagrangian RK) together produce a complicated and non-linear structure. As a result, the solution becomes laborious, loses precision, and costs more to compute.

Although the consistent time integration scheme (30) with the semi-discrete equation (27) form an accurate solution (Standard semi-Lagrangian RK). However, the combination of the above two equations forms a complex and non-linear structure. Thus, solving becomes cumbersome also, reduces accuracy, and increases computational cost. A novel approximation for the semi-Lagrangian framework is proposed to address the problem above, where the displacement, velocity, and acceleration approximations are interpolated by the same semi-Lagrangian RK shape functions with different generalized nodal coefficients, as

$$\begin{aligned} u^h &= \psi d, \\ v^h &= \psi \hat{v}, \\ a^h &= \psi \hat{a} \end{aligned} \quad (31)$$

where  $\hat{v}$ ,  $\hat{a}$  are nodal velocity and acceleration coefficients.

The new consistent semi-Lagrangian approximation equation (31) can be inserted into the consistent time integration (29) , and Galerkin weak form yields a simple and straightforward equation. The respective equation can be written as

$$\begin{aligned} d_{n+1} &\cong d_n + \hat{v}_n dt + (0.5 - \beta) \hat{a}_n dt^2 + \beta \hat{a}_{n+1} dt^2 \\ \hat{v}_{n+1} &\cong \hat{v}_n + (1 - \gamma) \hat{a}_n dt + \gamma \hat{a}_{n+1} dt \end{aligned} \quad (32)$$

$$\int_{\Omega} \rho \psi \psi d\Omega \hat{\mathbf{a}} + \int_{\Omega} \nabla \psi C_e \nabla \psi d\Omega \mathbf{d} = \int_{\Omega} \psi \mathbf{f}_b d\Omega + \int_{\Gamma_h} \psi \mathbf{h} d\Gamma \quad (33)$$

$$\mathbf{M} \hat{\mathbf{a}}_{n+1} + \mathbf{K} \mathbf{d}_{n+1} = \mathbf{F} \quad (34)$$

where  $\mathbf{M} = \int_{\Omega} \rho \psi \psi d\Omega$ ,  $\mathbf{K} = \int_{\Omega} \nabla \psi C_e \nabla \psi d\Omega$ , and  $\mathbf{F} = \int_{\Omega} \psi \mathbf{f}_b d\Omega + \int_{\Gamma_h} \psi \mathbf{h} d\Gamma$ .

The combination of the semi-discrete equation (34) with the consistent time integration technique (32)(Consistent semi-Lagrangian RK) avoids the need to explicitly compute the convection term, resulting in a simplified structure of the linear momentum equation. A straightforward equation of linear momentum and time integration that is compatible with each other is produced by the outcome of the above method. The resultant method improves computational efficiency in addition to accuracy, as the numerical example will demonstrate. The working of standard semi-Lagrangian RK method with inconsistent time integration, standard semi-Lagrangian RK method, consistent semi-Lagrangian RK method has been detailed in Appendix B. Table 1 shows a detailed comparison between the standard and consistent semi-Lagrangian RK formulation.

Table 1. Comparison of Standard and Consistent semi-Lagrangian RK formulation.

|                     | Standard semi-Lagrangian RK formulation                                                                                                                                                                                                                                                                                                     | Consistent semi-Lagrangian RK formulation                                                                                                                                                |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Approximations      | $u^h = \boldsymbol{\psi} \mathbf{d},$<br>$\mathbf{v}^h = \frac{D\mathbf{u}^h}{Dt} = \boldsymbol{\psi} \dot{\mathbf{d}} + \bar{\boldsymbol{\psi}} \mathbf{d},$<br>$\mathbf{a}^h = \frac{D^2\mathbf{u}^h}{Dt^2} = \boldsymbol{\psi} \ddot{\mathbf{d}} + 2\bar{\boldsymbol{\psi}} \dot{\mathbf{d}} + \bar{\bar{\boldsymbol{\psi}}} \mathbf{d}$ | $\mathbf{u}^h = \boldsymbol{\psi} \mathbf{d},$<br>$\mathbf{v}^h = \boldsymbol{\psi} \hat{\mathbf{v}},$<br>$\mathbf{a}^h = \boldsymbol{\psi} \hat{\mathbf{a}}$                            |
| Governing Equation  | $\mathbf{M} \ddot{\mathbf{d}}_{n+1} + \mathbf{C} \dot{\mathbf{d}}_{n+1} + \mathbf{K}_x \mathbf{d}_{n+1} = \mathbf{F}$                                                                                                                                                                                                                       | $\mathbf{M} \hat{\mathbf{a}}_{n+1} + \mathbf{K} \mathbf{d}_{n+1} = \mathbf{F}$                                                                                                           |
| Newmark beta method | $d_{n+1} = d_n + \dot{d}_n dt + (0.5 - \beta) \ddot{d}_n dt^2$<br>$\quad + \beta \ddot{d}_{n+1} dt^2$<br>$\dot{d}_{n+1} = \dot{d}_n + (1 - \gamma) \ddot{d}_n dt + \gamma \ddot{d}_{n+1} dt$                                                                                                                                                | $d_{n+1} = d_n + \hat{v}_n dt + (0.5 - \beta) \hat{a}_n dt^2$<br>$\quad + \beta \hat{a}_{n+1} dt^2$<br>$\hat{v}_{n+1} = \hat{v}_n + (1 - \gamma) \hat{a}_n dt + \gamma \hat{a}_{n+1} dt$ |

### 3.6.1. Verification of Consistent semi-Lagrangian RK Method

To demonstrate the working of the Inconsistent RK method (Inconsistent method), Standard RK method (Standard method) and Consistent semi-Lagrangian RK method (Consistent method), an example is manufactured in one dimensional. The length of the 1D beam is considered as 10, and the normalized support size of 2.5 with a linear basis and a 5-point Gauss quadrature employed as the domain integration.

$$\rho \frac{D^2 u}{Dt^2} = E \frac{\partial^2 u}{\partial x^2} + f_b \quad \text{on } \Omega \quad (35)$$

Boundary condition:  $u(0, t) = 0; \quad \frac{\partial u(L, t)}{\partial x} = k \cos(kl) (1 - e^{-wt})$

Initial condition:  $u(x, 0) = 0; \quad \frac{Du(x, 0)}{Dt} = w \sin(kx)$

Analytical solution:  $u(x, t) = \sin(kx) (1 - e^{-wt})$

where  $k = 0.25$ ,  $w = 2$ ,  $E = 1$ ,  $\rho = 1$ , and  $x$  is in the current configuration.

Figure 10 shows that, as a result of inconsistent time integration, the inconsistent method significantly deviates from the analytical solution. Although the standard method performs better,

however, there are some fluctuations in the stress state. While the consistent method exhibits superior performance, a stable solution, and good agreement with the analytical result. This example shows that when dealing with high-impact events, the consistent semi-Lagrangian RK formulation is required in the semi-Lagrangian framework. As the penetration simulation involves highly transient responses, temporal stability is essential to attain a reliable numerical solution. We investigate the temporal stability of the consistent semi-Lagrangian formulation in Appendix C.

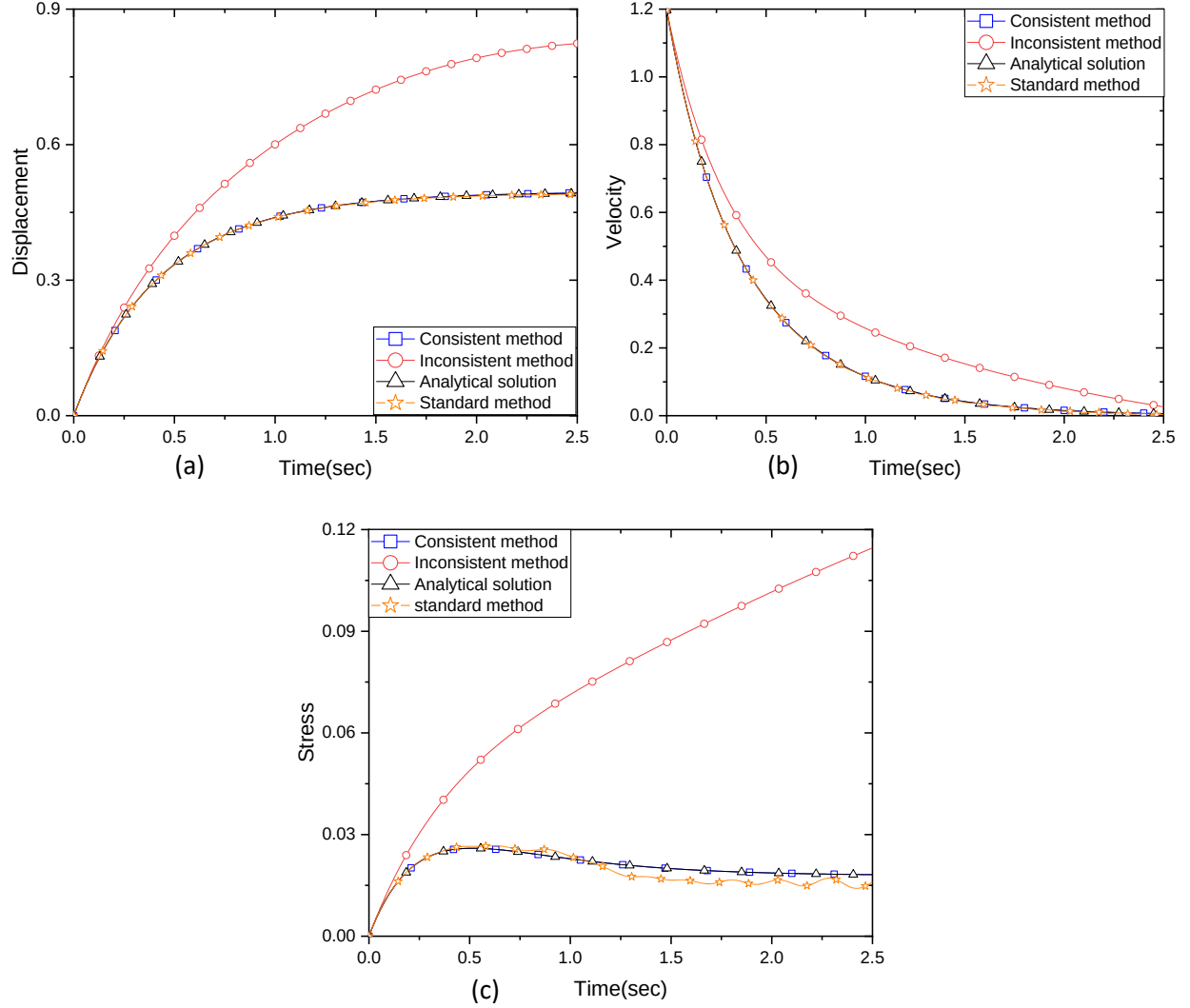


Figure 10. Time history of (a) displacement, (b) velocity at the end node of the 1D beam, and (c) Stress at the middle node of the 1D beam.

### 3.7. Absorbing Boundary Condition

One of the issues in the penetration into the earth is to model accurately and economically the semi-infinite domain. The spurious reflection due to the curtailed boundaries can adversely affect the computational accuracy of the solution. The most common way to restrict the infinite computation domain to a finite domain is to implement an absorbing boundary condition.

There have been several methods that can be implemented without much increment in the computational cost. The absorbing boundary condition can be formulated as local type and global type. Due to implementation difficulties of global type absorbing boundary conditions, local types are generally preferred. The simplest and most widely used local boundary is the standard viscous boundary of Lysmer and Kuhlemeyer (1969). This boundary can be described by two series of dashpots oriented normal and tangential to the boundary. The great advantage of this approach is that the absorption characteristics are independent of frequency. Thus the viscous boundary is suitable for penetration simulations, in which broadband waves are generated.

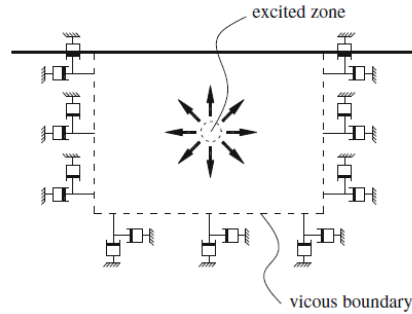


Figure 11. Schematic of standard viscous boundary condition.

The standard viscous boundary condition, as proposed by Lysmer and Kuhlemeyer (1969), is formulated in terms of the normal and tangential components of stress on the boundary as:

$$\begin{aligned} n + a\rho V_p \frac{\partial u}{\partial t} &= 0 \\ \tau + b\rho V_s \frac{\partial v}{\partial t} &= 0 \end{aligned} \quad \text{on the boundary } \Gamma_d \quad (36)$$

where  $V_p$  is the longitudinal velocity,  $V_s$  is the shear velocity,  $n$  is the normal stress,  $\tau$  is the tangential stress,  $\rho$  is the density of the material,  $u$  and  $v$  are the displacements in the normal and tangential directions, respectively, and ‘a’ and ‘b’ are the dimensionless parameters.

The Galerkin formulation of the equilibrium equation is then:

$$\int_{\Omega} w(\nabla \sigma + f_b) d\Omega = \int_{\Omega} w \rho \ddot{u} d\Omega \quad (37)$$

$$\int_{\Omega} \nabla w \sigma d\Omega + \int_{\Omega} w \rho \ddot{u} d\Omega = \int_{\Omega} w f_b d\Omega + \int_{\Gamma_c} w \sigma d\Gamma + \int_{\Gamma_d} w \sigma d\Gamma \quad (38)$$

$$\int_{\Omega} \nabla w \sigma d\Omega + \int_{\Gamma_d} w a V_p \frac{\partial u}{\partial t} d\Gamma + \int_{\Gamma_d} w b V_s \frac{\partial v}{\partial t} d\Gamma + \int_{\Omega} w \rho \ddot{u} d\Omega = \int_{\Omega} w f_b d\Omega + \int_{\Gamma_c} w \sigma d\Gamma \quad (39)$$

where  $\sigma$  is the Cauchy stress,  $f_b$  is the body force,  $w$  is the test function.

### 3.7.1. Simulations of Absorbing Boundary Condition

The standard viscous boundary condition was implemented and verified in the following benchmark problems. In these problems, self-consistent unitless parameters are employed for the purpose of verification.

#### 1D case

Consider the equilibrium equation with zero body force,

$$\rho \ddot{u} = E u_{,xx} \quad , \quad E = 1, \rho = 1, L = 4,$$

$$u(0, t) = 0, u(x, 0) = 0, v(0 < x < 0.5) = \sin\left(\frac{\pi x}{0.5}\right)$$

The schematic and discretization of this problem are given in Figure 12. This example was analyzed using SCNI method. The result in Figure 13 shows that the standard viscous boundary absorbs the impinging waves effectively. The reflected wave has a negligible amplitude.

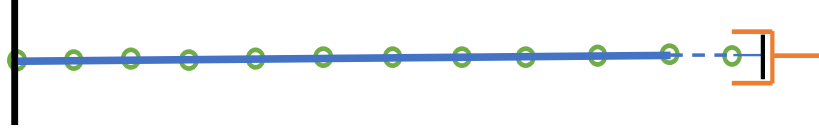


Figure 12. Schematic of a 1D beam with a damper on the boundary.

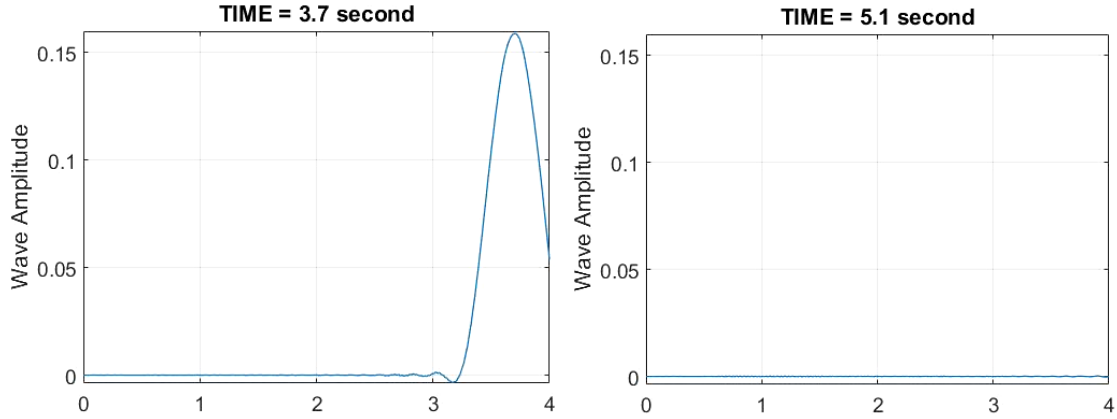


Figure 13. Wave solution of the 1D beam with 250 nodes at different time instances using absorbing boundary condition at  $x=4.0$ .

### 3D case

Consider the wave propagation with a concentrated external force and  $E = 4 \times 10^7$ ,  $\rho = 2 \times 10^3$ ,  $Lx = 1$ ,  $Ly = 1$ ,  $Lz = 1$ ,  $\text{poisson ratio} = 0.2$ . The schematic, discretization and boundary conditions are given in Figure 14. The problem was analyzed using MSNNI method with 20402 nodes. An impulse force is given at the top of the cube; the wave travels from the top, as shown in Figure 14. From simulation results in Figure 15, the standard viscous boundary absorbs the incoming waves to a certain extent, whereas the fixed boundary has more disturbance in the solution due to the reflection of the waves. It is well known that completely absorbing a broad-spectrum wave in 3D is not feasible. Nonetheless, the standard viscous boundary has performed well in absorbing waves without significantly disturbing the solution.

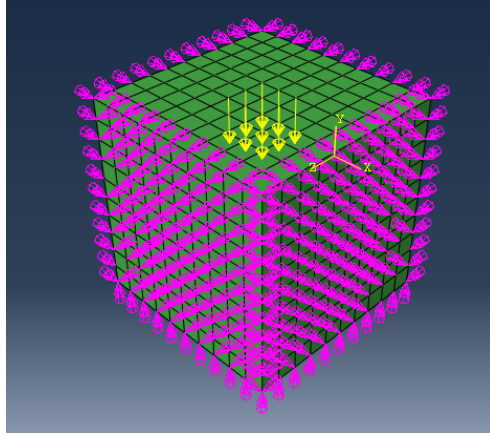


Figure 14. Schematic of a 3D cube with absorbing boundary condition.

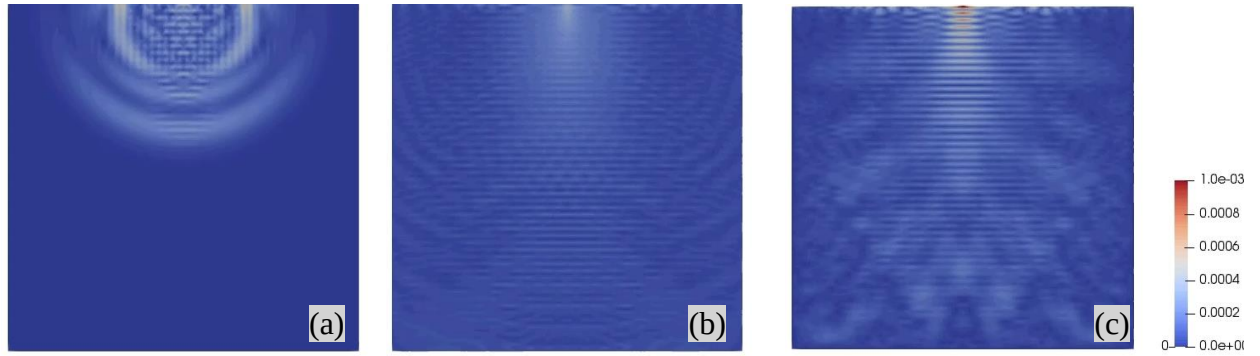


Figure 15. Stress contours of the 3D cube. (a) before the wave hits the boundary; after the wave hits the boundary, (b) using absorbing boundary condition and (c) without using absorbing boundary condition.

## 4. Task 2 – Soil Constitutive Model Improvement

### 4.1. Algorithm Improvement

We have implemented some improvements to the model to capture soil behavior more accurately. The basic formulation for the three-invariant plastic model we used is given in Appendix A for reference. We have reformulated and implemented the growth function for the cap surface so that it better reproduces experimental data (Figure 16). Previously, the shrinking of the yield surface can be seen when the cap softens. In the old cap function,

$$X(\kappa) = \kappa - RF_f(\kappa) \quad (40)$$

The size of the cap decreased as the cohesion and, hence,  $F_f$  decreased. The cap growth function is now

$$X = \hat{R}\kappa + S \quad (41)$$

which allows  $X$  to grow with  $\kappa$ . Both  $R$  and  $S$  are material constants.

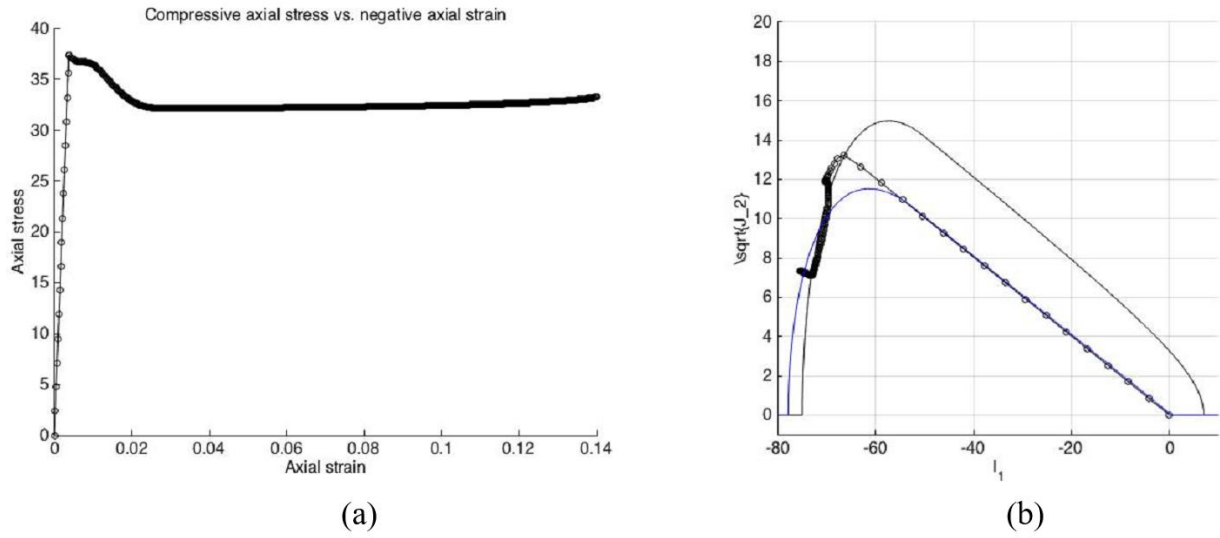


Figure 16. Triaxial compression test showing cap hardening. (a) axial strain vs. axial stress, compression positive. (b) first invariant of the stress tensor vs. the square root of the second. The first invariant is three times mean stress (tension positive), while the square root of the second invariant is a generalized measure of shear stress.

Simulations show that inelastic compaction at high mean stresses is more accurately captured. We have also split the viscosity formulation into volumetric and distortional (isochoric) parts with different fluidity parameters. In a single fluidity parameter Duvaut-Lions model, the stress calculation is a weighted average between the elastic trial and plastic solutions:

$$\boldsymbol{\sigma}_{n+1} = \frac{\eta}{\eta + \Delta t} \boldsymbol{\sigma}^{\text{tr}} + \frac{\eta}{\eta + \Delta t} \tilde{\boldsymbol{\sigma}} \quad (42)$$

Here, mean stress and deviatoric stress are calculated separately.

$$\begin{aligned} \mathbf{s}_{n+1} &= \frac{\eta_s}{\eta_s + \Delta t} \mathbf{s}^{tr} + \frac{\eta_s}{\eta_s + \Delta t} \tilde{\mathbf{s}} \\ p_{n+1} &= \frac{\eta_p}{\eta_p + \Delta t} p^{tr} + \frac{\eta_p}{\eta_p + \Delta t} \tilde{p} \end{aligned} \quad (43)$$

where the superscript *tr* indicates the trial elastic component, the tilde indicates the inviscid plastic solution,  $\eta$  is a fluidity parameter,  $\Delta t$  is the time step,  $\sigma$  is the total stress,  $\mathbf{s}$  is the deviatoric stress, and  $p$  is the mean stress. Hence the weighting uses different viscosity parameters for volumetric and shear deformation. We have also modified the plastic potential function so that plastic compaction of the soil may occur even as the shear strength increases. This behavior is more characteristic of most soils. The plastic potential is

$$g = \Gamma(\beta) \sqrt{J_2} - \sqrt{F_c^g F_f^g} \quad (44)$$

The change is in the function  $F_c^g$ , which is now

$$F_c^g = 1 - H(\kappa^g - I_1) \left( \frac{I_1 - \kappa^g}{X - \kappa^g} \right)^2 \quad (45)$$

The difference is that  $\kappa^g$  has replaced  $\kappa$ , allowing plastic compaction to occur at lower mean stresses. We have also implemented some algorithmic improvements that enhance the robustness and efficiency of the soil model implementation. These include calculating some products, where possible, in spectral form rather than using the full matrix. Changes to the way the tangent modulus is calculated have improved performance and speed. Numerical testing indicates optimal quadratic convergence over a wide range of parameter values and loading conditions (Figure 17).

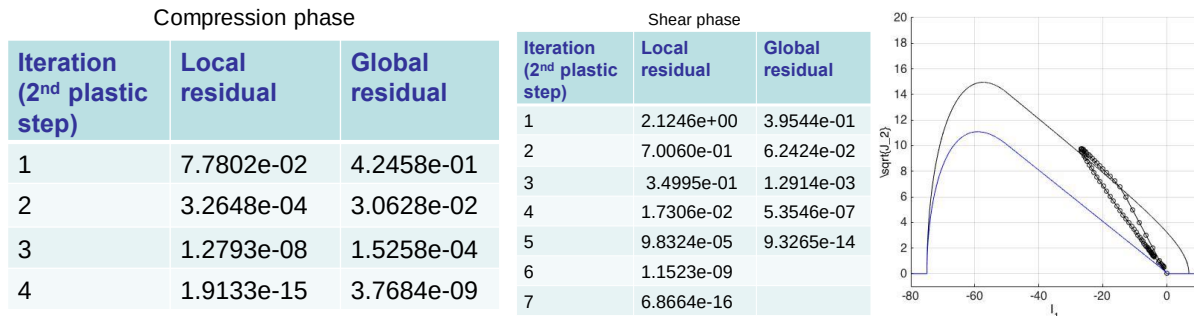


Figure 17. Local and global convergence output for a typical time step in a one-element test in combined compression and shear with the load path.

These improvements have been verified with previous versions of the model and simpler models such as the Drucker-Prager model (Figure 18). The meshfree implementation has been verified and matches the finite element for prescribed displacement (Figure 19). They are better at reproducing soil behavior than before.

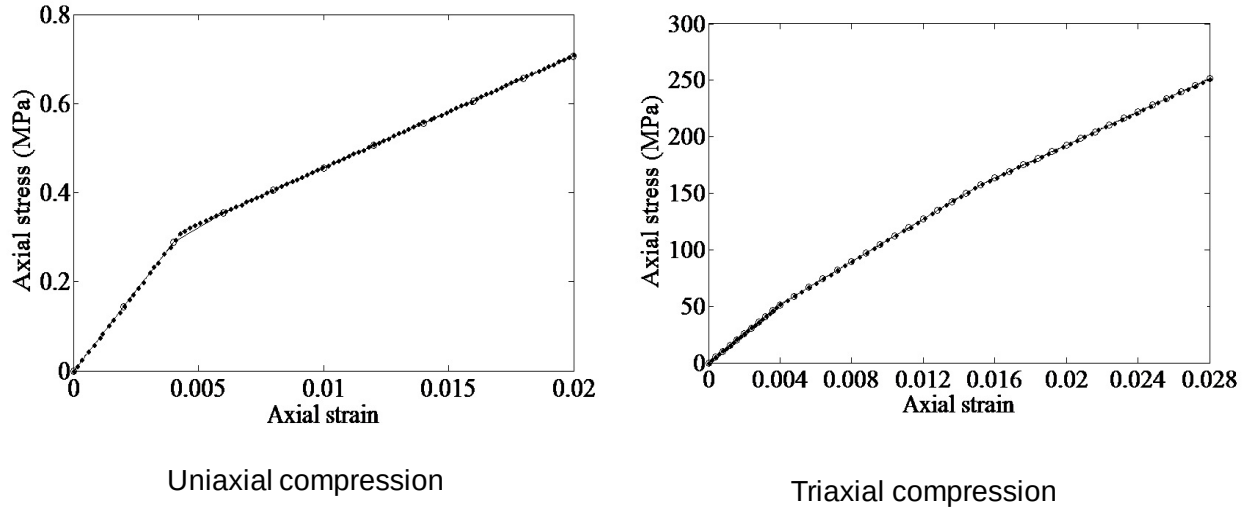


Figure 18. Verification of model vs. Drucker-Prager when the parameters match.

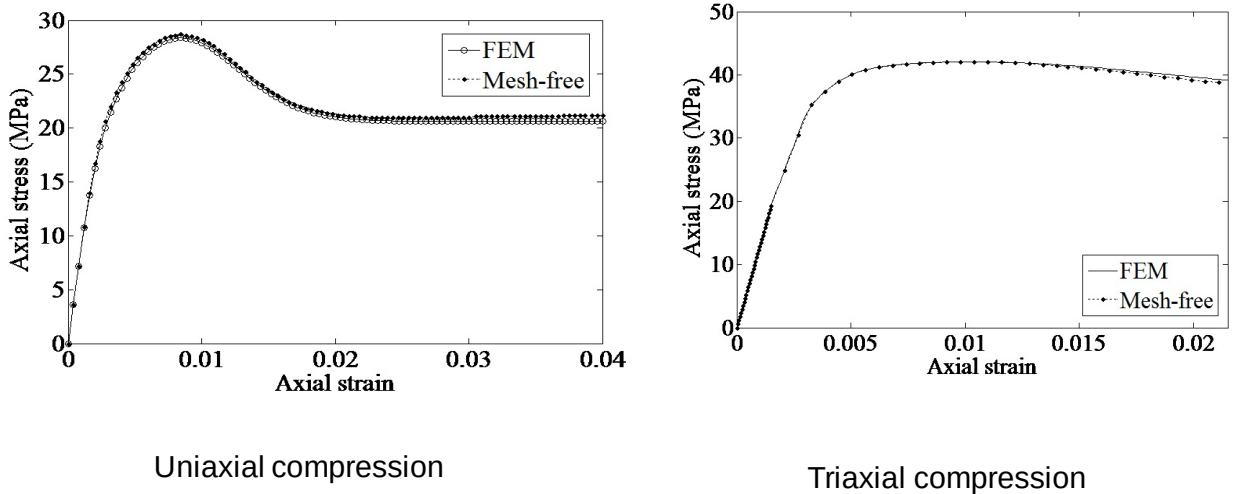


Figure 19. Verification between the meshfree and finite element implementations of the model. A slight difference can be attributed to the fact the meshfree code is dynamic while the finite element code is quasi-static.

The fully implicit formulation can be costly and have conditioning issues in the Newton-Raphson solution. A semi-implicit formulation [13] is faster and more robust; however, the method is only

conditionally stable. In applications such as the penetration problems run here with explicit global solvers, the time step requirement does not seem to impact the simulation.

The semi-implicit scheme uses an implicit formulation to solve the stress update equation, but the hardening law is explicit, using values from the previous time step. Thus,

$$\boldsymbol{\epsilon}_{n+1}^e = \boldsymbol{\epsilon}_n^e + \Delta\lambda \partial g / \partial \boldsymbol{\sigma}_{n+1} \quad (46)$$

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}(\boldsymbol{\epsilon}_{n+1}^e) \quad (47)$$

$$\boldsymbol{\alpha}_{n+1} = \boldsymbol{\alpha}_n + \Delta\lambda \mathbf{h}_n^\alpha \quad (48)$$

$$\kappa_{n+1} = \kappa_n + \Delta\lambda h_n^\kappa \quad (49)$$

$$f(\boldsymbol{\sigma}_{n+1}, \boldsymbol{\alpha}_{n+1}, \kappa_{n+1}) = 0 \quad (50)$$

Here  $\boldsymbol{\epsilon}^e$  is the elastic strain,  $\Delta\lambda$  is the incremental plastic multiplier,  $g$  is the plastic potential, and the  $\boldsymbol{\sigma}$  stress. The back stress  $\boldsymbol{\alpha}$  and the cap parameter  $\kappa$  have hardening functions  $\mathbf{h}^\alpha$  and  $h^\kappa$ , respectively, which are functions of the stress and internal state variables. The yield function is  $f$ . The subscript  $n$  or  $n+1$  denotes at which time step the quantities are evaluated.

Substituting the other equations into the yield function (50) results in a single-variable nonlinear equation that can be solved for  $\Delta\lambda$ . Once  $\Delta\lambda$  is solved, by Newton-Raphson in this code, the other variables are easily updated. This method can be applied with spectral decomposition, which may improve the efficiency modestly.

#### 4.2. Develop Material Constant Calibration Strategy

As the three-invariant soil model has several parameters, a rigorous strategy is necessary to fit the material parameters. The procedure developed for the soil model used in this project is adapted from the procedure for the original Sandia GeoModel. That procedure is described in [14].

Linear elastic material properties can be derived first. These can be calculated from the elastic straight-line portion of triaxial tests. It is important to measure lateral expansion to obtain Poisson's ratio. The failure surface can be determined using a least-square fit of a set of triaxial tests. Since the failure surface now contains a unified tension-shear surface, at least one tension test must be performed for cohesive soils. For non-cohesive soils, the tensile strength can be considered to be

zero or very close. It is recommended to perform at least three triaxial compression tests to ensure accuracy.

The ratio of triaxial extension to compression strength  $\psi$ , can be fitted with a triaxial extension experiment. If that is unavailable, the Mohr-Coulomb approximation can be used.

$$\psi = \frac{1}{1 + \sqrt{3} \left( M (C - M I_1) \left[ (C - M I_1)^2 - (C - M \Xi)^2 \right]^{-\frac{1}{2}} \right)} \quad (51)$$

The cap parameters can be fit using a hydrostatic compression test or very high yield strength triaxial tests. These need to be fit simultaneously. Currently, a trial-and-error approach is used and produces reasonable values quickly. The evolution parameters are fit next. These are fit, again with a trial-and-error procedure, to capture the plastic portion of the curve. Finally, the rate-dependent parameters are fit to higher strain-rate tests.

As an example, a clay tested by Xu and coworkers [15] is selected to verify the fitting procedure. The fit parameters are shown in Table 2. The fits to the data from these parameters are shown in Figure 20. Although the authors refer to the data as stress-strain, there is considerable inhomogeneity in the softening regime as a shear band forms. The model captures this formation as well, as shown in Figure 21.

*Table 2. Material parameters for the modified GeoModel fit to clay from Xu et al., 2018.*

|                                  |                                 |                             |
|----------------------------------|---------------------------------|-----------------------------|
| $E = 9733.3 \text{ kPa}$         | $a_t = 225.0$                   | $\gamma_C = 1.0$            |
| $\nu = 0.3$                      | $a_c = 225.0$                   | $\gamma_C^g = 1.0$          |
| $\psi = 1.0$                     | $a_s = 225.0$                   | $\gamma_\Xi = 1.0$          |
| $\hat{R} = 4.0$                  | $C_{max} = 39.8 \text{ kPa}$    | $C_0 = 39.7 \text{ kPa}$    |
| $S = 0.0$                        | $C_{max}^g = 39.8 \text{ kPa}$  | $C_0^g = 39.7 \text{ kPa}$  |
| $\kappa_0 = -1000.0 \text{ kPa}$ | $\Xi_{max} = 249.2 \text{ kPa}$ | $\Xi_0 = 248.7 \text{ kPa}$ |
| $M = 0.1598$                     | $C_{res} = 15.0 \text{ kPa}$    | $n_0 = 0.5$                 |
| $M_g = 0.1$                      | $C_{res}^g = 15.0 \text{ kPa}$  | $n_0^* = 0.5$               |
| $q_1 = 2.0$                      | $\Xi_{res} = 93.9 \text{ kPa}$  | $\tau = 0.3$                |

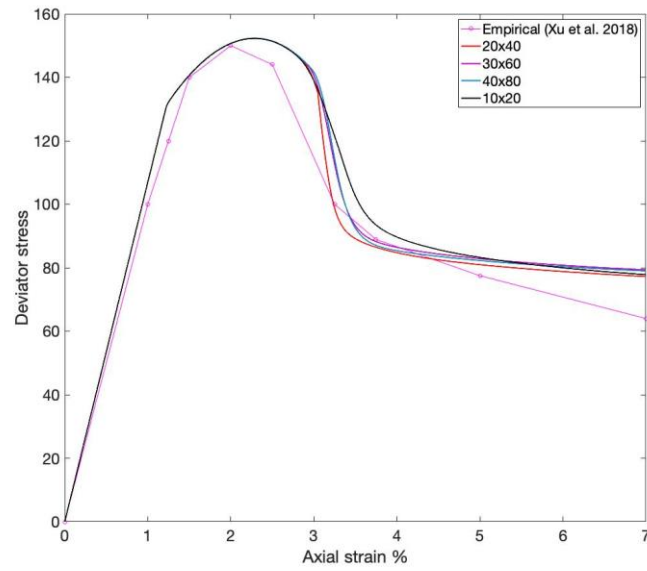


Figure 20. Fit of stress-strain data to tests from Xu et al., 2018.

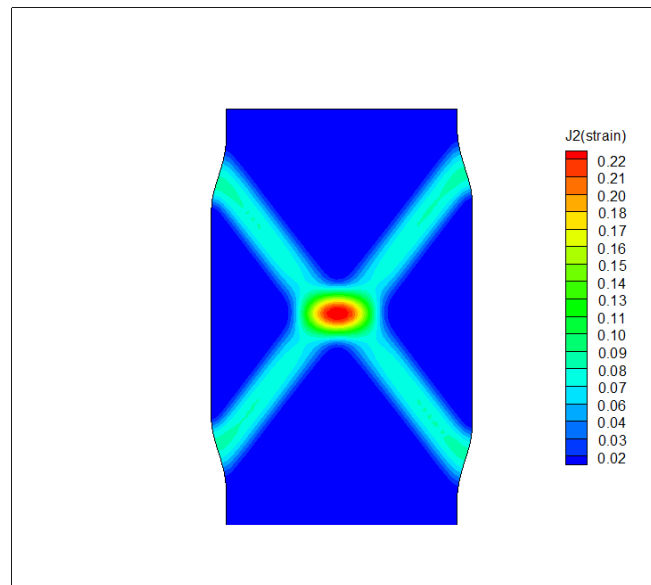


Figure 21. Equivalent plastic strain of clay sample after triaxial compression, showing conjugate shear bands.

### 4.3. Sensitivity Analysis for the Three-Invariant Soil Model

To enhance the efficiency of parameter calibration for the soil model, a Sobol variance-based sensitivity analysis [16] is performed to investigate the effect of each parameter on the simulation results. The general procedure of the sensitivity analysis is as follows: First, a set of input parameters is defined based on the problem of interest. A sequence of perturbed parameter sets is generated based on a sampling method with a defined range of variation and a probability distribution. The sequence of parameter sampling sets, referred to as input, is transferred as the inputs into the designated numerical model for analysis. The numerical results, referred to as output, are extracted for the sensitivity analysis. In the Sobol sensitivity analysis, the first-order and total Sobol sensitivity indices for each parameter can be obtained, and the predominant parameters can be identified.

The first-order index  $S_i$  is a direct variance-based measure. It indicates the contribution to the output variance of the main effect of  $X_i$  and measures the effect of varying  $X_i$  alone. The total effect index  $S_{Ti}$  accounts for the contribution to the output variance of  $X_i$ , including all variance caused by its interactions with any other input parameters. The first-order index and the total effect index can be calculated as follows.

$$S_i = \frac{V_{X_i}(E_{X_{\sim i}}(Y | X_i))}{V(Y)} \quad (52)$$

$$S_{Ti} = \frac{E_{X_{\sim i}}(V_{X_i}(Y | X_{\sim i}))}{V(Y)} = 1 - \frac{V_{X_{\sim i}}(E_{X_i}(Y | X_{\sim i}))}{V(Y)} \quad (53)$$

where  $X$  and  $Y$  are the input and output datasets, respectively,  $V(Y)$  represents the variance of the output,  $X_{\sim i}$  is the input except the  $i^{\text{th}}$  input,  $E_{X_i}$  is the effect of  $X_i$ , and  $E_{X_{\sim i}}$  is the effect of  $X_{\sim i}$ . Many Monte Carlo estimators are available for both indices. The following estimators are employed in this study:

$$V_{X_i}(E_{X_{\sim i}}(Y | X_i)) = \frac{1}{n} \sum_{j=1}^n f(A)_j (f(A_B^{(i)})_j - f(B)_j) \quad (54)$$

$$E_{X_{\sim i}}(V_{X_i}(Y | X_{\sim i})) = \frac{1}{2n} \sum_{j=1}^n (f(A)_j - f(A_B^{(i)})_j)^2 \quad (55)$$

Above for a  $k$  dimensional sampling Sobol sequence,  $\mathbf{A}$  and  $\mathbf{B}$  are two  $n \times k$  variance matrices generated by the Monte Carlo method.  $f(\mathbf{A})_j$  and  $f(\mathbf{B})_j$  represent the outputs for the  $j^{\text{th}}$  sample in  $\mathbf{A}$  and  $\mathbf{B}$ , respectively, and  $f(\mathbf{A}_B^{(i)})_j$  represents the output for the  $j^{\text{th}}$  sample by replacing the  $i^{\text{th}}$  column of  $\mathbf{A}$  with the  $i^{\text{th}}$  column of  $\mathbf{B}$ . The details can be found in Saltelli et al., 2010.

In this project, two scenarios are designed to determine critical parameters for the constitutive model. To expedite the analysis, the three-invariant soil model is converted into a user-defined subroutine for ABAQUS, a finite element analysis software. The numerical simulations are obtained by using ABAQUS.

#### 4.3.1. Sensitivity Analysis for Triaxial Compression Test

The schematic of the setting is shown in Figure 22. The baseline parameters for the model are given in Table 3. The parameters vary  $\pm 5\%$  in the sampling process in the sensitivity analysis. One hundred samples in total are randomly generated, and 3100 models are run in total to extract the outputs. The sensitivity indices of the maximum stress are calculated and shown in Figure 23 and Figure 24. Only four parameters have significant effects on the maximum stress in the triaxial compression test. Figure 24 also reveals that the initial yield stress  $\kappa_0$  has the most significant effect on the maximum stress.

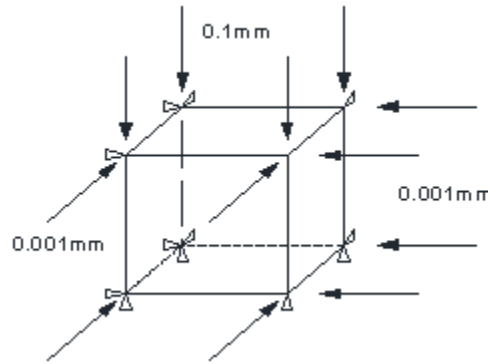


*Figure 22. Schematic of triaxial compression test for sensitivity analysis.*

Table 3. Baseline material parameters for sensitivity study.

|                      |                              |                          |
|----------------------|------------------------------|--------------------------|
| $E=9000.0$ MPa       | $a_t=100.0$                  | $\gamma_C^g=2.0$         |
| $\nu=0.15$           | $a_c=100.0$                  | $\gamma_\varepsilon=1.0$ |
| $\psi=0.8$           | $a_s=50.0$                   | $C_0=4.5$                |
| $\hat{R}=1.5$        | $C_{max}=5.5$ MPa            | $C_0^g=3.0$              |
| $S=0$                | $C_{max}^g=7.0$ MPa          | $\varepsilon_0=10.0$     |
| $R=0$                | $\varepsilon_{max}=11.0$ MPa | $n_0=0.2$                |
| $M_g=0$              | $C_{res}=2.0$ MPa            | $n_0^*=0.2$              |
| $\kappa_0=-50.0$ MPa | $C_{res}^g=0.5$ MPa          | $q_t=0.1$                |
| $M=0.2$              | $\varepsilon_{res}=7.0$ MPa  | $\tau=0.0$               |
| $M_g=0.05$           | $\gamma_C=1.0$               |                          |

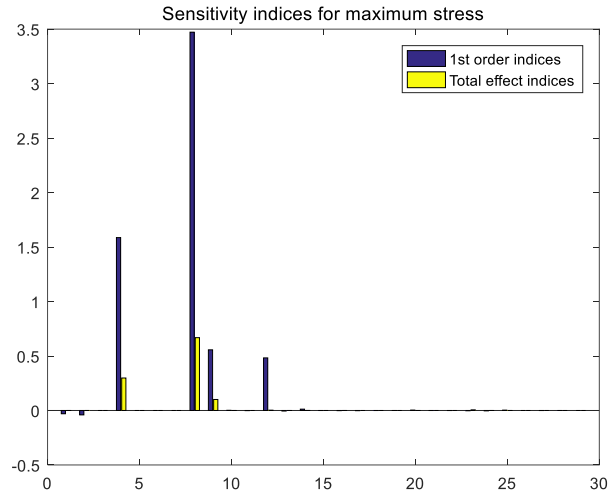


Figure 23. First-order indices and total effect indices for maximum stress in triaxial compression.

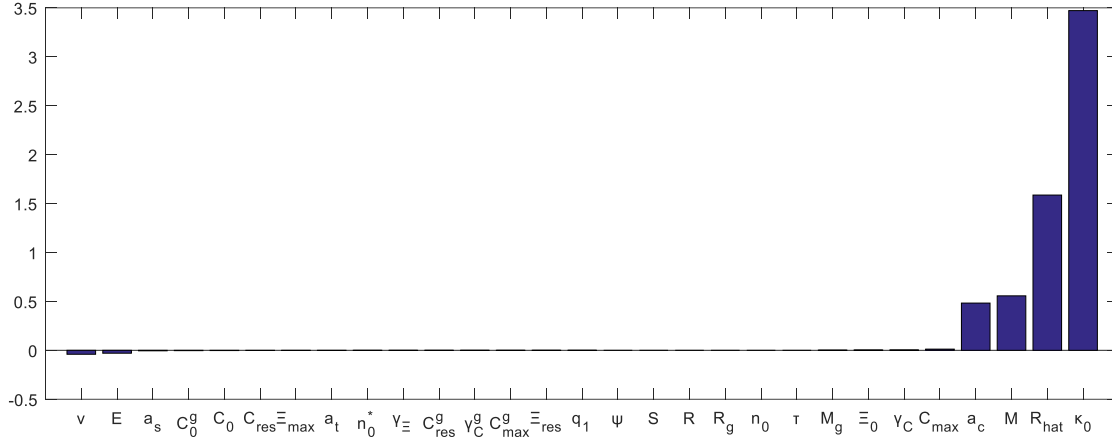


Figure 24. First-order sensitivity index ranking in the triaxial test.

#### 4.3.2. Sensitivity Analysis for Hertz Contact

To mimic the dominant deformation modes in the impact problem while accounting for computational efficiency, circular contact stress, extracted from the Hertz contact problem, is applied at the center of a cuboid in the quasi-static FE simulation. The finite element mesh, boundary conditions, and the contact stress distribution along the radial direction are given in Figure 25. The magnitude of the contact stress is obtained from the Hertz contact problem with the plastic effect neglected and can be expressed as  $p(r) = p_0 \sqrt{1 - r^2 / a^2}$ , where  $a$  is the contact radius,  $E$  represents the elastic modulus,  $\nu$  represents the Poisson's ratio, and  $p_0 = 2E(a/r)/\pi(1 - \nu^2)$ .

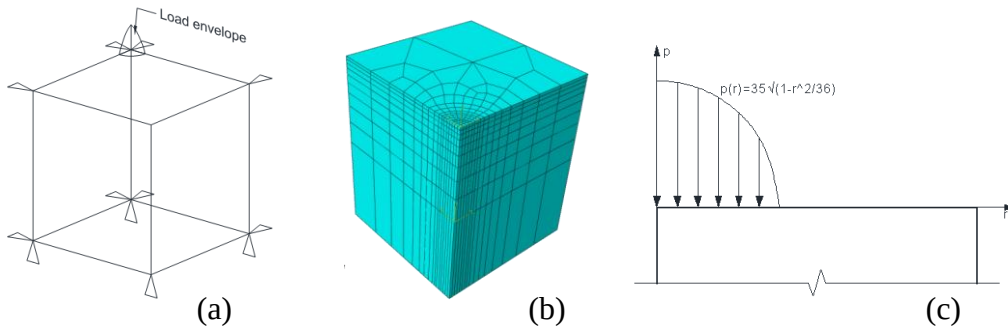


Figure 25. Hertz contact problem for sensitivity study. (a) schematic, (b) mesh configuration, and (c) contact stress distribution.

To evaluate the plastic deformation and indentation depth, three measured values are selected as the output parameters: 1) maximum indentation displacement, 2) the number of yielding integration points, and 3) plastic index. The maximum indentation displacement is a direct measurement of the deformation of interest, whereas counting the number of yielding integration points aims to evaluate the size of the plastic zone in the simulations. To further account for the level of plastic strain in the plastic zone, the plastic index is defined as follows.

$$PI = \frac{1}{nip} \sum_{i=1}^{nel} \sum_{j=1}^{nip} V_i \varepsilon_{e_j}^p \quad (56)$$

where  $V_i$  is the volume of the  $i$ -th element,  $\varepsilon_{e_j}^p$  denotes the equivalent plastic strain at  $j$ -th integration points in the  $i$ -th element,  $nel$  is the number of elements, and  $nip$  is the number of integration points in each element. In total, 800 samples are generated as the inputs and 24800 finite element models are analyzed. The sensitivity indices for the maximum displacement, number of yielding integration points, and plastic index are given in Figure 26 – Figure 31. The elastic modulus  $E$  has a predominant effect on the maximum displacement, followed by the initial shear strength  $C_0$  and the initial yield stress  $\kappa_0$ . On the other hand, the initial shear strength  $C_0$  and the shear yield surface parameter  $M$  contribute most to the plastic zone size and plastic index.

In summary, we conduct the Sobol sensitivity analysis to identify parameters that play an influential role in the three-invariant soil model, such as, the initial shear strength  $C_0$ , shear yield surface parameter  $M$ , the elastic modulus  $E$ , and the initial yield stress  $\kappa_0$ . This work may suggest specific experimental tests and fitting data strategies for the calibration of key parameters.

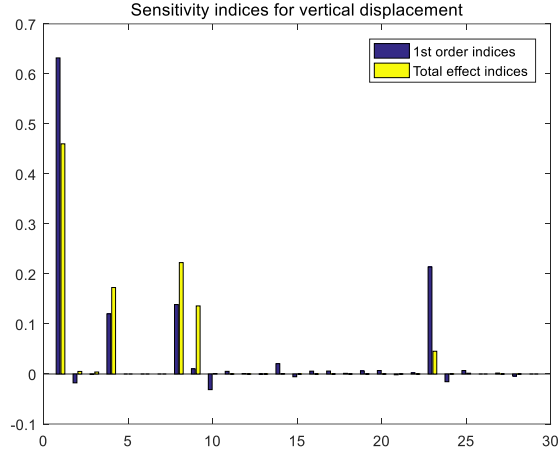


Figure 26. First-order indices and total effect indices for maximum displacement in the Hertz contact.

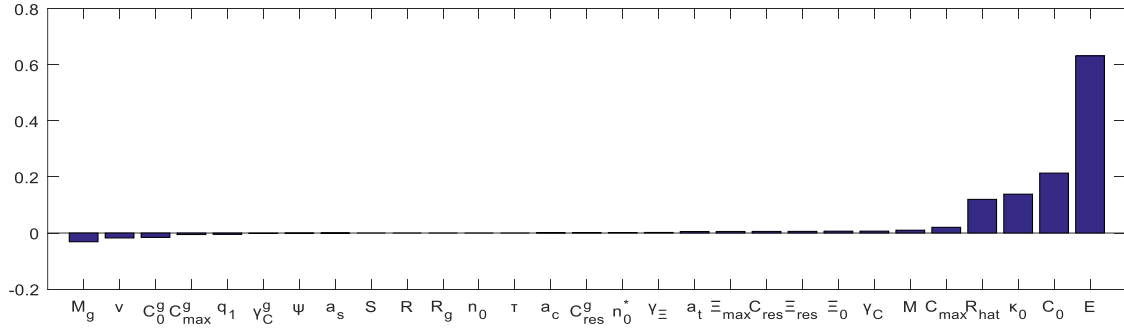


Figure 27. First-order sensitivity index ranking for maximum displacement in the Hertz contact.

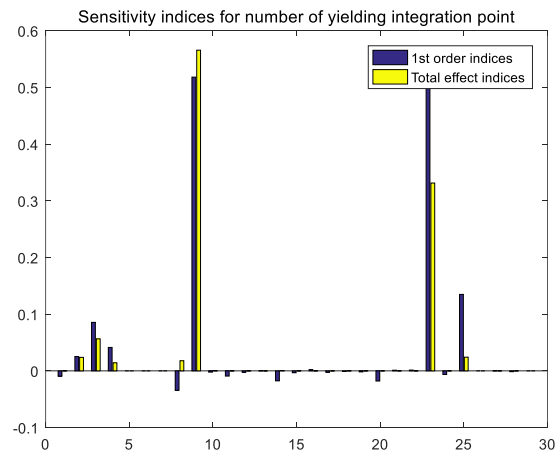


Figure 28. First-order indices and total effect indices for plastic zone size in the Hertz contact

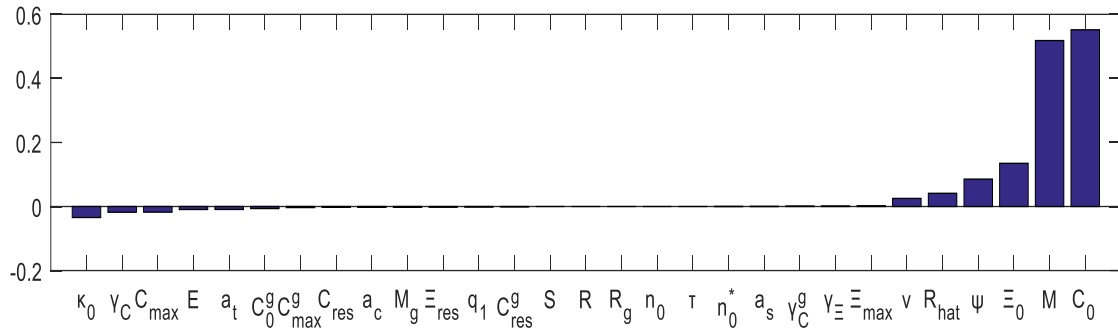


Figure 29. First-order sensitivity index ranking for plastic zone size in the Hertz contact

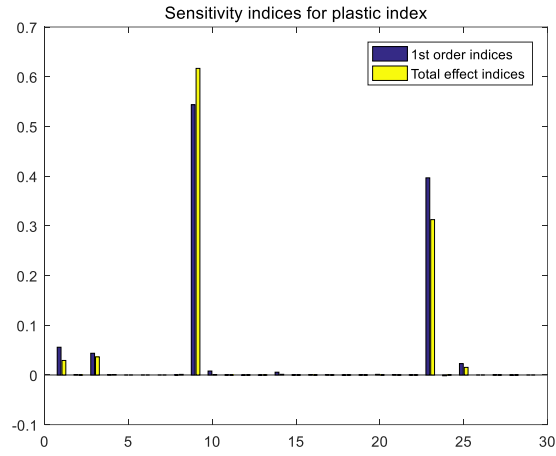


Figure 30. First-order indices and total effect indices for the plastic index in the Hertz contact

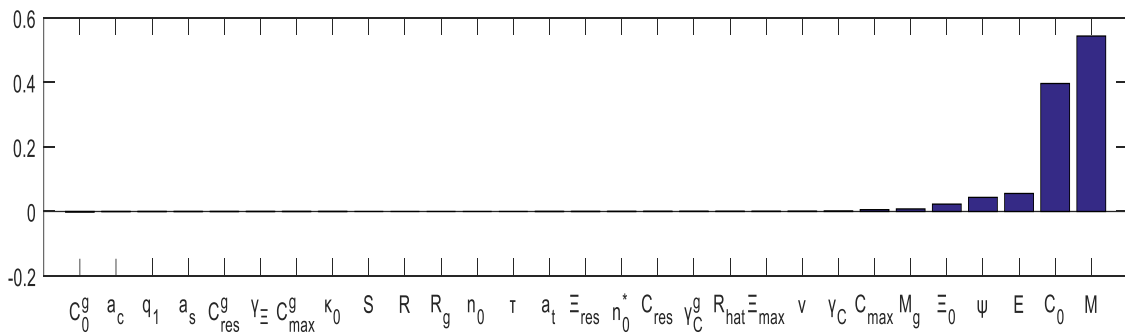


Figure 31. First-order sensitivity index ranking for the plastic index in the Hertz contact

#### 4.4. Partially Saturated Soil Model

We use a two-field formulation to model the behavior of partially saturated soil. The governing equations and boundary conditions are presented in Section 3.3. In this section, we describe the detailed constitutive relationship used in this project.

In partially saturated soil, the relative permeability  $k^{rw}$  in Equation (15) is a function of the degree of saturation  $\psi^w$ . The relationship between the degree of saturation and suction results from an experimental study. The mathematical model of the van Genuchten equation is utilized and given as:

$$\psi^w(P) = \psi_1 + (\psi_2 - \psi_1) \left[ 1 + \left( \frac{-P}{s_a} \right)^n \right]^{-m} \quad (57)$$

where  $\psi_1$  is the residual water retention,  $\psi_2$  is the maximum water saturation,  $s_a$  is scaling pressure, and  $n$  and  $m$  are empirical constants defining the shape of the degree of saturation-suction curve.  $n$  and  $m$  are not independent but are related by,

$$m = \frac{n-1}{n} \quad (58)$$

The water phase relative permeability is defined as,

$$k^{rw} = \theta^{\frac{1}{2}} \left[ 1 - \left( 1 - \theta^{\frac{1}{m}} \right)^m \right]^{22}, \quad (59)$$

$$\theta = \frac{\psi^w - \psi_1}{\psi_2 - \psi_1} \quad (60)$$

The variational equation (weak form) and implementation algorithms are given in Appendix E.1.

##### 4.4.1. Foundation Loading on Linear Elastic Clay

The response of a footing load on the linear elastic clay is analyzed using the model mentioned above. Because of the symmetry, only half of the domain is considered. The results are obtained

for both saturated and unsaturated soil domains by changing the level of the water surface. The schematic is shown in Figure 32. The load increases linearly to a maximum of 100 kN over 100 days and is then maintained.

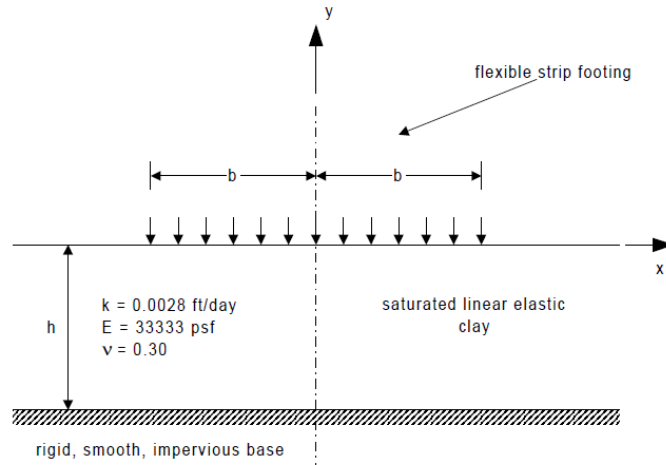


Figure 32. Schematic of a footing load on the linear elastic clay [17].

In the saturated case, the pore water pressure increases when the surface load is applied; then, the excess pore water pressure dissipates from the media, as shown in Figure 33a and Figure 34a. As a result of the dissipation of pore water pressure, there is a settlement, as depicted in Figure 33b and Figure 34b. In the unsaturated case, the pore water pressure is positive at the middle of the left edge, and it decreases since the sample is not saturated and no peak of water pressure is observed, as shown in Figure 34a and Figure 36a.

Table 4. Property values for the linear elastic clay.

| Description                         | Properties | Value     |
|-------------------------------------|------------|-----------|
| Shear Modulus                       | $\mu$      | 20000 KPa |
| Lamé's first parameter              | $\lambda$  | 80000 KPa |
| Scaling Pressure<br>(van Genuchten) | $S_a$      | 10 KPa    |
| Porosity                            | $n$        | 0.5       |
| Intrinsic Permeability              | $K$        | 5E-05 m/s |
| Coefficient (van Genuchten)         | $m$        | 0.5       |
| Coefficient (van Genuchten)         | $n$        | 2         |

Both saturated and unsaturated versions of the model are tested. The results for the saturated case are shown in Figure 33 and Figure 34. Figure 33a shows the pore water pressure at the end of the simulation, while Figure 33b shows the vertical displacement at the same time. Figure 34 shows the variation in time of pore pressure and displacement at nodes along the centerline, respectively. Displacement is shown for the node at the surface, while pore pressure is illustrated for the node 5 m below the surface.

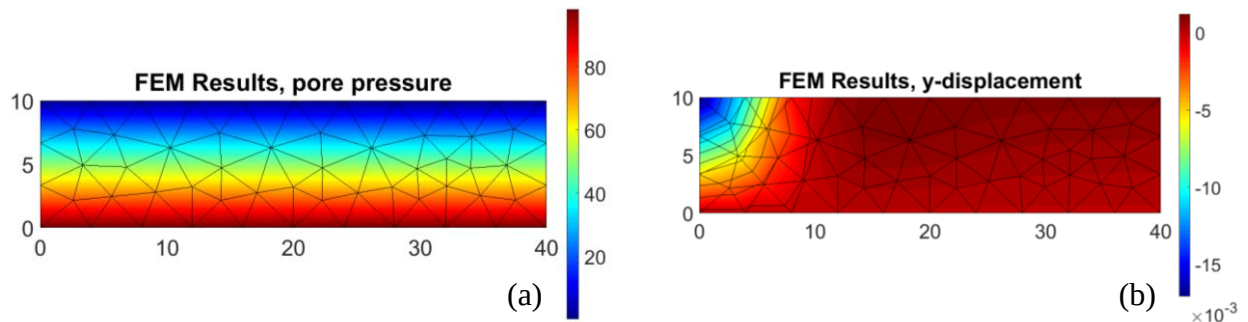


Figure 33. (a) Pore water pressure for the saturated case and (b) Vertical displacement for the saturated case

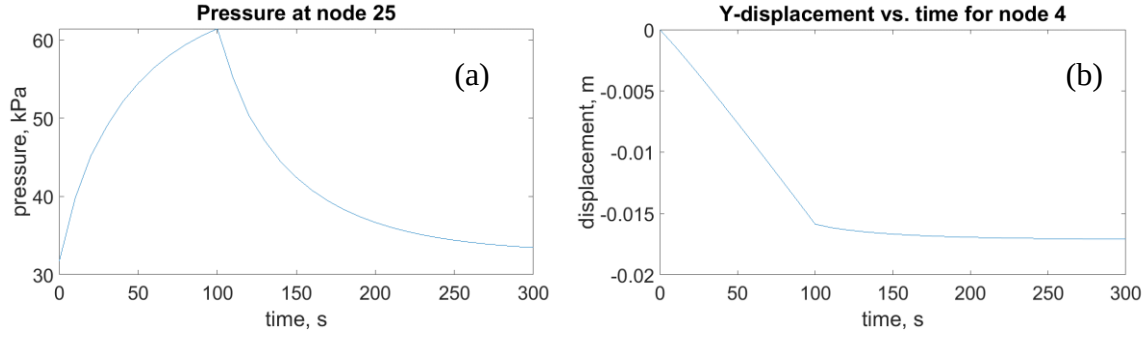


Figure 34. (a) pressure at the middle of the left edge for the saturated case and (b) Displacement at the middle of the left edge for the saturated case.

Figure 35 and Figure 36 show results for the unsaturated case. The loading is the same as for the saturated case, but the water table is lowered to 3 m below the surface of the water. The vertical displacement and pore pressure at the end of the simulation are shown in Figure 35. The water pressure is negative for the unsaturated case, which demonstrates the effects of suction pressure. The vertical displacement and pore pressure for nodes directly under the loading are shown in Figure 36. As in the saturated case, displacement is shown for the node at the surface, while pore pressure is illustrated for the node 5 m below the surface. The properties of the material are presented in Table 4.

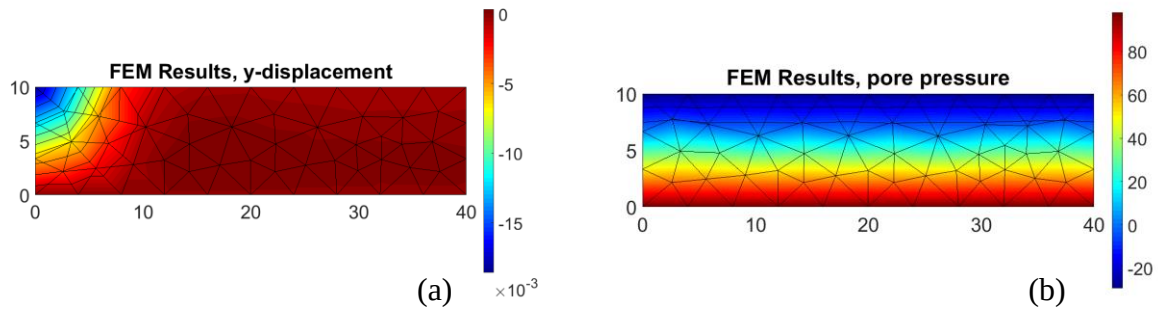


Figure 35. (a) vertical displacement for the unsaturated case and (b) pore pressure for the unsaturated case

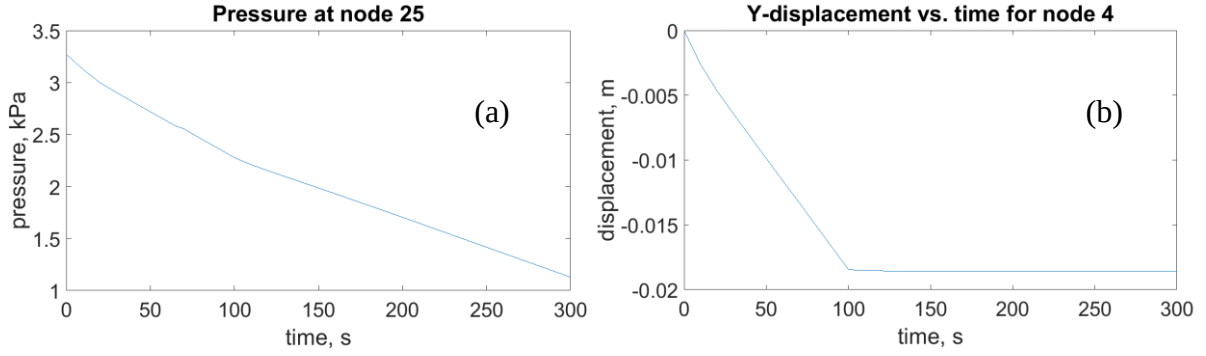


Figure 36. (a) pore pressure at the middle of the left edge for the unsaturated case and (b) displacement at the middle of the left edge for the unsaturated case

The partially saturated soil model has been implemented. Steady-state models have been verified and match the literature. However, currently, the material consolidates too quickly, as shown in. Figure 37. A benchmark problem is chosen from the analytical solution developed by [17]. The geometry of the problem is given in Figure 32, and because of the symmetry of the problem, only half of the geometry is considered. The properties of the material are given in Table 4. Surface loading is applied using a linear ramp function for the first 0.05 day. To verify the saturated formulation, the vertical settlement at the middle of the foundation is considered. The results are demonstrated in Figure 37.

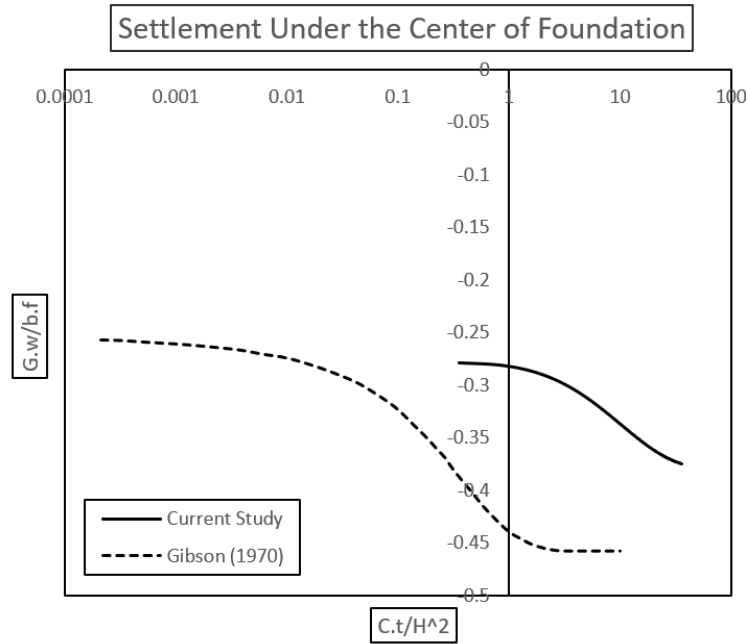


Figure 37. Benchmark comparison with Gibson (1970) [17] showing the settlement under the center of the foundation.

#### 4.5. Incorporation into Meshfree Code

The soil plasticity model has been incorporated into the meshfree code. The implementation has been verified, as shown in Figure 38. Slight differences arise in that the finite element simulation is quasistatic, while the meshfree simulation is dynamic. At slow loading rates, though, the results are very similar.

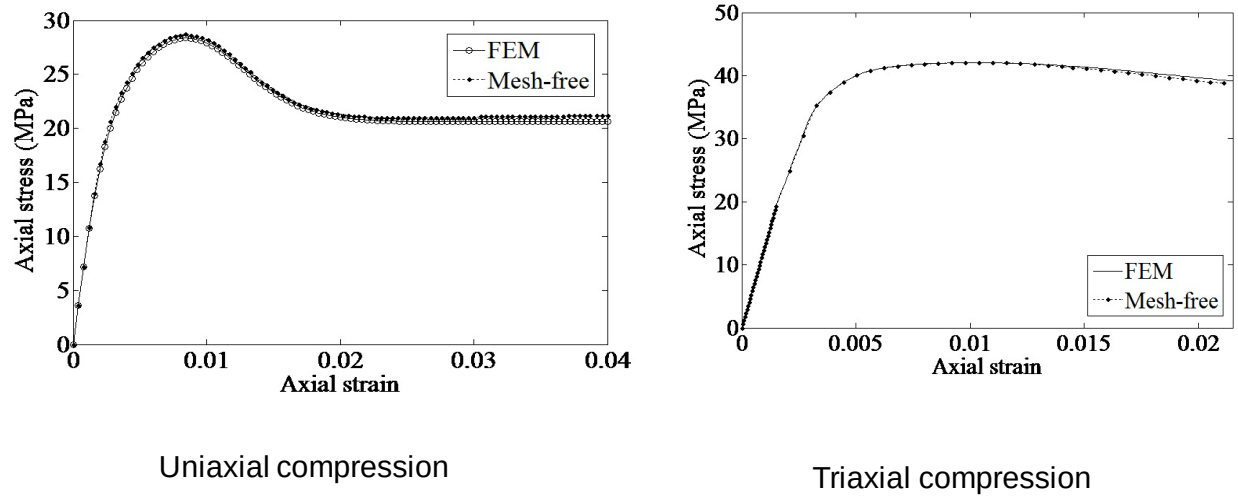


Figure 38. Verification between the meshfree and finite element implementations of the model.

## 5. Task 3 – Soil Tests and Penetration Tests

### 5.1. Soil Tests for Calibration Soil Mechanical Properties

The US SILICA™ Flint #12 sand was selected as the granular material for the laboratory tests and validations. Some of the mechanical properties of the sand have been reported in the previous reporting years, including density determination, specific gravity determination by pycnometer, grain size analysis, etc. The test results are summarized in Appendix F. Moreover, Consolidated Drained Triaxial Tests (CD) were conducted for dry, unsaturated, and fully saturated sand samples to determine the shear strength parameters under different saturation levels. A downhill simplex scheme was employed for extracting the material parameters for simulations.

#### 5.1.1. Consolidated Drained Triaxial Tests (CD) for Dry, Unsaturated, and Fully Saturated Sand

Following the ASTM D7181-11 code, CD triaxial tests were performed on the sand with various saturation ratios (SR), corresponding to nominal SRs of 0, 0.5, and 1. The actual SR was controlled between 0.45 to 0.55 by weight for the unsaturated sand and between 0.9 to 1.0 for the fully saturated sand. By applying 5-psi backpressure, the soil sample was soaked by applying 5-psi backpressure, and the designed SR was achieved by measuring the weight of water and sand. After carefully filling the cavity unfilled during the transporting, the sample is ready for testing. A 15-psi cell pressure was applied as the confinement pressure, and the loading machine was set to the

desired loading speed. Three loading speeds, 13, 6.5, and 1.3 mm/min, were tested, corresponding to 10%, 5%, and 1% strain per minute. The stress versus displacement for the fully saturated and unsaturated soil under the three strain rates is plotted in Figure 39-Figure 44. From the test results, the shear strength, the average maximum total stress, and the average frictional angle for the sand with various saturation ratios were determined and compared in Figure 45 and Figure 46.

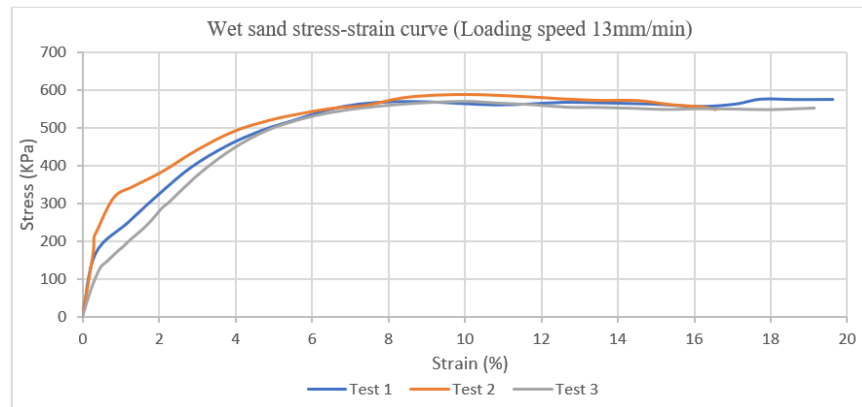


*Figure 39. Stress- displacement for fully saturated sand at 10% strain rate.*

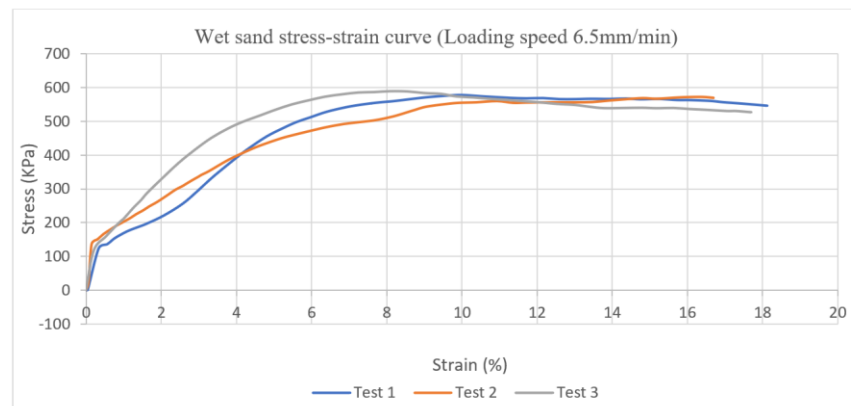


*Figure 40. Stress- displacement for fully saturated sand at 5% strain rate.*

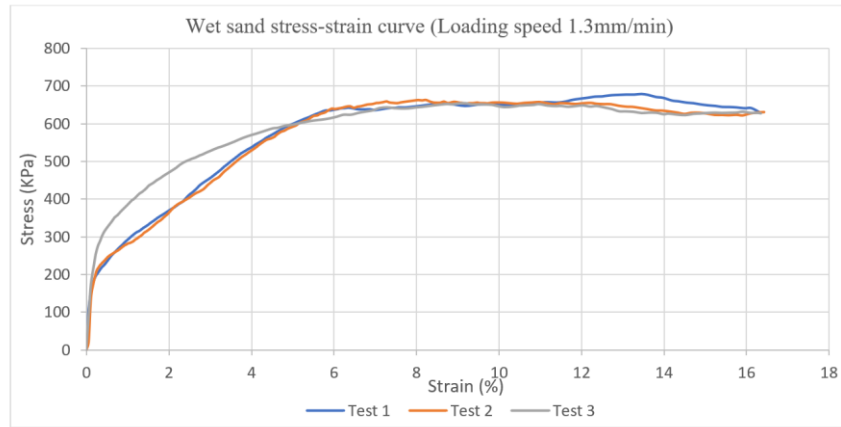


Figure 41. Stress- displacement for fully saturated sand at 1% strain rate.

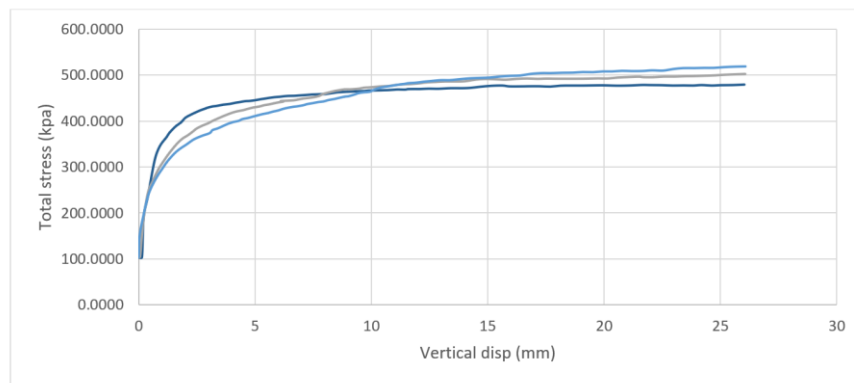


Figure 42. Stress- displacement for unsaturated sand at 10% strain rate.

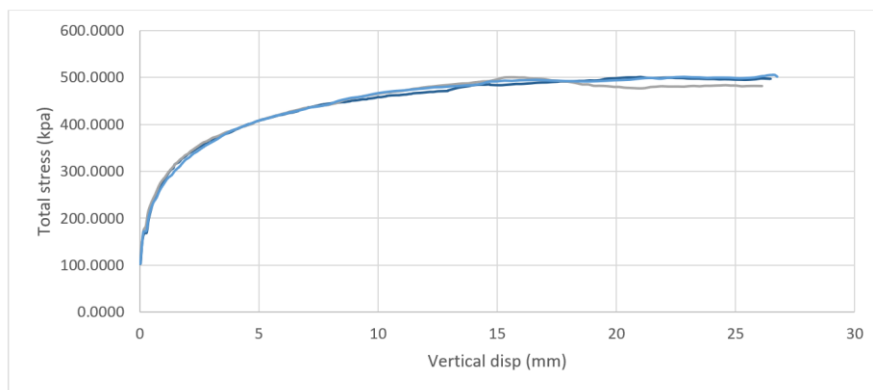


Figure 43. Stress- displacement for unsaturated sand at 5% strain rate.

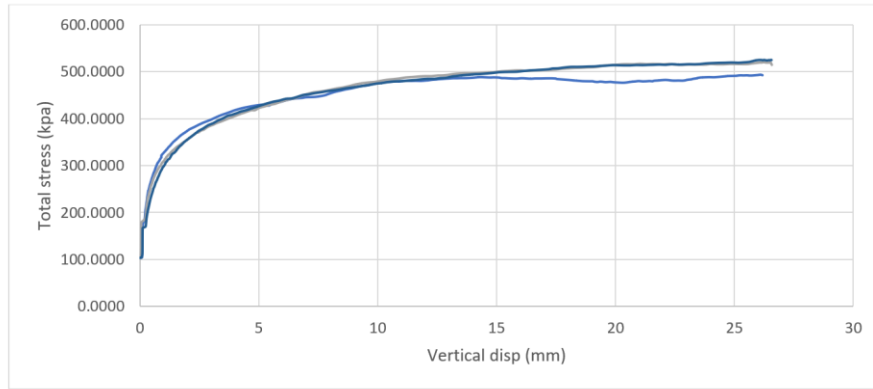


Figure 44. Stress- displacement for unsaturated sand at 1% strain rate.

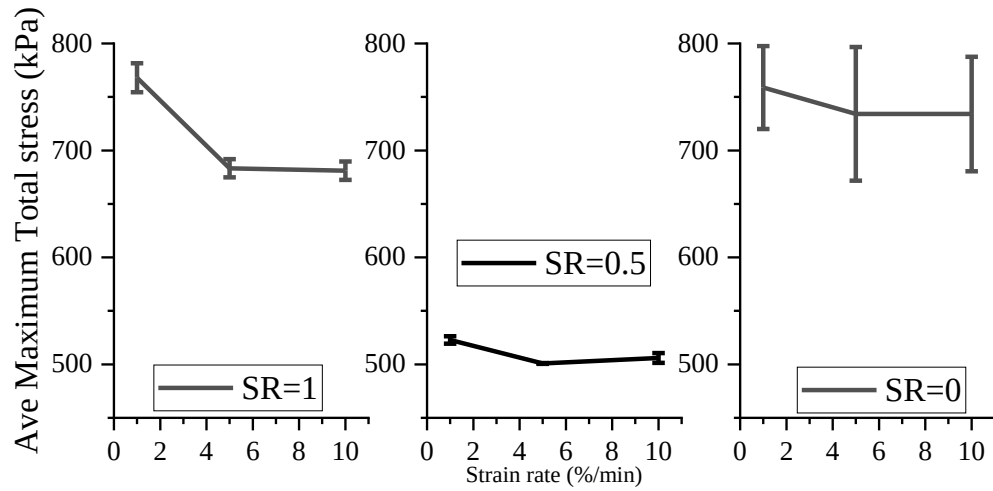


Figure 45. Average maximum total stress with various SR.

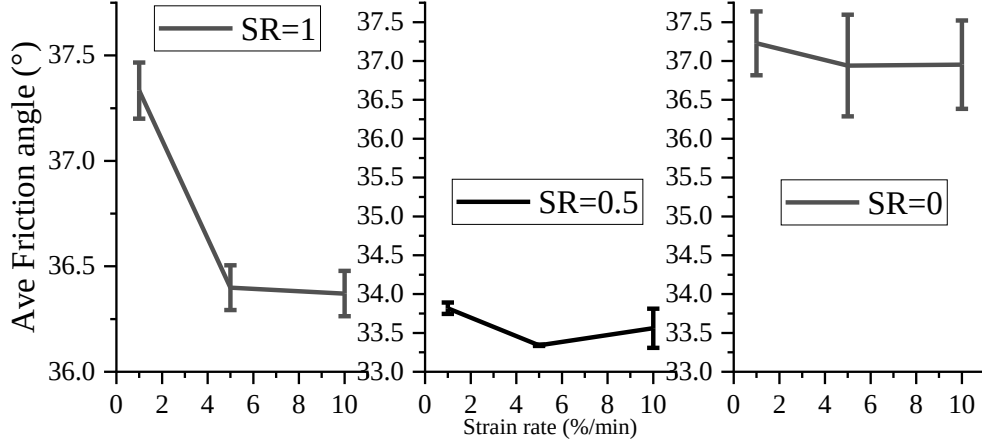


Figure 46. Average frictional angle with various SR.

#### 5.1.2. Automatic Material Parameter Calibration

Despite the effort made to calibrate material parameters from the experimental data (described in Section 4.2), some parameters do not have a direct correlation with load-displacement response and, therefore, are difficult to calibrate directly from experimental data. We employed a nonlinear optimization technique called the downhill simplex algorithm [18], to automatically calibrate the parameters for numerical models from provided experimental data. To do so, a numerical model that replicates the experiment setting of triaxial tests described in Section 5.1 was constructed (Figure 47), and the same loading condition as in the experiment was applied to the numerical model to obtain the numerical load-displacement curve. Note that the numerical model here can be constructed using the semi-Lagrangian RK formulation described in Section 3.1. However, a finite element model was used for computational efficiency. The downhill simplex method minimizes the difference between the experimental and numerical data. The difference index is defined as:

$$E = \frac{\int_t (RF_{test} - RF_{numerical})^2}{\int_t RF_{test}} \quad (61)$$

where  $RF_{test}$  is the measured force history and  $RF_{numerical}$ , which is a function of the set of material parameters in the constitutive law,  $\{\mathbf{Y}\}$ , is the corresponding force history output from numerical

simulations. By a series of iterative search processes in the hyperspace of  $\{\mathbf{Y}\}$ , the closest set that minimizes  $E$  can be found.

As an example, the Drucker-Prager with damage model was used to represent the soil sample in Figure. The termination criterion was set to be less than 5% for  $E$  in (61), and the set of initial parameters for dry sand given in Table 5 was adopted. Through the iterative procedure,  $E$  was decreased from around 60% to less than 5%. The converged numerical force history was compared with the experimental force history in Figure 48. The converged material parameters for the sand with various saturation ratios are listed in Table 2.

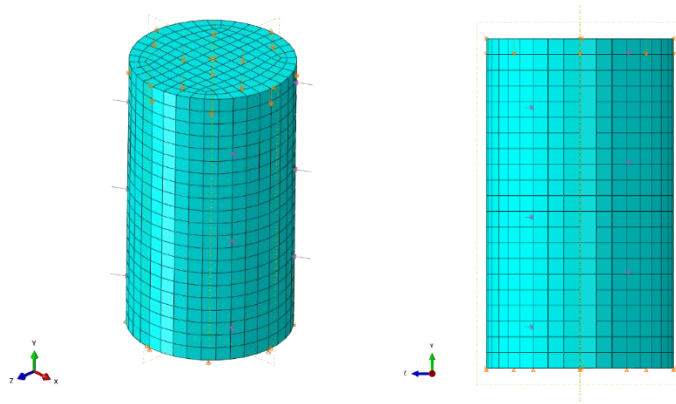


*Figure 47. Finite element model (ABAQUS) for simulating the triaxial test.*

Table 5. Initial material parameters for dry sand.

| Drucker-Prager soil properties                    |                                 |
|---------------------------------------------------|---------------------------------|
| Young's modulus ( $E$ )                           | $5.4 \times 10^6 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.35                            |
| Friction Parameter ( $\beta$ )                    | 0.14                            |
| Hardening                                         | $0.0 \frac{N}{m^2}$             |
| Cohesion strength ( $\alpha$ )                    | $1.0 \times 10^4$               |
| First parameter for damage accumulation function  | 0.05                            |
| Second parameter for damage accumulation function | 1.0                             |
| Density ( $\rho$ )                                | $1450 \frac{kg}{m^3}$           |

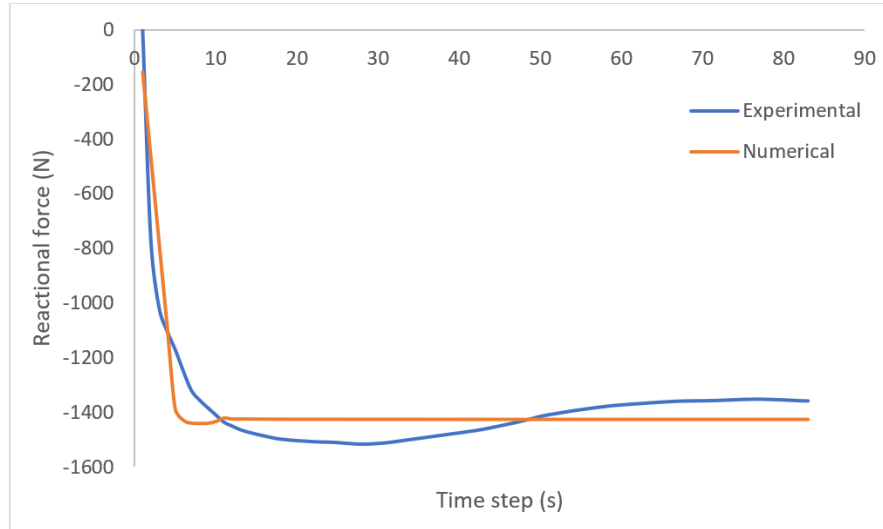


Figure 48. Comparison of numerical and experimental force histories after fitting.

Table 6. Material parameters for sand with various saturation ratios.

| Soil properties                                   | Dry                             | Unsaturated                      | Saturated                        |
|---------------------------------------------------|---------------------------------|----------------------------------|----------------------------------|
| Young's modulus ( $E$ )                           | $3.1 \times 10^7 \frac{N}{m^2}$ | $3.04 \times 10^7 \frac{N}{m^2}$ | $2.61 \times 10^6 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.2                             | 0.28                             | 0.1                              |
| Friction Parameter ( $\beta$ )                    | 0.38                            | 0.48                             | 0.46                             |
| Hardening                                         | $12.8 \frac{N}{m^2}$            | $651.5 \frac{N}{m^2}$            | $2.24 \times 10^5 \frac{N}{m^2}$ |
| Cohesion strength ( $\alpha$ )                    | 0.0001                          | 0.0001                           | 0.009                            |
| First parameter for damage accumulation function  | 0.0487                          | 0.044                            | 0.28                             |
| Second parameter for damage accumulation function | 0.96                            | 0.812                            | 0.46                             |
| Density ( $\rho$ )                                | $1450 \frac{kg}{m^3}$           | $1450 \frac{kg}{m^3}$            | $1450 \frac{kg}{m^3}$            |

## 5.2. Penetration Tests

### 5.2.1. Apparatus for Penetration Tests

An apparatus for performing penetration tests were built and placed in the UIC high-bay structural laboratory, see Figure 49a. This is the first apparatus for penetration tests at the UIC. The main goals of building the apparatus are to provide penetration testing data consistent with the soil testing data, validate the simulation results, and guide the modification of the current model. The system is powered by compressed air and can launch a 1-inch diameter bullet with different shapes up to 100 m/s. The system has a frame with a height of 10 feet and a width and depth of 40 inches and can accommodate a granular target up to 1.5 feet in width and 2 feet in depth. The system has a pneumatic projectile launcher and can launch a 1-inch diameter projectile by compressive air with a final speed desired up to 100 m/s. The final impact speed is measured by a chronograph, and the projectile penetration depth is measured by a digital caliper, as shown in Figure 49. After the shot is complete, the projectile is cleaned to wash off the sand residual, and a sample of the target is taken from a typical impact zone to analyze its water ratio in the laboratory.



*Figure 49. Penetration test apparatus. (a) frame, (b) chronograph, (c) depth gauge, (d) pneumatic launcher.*

#### 5.2.2. Penetration Tests for Dry, Unsaturated, Fully Saturated Sand

In each penetration test, the soil sample was prepared using the same conditions as in the soil tests described in Section 5.1. The depth gauge was zeroed referenced to the top surface of the soil target. After the projectile was shot in the soil target, a thin steel rod on the digital caliper was used to carefully probe the impact area and find the lowest reading before encountering resistance, which indicates the distance from the soil surface to the top surface of the projectile. This probing reading was further verified after the projectile was dug out, and the verified distance reading was recorded as the depth of penetration in this report. The penetration process was recorded with 240-fps videos to analyze the impact mechanics and validate the numerical simulations. After the measurement was complete, the projectile was cleaned to wash off the sand residual, and a sample of the soil target was taken from the impact zone to analyze the water ratio in the laboratory.

Over 100 tests were completed for dry, unsaturated, and fully saturated soil targets, of which the corresponding nominal saturation ratios (SR) were 0, 0.5, and 1, respectively. Thirty-eight tests, shown as green dots in Figure 50, were carried out for dry sand with the impact speed ranging from 7 to 71 m/s. As seen in Figure, the DOP increases linearly when the final speed is less than 25.3 m/s. The DOP increase rate decreases and reaches the maximum DOP at 53 m/s with an average DOP of 163.4 mm. It is observed that the DOP drops after the impact speed of 63.5 m/s, which may be attributed to the boundary effect from the bottom of the target. Overall, the DOP test data for dry sand show good consistency, of which the standard deviation is less than 4 mm, and the maximum deviation is less than 8 mm.

Fifty-three tests for the fully saturated sand, plotted as black dots in Figure 50, were carried out with the impact speed in the range of 7 to 63 m/s. The DOP is much lower compared with that of the dry sand under a similar impact speed. The DOP increases linearly with the increase in impact speed up to 40 m/s and achieves the maximum value at around 80 mm. However, the DOP results fluctuate in comparison to the ones for dry sand. The standard deviation of the DOP with a similar speed is around 5 mm, and the highest standard deviation is about 10 mm.

Due to the complexity of preparing partially saturated soil samples, only 12 tests were performed for sand with  $SR = 0.5$ , with the impact speed in the range of 30.8 to 66.4 m/s, shown as red dots in Figure 50. The DOP results fluctuate among all SR conditions due to the uncertainty of preparing samples. Nevertheless, excluding a seeming outlier, the mean DOP results lie between those for dry and fully saturated sand. Typical impact craters of soil samples for three different SRs are given in Figure 51.

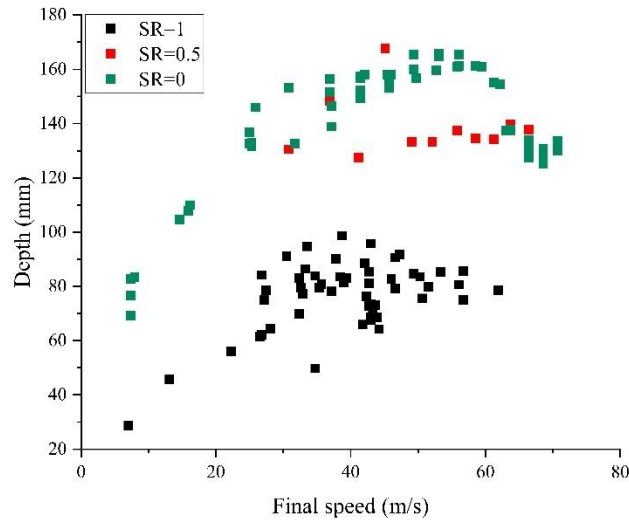


*Figure 50. DOP versus impact speed for sand with different saturation ratios.*



*Figure 51. Soil deformations after impact for (a) dry  $SR=0$ , (b) fully saturated  $SR=1$ , and (c) partially saturated  $SR = 0.5$ .*

Overall, penetration test results show that the DOP increases when the impact speed is low, reaches the maximum value in the mid-speed range, and decreases slightly in the higher speed range. The slight decrease in DOP in the higher speed range in our tests could be attributed to the confining effect of the container and the wave reflection from the bottom. These DOP data and deformation patterns were used to validate the simulation results in Section 6.5.

## **6. Task 4 – Verification and Validation**

### **6.1. Penetration of an Ogive Projectile into Dry Sand**

The experimental data in Hankins et al. [19] was considered to validate the developed meshfree code for modeling penetration into the soil. An ogive projectile of length 41.66 mm and radius 2.083 mm made of aluminum was projected with various velocities into the coarse sand. Several impact velocities ranging from 24 m/s to 170 m/s were simulated. The semi-Lagrangian framework was used with a normalized support size of 1.50, and the MPIM was employed for domain integration. Nearly 75100 nodes were used for the simulation, with an averaged nodal distance of 2.0 mm for both projectile and soil. The soil grain was modeled by the Drucker-Prager with damage model, and the aluminum projectile was modeled as linear elastic. The elastic material properties were adapted from the data reported in [19] and the plastic and damage parameters were calibrated using the penetration data at the impact velocity of 170 m/s. The calibrated material parameters were given in Table 7 and Table 8 for soil target and projectile, respectively. The model setup for this simulation is shown in Figure 52.

Table 7. Material properties of soil in simulation 6.1.

| Drucker-Prager soil properties                    |                                 |
|---------------------------------------------------|---------------------------------|
| Young's modulus ( $E$ )                           | $1.5 \times 10^7 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.32                            |
| Friction Parameter ( $\beta$ )                    | 0.1                             |
| Hardening                                         | $0.0 \frac{N}{m^2}$             |
| Cohesion strength ( $\alpha$ )                    | 0.0                             |
| First parameter for damage accumulation function  | 0.05                            |
| Second parameter for damage accumulation function | 1.0                             |
| Density ( $\rho$ )                                | $1450 \frac{kg}{m^3}$           |

Table 8. Material properties of the projectile in simulation 6.1.

| Linear Elastic Material   |                                     |
|---------------------------|-------------------------------------|
| Young's modulus ( $E$ )   | $7.82 \times 10^{10} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ ) | 0.32                                |
| Density ( $\rho$ )        | $2800 \frac{kg}{m^3}$               |

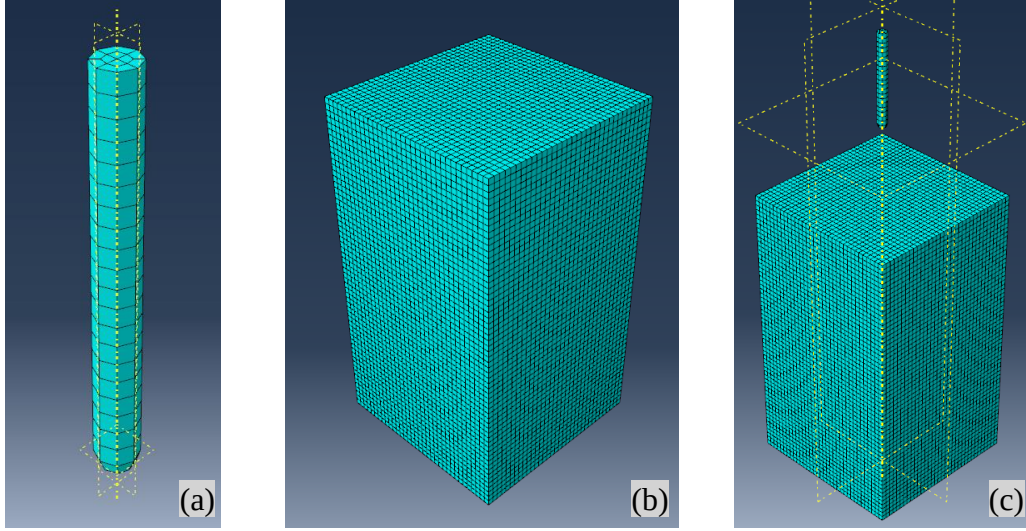


*Figure 52. (a) projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration in simulation 6.1.*

The simulation results for the projectile velocity of 170 m/s are shown in Figure 53. The color contour in Figure 53 indicates the level of damage (1 as 100% damaged). The numerical solutions using the MPIM remain stable throughout the simulation time, and the soil deformation in the simulation agrees with the experimental data in [19]. With the same set of parameters in Table 7 and Table 8, several impact velocities were simulated to attain the numerical prediction and form a trend line. The resulted trend line was validated with the experimental data and is shown in Figure 54. The numerical prediction qualitatively captures the tendency of the experimental data.

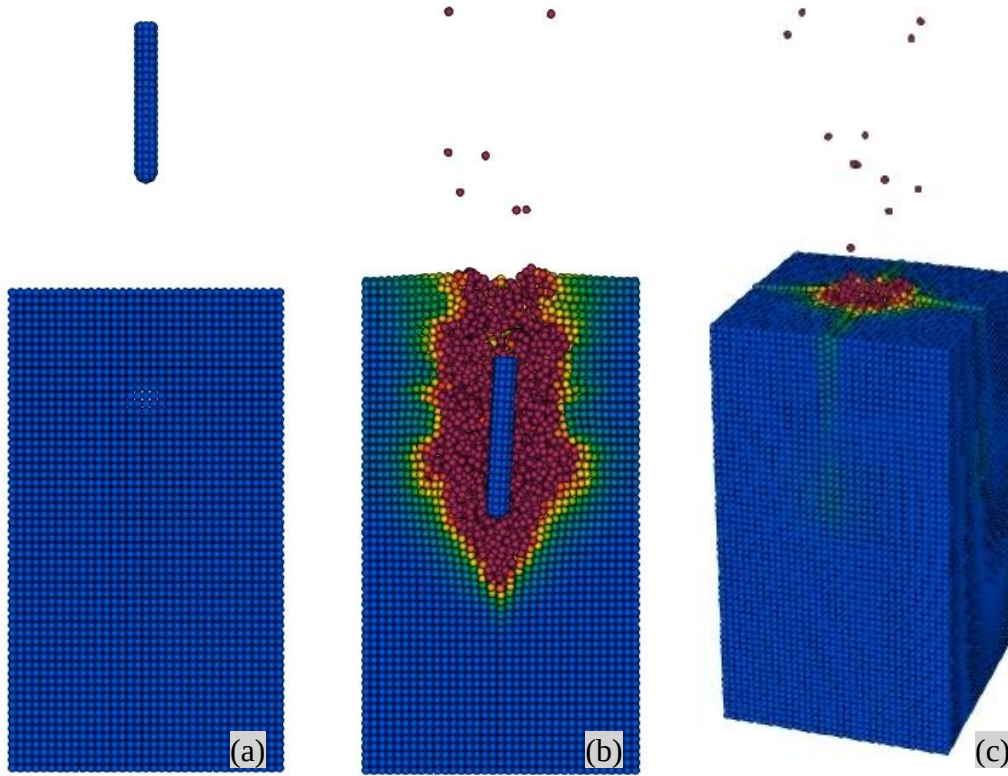


Figure 53. Penetration simulation results of case 6.1. (a) initial configuration, (b) sectional view of deformation of soil after bullet penetration, and (c) oblique view of deformation of soil after bullet penetration.

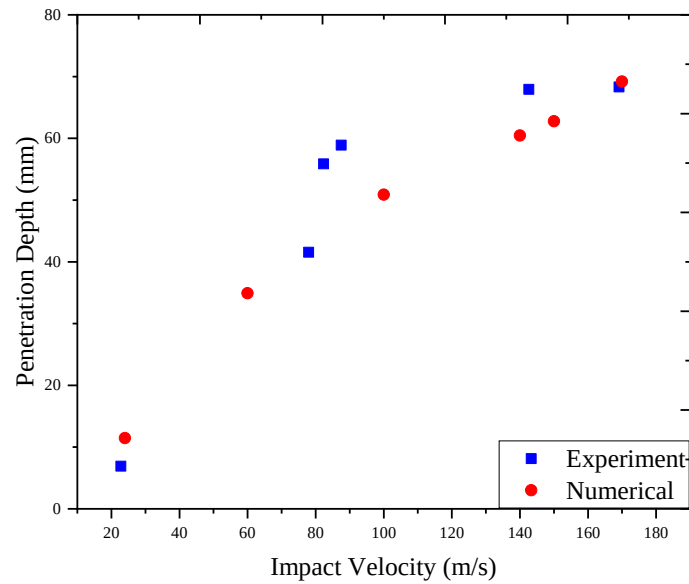


Figure 54. DOP versus impact velocities from experiments and numerical solutions of case 6.1.

## 6.2. Penetration of a Spherical Ball into Sand

The experimental data in Bivin et al. [20] was taken to validate the meshfree code for modeling penetration into dry soil. A spherical ball of radius 5.0 mm made of steel was projected with various velocities into the coarse sand target. Several impact velocities ranging from 40.66 m/s to 341.95 m/s were simulated. The semi-Lagrangian framework with a normalized support size of 1.50 and the MPIM was employed. About 101455 nodes were used to discretize the geometry of the model, with an averaged nodal distance of 1.8 mm for the projectile and 2.0 mm for the soil. The soil was modeled by the Drucker-Prager with damage model, and the steel ball was modeled as linear elastic. The elastic material properties were adapted from the data reported in [20], and the plastic and damage parameters were calibrated using the penetration data for the impact velocity of 162 m/s. The calibrated material parameters were listed in Table 9 and Table 10 for soil target and steel ball, respectively. The numerical setup for this simulation is shown in Figure 55.

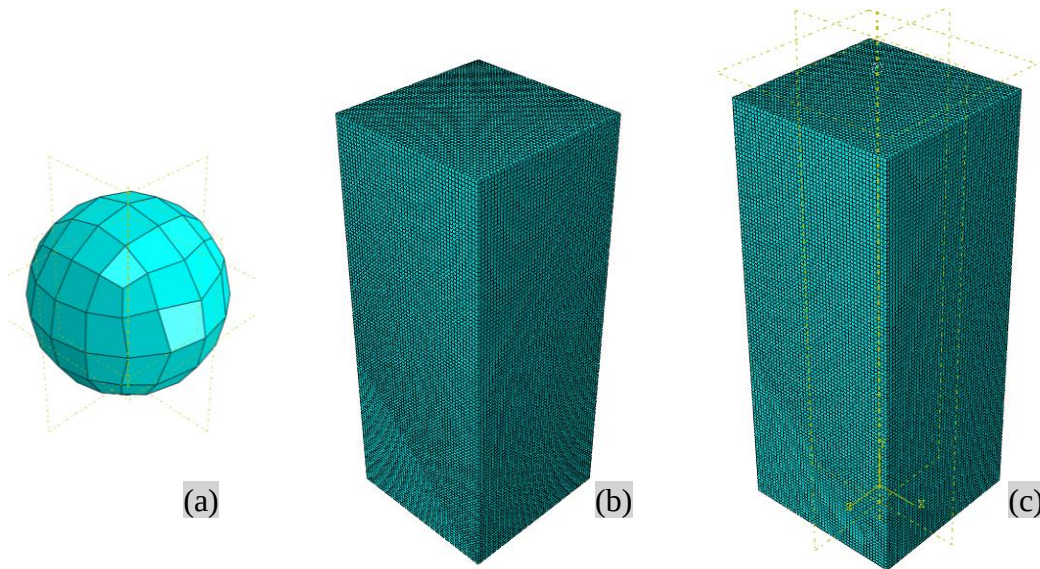


Figure 55. (a) spherical ball projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration in the numerical model of case 6.2.

Table 9. Material properties of soil in simulation 6.2.

| Drucker-Prager soil properties                    |                                 |
|---------------------------------------------------|---------------------------------|
| Young's modulus ( $E$ )                           | $5.4 \times 10^6 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.35                            |
| Friction Parameter ( $\beta$ )                    | 0.14                            |
| Hardening                                         | $0.0 \frac{N}{m^2}$             |
| Cohesion strength ( $\alpha$ )                    | 0.0                             |
| First parameter for damage accumulation function  | 0.05                            |
| Second parameter for damage accumulation function | 1.0                             |
| Density ( $\rho$ )                                | $1480 \frac{kg}{m^3}$           |

Table 10. Material properties of the projectile in simulation 6.2.

| Linear Elastic Material               |                                     |
|---------------------------------------|-------------------------------------|
| Young's modulus ( $E$ )               | $2.15 \times 10^{11} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )             | 0.30                                |
| Density ( $\rho$ )                    | $7850 \frac{kg}{m^3}$               |
| Mass proportional damping coefficient | 0.05                                |

Figure 56 shows the simulation results for the projectile velocity of 162 m/s. Figure 57 shows the DOP at a different impact velocity of experimental data and numerical solution. The numerical results marginally overpredict the penetration depth for lower and higher impact velocities. Nonetheless, the trend line of numerical results qualitatively agrees with the experimental data. It can also be noticed that the difference between the experimental mean penetration depth and numerical depth is under 10%.

Figure 58 shows the simulation result of a parametrical study by increasing Young's modulus 10 times the calibrated one. It is observed that the numerical DOP of various velocities is reduced when compared with the calibrated model. It is evident that by increasing Young's modulus, the model became stiffer and thus difficult to penetrate the target.

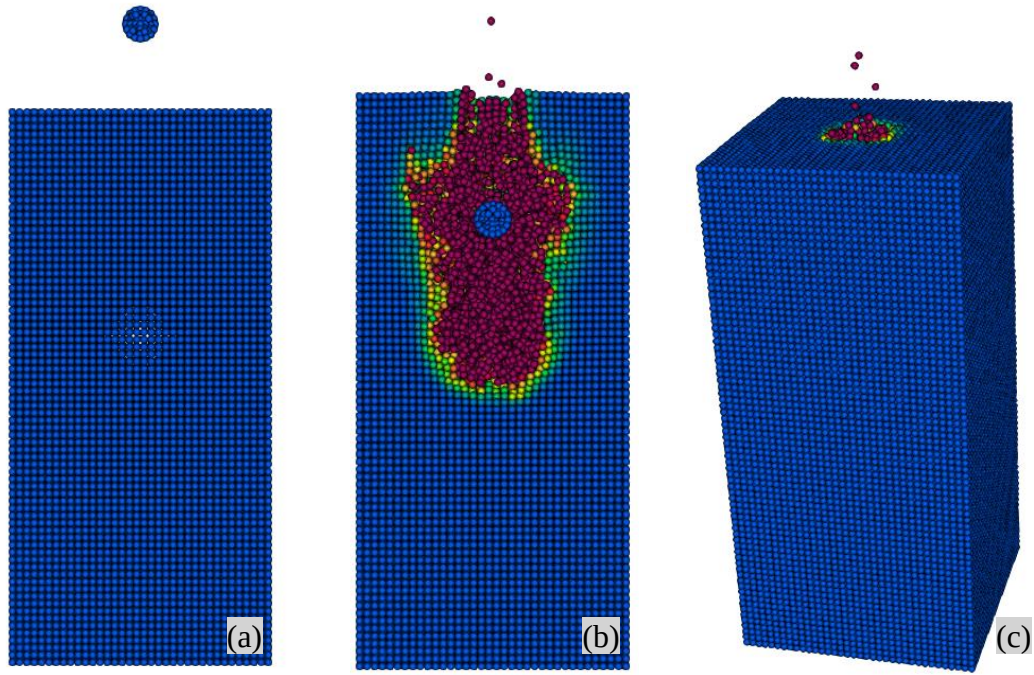


Figure 56. Penetration simulation results of case 6.2. (a) initial configuration, (b) sectional view of deformation of soil after bullet penetration, and (c) oblique view of deformation of soil after bullet penetration.

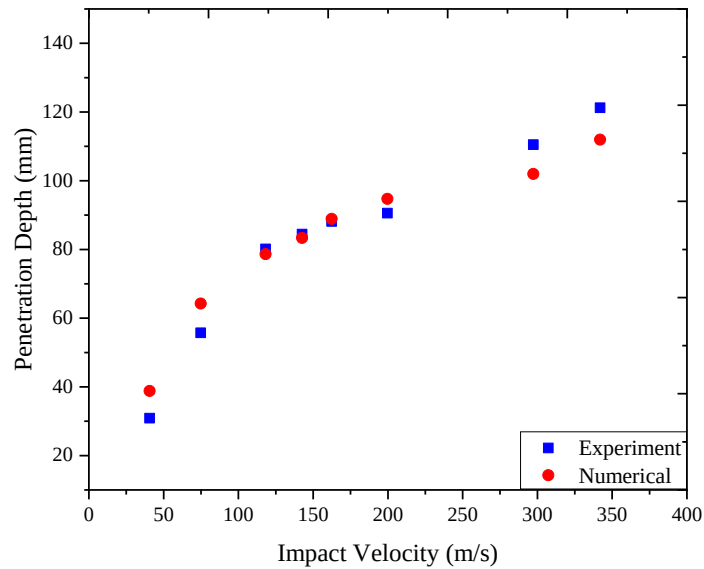


Figure 57. DOP versus various impact velocities from experiments and numerical solutions of case 6.2.

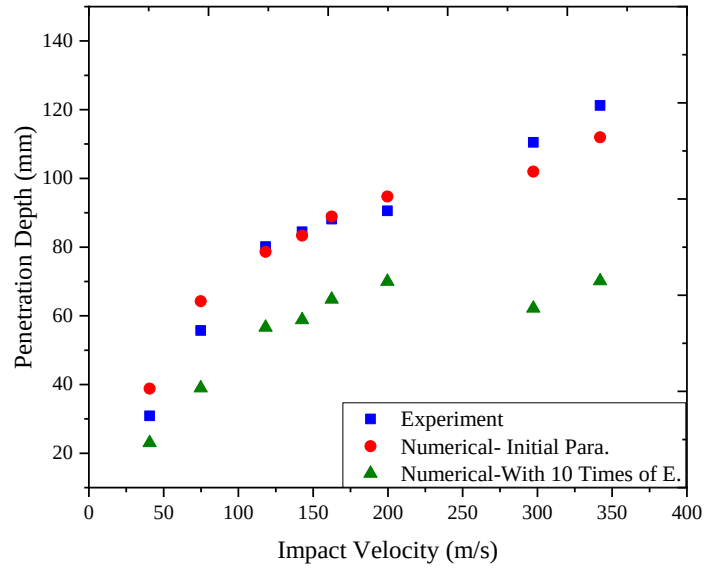


Figure 58. Effect of Young's Modulus on the DOP versus various impact velocities of case 6.2.

### 6.3. Ricochet of a Projectile on Sandy Soil

The experimental data in Xu et al. [21] was taken to validate the meshfree code for modeling penetration into dry soil with an impact angle. A spherical ball radius of 25.0 mm made of concrete was projected with an impacting velocity of 62.23 m/s into the coarse sand and an impact angle of 30°. The semi-Lagrangian framework with a normalized support size of 1.50 and the MPIM integration was employed. About 120413 nodes were used for the simulation, with an averaged nodal distance of 0.01m and 0.012m for projectile and soil, respectively. The soil grain was modeled by using the Drucker-Prager with damage model, and the concrete ball was modeled as linear elastic. The elastic material properties were adapted from the data reported in [21], and the plastic and damage parameters were calibrated using the penetration data for an impact velocity of 62.23 m/s with an impact angle of 30°. The calibrated material parameters are given in Table 11 and Table 12. The numerical setup for this simulation is shown in Figure 59.

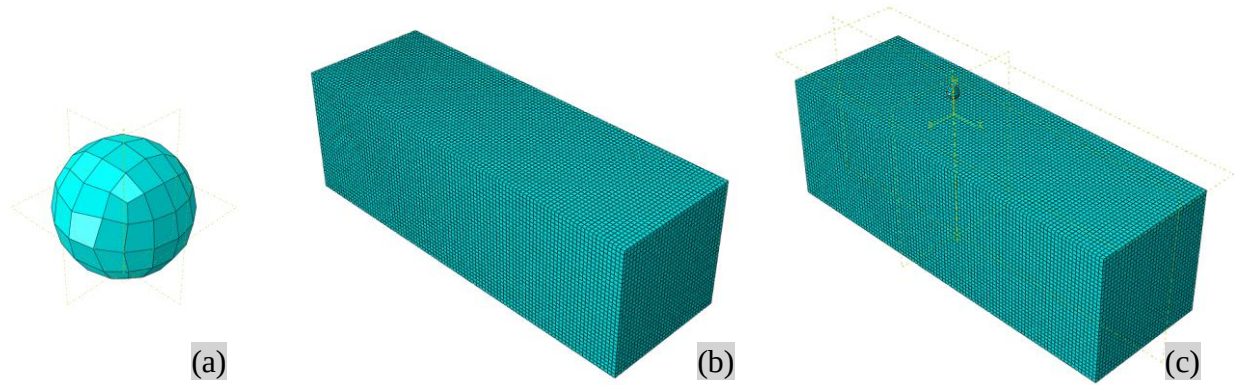


Figure 59. (a) the projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration in the numerical model of case 6.3.

Table 11. Material properties of soil in simulation 6.3.

| Drucker-Prager soil properties                    |                                 |
|---------------------------------------------------|---------------------------------|
| Young's modulus ( $E$ )                           | $1.2 \times 10^8 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.33                            |
| Friction Parameter ( $\beta$ )                    | 0.35                            |
| Hardening                                         | $1.0 \times 10^6 \frac{N}{m^2}$ |
| Cohesion strength ( $\alpha$ )                    | 0.0                             |
| First parameter for damage accumulation function  | 0.005                           |
| Second parameter for damage accumulation function | 0.5                             |
| Density ( $\rho$ )                                | $1724 \frac{kg}{m^3}$           |

Table 12. Material properties of the projectile in simulation 6.3.

| Linear Elastic Material   |                                     |
|---------------------------|-------------------------------------|
| Young's modulus ( $E$ )   | $3.00 \times 10^{10} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ ) | 0.18                                |
| Density ( $\rho$ )        | $2400 \frac{kg}{m^3}$               |

After calibrating the material properties and parameters, the rest of the cases were simulated for validation, and rebound angles and velocities were calculated and compared with the referred experimental results, as given in Table 13. Figure 60 shows the cut view and full view of the soil deformation after the ball rebounds from the soil surface, where the smooth contour indicates that the numerical is stable. The projectile trajectories for all cases are provided in Figure 61, showing that a higher impact angle and velocity result in a higher DOP. Overall, the numerical solution agrees with the experimental data. The maximum errors for exit velocity and exit angle are about 10% and 13%, respectively.

*Table 13. Comparison of numerical and experimental results of case 6.3.*

| Case no. |              | Incident Velocity (m/s) | Incident Angle with x-axis (°) | Exit Velocity (m/s) | Error (%) | Exit Angle with x-axis (°) | Error (%) | Penetration Depth (mm) |
|----------|--------------|-------------------------|--------------------------------|---------------------|-----------|----------------------------|-----------|------------------------|
| 1        | Experimental | 62.23                   | 30                             | 22.51               | -         | 18.72                      | -         |                        |
|          | Numerical    |                         |                                | 23.29               | 3.47      | 20.55                      | 9.78      | 9.07                   |
| 2        | Experimental | 92.31                   | 30                             | 34.29               | -         | 20.39                      | -         |                        |
|          | Numerical    |                         |                                | 32.37               | 5.60      | 22.30                      | 9.36      | 12.93                  |
| 3        | Experimental | 52.00                   | 40                             | 17.09               | -         | 17.17                      | -         |                        |
|          | Numerical    |                         |                                | 15.89               | 7.03      | 18.78                      | 9.38      | 15.63                  |
| 4        | Experimental | 72.20                   | 40                             | 19.42               | -         | 19.85                      | -         |                        |
|          | Numerical    |                         |                                | 17.37               | 10.56     | 22.48                      | 13.25     | 19.72                  |

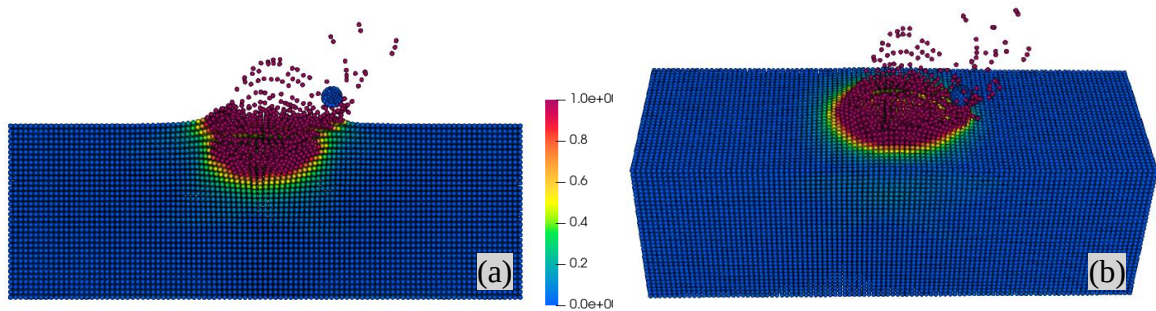


*Figure 60. Soil deformation and damage contours with impact velocity 62.23 m/s and impacting angle 30°. (a) x-y cut-view and (b) top view.*

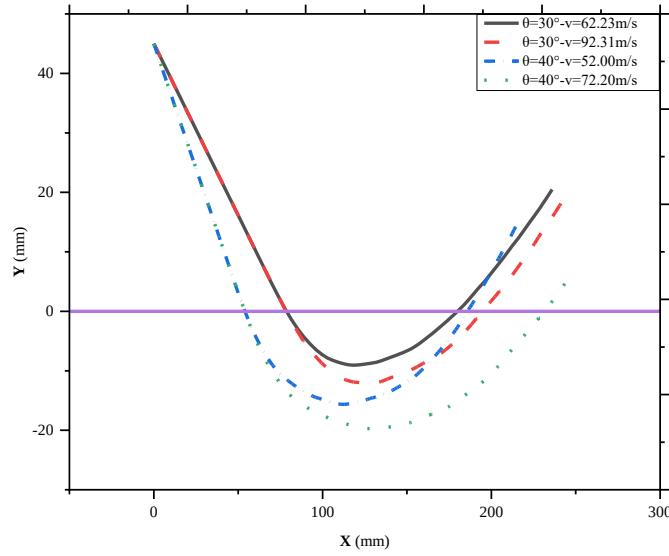


Figure 61. Concrete projectile trajectories of case 6.3.

#### 6.4. Penetration of Spherical Projectiles into Saturated Soil

The experimental data in Birch et al. [20] was taken to validate the numerical model for penetration into the saturated soil. The semi-Lagrangian framework with a normalized support size of 1.50 and the MPIM was employed. Around 119,000 nodes were used for the simulation, with an averaged nodal distance of 2 mm and 2.4 mm for projectile and soil, respectively. A spherical ball of a radius 6.5 mm made of steel was projected with an impacting velocity of 16.14 m/s and 24.02 m/s into the saturated sand. The soil grain was modeled by using the soil model described in Section 4, and the steel ball was modeled as linear elastic. The elastic material properties and basic soil parameters, such as Young's modulus, Poisson's ratio, and yield stress, were adapted from the data reported in the reference, while other soil parameters were calibrated from the penetration data for the impact velocity of 24 m/s. The calibrated material parameters for soil and projectile are given in Table 14 and Table 15, respectively. The numerical setup for this simulation is shown in Figure 62.

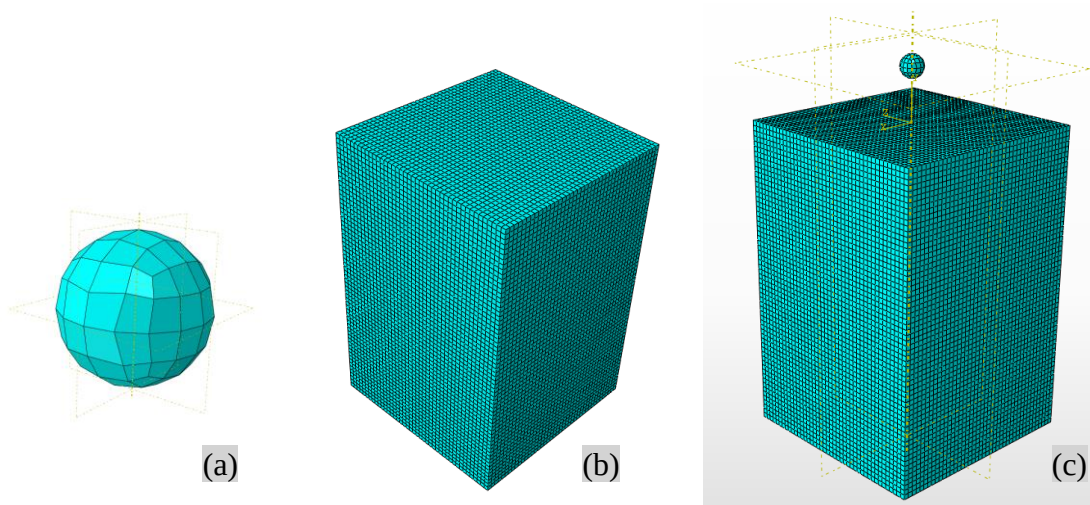


Figure 62. (a) the projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration in the numerical model of case 6.4.

Table 14. Material properties of the saturated soil of case 6.4.

| Wet soil properties                               |                                  |
|---------------------------------------------------|----------------------------------|
| Young's modulus ( $E$ )                           | $5.20 \times 10^8 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.34                             |
| Friction Parameter ( $\beta$ )                    | 0.25                             |
| Hardening                                         | $4.65 \times 10^6 \frac{N}{m^2}$ |
| Cohesion strength                                 | 0.0                              |
| First parameter for damage accumulation function  | 0.05                             |
| Second parameter for damage accumulation function | 0.5                              |
| Density ( $\rho$ )                                | $1850 \frac{kg}{m^3}$            |

Table 15. Material properties of the projectile of case 6.4.

| Linear Elastic Material   |                                     |
|---------------------------|-------------------------------------|
| Young's modulus ( $E$ )   | $2.15 \times 10^{11} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ ) | 0.30                                |
| Density ( $\rho$ )        | $7425 \frac{kg}{m^3}$               |

After calibrating the material parameters, other impacting velocities were simulated, and the DOPs were recorded and compared with the referred experimental results, as in Table 16. As observed from Table 16, the predicted penetration metrics from the meshfree simulation agree with experimental data with a maximum error of less than 10%. The soil deformations and damage contours are shown in Figure 63. The contour shows a smooth envelope near the impact wavefront, indicating that the numerical solution is stable. From the deformation in Figure 63, the soil debris shows a pastier behavior compared to the dry soil, e.g., Figure 60, which reflects the viscid nature of wet soil.

Table 16. Comparison of numerical and experimental results of case 6.4.

| Case no. | Impact Velocity (m/s) | Experimental Penetration Depth (mm) | Numerical Penetration Depth (mm) | Error (%) |
|----------|-----------------------|-------------------------------------|----------------------------------|-----------|
| #1       | 16.14                 | 17.07                               | 15.54                            | 8.96      |
| #2       | 24.02                 | 22.93                               | 25.1                             | 9.46      |

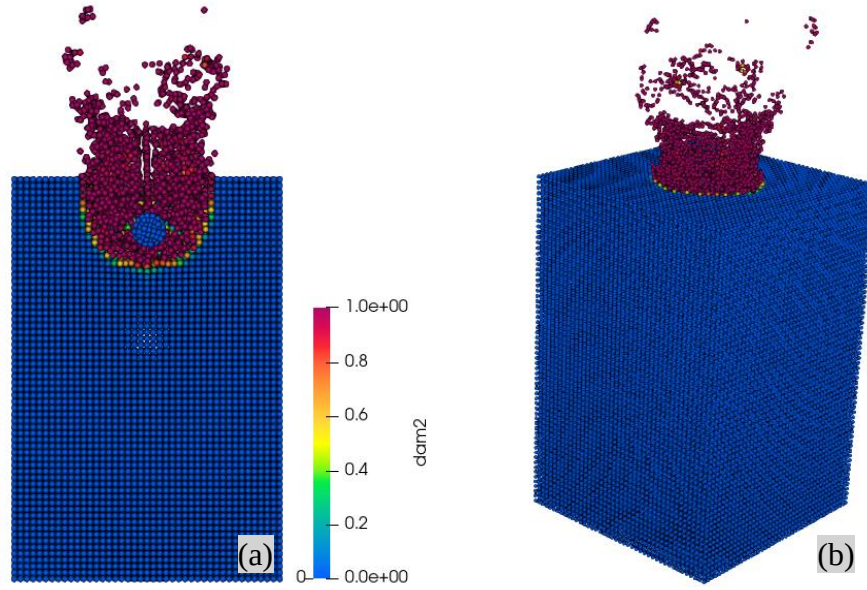


Figure 63. Soil deformation and damage contours with velocity 24.02m/s for case 6.4. (a) cut view, (b) full view.

### 6.5. Penetration of Spherical Projectiles into Sand with Different Saturation Ratios

The laboratory test results described in Section 5.2 were taken to validate the developed numerical modeling framework. A steel ball was shot into a circular bucket filled with sand with different SRs. The impact speed ranges from 20 m/s to 85 m/s. The semi-Lagrangian framework with a normalized support size of 1.5 was employed. About 127,000 nodes were used to discretize the model, which contains the bucket, sand target, and the projectile, with an averaged nodal distance of 12 mm, as shown in Figure 64. The Drucker-Prager with damage model and the soil model described in Section 4 were used for the sand target. The frictional kernel contact with a level set method [4] was employed to simulate the contact between the bucket and sand material. The bucket was set to be rigid, and gravity was applied to the sand target, allowing the sand target to contact the bucket before penetration. The steel ball was modeled as linear elastic with properties listed in Table 17. The properties and material parameters for dry, partially saturated, and fully saturated sand was calibrated from the soil test data (Section 5.1) by the downhill simplex algorithm (Section 5.1.2) and listed in Table 18. With the abovementioned model setup, simulations were completed

with nominal impact velocities of 30 m/s, 40 m/s, 50 m/s, and 60 m/s for saturation ratios of 0, 0.5, and 1.

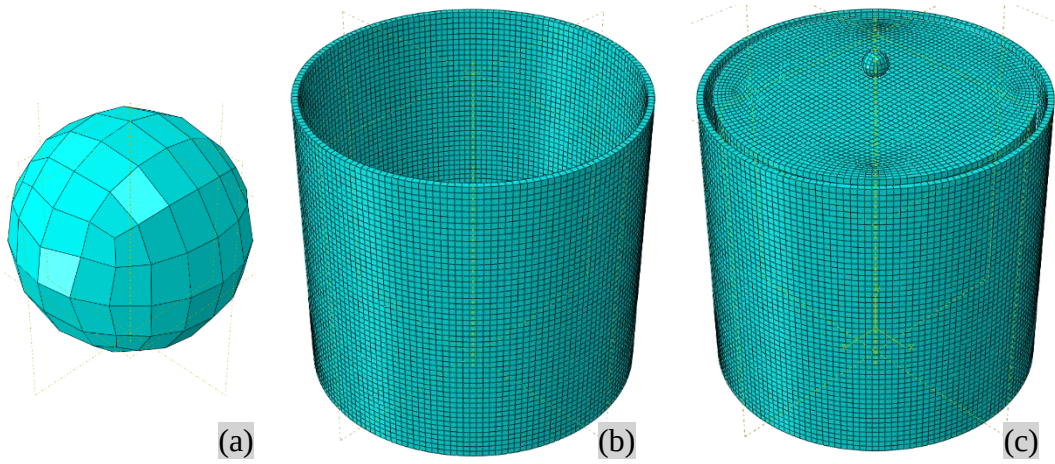


Figure 64. (a) the projectile mesh configuration, (b) bucket mesh configuration, and (c) overall initial configuration in the numerical model of case 6.5.

Table 17. Material properties of the projectile in simulation 6.5.

| Linear Elastic Material   |                                     |
|---------------------------|-------------------------------------|
| Young's modulus ( $E$ )   | $2.15 \times 10^{11} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ ) | 0.30                                |
| Density ( $\rho$ )        | $7850 \frac{kg}{m^3}$               |

Table 18. Material parameters for sand with various saturation ratios in simulation 6.5.

| Soil properties                                   | Dry                             | Unsaturated                      | Saturated                        |
|---------------------------------------------------|---------------------------------|----------------------------------|----------------------------------|
| Young's modulus ( $E$ )                           | $3.1 \times 10^7 \frac{N}{m^2}$ | $3.04 \times 10^7 \frac{N}{m^2}$ | $2.61 \times 10^6 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.2                             | 0.28                             | 0.1                              |
| Friction Parameter ( $\beta$ )                    | 0.38                            | 0.48                             | 0.46                             |
| Hardening                                         | $12.8 \frac{N}{m^2}$            | $651.5 \frac{N}{m^2}$            | $2.24 \times 10^5 \frac{N}{m^2}$ |
| Cohesion strength ( $\alpha$ )                    | 0.0001                          | 0.0001                           | 0.009                            |
| First parameter for damage accumulation function  | 0.0487                          | 0.044                            | 0.28                             |
| Second parameter for damage accumulation function | 0.96                            | 0.812                            | 0.46                             |
| Density ( $\rho$ )                                | $1450 \frac{kg}{m^3}$           | $1450 \frac{kg}{m^3}$            | $1450 \frac{kg}{m^3}$            |

The comparison of numerical and experimental DOP results for the fully saturated sand (SR = 1) is shown in Figure 65, where black and red markers denote experimental and numerical results, respectively. The numerical results fall within the cluster of experimental data and qualitatively agree with the trend line of experimental data. The deformation of the target and damage contour after impact with an impact velocity of 30 m/s is shown in Figure 66. The smooth contour indicates solution stability using the developed meshfree formulation. The numerical solution captures the deformation patterns observed in experimental tests for fully saturated sand, showing compacted clusters of fragmented material, the flat top surface of the target without a noticeable bulge, and a steep crater wall (Figure 66).

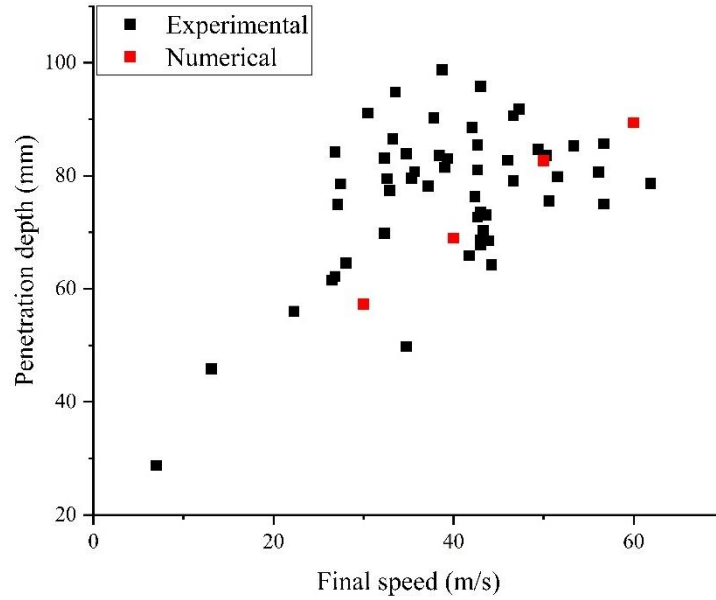


Figure 65. Comparison of numerical and experimental DOP for fully saturated sand ( $SR = 1$ ) in case 6.5.

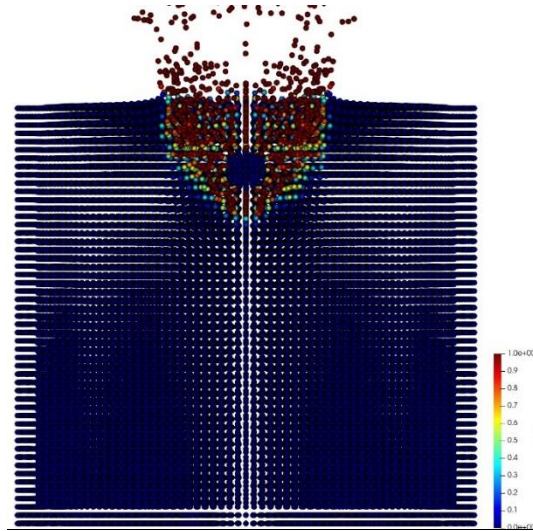


Figure 66. Soil deformation and damage contour for fully saturated sand ( $SR = 1$ ) with impact velocity 30 m/s in case 6.5.

The comparison of numerical and experimental DOP results for the partially saturated sand ( $SR = 0.5$ ) is shown in Figure 67. The DOP from the experiment remains almost constant within the speed range of 30-70 m/s at about 130 mm, while the simulation shows a linear increase from 30 m/s to 60 m/s. The numerical DOP with 60 m/s matches well with the experimental value, while with 50 m/s, the numerical result is 5% less than the test result. Overall, the difference between

the numerical DOP and the averaged experiment DOP is less than 15 % in this case. The deformation of the target and damage contour after impact with an impact velocity of 30 m/s is shown in Figure 68. The solution remains stable throughout the processing. The numerical solution shows an elliptic paraboloid crater with a tip angle of about  $60^\circ$  due to a lower hardening compared to the fully saturated case. The soil target's top surface marginally bulges upward after impact.

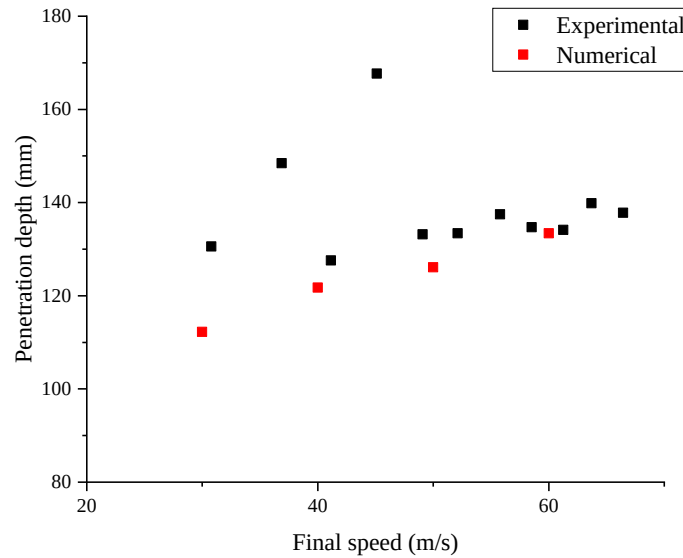


Figure 67. Comparison of numerical and experimental DOP for partially saturated sand ( $SR = 0.5$ ) in case 6.5.

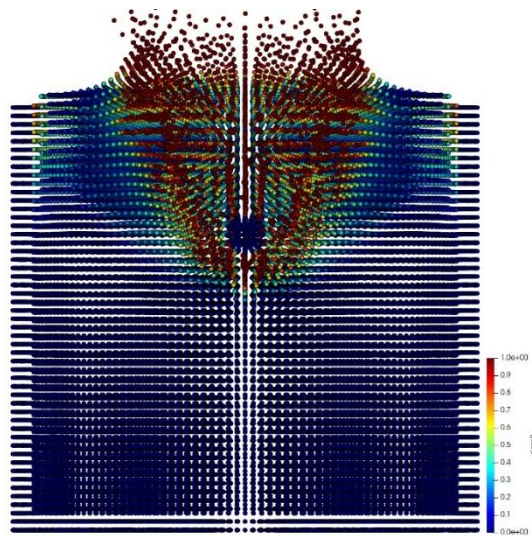


Figure 68. Soil deformation and damage contour for partially saturated sand ( $SR = 0.5$ ) with impact velocity 30 m/s in case 6.5.

The comparison of numerical and experimental DOP results for the dry saturated sand ( $SR = 0$ ) is shown in Figure 69. The DOP between 30 and 50 m/s increases linearly, while the DOP with 60 m/s drops, which matches the result of the experiment. The numerical DOP matches well with the averaged experimental DOP at impact velocities of 30 m/s and 60 m/s. Overall, the difference between the numerical DOP and the averaged experiment DOP is less than 10 % in this case. The deformation of the target and damage contour after impact with an impact velocity of 30 m/s is shown in Figure 70. Similar to the result of the unsaturated sand, the simulation result shows an elliptic paraboloid crater with a tip angle of about  $66^\circ$ . The wide range of sand grains fly upward and outward motion due to the impact, which forms a cavity along the path of the projectile. Similar to experimental observations, the target's top surface in the simulation bulges significantly with shattered fragments.

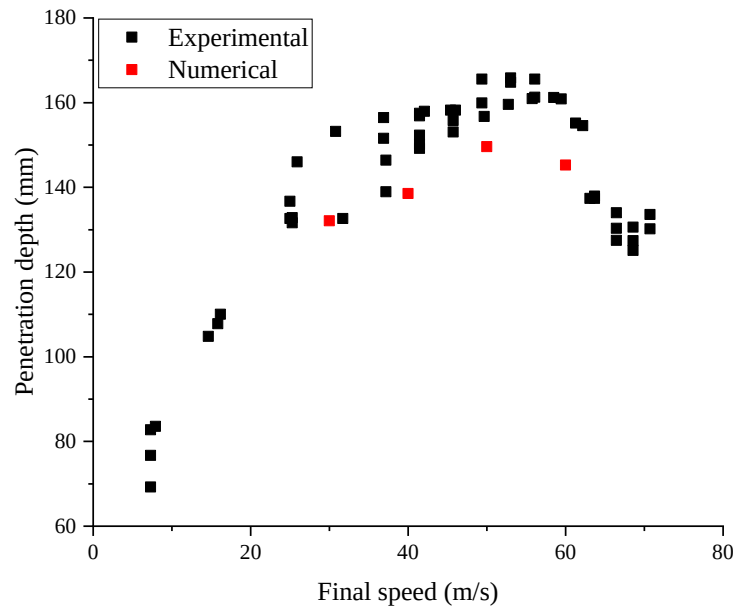


Figure 69. Comparison of numerical and experimental DOP for dry sand ( $SR = 0$ ) in case 6.5.

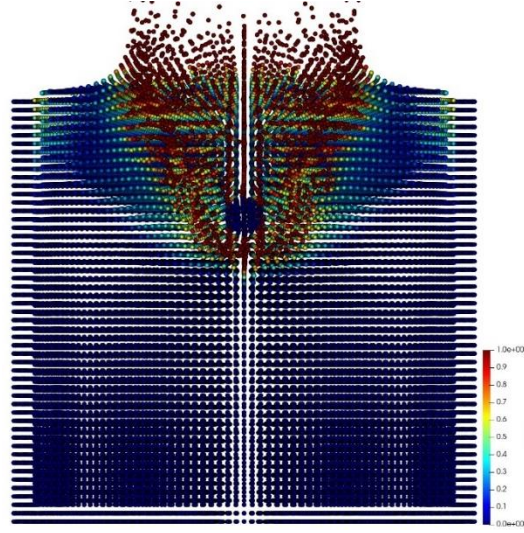


*Figure 70. Soil deformation and damage contour for dry sand ( $SR = 0$ ) with impact velocity 30 m/s in case 6.5.*

#### **6.6. Ricochet and Penetration of 25 mm Projectile into Sand**

The experimental data from Moxnes et al. [22] was taken to validate the developed numerical model for modeling high impact penetration into dry soil with an impacting angle. An ogive projectile of length 90.00 mm and radius of 12.5 mm made of aluminum was projected with an impacting velocity of 589 m/s and impact angles of  $25^\circ$  and  $12.5^\circ$  into the coarse sand. The semi-Lagrangian framework with a normalized support size of 1.50 and the MPIM was employed. 89550 nodes were used for the simulation, with an average nodal distance of 1.75 mm for projectile and 3 mm for soil. The soil grain was modeled by using the Drucker-Prager with damage model, and the projectile was modeled as linear elastic. The material properties of soil were obtained from [22]. The calibrated material parameters for soil and projectile are given in Table 19 and Table 20, respectively. The numerical setup for this simulation is shown in Figure 71.

##### Impact angle $25^\circ$ :

The simulation uses 231354 nodes with an averaged nodal distance of 3.5 mm. The sand bed's dimensions are 770 mm in length, 200 mm in height, and 60 mm in width. Figure 72 shows soil deformation at several time instances. There is a good agreement between the experimentally observed soil deformation profile and the numerical simulation. Several key parameters are also

included in Table 21 to support the numerical simulation based on the experiment data. The outcome demonstrates that the numerical solution has an error that falls within the acceptable range.

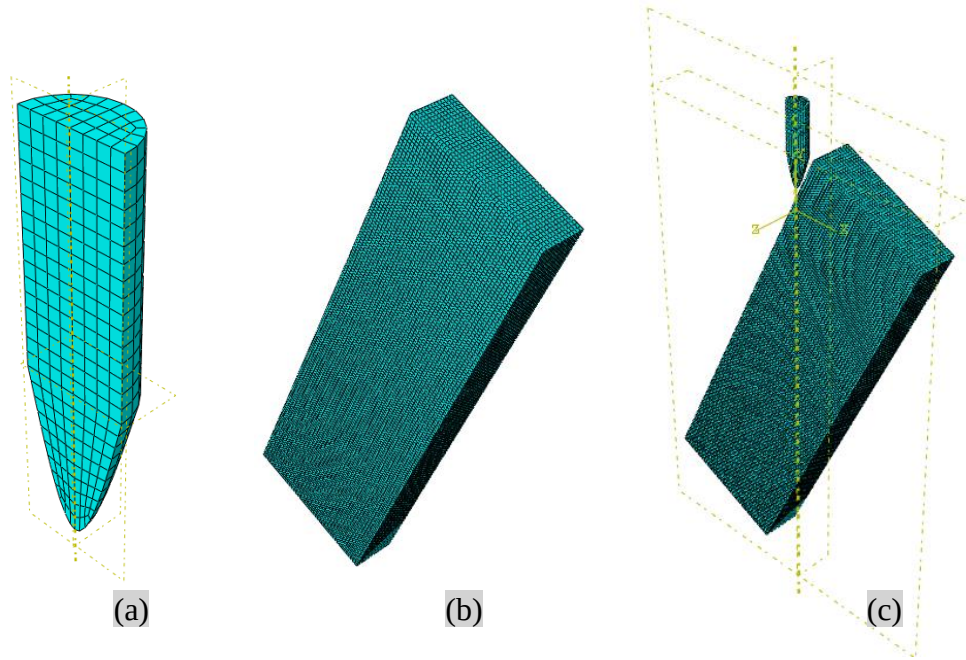


Figure 71. (a) projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration in the numerical model of case 6.6.

Table 19. Material properties of the soil of case 6.6.

| Soil properties                                   |                                 |
|---------------------------------------------------|---------------------------------|
| Young's modulus ( $E$ )                           | $2.5 \times 10^8 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.35                            |
| Friction Parameter ( $\beta$ )                    | 0.75                            |
| Hardening                                         | $1.0 \times 10^6 \frac{N}{m^2}$ |
| Cohesion strength ( $\alpha$ )                    | 0.0                             |
| First parameter for damage accumulation function  | 0.05                            |
| Second parameter for damage accumulation function | 1.0                             |
| Density ( $\rho$ )                                | $1670 \frac{kg}{m^3}$           |

Table 20. Material properties of the projectile of case 6.6.

| Linear Elastic Material   |                                     |
|---------------------------|-------------------------------------|
| Young's modulus ( $E$ )   | $2.15 \times 10^{11} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ ) | 0.27                                |
| Density ( $\rho$ )        | $7800 \frac{kg}{m^3}$               |

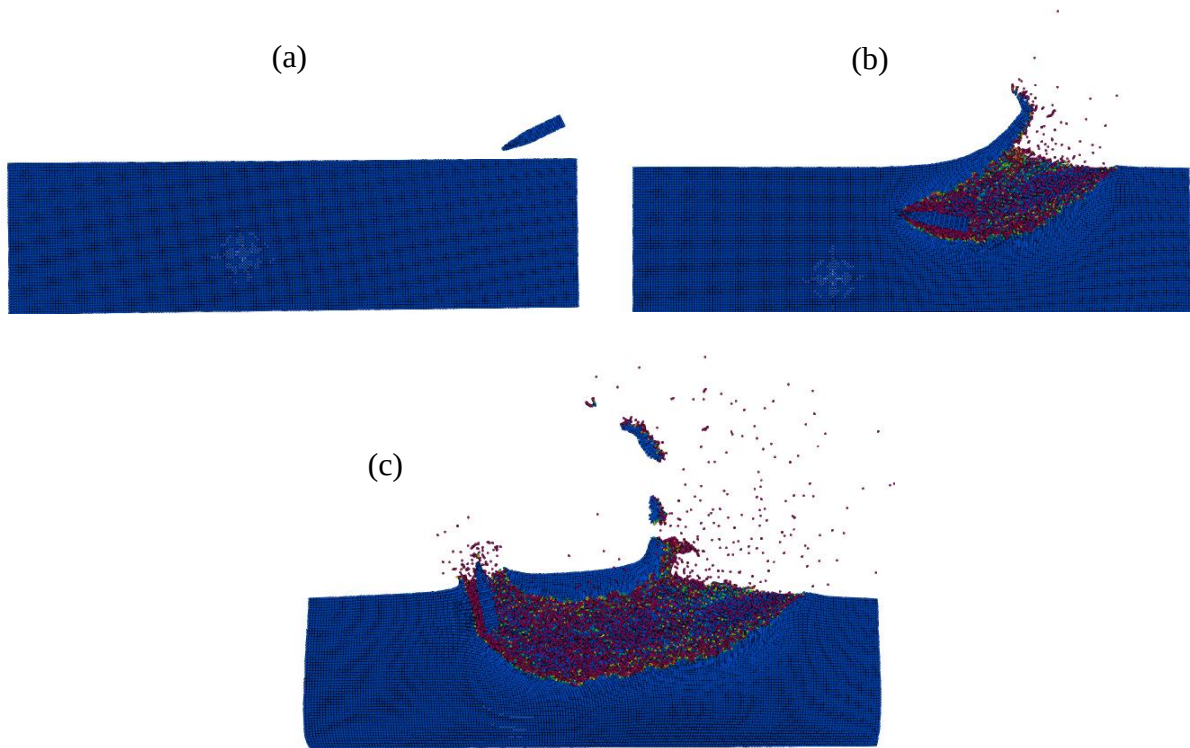


Figure 72. Soil deformation and damage contours for 25° impact angle in case 6.6. (a) at time = 0 ms, (b) at time = 0.70 ms, (c) at time = 1.80 ms.

Table 21. Comparison of numerical and experimental results for 25° impact angle in case 6.6.

|                                                        | Exit Velocity<br>(m/s) | Depth of<br>Cavity<br>(cm) | Penetration<br>Length<br>(cm) | Exit Time |
|--------------------------------------------------------|------------------------|----------------------------|-------------------------------|-----------|
| Experimental results                                   | 111                    | 10                         | 44                            | 3.7       |
| Reference numerical results<br>from Moxnes et al. 2016 | 154                    | 12                         | 41                            | 1.84      |
| RK numerical results                                   | 140.8                  | 11.78                      | 40.44                         | 2.15      |

Impact angle 12.5 °:

The simulation uses 162159 nodes with an average nodal distance of 2.875 mm. The sand bed's dimensions are 580 mm in length, 150 mm in height, and 40 mm in width. Figure 73 shows soil deformation at several time instances. An exit velocity is noted in the numerical simulation, and the parameter is compared in Table 22 with the experimental data. Overall, the numerical solution and experimental results are in good agreement.

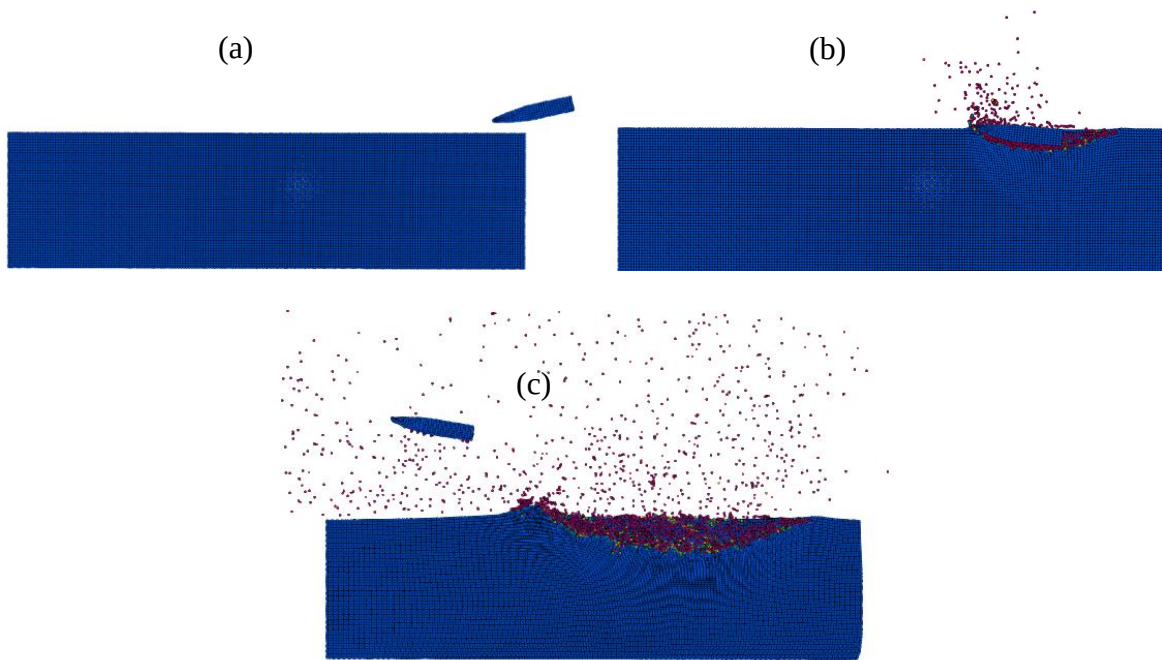


Figure 73. Soil deformation and damage contours for 12.5° impact angle in case 6.6. (a) at time = 0 ms, (b) at time = 0.30 ms, (c) at time = 1.0 ms.

Table 22. Comparison of numerical and experimental results for 12.5° impact angle in case 6.6.

|                                           | Exit Velocity<br>(m/s) |
|-------------------------------------------|------------------------|
| Experimental results                      | 440                    |
| Numerical results from Moxnes et al. 2016 | 408                    |
| RK Numerical results                      | 396.6                  |

## 6.7. Simulations of Some Realistic Projectiles Penetrating into Sand Ground

This section presents simulations of several realistic projectiles penetrating into dry and compacted sand ground, similar to the coarse sand in [22]. The dimensions and properties of projectiles are taken from the literature, and the impact velocities and angles are estimated from the muzzle velocities of the launching platform. The model setting, initial conditions, and boundary conditions are set as close as the realistic scenarios. Although the experimental data are not available for validation, the numerical presented here demonstrate that the meshfree method can simulate penetration processes of projectiles (UXOs) under realistic conditions. More importantly, the computational cost of these simulations provides insight into the costs of developing tabular DOP results for different sediments, projectiles, and impact trajectories.

### 6.7.1. Penetration of an APCBC/T Projectile into Sand Ground

A projectile of length 300.00 mm and radius of 38.0 mm is projected with an impact velocity of 500 m/s and 800 m/s into the sand, and the impact angle is 30° (Figure 74). A semi-Lagrangian framework is adopted with a normalized support size of 1.50, and the MPIM is employed. The soil grain is modeled by using the Drucker-Prager with damage model, and the bullet is modeled as linear elastic. An absorbing boundary is implemented on the boundaries of the soil grain to mimic an unbound domain.

*The simulation uses 241,899 nodes, with an average nodal distance of 9.00 mm. The sand bed's dimensions are 1750 mm in length, 700 mm in height, and 200 mm in width. The calibrated material parameters are given in*

Table 23 and Table 24. One hundred twenty-eight processors from UIC's Extreme HPC are used to obtain the following simulation results. The total computing time for each impact velocity case is about 92 hours.

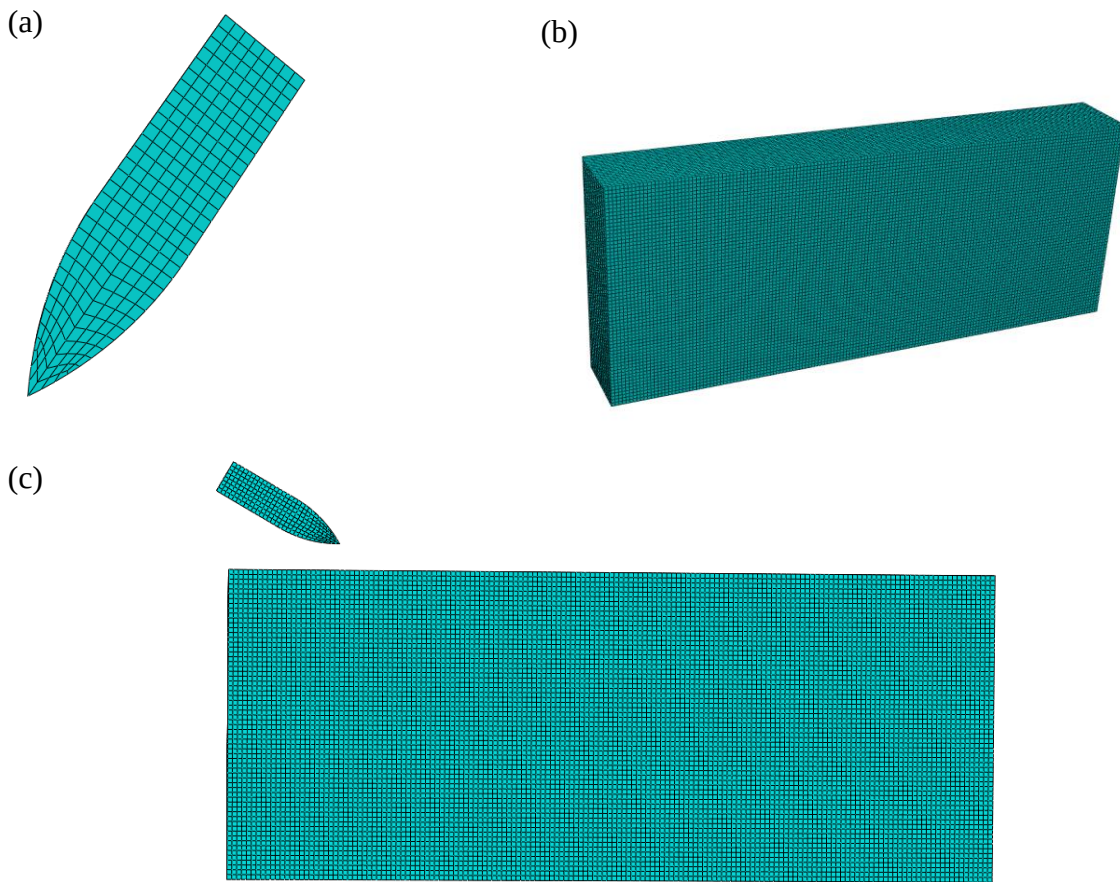


*Figure 74. APCBC/T penetration model. (a) projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration.*

Table 23. Material properties of the sand in case 6.7.1.

| Drucker-Prager soil properties                    |                                 |
|---------------------------------------------------|---------------------------------|
| Young's modulus ( $E$ )                           | $2.2 \times 10^8 \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ )                         | 0.35                            |
| Friction Parameter ( $\beta$ )                    | 0.294                           |
| Hardening                                         | $3.0 \times 10^5 \frac{N}{m^2}$ |
| Cohesion strength ( $\alpha$ )                    | 0.0                             |
| First parameter for damage accumulation function  | 3.0                             |
| Second parameter for damage accumulation function | 4.0                             |
| Density ( $\rho$ )                                | $1670 \frac{kg}{m^3}$           |

Table 24. Material properties of the projectile in case 6.7.1.

| Linear Elastic Material   |                                     |
|---------------------------|-------------------------------------|
| Young's modulus ( $E$ )   | $2.15 \times 10^{11} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ ) | 0.27                                |

Table 25. DOP simulation results of case 6.7.1.

| Impacting Velocity<br>(m/s) | Impacting Angle<br>(°) | Depth of Cavity<br>(mm) |
|-----------------------------|------------------------|-------------------------|
|-----------------------------|------------------------|-------------------------|

|     |    |        |
|-----|----|--------|
| 500 | 30 | 273.89 |
| 800 | 30 | 339.36 |

The maximum DOP is listed in Table 25. Figure 75 and Figure 76 show soil deformation at various time instances with impact velocities of 500 m/s and 800 m/s, respectively. Figure 77 shows von Mises stress contour at 1.5 ms for impact velocities of 500 m/s and 800 m/s. The smooth stress wavefront indicates that the numerical solutions are stable, and the absorbing boundary condition eliminates reflective waves and appropriately mimics the unbounded domain. Figure 78 compares the trajectory path of the projectile tip for impacting velocities of 500 m/s and 800 m/s with an impacting angle of 30°. The soil and projectile behaviors from the numerical simulation are similar to the validated results presented in previous sections.

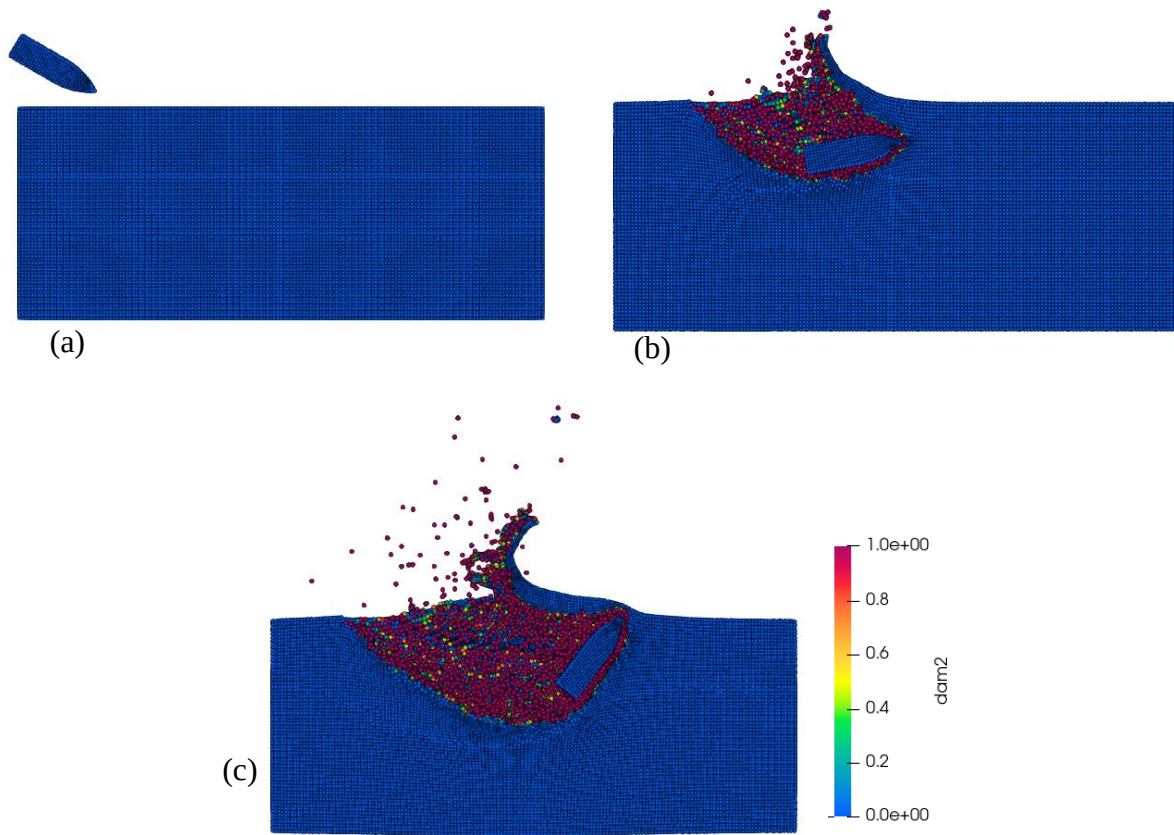


Figure 75. Soil deformation and damage contours for impact velocity 500 m/s in case 6.7.1. (a) at 0 ms, (b) at 1.74 ms, and (c) at 3.6 ms.

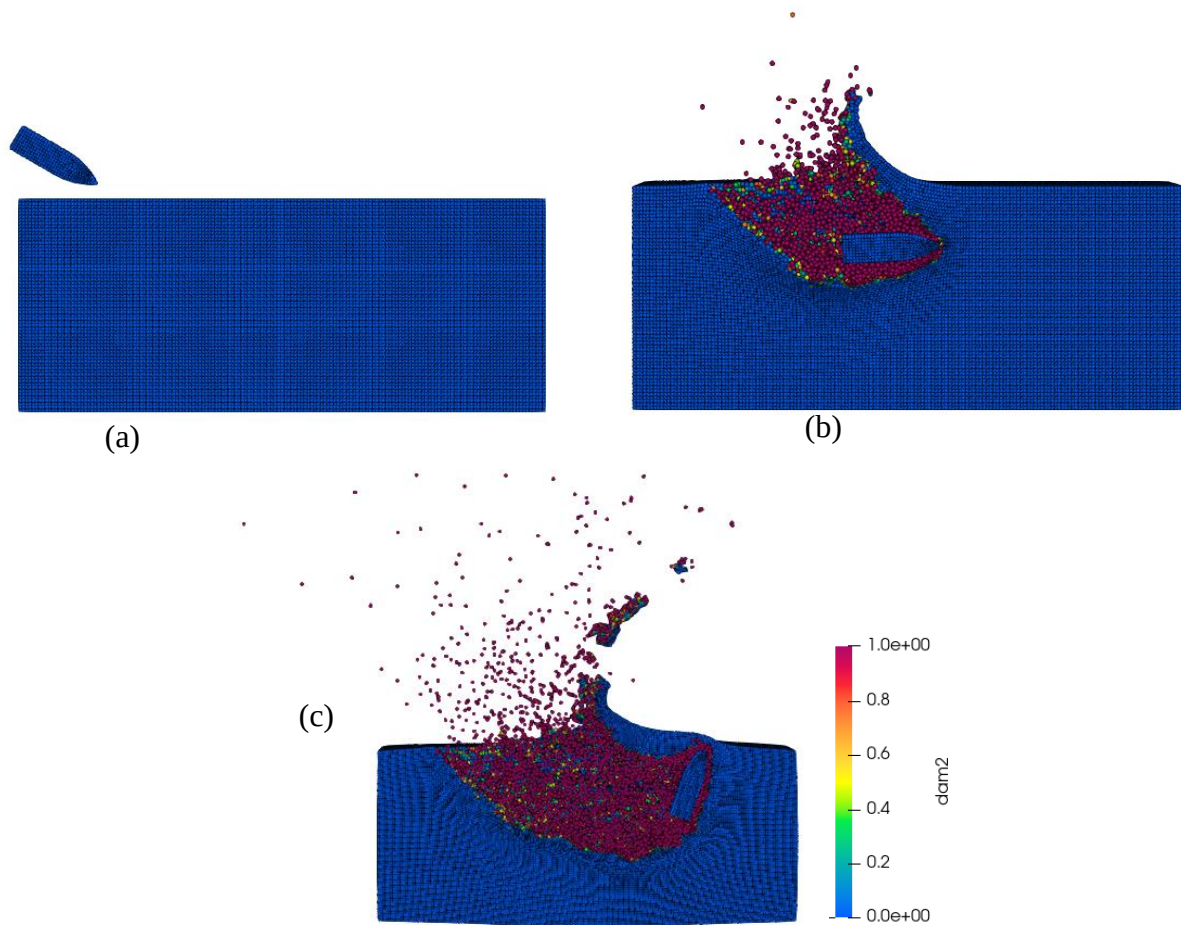


Figure 76. Soil deformation and damage contours for impact velocity 800 m/s in case 6.7.1. (a) at 0 ms, (b) at 1.30 ms, and (c) at 3.78 ms.

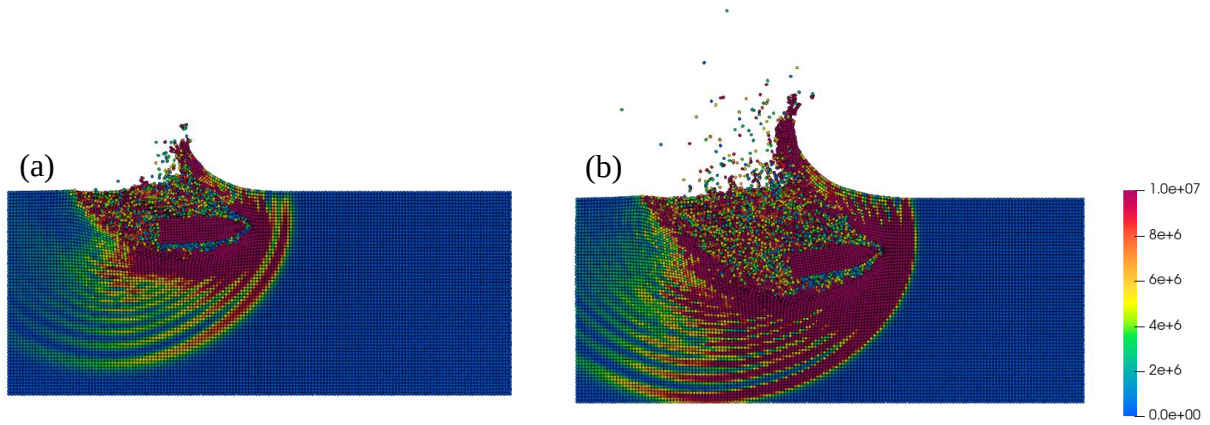


Figure 77. Soil deformation and von Mises stress contours at 1.5 ms for impact velocity (a) 500 m/s and (b) 800 m/s in case 6.7.1.

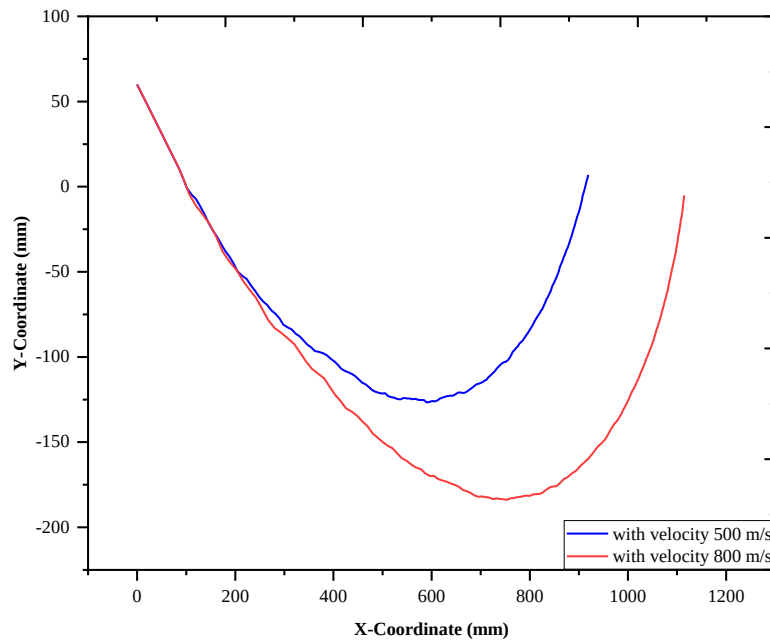


Figure 78. Projectile tip trajectories with impact velocities 500 m/s and 800 m/s in case 6.7.1.

#### 6.7.2. Penetration of an M821 Projectile into Coarse Sand Ground

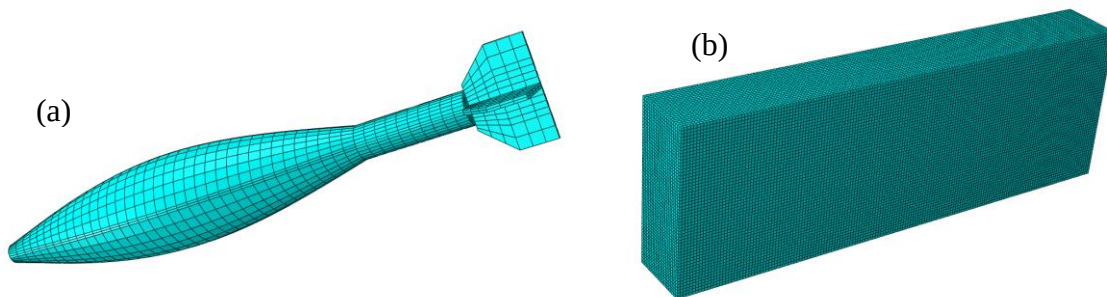
A projectile of length 508.00 mm and radius of 40.62 mm is projected with an impact velocity of 300 m/s into the sand, and the impact angle is 30° (Figure 79). A semi-Lagrangian framework is adopted with a normalized support size of 1.50, and the MPIM is employed. The soil grain is

modeled by using the Drucker-Prager with damage model, and the bullet is modeled as linear elastic. An absorbing boundary is set up on the boundaries of the soil grain.

*The simulation uses 116221 nodes, with an average nodal distance of 9.00 mm. The sand bed's dimensions are 1750 mm in length, 500 mm in height, and 200 mm in width. The calibrated material parameters are given in*

Table 23 and Table 26. The model setup for this simulation is shown in Figure 79. Simulation results were obtained using 128 processors from UIC's Extreme HPC. The total time cost to obtain the solution is 34 hours. The maximum depth of the crater is 123.02 mm at 4.64 ms, and afterward, the projectile started protruding out nearly at 6.4 ms.

Figure 80 shows soil deformation and damage contours at several time instances. Figure 81 shows the smoothed envelope of the von Mises stress contour, indicating that the simulation results are stable. Also, the stress waves do not reflect near the boundaries, indicating that the absorbing boundary condition works well with the developed meshfree method. Figure 82 shows the projectile tip trajectory with an impacting velocity of 300 m/s. The soil and projectile behaviors from the numerical simulation are stable for the entire time.



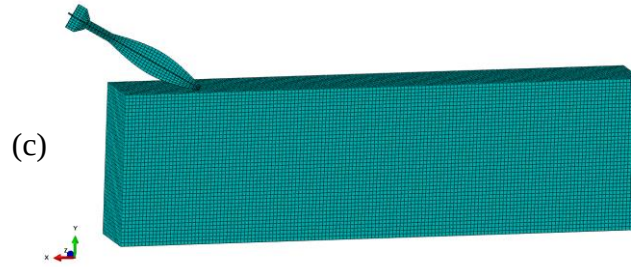


Figure 79. M821 penetration model. (a) projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration.

Table 26. Material properties of the projectile in case 6.7.2.

| Linear Elastic Material   |                                     |
|---------------------------|-------------------------------------|
| Young's modulus ( $E$ )   | $3.15 \times 10^{10} \frac{N}{m^2}$ |
| Poisson's ratio ( $\nu$ ) | 0.27                                |

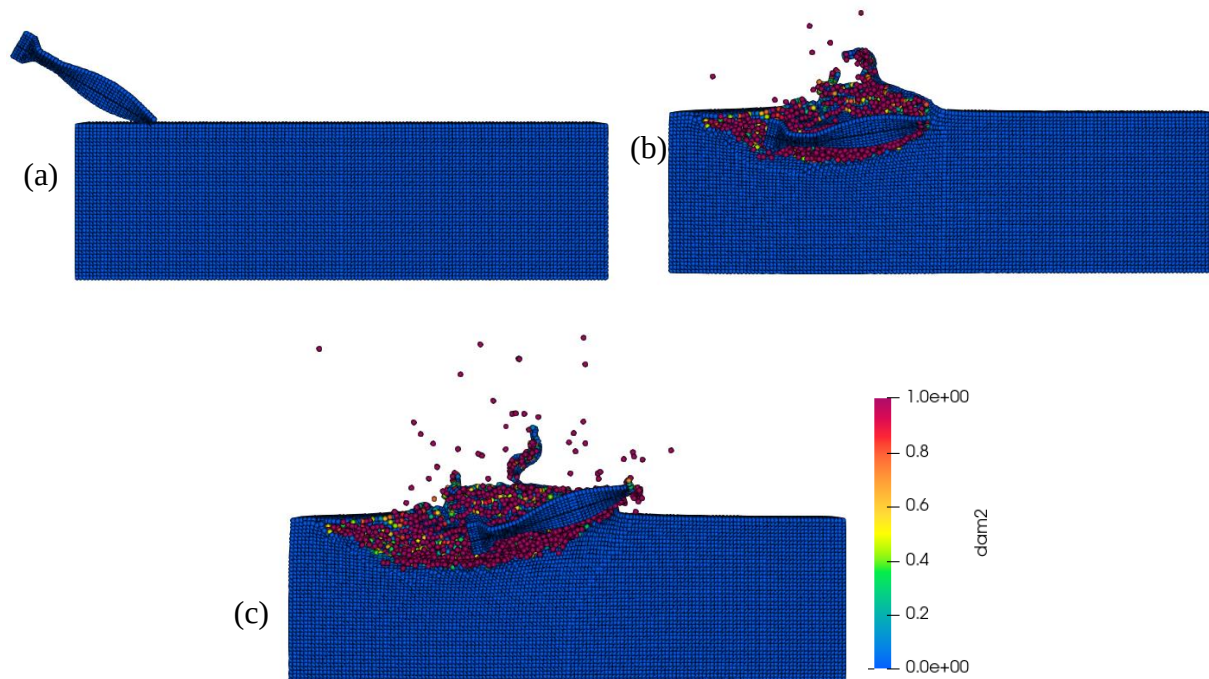


Figure 80. Soil deformation and damage contours with impact velocity 300 m/s in case 6.7.2. (a) at 0 ms, (b) at 3.20 ms, and (c) at 6.40 ms.

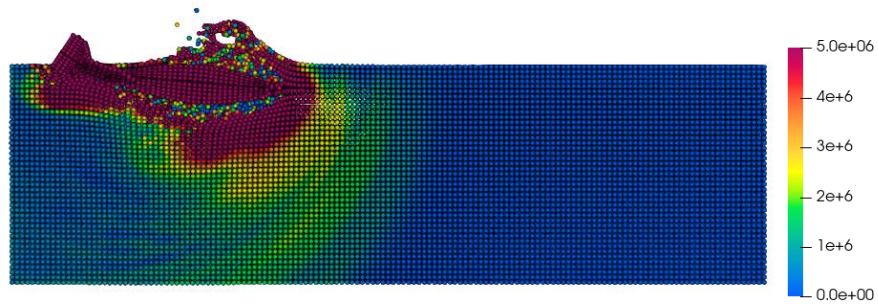


Figure 81. Soil deformation and von Mises stress contours in case 6.7.2. at 1.60 ms.

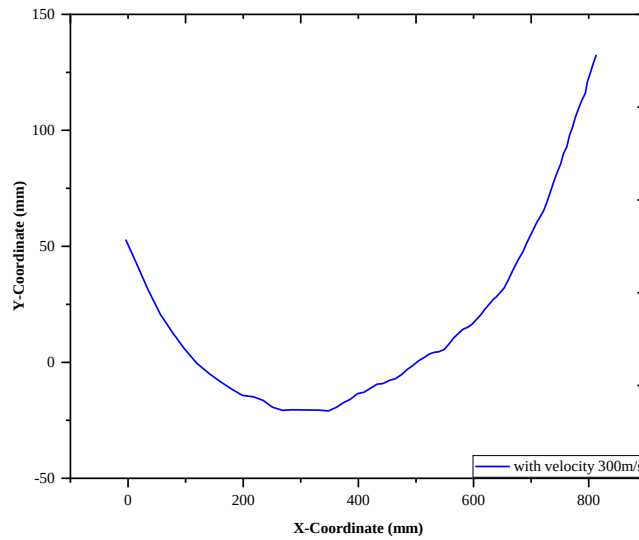


Figure 82. Projectile tip trajectory in case 6.7.2.

### 6.7.3. Penetration of an APDS Projectile into Coarse Sand Ground

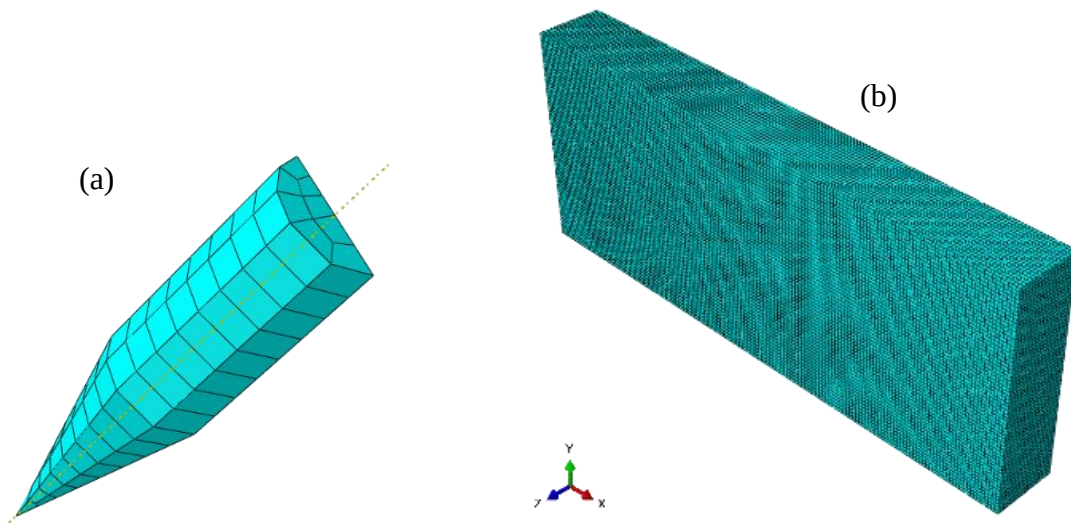
A projectile of length 210.00 mm and radius of 50 mm is projected with an impact velocity of 1120 m/s into the sand, and the impact angle is 30°. A semi-Lagrangian framework is considered with a normalized support size of 1.50, and the MPIM is employed. The soil grain is modeled by

using the Drucker-Prager with damage model, and the bullet is modeled as linear elastic. An absorbing boundary is implemented on the boundaries of the soil grain.

*The simulation uses 186000 nodes, with an average nodal distance of 11.00 mm. The sand bed's dimensions are 1750 mm in length, 800 mm in height, and 240 mm in width. The calibrated material parameters are given in*

Table 23 and Table 24. The numerical setup for this simulation is shown in Figure 83.

The simulation takes 91 hours with 128 processors from UIC's Extreme HPC. Figure 84 shows the soil deformation and damage contours at different time instances. The von Mises stress contours in Figure 85 show that the solution is stable throughout the process. The final penetration depth is 533 mm at 3.40 ms. The absorbing boundary properly absorbed the majority of incoming waves, though a few visible waves reflected into the domain of interest.



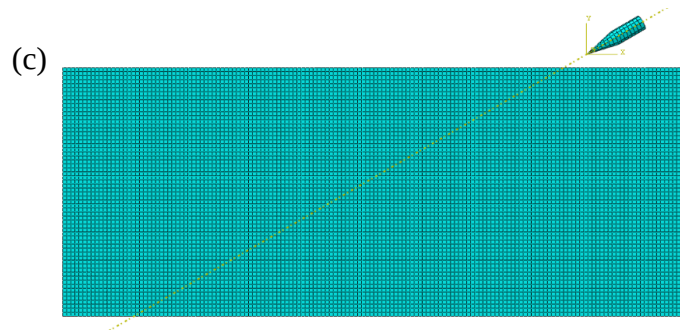


Figure 83. APDS penetration model. (a) projectile mesh configuration, (b) soil mesh configuration, and (c) initial configuration.

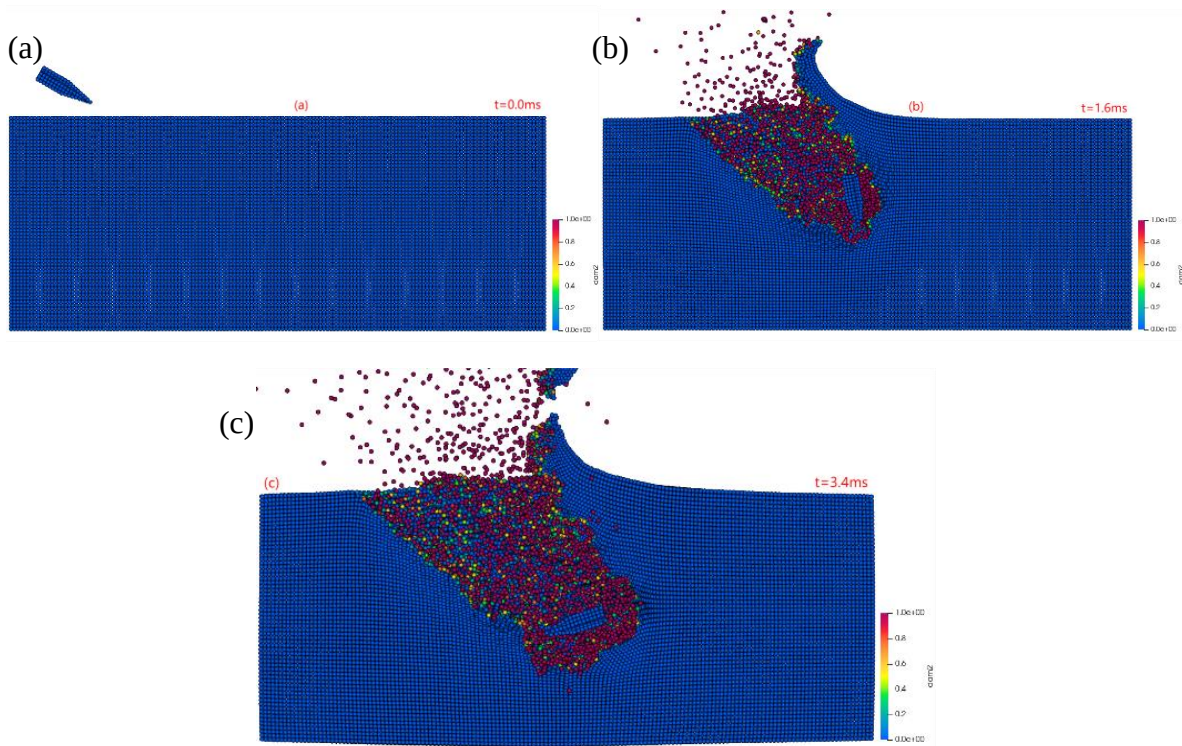


Figure 84. Soil deformation and damage contours with impact velocity 1120 m/s in case 6.7.3 (a) at 0 ms, (b) at 1.60 ms, and (c) at 3.40 ms.

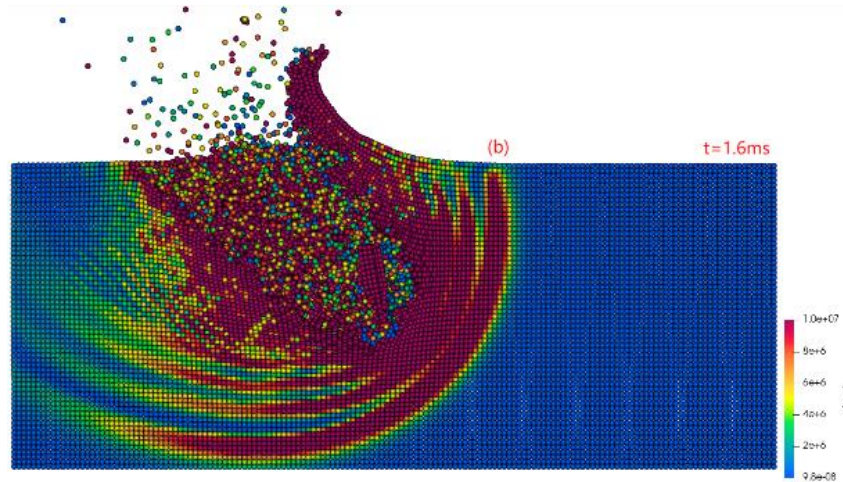


Figure 85. Soil deformation and von Mises stress contours in case 6.7.3 at 1.60 ms.

## 7. Lessons Learned

- The developed semi-Lagrangian meshfree code effectively offers an accurate numerical solution for soil penetration. In addition to bulk penetration measurements, such as DOP, the numerical solution accurately predicts the soil damage pattern and crater shape and size. Nevertheless, the computational cost remains high. Model reduction techniques or machine learning techniques combined with extensive offline simulation data will be needed to expedite the simulation.
- The data from penetration experiments using dry, saturated, and partially saturated sands provides insight into how saturation affects the depth of penetration and other penetration measurements. These data, along with simulations of three experiments and from the literature, suggest the meshfree modeling of penetration of smaller projectiles into sand is realistic.
- The mixed field formulation for modeling soil under different saturation conditions has been developed and validated. However, since the penetration process typically completes within a few hundredths of a second, the fluid in the pore does not have sufficient time to dissipate or transport in space. The continuity equation in such a case plays a minor role in predicting the overall penetration mechanics. Therefore, a single field formulation with a properly calibrated soil model may suffice for the DOP prediction for computational efficiency.

- While implicit implementation of constitutive models offers unconditionally stability, complicated formulas and conditioning issues in the tangent matrix can lead to issues in robustness. Semi-implicit methods improve robustness and, for applications that require small time steps, improve efficiency as well. Explicit methods can also be attempted, but the solutions tend to drift away from the yield surface or produce saw-tooth patterns that could reduce global stability for dynamic problems.
- In complex material materials with many parameters, fitting can be a challenge. It helps to create a model with physically motivated parameters that can be fit sequentially from experiments. Sensitivity analysis can also greatly aid in determining which parameters are most important to a given set of simulations.

## **Acknowledgments**

The support of this work by SERDP to the University of Illinois at Chicago under contract agreement W912HQ18C0099 and the SEED funding under W912HQ16P0002 are greatly acknowledged.

## **Appendix A.**

### **A.1. Eigen Analysis for Examining Stability of the MPIM**

An eigenvalue analysis is employed to assess the stability of the MPIM. The stiffness matrix of a 1D elastic rod discretized into 200 nodes is obtained, and its eigenvalues and eigenmodes are examined. The results by MPIM are compared with those by DNI, SCNI, 5-point Gauss integration, and Finite Element Method (FEM) in Figure 86. As can be seen in Figure 86a, the spectrum of the MPIM is comparable to and slightly higher than those of 5-point Gauss integration and fully integration FEM. Moreover, no spurious, oscillatory eigenmodes are observed in the MPIM, whereas the well-known SCNI method exhibits a spurious mode associated with the ninth-lowest eigenvalues (Figure 86b).

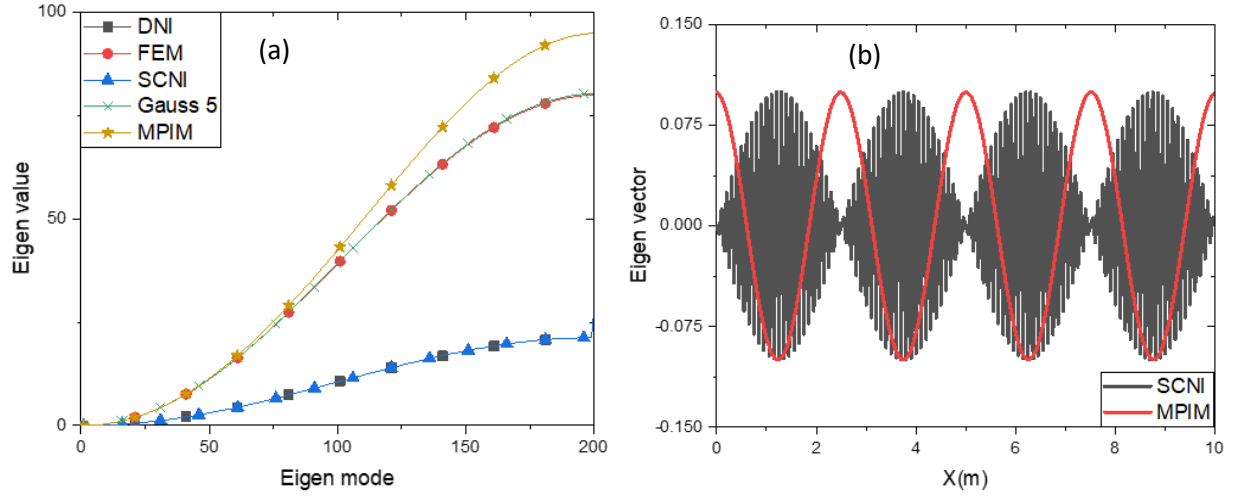


Figure 86. (a) eigen spectrum and (b) mode shape of the 9<sup>th</sup> mode.

## A.2. Cantilever Beam Under a Shear Load

A cantilever beam in Figure 87 subjected to a shear force is employed to investigate the convergence of the MPIM. A normalized support size of 2.0 is considered, and the elastic material properties are Young's modulus = 1.0 and Poisson's ratio = 0.3. From Figure 88, it is observed that DNI and SNNI perform well, with a convergence rate near 2.0, when uniform discretization is used; however, they lose convergence when non-uniform discretization is used. On the other hand, 5-point Gauss integration provides good accuracy but a low convergence rate. The MPIM shows good accuracy and a convergence rate of 2.0 in both uniform and non-uniform discretization.

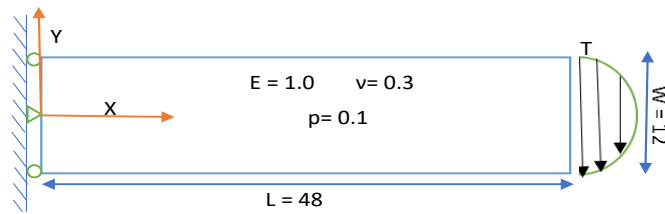


Figure 87. Schematic of a cantilever beam under a shear load.

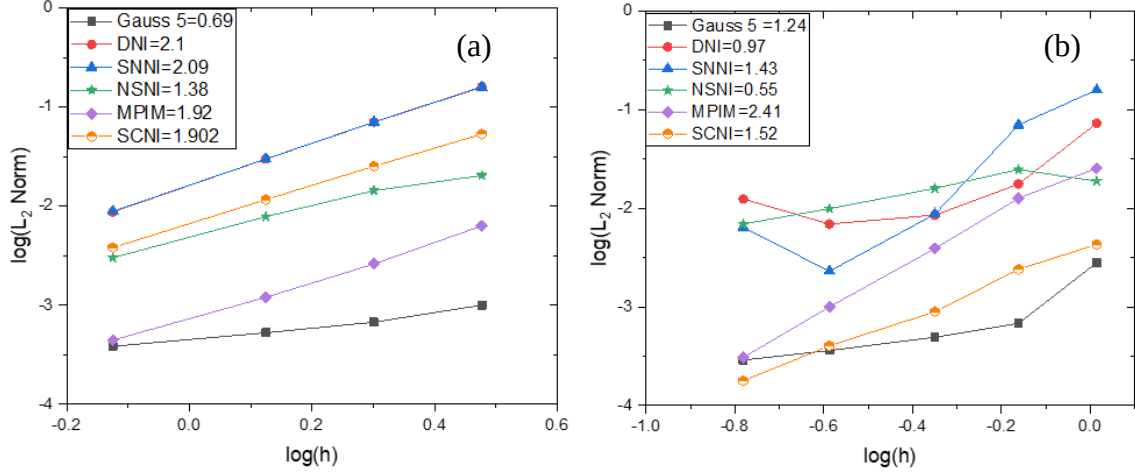


Figure 88. Convergence study using (a) uniform and (b) non-uniform discretization.

## Appendix B.

In this section, a detailed comparison of different semi-Lagrangian formulations is presented. Consider

$$\rho \frac{D^2 u}{Dt^2} = E \frac{\partial^2 u}{\partial x^2} + f_b \quad \text{on } \Omega \quad (\text{A1})$$

Boundary condition:  $u(0, t) = u_g; E \frac{\partial u(L, t)}{\partial x} = h$

Initial condition:  $u(x, 0) = 0; v = \frac{Du(x, 0)}{Dt} = v_0$

### Standard Semi-Lagrangian RK Formulation (Inconsistent time integration):

The approximations of displacement, velocity, and acceleration accounting for the time rate change of semi-Lagrangian RK can be written as

$$\begin{aligned} u^h &= \psi d, \\ v^h &= \frac{Du^h}{Dt} = \psi \dot{d} + \bar{\psi} d, \\ a^h &= \frac{D^2 u^h}{Dt^2} = \psi \ddot{d} + 2\bar{\psi} \dot{d} + \bar{\bar{\psi}} d \end{aligned} \quad (\text{A2})$$

The above equation (A1) , after applying Galerkin weak form, can be written as

$$\begin{aligned} \int_{\Omega} \rho \psi \psi d\Omega \ddot{\mathbf{d}} + \int_{\Omega} \rho \psi \bar{\psi} d\Omega \dot{\mathbf{d}} + \left( \int_{\Omega} E \psi_{,x} \psi_{,x} d\Omega + \int_{\Omega} \rho \psi \bar{\psi} d\Omega \right) \mathbf{d} \\ = \int_{\Gamma} E \psi \frac{\partial u^h}{\partial x} n d\Gamma + \int_{\Omega} \psi f_b d\Omega \end{aligned} \quad (A3)$$

$$\Rightarrow \mathbf{M} \ddot{\mathbf{d}}_{n+1} + \mathbf{C} \dot{\mathbf{d}}_{n+1} + \mathbf{K}_x \mathbf{d}_{n+1} = \mathbf{F}$$

where  $\mathbf{M} = \int_{\Omega} \rho \psi \psi d\Omega$  ,  $\mathbf{C} = \int_{\Omega} \rho \psi \bar{\psi} d\Omega$  ,  $\mathbf{K}_x = \int_{\Omega} E \psi_{,x} \psi_{,x} d\Omega + \int_{\Omega} \rho \psi \bar{\psi} d\Omega$  , and

$$\mathbf{F} = \int_{\Gamma} E \psi \frac{\partial u^h}{\partial x} n d\Gamma + \int_{\Omega} \psi f_b d\Omega .$$

The numerical solution is obtained after applying the inconsistent Newmark beta method (A4) to the semi-discrete equation (A3).

$$\begin{aligned} \mathbf{d}_{n+1} &= \mathbf{d}_n + \dot{\mathbf{d}}_n \Delta t + (0.5 - \beta) \ddot{\mathbf{d}}_n \Delta t^2 + \beta \ddot{\mathbf{d}}_{n+1} \Delta t^2 \\ \dot{\mathbf{d}}_{n+1} &= \dot{\mathbf{d}}_n + (1 - \gamma) \ddot{\mathbf{d}}_n \Delta t + \gamma \ddot{\mathbf{d}}_{n+1} \Delta t \end{aligned} \quad (A4)$$

#### Standard Semi-Lagrangian RK Formulation:

Standard semi-Lagrangian can be obtained using the semi-discrete equation (A3) obtained in the above section and combined with the Newmark beta time integration method (A5).

$$\begin{aligned} \psi \mathbf{d}_{n+1} &= u_n^h + v_n^h \Delta t + (0.5 - \beta) a_n^h \Delta t^2 + \beta \psi \ddot{\mathbf{d}}_{n+1} + 2\bar{\psi} \dot{\mathbf{d}}_{n+1} + \bar{\bar{\psi}} \mathbf{d}_{n+1} \Delta t^2 \\ \psi \dot{\mathbf{d}}_{n+1} + \bar{\psi} \mathbf{d}_{n+1} &= v_n^h + (1 - \gamma) a_n^h \Delta t + \gamma \psi \ddot{\mathbf{d}}_{n+1} + 2\bar{\psi} \dot{\mathbf{d}}_{n+1} + \bar{\bar{\psi}} \mathbf{d}_{n+1} \Delta t \end{aligned} \quad (A5)$$

#### Consistent Semi-Lagrangian RK Formulation:

The consistent approximations of displacement, velocity, and acceleration can be written as

$$\begin{aligned} u^h &= \psi \mathbf{d}, \\ v^h &= \psi \hat{\mathbf{v}}, \\ a^h &= \psi \hat{\mathbf{a}} \end{aligned} \quad (A6)$$

The above equation (A1) , after applying Galerkin weak form with the introduction of consistent approximation, can be expressed as

$$\int_{\Omega} \rho \psi \psi d\Omega \hat{\mathbf{a}} + \int_{\Omega} E \psi_{,x} \psi_{,x} d\Omega \mathbf{d} = \int_{\Gamma} E \psi \frac{\partial u^h}{\partial x} n d\Gamma + \int_{\Omega} \psi f_b d\Omega \quad (A7)$$

$$\Rightarrow \mathbf{M} \hat{\mathbf{a}}_{n+1} + \mathbf{K} \mathbf{d}_{n+1} = \mathbf{F}$$

where  $\mathbf{M} = \int_{\Omega} \rho \psi \psi d\Omega$ ,  $\mathbf{K}_x = \int_{\Omega} E \psi_{,x} \psi_{,x} d\Omega$ , and  $\mathbf{F} = \int_{\Gamma} E \psi \frac{\partial u^h}{\partial x} n d\Gamma + \int_{\Omega} \psi f_b d\Omega$ .

After the application of consistent Newmark beta method (A8) to the semi-discrete equation (A7), one can obtain the consistent semi-Lagrangian RK solution.

$$\begin{aligned} d_{n+1} &= d_n + \hat{v}_n dt + (0.5 - \beta) \hat{a}_n dt^2 + \beta \hat{a}_{n+1} dt^2 \\ \hat{v}_{n+1} &= \hat{v}_n + (1 - \gamma) \hat{a}_n dt + \gamma \hat{a}_{n+1} dt \end{aligned} \quad (A8)$$

## Appendix C.

### C.1. Stability Analysis and Time Step for the semi-Lagrangian Framework

To investigate the stability of the equation of motion in the semi-Lagrangian framework, a one-dimensional linear momentum equation is employed. When the considered time step is smaller than the critical time step, a stable solution for the dynamic analysis can be established. In this analysis, a stable time step for the explicit time integration scheme is determined using the Courant-Friedrichs-Lewy (CFL) condition by evaluating the eigenvalue of the assemblage system of equation [12]. The maximum eigenvalue of the system can be evaluated using the eigenvalue inequality theorem. The theorem was first developed for elements and later expanded to include the quadrature of the domain integration cell [23] [24] [25]. According to the inequality, evaluating the eigenvalue at the quadrature point yields the maximum eigenvalue, which is stated below:

$$\Delta t_{critical} = \frac{2}{\sqrt{\max_{Q=1}^{NQ} (\lambda_{max}^Q)}} \quad (A9)$$

where  $\lambda_{max}^Q$  is the maximum eigen values at the quadrature point, ‘NQ’ is the number of quadrature points in the entire domain.

To obtain the eigenvalue of the system, a Galerkin weak form is applied to the linear momentum equation under the semi-Lagrangian framework with the test function  $\delta u$ , leads to

$$\int_{\Omega} \rho \delta u \mathbf{a} d\Omega + \int_{\Omega} \nabla_x \delta u \boldsymbol{\sigma} d\Omega = \int_{\Omega} \delta u \mathbf{f}_b d\Omega + \int_{\Gamma_h} \delta u \mathbf{h} d\Gamma \quad (\text{A10})$$

To simplify the formulation, the external force is ignored from the above equation (A10) results in

$$\int_{\Omega_0} \rho_0 \delta u \mathbf{a} d\Omega + \int_{\Omega_0} \nabla_x \delta u F^{-1} \boldsymbol{\sigma} J d\Omega = 0 \quad (\text{A11})$$

where  $J = \det(F)$  is the Jacobian matrix,  $F$  is the deformation gradient, ' $X$ ' is the initial configuration, ' $x$ ' is the current configuration, ' $\Omega_0$ ' is the referential domain, ' $\Omega$ ' is the current domain,  $\boldsymbol{\sigma}$  is the Cauchy stress,  $\rho, \rho_0, \mathbf{f}_b, \mathbf{h}$  represents density in the current configuration, density in the initial configuration, body force, and the traction on the Neumann boundary, respectively. Linearizing the above equation (A11) can be transformed to

$$\overline{\int_{\Omega_0} \rho_0 \delta u \mathbf{a} d\Omega} + \overline{\int_{\Omega_0} \nabla_x \delta u F^{-1} \boldsymbol{\sigma} J d\Omega} = 0 \quad (\text{A12})$$

where  $\overline{p}$  operator represents, linearization of function ' $p$ ', expanding the above equation (A12) gives

$$\begin{aligned} & \int_{\Omega_0} \rho_0 \delta u \overline{\mathbf{a}} d\Omega + \int_{\Omega_0} \nabla_x \delta u \overline{F^{-1} \boldsymbol{\sigma} J} d\Omega + \int_{\Omega_0} \nabla_x \delta u F^{-1} \overline{\boldsymbol{\sigma}} J d\Omega + \\ & \int_{\Omega_0} \nabla_x \delta u F^{-1} \boldsymbol{\sigma} \overline{J} d\Omega = 0 \end{aligned} \quad (\text{A13})$$

Upon linearizing the above equation in the semi-Lagrangian framework, it yields the following expression

$$\int_{\Omega} \rho \boldsymbol{\psi} \overline{\boldsymbol{\psi}} \mathbf{a} d\Omega + \int_{\Omega} \rho \boldsymbol{\psi} Q F \boldsymbol{\psi} d\Omega \overline{\mathbf{d}} + \int_{\Omega} \nabla_x \boldsymbol{\psi} \overline{\boldsymbol{\sigma}} d\Omega = \mathbf{0} \quad (\text{A14})$$

The linearized Cauchy stress is replaced by the Truesdell rate as given in the below equation

$$\overline{\boldsymbol{\sigma}} = (C^{\sigma T} + \boldsymbol{\sigma})(F \nabla_x \boldsymbol{\psi} \overline{\mathbf{d}} + F^2 \frac{\partial^2 u}{\partial x^2} \boldsymbol{\psi} \overline{\mathbf{d}}) \quad (\text{A15})$$

$$\mathbf{M} \bar{\mathbf{a}} + \mathbf{K} \bar{\mathbf{d}} = \mathbf{0} \quad (\text{A16})$$

where  $\mathbf{K} = \int_{\Omega} \nabla_x \psi (\sigma + C^{\sigma T}) (F \nabla_x \psi + F^2 \frac{\partial^2 u}{\partial x^2} \psi) d\Omega + \int_{\Omega} \rho \psi Q F \psi d\Omega$ .

Using the eigenvalue theorem on the equation by substituting the mass and stiffness matrix, applying the eigenvectors  $\mathbf{v}^p$  to the mass and stiffness matrix yields the following equation.

$$\lambda^p = c^2 F \sum_{I=1}^n \frac{\psi_{I,x}^2(x_p)}{\psi_I(x_p)} + QF \Big|_{x_p} \quad (\text{A17})$$

Further approximation yields the following equation.

$$\lambda_{\max}^p = \frac{12c^2}{Fh^2} + QF \Big|_{x_p} \quad (\text{A18})$$

where,  $c = \sqrt{\frac{C^{\sigma T} + \sigma}{\rho}}$  is the instantaneous wave speed, ‘n’ is the total number of nodes.

The estimated eigenvalue is substituted into the equation (A9) to achieve the critical time step required for the semi-Lagrangian framework as described in the below equation

$$\Delta t_{\text{critical}} = \frac{2}{\sqrt{\max(\frac{12c^2}{Fh^2} + QF)}} \quad (\text{A19})$$

## C.2. Analysis of Maximal Eigenvalue Analysis

The analytical eigenvalue estimation can be verified using a numerical 1D example. The total length of the 1D beam is 40 m, Young's modulus is 10 kPa, density is 1.0 kg/cum, and a total number of 41 nodes with a linear basis is considered for the example. The numerical eigenvalues are evaluated in the semi-Lagrangian framework with a support size of 1.5. The analytical eigenvalues are estimated using the equation (A19); for this case, the gradient of acceleration is considered to be zero. Figure 89 shows a comparison between the numerical and analytical eigenvalues for various deformation gradients with a support size of 1.50. Since the analytical eigenvalues give an upper bound for the eigenvalues, the analytical eigenvalue remains greater

than the numerical eigenvalue. A graph is plotted between the normalized eigenvalue vs deformation gradient to observe the effectiveness of the approximation made in Equation (A18) of the analytical solution. It is observed that the trend line agrees well with the analytical solution along with few discrepancies

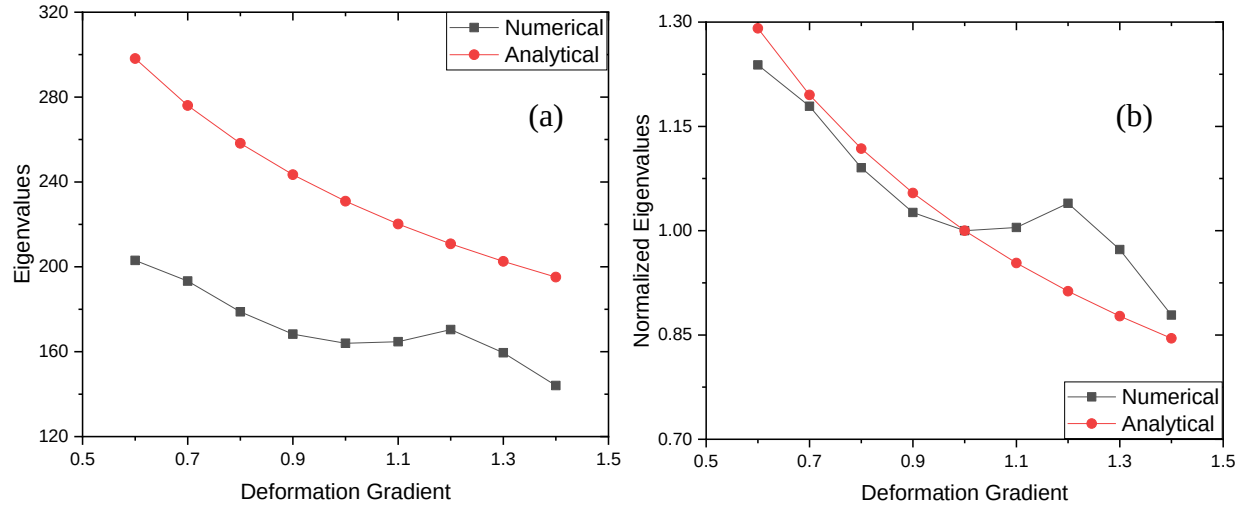


Figure 89. (a) eigenvalues vs deformation gradient (b) normalized eigenvalues vs deformation gradient with support size of 1.50 for numerical and analytical solution.

### C.3. Verification of Stability Analysis

To assess the performance of the above empirical solution to obtain the critical time step for the semi-Lagrangian framework, a sine wave is applied with varying frequency at the end of the 1D beam. The length of the 1D beam is considered 5.0 m, and the normalized support size of 1.5 with a linear basis and a 3-point Gauss quadrature is employed as the domain integration.

$$\rho \frac{D^2 u}{Dt^2} = E \frac{\partial^2 u}{\partial x^2} + f_b \quad \text{on } \Omega \quad (\text{A20})$$

Boundary condition:  $u(0, t) = 0; \frac{\partial u(L, t)}{\partial x} = EPk \cos(kl) \sin(\omega t)$

Initial condition:  $u(x, 0) = 0; \frac{Du(x, 0)}{Dt} = 0$

where  $w = k\sqrt{E/\rho}$ ,  $k = \Pi/4L, \Pi/8L, \Pi/16L, \Pi/32L, \Pi/64L$  for several sets of frequencies,  $E = 1 \text{ MPa}$ ,  $\rho = 1 \text{ kg/m}^3$ ,  $P = \text{scaling factor}$ , and  $x$  is in the current configuration.

The numerical system has been analyzed till the deformation gradient remains a monotonic function. By lowering the time step until the instability appears, the critical time step for the numerical solution is found. Stable time increment vs. deformation gradient is shown in Figure 90(a), and it can be seen that the numerically obtained critical time step achieves larger than the analytically predicted critical time step. This guarantees that the solution remains stable under the analytical estimated critical time step. Figure 90(b) plots normalized stable time increment vs deformation gradient; the trend line demonstrates that the analytical and numerical evaluated solutions are quite similar. This shows that the analytical solution effectively captured the parameters necessary to arrive at a stable solution.

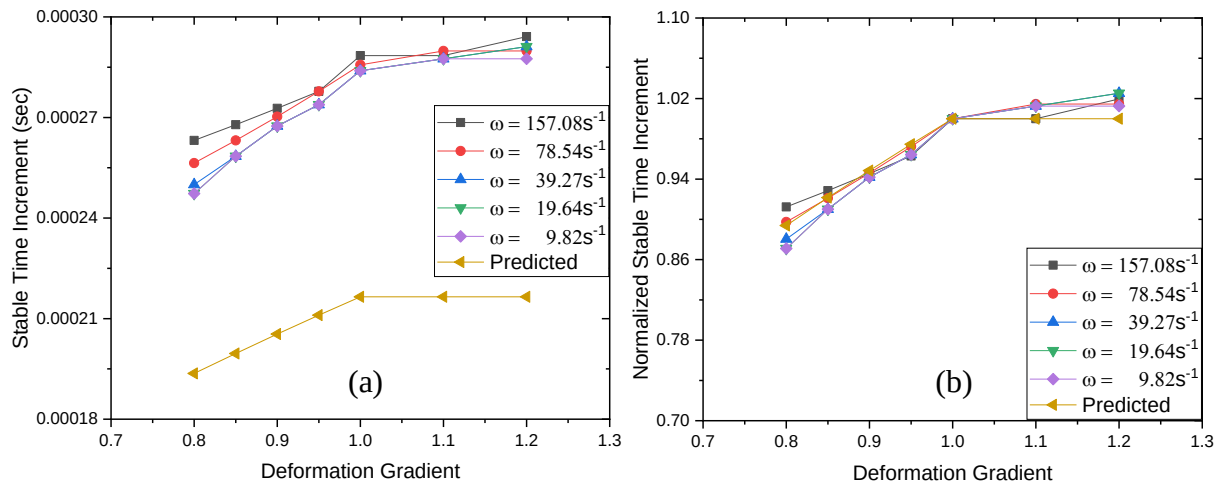


Figure 90. (a) stable time increment vs. deformation gradient (b) normalized stable time increment vs. deformation gradient of various frequencies for numerical solution and the predicted time (analytical critical time step).

## Appendix D.

The basic equations of the soil model we modified and used in this project are briefly described here to provide the necessary information to understand the tasks we performed in Section 4.1.

The model is a smooth cap plasticity model, originally based on the Sandia GeoModel [14] [26]. While earlier modifications added a tensile cap [27], the current version of the model integrates the shear and tensile surfaces, following a hyperbolic shear yield surface inspired by [28]

$$F_f = \sqrt{(C - MI_1)^2 - (C - M\Xi)^2} \quad (\text{A21})$$

Here  $C$  is a cohesion-like internal state variable (ISV),  $\Xi$  is a tensile strength ISV indicating the maximum elastic value of  $I_1$ , the first stress invariant. The material parameter  $M$  is a friction coefficient-like parameter measuring the slope of the shear surface at high compressive stresses. At very high compression, inelastic compaction can occur, and this is modeled using an elliptical compression cap.

$$F_C = 1 - H(\kappa - I_1) \left( \frac{I_1 - \kappa}{X - \kappa} \right)^2 \quad (\text{A22})$$

Here  $H$  is the Heaviside function, and  $\kappa$  and  $X$  are ISVs related by the equation

$$X = \hat{R}\kappa + S \quad (\text{A23})$$

where  $\hat{R}$  and  $S$  are material parameters dictating the shape of the ellipse as it evolves. There is also a third-invariant modifying function to account for differences in the Gudehus form [21]

$$\Gamma = \frac{1}{2}(1 + \sin(3\beta)) + \frac{1}{\psi}(1 - \sin(3\beta)) \quad (\text{A24})$$

where  $\psi$  is the difference in triaxial extension to compression strength (considered to be constant here), and

$$\beta = -\frac{3\sqrt{3}J_3}{2(J_2)^{1.5}} \quad (\text{A25})$$

is the Lode angle, with  $J_2$  and  $J_3$  being invariants of the deviatoric stress tensor. These components are assembled to a full yield function given by

$$f = \Gamma(\beta)\sqrt{J_2} - \sqrt{F_C F_f} \quad (\text{A26})$$

More details can be found in [26] [27].

## Appendix E.

### E.1. Strong form, Weak form, and Discrete Equations of u-p Mixed Formulation for Partially Saturated Soil

The strong form of governing equations comes from the balance of linear momentum and fluid mass. The balance of momentum equation can be written

$$\nabla \cdot \boldsymbol{\sigma} + \bar{\rho} \mathbf{g} = 0, \quad (\text{A27})$$

$$\bar{\rho} = \phi^s \rho_s + \phi^w \rho_w \quad (\text{A28})$$

where  $\phi^s$  and  $\phi^w$  are the volume fraction of solid and liquid, respectively.  $\rho_s$  is the solid mass density and  $\rho_w$  is the liquid mass density.  $\bar{\rho}$  is the mixture density of porous media consisting of water and solid,  $\psi^w$  is the degree of saturation, and  $\mathbf{g}$  is the gravitational acceleration.

The conservation of liquid mass equation is given as,

$$(1 - \phi^s) \frac{d\psi^w}{dt} + \psi^w \nabla \cdot \mathbf{v} + \nabla \cdot \mathbf{q} = 0 \quad (\text{A29})$$

where  $\mathbf{v}$  is the velocity of the solid matrix, and  $\mathbf{q}$  is the relative discharge velocity determined by Darcy's law as,

$$\mathbf{q} = -k^{rw} \mathbf{K} \cdot \nabla \left( \frac{p}{\rho_w g} + y \right) \quad (\text{A30})$$

where  $y$  is the vertical coordinate,  $\mathbf{K}$  is the saturated hydraulic conductivity,  $k^{rw}$  is the relative permeability given in Section 4.4.

The boundary condition should be defined to solve the aforementioned governing equations:

$$u = \bar{u} \quad \text{on } \Gamma_u \quad (\text{A31})$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \Gamma_t \quad (\text{A32})$$

$$p = \bar{p} \quad \text{on } \Gamma_p \quad (\text{A33})$$

$$-\mathbf{n} \cdot \mathbf{q} = \bar{q} \quad \text{on } \Gamma_q \quad (\text{A34})$$

Initial conditions for  $t = 0$  are given as  $(u_0, p_0)$ .

$\Gamma_u$  is solid displacement boundary,  $\Gamma_t$  depicts solid traction boundary,  $\Gamma_p$  is fluid pressure boundary and  $\Gamma_q$  denotes the fluid flux boundary.

The weak form is to find  $\{\mathbf{u}, p\} \in S_u \times S_p$  such that for all  $\{\boldsymbol{\omega}, \eta\} \in V_u \times V_p$

$$\int_{\Omega} \boldsymbol{\omega} \cdot (\nabla \cdot \boldsymbol{\sigma}) \, d\Omega + \int_{\Omega} \boldsymbol{\omega} \cdot (\bar{\rho} \mathbf{g}) \, d\Omega = 0 \quad (\text{A35})$$

$$\int_{\Omega} \eta (1 - \phi^s) \frac{d\psi^w}{dt} + \int_{\Omega} \eta \psi^w \nabla \cdot \mathbf{v} + \int_{\Omega} \eta \nabla \cdot \mathbf{q} = 0 \quad (\text{A36})$$

Galerkin form is obtained from the weak form by approximating the trial spaces  $S_u$  and  $S_p$  by  $S_u^h$  and  $S_p^h$ , respectively, and weighting spaces  $V_u$  and  $V_p$  by  $V_u^h$  and  $V_p^h$ , respectively. Also, function  $u$ ,  $p$ ,  $\omega$ , and  $\eta$  are substituted by  $u^h \in S_u^h$ ,  $p^h \in S_p^h$ ,  $\omega^h \in V_u^h$ , and  $\eta^h \in V_p^h$ . The Galerkin form is to find  $\{u^h, p^h\} \in S_u^h \times S_p^h$  such that for all  $\{\omega^h, \eta^h\} \in V_u^h \times V_p^h$

$$\int_{\Omega} \boldsymbol{\omega}^h \cdot (\nabla \cdot \boldsymbol{\sigma}) \, d\Omega + \int_{\Omega} \boldsymbol{\omega}^h \cdot (\bar{\rho} \mathbf{g}) \, d\Omega = 0 \quad (\text{A37})$$

$$\int_{\Omega} \eta^h (1 - \phi^s) \frac{d\psi^w}{dt} \, d\Omega + \int_{\Omega} \eta^h \psi^w \nabla \cdot \mathbf{v} \, d\Omega + \int_{\Omega} \eta^h \nabla \cdot \mathbf{q} \, d\Omega = 0 \quad (\text{A38})$$

After introducing reproducing kernel approximation described in Section 3.1 for trial and test functions, the discrete equations can be obtained. The detail can be found in [29] [3].

## Appendix F.

### F.1. Soil Tests for Parameter Calibration of Dry Sand

The mechanical properties of the dry sand (US SILICA™ Flint #12) were determined by the following soil experiments, including density determination, specific gravity determination by pycnometer, the grain size analysis, and the Unconsolidated Undrained triaxial test (UU).

#### F.1.1. Density (Unit Weight) Determination

The density was determined by the Drive-Cylinder Method according to ASTM D2937-10. Six samples were tested, and experimental results are given in Table 27. The mean density is 1.70 g/cm<sup>3</sup>, which is used in the numerical simulations.

*Table 27. Experimental results of density for sample sand.*

| No.                               | $M_c$ | $M_{cs}$ | $M_t$ | $\rho_d$ |
|-----------------------------------|-------|----------|-------|----------|
| 1                                 | 746.6 | 850      | 103.4 | 1.703    |
| 2                                 | 746.5 | 852.3    | 105.8 | 1.742    |
| 3                                 | 746.5 | 849.8    | 103.3 | 1.701    |
| 4                                 | 557.2 | 659.5    | 102.3 | 1.685    |
| 5                                 | 557.2 | 660.5    | 103.3 | 1.701    |
| 6                                 | 557.2 | 659.6    | 102.4 | 1.686    |
| Average                           |       |          |       | 1.70     |
| Length of Soil Sample: 7.059 cm   |       |          |       |          |
| Diameter of Soil Sample: 3.310 cm |       |          |       |          |
| Volume: 60.725 cm <sup>3</sup>    |       |          |       |          |
| Mass of empty can ( $M_c$ )       |       |          |       |          |
| Mass of total ( $M_{cs}$ )        |       |          |       |          |
| Mass of dry soil sample ( $M_t$ ) |       |          |       |          |
| Dry density ( $\rho_d$ )          |       |          |       |          |

#### F.1.2. Specific Gravity Determination by Pycnometer

Specific gravity is the density of the soil solid at a stated temperature divided by the density of water at the same temperature. The specific gravity of given sand is determined by the average of two repeated experiments using a water pycnometer, according to ASTM D854-00 standard. The results at 20°C are shown in Table 28, and the specific gravity is determined as **2.72**.

*Table 28. Experimental results of specific gravity of sand.*

| No.           | $M_p$ | $M_{ps}$ | $M_b$ | $M_a$ | $M_0$ | $G_s$       |
|---------------|-------|----------|-------|-------|-------|-------------|
| 1             | 48.8  | 59.7     | 153.7 | 146.8 | 10.9  | 2.725       |
| 2             | 47.5  | 62.7     | 156.4 | 146.8 | 15.2  | 2.71        |
| Average $G_s$ |       |          |       |       |       | <b>2.72</b> |

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$M_0 = M_{ps} - M_p$ : Mass of the Oven-dry Soil Sample

$M_p$ : Mass of Empty, Clean Pycnometer, (g)

$M_{ps}$ : Mass of Empty Pycnometer and Dry Soil, (g)

$M_b$ : Mass of Pycnometer, Dry Soil, and Water, (g)

$M_a$ : Mass of Pycnometer and Water, (g)

$G_s$ : Specific Gravity of Soil

$$G_s = \frac{M_0}{M_0 + M_a - M_b}$$

---

#### F.1.3. Grain Size Analysis - Sieve and Hydrometer Analysis

The goal of this measurement is to determine the distribution of grain sizes in the sand sample by mass. The larger particles are separated by sieve analysis, and the finer ones are tested using a hydrometer. According to ASTM D422 standard test, a total of 450.2g grain sand was sieved by the sieve shaker for 10 mins in the set of sieves (#4 to #200 set, standard test sieve made under ASTM E-11 Specification by ELE International.) The size distribution is shown in Table 29 and Figure 91.

*Table 29. Grain size analysis for the sand sample.*

| Sieve Number | Diameter (mm) | Mass Empty Sieve (g) | Mass Sieve & Soil Retained (g) | Mass Soil Retained (g) | Percent Retained | Percent Passing |
|--------------|---------------|----------------------|--------------------------------|------------------------|------------------|-----------------|
| 4            | 4.75          | 516.6                | 516.6                          | 0                      | 0.00             | 100.00          |

|     |       |       |           |       |       |        |
|-----|-------|-------|-----------|-------|-------|--------|
| 10  | 2     | 659.9 | 659.9     | 0     | 0.00  | 100.00 |
| 20  | 0.85  | 454.2 | 456.9     | 2.7   | 0.60  | 99.40  |
| 40  | 0.425 | 372.6 | 793.2     | 420.6 | 93.45 | 5.95   |
| 60  | 0.25  | 366.1 | 392.1     | 26    | 5.78  | 0.18   |
| 140 | 0.106 | 345.1 | 345.7     | 0.6   | 0.13  | 0.04   |
| 200 | 0.075 | 315.8 | 315.9     | 0.1   | 0.02  | 0.02   |
| pan | 0     | 374.5 | 374.6     | 0.1   | 0.02  | 0.00   |
|     |       |       | Total (g) | 450.1 |       |        |

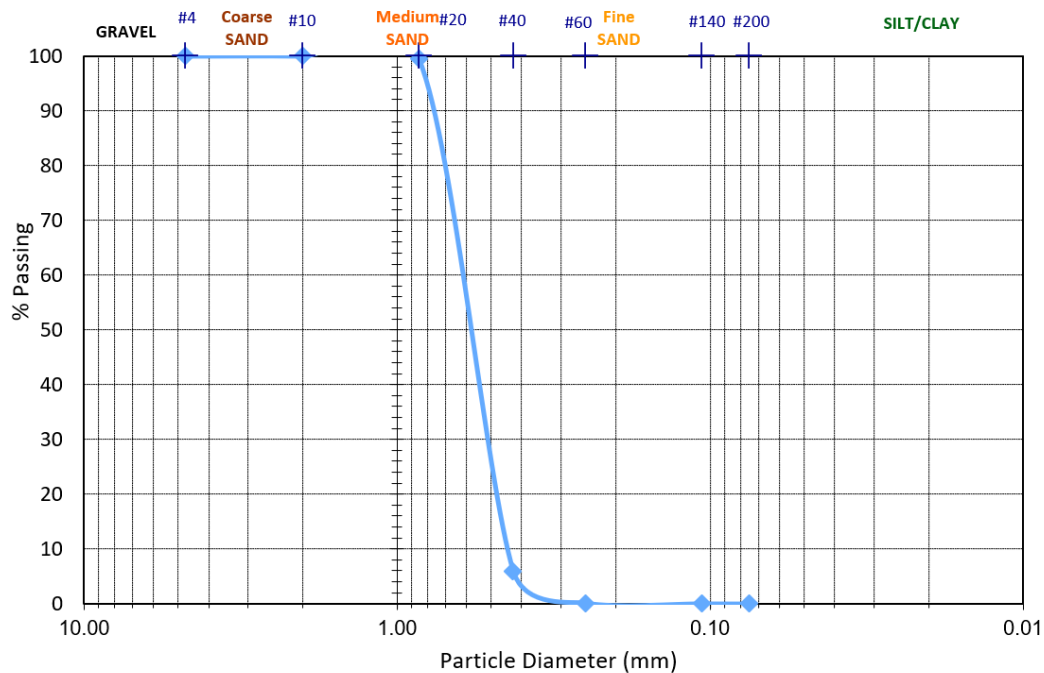


Figure 91. Grain size versus passing percent of the sand sample

#### F.1.4. Unconsolidated Undrained Triaxial Test (UU)

The Unconsolidated Undrained Triaxial Test (UU) was conducted to determine the shear strength parameters of the sand sample. In this test, the sand sample was not allowed to consolidate at any time. According to ASTM D2850, two loading speeds (1% and 10% of strain per minute) were applied to obtain stress-strain relationships under 15 psi confining pressure.

Cylinder specimens (diameter:70.000 mm, height: 122.9mm) were confined with membrane and assembled on the permeability cell, as shown in Figure 92. The measurement was carried on using a displacement-controlled device at two deformation speeds, 1% and 10% strain per minute.

Engineering stress and strain histories were recorded until the sample failure was noticed, as shown in Figure 93 and Figure 94. The obtained load-displacement relationship was used in conjunction with test data described in Section 5.1.1 to calibrate material parameters and validate the constitutive model.

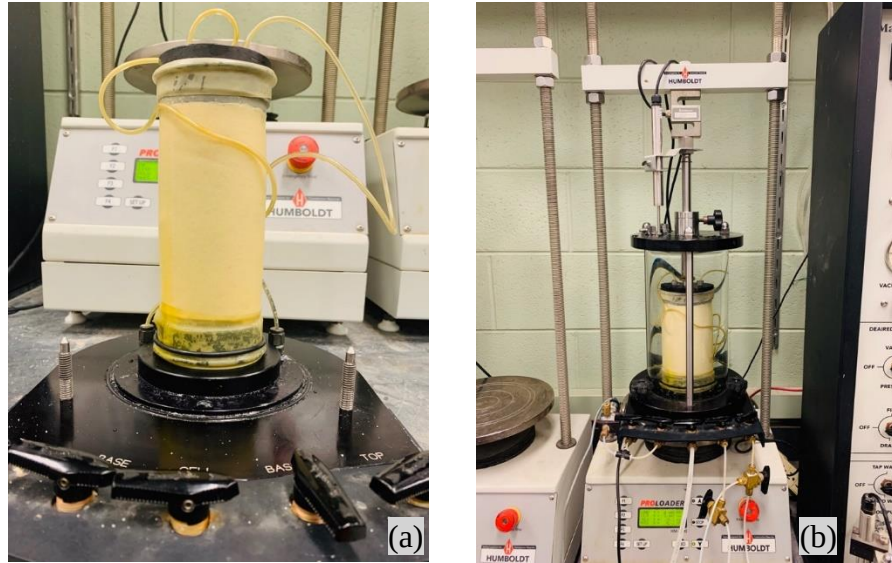


Figure 92. (a) Sand specimen and (b) experiment equipment for unconsolidated undrained triaxial test

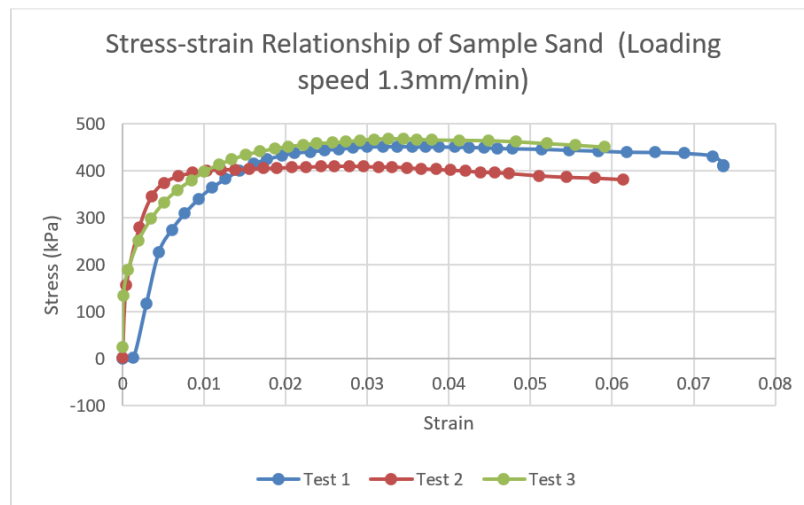


Figure 93. Stress-strain relationship of sand under unconsolidated undrained triaxial test with a 1% loading strain rate

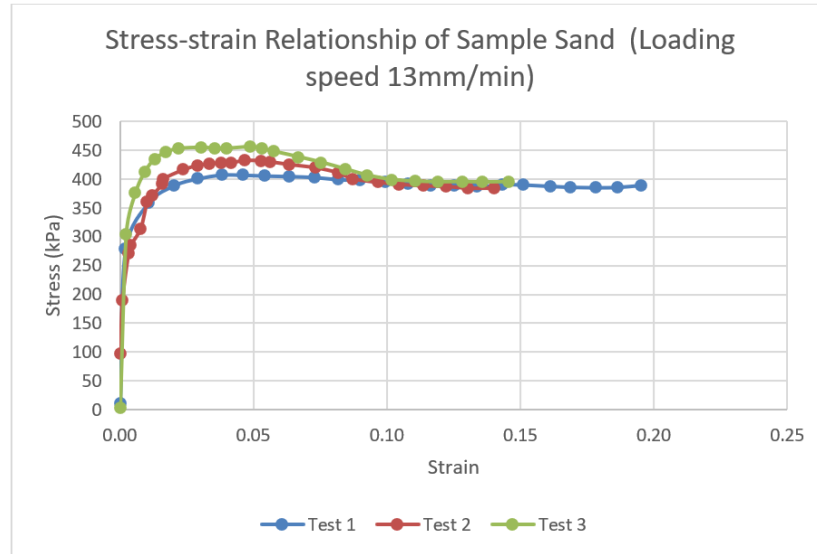


Figure 94. Stress-strain relationship of sand under unconsolidated undrained triaxial test with a 10% loading strain rate

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