NRL REPORT NO. R-3215

FR-3215

A GRAPHICAL METHOD FOR COMPUTING SHORT IMPEDANCE MATCHING TRANSFORMERS

C. B. Barker

Approved by:

L. C. Van Atta, Head, Antenna Research Section
J. M. Miller, Superintendent, Radio Division I

Problem No. 34R09-26 January 5, 1948

NAVAL RESEARCH LABORATORY
CAPTAIN H. A. SCHADE, USN, DIRECTOR
WASHINGTON, D.C.

DISTRIBUTION

ONR	
Attn: Mr. Harrison, Code 427	(1)
Attn: Code N482	(2)
BuAer	
Attn: Mr. E. P. McDowell	(1)
Attn: Code TD-4	(1)
BuOrd	
Attn: Mr. A. D. Bartelt, Re4f	(1)
BuShips	
Attn: Lt. Comdr. J. W. Steidley, Code 931	(1)
Attn: Mr. Alfred W. Andrews	(1)
Dir., USNEL	
Attn: Dr. T. J. Keary, Code 300	(2)
Dir., USNUSL	
Attn: Mr. C. M. Dunn	(1)
Ch. of Staff, USAF	
Attn: Mr. Joseph Weichbrod, AC/AS-4	(1)
OCSigO	
Attn: Mr. J. J. Kelleher, SIGGC-R-2	(1)
Attn: Mr. A. R. Beach, SIGOL-2	(1)
Attn: Ch. Eng & Tech. Div., SIGTM-S	(1)
CO, SCEL	
Attn: Dir. of Engineering	(2)
Dir., ESL	
Attn: Mr. O. C. Woodyard	(1)
Attn: Mr. Leonard Moore	(1)
CG, AMC, Wright Field	
Attn: Major J. L. Lambert, TSELP-2	(1)
Attn: Mr. T. J. Gibbons, TSELR-7B	(1)
Attn: Mr. A. R. Bell, TSELO	(1)
CO, AMC, Watson Labs., Cambridge	
Attn: Dr. R. C. Spencer, WLCAL	(1)
Attn: Mr. R. M. Barrett, WLCAL-4	(1)

CO, AMC, Watson Labs., Red Bank Attn: WLENG	(1)
BuOrd for Dir., NBS Attn: Dr. Harold Lyons, Ch.	(1)
INSMAT, N. Y. for RCA Labs., Long Island Attn: Mr. Phillip S. Carter	(1)
BAR, Akron for Ohio State University, Columbus Attn: Mr. Robert B. Jacques	(1)
ONR, San Francisco for University of California, Berkeley Attn: Professor Lester E. Reukema	(1)
BAR, N. Y. for Airborne Instruments Lab., Mineola Attn: Dr. R. S. Wehner	(1)
Secretariat, Aeronautical Board Attn: Mr. L. W. Sieck, Recorder	(1)
RDB Attn: Library Attn: Navy Secretary Attn: Committee on Electronics	 (2) (1) (3)
Science and Technology Project Attn: Mr. J. H. Heald, Ch.	(2)
Cruft Lab.. Harvard U	2
NRL: Code 109	1
" 110	1
" 186	3
" 187	1
" 1302	2
" 1380	2
" 188	60

CONTENTS

Abstract	vi
Problem Status	vi
INTRODUCTION	1
GENERAL PROPERTIES OF SLEEVE TRANSFORMERS	1
QUARTER-WAVE SLEEVE TRANSFORMERS	1
GRAPHICAL METHOD FOR COMPUTING NON-QUARTER-WAVE TRANSFORMERS	2
EXPERIMENTAL VERIFICATION	3

ABSTRACT

This report describes a simple method for computing the dimensions of a sleeve impedance transformer for a coaxial line feeding an antenna. Other means of making this calculation are well known, but the method described here possesses two distinct advantages. In the first place, the actual distance between the radiating element and the generator end of the transformer is reduced to an absolute minimum, thus lowering the frequency sensitivity of the system. In the second place, the computation is very rapid, being a simple graphical process.

PROBLEM STATUS

This report concludes the work on this phase of problem 34R09-26. Work will continue on the basic problem.

A GRAPHICAL METHOD FOR COMPUTING SHORT IMPEDANCE MATCHING TRANSFORMERS

INTRODUCTION

During the investigations arising from NRL Problem No. 34R09-26, it became necessary to design a sleeve transformer for the coaxial line feeding the antenna. Since there was a strict size limitation, the conventional quarter-wave sleeve could not be used. This limitation made it necessary to place the transformer adjacent to the radiating element and at the same time required the length of the transformer to be minimized. The method for computing such a transformer, as described in this report, was developed and successfully utilized.

GENERAL PROPERTIES OF SLEEVE TRANSFORMERS

Point A on the impedance chart of Figure 1 represents the impedance of any point on a coaxial line-fed antenna. It is well known that if a sleeve transformer is placed at this point, the impedance will move along the line OA, away from O, to some point B. The distance OB is equal to OA times Z_o/Z_t where Z_o is the characteristic impedance of the line and Z_t is the impedance of the section of line containing the transformer. As we now move along the transformer toward the generator, the impedance moves clockwise along a power standing wave ratio circle to some point C. The number of degrees it moves is determined by the formula,

$$\theta = \frac{L \times 720^\circ}{\lambda},$$

where L is the length of the transformer and λ is the wave length, both measured in like units. As we go from the transformer back to the original line, the impedance moves toward O along the line CO, to some point D. This point D is located by the fact that OD is equal to OC times Z_t/Z_o. The impedance represented by D is that of the antenna and transformer system, measured at the generator end of the transformer. The antenna is then said to possess the power standing wave ratio which is determined by the power standing wave ratio circle upon which D falls. (In the case shown in Figure 1, the PSWR is 1.6.)

QUARTER-WAVE SLEEVE TRANSFORMERS

The common method of utilizing a sleeve transformer to secure an optimum power standing wave ratio in the antenna system makes use of a quarter-wave sleeve transformer in the following manner. The impedance is first measured at some convenient reference point on the antenna, usually at the feed point of the radiating element nearest the generator. The diameter of the transformer is computed and placed on the line with the load end at a minimum voltage point. This procedure is illustrated in Figure 2, where A represents the impedance of the antenna at the feed point of the radiating element nearest the generator. In order to reach the first minimum voltage point down the line, we must move clockwise around the power standing wave ratio circle through A to the point B, which lies on the real axis. The sleeve transformer now takes the impedance from B to C, then around the power standing wave ratio circle through C, 180° to D, and finally as we go back to the original coaxial line, the impedance moves back to M, the point representing a power standing wave ratio of unity. Note that in the case just illustrated, the total distance from the radiating element to the generator end of the transformer is $11\,\lambda/16$.

GRAPHICAL METHOD FOR COMPUTING NON-QUARTER-WAVE TRANSFORMERS

With the previous discussions in mind, we can now describe the new graphical method for computing a non-quarter-wave transformer. As in the previous case, the impedance of the antenna is measured at the feed point of the first radiating element and is then plotted as A of Figure 3. The line joining O to A is extended as is the line OM, and then the line AM is drawn. The points B and C, on OA and OM, respectively, are now located to satisfy the following two conditions:

1. The straight line BC is parallel to AM.

2. Both B and C lie on the same PSWR circle.

Let us now consider a sleeve transformer, placed at the point represented by A, whose diameter is determined by the condition that

$$Z_t = \frac{OA}{OB} Z_o ,$$

and whose length is

$$L = \frac{\theta}{720^\circ} \lambda ,$$

where θ is the angle between the points B and C, measured toward the generator. It is clear that such a transformer will first move the impedance from A to B. Then due to the transformer length, the impedance will move around the power standing wave ratio circle through B to the point C. Finally, since the two triangles OAM and OBC are similar,

$$\frac{OB}{OA} = \frac{OC}{OM} ,$$

so that

$$OM = \frac{Z_t}{Z_o} OC ,$$

and the impedance will thus move to M. This is the new graphical procedure for computing sleeve transformers. It is worthwhile to mention in passing that the points A in both Figure 2 and Figure 3 are identical, the distance from the radiating element to the generator end of the transformer computed in Figure 3 is less than $\lambda/8$, while the same distance using a conventional quarter-wave sleeve as in Figure 2 is $11\,\lambda/16$.

The short method described above fails in the event that the measured impedance point does not fall in the shaded area of Figure 4. However, in this case, if we merely go down the coaxial line toward the generator until the point does come into the shaded area, the resulting transformer will still be found to give the minimum distance from the point of measurement to the generator end of the transformer. In general, it has been found that the optimum results are obtained when the point has an angle 20° and 70°, measured from the real axis toward the generator.

An additional feature can be pointed out—if the measured impedance falls in the area between -90° and 180°, this short method can be employed to compute an undercut transformer on the inner conductor.

In the following tabulation, Column 1 gives the distance from the point of measurement of the impedance to the generator end of the transformer for the points A, B, C, D, and E of Figure 4 using the short method described above. Column 2 gives this same distance if a conventional quarter-wave sleeve is employed.

Point	Column I	Column 2
A	.125 λ	.688 λ
B	.563 λ	.625 λ
C	.458 λ	.542 λ
D	.375 λ	.438 λ
E	.250 λ	.313 λ

EXPERIMENTAL VERIFICATION

In Figure 5, experimental verification of the utility of this method is given. Points A and B represent the impedance of two different antennas as actually measured at the feed point of the radiating element nearest the generator. Points A′ and B′ represent the impedance as measured at the generator end of the transformer. In both cases, the transformer was computed by the method described here, and was placed at the specified point on the antenna.

* * *

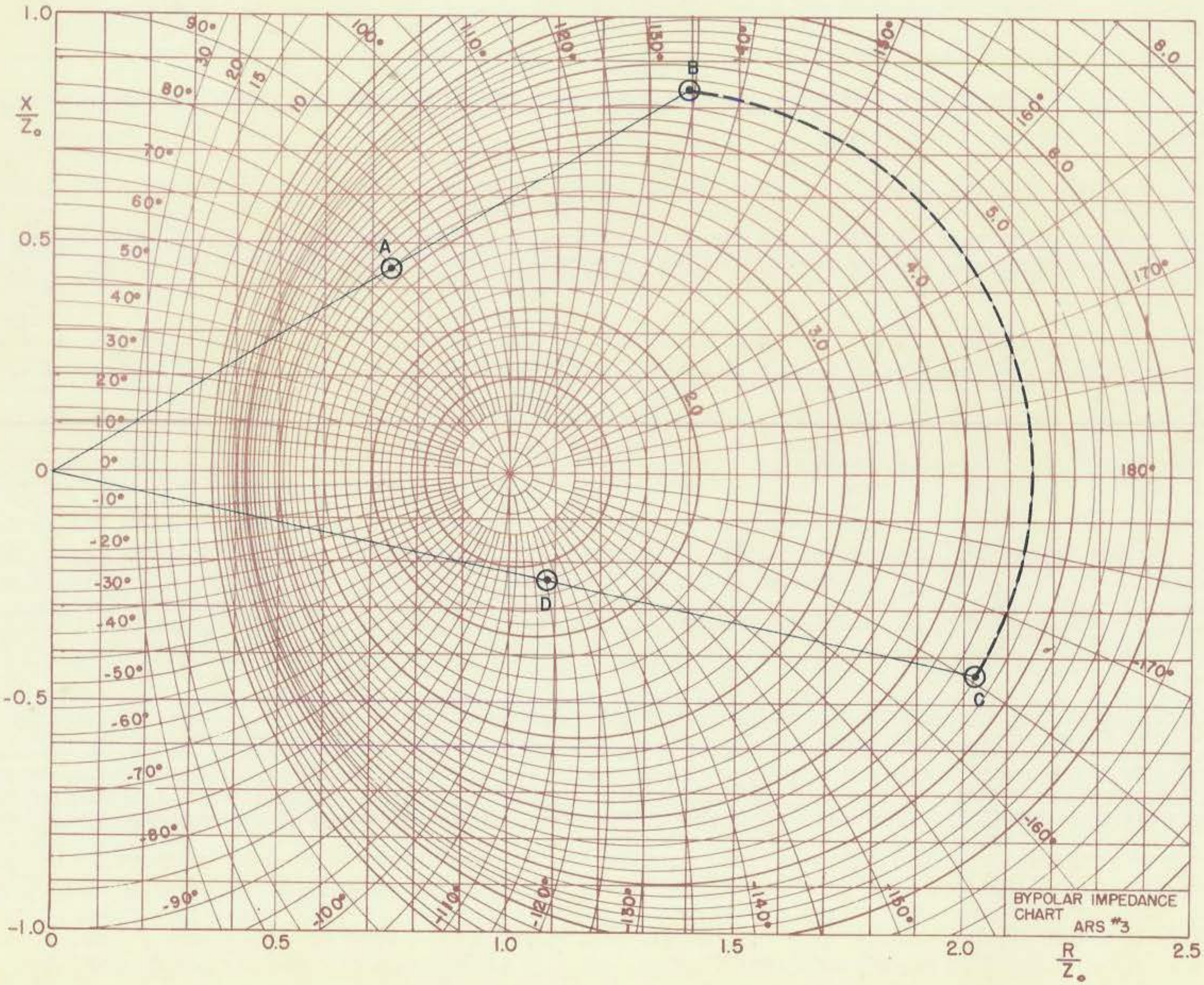


Figure 1

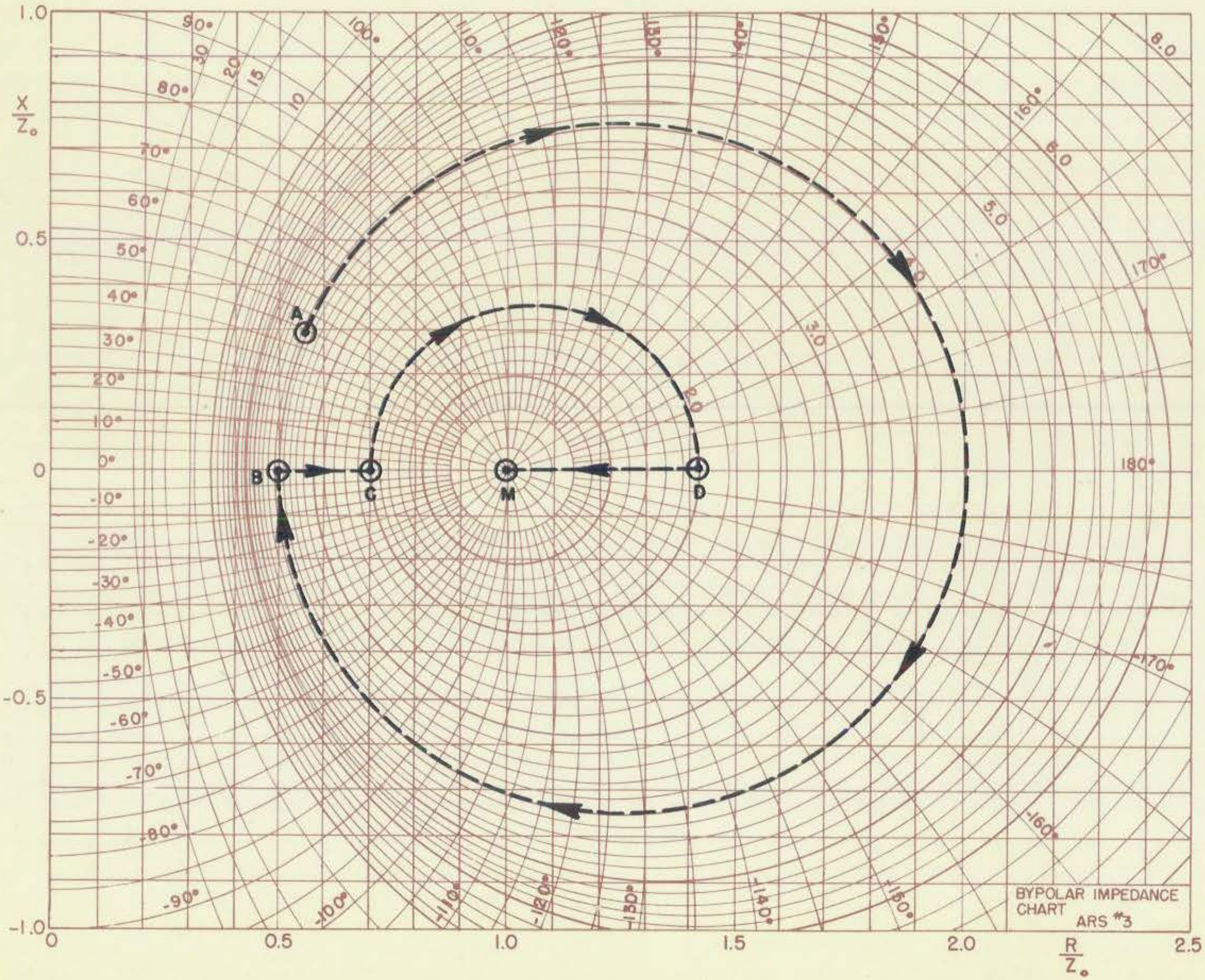
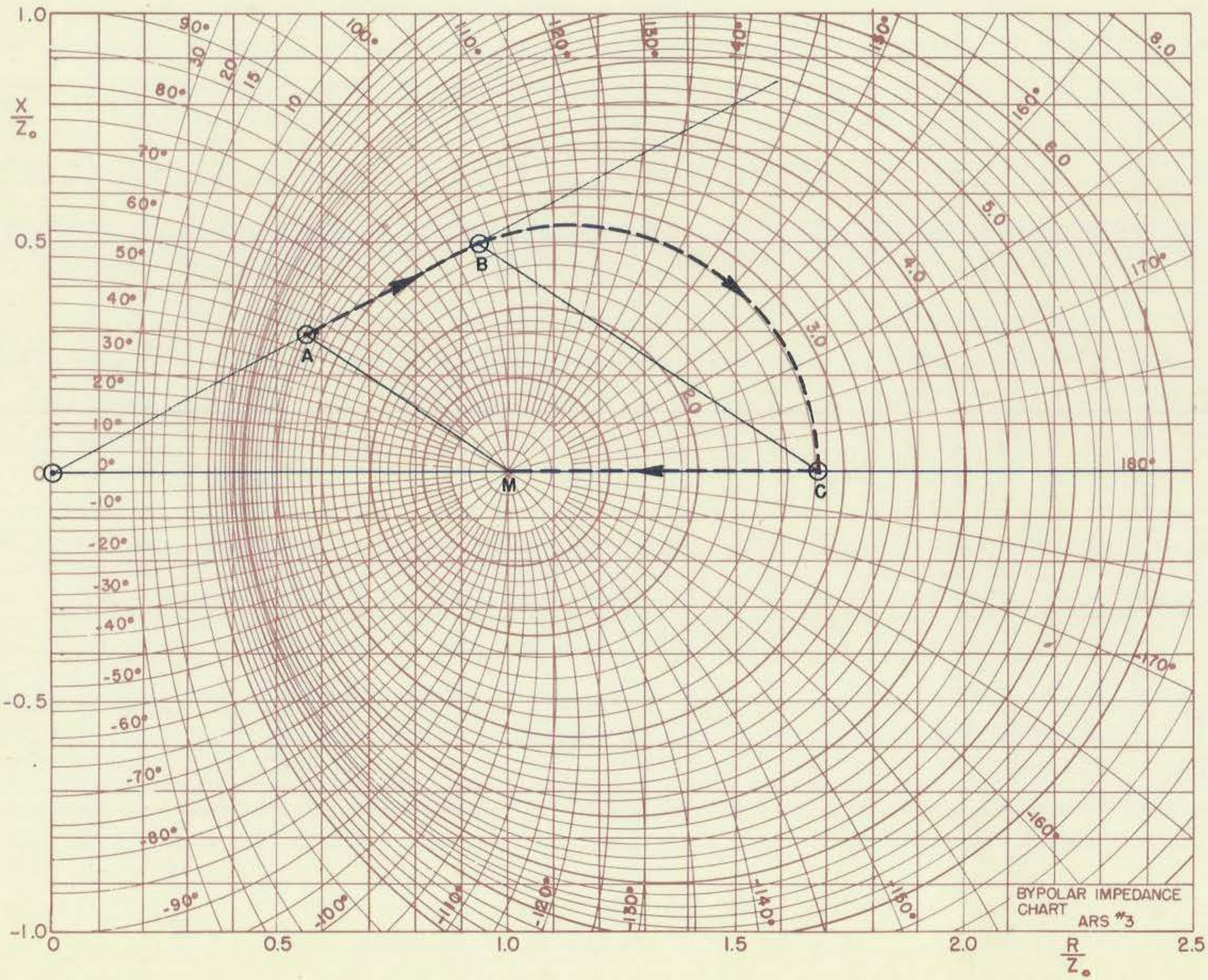

BYPOLAR IMPEDANCE
CHART
ARS #3
A
B
C
M
D
$\frac{X}{Z_o}$
$\frac{R}{Z_o}$
1.0
0.5
0
-0.5
-1.0
0
0.5
1.0
1.5
2.0
2.5

Figure 2



1.0
0.5
0
-0.5
-1.0
0.5
1.0
1.5
2.0
2.5
X/Z₀
R/Z₀
A
B
C
M
BYPOLAR IMPEDANCE CHART ARS #3

Figure 3

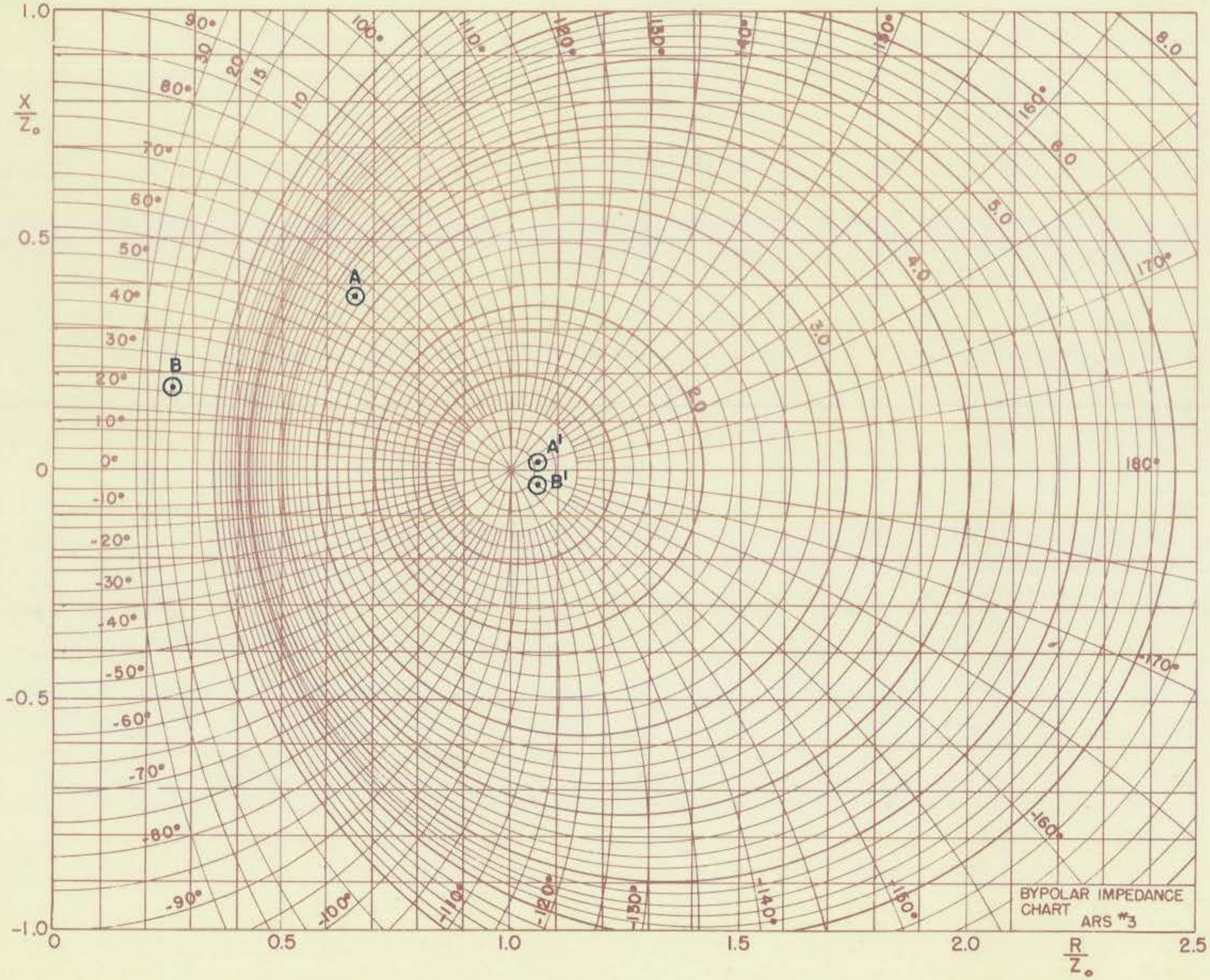

BYPOLAR IMPEDANCE
CHART
ARS #3
X/Z₀
R/Z₀
A
B
A'
B'
1.0
0.5
0
-0.5
-1.0
0
0.5
1.0
1.5
2.0
2.5

Figure 5