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FURTHER DESIGN AND DEVELOPMENT OF COMPONENTS FOR SIMULTANEOUS LOBING RADAR TAB

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FURTHER DESIGN AND DEVELOPMENT OF COMPONENTS FOR SIMULTANEOUS LOBING RADAR TAB

H. L. Gerwin and A. E. Hastings

Approved by:

Mr. J. E. Meade, Head, Radar I Section
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Problem No. 36R05-01 January 13, 1948

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CONTENTS

ABSTRACT

This report, the fourth on the amplitude-comparison simultaneous-lobing radar TAB, discusses the antenna feed and r-f plumbing and the discriminator in the i-f-to-video converter used in a system which has been built and operated in fully automatic tracking in bearing, elevation, and range since 22 July 1947.

PROBLEM STATUS

This is an interim report on this development; work is continuing.

AUTHORIZATION

This development was undertaken in accordance with Bureau of Ordnance Request No. 0-20.1 AR-C. The NRL Problem Number is R05-01.

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FURTHER DESIGN AND DEVELOPMENT OF COMPONENTS FOR SIMULTANEOUS LOBING RADAR TAB

INTRODUCTION

The development of a simultaneous lobing radar has been the subject of previous reports. The first[*] gave a complete discussion of the expected advantages of simultaneous lobing and suggested a novel method of carrying out the principle. Another[†] described the equipment designed to test this method and included analyses of the methods of obtaining the null and the indication of sense based on a 5-horn antenna feed. A third report[‡] described a modification for tracking in one angle only, using a 2-horn feed and rat-races for combining r-f channels, and included an analysis of the effects of phase shift in the r-f circuits. Since these reports have appeared, a system using a 4-horn feed has been built and operated with fully automatic tracking in bearing, elevation, and range. This report describes in particular the antenna feed and the i-f-to-video converter employed. The only other unconventional component, the Pisa indicator, has already been described in detail.[†]

DESCRIPTION OF SYSTEM

The present system operates in principle exactly as originally outlined.[*] There are four tilted antenna lobes for reception just as in the conventional lobe-switched radar. Angle-off-target is determined by subtracting at r-f the signals received simultaneously in opposed lobes. Thus the on-target condition is essentially an r-f amplitude null. A phase comparison at i-f of the lobe-difference signal with the signal from a fifth untilted lobe gives an indication of sense or direction off target. This comparison, provided by the discriminator in the i-f-to-video converter, results in a video signal with amplitude proportional to angle off-target and with polarity (±) indicating direction off target. The fifth lobe is also used for transmission and for range reception.

The first experimental system was planned with five separate feed horns, one for each of the lobes described. Such a feed proved difficult to design with suitable crossover and side-lobe characteristics, and a four-horn feed was substituted. The required five lobes are obtained by connecting the horns through an arrangement of rat-races to the receivers and to the transmitter.

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[*] Page, R. M., "Accurate Angle Tracking by Radar," NRL Report RA 3A 22A, 28 December 1944

[†] Trevor, J. B., and Hastings, A. E., "Analysis and Specifications of Simultaneous Lobing System TAB," NRL Report R-2554, 1 July 1945

[‡] Gerwin, H. L., "Design and Development of Antennas and R-F Components for Simultaneous Lobing Radar," NRL Report R-3042, 3 January 1947

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THE ANTENNA AND R-F PLUMBING

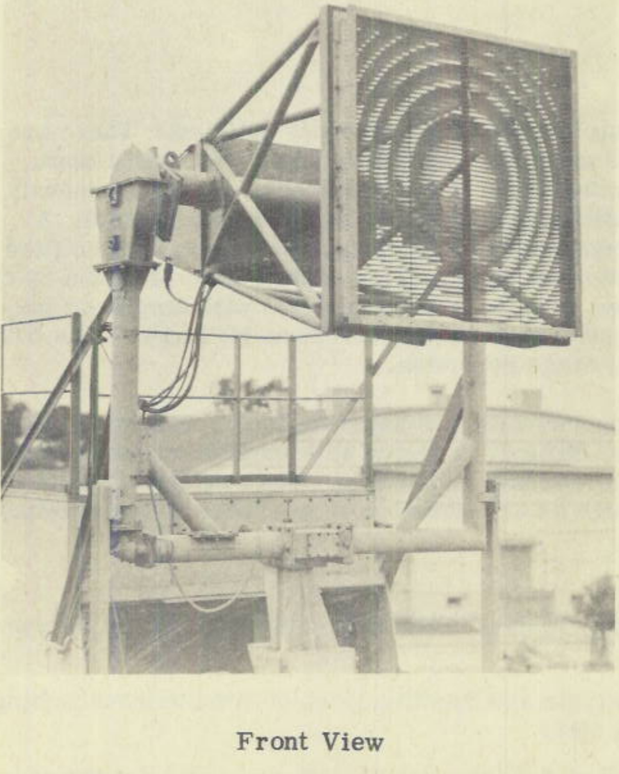
The antenna consists of a four-foot lens illuminated by a feed assembly of four horns located at the 48-inch focal position. The r-f plumbing, local oscillator, balanced converters, i-f preamplifiers, and transmitter are located in a waterproof housing behind the four-horn feed assembly. The entire assembly is suspended in a yoke support mounted on a modified Gun Director Mark 50. Figures 1 and 2 show the mount and the antenna assembly.

Side View

Fig. 1 - Mount and Antenna Assembly

FOUR-HORN FEED ASSEMBLY

The four horns used to illuminate the lens are connected together by three rat-races and one Y-junction. The r-f output signals from this assembly are range or reference signal, elevation angular error signal, and train angular error signal. Angular direction off target is not given by these error signals but is obtained in the i-f-to-video converter. Figures 3 and 4 are photographs of the assembly, and Figure 5 is a schematic of its waveguide plumbing.

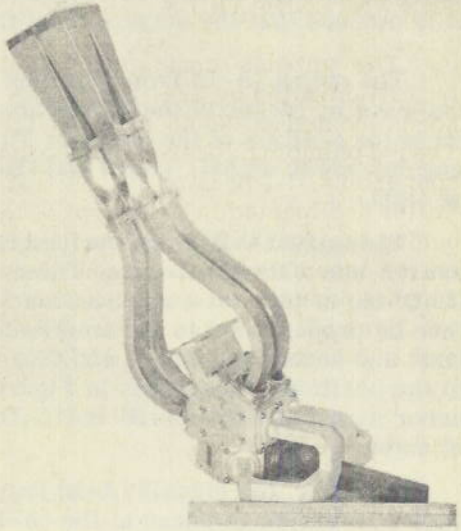


Front View

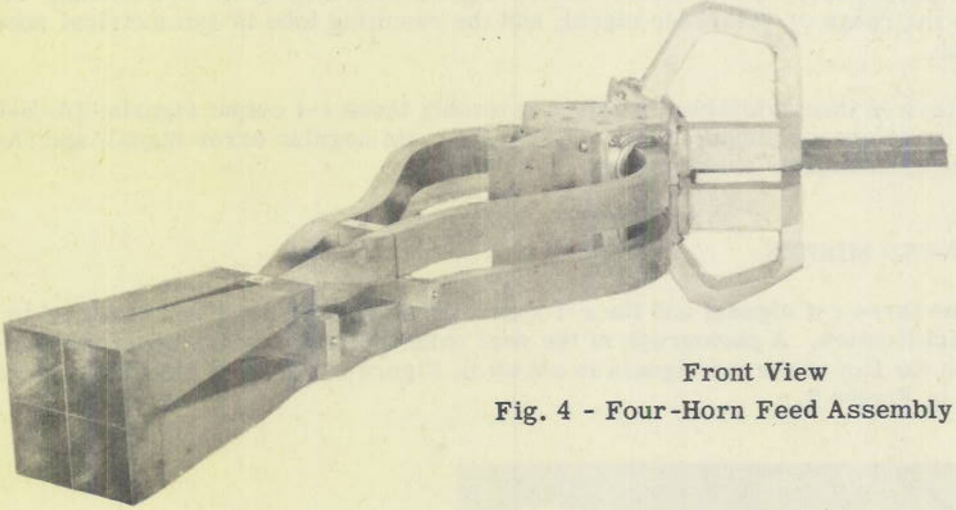
Fig. 2 - Mount and Antenna Assembly

In describing the operation it is necessary to consider first the signals received by horns A and B, see Figure 5. Because of the 90-degree opposed twists in the waveguide sections connecting the horns to the rat-race junction, the signals received in the two horns will arrive at the (A-B) leg of the rat-race junction 180-degrees out of phase. The resulting signal will then be proportional to the amplitude difference between the signals received in horns A and B and is thus labeled (A-B). Similarly, the signals from the horns will arrive at the (A+B) leg of the rat-race junction in phase, and the resulting signal in this leg will be proportional to the sum of the signals received in horns A and B and is thus labeled (A+B).

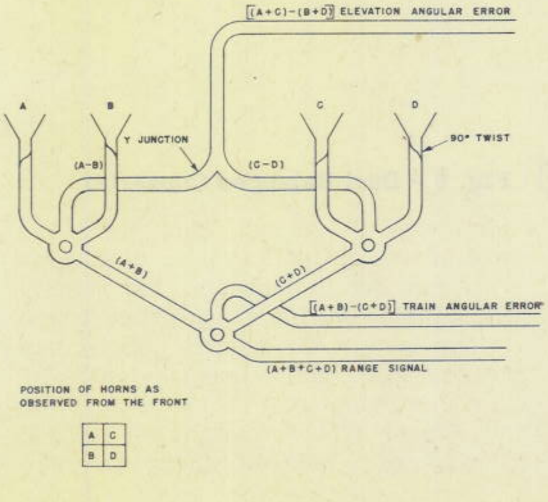
The signals received by horns C and D are also fed into their rat-race



Side View
Fig. 3 - Four-Horn Feed Assembly ⇨



Front View
Fig. 4 - Four-Horn Feed Assembly



Schematic Diagram
⇦ Fig. 5 - Four-Horn Feed Assembly

junction through 90-degree opposed waveguide sections. By following the above reasoning it is obvious that the outputs from this rat-race junction are (C-D) and (C+D).

The output (A-B) from the first rat-race is added to the output (C-D) from the second rat-race by means of the Y-junction. This sum may now be written (A+C) - (B+D), and from the position of the horns in Figure 5, it is evident that this signal is the elevation angular error signal. (A+C) and (B+D) form two lobes displaced vertically from the line of sight.

The output (A+B) from the first rat-race and the output (C+D) from the second rat-race are fed into a third rat-race. These signals, (A+B) and (C+D), will arrive at the (A+B) - (C+D) leg of this rat-race junction 180-degrees out of phase. The resulting signal will then be proportional to the amplitude difference between the signals received from the first and second rat-races, and this signal then becomes (A+B) - (C+D). Again by referring to the position of the horns in Figure 5, it is evident that this signal is the train angular error signal and that (A+B) and (C+D) form two lobes displaced horizontally from the line of sight.

Similarly, the signals (A+B) from the first rat-race and (C+D) from the second rat-race will arrive in phase at the (A+B+C+D) leg of the rat-race, and the resulting signal will be proportional to the sum of all the signals received by the four horns. Thus, it becomes the range or reference signal, and the resulting lobe is symmetrical about the line of sight.

There is then available from this assembly three r-f output signals: (A+B+C+D), the range or reference signal; (A+B) - (C+D), the train angular error signal; and (A+C) - (B+D), the elevation angular error signal.

BALANCED MIXERS

The three r-f signals and the r-f signal used for AFC must be converted to i-f signals for amplification. A photograph of the dual balanced mixer which accomplishes this conversion for two of the r-f signals is shown in Figure 6. The r-f plumbing schematic is shown in Figure 7.

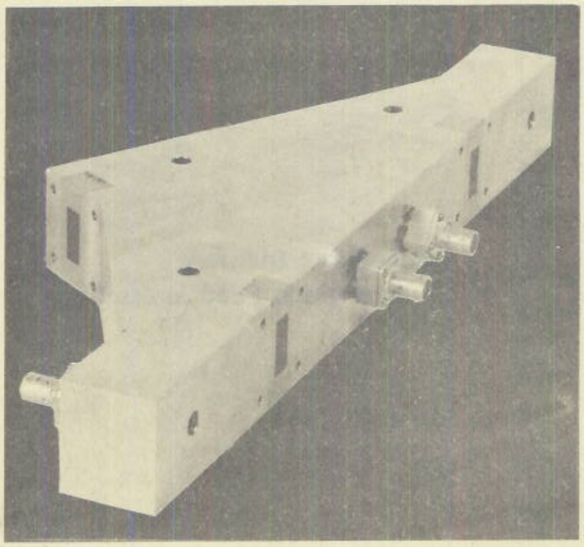


Fig. 6 - Dual Balanced Converter

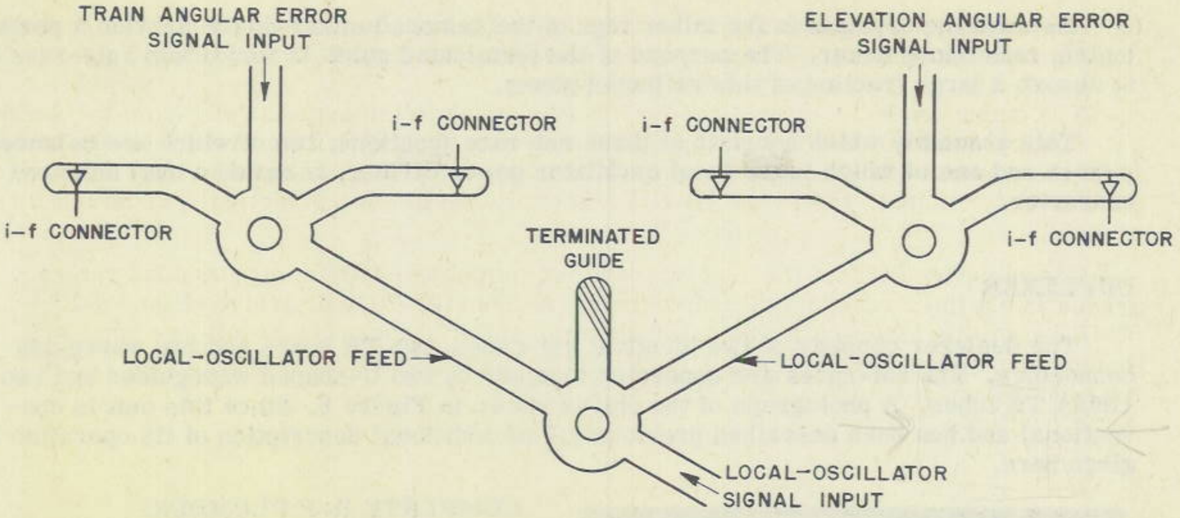


Fig. 7 - Dual Balanced Converter - Schematic Diagram

Balanced mixers are used to reduce noise signals originating in the local oscillator and to reduce coupling between the r-f waveguide inputs through the local oscillator feed connections. The operation of the balanced mixer can be best understood by considering first the train error-signal to its rat-race junction, Figure 7. This input signal divides equally, half going to each of the crystal mixer legs, which are the guides terminated with crystals. Also, the signals from the r-f input arrive at the crystals 180-degrees out of phase (i.e., in phase opposition). Essentially no r-f power will enter the fourth leg which is the local oscillator feed input, since the input signals going around the ring section of the rat-race junction arrive at this leg in phase opposition and cancel out.

Power from the local oscillator, which feeds through the fourth leg labeled local oscillator feed, enters the rat-race junction and divides equally, half going into each crystal mixer leg of the balanced mixer. The local oscillator signals which arrive at the crystals in the mixer legs have a like phase.

With the local oscillator power and the r-f error signals superimposed at the crystals and rectified, heterodyning occurs and i-f signals are produced. The original r-f signal is now divided into two i-f signals.

For the type of i-f remixing used, it was necessary to have the i-f signals from the two crystals of the balanced mixer of a like phase. Since the r-f signals arrive at the crystals in phase opposition, it is necessary to make the i-f output connections on opposite ends of the crystals. This is diagrammatically illustrated in Figure 7. In addition to providing the correct i-f phase relation for the desired signal, connecting to opposite ends of the crystals causes any noise signals originating in the local oscillator and feeding through the crystals to arrive at the i-f remixing tube in phase opposition and thereby to cancel out.

Since the operation of the rat-race balanced mixer for the elevation angular error-signal input is identical with the operation for the train angular error-signal input, no description is necessary.

To provide local oscillator power to both balanced mixers, the bottom rat-race junction is inserted and operates as a power divider. The local oscillator power is fed into the fourth leg of this rat-race junction.

Because the crystals in the mixer legs of the balanced mixer do not provide a perfect match, reflections occur. The purpose of the terminated guide in the bottom rate-race is to absorb a large fraction of this reflected power.

This assembly which consists of three rat-race junctions, two of which are balanced mixers and one of which is the local oscillator power divider, is called a dual balanced converter.

DUPLEXER

The duplexer consists of two identical rat-races, two TR boxes and two waveguide connectors. The rat-races are connected together by two U-shaped waveguides and two 1B63A TR tubes. A photograph of the unit is shown in Figure 8. Since this unit is conventional and has been described previously,§ no additional description of its operation is given here.

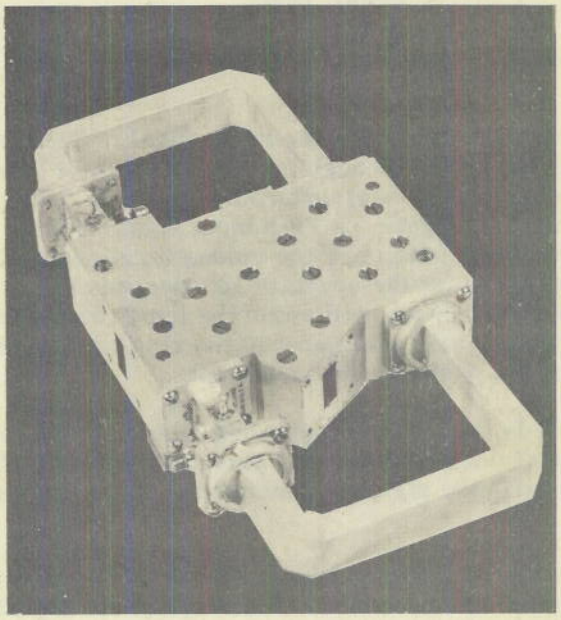


Fig. 8 - Duplexer Assembly

COMPLETE R-F PLUMBING

The complete r-f plumbing schematic is shown in Figure 9, and a photograph of the assembly is shown in Figure 10. The assembly is made up of the four-horn feed assembly, two dual balanced mixer assemblies, and a duplexer assembly with additional TR tubes, waveguide connecting sections, and the local oscillator.

The elevation error signal (A+C) - (B+D) is connected through a TR tube to one balanced mixer, and the train error signal (A+B) – (C+D) is connected through a TR tube to another balanced mixer of the dual balanced converter #5a shown in Figure 9. The range signal (A+B+C+D) is connected through the duplexer to a third balanced mixer of the dual balanced mixer #5b.

The fourth balanced mixer is used for the i-f conversion for automatic frequency control. The transmitter power is coupled out of the range reference channel by means of a directional coupler and is fed to the balanced mixer through a coaxial cable.

To provide local oscillator power to both dual balanced mixers, they are coupled to gether by rat-race #1. Again the purpose of the terminated guide in this rat-race is to absorb signals which are reflected because the dual balanced mixers are not always a perfect match.

To retain broad band characteristics, it is necessary that the phase relation of the local-oscillator signals arriving at the balanced mixers remains a constant. If this characteristic is not retained or if a differential phase shift does occur with a change in local

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§ Reed, J., "Rat-Race Duplexing," MIT Radiation Laboratory Report 885, 2 April 1946

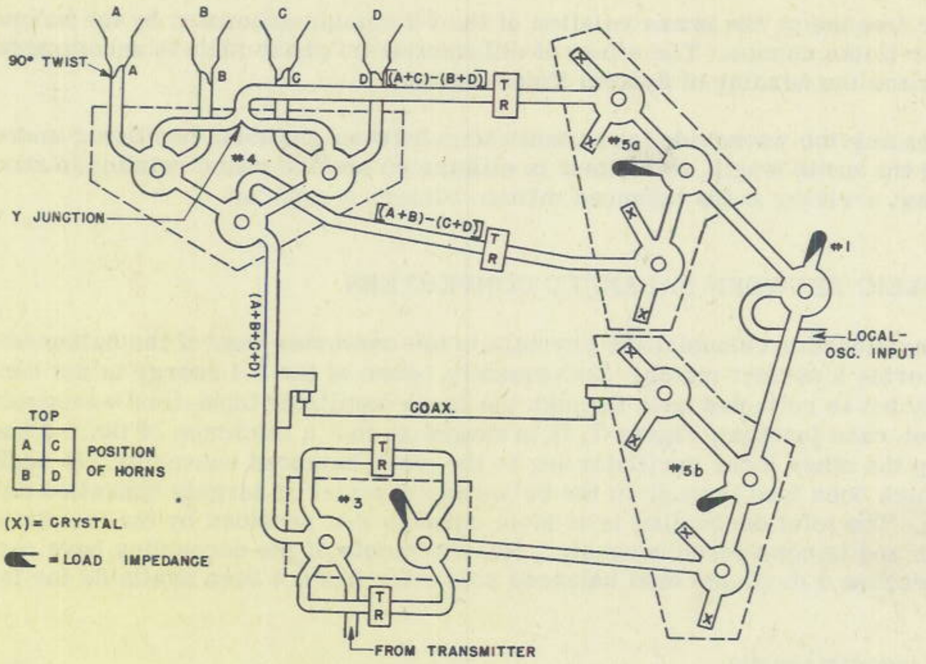

Fig. 9 - R-F Plumbing - Schematic Diagram

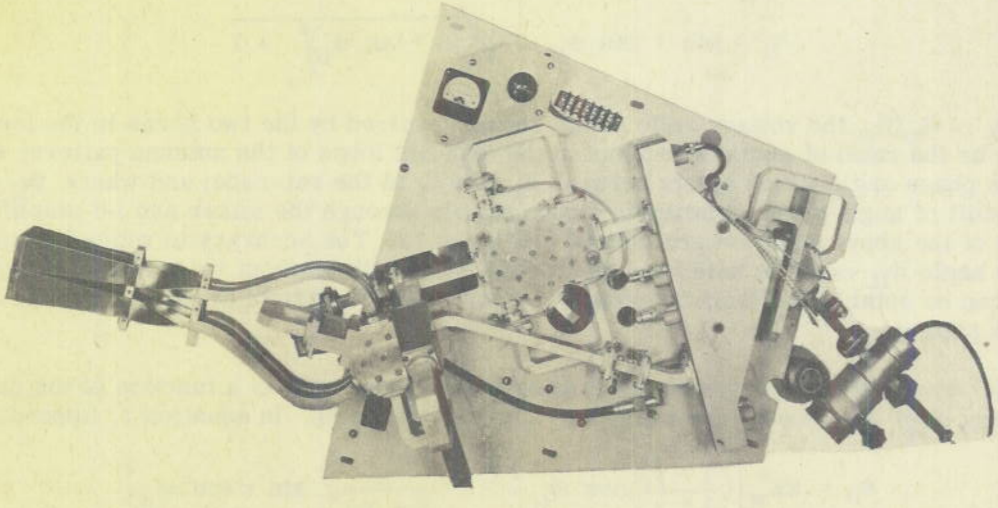

Fig. 10 - R-F Plumbing

oscillator frequency, the phase relation of the i-f signals generated by the balanced mixers will suffer a like change. The effect of differential i-f phase shift is undesirable as is explained under the heading of System Tolerances.

By making the waveguide feed connections between the local oscillator and each balanced mixer all the same length, this effect is eliminated and the phase relation of the local oscillator signal arriving at the balanced mixers remain a constant.

DECOUPLING BETWEEN BALANCED CONVERTERS

As mentioned previously, the crystals in the converter legs of the balanced converter do not provide a perfect match. Consequently, some of the r-f energy is not converted to i-f energy but is reflected back through the local-oscillator input-feed waveguide. The bottom rat-race junction, Figure 7, is arranged so that a minimum of the r-f energy is reflected up the other local oscillator leg to the other balanced converter. In addition, any signal which does feed through to the balanced converter is largely cancelled out in the i-f remixing. The total decoupling is at least equal to that provided by the isolation of three rat-races and is considered adequate. Measurements of the decoupling have not been made to date because a duplicate dual balanced converter has not been available for tests.

SYSTEM TOLERANCES

Tolerances for this system may be divided as follows: (a) boresight error as a function of r-f and i-f phase shift, (b) sensitivity as a function of i-f phase shift, and (c) r-f impedance match as it affects the transmitter.

The boresight error will be taken up first. For simplicity a two-horn feed assembly will be considered. In Appendix I it is shown that the servo will rotate the radar antenna to a position determined by the following equation (Equation 2, Appendix I):

$$y_1 = \sin\tau \tan\phi_{if} + \sqrt{\left(\sin\tau \tan\phi_{if}\right)^2 + 1}$$

where $y = E_2/E_1$, the voltage ratio of the signal received by the two horns in the feed assembly or the ratio of signal strengths in the opposed lobes of the antenna pattern; where τ is the phase angle which exists between E_1 and E_2 at the rat-race; and where ϕ_{if} is the phase shift of angle error relative to range signals through the mixer and i-f amplifiers. Graphs of the above equation are plotted in Figure 11. The accuracy to which the angle τ and the angle ϕ_{if} must be held for a particular system throughout its operating frequency range can be established from the graph as soon as the boresight error is fixed for a given antenna pattern.

The sensitivity of the system as it affects servo loop gain is a function of the differential phase shift ϕ_{if} between the range and angle i-f channels. In equation 1, Appendix I.

$$e_o = kE_R^2\left[\left(\frac{1-y_1}{1+y_1}\right)\cos\phi_{if} + 2\,\frac{y_1}{(1+y_1)^2}\,\sin\tau\sin\phi_{if}\right]$$

e_o = the video output voltage from the discriminator, and $E_R = E_1 + E_2$, which is maintained constant by the automatic-gain-control circuits of the radar. It can be seen from an inspection of this equation that the sensitivity for all practical purposes varies as the cosine of

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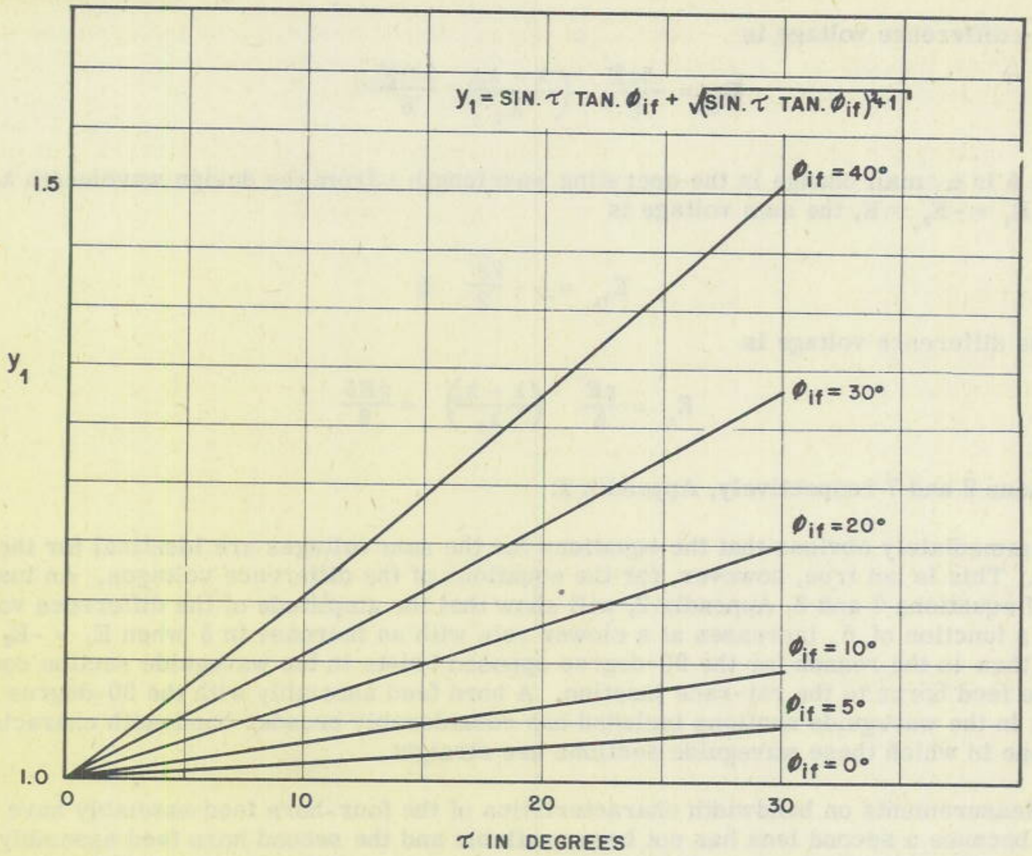

Fig. 11 - Graph of Equation 1

ϕ_{if}. The second term is insignificant from the sensitivity standpoint because both τ and ϕ_{if} are sine functions and for small angles these functions are small.

The i-f phase shift ϕ_{if} is dependent on amplifier tube and circuit characteristics. Experimental data which have been compiled on i-f amplifier strips indicate that the phase shift can be held to less than ± 30 degrees.

The r-f phase shift τ is a function of the accuracy of the initial adjustment and the operating frequency range. The method of making the initial adjustment of τ has been described in NRL Report R-3042. The effect of frequency change has not been measured.

Appendix 2 is a detailed mathematical study of the bandwidth characteristics of the type of rat-race junction used in the four-horn feed assembly. Again, for simplicity, a two-horn feed assembly is considered. The operating point under consideration is that established when $|E_1|$ is equal to $|E_2|$. There are two conditions which are to be compared. The first is when $E_1 = E_2$, the second is when $E_1 = -E_2$. Suppose $E_1 = E_2 = E$. From equations 6 and 8, Appendix 2, the sum voltage is

$$E_c = j\,\frac{\sqrt{2}}{2}\,E$$

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and the difference voltage is

$$E_b = \frac{5\pi E}{8} \left(\frac{\lambda - \lambda_0}{\lambda_0}\right) = \frac{5\pi E\delta}{8}$$

where δ is a small change in the operating wavelength λ from the design wavelength λ_0. When $E_1 = -E_2 = E$, the sum voltage is

$$E_b = j \, \frac{\sqrt{2}}{2} \, E$$

and the difference voltage is

$$E_c = \frac{\pi E}{8} \left(\frac{\lambda - \lambda_0}{\lambda_0}\right) = \frac{\pi E\delta}{8}$$

equations 9 and 7 respectively, Appendix 2.

It is immediately obvious that the equations for the sum voltages are identical for the two cases. This is not true, however, for the equations of the difference voltages. An inspection of equations 7 and 8, Appendix 2, will show that the amplitude of the difference voltage, while a function of δ, increases at a slower rate with an increase in δ when $E_1 = -E_2 = E$. This, then is the reason for the 90-degree opposed twists in the waveguide section connecting the feed horns to the rat-race junction. A horn feed assembly with the 90-degree opposed twists in the waveguide sections included has considerably broader bandwidth characteristics than one in which these waveguide sections are straight.

Measurements on bandwidth characteristics of the four-horn feed assembly have not been made because a second lens has not been available and the second horn feed assembly has not been completed. Impedance measurements, as they affect the transmitter, have been made on the assembly including the three rat-races and one Y-junction but excluding the horn feeds. These measurements indicate that a bandwidth of 9 percent is possible with a voltage standing wave ratio of less than 1.15.

I-F SIGNALS

As mentioned previously the r-f signal input to a balanced converter is divided into two in-phase i-f output signals. These two i-f signals are fed through pi-matching networks to separate grids of a dual triode. Addition of the two i-f signals is obtained by the common plate load for the dual triode. Additional i-f gain required is obtained by using 6AK5 pentode tubes with unity coupled interstage transformers.

The amplified train and elevation error signals and the range reference signal are fed to the i-f-to-video converter through coaxial cables. To provide the tube-to-cable match, pi-matching networks are used throughout. To provide the correct phase relation between the i-f angular error signals and the range reference signal, the coaxial cables between the i-f amplifiers and the i-f to video-converter were cut to the proper length.

DISCRIMINATOR

The purpose of the discriminator is to convert the angular error i-f signal to an angular error video signal. This is accomplished by using the range i-f signal as a reference in the following manner. First the angular error signal is converted into 180-degree out-of-phase voltages by means of the cathode-coupled amplifier VI, Figure 12. The two out-of-phase voltages are then fed into the back-to-back-connected diode detectors of V2. The range reference i-f signal is applied to the plate of V2a and to the cathode of V2b by means of amplifier tube V3.

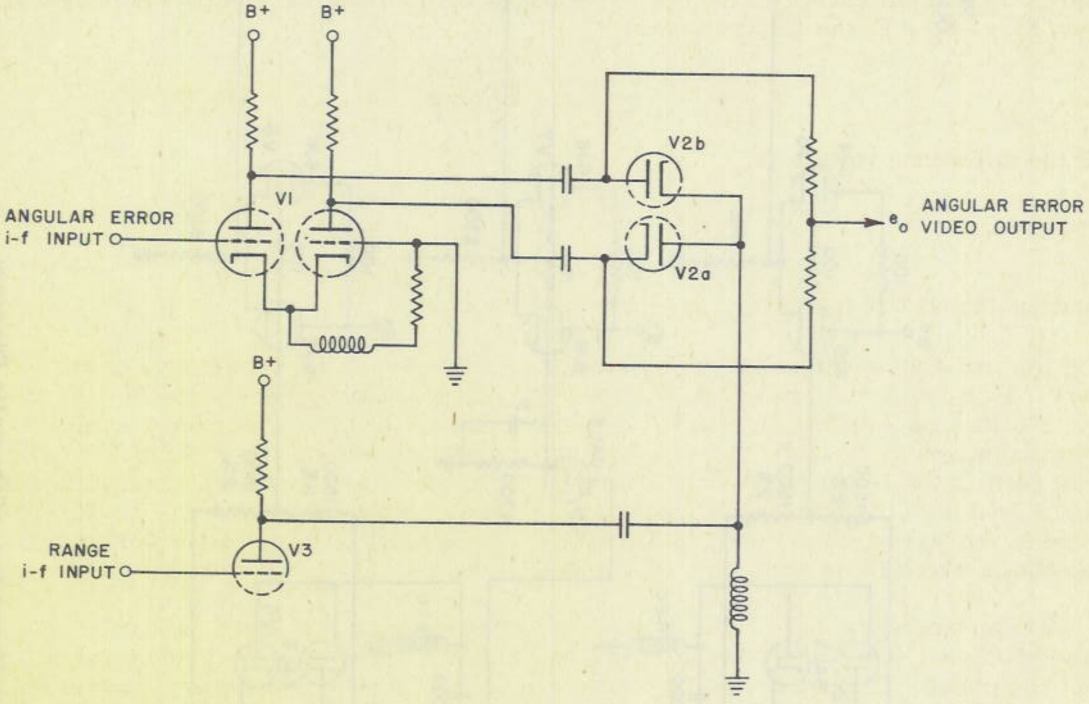

Fig. 12 - Discriminator - Schematic Diagram

When the angular error signal voltage is zero or when it is 90-degrees out of phase with the range reference signal, the output signal is zero. When this condition exists, the current drawn by the cathode of V2a during the negative half of the i-f alternating-current wave will just equal the current drawn by the plate of V2b during the positive half. Since these currents are of opposite polarity, cancellation results and the output signal is zero.

If an angular error voltage is applied which has a component in phase with the range reference signal, the output signal will no longer be zero. When this condition exists, the current drawn by the cathode of V2a during the negative half of the i-f alternating-current wave will be less than the current drawn by the plate of V2b during the positive half. This occurs because during the negative half of the alternating-current wave the cathode-to-plate voltage of V2a is less and during the positive half of the alternating-current wave the plate-to-cathode voltage of V2b is greater than it was when no error signal existed. The unbalanced current flow will produce an output voltage which is proportional to the angular error voltage.

This is true as long as the amplitude of the angular error voltages is small compared to the range reference voltage. When the component of angular error voltage and the range reference voltage are of reverse polarity, the current unbalance is in the opposite polarity. In this manner the discriminator will develop video angular error signals whose amplitude will be a function of the amplitude of the angular error signal and whose polarity will be determined by the phase relation of the angular error signal to the range or reference signal. A mathematical analysis of the operation of the discriminator is given in Appendix 3.

[redacted]

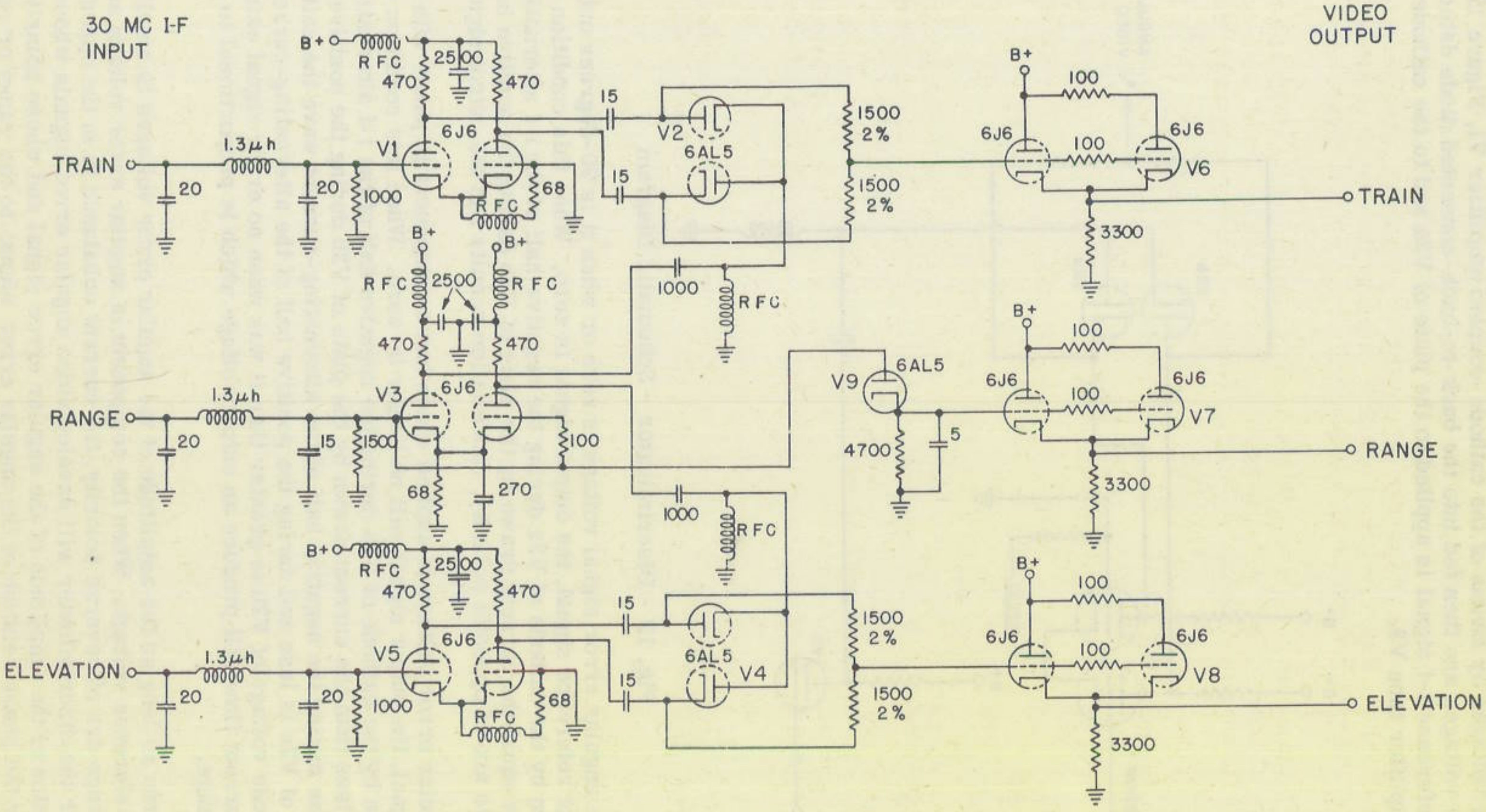

Fig. 13 - I-F-to-Video Converter - Schematic Diagram

I-F-TO-VIDEO-CONVERTER

Figure 13 is a complete circuit diagram of the i-f -to-video converter used with the system. The circuit contains two discriminator circuits and one conventional diode detector. V6, V7, and V8 are conventional cathode-follower output stages.

The conventional diode detector, V9, is used to convert the i-f range reference signal to a video range signal. This signal is applied as a deflection voltage along with angular error signals for the Pisa indicators, and as a video signal for automatic tracking of a target in range.

The video train and elevation error signals developed by the two discriminator circuits, after gating and pulse-lengthening to produce d-c error voltages, are applied to servo circuits through equalizing networks to position the antenna mount in train and elevation.

FUTURE PLANS

The present experimental amplitude-comparison simultaneous-lobing radar is to be used to conduct tracking tests. These tests are for the purpose of evaluating the system.

A narrow-beam radar system inherently has poor target-acquisition characteristics. To overcome this difficulty, a scanning mechanism which will cover a large field is necessary. The development of an acquisition scanning system is then the next logical step in the progress of TAB.

Components are also being developed to operate the system with one megawatt of transmitter power when magnetrons of this power are available.

A comparison of different types of r-f feed assemblies and a comparison of dual balanced mixers are to be made, taking into account both electrical characteristics and fabrication problems.

ACKNOWLEDGMENTS

Mr. J. E. Meade is credited with valuable aid in planning the experimental work and in carrying out the analyses. Acknowledgment is also due Mr. J. P. Spalding for his development work on the i-f circuits.

[redacted]

APPENDIX 1

Boresight Error as a Function of R-F and I-F Phase Shifts

Consider two horns receiving in one angle only. Let $e_1 = E_1 \underline{/0}$ be the voltage in one horn and $e_2 = E_2 \underline{/\tau}$ the voltage in the other. The sum e_r and the difference e_a of these voltages are formed as in Figure 14.

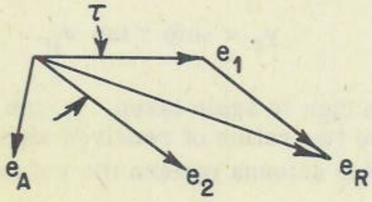

Fig. 14 - Vector Diagram

$$e_a = e_1 - e_2 = E_1 - E_2 \underline{/\tau}$$

$$e_r = e_1 + e_2 = E_1 + E_2 \underline{/\tau}$$

Let e_a alone suffer a shift of ϕ_{if} degrees through the mixer and i-f amplifier of gain unity. Then

$$\begin{aligned} e'_a &= e_a \underline{/\phi_{if}} = E_1 \underline{/\phi_{if}} - E_2 \underline{/(\tau + \phi_{if})} \\ &= E_1 \cos \phi_{if} - E_2 \cos \left(\tau + \phi_{if}\right) + j \left[E_1 \sin \phi_{if} - E_2 \sin \left(\tau + \phi_{if}\right)\right] \\ e'_r &= e_r = E_1 + E_2 \cos \tau + j E_2 \sin \tau \end{aligned}$$

The discriminator forms the scalar product (See Appendix 3), assuming a conversion factor k,

$$\begin{aligned} e_o = k\, e'_a \cdot e'_r = k &\Big[E_1^2 \cos \phi_{if} - E_1 E_2 \cos\left(\tau + \phi_{if}\right) + E_1 E_2 \cos \tau \cos \phi_{if} \\ &- E_2^2 \cos \tau \cos \left(\tau + \phi_{if}\right) + E_1 E_2 \sin \phi_{if} \sin \tau - E_2^2 \sin \tau \sin \left(\tau + \phi_{if}\right)\Big] \\ = k &\left[\left(E_1 + E_2\right)\left(E_1 - E_2\right) \cos \phi_{if} + 2 E_1 E_2 \sin \tau \sin \phi_{if}\right] \end{aligned}$$

Let $E_2/E_1 = y_1$ and $\left(E_1 + E_2\right) = E_R$. Then

$$e_o = k E_R^2 \left[\frac{1 - y_1}{1 + y_1} \cos \phi_{if} + 2 \frac{y_1}{(1 + y_1)^2} \sin \tau \sin \phi_{if}\right] \tag{1}$$

Since the radar servo rotates the antenna, varying y_1 until $e_o = 0$,

$$(1 - y_1^2) \cos \phi_{if} + 2 v_1 \sin \tau \sin \phi_{if} = 0$$

Solving for y_1,

$$y_1 = \sin \tau \tan \phi_{if} \pm \sqrt{(\sin \tau \tan \phi_{if})^2 + 1}$$

Since y is always a positive quantity, the plus sign must be taken, and

$$y_1 = \sin\tau \tan\phi_{if} + \sqrt{(\sin\tau \tan\phi_{if})^2 + 1} \tag{2}$$

This is plotted in Figure 11. If y had been defined as E_1/E_2, the result would have been

$$y_2 = -\sin\tau \tan\phi_{if} + \sqrt{\left(\sin\tau \tan\phi_{if}\right)^2 + 1}$$

Here the plus sign is again taken. y_1 can be shown to be the reciprocal of y_2. Then y_1 and y_2 are the two ratios of received signals which result from phase shifts τ and ϕ_{if}. From the radar antenna pattern the y's can be related to the boresight error.

The difference voltage has the absolute value

$$\left|e_a\right| = \left|E_1 - E_2 \cos\tau - j\,E_2 \sin\tau\right| = \frac{E_R}{(1+y_1)} \sqrt{y_1^2 - 2\,y_1 \cos\tau + 1}$$

If y_1 is varied to make $\left|e_a\right|$ a minimum, it is found that this occurs when $y_1 = 1$. Then the minimum value of $\left|e_a\right| / E_R$ obtained as the antenna is rotated is

$$\frac{\left|e_a\right|}{E_R} = \sin\frac{\tau}{2}$$

An equation approximately similar to this is plotted in db in Plate 9 of NRL Report R-3042.

APPENDIX 2

Bandwidth of 4-Guide Rat–Race **

The rat-race is a complex arrangement of multiple quarter-wave-length sections of wave guide. It can be studied more easily by replacing each section with an equivalent four-terminal network and by solving the resulting circuit by standard methods.†† The symmetrical T-section equivalent to a section of line or wave guide of impedance Z_a in which the phase shift is ϕ radians and in which there is no loss is shown in Figure 15. If l is the length of the section and λ the wavelength,

$$\phi = \frac{2\pi l}{\lambda}$$

In the present application, l is choosen so that all sections are integral multiples N of $\lambda/4$ at some wavelength λ, or

$$l = \frac{n\lambda_0}{4}$$

Then

$$\phi = \frac{n\pi\lambda_0}{2\lambda}$$

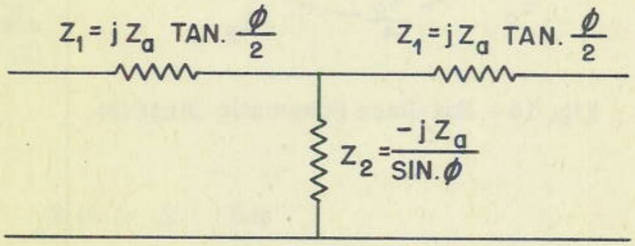

Fig. 15 - Equivalent Symmetrical T-Section

If λ is allowed to vary a small amount about λ_0, let $\lambda = \lambda_0\,(1+\delta)$, where $\delta = \frac{\lambda-\lambda_0}{\lambda_0} \ll 1$. Then

$$\phi = \frac{N\pi}{2(1+\delta)} \cong \frac{N\pi}{2} - \frac{N\pi\delta}{2}$$

If N = 1, 3, 5, etc., the impedance of the equivalent symmetrical T are

$$Z_1 = j\,Z_a \tan\left(\frac{N\pi}{4} - \frac{N\pi\delta}{4}\right) \cong j\,Z_a\left[(-1)^{\frac{N-1}{2}} - \frac{N\pi\delta}{2}\right] \qquad (1)$$

and

$$Z_2 = \frac{-j\,Z_a}{\sin\left(\frac{N\pi}{2} - \frac{N\pi\delta}{2}\right)} \cong -j\,Z_a\,(-1)^{\frac{N-1}{2}} \qquad (2)$$

If N = 2, 4, 6, etc., the section can be considered as made up of smaller half- and full-wavelength sections, leaving only the shift $N\pi\delta/2$ to be considered beside the reversal

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** This analysis follows that outlined in Chapter 10, Section 3, of BTL Technical Report, "Project NIKE," dated 15 January 1947.

†† Guillemin, E. A., "Communication Networks," Vol.1, Chap. 4. S. Wiley and Sons Inc. (1931-1935)

of terminals if N = 2, 6, 10, etc. The impedances are then

$$Z_1 = j\,Z_a \tan\left(-\frac{N\pi\delta}{4}\right) \cong j\,\frac{Z_a N\pi\delta}{4} \qquad (3)$$

and

$$Z_2 = \frac{-j\,Z_a}{\sin\left(-\frac{N\pi\delta}{2}\right)} \cong j\,\frac{2\,Z_a}{N\pi\delta} \qquad (4)$$

The 4-guide rat-race is shown in Figure 16. All wave guide taps are terminated either in a generator impedance or a load impedance Z_b. The equivalent symmetrical T for a $\lambda_0/4$ section (N = 1) has the impedance from (1) and (2)

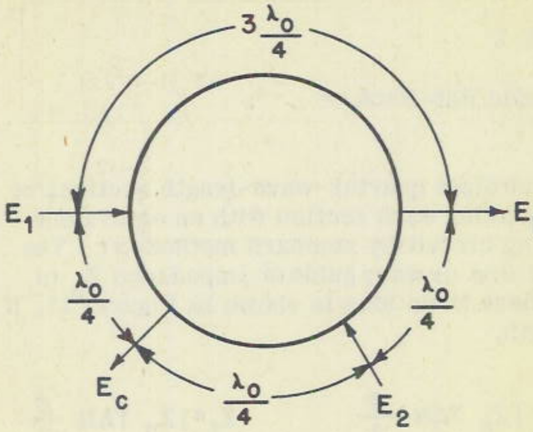

Fig. 16 - Rat-Race Schematic Diagram

$$Z_1 = j\,Z_a\left(1-\frac{\pi\delta}{2}\right)$$

and $Z_2 = -j\,Z_a$

For the $3\,\lambda_0/4$ section (N = 3), also from (1) and (2),

$$Z_1 = -j\,Z_a\left(1+\frac{\pi\delta}{2}\right)$$

and $Z_2 = j\,Z_a$

If E_1 and E_2 are the voltages from generators matched to the lines, this can be represented by series impedances Z_b with E_1 and E_2 placed in the circuit. The voltages E_b and E_c are developed across the terminating impedances Z_b. Then the equivalent network becomes that shown in Figure 17.

The network determinant after simplification is

$$D = \begin{vmatrix} \left(Z_b - j\,2\,Z_a\,\pi\delta\right) & j\,Z_a & 0 & -j\,Z_a \\ j\,Z_a & \left(Z_b - j\,Z_a\,\pi\delta\right) & j\,Z_a & 0 \\ 0 & j\,Z_a & \left(Z_b - j\,Z_a\,\pi\delta\right) & j\,Z_a \\ -j\,Z_a & 0 & j\,Z_a & \left(Z_b - j\,2\,Z_a\,\pi\delta\right) \end{vmatrix} \cong \left(Z_b^2 + 2\,Z_a^2\right)^2 - j\,9\,Z_a Z_b\,\pi\delta\left(Z_b^2 + Z_a^2\right)$$

where powers of δ higher than the first have been neglected.

The impedance at E_1 is given by the ratio of D to B_{11}, where B_{11} is the minor of D obtained by eliminating the first row and the first column. Then, if $\delta = 0$,

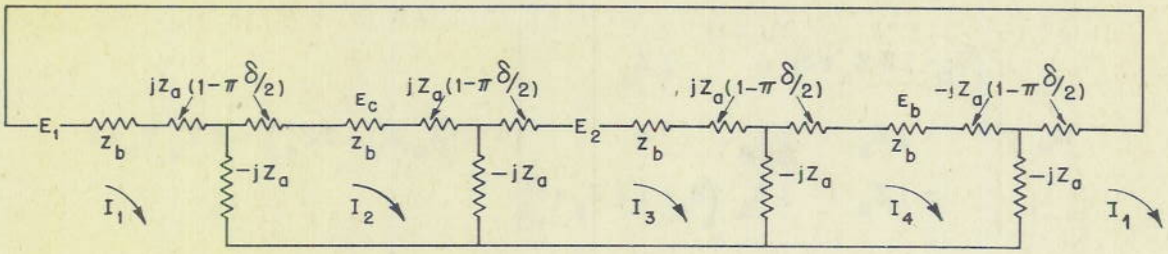

Fig. 17 - Network Equivalent to Rat-Race

$$B_{11} = \begin{vmatrix} Z_b & j\,Z_a & 0 \\ j\,Z_a & Z_b & j\,Z_a \\ 0 & j\,Z_a & Z_b \end{vmatrix} = \left(Z_b^2 + 2\,Z_a^2\right) Z_b$$

and D becomes

$$D = \left(Z_b^2 + 2\,Z_a^2\right)^2$$

so that

$$\frac{D}{B_{11}} = \frac{Z_b^2 + 2\,Z_a^2}{Z_b}$$

The input impedance at E_1 is this quantity less Z_b, since Z_b is in the generator. This difference must equal the impedance Z_b of the input wave guides for matching at the junction. Then

$$\frac{Z_b^2 + 2\,Z_a^2}{Z_b} - Z_b = Z_b \quad \text{or} \quad Z_a = \frac{Z_b}{\sqrt{2}} \tag{5}$$

which gives the relation between impedances of input wave guide and rat-race.

Again when $\delta \neq 0$, the output voltage E_c is given by

$$E_c = I_2\,Z_b = \frac{Z_b}{D}\left(B_{12}\,E_1 + B_{32}\,E_2\right)$$

From D is obtained the minors

$$B_{12} = \begin{vmatrix} j\,Z_a & j\,Z_a & 0 \\ 0 & \left(Z_b - j\,Z_a\,\pi\,\delta\right) & j\,Z_a \\ j\,Z_a & j\,Z_a & \left(Z_b - j\,2\,Z_a\,\pi\,\delta\right) \end{vmatrix} \cong j\left(Z_a\,Z_b^2 + 2\,Z_a^3\right) + 3\,Z_a^2\,Z_b\,\pi\,\delta$$

$$B_{32} = \begin{vmatrix} \left(Z_b - j\,2\,Z_a\pi\delta\right) & 0 & -j\,Z_a \\ j\,Z_a & j\,Z_a & 0 \\ -j\,Z_a & j\,Z_a & \left(Z_b - j\,2\,Z_a\pi\delta\right) \end{vmatrix} \cong j\left(Z_a Z_b^2 + 2Z_a^3\right) + 4\,Z_a^2 Z_b \pi\delta$$

where quantities involving powers of δ higher than the first are neglected. If there is matching at the junctions, so that

$$B_{12} \cong Z_a^3\,(j4 + 3\sqrt{2}\,\pi\delta)$$

$$B_{32} \cong Z_a^3\,(j4 + 4\sqrt{2}\,\pi\delta)$$

$$D \cong Z_a^4\,(16 - j\,27\sqrt{2}\,\pi\delta)$$

then

$$E_c \cong \frac{\sqrt{2}\left[\left(j4 + 3\sqrt{2}\,\pi\delta\right)E_1 + \left(j4 + 4\sqrt{2}\,\pi\delta\right)E_2\right]}{16 - j\,27\sqrt{2}\,\pi\delta}$$

If $E_2 = E_1 = E$,

$$E_c \cong j\,\frac{\sqrt{2}}{2}\,E \tag{6}$$

If $E_2 = -E_1 = E$,

$$E_c \cong \frac{\pi\delta E}{8} = \frac{\pi E}{8}\left(\frac{\lambda - \lambda_0}{\lambda_0}\right) \tag{7}$$

The output E_b is given by

$$E_b = I_4\,Z_b = \frac{Z_b}{D}\left(B_{14}\,E_1 + B_{34}\,E_2\right)$$

Then

$$B_{14} = \begin{vmatrix} j\,Z_a & \left(Z_b - j\,Z_a\pi\delta\right) & j\,Z_a \\ 0 & j\,Z_a & \left(Z_b - j\,Z_a\pi\delta\right) \\ -j\,Z_a & 0 & j\,Z_a \end{vmatrix} = -j\left(Z_a Z_b^2 + 2Z_a^3\right) + 2Z_a^2 Z_b \pi\delta$$

$$B_{34} = \begin{vmatrix} \left(Z_b - j\,2\,Z_a\pi\delta\right) & j\,Z_a & 0 \\ j\,Z_a & (Z_b - j\,Z_a\pi\delta & j\,Z_a) \\ -j\,Z_a & 0 & j\,Z_a \end{vmatrix} = j\left(Z_a Z_b^2 + 2Z_a^3\right) + 3Z_a^2 Z_b \pi\delta$$

If again $Z_b = \sqrt{2}\ Z_a$,

$$B_{14} \cong Z_a^3\ (-j4 + 2\sqrt{2}\ \pi\delta)$$

$$B_{34} \cong Z_a^3\ (j4 + 3\sqrt{2}\ \pi\delta)$$

Then

$$E_b = \frac{\sqrt{2}\ \left[\left(-j4 + 2\sqrt{2}\ \pi\delta\right) E_1 + \left(j4 + 3\sqrt{2}\ \pi\delta\right) E_2\right]}{16 - j\ 27\sqrt{2}\ \pi\delta}$$

If $E_2 = E_1 = E$,

$$E_b = \frac{5\pi\delta E}{8} = \frac{5\pi E}{8}\left(\frac{\lambda - \lambda_0}{\lambda_0}\right) \qquad (8)$$

If $E_2 = -E_1 = E$,

$$E_b = j\ \frac{\sqrt{2}}{2}\ E \qquad (9)$$

If $E_2 = E_1 = E$ and if also $\delta = 0$,

$$E_b = j\ \frac{\sqrt{2}E}{2} \quad \text{and} \quad E_c = 0$$

The power relationships are given by

$$P_{in} = \frac{2\left(\frac{E}{2}\right)^2}{Z_b} = \frac{E^2}{2Z_b}$$

and

$$P_{out} = \frac{E_b^2}{Z_b} + \frac{E_c^2}{Z_b} = \frac{E^2}{2Z_b}$$

APPENDIX 3

Analysis of the Discriminator

The i-f-to-video converter forms the sum $(e_a + e_r)$ and the difference $(e_a - e_r)$ of the error and reference voltages. These are passed through non-linear elements with characteristics assumed as a power series and subtracted. The difference is then

$$e_o = \sum_{n=o}^{\infty} a_n \left[\left(e_a + e_r\right)^n - \left(e_a - e_r\right)^n \right]$$

where the $\underline{a}$'s are the coefficients of the power series, assumed identical for each non-linear element. Writing the binomial expansions as summations,

$$e_o = \sum_{n=0}^{\infty} a_n \sum_{k=0}^{n} \left[\frac{n!}{k!(n-k)!} e_a^{n-k} e_r^{k} - \frac{n!(-1)^k}{k!(n-k)!} e_a^{n-k} e_r^{k} \right]$$

$$= \sum_{n=1}^{\infty} a_n \sum_{k=1}^{n} \frac{2n!}{k!(n-k)!} e_a^{n-k} e_r^{k}$$

(k odd)

Let

$$e_a = E_a \sin (m\omega\tau + \phi) = E_a \sin x = -\frac{iE_a}{2} \left(e^{ix} - e^{-ix}\right)$$

$$e_r = E_r \sin \omega\tau = E_r \sin y = -\frac{iE_r}{2} \left(e^{iy} - e^{-iy}\right)$$

If these e's are substituted, and if the output $\underline{e_o}$ is passed through a low pass filter which removes all a-c components, the remaining d-c is obtained after some manipulation as

$$e'_o = 4 \sum_{n=2}^{\infty} \frac{n!}{2^n} a^n \sum_{k=1}^{n} E_a^{n-k} E_r^{k} \sum_{p=1}^{n-k} \frac{\cos m p \phi}{\left(\frac{n-k+p}{2}\right)! \left(\frac{n-k-p}{2}\right)!} \sum_{q=mp}^{k} \frac{1}{\left(\frac{k+q}{2}\right)! \left(\frac{k-q}{2}\right)!}$$

($\underline{n}$ even; k, p, q odd; $\underline{mp} \leqq k$)

If this is expanded for m = 1 (error and reference frequencies equal),

$$e_o' = \Big[2a_2 E_a E_r + 3a_4 \left(E_a^3 E_r + E_a E_r^3\right) + a_6\left(\frac{5}{2} E_a^5 E_r + \frac{15}{2} E_a^3 E_r^3 + \frac{5}{2} E_a E_r^5\right)$$

$$+ a_8\left(\frac{7}{8} E_a^7 E_r + \frac{21}{4} E_a^5 E_r^3 + \frac{21}{4} E_a^3 E_r^5 + \frac{7}{8} E_a E_r^7\right) + \text{------}\Big] \cos\phi$$

$$+ \Big[\frac{5}{6} a_6 E_a E_r^3 + a_8 \left(\frac{7}{8} E_a^5 E_r^3 + \frac{7}{8} E_a^3 E_r^5\right) + \text{----}\Big] \cos 3\phi + \text{----}$$

The a's ordinarily decrease rapidly with n, so that the expression can be approximated by

$$e_o' \cong 2a_2 E_a E_r \cos\phi$$

Then the output is proportional to the rate of change of slope of the input-output voltage characteristic of the nonlinear elements, to the amplitudes of error and reference voltages, and to the cosine of the phase angle between these voltages. In vector notation this is equivalent to a scalar product

$$e_o' \cong 2a_2 \widetilde{E}_a \cdot \widetilde{E}_r$$

where the E's are now in complex form as in $e_a = \widetilde{E}_a \sin \omega\tau$; $e_r = \widetilde{E}_r \sin \omega\tau$.

There is no output if the frequency of the error voltage is an even multiple of that of the reference voltage (m even). The output for odd values of m can be calculated from the general expression for e_o', and for m = 3, this is

$$e_o' \cong a_4 E_a E_r^3 \cos 3\phi$$

which is relatively small compared to the output for m = 1.

* * *