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RESPONSE OF AN AIRFRAME TO SINUSODIAL WING FLAP DEFLECTION

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RESPONSE OF AN AIRFRAME TO SINUSODIAL WING FLAP DEFLECTION

James W. Titus

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March 5, 1948

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Approved by:

Mr. Peter Waterman, Head, Equipment Research Section
Dr. R. M. Page, Superintendent, Radio Division III

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ABSTRACT

The frequency response of an airframe to wing flap deflections is derived analytically from the simplified equations of motion of the airframe in a plane. The response curves are put into a form convenient for their use in the overall design of an automatic aircraft control system. The effects upon the frequency response of changing the aerodynamic and physical parameters are considered briefly.

PROBLEM STATUS

This report represents completion of one study under problem RO5-16.

AUTHORIZATION

NRL Problem No. R05-16 (BuAer A-156).

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RESPONSE OF AN AIRFRAME TO SINUSOIDAL WING FLAP DEFLECTION

INTRODUCTION

In general, the frequency response of a mechanism is the ratio of the amplitude of some "output" quantity, or dependent variable, to the amplitude of a sinusoidal "input", or independent variable, expressed as a function of the frequency of the "input". In the case of an airframe which may roll, pitch, yaw, and may be controlled by wing flaps, ailerons and tail flaps, a sinusoidal motion of some control surface would normally be taken as the input quantity, while one of the resulting maneuvers would be considered as the output. Thus, frequency responses of interest could be the response in airframe rotation to wing flap deflections, or the response in airframe flight path angle to tail flap deflections.

A knowledge of certain frequency response characteristics of an airframe is prerequisite to an analytical determination of the proper equalization required to give stable auto-pilot control.* The Equipment Research Section of Radio Division III, NRL, has studied the frequency response of a missile airframe to sinusoidal displacements of its wing flaps. The missile considered has horizontal and vertical wings and a cruciform tail. Previous results obtained by the Reeves Instrument Corporation, using the D. C. analyzer built for Project Cyclone were reported in the Lark-Wasp Missile Seminar of April 1947.† Earlier works by Harmon‡ and Greenberg§ of the National Advisory Committee for Aeronautics determine the dynamic stability of airplanes with automatic controls, using frequency response methods, without isolating the airframe characteristics.

In this report, the simplified equations of motion of the airplane are solved analytically to obtain general expressions for the frequency response of airframe rotation in pitch to wing flap control, and the frequency response of airframe flight path direction to wing flap control.

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* Gaylord, Russell E., "Development of an Aircraft Control System," NRL Report R-3108, Appendix II of Lark-Wasp Guided Missile Seminar, April 1947

† Waterman, Peter and Titus, James, W., "KAQ-1 Lark Airframe Frequency Response," NRL Report R-3108, Appendix I of Lark-Wasp Guided Missile Seminar, April 1947

‡ Harmon, S., "Aerodynamic Frequency Response Curves and Automatic Dynamic Stability Boundaries for the Navy Lark Missile," MR-L5J05A, NACA TED 2380, Confidential, 29 October, 1945

§ Greenburg, Harry, "Frequency Response Method for Determination of Dynamic Stability Characteristics of Airplanes with Automatic Controls," NACA Tech. Note 1229, March, 1947

SYMBOLS

The symbols used herein are as follows: Standard NACA terminology is followed, where possible.

a = linear acceleration ft sec^{-2}

c = wing chord, ft

C_L = coefficient of lift, $C_L = \frac{L}{qS}$

$C_{L\alpha} = \frac{d\,C_L}{d\,\alpha}$

$C_{L\delta} = \frac{d\,C_L}{d\,\delta_F}$

C_m = coefficient of pitching moment $C_m = \frac{\text{(pitching moment)}}{qSc}$

$C_{m\alpha} = \frac{d\,C_m}{d\alpha}$

$C_{m\delta} = \frac{d\,C_m}{d\,\delta_F}$

$C_{mq} = \frac{d\,C_m}{d\text{ (angular velocity in pitch)}}$

f = frequency

I_y = moment of inertia about pitch axis

$j = \sqrt{-1}$

K = radius of gyration, ft

L = lift, #.

m = mass, # sec^2 ft^{-1}

M = Mach number

$p = \frac{d}{dt}$ or $j\omega$

q = dynamic pressure # ft^{-2}

S = wing area ft^2

[redacted]

V = velocity ft sec^{-1}

α = angle of attack, radians

δ_F = angle of wing flap, radians

δ_T = angle of tail flap, radians

γ = flight path angle, radians

ρ = density of air # sec^2 ft^{-4}

θ = angle of pitch, radians

ω = angular velocity of input signal, radians sec^{-1}

ASSUMPTIONS AND BASIC DATA

The cruciform type of airframe under consideration is shown on Figure 1. It may be noted that the vertical wing section is a symmetrical airfoil NACA 16-009, while the horizontal wing section is an unsymmetrical airfoil NACA 16-209. Control in pitch and yaw is obtained with full span wing flaps. Tail flaps are provided for an angle of attack control. The following analysis assumes the tail flaps fixed, (i.e., no angle of attack control). The missile is automatically stabilized in roll by a servo system which operates small extensible airfoils on the vertical wing tips. It is assumed that this system holds the angle of roll to values sufficiently small that interaction between roll and other motions can be neglected. Because of the symmetrical construction of the missile it should be noted that, for small displacements, the control characteristics for pitch should be almost identical with those for yaw. The airframe is assumed to have a constant forward velocity. The components of gravitational force acting along the longitudinal axis are neglected.

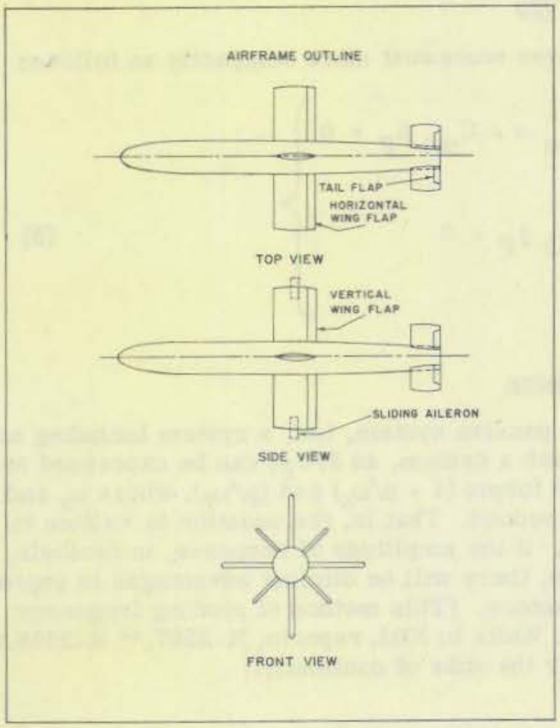

Fig. 1 - Airframe Outline

RESPONSE OF AIRFRAME ATTITUDE TO WING FLAP DEFLECTION

Angles are measured as indicated in Figure 2.

With the assumptions above, simplified equations of motion for the airframe can be written. The summation of moments about an axis in the plane gives:

$$-p^2 \theta I + p\theta C_{mq} qSc + C_{m\alpha} \alpha qSc + C_{m\delta} \delta_F qSc = 0 \qquad (1)$$

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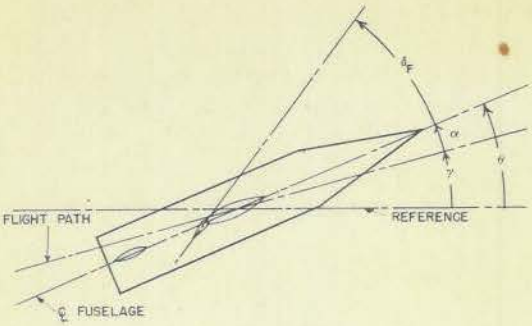

Fig. 2 - Angle Rotation

The summation of forces perpendicular to the flight path axis gives:

$$ma - C_{L\alpha}\,\alpha\, qS - C_{L\delta}\,\delta_F\, qS = 0 \qquad (2)$$

By definition,

$$\alpha = \theta - \gamma \qquad (3)$$

But $\quad a = \frac{V^2}{r}\,;\ \frac{V}{r} = p\,\gamma$

Then ma = mV pγ and (2) becomes,

$$mV\, p\gamma - C_{L\alpha}\,\alpha qS - C_{L\delta}\,\delta_F\, qS = 0 \qquad (4)$$

$$\text{Let} \qquad \omega_0^2 = \frac{qSc}{I}\,;$$

$$\omega_1 = \frac{1}{C_{mq}}$$

$$\omega_2 = \frac{qS}{mv}$$

Then equations (1), (3), and (4) can be written somewhat more compactly as follows:

$$\left.\begin{aligned} -\frac{p^2}{\omega_0^2}\,\theta + \frac{p}{\omega_1}\,\theta + C_{m\alpha}\,\alpha + C_{m\delta}\,\delta_F &= 0 \\ \frac{p}{\omega_2}\,\gamma - C_{L\alpha}\,\alpha - C_{L\delta}\,\delta_F &= 0 \\ \alpha &= \theta\ \gamma \end{aligned}\right\} \qquad (5)$$

METHOD OF PLOTTING FREQUENCY RESPONSE

These equations (5) treat the airframe as a passive system, i.e., a system including no sources of energy. A frequency response of such a system, as θ/δ_F, can be expressed as the product of a constant and several factors of the forms $(1 + p/\omega_x)$ and (p/ω_y), where ω_x and ω_y are constants with the dimensions radians per second. That is, the equation is written in factored form, where $(-\omega_x)$ and $(-\omega_y)$ are roots. If the amplitude of response, in decibels, be plotted versus frequency on a logarithmic scale, there will be distinct advantages in expressing the frequency response in terms of these factors. (This method of plotting frequency response curves is discussed in detail by C. F. White in NRL reports, R-2587,** R-2668,†† and R-3167.‡‡ A brief summary is given here for the sake of continuity.)

**White, C. F., " Simplified Analysis of R-C and R-L Networks," NRL Report R-2668, 15 October, 1945.

††White, C. F., "Resistance-Capacitance Low-and High-Pass Filters," NRL Report R-2587, 22 February, 1945.

‡‡White, C. F., " Transfer Characteristics of a Bridged Parallel-T Network," NRL Report R-3167, September, 1947.

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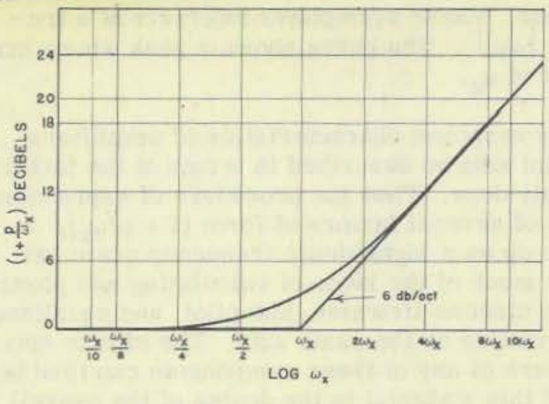

Fig. 3 Logarithmic Plot of the Function $(1 + p/\omega_x)$

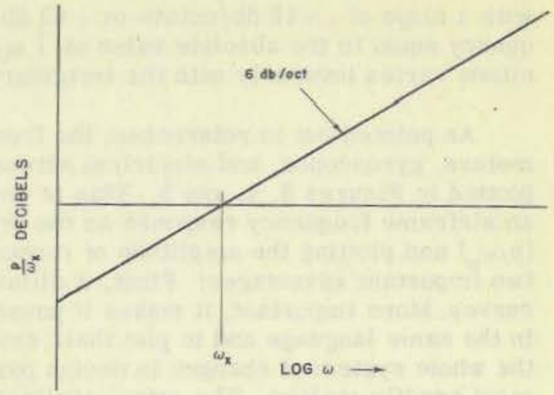

Fig. 4 Logarithmic Plot of the Function (p/ω_x)

Let $A = \left|(1 + p/\omega_x)\right|$ where $\left|Q\right|$ denotes "the absolute value of Q," and ω_x is positive real. Then $A_{db} = 20 \log \left|(1 + p/\omega_x)\right|$.

Where A_{db} is read: "A, expressed in decibels".

This expression is plotted versus log frequency on Figure 3. It can be seen that the curve is asymptotic to straight lines at high and at low frequencies. These asymptotes intersect at frequency ω_x. The low frequency asymptote has zero slope. Let an octave be defined as a frequency interval of two to one, and a decade be defined as a frequency interval of ten to one, that is, one cycle of graduations on a sheet of logarithmic graph paper. Then the slope of the high frequency asymptote may be described as + 6 db per octave or + 20 db per decade.

Let $$A = \left|\frac{p}{\omega_y}\right|$$

Then $$A_{db} = 20 \log (p/\omega_y)$$

This expression is plotted versus logarithmic frequency on Figure 4. On these axes the expression is a straight line with a slope of + 6 db/octave.

Let $A = \left|(1 + p/\omega_u)(1 + p/\omega_v)\right|$ where the constants, ω_u and ω_v, are a complex conjugate pair with positive real parts (corresponding to complex conjugate roots with negative real parts).

This expression is plotted versus logarithmic frequency on Figure 5.

It will be noticed that at low frequencies the curve is asymptotic to a straight line with zero slope, while at high frequencies the curve is asymptotic to a straight line

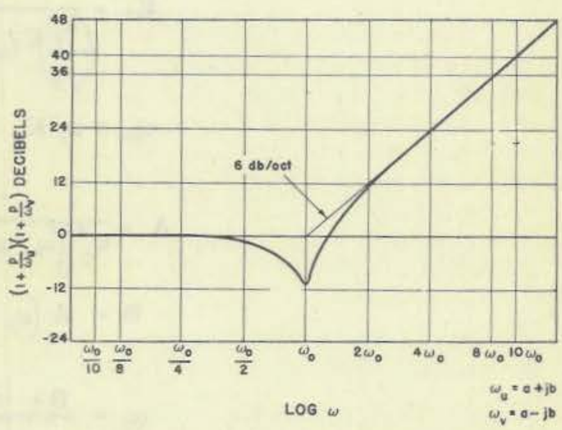

Fig. 5 Logarithmic Plot of the Function $(1 + p/\omega_u)(1 + p/\omega_v)$

with a slope of +12 db/octave or +40 db/decade. These asymptotes intersect at a frequency equal to the absolute value of $|\omega_u| = |\omega_v|$. The curve shows a peak whose magnitude varies inversely with the imaginary part of ω_u.

As pointed out in references, the frequency response characteristics of amplifiers, motors, gyroscopes, and electrical networks can also be described in terms of the factors plotted in Figures 3, 4, and 5. This is commonly done. Thus the procedure of expressing an airframe frequency response as the product of several factors of form $(1 + p/\omega_x)$, (p/ω_y) and plotting the amplitude of response in db on a logarithmic frequency scale has two important advantages: First, it eliminates much of the labor of calculating and plotting curves. More important, it makes it possible to discuss airframe, autopilot, and equalizer in the same language and to plot their characteristics on the same axis. The effects upon the whole system of changes in design parameters of any of these components can then be most readily studied. The actual application of this material to the design of the overall system, however, is beyond the scope of this report.

Equation (5) is solved simultaneously for the ratio θ/δ_F.

Then
$$\frac{\theta}{\delta_F} = \frac{K_2(1 + p/\omega_3)}{\frac{p}{\omega_2}(p^2 A + pB + 1)} \qquad (6a)$$

or the alternate form:
$$\frac{\theta}{\delta_F} = \frac{(1 + p/\omega_3)}{\frac{p}{\omega_{10}}(1 + p/\omega_4)(1 + p/\omega_5)} \qquad (6b)$$

where
$$\omega_3 = -\frac{\omega_2 K_1}{C_{m\delta}}$$

$$K_1 = C_{m\alpha} C_{L\delta} - C_{m\delta} C_{L\alpha}$$

$$K_2 = \frac{K_1}{\left(\frac{\omega_2}{\omega_1}\right) C_{L\alpha} + C_{m\alpha}}$$

$$\omega_{10} = \omega_2 K_2$$

$$A = \frac{-1}{\omega_0^2\left[\left(\omega_2/\omega_1\right) C_{L\alpha} + C_{m\alpha}\right]}$$

$$B = A\left(\omega_2 C_{L\alpha} - \omega_0^2/\omega_1\right)$$

$$\omega_4 = \frac{B + \sqrt{B^2 - 4A}}{2A} = \frac{B}{2A} + \sqrt{\frac{B^2}{4A^2} - \frac{1}{A}}$$

$$\omega_5 = \frac{B - \sqrt{B^2 - 4A}}{2A} = \frac{B}{2A} - \sqrt{\frac{B^2}{4A^2} - \frac{1}{A}}$$

It will be noted that the factors of equation (6) are all of the forms: $(1 + p/\omega_x)$ and (p/ω_x). Since the frequency response curve corresponding to each of these factors is known, the frequency response characteristic of equation (6) can be readily sketched on semilog paper by first sketching the curve corresponding to each of the well known factors, and then adding these curves graphically. (The two factors in the denominator are a complex conjugate pair, and are conveniently plotted together, as discussed in paragraph 10).

Equation (6) then has the form shown in Figure 6. Let $\omega_6 = \sqrt{1/A}$

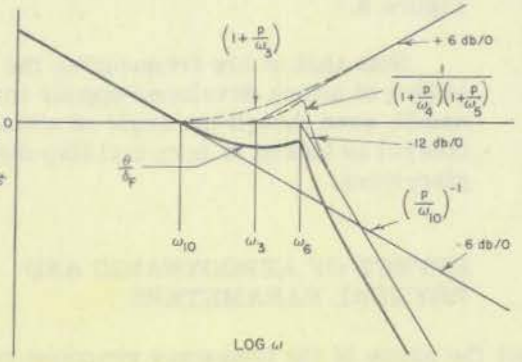

Fig. 6 - Frequency Response, Airframe Rotation

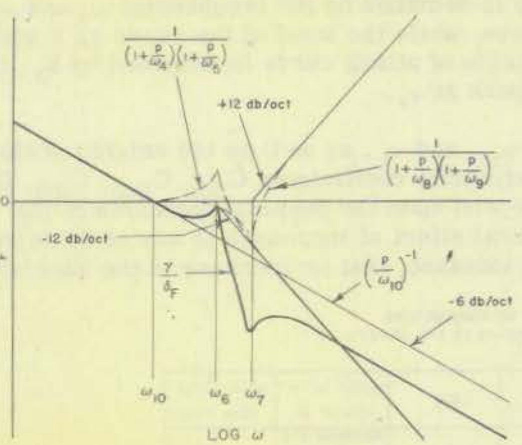

Fig. 7 - Frequency Response, Flight Path Angle

RESPONSE OF FLIGHT PATH ANGLE TO WING FLAP DEFLECTION

Equations (1), (3), and (4), are solved simultaneously for γ/δ_F. The result can be put into the form:

$$\frac{\gamma}{\delta_F} = K_2 \frac{(p^2 C + pD + 1)}{\frac{p}{\omega_2}(p^2 A + pB + 1)} \qquad (7)$$

or

$$\frac{\gamma}{\delta_F} = \frac{(1 + p/\omega_8)(1 + p/\omega_9)}{\frac{p}{\omega_{10}}(1 + p/\omega_4)(1 + p/\omega_5)} \qquad (8)$$

where

$$C = -\frac{C_{L\delta}}{\omega_0^2 K_1}$$

$$D = \frac{C_{L\delta}}{\omega_1 K_1}$$

$$\omega_8 = \frac{D}{2C} + \sqrt{\frac{D^2}{4C^2} - \frac{1}{C}}$$

$$\omega_9 = \frac{D}{2C} - \sqrt{\frac{D^2}{4C^2} - \frac{1}{C}}$$

Equation (8) has the form shown on Figure 7, where $\omega_7 = \sqrt{1/C}$

RESPONSE OF ANGLE OF ATTACK TO WING FLAP DEFLECTION

Equations (1), (3), and (4) can also be solved for α/δ_F. The result can be put into the form:

$$\frac{\alpha}{\delta_F} = K_3 \frac{(1 + p/\omega_{11})}{p^2 A + pB + 1} \qquad (9)$$

or

$$\frac{\alpha}{\delta_F} = K_3 \frac{(1 + p/\omega_{11})}{(1 + p/\omega_4)(1 + p/\omega_5)} \qquad (10)$$

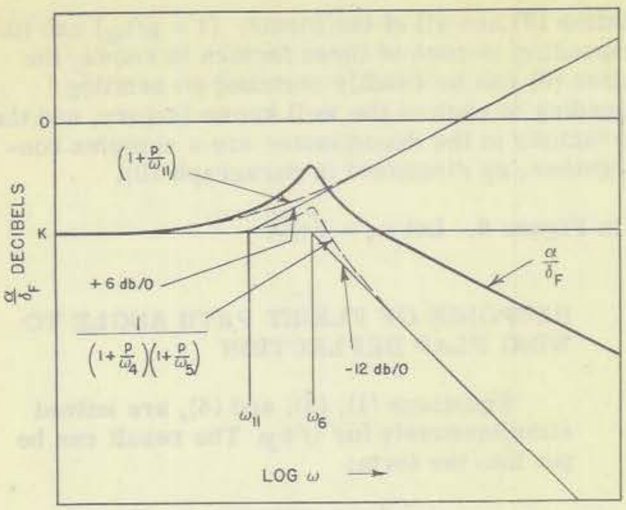

Fig. 8 - Frequency Response, Angle of Attack

where

$$K_3 = -\frac{C_{L\delta}\left(\omega_2/\omega_1\right) + C_{m\delta}}{C_{m\alpha} + C_{L\alpha}\left(\omega_2/\omega_1\right)}$$

$$\omega_{11} = -\omega_0^2\left[C_{mq} + \left(C_{m\delta}/\omega_2\; C_{L\delta}\right)\right]$$

Equation (10) has the form shown on Figure 8.

Note that at low frequencies the angles of attack developed appear to be small, even though the angle of attack control is locked at zero tail flap displacement.

EFFECT OF AERODYNAMIC AND PHYSICAL PARAMETERS

It can be seen from the paragraphs above that the shape of the frequency response curve for airframe rotation can be described by the frequencies ω_3 and ω_6 and by the height of the peak at ω_6; while the level of the curve as a whole is determined by ω_{10}. Also, the shape of the curve of flight path angle frequency response is decribed by the frequencies ω_6 and ω_7, and by the heights of the peaks at these frequencies, while the level of the curve as a whole is determined by ω_{10}. Finally, the shape of the angle of attack curve is decribed by K_3, the frequencies ω_{11}, and ω_6, and by the height of the peak at ω_6.

The five significant frequencies, ω_3, ω_6, ω_7, ω_{10}, and ω_{11}, as well as the heights of the peaks have been expressed in terms of five aerodynamic coefficients $C_{m\delta}$, $C_{m\alpha}$, C_{mq}, $C_{L\delta}$, and $C_{L\alpha}$, and two parameters ω_0 and ω_2 which depend upon the physical constants of the missile, its speed, and the air density. The general effect of increases in any of these parameters are given in Table 1. It can be seen, for instance, that an increase in the absolute

APPROXIMATE EFFECT OF PARAMETERS
NUMBER IN BLOCK INDICATES POWER OF THE EFFECT
"0" INDICATES NO EFFECT

INCREASE IN MAGNITUDE OF PARAMETER / EFFECT OF INCREASE ON	ω_{10}	ω_3	ω_6	ω_7	HEIGHT OF θ PEAK AT ω_6	HEIGHT OF γ PEAK AT ω_7
ω_0	0	0	+1	+1	MAXIMUM FOR $\omega_0 = \left(\frac{\omega_2 C_{L\alpha}}{C_{mq}}\right)^{\frac{1}{2}}$	-1
ω_2	+1	+1	$+\frac{1}{2}$	0	-1	0
C_{mq}	-1	0	$+\frac{1}{2}$	0	-1	-1
$C_{m\alpha}$	SMALL EFFECT -1	+1	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	+1
$C_{m\delta}$	+1	-1	0	$+\frac{1}{2}$	0	+1
$C_{L\alpha}$	SMALL EFFECT +1	+1	$+\frac{1}{2}$	$+\frac{1}{2}$	-1	+1
$C_{L\delta}$	+1	+1	0	$-\frac{1}{2}$	0	INCREASE FOR $C_{L\delta} > \frac{C_{m\delta} C_{L\alpha}}{C_{m\alpha}}$

Table I

magnitude of $C_{m\delta}$ would increase ω_{10} and the height of the depression in the γ curve at ω_7 in proportion to the + 1 power of $C_{m\delta}$, while ω_3 would change with the -1 power. ω_6 and the height of the peak in the θ curve occurring at ω_6 would be unaffected, while ω_7 would increase with the + 1/2 power.

FLIGHT CONDITIONS

The air temperature and density, and the speed of a missile affect the frequency response through the parameters ω_0 and ω_2. Repeating the definitions of these quantities here for convenience, and rearranging:

$$\omega_0 = \sqrt{\frac{qSc}{I}} = \sqrt{\frac{\rho V^2 Sc}{2I}} = V\sqrt{\frac{\rho Sc}{2I}}$$

$$\omega_2 = \frac{qS}{mV} = \frac{\rho V^2 S}{2mV} = \frac{\rho V S}{2m}$$

Thus both ω_0 and ω_2 are directly proportional to V. ω_0 increases as the square root of ρ, while ω_2 increases directly with ρ.

Since the mach number of a body traveling in air of a given temperature is directly proportional to the velocity, this quantity could affect the frequency response through any aerodynamic coefficients which are functions of mach number. The wind tunnel data available for the KAQ-1 Lark shows the effect of mach number upon $C_{L\alpha}$, $C_{L\delta}$, and $C_{m\delta}$ to be negligible. At small flap deflections, $C_{m\alpha}$ has only a small increase with mach number; however, the magnitude of C_{mq} decreases appreciably with mach number. (22 percent variation for $0.59 < M < 0.86$). In the rough inspection of the general trends, these effects will be neglected. It can then be said that ω_3, ω_7, and ω_{10} increase linearly with speed, ω_6 increases with the 3/2 power of speed, while the amplitude of the minimum occuring in the γ/δ_F curve at ω_7 is proportional to the reciprocal of the speed. In the range of speeds of the Lark, the height of the peak in the θ/δ_F curve decreases with increasing speed.

For a given speed, the mach number is proportional to the square root of the ambient air temperature. Thus the temperature of the air has a slight effect upon the frequency response of the airplane, through the aerodynamic coefficients $C_{m\alpha}$ and C_{mq} which vary with mach number.

EFFECT OF VARIATION OF MASS

Since the available data show the radius of gyration, K, to have little change with the mass, m, of the airplane, it will be considered that the moment of inertia, I, varies directly with the mass. It can then be seen that ω_0 varies with $m^{-1/2}$, while ω_2 varies with m^{-1}. Accordingly, the frequencies ω_3, ω_6, ω_7, ω_{10}, are all decreased with an increase in mass, while the heights of the peaks are increased.

FREQUENCY RESPONSE CURVES

Figure 9 gives curves of frequency response of a missile in airframe rotation (θ/δ_F) for three different conditions of flight. Curve A corresponds to parameters selected to

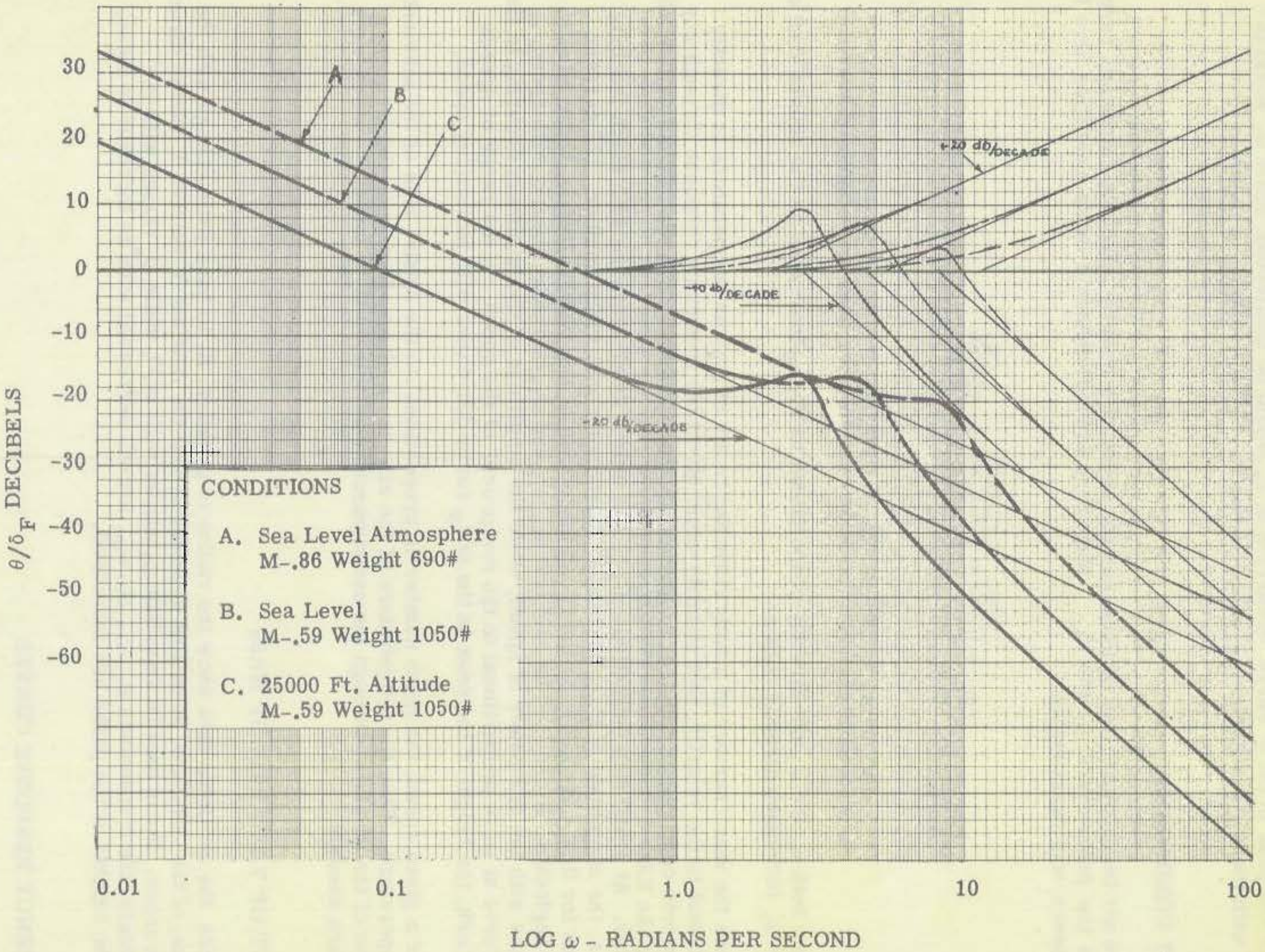

Fig. 9 - Frequency Response, Airframe Rotation

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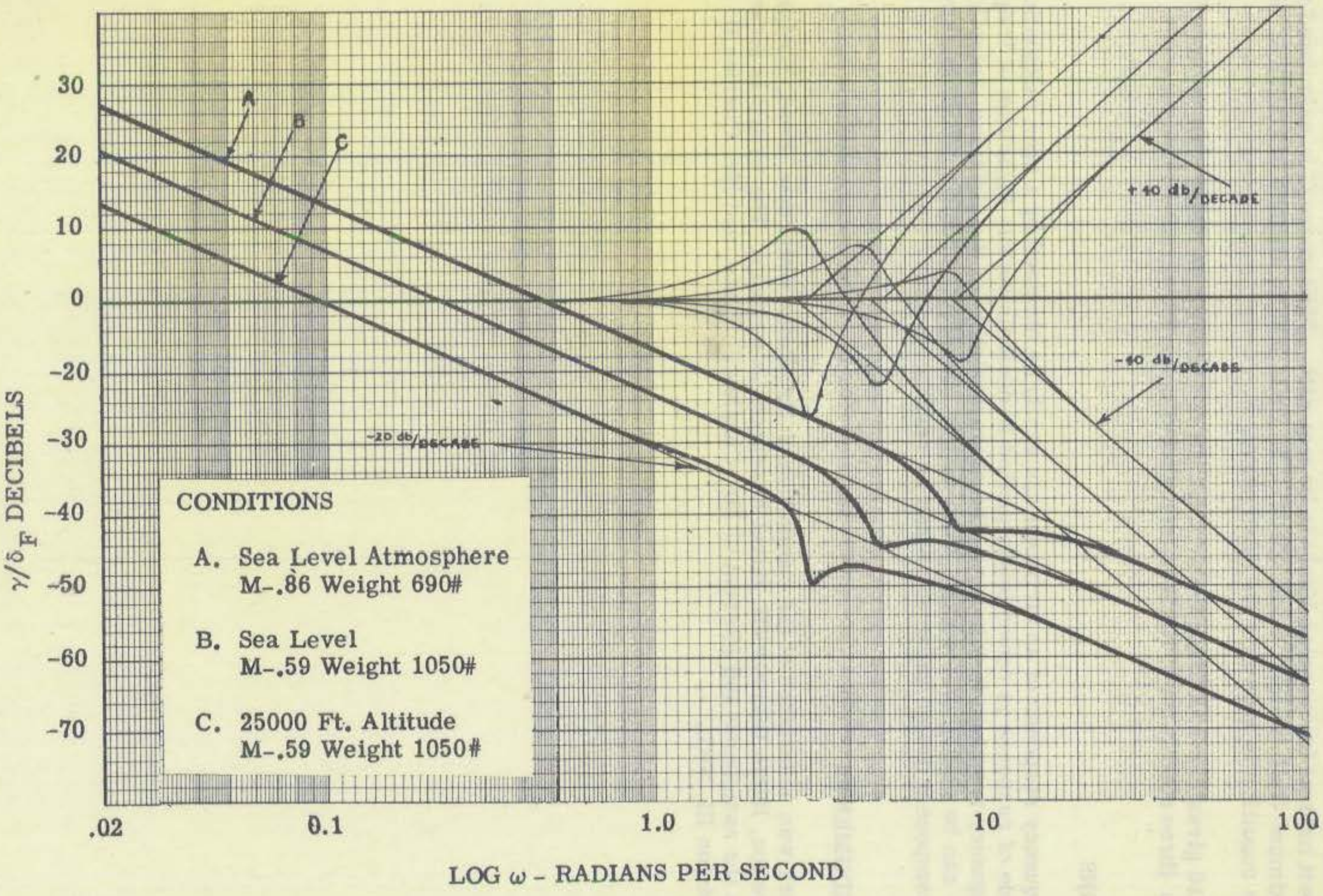

Fig. 10 - Frequency Response, Flight Path Angle

give the highest natural frequencies likely to be met with. Thus minimum weight, and maximum speed and sea level air density are assumed. Curve C has parameters selected to give the lowest natural frequencies likely to be met with. Thus curve C is calculated on the basis of maximum (full) weight, a low speed of M = 0.5, and an altitude of 25,000 ft. An intermediate condition is shown on the third curve, B.

Figure 10 gives curves of frequency response of a missile in flight path angle (γ/δ_F) for the same three conditions of flight selected for Figure 9.

CONCLUSIONS

The frequency response of an airframe can easily be obtained by a hand calculation. The amplitude of response is conveniently plotted in decibels versus logarithm of frequency. With the frequency response in this form, the characteristics of the airframe, autopilot, and equalization can be plotted on the same axis, so that the effects upon the whole system of changes in component design parameters can be readily studied.

ACKNOWLEDGMENTS

This work was undertaken at the suggestion of Mr. Peter Waterman, Head, Equipment Research Section, Radio Division III, Naval Research Laboratory. The writer is pleased to acknowledge the assistance and interest of Mr. Charles F. White, and Mr. Charles H. Dodge, of Radio Division III, NRL.

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PRNC-6200-6-22-48-200