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THE FRESNEL ZONE PLATE ANTENNA

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ABSTRACT

The Fresnel zone plate antenna utilizes as a focusing objective a plate which discards the radiation from alternate Fresnel zones or reverses its phase. An antenna feed placed at the reference point for the zones receives radiation from all parts of the aperture in phase within $\pm \pi/2$.

There is a loss in gain of 3 db because of phase variations resulting from the stepped nature of the phase correction, of 2 db because of nonuniform amplitude distribution, and an additional loss of 6 db if alternate zones are made reflecting. Pattern characteristics and off-axis gain performance are comparable with paraboloid antennas. The impedance reaction of the zone plate on the feed is very small.

Zone plates with all zones transmitting can be made of a single refracting medium by the use of different thicknesses for alternate zones, or of a single thickness with different refracting mediums for alternate zones. In either case the zone plates can be designed to achieve cancellation of reflections.

To obtain optimum performance the refractive indices should not depart too widely from unity, the plate should be kept thin, and all zones should have radial dimensions greater than a wavelength.

PROBLEM STATUS

This is an interim report. Further work on this phase of the problem is proposed and outlined in the report.

AUTHORIZATION

NRL Problem No. R26-09.

THE FRESNEL ZONE PLATE ANTENNA

INTRODUCTION

Microwave antennas of high directivity frequently utilize optical instrument design; that is, they consist of a focusing objective and an "eye-piece" or feed. The focusing action in reception consists of converting a portion of a plane wave into a concave spherical wave converging on the feed. This is accomplished by equalizing the optical path lengths from all parts of the aperture to the feed point. The focusing objective has commonly been a paraboloid reflector, though dielectric and metal-plate lenses have had limited use.

It is almost inevitable that a highly directive antenna will be required to scan its beam in order to realize the necessary angular coverage. When the required scan rate is too rapid to be achieved by mechanical rotation of the entire antenna, some electrical scanning technique must be employed. The most promising approach to the electrical scanning problem again utilizes optical techniques; the feed of a two-piece antenna is moved with respect to the focusing objective to cause the beam to scan. Optical scanning imposes off-axis focusing requirements on the objective which a fixed feed does not.

The focusing properties of a paraboloid reflector degenerate rapidly as the feed is moved off axis. Furthermore, the bulky mechanical system for scanning the feed tends to obstruct the aperture. The metal plate lens which has been proposed* to obviate these difficulties leaves something to be desired in electrical performance and creates serious constructional and weight problems. As a result of these difficulties, there is need for much research on microwave focusing objectives: improved reflector shapes, new refracting media, new types of focusing objectives.

One method for achieving approximate focus utilizes Fresnel's concept of half-wave zones. A plane aperture can be divided into circular zones with reference to a chosen focal point on the basis that all radiation from that zone arrive at the point in phase to within $\pm \pi/2$. If the radiation from alternate zones is suppressed or shifted in phase by π, an approximate focus is obtained. Because of the importance of developing improved focusing objectives, the focusing characteristics of the Fresnel zone plate have been investigated. Whereas the zone plate has obvious limitations as a focusing device, it offers the possibility of improved off-axis performance and simpler constructional problems.

THEORY OF THE FRESNEL ZONE PLATE

The Fresnel zone plate (Figure 1) develops naturally from the knowledge of the character of the disturbance caused by an electromagnetic plane wave at a point F at a distance

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* Kock, W. E., Proc. Inst. Radio Engrs. 34; 828-836 (1946)

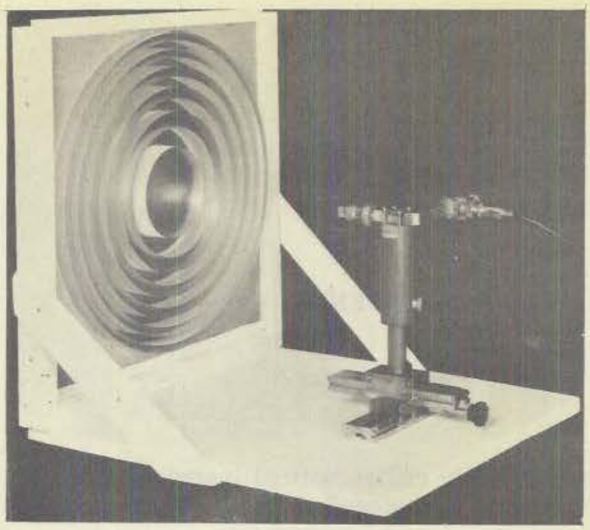
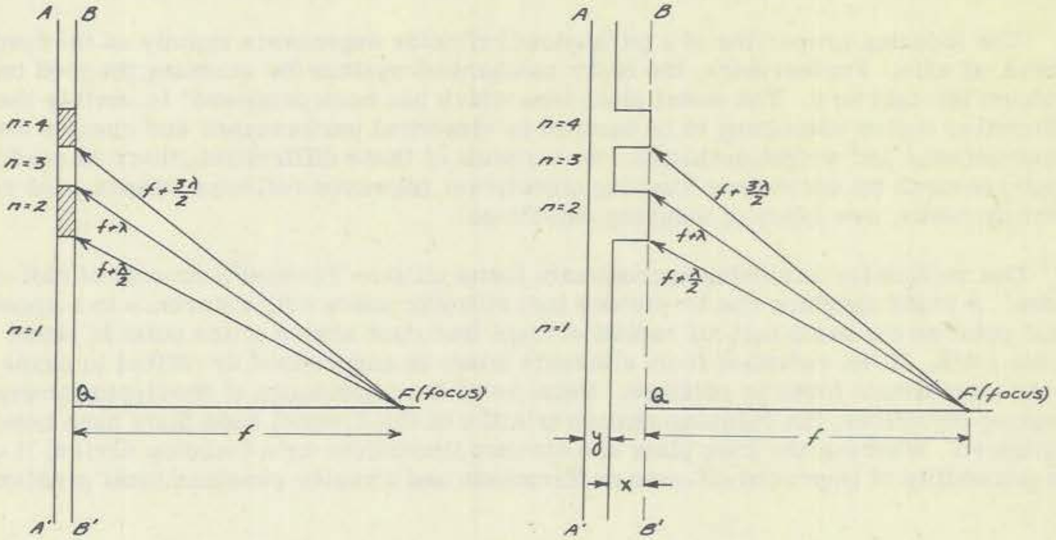


Fig. 1 - Fresnel Zone Plate with Feed Horn on Adjustable Mount

f in advance of a wave front (Figure 2a). The method employed in this report in determining the character of the disturbance at F is the method of physical optics. The electromagnetic plane wave is interpreted by a scalar wave function u and the disturbance at F is determined by using the half-period zones of Fresnel and Huyghens-Fresnel principle.

Consider a plane wave approaching the point F from the left. Partition the wave front BB′ (Figure 2a) into the half-period zones of Fresnel by describing circles on BB′ so that every point on the nth circle is at the distance $f + n\lambda / 2$ from F, where λ is the free space wavelength of the plane wave. All circles have the common center Q, at the foot of the perpendicular from F to BB′. The radius of the nth circle is $\sqrt{nf\lambda + (1/4)n^2\lambda^2}$, (n = 1, 2, 3, -------). The nth zone is the annular region between the (n-1)st and nth circles.

According to the Huyghens-Fresnel Principle each point of the wavefront BB′ may be considered as the source of a disturbance that is propagated as a spherical wavelet whose amplitude in the direction θ with the forward normal to the wavefront is proportional to the obliquity factor $1 + \cos\theta$. At the point F the phase difference of the disturbance due to any two points of BB′ is given by the difference of the distances of the points from F.



(a) Even Zones Blocked Out with Reflecting Material

(b) Even Zones Thicker than Odd Zones

Fig. 2 - Two Types of Fresnel Zone Plates

Because of the method of construction of the half-period zones, the difference between the distances from F to any two points in the same zone is not greater than $\lambda/2$, and the mean distance from the nth zone to F is greater by $\lambda/2$ than the mean distance from the (n-1)st zone to F. Consequently, the resultants of the disturbances at F due to the sources in the individual even zones are in phase with each other and 180° out of phase with each of the resultant disturbances arising from sources in the individual odd zones. It follows that if u_n is the amplitude of the resultant disturbance at F due to the sources in the nth zone, the amplitude of the total disturbance at F due to the wavefront BB′ is

$$u_1 - u_2 + u_3 - u_4 + \text{-------}. \tag{1}$$

The Fresnel Zone Plate with Alternate Zones Reflecting

If the even (odd) half-period zones of the wavefront are blocked out so that energy will not be transmitted through them, then the negative (positive) terms vanish from Equation 1 and the disturbance at F has a greatly increased amplitude. F acts as a focus for the plane wave. This statement is the basis for the construction of a Fresnel zone plate with alternate reflecting zones.

A Fresnel zone plate with alternate reflecting zones is constructed from a plane dielectric sheet with plane faces AA′ and BB′ (Figure 2a). A distance is chosen for the focal length f and the half-period zones are drawn on BB′. Then either the even or odd zones are blocked out with a reflecting material. The number of zones to be used depends on the diameter necessary for the required beamwidth.

The amplitude at the focus F that results from blocking out alternate zones represents a low gain when compared to an aperture of the same dimensions but with uniform phase and amplitude distribution. The total loss in gain due to the presence of the reflecting rings consists of 3 db arising from the outright rejection of one-half the incident energy and of 3 db due to the reduction of the effective aperture area. In addition, there is a loss of approximately 5 db due to nonuniform phase and amplitude distribution and 0.25 db due to reflections.

The Fresnel Zone Plate with All Zones Transmitting

The loss in gain caused by blocking out alternate zones may be reduced by designing a Fresnel zone plate with all zones transmitting and with a phase shift of $(2p + 1)\pi$ on adjacent zones, where p is a positive integer. Because the difference of the mean distances from adjacent zones to F is $\lambda/2$, equivalent to a phase shift of π, the resultant disturbances from all zones will arrive at F in phase and the resultant amplitude at F will be the arithmetic sum of the individual amplitudes.

<u>Single-Medium Zone Plate</u> - The $(2p + 1)\pi$ phase delay on alternate zones over the face BB′ (Figure 2b) may be achieved by selecting a uniform thickness of dielectric for all odd zones and a different but also uniform thickness of dielectric for all even zones so that different degrees of phase delay will occur on adjacent zones. To determine the proper difference in the dielectric thicknesses, it is merely necessary to solve the following equation, expressing the required relation between the phase shifts through adjacent zones, for the distance x (Figure 2b):

$$\frac{2\pi}{\lambda}\, n(x + y) - \frac{2\pi}{\lambda}(ny + x) = (2p + 1)\pi\,,\ (p = 0,\ 1,\ 2.........) \tag{2}$$

where n is the index of refraction of the dielectric. The solution is $x = \frac{2p + 1}{n-1}\,\frac{\lambda}{2}$ (3)

The dimension y (Equation 2) obviously does not affect the relative phase over BB′. Y may be chosen to give mechanical support to the antenna and to cancel undesirable reflections from the front and back surfaces of the dielectric, reflections that could give rise to a serious back lobe. To achieve the required cancellation of reflections, the dielectric thickness of each zone must be an integral number of one-half wave lengths measured in the dielectric.† Consequently, the dimensions x and y must satisfy the relations

$$x + y = k_1 \frac{\lambda_D}{2} \text{ and } y = k_2 \frac{\lambda_D}{2}, \tag{4}$$

where $\lambda_D \left(=\frac{\lambda}{n}\right)$ is the wavelength in the dielectric and k_1 and k_2 are positive integers. Equations 4 may be rewritten as follows:

$$x + y = \frac{k_1 \lambda}{2n} \text{ and } y = \frac{k_2 \lambda}{2n}. \tag{5}$$

In order to achieve a phase shift of π on all zones and a phase shift of $(2p + 1)\pi$ on adjacent zones, Equations 3 and 5 must be satisfied. Elimination of x and y from Equations 3 and 5 yields

$$k_1 - k_2 = \left(\frac{n}{n-1}\right)(2p + 1), \tag{6}$$

where $k_1 - k_2$ must be a positive integer. This equation must hold if a Fresnel zone plate is to have all zones transmitting and cancellation of front and back reflections on each zone.

Two-Medium Zone Plate - A phase shift of $(2p + 1)\pi$ over adjacent zones may be achieved by alternately retarding and advancing the phase of successive zones. The phase delay may be achieved with a dielectric medium and the phase advance with a metal-plate medium.

To determine the thicknesses of the different media, let x be the difference in the thicknesses, let y be the smaller thickness, and choose the dielectric to be of thickness y. If n_d and n_m are the indices of refraction of the dielectric and metal-plate media, respectively, then the required phase shift over adjacent zones is achieved with dimensions satisfying the following equation:

$$\frac{2\pi}{\lambda} y\, n_d + \frac{2\pi}{\lambda} x - \frac{2\pi}{\lambda} n_m (x + y) = (2p + 1)\pi, \tag{7}$$

where p is a positive integer or zero. To cancel reflections from the front and back surfaces of each zone, it is necessary that x and y satisfy

$$x + y = \frac{\lambda_m}{2} k_m = \frac{\lambda}{2} \frac{k_m}{n_m},$$

$$y = \frac{\lambda_d}{2} k_d = \frac{\lambda}{2} \frac{k_d}{n_d} \tag{8}$$

where k_m and k_d are positive integers and λ_m and λ_d are the wavelengths in the metal-plate and dielectric media, respectively. Elimination of x and y from Equations 7 and 8 yields

$$k_d \left(1 - \frac{1}{n_d}\right) + k_m \left(\frac{1}{n_m} - 1\right) = 2p + 1. \tag{9}$$

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† Stratton, J. A., Electromagnetic theory, pp. 490-497 McGraw-Hill Book Co., (1941).

Equation (9) must be satisfied if the $(2p+1)\pi$ phase shift is to be achieved on adjacent zones and if reflections from the front and back surfaces of each zone are to cancel.

From the mechanical standpoint it is obviously desirable to design the two-medium zone plate with uniform thickness. In addition, uniform thickness eliminates the problem of compensating for the undesirable waveguide effect which will be discussed in under "Absolute Gain." The uniform thickness is found by letting x equal zero in Equations (7) and (8). The elimination of y from the new equations yields

$$k_d - k_m = 2p + 1. \tag{10}$$

Then y may be obtained from either equation 7 or 8, under the restriction imposed by Equation 10.

The following are a few selected sets of values of p, k_m and k_d for a two-medium zone plate of uniform thickness and the corresponding value of n_m and the uniform thickness y for the index of refraction n_d of the dielectric equal to 1.6:

p	k_d	k_m	n_m	Thickness (y)
0	2	1	0.8	0.625 λ
1	4	1	0.4	1.250 λ
1	5	2	0.64	1.562 λ
1	6	3	0.80	1.875 λ
2	8	3	0.60	1.667 λ
2	9	4	0.71	2.817 λ

Of the sets of values listed above the first is to be recommended because the index of refraction n_m is close to unity and because the other sets of values result in excessive thicknesses.

An alternate way to cancel reflections from a two-medium zone plate of uniform thickness requires that

$$n_d \cdot n_m = 1 \tag{11}$$

and that the number of zones be even. With the removal of restrictions on thickness imposed to cancel reflections from the front and back surfaces of each zone (Equations 8), Equation 7 gives for the uniform thickness

$$y = \frac{2p+1}{n_d - n_m} \frac{\lambda}{2}. \tag{12}$$

A small plate thickness may be obtained by letting p equal zero and by making $n_d - n_m$ as large as practicable, say unity.

Equation 11 implies that the amplitude reflection coefficients,

$$\Gamma = \left|\frac{n-1}{n+1}\right|,$$

of the two media are equal. Therefore, because the zones are of equal area, the magnitudes of reflections from the different zones will be equal. Consequently, a plane wave normally incident upon the zone plate will not show a resultant reflection from the front surface because reflections from alternate zones will be equal in magnitude and 180° out of phase, due to phase reversal from zones whose index of refraction is greater than unity.* Furthermore, there will be no resultant reflection from the back surface because the difference in phase due to twice traversing the thickness of the plate is an integral multiple of 2π, and because of the phase reversal at the back surfaces of zones for which the index of refraction is less than unity.

Similar considerations show that the reflection of radiation reaching the zone plate from a point source at the focus will not be focused back upon the source, but will be radiated as a plane wave in the backward direction. Cancellation of reflections at the feed is not exact but approximate, because the illumination of successive zones is not uniform but smoothly tapered. Thus the transmitter is protected from an impedance mismatch but the radiation pattern of the antenna is afflicted with an appreciable back lobe.

A third way to cancel reflections from a two-medium zone plate of uniform thickness is to combine the previous two methods; that is, require that Equations 8 and 11 be satisfied and that the number of zones be even. This insures the cancellation of reflections between successive zones as well as on the front and back surfaces of the same zone. Moreover, the use of Equation 11 reduces the average reflection coefficient for the two media.

The correct thickness is obtained by substituting Equations 10 and 11 into Equation 9 and obtaining

$$n_d = \frac{1}{n_m} = \sqrt{\frac{k_d}{k_m}}. \tag{13}$$

Then Equation 8 for the uniform thickness becomes

$$y = (\lambda/2)\sqrt{k_d k_m}. \tag{14}$$

The following are a few selected sets of values of p, k_d and k_m and the corresponding values of n_d, n_m and y for a two-medium zone plate.

set	p	k_d	k_m	n_d	n_m	y
1	0	2	1	1.41	0.71	0.71λ
2	0	3	2	1.22	0.82	1.22λ
3	1	4	1	2.00	0.50	1.00λ
4	1	5	2	1.58	0.63	1.58λ
5	2	7	2	1.87	0.54	1.87λ
6	2	8	3	1.63	0.61	2.45λ

Of the several sets of values listed above, sets 1 and 2 are to be recommended because they utilize indices of refraction not too far removed from unity and result in a zone plate of small thickness. Sets 4 and 6 require indices near those usually encountered in common plastics and in metal plate media in general use, but result in a zone plate of excessive thickness and weight.

DESCRIPTION OF FRESNEL ZONE PLATES DESIGNED AND TESTED

Three Fresnel zone plates have been designed and tested. In each case the operating wavelength was 1.27 cm, the focal length was 10 in., and the number of zones, eleven.

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* Stratton, J. A., Ibid, pp. 490-497.

Consequently, the inner and outer radii of corresponding zones were the same. The dielectric was Lucite (methyl methacrylate). Polystyrene has a somewhat lower loss, but it presented procurement difficulties. Figure 1 shows one of the zone plates together with the feed horn and a means for positioning it with respect to the focal point.

(A) Alternate Zones Reflecting

The even zones were coated with a reflecting material, Dupont silver paste 4132. In order to facilitate mounting of the antenna the outer edges of the dielectric formed a square, with the outer edge of the eleventh zone tangent to the edge of the square. The corners of the square beyond the eleventh zone were coated with the reflecting material.

(B) All Zones Transmitting with Cancellation of Front and Back Reflection

The dimensions x and y (Figure 2b) were chosen in accordance with Equations 3 and 5. The index of refraction n of Lucite is approximately 1.6 at 1.27 cm, so $k_1 - k_2$ of Equation 6 is a positive integer when $2p + 1$ is a multiple of 3. In order to make the thickness of the dielectric as small as possible and at the same time assure adequate mechanical support, p was chosen equal to unity. The minimum dielectric thickness was 5/8 in., and the maximum 1-7/8 in. The odd zones were the thicker zones. The corners of the square zone plate were coated with the reflecting paint to give a good comparison with the first zone plate.

(C) All Zones Transmitting with Cancellation of Front and Back Reflections on the Even Zones Only

This antenna was designed to be light in weight and with as many of the desirable features of the previous one as could be retained. The dimension x was calculated from Equation 3 with m equal to unity and it was found to be 0.417 in. The odd zones were made the thicker ones. The dielectric thickness of the even zones was chosen so that front and back reflections would cancel on those zones. This minimum thickness was 0.156 in. so that the maximum was 0.573 in.

Primary Feed—The same primary feed was used with all three antennas. The primary feed was a horn with rectangular aperture, designed to have a 1/10 power width of 86° in both the E and the H planes. The primary feed pattern was found experimentally to have a 1/10 power width of 87° in the H-plane and 92° in the E-plane; the 1/2 power widths were 46° and 56°, respectively.

Throughout this report the antennas will be referred to by the letters A, B, C, which designate the subheadings of this section.

ABSOLUTE GAIN

The gain characteristics of the three Fresnel zone plate antennas (A, B, and C) described in the previous section are summarized below:

	A	B	C
Gain factor	0.072	0.094	0.204
Gain in db	28.7	29.8	33.2
*db down from ideal	11.38	10.28	6.88

*(db loss in gain compared to an identical aperture with uniform phase and amplitude distribution).

The gain, defined as the maximum of the gain function

$$G(\theta,\phi) = \frac{4\pi P(\theta,\phi)}{P_T} ,$$

where P_T is the total power radiated and $P(\theta,\phi)$ is the power radiated per unit solid angle in the (θ,ϕ) direction, was obtained by comparison with a gain standard. The gain factor is the ratio of the gain of the antenna to the gain of an identical aperture with uniform phase and amplitude distribution.

On the basis of a rough estimation of the losses due to a number of factors the results obtained are not unreasonable. The following factors were considered as possible sources of loss in gain: (a) attenuation and reflection due to the dielectric; (b) nonuniform amplitude distribution across the aperture; (c) nonuniform phase distribution across the aperture; (d) rejection of one-half the incident energy due to the presence of reflecting zones; (e) waveguide effect resulting in the defocusing of the antenna. Table 1 summarizes for the three antennas the estimated losses due to each of the above factors.

Table 1. Estimated loss in gain (db), due to factors (a) - (d), for three zone plate antennas, compared to an aperture with identical dimensions and with uniform phase and aperture distribution.

TABLE 1

	Antenna		
Source of Loss	A	B	C
a	0.25	0.5	0.5
b	2	2	2
c	3	3	3
d	6	0	0
Total	11.25	5.5	5.5
Observed	11.38	10.28	6.88
Discrepancy (Attributed to e)	---	4.8	1.4

The estimates for factor (a) were based entirely upon dielectric reflection since attenuation due to dielectric loss is negligible compared with other losses. For antenna A the reduction in transmission through the transmitting zones could be calculated with some confidence from the dielectric constant and thickness of the sheet. Whereas antenna B was designed by proper choice of dielectric thickness to be nonreflecting, account was not taken of the fact that most of the zones are smaller in radial dimension than a wavelength so that the wavelength of the radiation traversing these zones will not be that calculated from the nominal dielectric constant of the material. Therefore, in estimating factor (a) for antennas B and C, it was necessary to accept the statistical result obtained by adding the amplitude reflections from the front and back surfaces in quadrature.

The estimate for factor (b) ignored the nonuniformity of the phase distribution. The amplitude distribution across the aperture was determined from the pattern of the primary feed and its distance from the zone plate. The amplitude distribution was compared to the distributions considered by R. C. Spencer.* The effect of spillover was found by estimating the ratio of aperture area to illuminated area. The estimated 2 db loss was then read from Figure 7 of Spencer's report.

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* Spencer, R. C., Radiation Laboratory Report T-7, October 21, 1942.

The effect of nonuniform phase distribution (c) was estimated by ignoring the nonuniformity of the amplitude distribution. The phase distribution for the nth zone of antenna C is (disregarding the effect that the small radial dimensions of the zones have on the wavelength)

$$\phi_n(x) = \frac{2\pi}{\lambda}\left[f + \frac{n\lambda}{2} - \sqrt{f^2 + x^2}\right] ,$$

with $\phi_n(x)$ vanishing outside the nth zone. Figure 3 shows the graph of the phase along the intersection of the antenna C with a plane through the focus and perpendicular to the aperture. The same distribution was assumed for antenna B. Assuming saw-tooth variation in phase and considering all vectors of the same phase to be added, an estimate of the phase distribution is a single saw-tooth. The vectors forming this saw-tooth inscribe a semicircle, because the end vectors are 180° out of phase, and their vector sum is the diameter of the circle. The decrease in amplitude from the amplitude for uniform phase distribution is the ratio of the diameter of the circle to the semi-circumference, or $2/\pi$. The reduction in gain is 4 db. However, assuming the actual phase distribution instead of the saw-tooth distribution, it is evident that the phase will change more rapidly at one end of the resultant phase distribution curve than at the other end. A graphical construction indicates that the gain is then approximately 3 db below that of an aperture with the same dimensions but with uniform phase and amplitude distribution.

Obviously only antenna A is affected by the presence of reflecting rings. The 6 db loss in gain for antenna A consists of a 3 db loss due to the outright rejection of one-half the incident energy and a 3 db loss due to the reduction of the effective aperture area.

For antennas B and C the discrepancies (Table 1) between the estimated losses in gain due to factors (a) - (d) and the observed losses must be attributed to faulty design. The faulty design is in those portions of the antennas where shells of dielectric and air exist side by side on alternate zones. Due to this construction and the fact that the widths of the outer five zones are less than the free space wavelength, the actual wavelength will differ from the free space wavelength in the air filled shells and from the calculated wavelength in the other shells. Except for the central zone, which has a comparatively large radius, each zone presents to the E vector of the dominant TE_{10} mode of propagation a varying width perpendicular to that vector. This undoubtedly results in different wavelengths through different portions of even the same zone. These combined effects are referred to as the waveguide effect of factor (e).

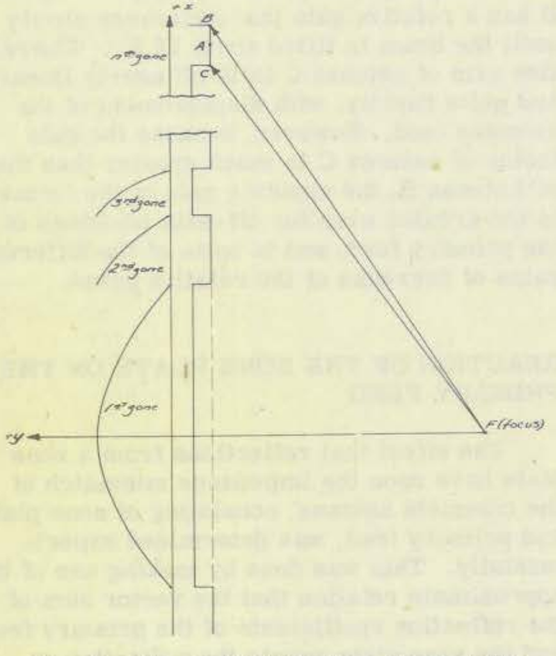

Fig. 3 - Phase Distribution Across the Aperture with Feed at the Focus F for Antenna C

To estimate the reduction in gain due to the waveguide effect would be extremely difficult. However, because the shells of antenna B have the same lateral dimensions as those of antenna C and are three times as deep, the phase errors due to the waveguide effect will be three times as great in antenna B as in antenna C. Therefore, the discrepancies in gain (Table 1) of 4.8 db for antenna B and 1.4 db for antenna C are not inconsistent in relative magnitude with this explanation.

Losses in gain due to the waveguide effect could be eliminated by improved design with sufficient information about the resultant wavelength in such constricted channels. However, the waveguide effect is an argument against deep channels and narrow zones.

POWER PATTERNS

Figure 4 shows the relative gain as a function of beam tilt for antennas B and C. The pattern characteristics of the antennas A, B, and C are summarized in Tables 2 to 4 and Figures 5 to 9.

The half-power width, $\theta_{\frac{1}{2}}$, with the primary feed located at the focus varies from 2° to 2.2° for the three antennas.[2] For an aperture of the same dimensions with uniform amplitude and phase distribution, the half-power width is 1.85°. The variation in $\theta_{\frac{1}{2}}$ as the primary feed is moved away from the focus in a direction perpendicular to the axis is less than 5 percent for beam tilts up to at least 9.5° or approximately 5 beamwidths.

The side lobe level of antenna B is nearly constant for beam tilts up to 9.5°. However, this is probably due to the phase distortion caused by the waveguide effect; it is this factor that reduces the antenna gain to its low value on axis. Antenna C has a side lobe level that increases by at least 8 db for a beam shift of 10°. The front to back lobe ratios are about as expected, being high for antenna A with its alternate reflecting zones, and down to 13.2 db and 15.52 db for antennas B and C, respectively.

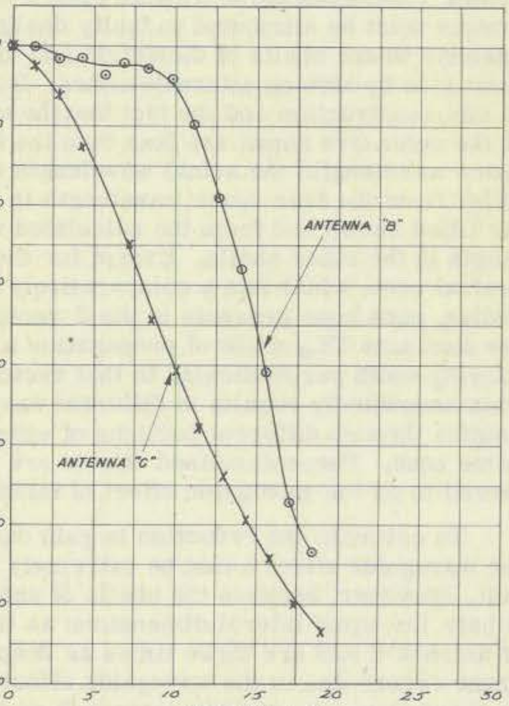

Fig. 4 - Variation of Gain with Beam Tilt ($\theta_{\frac{1}{2}}$ is the Full Beamwidth at Half Power)

The relative gain (compared to maximum gain) as a function of normalized beam tilt, achieved by the displacement of the primary feed from the focus in a direction perpendicular to the axis, is given in Figure 4. Antenna B has a relative gain that decreases slowly until the beam is tilted about 13.5°. The relative gain of antenna C falls off nearly linearly, and quite rapidly, with displacement of the primary feed. However, because the gain factor of antenna C is much greater than that of Antenna B, the absolute gain of the former is the greater even for off-axis positions of the primary feed, and in spite of the different rates of decrease of the relative gains.

REACTION OF THE ZONE PLATE ON THE PRIMARY FEED

The effect that reflections from a zone plate have upon the impedance mismatch of the complete antenna, consisting of zone plate and primary feed, was determined experimentally. This was done by making use of the approximate relation that the vector sum of the reflection coefficients of the primary feed and the zone plate equals the reflection coefficient of the complete antenna. The reflection coefficients for the complete antenna

TABLE 2

Pattern Characteristics of Antenna A (See P. 7)

Position of Horn Off Axis	Beamwidth		Side Lobe Level (db)		Beam Shift			
					Degrees		Beamwidths	
	H	E	H	E	H	E	H	E
0 cm	2°	--	16.5	--	0°	--	0	--
2.5	2°	--	12.25	--	6.1°	--	3.1	--
5.0	2.1°	--	11.0	--	11.1°	--	5.6	--
7.5	2.75°	--	12.0	--	14.5°	--	7.3	--

H-plane front to back lobe ratio: 2.28 db

TABLE 3

Pattern Characteristics of Antenna B (See P. 7)

Position of Horn Off Axis	Beamwidth		Side Lobe Level (db)		Beam Shift			
					Degrees		Beamwidths	
	H	E	H	E	H	E	H	E
0 cm	2.2°	2.1°	17	18	0°	0°	0	0
2.5	2.2°	2.1°	18	17.5	5°	5.1°	2.3	2.4
5.0	2.3°	--	17	--	9.5°	--	4	--
7.5	2.75°	2.75°	11.5	9	13.5°	14.6°	6.1	7

H-plane front to back lobe ratio: 13.2 db

TABLE 4

Pattern Characteristics of Antenna C (See P. 7)

Position of Horn Off Axis	Beamwidth		Side Lobe Level (db)		Beam Shift			
					Degrees		Beamwidths	
	H	E	H	E	H	E	H	E
0 cm	2.2°	2.1°	20	22	0°	0°	0	0
2.5	2.2°	2.1°	17.5	14	5°	5.4°	2.3	2.5
5.0	2.2°	2.1°	12	10.5	10.3°	11.2°	4.7	5.3
7.5	3.0°	3.0°	10.5	9	15°	16.5°	6.8	7.9

H-plane front to back lobe ratio: 15.5 db

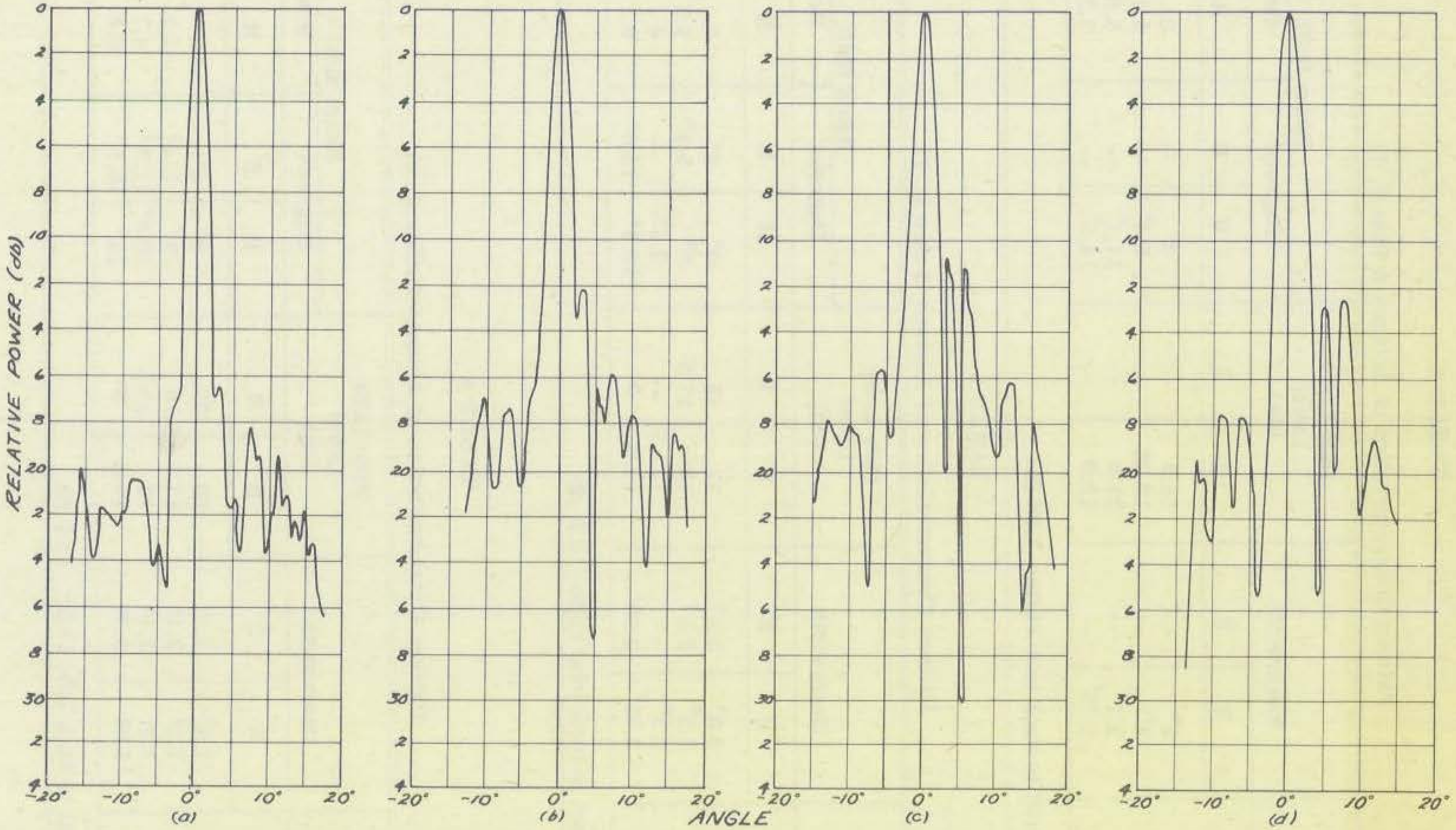

Fig. 5 - H-Plane Patterns of Antenna A with the Feed Various Distances Off Axis:
(a) 0 cm; (b) 2.5 cm; (c) 5.0 cm; (d) 7.5 cm

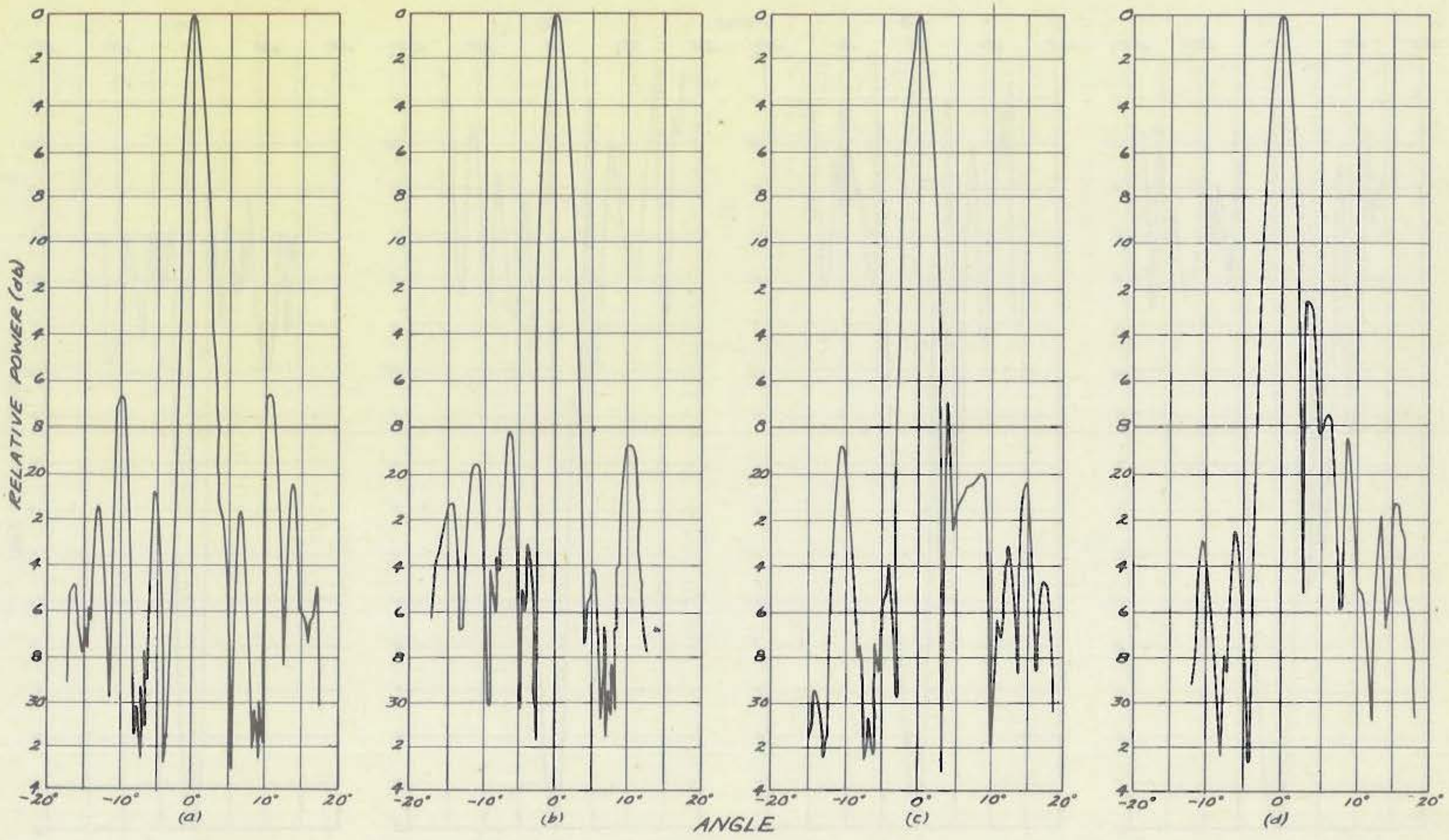

Fig. 6 - H-Plane Patterns of Antenna B with the Feed Various Distances Off Axis: (a) 0 cm; (b) 2.5 cm; (c) 5.0 cm; (d) 7.5 cm

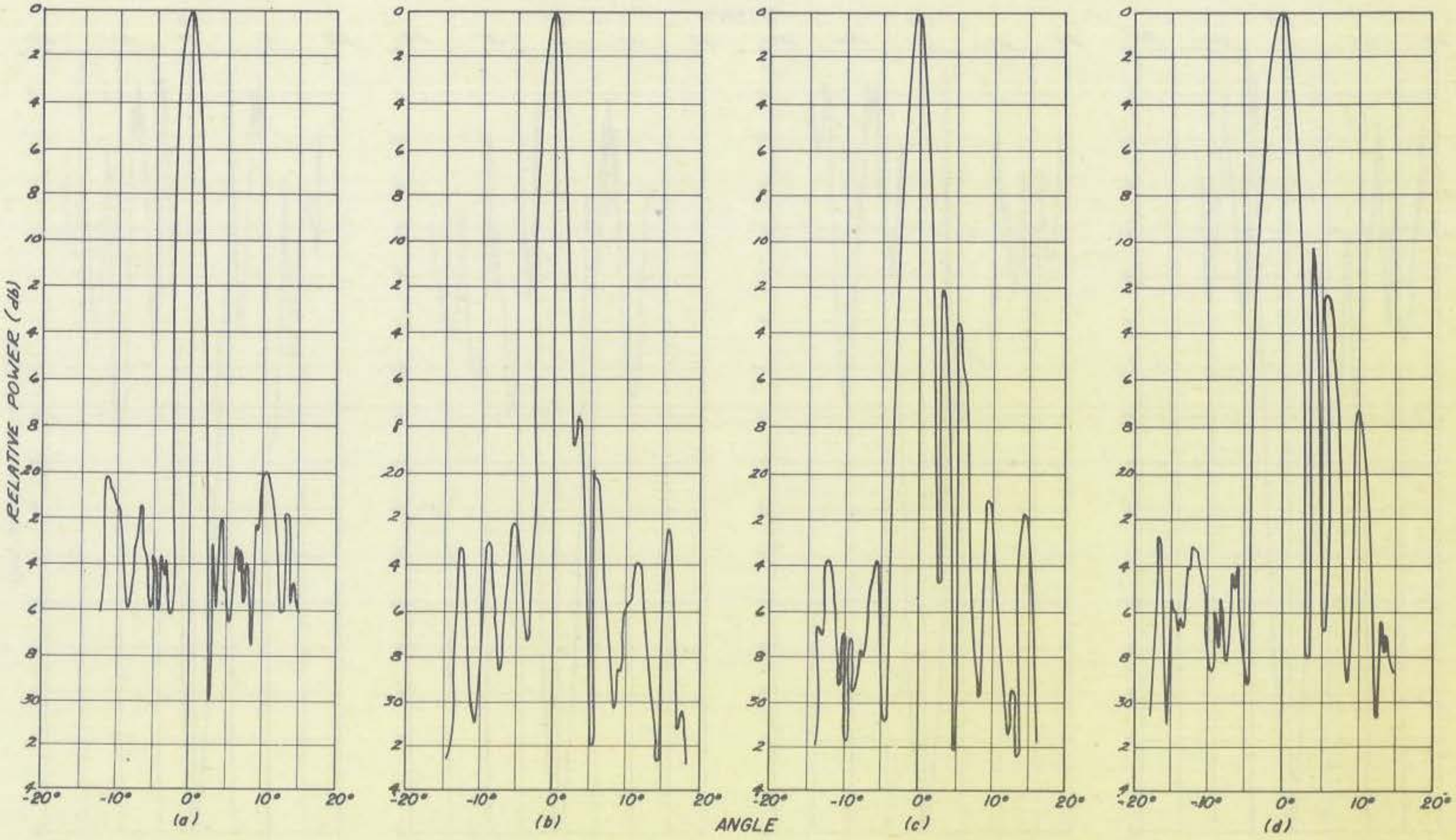

Fig. 7 - H-Plane Patterns of Antenna C with the Feed Various Distances Off Axis: (a) 0 cm; (b) 2.5 cm; (c) 5.0 cm; (d) 7.5 cm

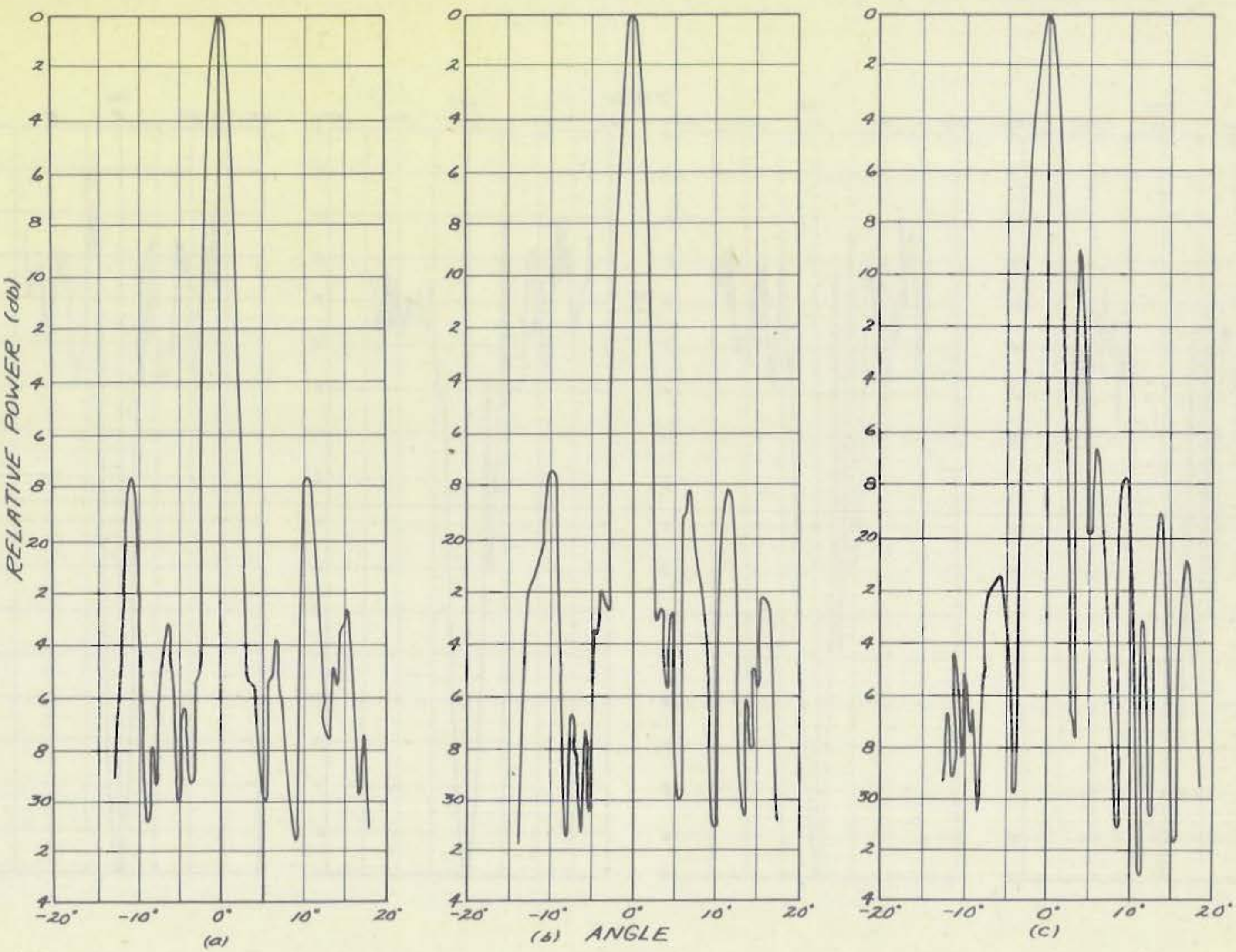

Fig. 8 - E-Plane Patterns of Antenna B with the Feed Various Distances Off Axis:
(a) 0 cm; (b) 2.5 cm; (c) 5.0 cm; (d) 7.5 cm

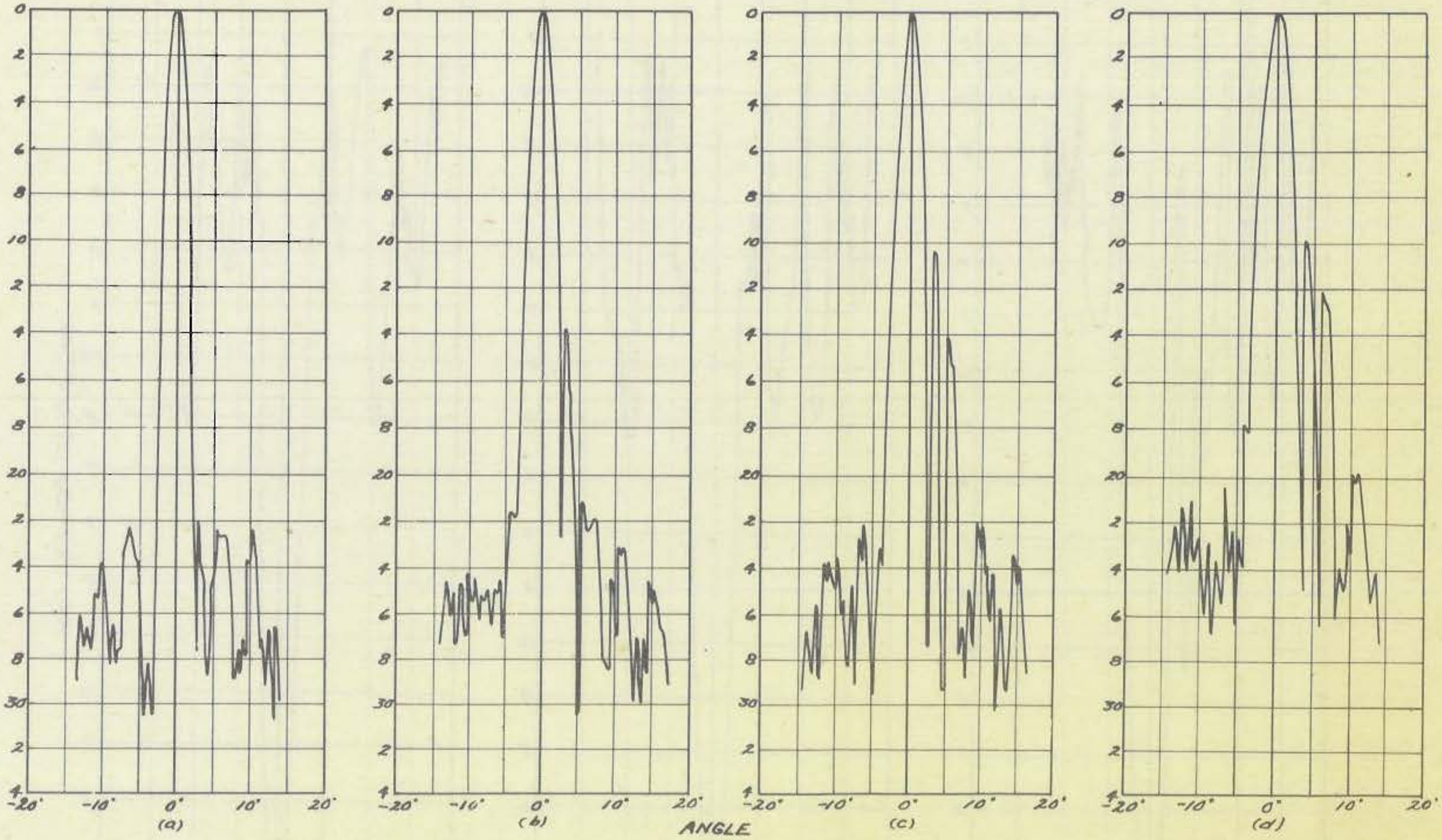

Fig. 9 - E-Plane Patterns of Antenna C with the Feed Various Distances Off Axis:
(a) 0 cm; (b) 2.5 cm; (c) 5.0 cm; (d) 7.5 cm

were determined for a number of positions on the axis of the zone plate and in the neighborhood of the focus. The reflection coefficient of the primary feed in free space was determined. For each position the absolute value, $|\Gamma|$, of the reflection coefficient of the zone plate was calculated. For each of the three zone plates the absolute value of the reflection coefficient was less than 0.02. Because the values of $|\Gamma|$ were so small, the mismatch of the primary feed may, for most ourposes be considered to be the mismatch of the complete antenna. All measurements were taken at the design frequency of 1.27 cm.

SUMMARY

A microwave antenna with a Fresnel zone plate as the focusing device produces radiation patterns comparable with other types of aperture antennas. Antenna C (one-medium zone plate with all zones transmitting) showed a loss in gain below an ideal aperture of about 7 db, which could be reduced to about 5 db by elimination of the waveguide effect and reduction of reflections. The off-axis performance of antenna C, when expressed as a function of normalized beam tilt (Figure 4), is comparable with that of paraboloid and lens antennas.

A two-medium zone plate of uniform thickness, designed according to either of two straight-forward procedures may be expected to have performance and constructional advantages over the one-medium zone plate.

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