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GUIDED MISSILE SEMINAR

SEPTEMBER 1947

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GUIDED MISSILE SEMINAR

SEPTEMBER 1947

A. S. Locke

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July 19, 1948

Mr. A. S. Locke, Consultant, Radio Division III
Dr. R. M. Page, Superintendent, Radio Division III

DEPARTMENT OF THE NAVY OFFICE OF NAVAL RESEARCH

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INTRODUCTION

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ABSTRACT

An interim report on the research and development of the Wasp system of mid-course guidance for the Lark missile is given. An interim report on the progress on the terminal guidance system for the Skylark program is presented. The following papers: "Discussion of FM-CW Oscillator for Wasp Missile Control", "Discussion of The Effect of Random Noise on Missile Guidance" and "Determination of Proper Bandpass for Automatic Correction of Range for Lark" are included as part of this report.

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INTRODUCTION

PURPOSE OF PROBLEM A-156R-C, LARK MISSILE CONTROL

The primary purpose of Problem A-156R-C, assigned by BuAer is: "to modify the SP shipborne radar and, with a radar beam control receiver, afford radar beam control of the Lark missile".

The Lark is a subsonic missile operating at approximately 0.85 Mach number. Two versions are under development: XSAM-2 Lark, designed and manufactured by the Fairchild Pilotless Plane Division; and XSAM-4 Lark, designed and manufactured by Consolidated Vultee Aircraft Company.

The specifications for the beam rider system provide that the operating range of the system shall be at least 90,000 yards; that the control shall be operable to at least 40,000 feet altitude. The target shall be considered to be comparable to a medium bomber. The SP shipborne radar shall be modified to incorporate automatic tracking of the target and shall transmit control intelligence to the receiver in the Lark by pulse time modulation.

The receiver shall be dimensioned as required to be located in the Lark; it shall accept and interpret the information received from the radar beam so as to furnish proportional control to the missile to maintain it on the line of sight between the radar and the target.

Fifty beam riding receivers with correlated antennae and beacons will be manufactured, under Laboratory jurisdiction, for use in Lark flight research. The Naval Research Laboratory will participate in the flight research program as consultants on beam riding guidance research.

PURPOSE OF PROBLEM O(A)-126R-C, PROJECT WASP.

The primary purpose of Problem O(A)-126R-C, assigned by BuOrd, is: "to develop and test a system for control of a guided missile by means of a radar beam modulated at different frequencies in different lobes".

NRL PLANS FOR DEVELOPING BEAM RIDING GUIDANCE SYSTEMS

The problem of providing beam riding control for the Lark and developing the Wasp system of guidance are precisely similar in basic principle. The NRL plan for developing a beam riding guidance system has been based upon utilization of the implementation specified for control of the Lark missile.

[redacted]

An SP shipborne radar, using S-band carrier frequency scanned at 24 c.p.s., has been modified to provide pulse time modulated signals to furnish the reference for the airborne receiver. The construction of the new servo systems in the SP radar mount, designed both for automatic tracking and better performance, has been completed. Provision has been made for target acquisition by use of an optical handstand mount. The SP radar has been installed at the Chesapeake Bay Annex of the Naval Research Laboratory.

The shipborne radar, because of weight and other physical restrictions, can not be considered adaptable for field test. A truck-mounted mobile radar, the SP-1M, differing from the SP only in that the dish is six feet in diameter as compared to eight feet and that no stabilization or cross level axis is provided, is undergoing modification for field-test use. The modification of the SP-1M, to improve the mount structure and to achieve servo performance comparable to the redesigned SP, is underway.

Several designs of the receiver, designated officially as AN/APW-4, have been investigated, tested and discarded. Two receivers are now being flight tested; both are superheterodynes, differing in demodulator and other circuit details. The research work with one receiver is being concentrated on fundamental propagation problems and long-term receiver details. The flight tests with the second receiver are directed toward the immediate accomplishment of a beam rider receiver for the Lark missile.

Preliminary flights have been conducted with a human pilot guiding the plane from cross-pointer meters which indicate departure of the plane from the line of sight of the radar beam. Outputs from the receiver to the cross-pointer meter have been modified by equalizing circuits in such manner that the cross-pointer meter presents to the pilot a combination of the error between the line of sight of the radar and the plane and the first derivative of the error. This device results in the exponential reduction of error without oscillatory flight. These flights were successful.

Flight research to determine noise and dispersion has been completed. In these flights, the pilot flew on the beam by use of the cross-pointer meters. The outputs of the receiver were fed to an analogue computer simulating the known transmission characteristics of the autopilot and the airframe. Frequency response tests of the autopilot and the airframe have been made and reported at previous seminars. The outputs from the analogue computer were recorded; this recorded data represents angular error from the radar line of sight. The angular error recorded represents pilot error plus system noise. Pilot error is known, inasmuch as the position of the plane with respect to the radar line of sight is recorded photographically.

[redacted]

Subsequent to the noise flight tests, the receiver outputs were fed to the autopilot and used to guide the plane automatically along the line of sight of the radar beam. Flight tests to date have been accomplished using an SP-1M radar modified for correct RF transmission only as an interim measure until the SP radar is operative. Lack of automatic tracking in the SP-1M and satisfactory tracking servos have been a handicap in the flight tests. Difficulty has been experienced in engaging the receiver with the correct crossover, as predicted in previous seminars.

The plane used for the flight tests was an SNB in which a modified P-1 autopilot was installed. Considerable maintenance difficulty developed with this plane; ultimately it became unsafe for continued test work. For lack of a satisfactory vehicle test work was halted after several automatically guided flights. Difficulty with this airplane has been the greatest single deterrent to the satisfactory pursuit of the test work for this program. Arrangements have been underway for some time to procure and equip another SNB in order that flight tests may continue.

The automatically guided flights which were accomplished were sufficient to determine that the receiver should be satisfactory for use in the Lark vehicle. Specifications were prepared and bids received on the construction of fifty beam rider receivers AN/APW-4. The work will be accomplished in two stages: the first models will be tested for shock, vibration, temperature, and altitude variation, effect of Lark motor on received signals and operational airplane flight tests; the remaining receivers will be built to a standard design developed from the model which satisfies all test demands. The contract is to be awarded to the Reeves Instrument Company of New York City.

Throughout the entire development of the beam riding control, it is planned to examine fully all parameters of each phase of the problem, building step-by-step upon known quantities and determined facts. This procedure has entailed studies beyond the original scope of the problem and has clearly illustrated the need for intense coordination of the efforts of the Laboratory with the contractors for the control equipment and the airframe of the Lark, and with the representatives of the cognizant Naval Bureaus.

A beam rider simulator, operating on a 1:1 time scale is being constructed. This is an analogue computer and will be used to examine the dynamic behavior of the airframe and control components in simulated flight, as a closed loop. The simulator may also be used as a tester for some control components. The simulator is designed for use for study of any Wasp-type beam rider; it is not limited to the specific problem of control of the Lark.

Many of the problems encountered in the development of the beam riding guidance system are of scope beyond the strict bounds of the assigned problem, but so closely related to the successful conclusion of the project that independent investigations are necessary. In order to disseminate the information obtained as a result of these studies, the major portion of the Guided Missile Seminars are devoted to the presentation of technical papers or talks on the studies. The studies include: the attenuation and modulation of signals caused by the propelling flame; the effects of ground reflection on the control line; the magnitude and effects of noise; problems of launching and capture by the guiding radar beam; optimum crossover for a beam rider guidance system; servo considerations for a beam riding airplane and the missiles; tactical problems vs transmission bandwidth and many others. Work on these investigations is continuing; additional studies will be started and similarly reported as the need arises.

The beam riding guidance system is but a part of the Lark control system and its characteristics are dependent upon dynamic behavior of the control elements and the airframe. In order to maintain the close coordination necessary between the Laboratory, the contractors supplying the airframes and control components and the cognizant Naval Bureaus, these agencies have given technical papers or talks upon their problems at the Guided Missile Seminars. Groups engaged on similar guidance and control problems for other pilotless aircraft and guided missiles have discussed the development of their equipment, the problems encountered, and the methods utilized in overcoming these problems. This has resulted in a clearer appreciation of the magnitude and parameters of the interrelated problems to the mutual satisfaction of the contributing groups.

PURPOSE OF PROBLEM A-313R-C, THE SKYLARK PROGRAM

The purpose of Problem A-313R-C, assigned by BuAer, is to develop "a command mid-course guidance and passive terminal guidance for the XSAM-2 and XSAM-4 Lark Pilotless Aircraft". The Naval Research Laboratory has been requested to serve as consultant to the Pilotless Plane Division of the Fairchild Engine and Airplane Corporation, the contractors for this program.

There are no definite specifications established by BuAer as to the precise type of mid-course command and passive terminal guidance system. The system is to be capable of guiding the XSAM-2 and XSAM-4 Lark Pilotless Aircraft, within the maneuverability limitations of the aircraft, from launching through mid-course and terminal phases to effect a collision with an aircraft target having the following characteristics:

Radar cross section area:	Equivalent to SNB. (3 sq. meters assumed for computation)
Speed:	300 m.p.h.
Maneuverability:	3g
Maximum Altitude:	25,000 feet

[redacted]

NRL PLANS FOR DEVELOPING SKYLARK

By conference with BuAer and Fairchild, and in view of the short time allotted for this development, a division of the work between NRL and Fairchild, for the initial phase of the contract, was decided upon. The terminal guidance system, at the request of BuAer, was given initial priority. Laboratory personnel have concentrated their studies upon the application of simultaneous lobe comparison systems for terminal guidance; Fairchild personnel have concentrated upon sequential lobing type terminal guidance systems. In order to weigh the comparison between homing systems a standard of merit was decided upon to permit a choice of system. The conclusions reached in the initial study phase upon the type of passive terminal guidance system to be utilized for Skylark, are contained in this report.

The Naval Research Laboratory has reoriented the work on the Hornet system of command guidance for possible application to the Skylark program. One version of the Hornet system, as presently envisioned and previously described in past seminars, is designed to guide the missile automatically along the radar line of sight. The ground radar automatically tracks the target; the missile, using a beacon, is simultaneously gated. When the missile departs from the line of sight of the radar, the error, observed at the ground radar, automatically codes the radar pulses with intelligence to command the missile to correct the error.

A premodulator, designed to be utilized with the SP radar, located at the Chesapeake Bay Annex of the Naval Research Laboratory, is being constructed. Preliminary flight research has been started to examine many parameters of the system under controlled conditions (i.e., propagation phenomena, fading, noise effects, etc.). The Hornet receiver will then be installed in an SNB aircraft, with the receiver outputs indicating the deviation from line-of-sight path on a cross-pointer meter. The pilot, by use of the cross-pointer meter, will attempt to fly by command from the ground radar. Subsequent to these tests, the receiver will be installed in the SNB containing the modified autopilot and the plane will be flown automatically by the command link. It is planned to follow, where true comparisons exist, the test procedure outlined for the Wasp system to obtain the greatest advantage from past experience and existing implementation. Progress on the development of the Hornet system as a potential mid-course command guidance system for the Skylark will be reported in each seminar report.

[redacted]

SEMINAR MEETING OF 18 SEPTEMBER 1947

PLACE OF MEETING

Naval Research Laboratory, Building 27, Room 101.

ATTENDANCE

Laboratory personnel of Radio Division III:

Dr. R. M. Page
A. H. Schooley
P. Waterman
J. J. Fleming
A. M. King
J. H. Campagna
J. E. Meade
G. P. Walker
R. R. Riley
G. C. Collins

Dr. D. C. Harkin
L. R. Sneiderman
R. J. Mackey
H. C. Humiston
C. M. Morrow
W. L. Krewson
M. S. McVay
C. W. Stoops
L. F. Gilchrist
C. E. Francis

F. M. Gager
J. P. Spalding
C. L. Key
C. H. Dodge
J. W. Titus
C. E. Corum
C. R. Ahern
R. E. Gaylord
W. C. Hodgson
A. S. Locke

Visitors:

Capt. Cogswell	CNO	R. H. Jones	BuShips
Comdr. Herold	CNO	R. O. Mather	AAF
Lt.Cdr. F.M. Ralston	BuAer	A. Shostak	ONR
A. Weinstein	BuAer	W. M. Richards	ONR
H. F. McPherson	BuAer	D. B. Houghton	Franklin Inst.
J. H. Phillips	B.S.O.	R. H. Larson	APL-JHU
C. D. Perrine, Jr.	Fairchild	G. B. Harris	APL-JHU
V. J. Profumo	Fairchild	W. A. Good	APL-JHU
J. G. Barry	Fairchild	R. Powelson	Consolidated-Vultee
H. Blackstone	Servo Corp.		
F. G. Willey	Servo Corp.		

[redacted]

REPORTS OF PROGRESS ON LARK

Ground Control Stations: To implement the research, development, and testing program for the beam rider receiver for the Lark airframe, both an SP and an SP-1M radar are being modified to serve as ground control stations. The SP radar is located at the Chesapeake Bay Annex of the Naval Research Laboratory and will be utilized for airplane simulated missile flights for either beam rider or command type control. The SP-1M will be utilized as a ground control station for actual missile flights.

The SP radar has been modified to obtain automatic tracking in bearing and elevation. The modification is sufficiently great that the resulting equipment is completely redesigned radar. An automatic range unit will be installed at a later date. The antenna has been instrumented with boresighted cameras for optically recording the position of the beam rider airplane. Instrumentation has been provided for recording the position of the antenna in bearing and elevation. A cross-pointer meter is provided to indicate the position of the beam rider with respect to the beam; a photographic record of this is maintained. The installation of the modified SP at CBA was completed sufficiently for use as a ground control station in August; the emphasis of modification work has been put upon the SP-1M in order to meet BuAer requirements for missile guidance.

The SP-1M, previously utilized for flight tests, has been returned to Radio Division III at the Naval Research Laboratory. It is estimated that this equipment will be modified for use as a ground control station by the spring of 1948. The modification of the SP-1M requires more work than the SP inasmuch as two receivers and a double range gate will be required for missile tests. The SP-1M will automatically track a target with the second receiver gated on the missile beacon to indicate error from the radar beam. This equipment must be instrumented fully to serve its purpose as a mobile ground control station for missile test flights.

Beam Rider Receiver AN/APW-4: On 17 July 1947, the first fully automatic flight of the NRL beam rider receiver was made in an SNB airplane using a modified P-1 autopilot. The radar beam source was the modified SP-1M located at the Chesapeake Bay Annex. The SNB was flown automatically to a distance of 45 miles on the initial flight.

It is particularly significant that this and succeeding flights were made over water at radar beam elevation angles under four degrees, at which angles radar tracking with the SP-1M is difficult because of surface reflection. The guiding was sustained through areas in which the missile echo at the radar was obliterated by surface echo clutter.

[redacted]

In making these tests, the SNB was flown at a fixed bearing and a constant altitude of approximately 1000 feet over the ground control station to a distance of from three to five miles. The flight was tracked by the SP-1M until the plane was at the desired elevation angle with respect to the ground control station. At a given signal, the SP-1M beam was held stationary and the beam rider control was made automatic. The plane then continued out the beam in fully automatic control. In some of the runs the radar beam was moved, both in bearing and elevation, to observe the behavior of the automatically controlled SNB under dynamic conditions.

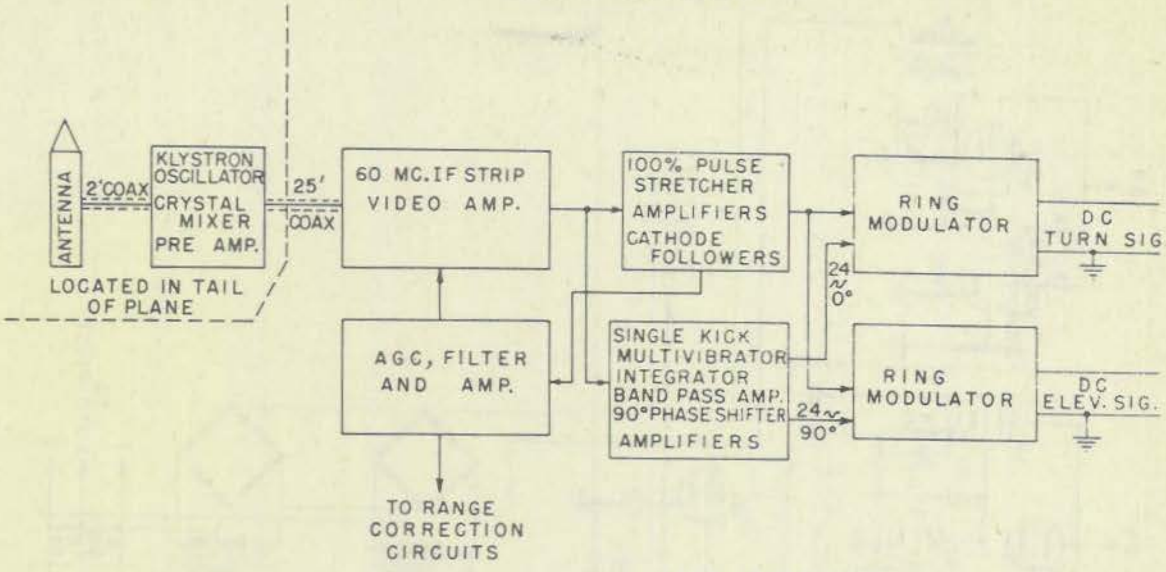
The outputs of the receiver, in addition to controlling the SNB through the P-1 autopilot, were fed to an analogue computer network shaped to conform to the transmission characteristics of the XSAM-2 Lark. The outputs of the computer network were then recorded. This recorded information qualitatively represents the response or dispersion positions of the Lark missile in flight with respect to the radar beam.

In addition to recording the simulated missile errors in the SNB, the plane was to have been photographed by cameras, mounted both on the radar dish and on a manually tracking handstand. Numerous operationally successful runs were made but no significant quantitative photographic data has been obtained. The maximum range of photographic data during these tests has been under 10,000 yards, because of atmospheric conditions. Continued failures of the photographic instrumentation has rendered the procurement of quantitative data extremely difficult.

The data recorded in the plane indicate the realistic error existing between the missile and the guiding beam. By displacing the radar beam a known amount a partial calibration of the error may be obtained. The SP-1M radar, used to date for these tests, has not had the servos modified to make the calibration of airborne recorded errors satisfactorily accurate. The immediate aim of the program is to procure satisfactory quantitative data.

<u>Receiver Circuits</u>: A block diagram, see Figure 1, and a circuit diagram, Figure 2, show the receiver to be essentially that previously reported at NRL Guided Missile Seminars for use with "WASP" simulator, with the exception of the additional ten-centimeter circuits.

[redacted]



LARK RECEIVER
BLOCK DIAGRAM

Figure 1

A dielectric type antenna is used and is coupled by a short length of coaxial cable to the 1N21B crystal mixer (X1) and the 726B Klystron (V1) ten-centimeter local oscillator (see Figure 2). This portion of the receiver was located in the tail of the plane.

In order to eliminate the necessity for a separate power source to supply the negative repeller voltage the circuit Figure 3 was developed. To compensate for varying plate voltage a VR tube, type OB2, is used in the cathode circuit to hold the repeller voltage constant. To compensate for varying filament voltage the resistance R72 was included in series with the OB2 regulator tube. On Figure 4, is shown a plot of the resulting frequency vs power supply variation. As can be seen, the frequency is held within the bandwidth of the intermediate frequency amplifier following, with a power supply variation of ± 10 o/o or from a maximum voltage down to 77.5 o/o of starting voltage. Temperature, pressure, and warm-up time vs. frequency measurements were made on an unselected 726B.

The 60 mc/s best frequency output from the crystal is fed to a 6AK5 (V2) preamplifier through a pi coupling network and the amplified signal is coupled by an output transformer through a 25-foot section of coaxial cable to the receiver proper.

[redacted]

LARK RECEIVER

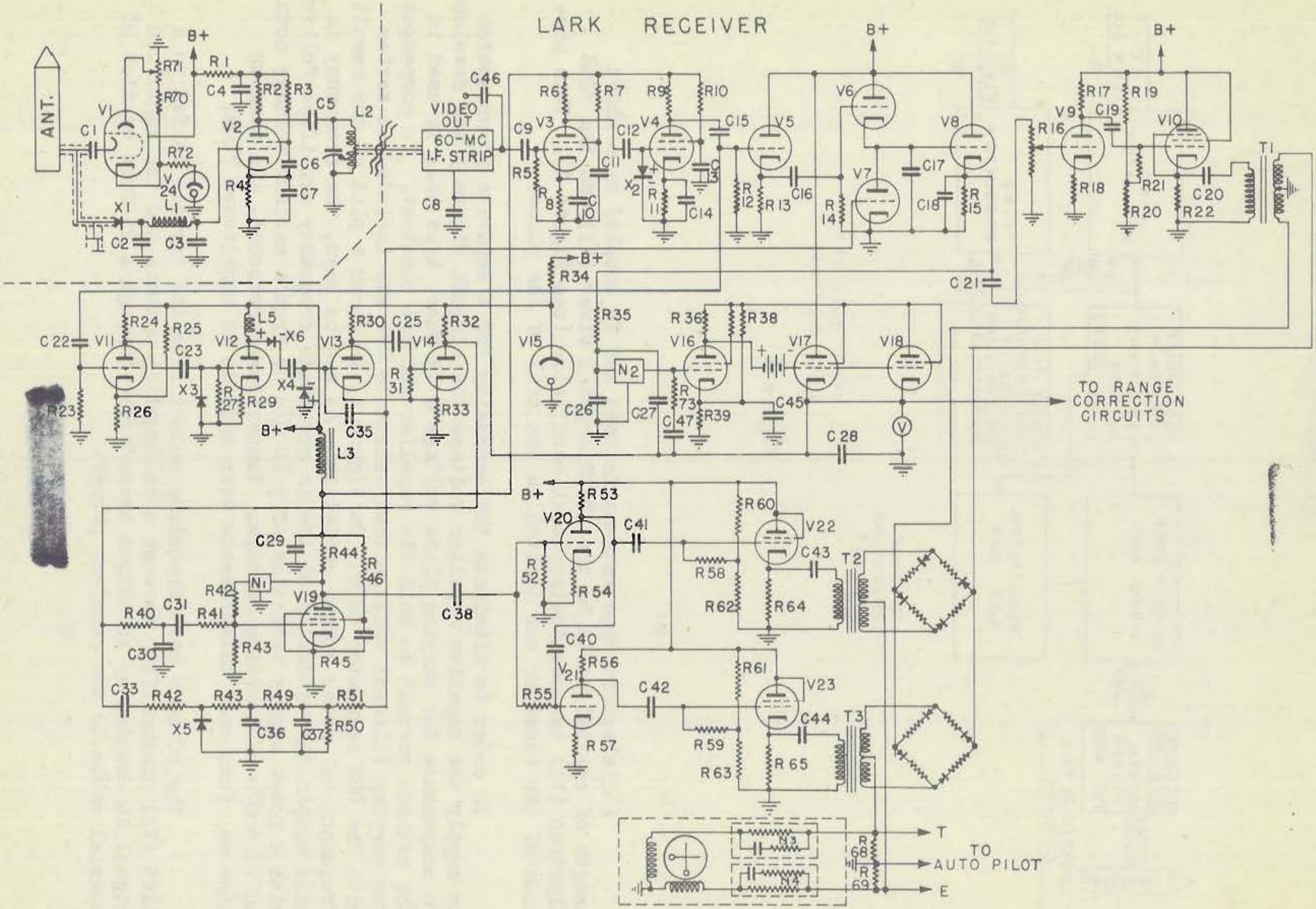


Figure 2

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726 B KLYSTRON
CIRCUIT CONFIGURATION

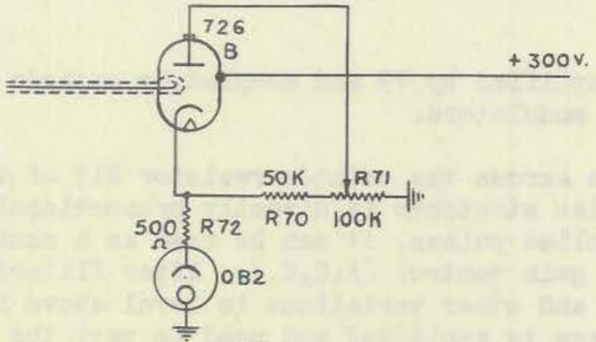

Figure 3

726 B KLYSTRON
FREQUENCY VS. TOTAL SUPPLY
VARIATION
(AT 25°C, 100% E_P=300v, 100% E_F=6.3v)

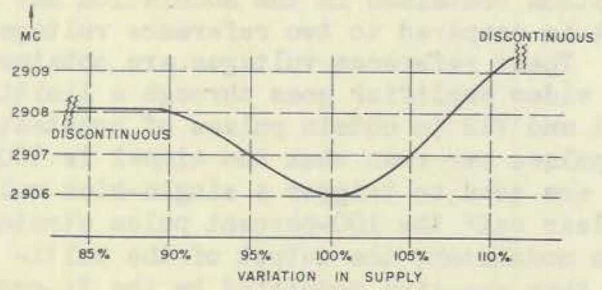

Figure 4

The signal from the coax is amplified in a commercially built 60-mc IF amplifier strip. The gain of the IF strip is controlled by varying the applied plate voltage in such a manner that the average or unmodulated pulse output is held constant within one db with a change of input signal from 20 microvolts to over 20 millivolts. The time delay of the AGC circuit is such that it holds the gain constant for relatively short fade-outs of the signal and thus prevents large increases in noise that would otherwise occur. The time delay must also be short enough to take care of the rate of change of signal as the missile goes out the beam.

After amplification in a two-stage video amplifier (V3 and V4) the signal is routed simultaneously through a cathode follower (V5) to a 100-percent pulse stretcher (V6 and V7) and through a clipping and differentiating amplifier (V11 and V12) to a single-kick multivibrator (V13 and V14). Considering first the operation of the 100-percent pulse stretcher it will be noted that any particular pulse causes to be impressed on condenser C17 a charge which is directly proportional to the amplitude of the pulse. The charge remains on the condenser until a fraction of a microsecond before the next pulse arrives. It is "cleared out" or discharged as the result of the arrival of a sharp positive pulse on the grid of the normally non-conducting section of the tube (V7). Bias for this tube is gotten by rectification of part of the square-wave output from the single-kick multivibrator by the crystal X5. The output after filtering, about six volts, holds V7 normally cut off. When the received signal is absent, this bias voltage disappears and V7 tends to hold capacitor C17 discharged.

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The voltage across R15 is a minimum as a consequence and the AGC voltage is maximum making the receiver sensitivity also maximum. From this pulse stretcher a continuous 24-cycle amplitude modulation on the received signal is derived from the video pulses. The modulation is the result of the missile's being out of the beam center of the conically scanned lobe. The correct interpretation of this information depends upon comparison with a reference signal whose phase is fixed with respect to angular position from beam center.

The 24-cycle signal is amplified by V9 and coupled by cathode follower V10 to the crystal ring modulators.

Since the average voltage across the cathode resistor R15 of the tube V8 following the 100 % pulse stretcher is directly proportional to the average amplitudes of the applied pulses, it can be used as a source of control voltage for automatic gain control (A.G.C.). After filtering out the 24-cycle component by N2 and other variations in level above four cycles by R35 and C26, this voltage is amplified and used to vary the resistance of a pair of 6AK6's (V17 and V18). The voltage at the cathodes of these tubes is applied as the plate voltage for the six-tube IF amplifier controlling the gain in such a manner to hold the average amplitudes of the pulses constant. The percentage of amplitude modulation remains unaffected.

In order that the information contained in the modulation may be applied to its proper channel, it is compared to two reference voltages that are 90° apart electrically. These reference voltages are obtained as follows: The signal from the video amplifier goes through a limiting and differentiating amplifier V11 and V12 to obtain pulses of constant amplitude except for one or two pulses per scan when the signal is 100-percent modulated. These pulses are used to trigger a single-kick multivibrator (v13 and V14) and to "clear out" the 100-percent pulse stretcher V7. When the pulses are not time modulated, the output of the multivibrator is symmetrical and when they are time modulated by the 24-cycle reference signal, the output changes its symmetry 24 times per second. After integration and amplification by a feedback amplifier (V19) with a 24-cycle rejection network (N1) in the feedback path the signal is an undistorted constant amplitude 24-cycle voltage in time phase with the lobing generator at the transmitter. The use of a parrallel "T", or similar high rejection network in the feedback circuit to give a high "Q" bandpass amplifier is necessary in order that a reference signal will be continually available during short periods of signal fade-outs. In this circuit the "Q" was 12 and the signal decreased to half amplitude in approximately three cycles.

The resultant 24-cycle signal is split in phase to two 90° phased signals in V20 and V21, one of which is in correct phase for horizontal

[redacted]

reference and the other for vertical reference. Cathode followers V22 and V23 couple these to the corresponding ring modulators X7 and X8.

The outputs from the amplitude and phase sensitive ring modulators are the D. C. error signals; one being for train and the other for elevation.

Analogue-Computer Network: In order that the measured voltages give a true picture of the missile's position in flight as the controls respond to noise in the signal an analogue computer network must be inserted between the receiver output and the recording meters, and the receiver gain increased with range. The network very closely gives the same response curve as that calculated for the airframe. The network is of the form:

$$\alpha = \frac{(1\ p/.70715)}{2.0849\ (1\ p/1.3093)(1\ p/2.2588)(1\ p/5.0419)}$$

Receiver Performance: Flight tests have shown the receiver operates over a range of 1000 yards to 45 miles, the specified required range, with no signs of saturation at close range and well within the maximum sensitivity of the receiver at 45 miles. Extrapolation of sensitivity curves (see Figure 5) show that equally satisfactory operation should be available to 100 miles.

A frequency response measurement of the overall error channel of the receiver to sinusoidal variations of the percentage of 24-cycle modulations on the carrier shows an attenuation of less than 3 db at $\omega = 75$ or frequency of 12 cycles.

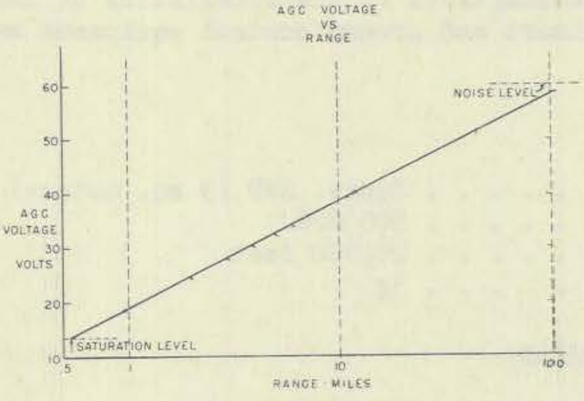

Figure 5

The response of the AGC circuit drops 3 db at a signal variation of 75 radians per second and the combined response of the AGC and the error channel is down 3 db at 15 radians.

Receiver Production Contract: Specifications were prepared for the beam rider receiver AN/APW-4 and competitive bids were obtained for the manufacture of fifty receivers. The specifications provided for two phases of receiver manufacture: phase one, the construction of ten receivers, reduced in dimensions as required for the Lark, for test and developmental use leading to a standardized design; phase two, the manufacture of forty receivers to the standard design arrived at by phase one. The Reeves Instrument Company, N.Y.C., were the successful bidders. The contract is under preparation at the Office of Naval Research and is expected to be awarded in the immediate future.

REPORTS OF PROGRESS ON SKYLARK

The initial phase of the Skylark contract is one of study and investigation leading to a choice of system development for the homing and mid-course guidance equipment. The terminal guidance system, at the request of the Bureau of Aeronautics, was given priority.

Objectives: The objectives of the development program for the terminal guidance system are the attainment of a passive homing test flight by 1 June 1948, and an increase in homing range over that predicted for the present AN/APN-23 active radar target seeker.

During the Skylark Conference of 20 April 1947, it was decided that both Fairchild and NRL would initiate a study of the limitations of various radar homing equipments having possible applications to the Skylark missile. The study at NRL was to be devoted to a comparison of the TAB and Meteor simultaneous lobe comparison systems as passive radar homing devices operating at S-Band frequency. The Fairchild study was to be devoted to scanning type radar homing devices operating at S-band frequency. It was agreed that on 18 September 1947, the study of the various radar homing systems would be evaluated and a selection would be made of one system for development for Skylark.

The following fundamental assumptions of characteristics of the missile, target, homing radar equipment and ground control equipment were made:

Characteristics of Target

Assumed Target	Equiv. SNB (3 sq. meters)
Target Velocity.	300 MPH
Altitude Target.	25,000 feet
Maneuvers of Target.	3g

Characteristics of the Missile

Configuration, spacing, etc.	The Lark Missile
Velocity.	Mach No. 0.85

[redacted]

Characteristics of Radar Homing Equipment

Frequency	S-Band
Type of homing	Passive type
Maximum lead angle	30°

Characteristics of SP-1M Ground Control Station

Ground Radar	SP-1M (as modified by NRL)
Frequency	S-Band
Peak Power	800 KW
Pulse width	1 microsecond
Pulse repetition frequency	576 pps
Scanning rate	24 cps
Antenna diameter	8 ft.
Crossover (down)	1.4 db 1 way
Approximately 100 o/o modulation	3° from crossover
Train and elevation servo bandwidth	below 1/2 cycle
Range gates automatic	± 10 yards accuracy

Criteria in the following order of importance were decided upon a basis for comparison of various homing systems:

a. Maximum Range

Curves shall be plotted of range from radar to target vs maximum homing range using established standard target and method outlined.

b. Availability of System Equipment

Under this item shall be listed the stage of research upon, design of or status of manufacture of such systems or components of systems being compared.

c. Bandwidth Requirements

This will involve a study of the bandwidth requirements of the overall guidance loop and the internal loop of airframe control in the absence of guiding signals. The terminal trajectory will determine these requirements in part; and the outputs of the homing receiver may be a determining factor for the type of trajectory. Susceptibility to jamming and extraneous noise will be considered.

[redacted]

d. Economics

Under this item will be considered:

1. The complexity of ground equipment; the time required to accomplish construction; and maintenance difficulties.
2. Complexity of airborne equipment - weight, space, servo power requirements, electrical power requirements, efficiency of operation, ease of installation, ease of design, etc.
3. System stability - including requirement to adjust and trim for operation.

e. Transition to Homing

Particular reference to test rather than tactical conditions will be observed in this consideration.

f. Target Discrimination

Both angular and range discrimination and derivatives of each shall be considered.

g. Detonation

The ability of the homing system to detonate the missile within 50 feet of the target will be considered.

TAB (Amplitude) Simultaneous Lobe Comparison System: The theory of operation and design of this system was studied in an effort to determine the possibility of adopting the present NRL X-Band TAB system to S-Band, and, secondly, to investigate the possible problems which may be encountered in the development of such a system for passive radar homing. It was immediately concluded that the system could be converted for S-Band use, but to adapt the system to such a missile as the Skylark presented many difficult problems. The major problem to be encountered in such a development is that of antenna design. Since the maximum diameter of the missile at the forward section is 11 inches, the receiving antenna in the missile is limited to 9.25 inches or less. The amplitude comparison system requires that four horns, or dipoles, be placed together as a point source, located at the focal point of a parabolic or lens antenna. Investigations indicated that because of the relatively large area of the horns with respect to the small area of the dish, (particularly in view of the use of S-Band frequency), such an antenna arrangement would introduce a considerable amount of r-f phase change along with the amplitude change.

In this case, where the focal length is short and the source of energy (area of the feed horns) is about 60 o/o of the size of the antenna aperture, a non-uniform r-f phase front will exist across the face of the antenna system, resulting in an r-f phase change along with the amplitude change. Previous studies made in the development of the TAB system indicate that although a system can be developed under these conditions (phase with amplitude), such a system will have a loss of angle sensitivity and will possess a high amplitude first reverse lobe with attending ambiguity.

Consideration was given to a wave guide filled with material having a high dielectric-constant to reduce the feed area, but it was concluded that the use of such a technique was a problem of long development having questionable end results.

It was therefore concluded that a pure amplitude comparison system at S-band frequency was not immediately practical for the Skylark application.

Meteor (Phase) Simultaneous Lobe Comparison System: In order to discuss the problems of developing an S-band phase type simultaneous lobe comparison system as a radar homing system for Skylark missile, it is necessary first to describe the theory of operation of this system. The Meteor system requires a receiver utilizing two channels for each plane. One local oscillator feeds two balanced mixers. The antennas are spaced a known distance apart. The r-f phase of the signals arriving at the antennas varies, depending on the relative positions of the target and the antennas. The changing phase is compared in a sensitive phase detector following the IF amplifiers. The output voltage is therefore proportional to the phase difference between the two antennas of each plane. As the receiver is driven to phase balance (null output from the phase detector) the coupled shaft of the servomechanism will rotate at a velocity proportional to the rotation of the tracking line.

Since the antennas are separated and fixed to the missile, a device is required in the Meteor homing system to remove the effects of the motion of the missile. The MIT staff, at the present time, is working on the development of two alternate devices to be utilized in the solution of this problem - a single sideband modulator and a goniometer operating at 30 megacycles per second. The Meteor terminal guidance system is in the developmental stage with considerable work remaining to be accomplished. It is believed that the time required to adapt this system to Skylark would not be compatible with the time allotted for the Skylark development. Further, since an extremely capable MIT staff is engaged in a complete exploration of this type of homing system, the duplication of this effort is not warranted. Therefore, the phase comparison system was not recommended for use in the Skylark program.

[redacted]

High Speed Electronic Lobe Switching System: The high speed electronic lobe switching system, using r-f switch tubes, developed by NRL, was studied for possible application to the Skylark program. Investigation indicated that a high-speed lobing system would have advantages, but the tubes now in use are for X-band; no assurance could be obtained that S-band switch tubes would be developed in time to meet the requirements of the Skylark schedule.

Conclusions: Because of the contractural requirement of producing a working radar homing equipment within eight months, it is concluded that neither the amplitude nor phase types of simultaneous lobe comparison systems, nor a high-speed electronic lobe switching system can be adapted for the Skylark program within this time.

Recommendations: It is recommended that a conical scanning type radar homing equipment, utilizing standard r-f and receiver components, be adapted for the Skylark program.

It is recognized that, depending upon configuration and size of guided missiles in future studies, advantages of fixed antennas and/or reduction of moving parts may accrue to the favor of simultaneous lobe comparison, and electronically scanned radar homing equipments. It is therefore recommended that, when the workload of the Radio Division III, NRL, permits, a fundamental, mathematical, and theoretical study to determine the advantages of each of these equipments for use with radar homing systems be initiated. If sufficient advantages should accrue from this study in favor of one or more of the systems, it is then recommended that suitable work projects be set up to prove the theoretical analyses.

Conical Scanning Radar Homing Equipment: In view of the foregoing conclusions and recommendations, it was deemed necessary to study conical scanning radar equipments for the specific use of the Skylark homing system. The investigations made are preliminary in character and three-fold in nature: (a) maximum antenna dish diameter and estimated space requirements for antenna gimbals and servos, (b) circuit details and weight estimates, and (c) calculations to determine the theoretical maximum tracking ranges at various target to illuminating radar ranges.

Antenna Design: The preliminary investigations for the design of the antenna indicate that the antenna dish may be a maximum of approximately 9" in diameter (this corresponds to a beamwidth of 33° with a gain of 27). The antenna design investigation indicated that it was possible to spin the dish at 10,000 R.P.M. with the dish offset at such an angle as to provide a lobe crossover point at 3 db down (one-way). Considerations of the tactical problem indicate that the antenna must be capable of a minimum of 30° lead angle in any direction from the forward heading of the missile. It is estimated that the entire antenna and scanning assembly including gimbals and servo motors can be designed to weigh less than thirty (30) pounds.

Circuit Details and Weight Estimates: The table below is a list of estimated weights and tube complements for the various circuits of the receiver. The estimates are based upon the use of miniature-type tubes and are subject to weight reductions if the use of sub-miniature tubes is found practicable. The block diagram of a typical receiver is shown in Figure 6.

Components	Weights	Tubes
Local Oscillator	3.0 lb.	1
Preamplifier (if necessary)	2.0 lb.	2
I-F amplifier.	3.0 lb.	6
Detector, Video.	2.0 lb.	3
Automatic Frequency Control.	3.0 lb.	4
Automatic Gain Control	2.0 lb.	2 to 3
Gate Circuit (to remove transmitter pulse)	3.0 lb.	
Total.	18.0 lbs.	19 tubes approx.

TYPICAL RECEIVER BLOCK DIAGRAM
FOR A
PASSIVE HOMING SYSTEM

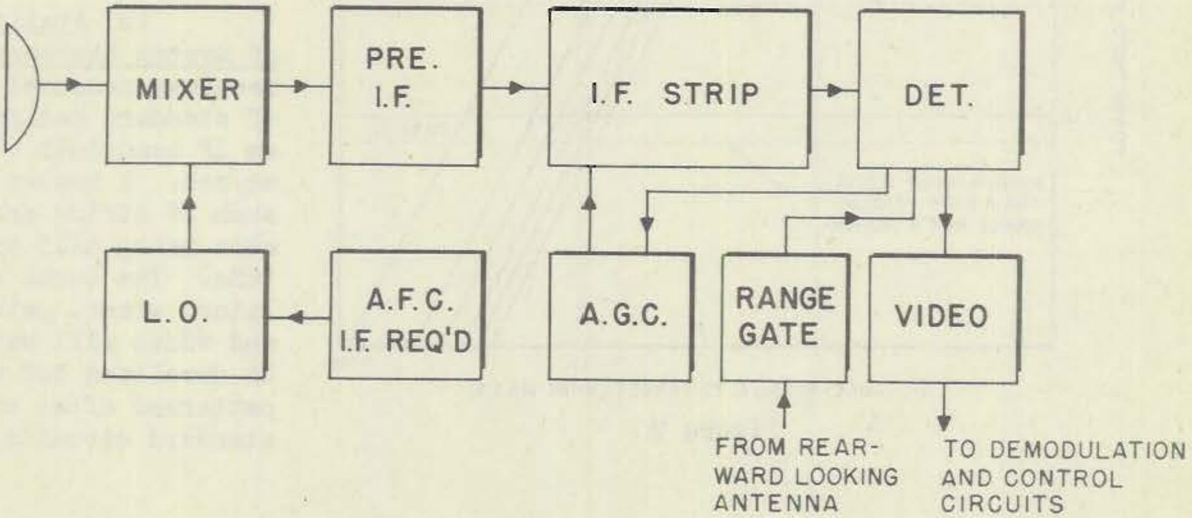

Figure 6

[redacted]

A number of criteria were proposed for consideration in this survey. Although these criteria have not been considered completely, the following comments are appropriate at this time.

(a) Maximum Range. The theoretical maximum homing range of this proposed passive target seeker shown in Figure 7 is based on the use of the SP-1M modified radar illuminating a target having a reflecting area of three square meters. Two empirical limits of receiver design were chosen as Case I, illustrative of conservative theoretical design, and Case II, illustrative of theoretical high performance operation. From previous experience, it would be expected that normal operating performance would fall within these limits.

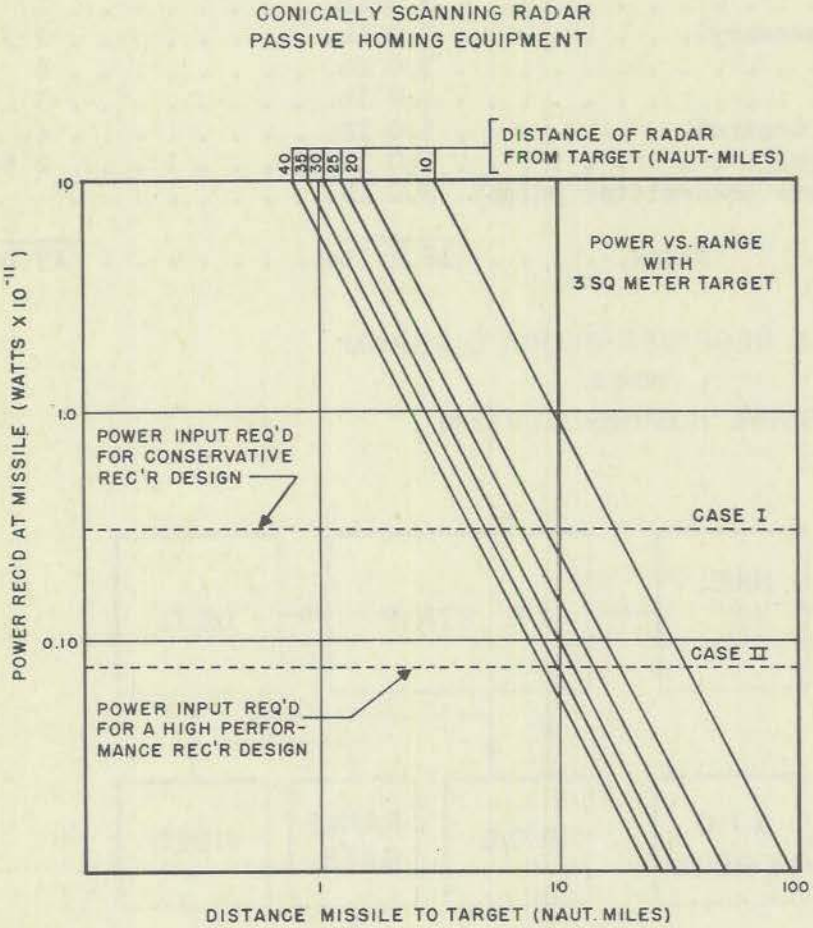

Figure 7

These curves indicate that with a target-to-ground radar range of 30 miles, it will be possible to begin tracking with the homing system at about 6 miles range. A number of conservative assumptions were made when calculating this figure. With more optimistic assumptions it appears possible to begin tracking the target at 12 miles missile-to-target range.

(b) Availability of System Equipment. The receiver proposed is one of standard design with an IF bandwidth of 1.5 mc/sec. A number of such IF strips are available using 6AK5 type tube. The local oscillator, mixer, gate circui and video will have to be developed but will be patterned after existing standard circuits.

(c) Bandwidth Requirements. The servo bandwidth requirements of the overall guidance loop are now under study as part of the control system. The susceptibility to jamming and extraneous noise is a problem common with that of all S-band radars during this past war. The best available designs will be incorporated into this system.

(d) Target Discrimination. This system shall be designed so that it may discriminate against targets separated 200 yards nominally, in radar range. This figure is based on the characteristics of the SP-1M radar.

DISCUSSION

Mr. Joseph P. Spalding, Radio Division III, Naval Research Laboratory, presented a paper entitled "Discussion of FM-CW Oscillator for Wasp Missile Control". A copy of this paper forms Appendix 1 of this report.

Mr. W. S. Ament, Radio Division I, Naval Research Laboratory, has written a memorandum entitled "Discussion of the Effect of Random Noise on Missile Guidance". This memorandum is included in this report as Appendix 2 by virtue of its relationship to the report of noise tests given in the text.

Mr. C. E. Corum and Mr. C. L. Key, Jr., Radio Division III, Naval Research Laboratory have written a paper entitled "Determination of Proper Bandpass for Automatic Correction of Range for Lark". This paper is included in this report as Appendix 3 as relevant to the report on the beam rider receiver.

[redacted]

J. P. Spalding
Guided Missile Seminar
18 September 1947

APPENDIX 1

DISCUSSION OF FM-CW OSCILLATOR FOR WASP MISSILE CONTROL

Original specifications for the beam rider system required transmission of control intelligence to the Lark receiver by pulse time modulation from the SP shipborne radar. Receivers now under test are designed to accept and to provide guiding intelligence for the Lark missile from pulse information in the beam. From flight tests conducted in the past, sufficient information has been obtained to verify the basic soundness of design concept, and work is still proceeding on precise evaluation of receiver performance.

Although a discussion of present missile receiver design was given in the Lark Seminar of June 12, a brief review will help to clarify the purpose of this discussion. Conversion of information from S-band to video frequencies is accomplished with a conventional superheterodyne receiver. Guiding intelligence for the Lark is then extracted from information carried by very short pulses. As a result, fairly complex circuits are required to convert these pulses into 24-cycle sine-wave signals of correct phase, waveform, and level. The AGC problem adds further complication in circuitry by the necessity of securing tight AGC control from a short duty cycle repetition rate.

To perform its basic functions with pulse operation, at least 25 tubes are required by the present missile receiver.

In addition to receiver complexity, certain modifications are necessary in order to adapt the SP radar for radar beam control of the Lark missile. To provide pulse time modulation a timing generator and thyratron modulator were required to replace the spark gap modulator.

It is the purpose of this discussion to suggest an alternative method of missile control which should simplify considerably modification of the SP radar and design of the missile receiver.

In this proposed system, a CW oscillator will be incorporated into the SP radar without interfering with its present operation. The CW oscillator will be frequency modulated at the lobing rate for reference voltage. Error voltages will be produced by the normal antenna scanning process. The frequency separation between the CW oscillator and the radar will be sufficient to provide pulse rejection at the receiver.

The bird receiver design would then be considerably simplified by elimination of circuits necessary for conversion of pulse into sine-wave information. It is estimated that approximately 15 tubes can be eliminated by the use of CW operation.

Pulse rejection will be accomplished in the head end of the receiver by a resonant cavity. The IF portion will consist of three stages of 30-mc IF amplification, followed by limiter and discriminator stages. AM modulation or error signal will be recovered by grid-leak detection in the limiter stage, and FM modulation or reference signal by a conventional FM discriminator. Receiver stability will be maintained by automatic frequency control of the local oscillator. Conversion of the 24-cycle sine-wave error and reference signals into guiding information for beam rider control will be accomplished by circuits similar to those in the present receiver.

To provide the CW power necessary for reliable operation of the Lark receiver, the following calculations indicate that 10 watts of oscillator power will provide a signal level of 320 volts across the 50-ohm receiver input, at a fifty-mile range. This signal strength is more than is required to insure noise-free operation.

Calculations:

Transmitting power gain . . .	2900
Receiving antenna gain. . . .	10
Transmitter power.	10 watt

$$P_i = \frac{P_t A_e{*}}{4\pi D^2}$$

$$= \frac{2.9 \times 10^4 \times 12}{4\pi(100{,}000 \times 36)^2}$$

$$= \frac{3.48 \times 10^4}{12.5 \times 13 \times 10^{12}}$$

$$= 2.1 \times 10^{-9} \text{ watts}$$

$$A_e = \frac{G\lambda^2}{4\pi}$$

$$= \frac{10 \times 100}{4 \times 3.14}$$

$$= 80 \text{ sq. cm or } 12 \text{ sq. in.}$$

At 50-ohm input to receiver

$$P = \frac{E^2}{R}$$

* BTL radar range calculator[redacted]

[redacted]

$$E = 2.1 \times 5.0 \times 10^{-8}$$

$$= 320 \quad \text{volts}$$

Symbols: G = receiving antenna gain
A_e= effective receiving antenna area

P_t= transmitting power gain

The principle of utilizing a CW oscillator ror missile control provides a technique that can be applied with slight modification to any radar system employing lobing for angular information. It eliminates the necessity of changing spark gap modulators to electronically keyed modulators for pulse time modulation. With such systems as the SCR 584 which employ electronically keyed modulators, the CW FM system will still simplify receiver design.

This type of guidance could also be added to our new type radars known as High-Speed lobing and TAB systems. In the High-Speed lobing system, angular tracking information is obtained by receiver lobe-switching because of the low power-handling ability of present gas switch tubes. This obstacle to Wasp control functions can be eliminated by the introduction of low-power CW into the receiving wave guide systems. In the TAB simultaneous lobing system, guiding intelligence could be similarly transmitted by modulating the beam radiated by the CW oscillator instead of the radar transmitter. The four beams projected for tracking purposes could be modulated at four different rates for missile control.

Another application for the use of low power CW for short ranges would be to transmit on two CW frequencies which were separated by the IF frequency of the missile receiver. This would eliminate the need for a local oscillator in the receiver with the attendant advantages of stability and elimination of operational adjustments prior to launching.

Another advantage of the CW system would be to eliminate the conflict in optimum beam crossover points which arises from the use of a radar system for both tracking and beam riding functions. This difficulty can be overcome by using the CW system at 3 cm and tracking system at 10 cm.

The use of CW transmission thus provides a versatile tool for addition of missile control functions to tracking radar systems.

Two additional applications for CW transmission have been suggested. One would be to provide target discrimination for passive homing information

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by comparing in the receiver the doppler shift between transmitter signals and signals reflected from the target. This could also be accomplished by the FM system of range detection.

The second suggestion is the use for long range guidance. Narrow-band CW transmission may require simpler instrumentation for missile control.

At the present time, investigation is being made of the frequency stability of currently available oscillators in order to determine their modulation index characteristics. This information will dictate the bandwidth requirements for a test missile receiver. Preliminary results indicate wide bandwidths will be necessary with available klystron oscillators. Although narrow-band FM would be more desirable, work is already underway to construct or modify equipment suitable for proper evaluation of missile control by CWFM techniques.

W. S. Ament
Radio Division I
Guided Missile Seminar
18 September 1947

APPENDIX 2

DISCUSSION OF THE EFFECT OF RANDOM NOISE ON MISSILE GUIDANCE

This discussion is intended to call attention to the fact that the effect of random electrical noise in causing a guided missile to wander from its ideal course can be at least partially predicted by well-known mathematical methods. A full discussion of such methods applying to the linear case is to be found in the article "Stochastic Processes in Physics and Astronomy" by S. Chandrasekhar, Reviews of Modern Physics, Vol. 15 No. 1, January 1943. Radiation Laboratory Report No. 129 (V-16S), "Response of a Non-Linear Device to Noise", by N. Wiener, 6 April 1942, contains concepts applying to the non-linear case, if this case need be examined. (The concepts 'linear' and 'non-linear', as applying to the missile problem, will be qualitatively defined later in this discussion).

Here are statements of three types of noise problems which may be completely or partially solved by the above mathematics:

First, suppose it has been experimentally determined that a sinusoidal signal, of power P and frequency f, causes the missile to follow a sinusoidal path in the beam, a path whose maximum distance from the center line of the beam is found to be $A(f)\sqrt{2P}$. Suppose the noise power per unit bandwidth which is expected to act on the missile is $N(f)$. $N(f)$ is the so-called noise spectrum. Then the solution for the expected distance of the missile from the center line of the beam it will ride is given as follows: - determine the quantity $\langle\bar{x}^2\rangle$ according to the formula:

$$\bar{x}^2 = \int_0^\infty A^2(f)\, N(f)\, df \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

Then, the probability of finding the missile in a range dx at a distance x from the center line of the beam is given by:

$$\frac{dx}{\sqrt{2\pi\langle\bar{x}^2\rangle}}\, e^{\frac{-x^2}{2\langle\bar{x}^2\rangle}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

In the above solution the missile path was considered as lying in, say, a horizontal plane. It is a mathematically simple step to combine the formula for vertical deviation, in order to obtain the probable radial displacement of the missile from the center of the beam.

This problem is also <u>linear</u> in the sense that the amplitude of the lateral oscillation of the missile to a voltage of a given frequency (this voltage being superimposed on the voltage field composing the beam) is proportional to that voltage.

A second problem which is mathematically tractable is the following: Suppose that on a test-bench set-up, one measures the deflection of the control surfaces of the missile control unit as function of the amplitude and frequency of the input signal. The aerodynamic assumption is now made that the lateral acceleration of the missile is as a known function of the deflection of the control surfaces, and that the acceleration is a linear function of the deflection for small enough deflections. Assignments are made of probable values of error voltage as function of lateral displacement from the control line, and of the expected noise spectrum. Sufficient data are now at hand to calculate the probability of finding the missile at a given lateral distance from the control line as function of time of flight. It turns out that after a while the effect of the initial position and velocity of the missile will have died out, and the probability distribution of the missile's lateral position is like formula (2). But it is also possible to compute, as function of time, the wandering of the missile away from the ideal path in the noise-free beam. Mathematics for this problem is completely worked out for the linear first approximation (in connection with another problem) in Chapter II of the Chandrasekhar article.

A third problem arises when the deflection of the control surfaces is bounded by the installation of stops in the missile. It is believed that a full mathematical treatment can be given to finding the random and controlled wandering of the missile in the linear first approximation. In this case, by linear is meant that, for a given frequency, the deflection of the control surfaces is proportional to the input voltage up to the maximum deflection allowed, and that the lateral acceleration of the missile is at all times proportional to the deflection of the control surfaces. Mathematics bearing on this problem is to be found in Chapters I and II of Chandrasekhar's article.

The fact that the noise power may persist, whereas the controlling signal fades out as the missile flies away from the radar, will force inclusion of a time-dependent term in the equations of Chapter II of Chandrasekhar's article, but it is believed that the resulting equations can be solved by an extension of the methods of that Chapter.

These problems will become very, if not excessively, complicated if taken too far beyond their linear first approximations. This will be the case if a signal persists long enough to cause the missile axis to make a large angle with respect to the control line, in which case the lateral acceleration (acceleration of the center of the missile away from the control line) may no longer be considered a linear function of the signal voltage. Aerodynamical slippage and mechanical looseness also contribute to the non-linearity. The mathematics of the Wiener report will probably

[redacted]

prove helpful when certain types of non-linearities are introduced, but undoubtedly not for all types. However, when the expected noise powers at the important frequencies are small, the linear approximation should be quite sufficient.

C. E. Corum
C. L. Key, Jr.
Guided Missile Seminar
18 September 1947

APPENDIX 3

DETERMINATION OF PROPER BANDPASS FOR AUTOMATIC CORRECTION OF RANGE FOR LARK

It is desirable that the automatic range control system for the Lark receiver have as narrow a bandwidth as possible, in order to keep out of the ARC voltage as large a percentage as possible of signal fluctuations due to missile maneuvers, target aspect changes, and random changes in propagation characteristics.

Assuming that a simple low-pass filter is used, the following relationship holds:

$$E_0 = \frac{\omega_1}{\omega_1 + p} E \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where E_0 = output voltage

E_{in} = input voltage

ω_1 = corner frequency of filter

p = operator d/dt.

This equation may be rewritten as a conventional differential equation:

$$\frac{dE_0}{dt} + \omega_1 E_0 = \omega_1 E_{in} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

This is a linear first-order differential equation. Consequently, we may write the solution immediately:

$$E_0 = Ke^{-\omega_1 t} + e^{-\omega_1 t} \int E_{in}\, \omega_1 e^{-\omega_1 t} dt \quad \ldots\ldots\ldots\ldots \quad (3)$$

The input voltage for the Lark receiver, as determined both theoretically and empirically under the direction of Mr. Peter Waterman, is a logarithmic function of range between the limits 1 and 45 miles. If the missile travels at constant speed in a straight line, the input voltage is a logarithmic function of time also, and may be written in the form

$$E_{in} = A + B \ln (Ct+1) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

assuming t = 0 at 1 mile range, and using $\ln$ to denote the natural logarithm. The constants may be evaluated from the empirical data.

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[redacted]

Substituting this function into equation (3), we find that it is necessary to evaluate the integral

$$\int \omega_1 \left[A + B \ln(Ct+1)\right] e^{\omega_1 t}\, dt$$

$$= \int \omega_1 A e^{\omega_1 t}\, dt + \int B \ln(Ct+1) \cdot \omega_1 e^{\omega_1 t}\, dt$$

$$= A e^{\omega_1 t} + B \int \omega e^{\omega_1 t} \ln(Ct+1)\, dt \quad \ldots\ldots\ldots\ldots \quad (5)$$

To evaluate the remaining integral, it is necessary to integrate by parts:

$$\int \omega_1 e^{\omega_1 t} \ln(Ct+1) \cdot dt = e^{\omega_1 t} \ln(Ct+1) - \int \frac{e^{\omega_1 t} \cdot C\,dt}{Ct+1}$$

$$= e^{\omega_1 t} \ln(Ct+1) - \int \frac{e^{\omega_1 t}\, dt}{t + \frac{1}{C}} \quad \ldots\ldots\ldots\ldots \quad (6)$$

The last integral here is evaluated by the substitution $X = \omega_1 (t + \frac{1}{C})$ and becomes

$$e^{-\omega_1/C} \int \frac{e^X}{X}\, dx \quad \ldots\ldots\ldots\ldots \quad (7)$$

which may be found in the tables or integrated by expanding e^x in series. It is found that

$$\int \frac{e^X}{X}\, dx = \ln X + \sum_1^{\infty} \frac{x^{\eta}}{\eta!\,\eta} \quad \ldots\ldots\ldots\ldots \quad (8)$$

The general solution of (2) then becomes

$$E = K e^{-\omega_1 t} + A + B \ln(Ct+1) - B e^{\omega_1 (t + \frac{1}{C})} \left[\ln \omega_1 (t + \tfrac{1}{C}) + \sum_1 \omega_1^{\eta} \frac{(t + \frac{1}{C})^{\eta}}{\eta!\,\eta}\right]. \quad \ldots (9)$$

Using the initial conditions, K may be expressed as

$$K = B e^{-\omega_1/C} \left\{ \ln \frac{1}{C} + \sum_1^{\infty} \left[\left(\frac{\omega_1}{C}\right)^{\eta} \frac{1}{\eta!\,\eta} \right] \right\} \quad \ldots\ldots\ldots\ldots \quad (10)$$

On consulting the introduction to the WPA "Tables of Sine, Cosine and Exponential Integrals", Vol. I, we find that

$$E_i(X) = \sum_1^{\infty} \frac{X^{\eta}}{\eta!\,\eta} - \ln X - \gamma \quad \ldots\ldots\ldots\ldots \quad (11)$$

where $E_i(x) = \int_{-\infty}^{x} \frac{e^t}{t}\, dt =$ exponential integral of x, and γ .577216 = Euler's

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constant. This provides a means of evaluating the series for values of x up to 10, and, to less accuracy, up to 15.

If it is desired to evaluate the series for larges values of x, the necessary values of $E_i(x)$ may be obtained from the asymptotic series

$$E_i(x) = e^x \left(\frac{1}{x} + \frac{1}{x^2} + \frac{2!}{x^3} + \frac{3!}{x^4} + \cdots \right) \qquad \ldots\ldots \quad (12)$$

The terms should be summed until they stop decreasing. The error is of the order of magnitude of the last term used.

Graphs of the solution for $\omega_1 = 1.$ .2 and .02 radians per second and a missile speed of 935 feet per second are given as Figure 8 and 9.

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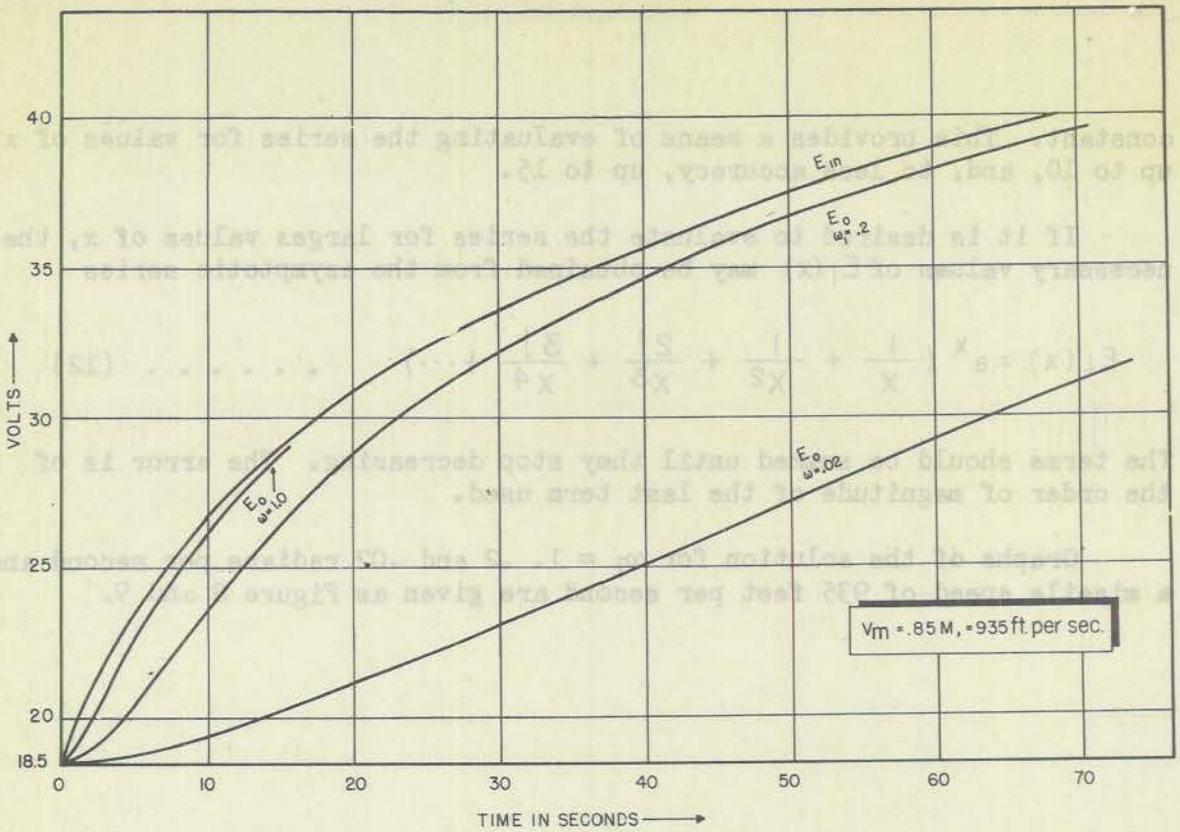

Figure 8

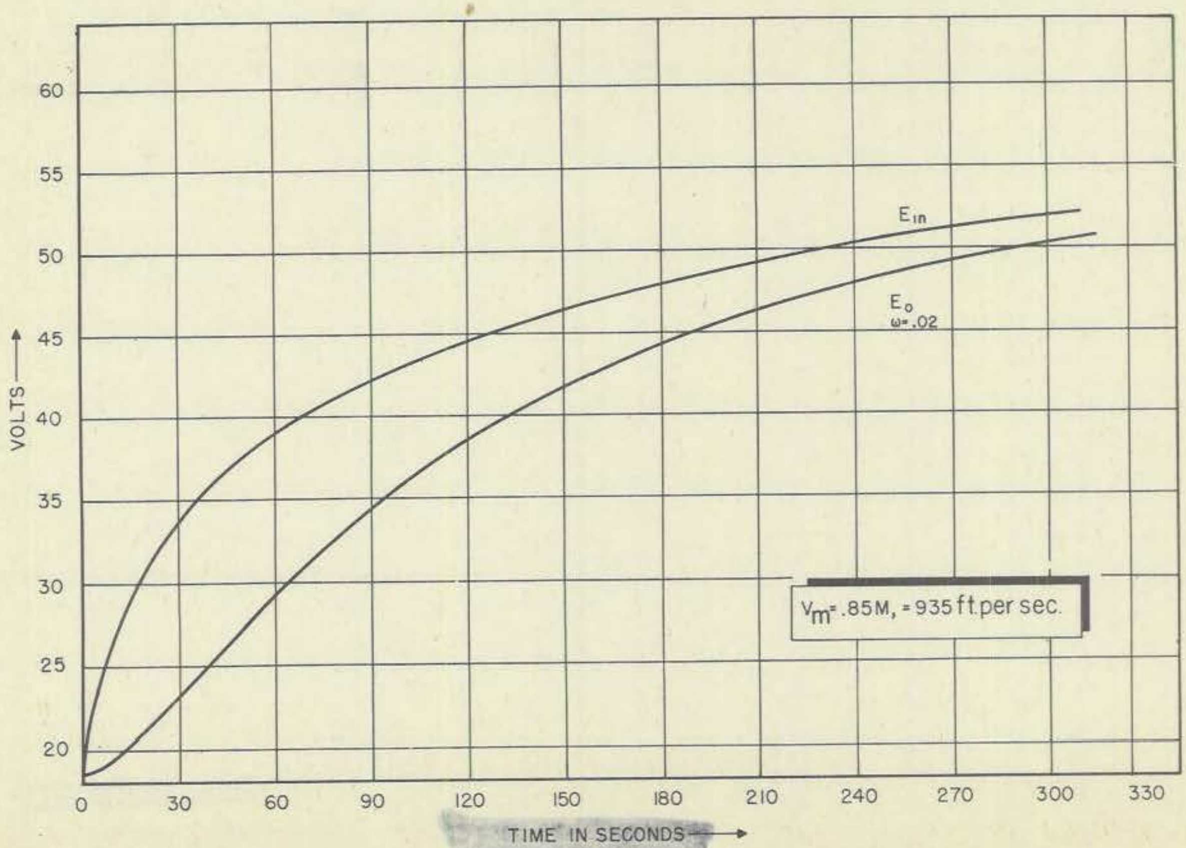

Figure 9