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ANTENNA FEEDS FOR TRACKING RADARS - II

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NRL REPORT R-3334

ANTENNA FEEDS FOR TRACKING RADARS - II

A. E. Hastings

August 13, 1948

Approved by:

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ABSTRACT

The study of the dependence of angle and range sensitivities of a tracking radar on the design of antenna feed is continued. A less approximate solution allows the specification of the size and position of the elements in the feed. Applications have been made to two types of experimental radars now operating. An application to a simple feed has been verified experimentally.

PROBLEM STATUS

This is an interim report on one phase of this problem.

AUTHORIZATION

NRL Problem No. R12-01D (BuOrd request No. 020).

ANTENNA FEEDS FOR TRACKING RADARS - II

INTRODUCTION

In a previous report[1], the influence of the antenna feed design on certain characteristics of a lobing radar was discussed. Of particular interest were the on-target angle and range sensitivities. The discussion assumed uniform intensity of illumination of a lens or reflector by the feed, expanded the expression for the secondary patterns of each lobe about the on-target position, and combined these expressions in a manner corresponding to the behavior of a number of radar systems with various types of antenna feeds. The results were applicable only close to the on-target position and for small primary feed apertures. The positions of feed aperture for certain desired characteristics were determined, but the study could not give any information about optimum size of primary aperture.

The more general analysis of the present report assumes uniform intensity of illumination, not of the lens or reflector, but of the primary aperture, such as a feed horn. The results are general enough to apply to any of the feeds previously described, and not being restricted to small angles about the on-target position, can predict the secondary antenna patterns and the angluar error signal over a wider range of angles. Such error signal functions have been calculated to determine the most desirable feed design for two radar systems of particular interest and for a simple two-aperture system to check with experimental measurements.

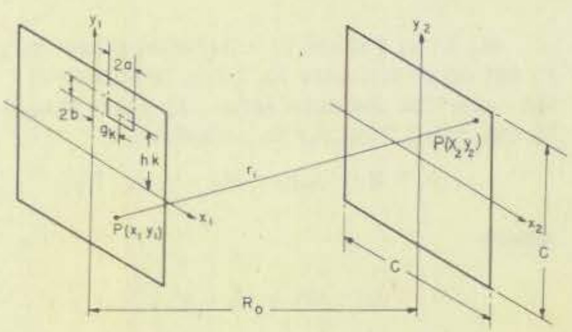

Fig. 1 - Primary and Secondary Apertures

SECONDARY PATTERN FROM A NUMBER OF PRIMARY APERTURES

Radiation of uniform intensity and wavelength λ falls on a number of rectangular apertures n with sides 2a x 2b in the x_1, y_1 plane, one aperture only being shown in Figure 1. The intensity at the apertures is such as to maintain constant total power through the plane, so that the intensity is proportional to $1/abn$. The field is then proportional to $1/\sqrt{abn}$. The centers of apertures are at g_k, h_k, where $k = 1,2,\ldots n$. The radiation may be completely out of phase at certain apertures; a factor α_k, having values of ± 1, allows this. If r_1 is the distance between any point $P(x_1, y_1)$ and any point $P(x_2, y_2)$ in a plane parallel to the x_1, y_1 plane and distant R_0 from it, the field at the point $P(x_2, y_2)$ due to the radiation from

[1] Hastings, A. E., "Antenna Feeds For Tracking Radars," NRL Report R-3268, 29 March 1948

 [redacted]

an element of area dS at x_1, y_1 in the x_1, y_1 plane is

$$\frac{\alpha}{\sqrt{abn}} e^{-2\pi i r_1/\lambda}\, ds.$$

But

$$r_1 = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + R_0^2\right]^{\frac{1}{2}}.$$

If $x_1 << R_0$ and $y_1 << R_0$, and if

$$R_1 = \left[R_0^2 + x_2^2 + y_2^2\right]^{\frac{1}{2}},$$

r_1 becomes with approximations

$$r_1 = R_1 - x_1 x_2 / R_1 - y_1 y_2 / R_1 .$$

If a lens is used with aperture c x c in plane x_2, y_2, a flat phase front is produced, and the electrical path R_1 is constant and equal to the focal length of the lens R_0. The effect of the radiation in the x_1, y_1 plane is then

$$E(x_2, y_2) = \sum_{k=1}^{n} \alpha_k \frac{e^{-2\pi i R_0/\lambda}}{\sqrt{abn}} \int_{g_k - a}^{g_k + a} e^{2\pi i x_1 x_2 / R_0 \lambda}\, dx_1 \int_{h_k - b}^{h_k + b} e^{2\pi i y_1 y_2 / R_0 \lambda}\, dy_1$$

$$= \frac{\alpha_k R_0^2 \lambda^2 e^{-2\pi i R_0/\lambda}}{4\pi^2 x_2 y_2 \sqrt{abn}} \left[e^{2\pi i x_2 (g_k + a)/R_0 \lambda} - e^{2\pi i x_2 (g_k - a)/R_0 \lambda}\right]$$

$$\left[e^{2\pi i y_2 (h_k + b)/R_0 \lambda} - e^{2\pi i y_2 (h_k - b)/R_0 \lambda}\right].$$

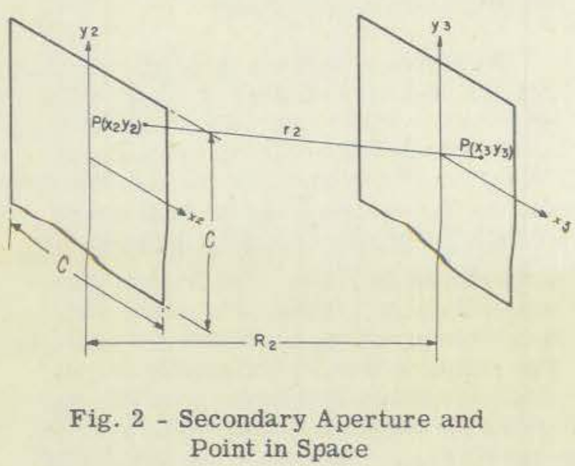

Fig. 2 - Secondary Aperture and Point in Space

If, as in Figure 2, a parallel plane x_3, y_3 is set up at distance R_2 from the plane of the lens, the distance from any point $P(x_2, y_2)$ to any point $P(x_3, y_3)$ is as before

$$r_2 = R_3 - x_2 x_3 / R_3 - y_2 y_3 / R_3 ,$$

where

$$R_3 = \left[R_2^2 + x_3^2 + y_3^2\right]^{\frac{1}{2}}.$$

If angular coordinates $\theta = x_3 / R_3$ and $\phi = y_3 / R_3$ are used, and if $x_3 << R_2$ and $y_3 << R_2$, the field pattern effectively being measured on a constant radius R_3, then

$$r_2 = R_3 - x_2 \theta - y_2 \phi .$$

[redacted]

The effect of the radiation from an element dS in plane x_2, y_2 is

$$E(x_2,y_2)\, e^{-2\pi i r_2/\lambda}\, dS = E(x_2 y_2)\, e^{-2\pi i R_3/\lambda}\, e^{2\pi i \theta x_2/\lambda}\, e^{2\pi i \phi y_2/\lambda}\, dx_2\, dy_2 \ .$$

Substituting for $E(x_2,y_2)$ the sum already derived, and neglecting phase factors and amplitude constants, the total effect at angles θ and ϕ is proportional to

$$E(\theta,\phi) = \sum_{k=1}^{n} \frac{\gamma_k R_0^2 \lambda^2}{2\pi i \sqrt{abn}}$$

$$\left[\int_{-c/2}^{c/2} e^{2\pi i x_2 (R_0\theta + g_k + a)/R_0\lambda}\, \frac{dx_2}{x_2} \int_{-c/2}^{c/2} e^{2\pi i x_2 (R_0\theta + g_k - a)/R_0\lambda}\, \frac{dx_2}{x_2} \right]$$

$$\left[\int_{-c/2}^{c/2} e^{2\pi i y_2 (R_0\phi + h_k + b)/R_0\lambda}\, \frac{dy_2}{y_2} \int_{-c/2}^{c/2} e^{2\pi i y_2 (R_0\phi + h_k - b)/R_0\lambda}\, \frac{dy_2}{y_2} \right].$$

The integrals are of the form

$$\int_{-c/2}^{c/2} e^{imx'}\, \frac{dx'}{x'} \ .$$

Let $x = mx'$, $dx = mdx'$. Then the limits become $\pm\, cm/2$, and the integral is

$$\int_{-cm/2}^{cm/2} e^{ix}\, \frac{dx}{x} = \int_{-cm/2}^{cm/2} \cos x \frac{dx}{x} + i \int_{-cm/2}^{cm/2} \sin x \frac{dx}{x}.$$

Since $\cos x$ is an even function, it has the same value at both limits, and the first integral is zero. Since $\sin x$ is odd, the result is

$$2i \int_{0}^{cm/2} \sin x \frac{dx}{x} = 2\, i\, si\, (cm/2)$$

Applying this simplification to the expression for $E(\theta,\phi)$,

$$E(\theta', \phi') = \sum_{k=1}^{n} \frac{\alpha_k\, c R_0 \lambda}{\sqrt{a'b'n}} \left[si\left(\theta' + g_k' + a'\right) - si\left(\theta' + g_k' - a'\right) \right]$$

$$\left[si\left(\phi' + h_k' + b'\right) - si\left(\phi' + h_k' - b'\right) \right] \qquad (1)$$

acteristics. Within the approximations, the analysis applies equally well to a radiating aperture in the x_2,y_2 plane.

SEQUENTIALLY-LOBED RADAR[3]

This radar uses the antenna feed shown in Figure 3, called a 5-aperture diamond. The center aperture is used for transmission, either the center aperture or the other apertures in succession for range reception, and the outer apertures in opposite pairs for angle reception. Three field patterns are then required. For the single central aperture, $n = 1$,

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[2] "Tables of Sine, Cosine, and Exponential Integrals," 3 Vols., Federal Works Agency (1940).

[3] Foin, Jr., O. F. and Allen, P. J., "An Electronically-Lobed Tracking and Guiding Radar," NRL Report R-3116, June 1947.

The effect of the radiation from an element dS in plane x_2, y_2 is

$$E(x_2,y_2)\, e^{-2\pi i r_2/\lambda}\, dS = E(x_2 y_2)\, e^{-2\pi i R_3/\lambda}\, e^{2\pi i \theta x_2/\lambda}\, e^{2\pi i \phi y_2/\lambda}\, dx_2\, dy_2 \;.$$

Substituting for $E(x_2,y_2)$ the sum already derived, and neglecting phase factors and amplitude constants, the total effect at angles θ and ϕ is proportional to

$$E(\theta,\phi) = \sum_{k=1}^{n} \frac{\gamma_k R_0^2 \lambda^2}{2\pi i \sqrt{abn}}$$

$$\left[\int_{-c/2}^{c/2} e^{2\pi i x_2 (R_0\theta + g_k + a)/R_0\lambda} \frac{dx_2}{x_2} \int_{-c/2}^{c/2} e^{2\pi i x_2 (R_0\theta + g_k - a)/R_0\lambda} \frac{dx_2}{x_2}\right]$$

$$\left[\int_{-c/2}^{c/2} e^{2\pi i y_2 (R_0\phi + h_k + b)/R_0\lambda} \frac{dy_2}{y_2} \int_{-c/2}^{c/2} e^{2\pi i y_2 (R_0\phi + h_k - b)/R_0\lambda} \frac{dy_2}{y_2}\right].$$

The integrals are of the form

$$\int_{-c/2}^{c/2} e^{imx'} \frac{dx'}{x'} \;.$$

Let $x = mx'$, $dx = mdx'$. Then the limits become $\pm\, cm/2$, and the integral is

$$\int_{-cm/2}^{cm/2} e^{ix} \frac{dx}{x} = \int_{-cm/2}^{cm/2} \cos x \frac{dx}{x} + i \int_{-cm/2}^{cm/2} \sin x \frac{dx}{x}.$$

Since $\cos x$ is an even function, it has the same value at both limits, and the first integral is zero. Since $\sin x$ is odd, the result is

$$2i \int_0^{cm/2} \sin x \frac{dx}{x} = 2\, i\, \mathrm{si}\,(cm/2)$$

Applying this simplification to the expression for $E(\theta,\phi)$,

$$E(\theta',\phi') = \sum_{k=1}^{n} \frac{\alpha_k\, c R_0 \lambda}{\sqrt{a'b'n}} \left[\mathrm{si}\left(\theta' + g_k' + a'\right) - \mathrm{si}\left(\theta' + g_k' - a'\right)\right]$$

$$\left[\mathrm{si}\left(\phi' + h_k' + b'\right) - \mathrm{si}\left(\phi' + h_k' - b'\right)\right] \qquad (1)$$

where

$$\theta' = \frac{\pi c}{\lambda}\theta, \qquad g_k' = \frac{\pi c}{\lambda R_0} g_k , \qquad a' = \frac{\pi c}{\lambda R_0} a ,$$

$$\phi' = \frac{\pi c}{\lambda}\phi , \qquad h_k' = \frac{\pi c}{\lambda R_0}\, h_k , \qquad b' = \frac{\pi c}{\lambda R_0} b .$$

In certain types of feeds the sides of the primary rectangular apertures are not parallel to one of the radar coordinate axes but are oriented at an angle β. The preceding analysis can be used with a change of coordinates. If g_k and h_k are specified as before with reference to axes x_1 and y_1 parallel to the aperture sides, results can be obtained in new coordinates θ'' and ϕ'', rotated at an angle β with respect to θ' and ϕ'. Let $\psi = \theta' + i\phi'$ and $\psi'' = \theta'' + i\phi''$. But $\psi'' = \psi' e^{-i\beta}$ and

$$\theta'' + i\phi'' = (\phi' \sin\beta + \theta' \cos\beta) + i\,(\phi' \cos\beta - \theta' \sin\beta) .$$

Then

$$\theta'' = \phi' \sin\beta + \theta' \cos\beta$$

and

$$\phi'' = \phi' \cos\beta - \theta' \sin\beta .$$

If $\beta = \pi/4$, as is common,

$$\theta'' = (\phi' + \theta')/\sqrt{2}$$

and

$$\phi'' = (\phi' - \theta')/\sqrt{2} .$$

If $\phi'' = 0$, $\theta' = \phi' = \theta''/\sqrt{2}$. Equivalent transmitting patterns as a function of angle θ off target can be found by making this substitution in the previous analysis.

The parameters g_k', h_k', a', and b' generalize the description of a feed by limiting the effect of the factors c, R_1, and λ to one of pattern amplitude only. The expression (1) for the field pattern can be applied to a particular arrangement of primary apertures in an antenna feed. The sine integrals can easily be evaluated numerically from tables,[2] curves can be plotted, and values of the parameters can be found for desired radar characteristics. Within the approximations, the analysis applies equally well to a reflecting aperture in the x_2, y_2 plane.

SEQUENTIALLY-LOBED RADAR[3]

This radar uses the antenna feed shown in Figure 3, called a 5-aperture diamond. The center aperture is used for transmission, either the center aperture or the other apertures in succession for range reception, and the outer apertures in opposite pairs for angle reception. Three field patterns are then required. For the single central aperture, $n = 1$,

---

[2] "Tables of Sine, Cosine, and Exponential Integrals," 3 Vols., Federal Works Agency (1940).

[3] Foin, Jr., O. F. and Allen, P. J., "An Electronically-Lobed Tracking and Guiding Radar," NRL Report R-3116, June 1947.

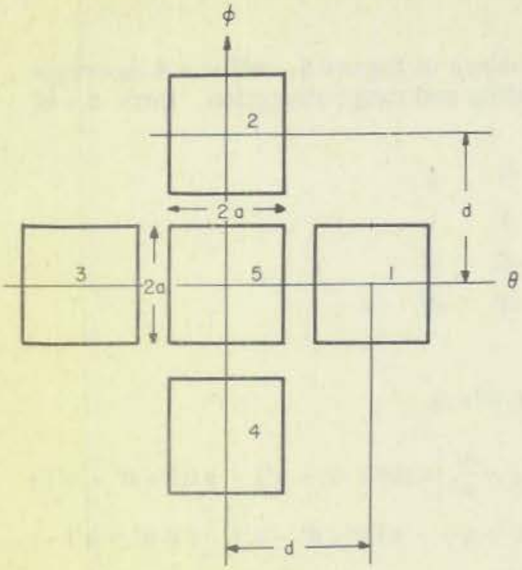


Fig. 3 - Five-Aperture Diamond Feed

$g_k = h_k = 0$, $\alpha = 1$, $a = b$. Then neglecting parameters c, R_0, and λ, a substitution in (1) gives

$$E_5 = \frac{2}{a'}\left[\mathrm{si}\,(\theta' + a') - \mathrm{si}\,(\theta' - a')\right] \mathrm{si}\,(a') .$$

Since the variation with ϕ' is the same as that with θ', ϕ' is taken as zero. On-target signal is then proportional to E_5^2 for $\theta' = 0$, or to

$$E_r = \frac{16}{a'^2}\, \mathrm{si}^4(a') .$$

If range is received on the outer apertures in sequence, the on-target received pattern is obtained from the substitutions $g_k = d'$, $h_k = 0$, $\alpha_k = 1$, $\phi' = \theta' = 0$. This gives an equivalent transmitted pattern

$$E_1 = \frac{2}{a'}\left[\mathrm{si}\,(d' + a') - \mathrm{si}\,(d' - a')\right] \mathrm{si}\,(a') .$$

The range signal is proportional to $|E_5\ E_1|$ for $\theta = 0$ or to

$$E_r = \frac{8}{a'^2}\left|\mathrm{si}\,(d' + a') - \mathrm{si}\,(d' - a')\right| \mathrm{si}^3(a') .$$

For angle reception in the θ' coordinate, apertures 1 and 3 are used, $n = 2$, and the parameters are given by

Aperture	1	3
α_k	1	-1
g'_k	d'	-d'
h'_k	0	0 .

From (1) the equivalent transmitted pattern is

$$E_{1-3} = \frac{\sqrt{2}}{a'}\left[\mathrm{si}(\theta' + d' + a') - \mathrm{si}(\theta' + d' - a') - \mathrm{si}(\theta' - d' + a') + \mathrm{si}(\theta' - d' - a')\right] \mathrm{si}(a') .$$

The angular error signal is proportional to the product $|E_5|E_{1-3}$ or to

$$E_a = \frac{2\sqrt{2}}{a'^2}\left[\mathrm{si}(\theta' + d' + a') - \mathrm{si}(\theta' + d' - a') - \mathrm{si}(\theta' - d' + a') + \mathrm{si}(\theta' - d' - a')\right]$$

$$\left|\mathrm{si}(\theta' + a') - \mathrm{si}(\theta' - a')\right| \mathrm{si}^2(a') .$$

SIMULTANEOUSLY-LOBED RADAR[4]

This radar at present uses the antenna feed shown in Figure 4, called a 4-aperture square. All four apertures are used for transmitting and range reception. Here a = b, n = 4, and the other parameters are given by

Aperture	1	2	3	4
α_k	1	1	1	1
g'_k	d′	-d′	-d′	d′
h'_k	d′	d′	-d′	-d′

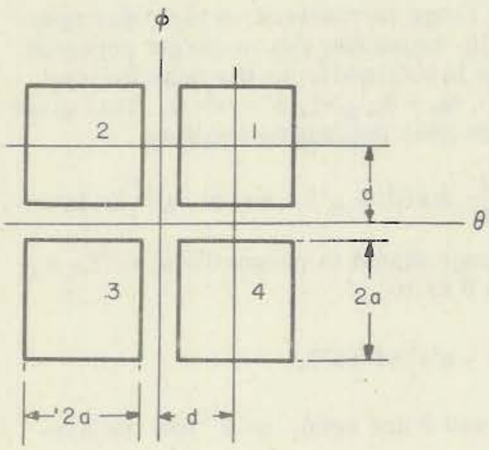


Fig. 4 - Four-Aperture Square Feed

Again for $\phi' = 0$,

$$E_{1+2+3+4} = \frac{1}{a'}\Big[si(\theta' + d' + a') - si(\theta' + d' - a') + si(\theta' - d' + a') - si(\theta' - d' - a')\Big]\Big[si(d' + a') - si(d' - a')\Big]$$

On-target range signal is then proportional to

$$E^2_{1+2+3+4}$$

for $\theta' = 0$ or to

$$E_r = \frac{4}{a'^2}\Big[si(d' + a') - si(d' - a')\Big]^4.$$

For angle reception all four apertures are again used for the θ' coordinate according to

Aperture	1	2	3	4
α_k	1	-1	-1	1
g'_k	d′	-d′	-d′	d′
h'_k	d′	d′	-d′	-d′

This gives an equivalent transmitted pattern of

$$E_{1-2-3+4} = \frac{1}{a'}\Big[si(\theta' + d' + a') - si(\theta' + d' - a') - si(\theta' - d' + a') + si(\theta' - d' - a')\Big]$$
$$\Big[si(d' + a') - si(d' - a')\Big].$$

[4] Gerwin, H. L. and Hastings, A. E., "Further Design and Development of Components for Simultaneous Lobing Radar TAB," NRL Report R-3221, 13 January 1948.

The radar system which uses this feed has an angular error signal proportional to $E^3_{1+2+3+4}\, E_{1-2-3+4}$, or to

$$E_a = \frac{1}{a'^4}\left[si(\theta' + d' + a') - si(\theta' + d' - a') + si(\theta' - d' + a') - si(\theta' - d' - a')\right]^3$$

$$\left[si(\theta' + d' + a') - si(\theta' + d' - a') - si(\theta' - d' + a') + si(\theta' - d' - a')\right]\left[si(d' + a') - si(d' - a')\right]^4.$$

Another feed, called a 4-aperture diamond, which has been proposed for use with this radar, is shown in Figure 5. As before, all four apertures are used for transmission and range reception, and the same expression for range signal applies. For transmission, $a' = b'$, $n = 4$, and $\theta' = \phi' = \theta''/\sqrt{2}$. The other parameters are given by

Aperture	1	2	3	4
α_k	1	1	1	1
g'_k	d'	-d'	-d'	d'
h'_k	d'	d'	-d'	-d'

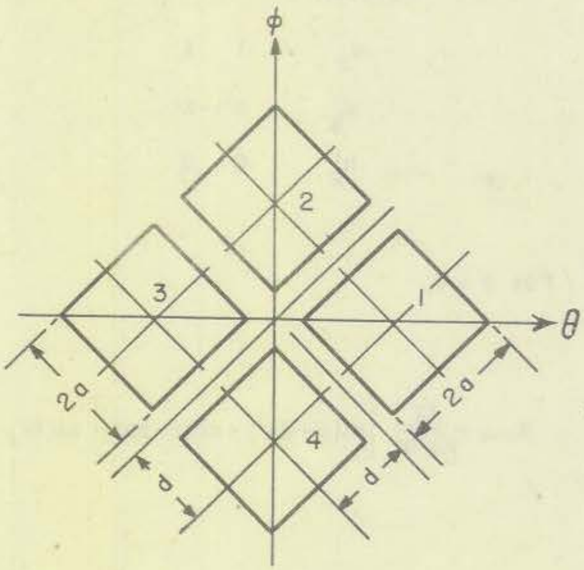


Fig. 5 - Four-Aperture Diamond Feed

The transmitted pattern is then

$$E_{1+2+3+4} = \frac{1}{2a'}\left[si(\theta''/\sqrt{2} + d' + a') - si(\theta''/\sqrt{2} + d' - a') + si(\theta''/\sqrt{2} - d' + a') - si(\theta''/\sqrt{2} - d' - a')\right]^2.$$

For angle reception in the θ'' coordinate, apertures 1 and 3 are used, $a' = b'$, $n = 2$, $\phi'' = 0$, and $\theta' = \phi' = \theta''/\sqrt{2}$. The other parameters are given by

Aperture	1	3
α_k	1	-1
g'_k	d'	-d'
h'_k	d'	-d'.

The equivalent transmitted pattern is

$$E_{1-3} = \frac{1}{\sqrt{2}\,a'}\left\{\left[si(\theta''/\sqrt{2}+d'+a') - si(\theta''/\sqrt{2}+d'-a')\right]^2 - \left[si(\theta''/\sqrt{2}-d'+a') - si(\theta''/\sqrt{2}-d'-a')\right]^2\right\}.$$

The angle signal is proportional to $E^3_{1+2+3+4}\, E_{1-3}$, or to

$$E_a = \frac{1}{8\sqrt{2}\,a'^4}\left[si(\theta''/\sqrt{2}+d'+a') - si(\theta''/\sqrt{2}+d'-a') + si(\theta''/\sqrt{2}-d'+a') - si(\theta''/\sqrt{2}-d'-a')\right]^6$$

$$\left\{\left[si(\theta''/\sqrt{2}+d'+a') - si(\theta''/\sqrt{2}+d'-a')\right]^2 - \left[si(\theta''/\sqrt{2}-d'+a') - si(\theta''/\sqrt{2}-d'-a')\right]^2\right\}.$$

 [redacted]

Still another proposed feed includes a fifth aperture for transmission and angle reception as shown in Figure 6. The transmitting pattern and the range signal are then as described in the sequentially-lobed radar system. Angle reception is as in the 4-aperture, square feed just described. Then the angle signal is proportional to $E_5^3\, E_{1-2-3+4}$, or to

$$E_a = \frac{8}{a'^4}\left[si(\theta'+a') - si(\theta'-a')\right]^3 \left[si(\theta'+d'+a') - si(\theta'+d'-a') - si(\theta'-d'+a') + si(\theta'-d'-a')\right]$$

$$\left[si(d'+a') - si(d'-a')\right]\; si^3(a') .$$

EXPERIMENTAL TEST RADAR

A radar set up to check the results of this analysis uses the feed of Figure 7. Transmission occurs on both apertures in phase. Then n = 2, the apertures are in contact so that $d' = a'$, and the other parameters are given by

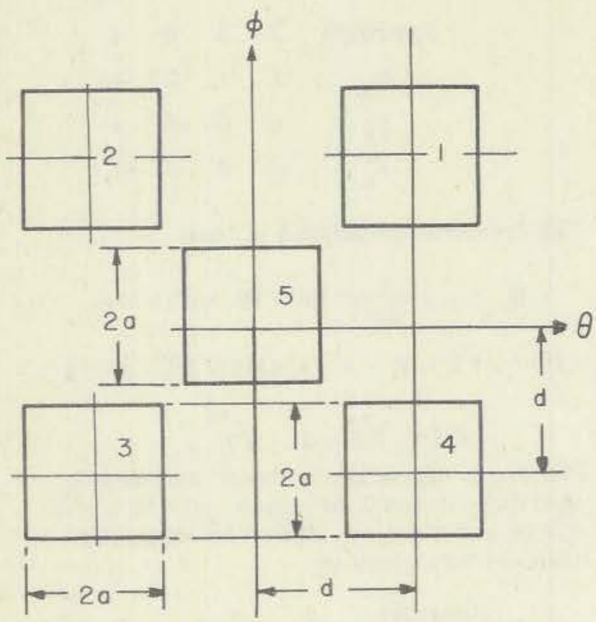


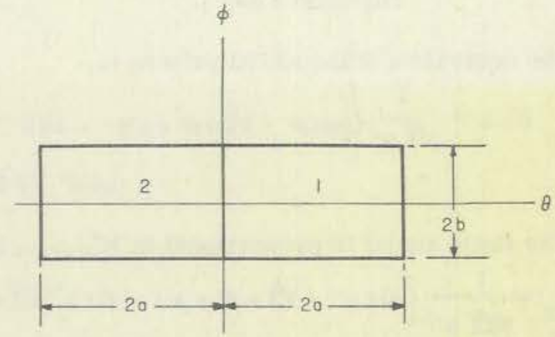
Fig. 6 - Five-Aperture Square Feed

Aperture	1	2
α_k	1	1
g'_k	a'	$-a'$
h'_k	0	0

For $\phi' = 0$,

$$E_{1+2} = \frac{\sqrt{2}}{\sqrt{a'b'}}\left[si(\theta'+2a') - si(\theta'-2a')\right] si(b') .$$

Fig. 7 - Two-Aperture Feed



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Angle reception occurs on both apertures, one out of phase with the other. Then the parameters are given by

Aperture	1	2
α_k	1	-1
g'_k	a′	-a′
h'_k	0	0.

For $\phi' = 0$, the equivalent transmitted pattern is

$$E_{1-2} = \frac{\sqrt{2}}{\sqrt{a'b'}} \left[si(\theta' + 2a') + si(\theta' - 2a') - 2\,si(\theta') \right] si(b').$$

The angular error signal is proportional to the product $|E_{1+2}|E_{1-2}$, and

$$E_a = \frac{2}{a'b'} \left[si(\theta' + 2a') + si(\theta' - 2a') - 2\,si(\theta') \right] \left| si(\theta' + 2a') - si(\theta' - 2a') \right| si^2(b').$$

IDEAL RADAR

A number of variations of feed arrangement have been suggested to improve the sensitivities of the radars described. One such variation uses dielectric rods to obtain the effect of overlapping primary apertures. The evaluation of this and other schemes is not easily carried out analytically without approximations of unknown degree. A valuable concept to have, then, is that of an ideal radar, which is not too limited by practice and which can give a measure of the maximum improvement that can be expected in existing radars. This ideal radar transmits and receives range on a single on-axis primary aperture. Since the only requirement of this aperture is that it produce maximum field E_t along the axis, its dimensions can be determined from

$$\frac{\partial E_t}{\partial a'} = \frac{\partial}{\partial a'} \left[\frac{4}{\sqrt{a'b'}} si(a')\, si(b') \right] = 0 = \frac{\partial E_t}{\partial b'}$$

If the differentiation is carried out under the integral sign and the result is integrated, the conditions require $si(a') = 2 \sin a'$ and $si(b') = 2 \sin b'$, or $a' = b' = 2.15$. The square aperture of this dimension results in a range sensitivity of 27, and $E_t = 5.17$. Angle reception in the ideal radar takes place on two opposite apertures, not restricted by each other or by the transmitting aperture. The equivalent transmitted pattern from these receiving apertures is

$$E_{1-2} = \sqrt{2/a'b'} \left[si(\theta' + d' + a') - si(\theta' + d' - a') - si(\theta' - d' + a') + si(\theta' - d' - a') \right] si(b').$$

This expression maximizes with b' for $b' = 2.15$. For small angles off target the angle signal becomes for a sequentially-lobed radar.

$$E_a = E_t \cdot E_{1-2} = \frac{8.33}{\sqrt{a'}} \left[si(\theta' + d' + a') - si(\theta' + d' - a') - si(\theta' - d' + a') + si(\theta' - d' - a') \right].$$

For a simultaneously-lobed radar the numerical constant is 223. If this expression is maximized for both d' and a', $d = 2.17$, $a' = 1.65$, and $E_a = 14.5$ for the sequentially-lobed radar and 387 for the simultaneously-lobed radar.

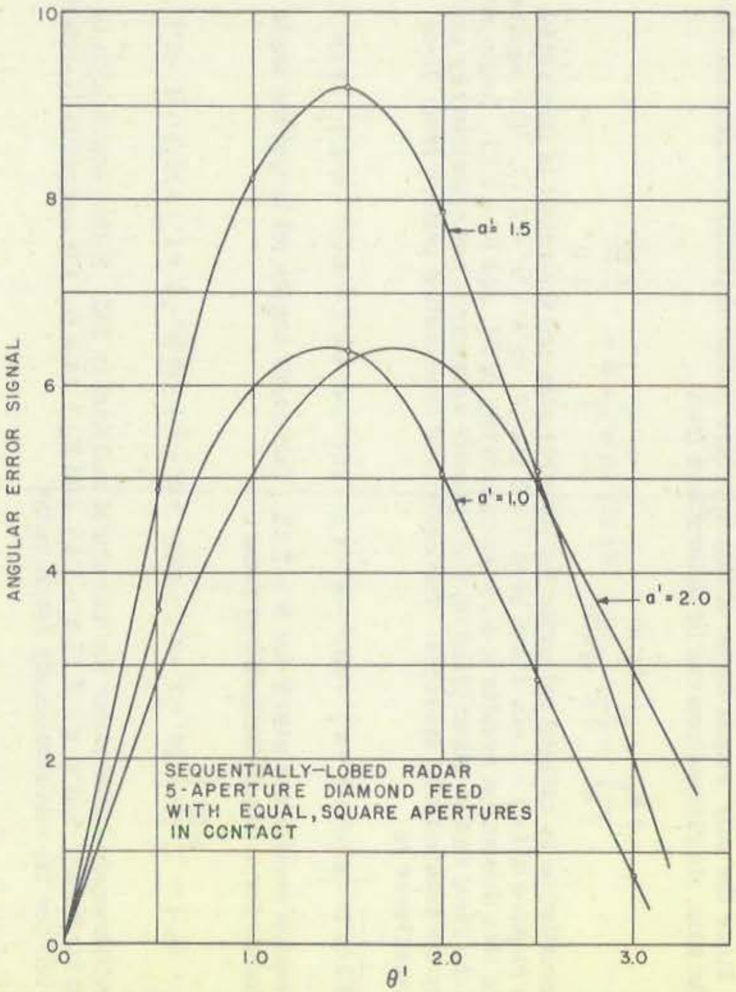

Fig. 8 - Error Signal vs Angle Off Target

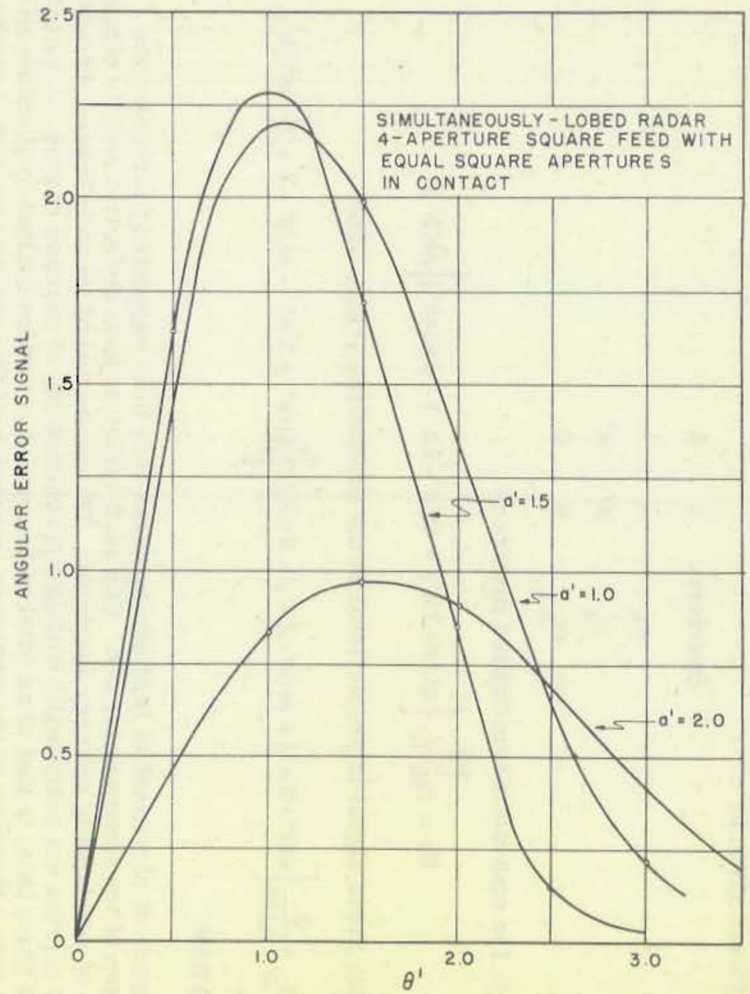

Fig. 9 - Error Signal vs Angle Off Target

RESULTS OF APPLICATIONS

Figures 8 and 9 show the calculated angular error signals of two radar systems as functions of angle off target for three values of a′, the parameter determining size of primary aperture. The spacing d of the primary apertures is chosen so that the edges of the apertures are in contact. The curves indicate for each system a value of a′ for which the angular sensitivity, defined as the slope of the curve at the origin, is a maximum. Figures 10 and 11 show the variation, expressed in db down from the values for the ideal radar, of angle and range sensitivities with a for the first two radars. Range sensitivity is defined as range signal on target.

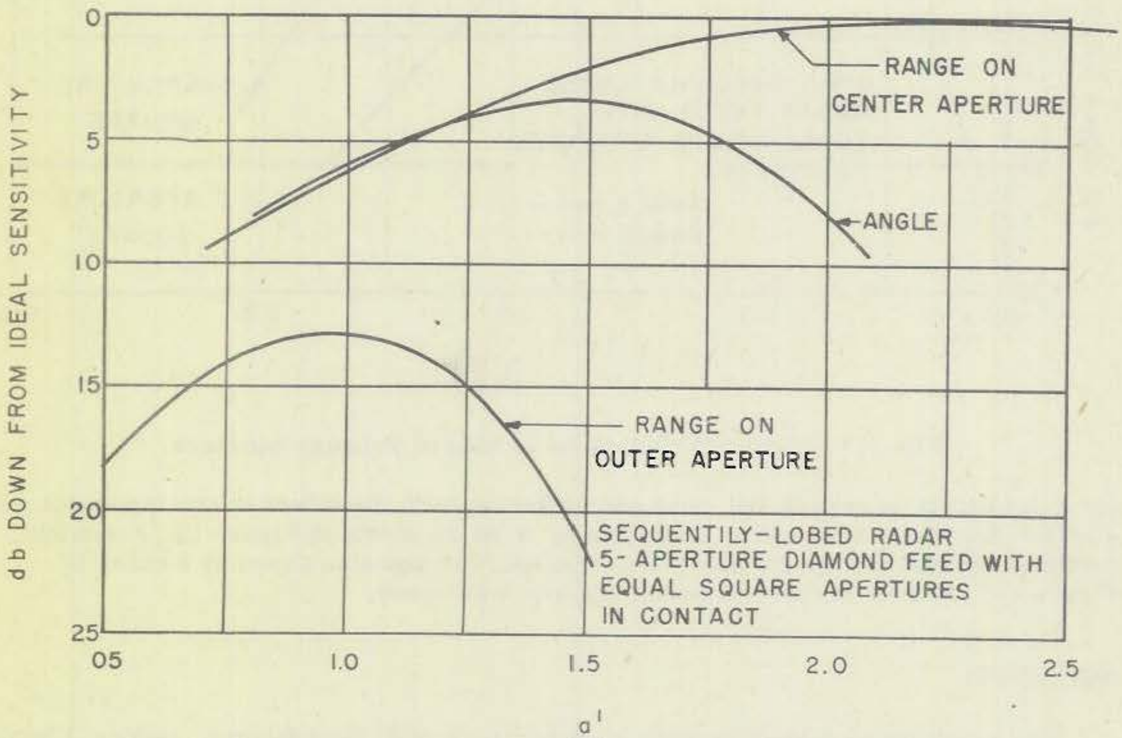

Fig. 10 - Relative Sensitivities vs Size of Primary Aperture

By further calculations, it has been shown for the system with 4-aperture, square feed that spacing of apertures exceeding that to produce contact results in a similar variation with a′, but with a lower maximum of angle sensitivity. Similarly the effect of increasing the size of the central aperture in the system with 5-aperture feed has been investigated for various aperture spacing, the apertures remaining in contact. The optimum arrangement for maximum angle sensitivity is shown to require all horns to be equal.

Measurements on an experimental system have been made to check the application of this analysis. Two rectangular horns, mounted at the focal point of a circular lens, were connected to a rat-race, and sum and difference r-f receiving patterns were measured. Measuring equipment was calibrated by the use of a standard receiving horn. Angular error signal was calculated from the sum and difference patterns. All this was repeated

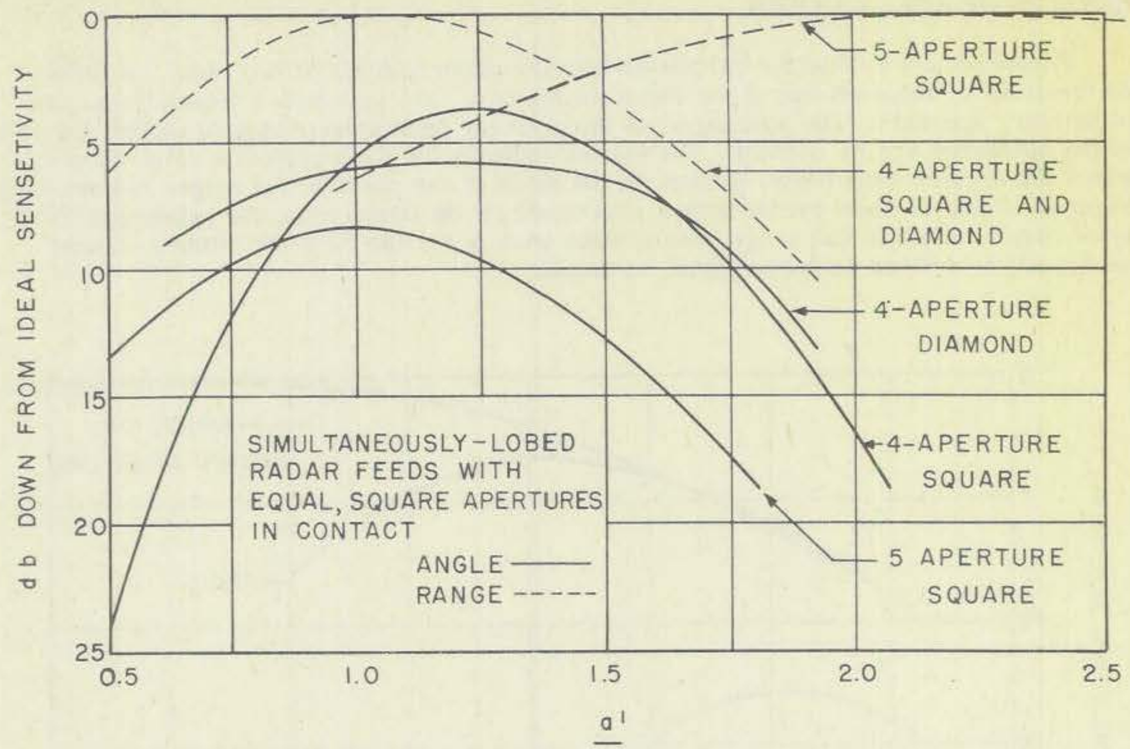

Fig. 11 - Relative Sensitivities vs Size of Primary Aperture

for a number of pairs of horns, each pair differing from the others in one dimension only. Angular sensitivity is shown as a function of a' by the points in Figure 12. A similar function, calculated from the analysis presented here and also shown as a curve in Figure 12, indicates agreement within experimental error.

DISCUSSION

Two assumptions have been made in order to simplify the analysis: uniform illumination and zero coupling of the apertures. The assumption of zero coupling, discussed in a previous report[5], appears to hold quite well, but uniform illumination of the primary apertures is not easily obtainable in both planes. Experimental secondary patterns indicate some degree of continuous phase shift with angle, which the analysis does not explain (except for reversal of sign at nulls). This phase shift may be a result of actual illumination or of some other cause such as lack of symmetry in the antenna assembly. The angular error function of an actual system may then differ considerably from the theoretical one at large angles.

The analysis applies strictly to a system with a square secondary aperture c x c, oriented with its sides parallel to those of the rectangular primary aperture. In practice, apertures are sometimes circular or differently oriented. Since this affects radiation

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[5] Hastings, A. E., op. cit.

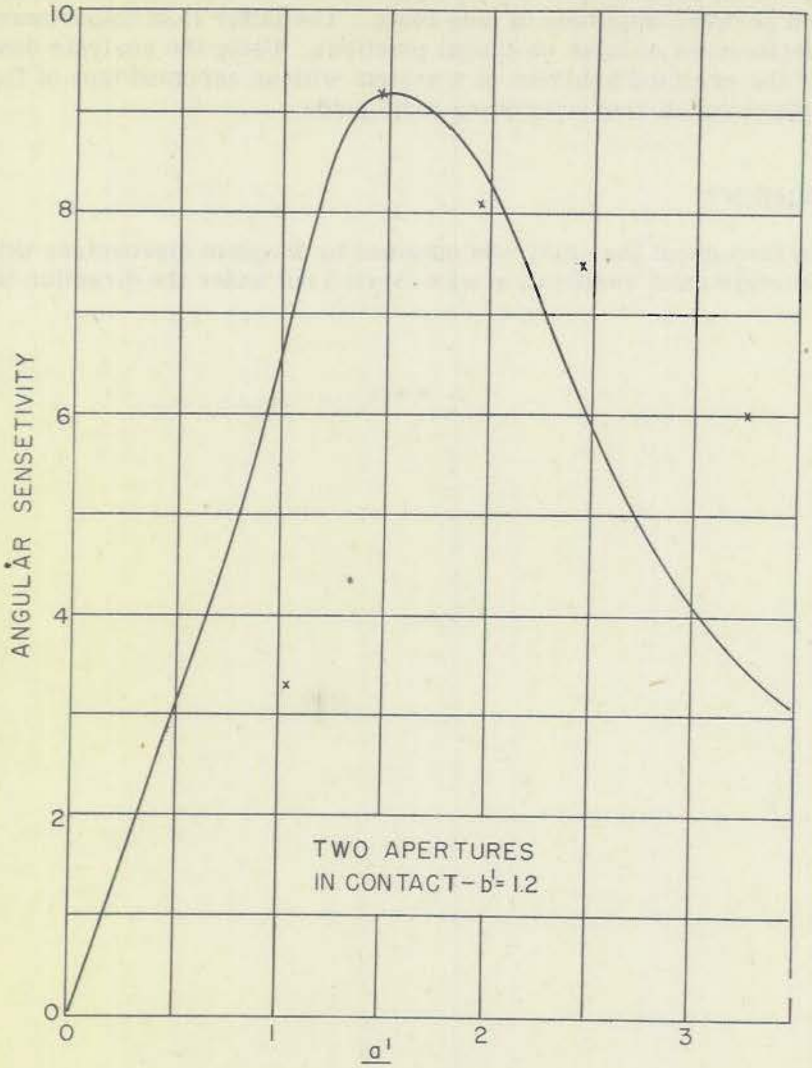

Fig. 12 - Angular Sensitivity of Experimental System vs Size of Primary Aperture

through a small part of the aperture only, the analysis probably applies quite well. The circular secondary aperture in the test system appears to have produced results close to those predicted for a square aperture.

In calculating range sensitivities, it has been assumed that all comparison of lobe signals in the radars is done simultaneously. To have included the actual operation of radar systems would have complicated the applications with signal-to-noise considerations, which are the subject of a separate study to be reported soon. This assumption does not allow comparison of widely different systems, as the first two described here, but the comparison of feed types of a single system given in either Figure 10 or Figure 11 is valid and useful.

The design of an antenna feed is a compromise involving angle sensitivity, range sensitivity, and perhaps amplitude of side lobes. The latter is of importance in determining the effectiveness of false on-target positions. Using the analysis described here, calculations of the expected behavior of a system with an assumed type of feed allow a suitable compromise of characteristics to be made.

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