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OVERINTERROGATION AND ASYNCHRONOUS REPLIES, AND THEIR RELATION TO DISPLAY LIMITATION AND TRAFFIC HANDLING CAPACITY IN AN IFF SYSTEM

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OVERINTERROGATION AND ASYNCHRONOUS REPLIES, AND THEIR RELATION TO DISPLAY LIMITATION AND TRAFFIC HANDLING CAPACITY IN AN IFF SYSTEM

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August 25, 1948

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ABSTRACT

Statistical methods are employed in obtaining equations and curves which may be used to calculate the maximum amount of transpondor count down which needs to be tolerated in an electronic identification and recognition system if the system is to operate, say for 99 percent of the time it is in use, without failure due to design limitations. Examples are included which give an idea of the order of magnitude of the count down to be expected in a hypothetical IFF system with characteristics closely approximating the Mark V IFF system. Statistical equations are also given for determining a maximum amount of "fruit" which needs to be tolerated under various conditions if the system is to operate without failure due to design limitations for a given percent of the time it is in use. Although calculations using these equations are laborious, those involving "fruit" tolerances being extremely so, it is shown that safe tolerance limits can be set by this statistical approach.

PROBLEM STATUS

This is an interim report; work is continuing.

AUTHORIZATION

NRL Problem R03-06R (BuShips Problem S1234X-S).

[redacted]

OVERINTERROGATION AND ASYNCHRONOUS REPLIES, AND THEIR RELATION TO DISPLAY LIMITATION AND TRAFFIC HANDLING CAPACITY IN AN IFF SYSTEM

INTRODUCTION

Operation of an electronic identification and recognition system using large numbers of interrogators and transpondors in a given region may result in a limitation of the readability of the reply or, if automatic devices are being used, in complete or partial failure to properly identify radar targets. The system designer needs to know what transpondor count down and responsor fruit rate must be tolerated if the system is to operate without failure due to design limitations.[1] There seems to be no experimental way of determining such specifications except by actually building, installing and trying out large numbers of equipments under expected operational conditions. The alternative is, of course, to make suitable assumptions regarding expected conditions and to calculate the values involved. In the past, such calculations have been made by assuming that the maximum number of interrogators and transpondors are operating under the worst possible conditions. Specifications based on such calculations are unduly severe.

It is shown in this report that, by making certain rather idealized assumptions regarding system characteristics and operating conditions, and by using statistical methods, one can obtain maximum count-down and fruit requirements which are considerably less severe than those based upon the worst possible situations. While the idealized assumptions made will at best only be approximated under actual conditions, the resulting maximum count-down and fruit requirements are safe design figures for any system in a practical situation.

By selecting a percentage known as the "design risk" we specify the fraction of total operating time of an IFF system during which functional failures due to inherent design limitations can occur. The derived relationships provide a statistical method of determining the maximum transpondor count down and responsor fruit rate which the system must tolerate for a given design risk. Curves are plotted for the equations involving transpondor count down which enable one to determine maximum count down which must be tolerated by a system for design risks of 1%, 5%, and 10%. Several examples are included to demonstrate the use of these curves and to give an idea of the order of magnitude of count down to be expected in certain hypothetical cases.

The equations involving responsor fruit require a great deal of laborious calculation in order to obtain the desired system specification. There does not appear to be much point, therefore, in performing this excessive work until an actual system is being designed with system characteristics and operational requirements more definitely known,

[1] Unsynchronized signals appearing on a radar screen are known as "fruit."

and no curves or examples involving these equations are included. The examples on count down do, however, include the maximum fruit which must be tolerated for 1% design risk assuming a single transpondor and 200 interrogators. While the fruit obtained from a single transpondor is of little practical significance in impairing system operation unless the number of interrogators is extremely high, the results thus obtained are useful in indicating what effect may be expected on fruit rates when other system parameters are varied.

ASSUMPTIONS AND DEFINITIONS

Operational Use of Systems Being Considered

The assumptions made in this report are for identification systems under wartime operation rather than for traffic control of air transport services. Therefore the results obtained may not be entirely applicable to a system whose primary purpose is other than the identification of radar targets.

"Our" Interrogator and "Foreign" Interrogators

The interrogator-responsor unit under observation for effects of display limitation will be known as "our" interrogator or "our" responsor. All other interrogators which are causing transpondor count down and/or generating unlocked fruit replies will be referred to as "foreign" interrogators.

Region of Operation and Interrogator Distribution

We define the region of operation as the area within a circle centered at "our" interrogator and of radius equal to the maximum range of "our" responsor receiver. For simplicity in calculations, it is necessary to assume that all transpondors in the region of operation are within triggering range of all interrogators in the same region. Such a condition could not be realized in practice with a random distribution of "foreign" interrogators of various maximum ranges. If, however, we assume that all "foreign" interrogators are bunched closely around "our" interrogator, approximating a point source of asynchronous interrogations, and if we further assume that the maximum range of each "foreign" interrogator is at least equal to the maximum range of "our" responsor, we shall clearly be considering the worst possible situation. The practical case of a fleet task force operating in conjunction with aircraft distributed around it in all directions is not a great deal removed from this assumption.

Interrogator Search

If a system were designed on the assumption that interrogators will "searchlight" transpondors in any prescribed way, it would be satisfactory for operation under conditions of severity equal to or less than the given conditions, but under more severe conditions would certainly fail. Further, a system so designed, when the likelihood of these conditions is not nearly so great as that of random searching, would demand excessive transpondor duty-cycle requirements and responsor fruit tolerance. On the other hand, randomness of interrogator search permits including the possibility that a number of interrogators will cover the same area simultaneously according to an associated probability. Complete randomness of interrogator search is a necessary assumption if each interrogation of a transpondor and the interrogation of the same or another transpondor by another interrogator are to be considered mutually independent events. This condition must exist in order that probability methods remain valid. It seems likely that in the

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design of a future IFF system, a radar setup consisting of a definite arrangement of radars and interrogator-responsors with different azimuth scanning rates all searching various azimuth sectors, must be taken into account. If the azimuth scanning rates of any of these radars were precisely equal, then a probability treatment of the problem would be invalid since the interrogation of a given transpondor by a given interrogator would then be a periodic function of other interrogations and could no longer be considered independent of them. It is unlikely, however, that such a condition would prevail under operating conditions, since there is no reason for attaining any such precision in azimuth scanning rates. Therefore, we shall assume that all interrogators are searching certain prescribed sectors of azimuth in a completely random fashion, i.e., that none of the azimuth scanning rates are absolutely constant, and that no two interrogators have precisely equal azimuth scanning rates. This assumption necessarily excludes the effects of any interrogators which are "searchlighting" targets in order to receive slow code information. In any given situation, the total number, n, of "foreign" interrogators must therefore exclude these "searchlighting" units and appropriate addition must be made to the resulting required tolerances if any "searchlighting" interrogators are present.

Transpondor Distribution

It is assumed that transpondors are located in a given region in a completely random manner; that is, there is equal likelihood that a transpondor will be located at any point in the region of operation. The region in which this randomness of distribution is assumed may be all of a given region of operation, or it may be confined to a certain specific portion of the entire region. The particular manner in which transpondors are distributed does not affect the count down to be experienced but will change the manner in which the fruit is calculated. The important thing to remember is that the assumption of randomness can be applied to restricted portions of an entire area, thus enabling this treatment to account for concentration of transpondors. This assumption is made in order to permit straightforward treatment of the problem by probability methods.

"Instant" of Time

In any operating system with random target distribution and interrogator scanning, the amount of unlocked fruit signals generated and/or the count down taking place in any transpondor is continuously varying. However, we are interested in values for fruit rate and/or count down only during the short period of time during which "our" interrogator-responsor is identifying a target. Let us choose this "instant" of time as one long enough to insure a positive identification of a locked reply and at the same time short enough so that during this "instant" of time all target positions and directions of interrogator antennas may be considered as sensibly constant. If we choose this "instant" as the total time that "our" interrogator is challenging a target we find that it will be

$$\alpha T_0$$

where T_0 is the repetition period of "our" interrogator and α is the number of interrogations arriving at the transpondor per antenna revolution (to the nearest integer) from "our" interrogator. If we let f_0 be "our" interrogation rate in cycles per second, and V_0 equal "our" interrogator's azimuth scan rate in revoultions per minute, then the number of degrees scanned per second is $V_0/60 \times 360 = 6V_0$. The number of interrogations per degree is $f_0/6V_0$. If the effective beam width of "our" interrogator antenna is θ_0 degrees, then the number of interrogations per azimuth scan is

$$\alpha = \frac{\theta_0 f_0}{6V_0}$$

and the length of the "instant" of time that "our" interrogator challenges a target is a function of "our" interrogator beam width and azimuth scanning rate given by

$$\alpha T_0 = \frac{\theta_0 f_0}{6V_0} \times T_0 = \frac{\theta_0}{6V_0}.$$

Antennas

It is assumed that all transpondor antennas are omnidirectional[2] and that all interrogator antennas and responsor antennas are directional.

Transpondor Triggering and Interrogator Beam Widths

The probability approach to this problem requires that we assume all interrogators to have an "ideal" antenna pattern devoid of any side lobes and that any transpondor in the region of operation which is not insensitive to challenge pulses due to dead time from a previous challenge will be triggered if it comes within the beam of an interrogator and will not be triggered if it lies outside such a beam. This assumption ignores the fact that the azimuth arc where transpondor triggering occurs is a function of transpondor range and height. The extreme complications which would be introduced by attempting to consider interrogator side lobes and transpondor range and height are thereby avoided. Relationships derived by using this assumption, however, should furnish an indication of the kind of relationships that would be expected from any other less ideal situation, even though the answers to numerical examples might not be the same in each case. It seems reasonable in applying this assumption to use an "effective" interrogator beam width which will more nearly approach the correct answers.

Generally, the arc width on a PPI display subtended by an IFF signal will be narrow at extreme range and will increase with decreasing range. At close range the IFF signal may be expected to show throughout the entire 360 degrees (assuming no gain-time-control). Furthermore, since the arc width is a function of signal strength at the responsor, the arc will also depend upon transpondor height. If data is available on the antennas for any proposed system similar to that obtained on the Mark 5 IFF antennas[3] AN/UPA-3A and AN/UPA-4, and in the recent Mark 5 IFF sea trials,[4] in which IFF arc width information is given graphically, then an effective antenna beam width may be selected by choosing the worst possible situation which might occur; i.e., aircraft at medium to short ranges and a predominance of wide-beam interrogator antennas. Such a choice for beam widths would lead to results of probable count down and/or fruit levels considerably in excess of those which would obtain from a value of specified antenna pattern at half-power points, but it is believed such a choice will give far more realistic numerical results.

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[2] A transpondor antenna system has been suggested by C. E. Cleeton, Naval Research Laboratory, which provides for omnidirectional reception and directional response aimed back at the source of the interrogation. Such a system would not alter the transpondor count down relationships as presented here, but of course, would result in less fruit. The fruit problem for such a system would depend entirely upon the characteristics of the system. It is a special case not easily adaptable to the probability treatment given here and will not be included.

[3] R. H. Brown, Notes on gain time control, Combined Research Group Technical Memo. No. 174, 9 July 1945.

[4] Evalulation of IFF Mark 5/UNB system, ComOpDevFor Second Partial Report on Project OP/S104/S67-6, 13 April 1948.

Symbols Used

A = Number of transponders within "our" antenna beam width during an "instant."

B = Total number of transponders in the region of operation.

d = Reply pulse width, seconds.

F = "Fruit" frequency received on "our" responsor.

f_{av} = Average pulse repetition frequency of all interrogators, pulses per second.

f_i = Transpondor interrogation rate, cycles per second.

f_o = Repetition rate of "our" interrogator, pulses per second.

n = Total number of "foreign" interrogators in a region.

P = Probability

p = Probability that a single interrogator will challenge a single transpondor.

r = Number of interrogators challenging a transpondor or group of transpondors. In general $o \leq r \leq n$.

s = Number of interrogators challenging a transpondor or group of transpondors when the total n "foreign" interrogators are operating on t interrogation frequency channels. In general $o \leq s \leq r \leq n$.

T_o = Repetition period of "our" interrogator, seconds.

t = Total number of interrogation frequency channels.

V_o = Azimuth scanning rate of "our" interrogator, in rpm.

W = Count down or fractional response, equal to the number of output signals divided by the total number of input challenges in a given interval of time.

α = Number of interrogations per azimuth scan, nearest integer.

γ = A specified azimuth sector in which one or more transpondors are located.

ψ = A specified interrogator-search sector.

$\epsilon = np$

θ = Interrogator antenna beam width, degrees.

θ_o = Beam width of "our" interrogator antenna pattern, degrees.

θ_r = Beam width of "our" responsor antenna pattern, degrees.

τ = Total transpondor "dead" time or recovery time, seconds.

DISCUSSION

Contributing Factors to Limitation of Reply Readability or Usefulness

Two distinct factors contribute to the limitation of display readability or usefulness resulting from the operation of large numbers of interrogators and/or transpondors in a given region.

(a) Transpondor count down is the ratio of output replies to total number of challenges. The effect of this factor on the PPI display is a "chopped up" or faint appearance of the reply. If the IFF reply is to be used in actuating some automatic device, count down in excess of a certain limiting value may cause complete failure of identification. This factor is a function of the number of interrogators and of the characteristics of the interrogators and transpondors operating, but is independent of the number of transpondors involved.

(b) The appearance of unlocked reply signals in the responsor. These unlocked replies, known as "fruit," appear as clutter on a display and if present in a responsor connected to actuate automatic devices, may cause a hostile target to be identified as friendly. The amount of fruit clutter present is a function of both the number and characteristics of the interrogators and transpondors operating.

These two factors are not independent of each other, and the relative effect of each varies with the conditions involved. The particular combination which will result in failure to identify would be determined by a "fruit"-to-signal-ratio which exceeds some critical value. Given certain system characteristics, it is possible to determine a value of fruit and/or count down at a given responsor such that the probability of exceeding this value is less than some fixed small amount which is regarded as negligible. The amount of fruit or count down thus determined represents the largest value which the system must be capable of tolerating for satisfactory operation.

Interrogation and Reply Coding

Interrogation coding may be accomplished by multiple pulse techniques or by the use of more than one interrogation frequency channel. With present transpondor techniques, dead time is longer than the duration of a pulse-pattern coded IFF signal, and thus multiple-pulse space-coded modes will have no different aspect for our purpose than single-pulse transmission. Interrogation coding by varying interrogation frequency will complicate the problem at hand; therefore first consideration will be a system using single-pulse interrogation on a single interrogation frequency. The effects of several interrogation frequencies will then be considered separately. Since a practical pulse-coded system which discriminates against random noise signals greatly reduces the effects of noise interference, our calculations will more nearly approximate this situation and at the same time will be greatly simplified if we neglect the possible effects of noise triggering in this single-pulse system.

Reply frequency coding which is done by varying the reply frequency channel in several discrete steps is no different for our purpose than no reply frequency coding at all, since all transpondors and responsors will be operating on the same channel at any given time whether the channel varies or not. Our first consideration will therefore be a system employing single-pulse reply signals on a single reply frequency channel. The effects of having several reply frequency channels, each one determined by a particular interrogation channel, and all used simultaneously to divide the traffic among channels, will be considered in connection with several interrogation channels.

DERIVATIONS - A SIMPLIFIED IFF SYSTEM

The system considered here is one employing single-pulse interrogation on a single interrogation frequency and single-pulse reply on a single reply frequency.

Single Transpondor. All Interrogator Beam Widths the Same

Maximum number r of interrogators to be considered. Suppose we have n "foreign" interrogators and one transpondor in a given region. We are interested in determining a number of "foreign" interrogators, r, such that the probability $P_n(\geq r)$ that r or more of the n interrogators challenge the transpondor during a given "instant" of time is some small value. The question arises: What value should be set for $P_n(\geq r)$ for satisfactory operation? If r is chosen large enough, $P_n(\geq r)$ will, of course, become vanishingly small. In the operation of any piece of equipment, however, there is a certain risk that it may not work at all because of mechanical or electrical failure, and this risk is not vanishingly small. Little advantage will result from making design risks much less than this ordinary risk of equipment failure. If experience shows that an equipment is likely to fail operationally one percent of the time, there is little advantage in designing equipment to operate for a value of $P_n(\geq r)$ much less than one percent. For all practical purposes then, we can say in this case, that if $P_n(\geq r)$, the probability that r or more of the n interrogators will challenge the transpondor, is one percent or less, than r is the maximum number of simultaneous interrogations which need be considered for design purposes.

If we assume that complete identification is not required in one' "instant", that is, if we can afford to wait for a second radar scan of the target if necessary, then the probability $P'_n(\geq r)$ that r or more out of the n interrogators will challenge the transpondor during both scans is

$$P'_n(\geq r) = \left[P_n(\geq r)\right]^2$$

Similarly if we can be allowed to wait for more radar scans for complete identification, the exponent in the above expression will be the number of radar scans allowed. Under such conditions, the value of $P_n(\geq r)$ can be allowed to be greater than for the case where identification must be complete in one "instant". Since we know future target speeds will increase, it seems wisest to base our calculations on the assumption that complete IFF identification is required in one "instant" of time.

In finding an expression for $P_n(\geq r)$ and a method of determining r for a given $P_n(\geq r)$, it is necessary to distinguish between the terms "interrogate" or "challenge", and "trigger." As used here, an interrogator "interrogates" or "challenges" a transpondor if its challenging signals arrive at the transpondor with sufficient amplitude for triggering, regardless of whether or not the transpondor is triggered. Thus the number of interrogations r is unaffected by transpondor dead time τ, while the number of times the transpondor is triggered may be less tha r if r and τ are both large enough.

In view of the assumptions of randomness and of the above definition of "interrogate", it is now reasonable to assume that the interrogation of the transpondor by any one interrogator is independent of the interrogation of the transpondor by any other interrogator. One may thus use a fundamental theorem of probability which is stated generally as follows.[5] Consider n independent events $A_1, A_2, A_3 \ldots . A_n$, whose respective probabilities are p_1,

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[5] G. Chrystal, Textbook of algebra, Vol. II, pp. 584-585, A. and C. Black, Ltd., London, (1931).

$p_2, p_3 \dots p_n$. The probability that exactly r of the n events happen is

$$P_n(r) = \Sigma\, p_1 p_2 p_3 \dots p_r \left(1-p_{r+1}\right)\left(1-p_{r+2}\right)\dots\left(1-p_n\right) \tag{1}$$

where the use of Σ in equation (1) denotes the sum of all the possible combinations of r out of the n events forming a product of the type in equation (1).[6]

If the probability of each single event is the same in all cases, equation (1) reduces to the binomial law

$$P_n(r) = C_r^n\, p^r (1-p)^{n-r} = \frac{n!}{r!(n-r)!} p^r (1-p)^{n-r} \tag{2}$$

where p is the probability that an event occurs and (1-p) is the probability that an event does not occur.

If n is large and p is small, the Poisson Law is a good approximation for equation (2):

$$P_n(r) = \frac{e^{-\epsilon}\epsilon^r}{r!} \tag{3}$$

where $\epsilon = np$ is called the expected or average number of times the event will occur in a large number of trials.[7] Examination of this approximation indicates that it is reliable for values of n substantially greater than 100 and of p not larger than 1/10. If we consider $n < 100$ and values of p as large as 1/10 the approximation is questionable and the exact equation (2) should be used. Therefore equations (2) and (3) express the probability that exactly r out of n interrogators will challenge the transpondor during a given "instant", where p is the probability that a single interrogator will challenge the transpondor. If θ is the interrogator antenna beam width (assuming all interrogators to have equal beam widths) then[8]

$$p = \frac{\theta}{360}$$

The probability that less than r of the n interrogators challenge the transpondor is

$$P_n(<r) = P_n(0) + P_n(1) + \dots + P_n(r-1) \tag{4}$$

If n is greater than approximately 100 and $p < \frac{1}{10}$ we can substitute the approximation of equation (3) in equation (4):

$$P_n(<r) = e^{-\epsilon}\left[\frac{1}{0!} + \frac{\epsilon}{1!} + \frac{\epsilon^2}{2!} + \frac{\epsilon^3}{3!} + \dots + \frac{\epsilon^{r-1}}{(r-1)!}\right] \tag{5}$$

If $n \leq 100$, by substituting (2) in (4) we have:

$$P_n(<r) = \frac{n!}{0! \times n!} p^0 (1-p)^n + \frac{n!}{1! \times (n-1)!} p^1 (1-p)^{n-1} +$$

$$\frac{n!}{2!(n-2)!} p^2 (1-p)^{n-2} + \dots + \frac{n!}{(r-1)!(n-r+1)!} p^{r-1} (1-p)^{n-r+1} \tag{6}$$

---

[6] See Appendix I for clarification.

[7] T. C. Fry, Probability and its engineering uses, pp. 214-215, Van Nostrand (1928).

[8] See Appendix II.

Both equations (5) and (6) contain 0! The value of 0! is shown to be unity[9] by writing

$$X! = X(X-1)!$$

and substituting the value X = 1.

The probability that r or more of the interrogators will challenge the transpondor is

$$P_n(\geq r) = 1 - P_n(<r). \tag{7}$$

For any desired design risk $P_n(\geq r)$ in a given system, equation (7) gives the corresponding value of $P_n(<r)$. If this figure is substituted in equation (5) or (6), depending on the size of n and p, r is determined by carrying out the series to the minimum number of terms to make the right side at least equal to the left. Figures 1, 2, and 3 have been

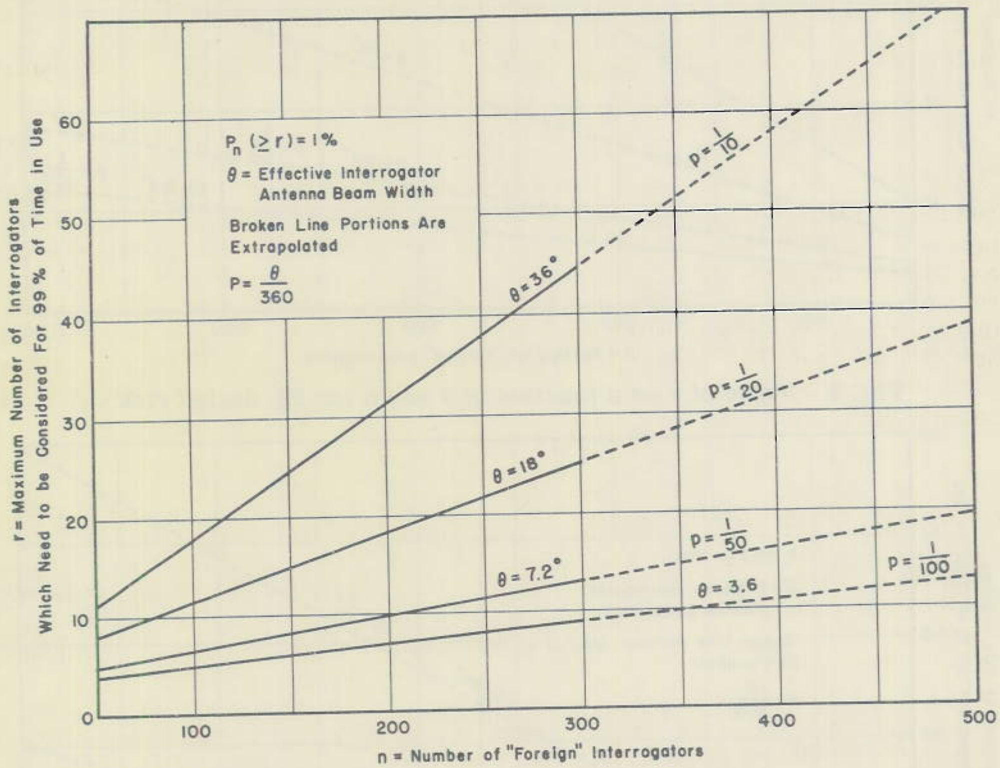

Fig. 1 - Value of r as a function of θ and n for 1 % design risk

prepared for convenience in determining r for specific examples. For values of $n \leq 100$, the ordinate r was obtained from equation (6) and for values of $n \geq 200$ the ordinate r was obtained from equation (5). Figure 1 is based on a design risk of 1%, that is, the value of r found for any given interrogator beam width θ and any number of interrogators n is such that only during 1% of the time of system operation will r or more of the n interrogators be challenging a transpondor. Figures 2 and 3 are based on design risks of 5%, and 10% respectively. Values of r should be read to the nearest integer. For values of $n<50$ the fruit and count down problem is not serious. For $n>500$ the graphs could be

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[9] T. C. Fry, Probability and its engineering uses, pp. 21-23, Van Nostrand (1928).

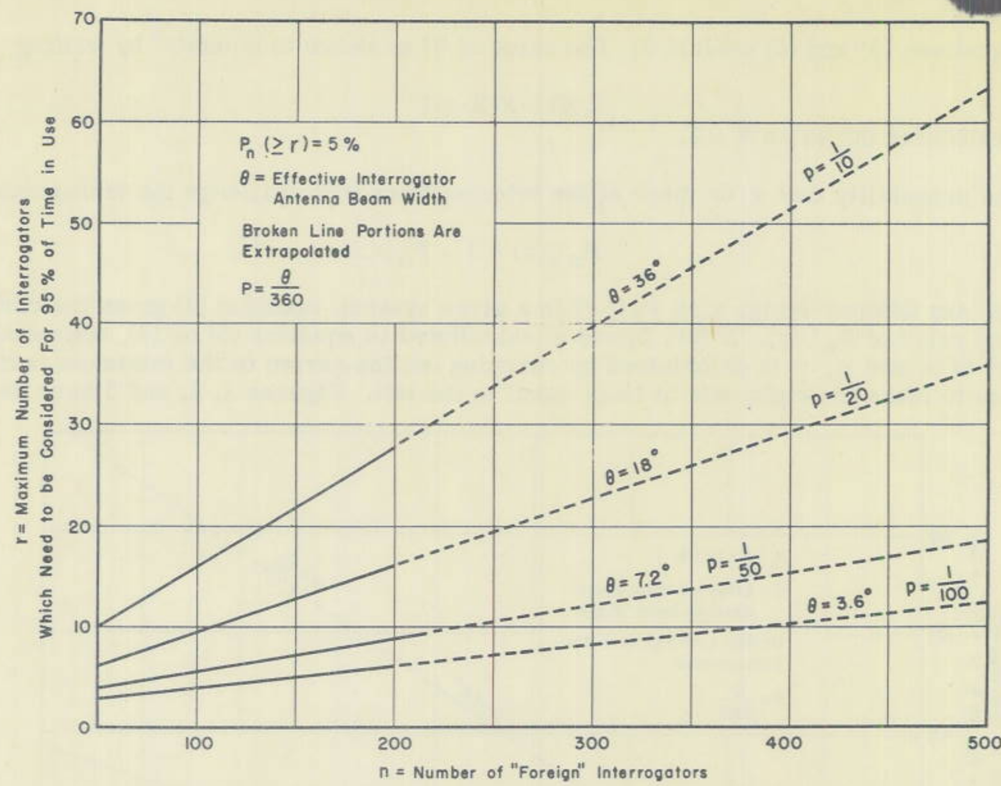

Fig. 2 - Value of r as a function of θ and n for 5% design risk

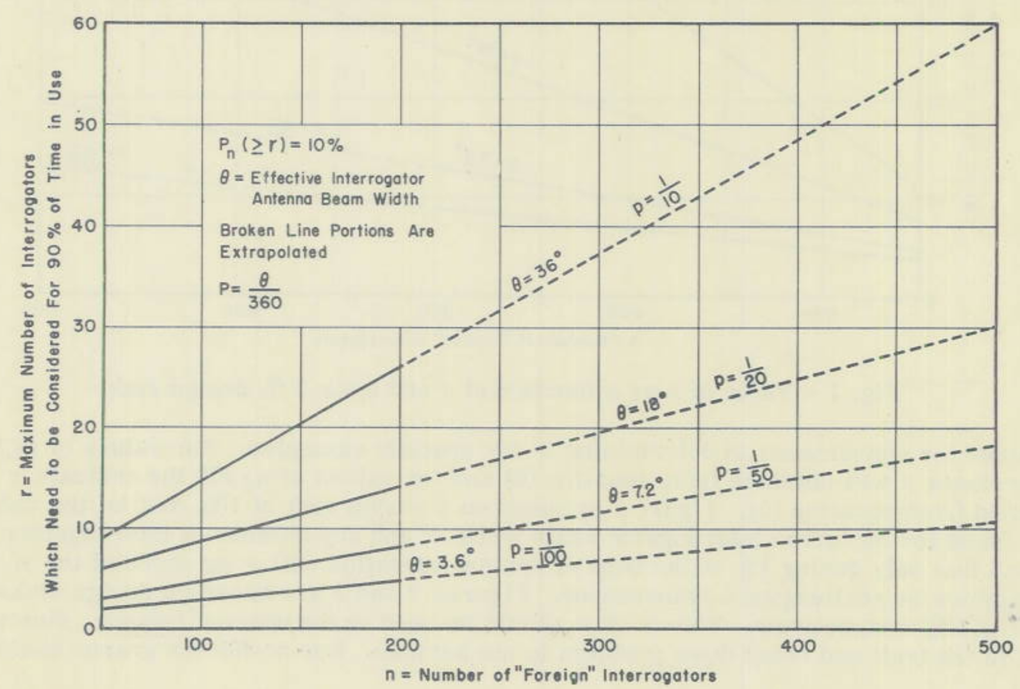

Fig. 3 - Value of r as a function of θ and n for 10% design risk

extrapolated. Intermediate values of θ can be interpolated if one remembers that the emphasis belongs on the order of magnitude of the result rather than on its exactness.

Figure 4, prepared entirely from equations (5) and (7), plots $P_n(\geq r)$ against r for several values of ϵ. Since the Poisson approximation is good only for large n ($n>100$), the use of Figure 4 for $n\leq 100$ should be limited to determining the order of magnitude of $P_n(\geq r)$ but not an exact value. This graph may be used to answer the following questions: Given an IFF system with a fixed value of ϵ, what is the probability that a certain number r of the interrogators will challenge a transpondor simultaneously? Or, what number r of the interrogators have an associated probability $P_n(\geq r)$?—that is, what is the number r, such that the probability that r or more of the interrogators challenging a transpondor simultaneously is represented by $P_n(\geq r)$?

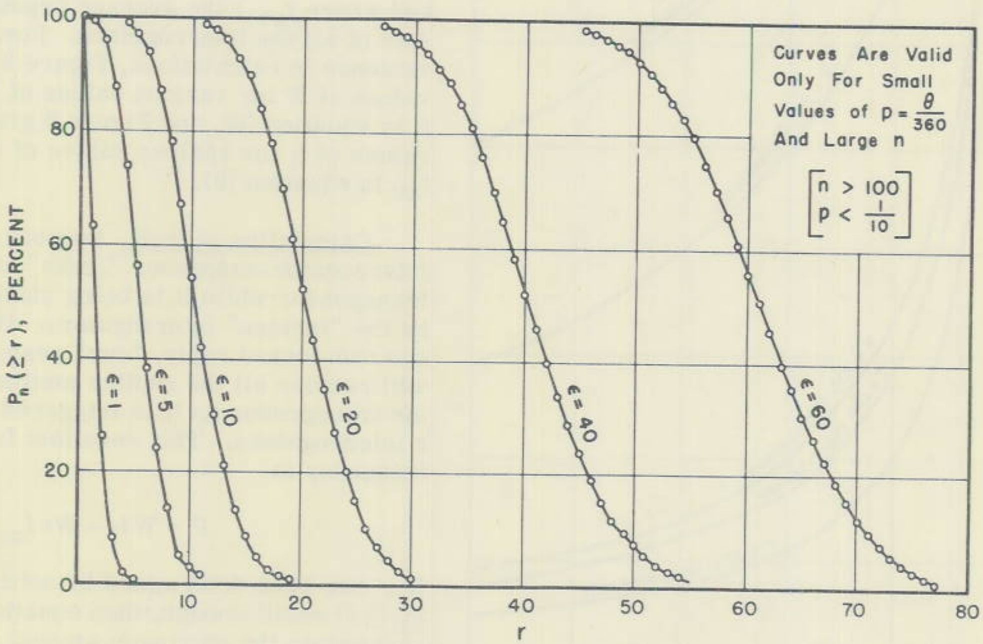

Fig. 4 - Relation between $P_n(\geq r)$ and r for several values of $\epsilon = np$ (Equations 5 and 7).

Calculation of transpondor count down. Since the transpondor may be expected to have a finite "dead" time, there will be a definite count down of replies for every interrogation rate. The maximum count down which needs to be considered is that which results from an interrogation rate due to r interrogators challenging during a given "instant" of time. If count down, or fractional response, is given by

$$W = \frac{\text{Reply Signals}}{\text{Total No. of Interrogations}}$$

then W represents the probability that an interrogation will result in a reply. Since the probability of not obtaining a reply is equal to the fraction of the time occupied by all the recovery periods, the maximum count down which needs to be considered for design

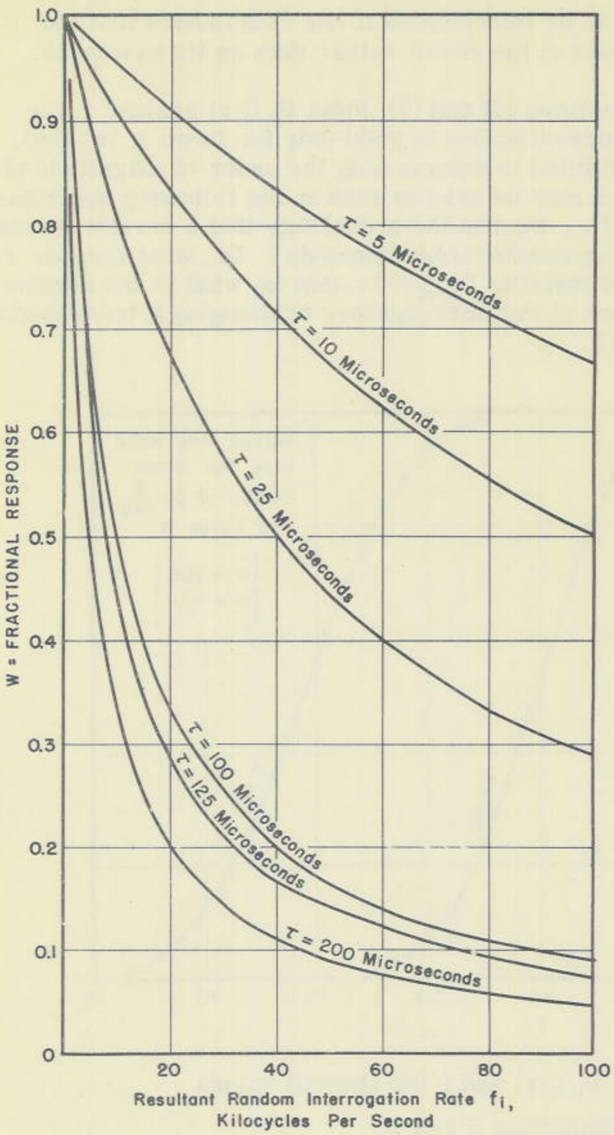

Fig. 5 - Value of transpondor count down as a function of interrogation rate, f_i and recovery time, τ

purposes is given by the relationship[10]

$$1 - W = W f_i \tau$$

or

$$W = \frac{1}{1 + f_i \tau} \qquad (8)$$

where τ is the transpondor dead time in seconds, f_i is the interrogation rate, given by the relationship

$$f_i = r f_{av} \qquad (9)$$

and where f_{av} = the average repetition rate of all the interrogators. For convenience in calculations, Figure 5 gives values of W for various values of τ and f_i in equation (8), and Figure 6 gives values of f_i for various values of r and f_{av} in equation (9).

<u>Calculation of fruit.</u> Suppose "our" interrogator-responsor "looks" at the transpondor while it is being challenged by the "foreign" interrogators. Besides our own locked reply, "our" responsor will receive all the replies emitted by the transpondor as it is triggered by the r interrogators. This resultant fruit frequency is

$$F = W f_i = W r f_{av}. \qquad (10)$$

If r has been determined by setting $P_n(\geq r)$ small enough, then equation (10) represents the maximum amount of fruit which we need to consider for the case n interrogators and one transpondor, even though the maximum amount possible is

$$F_m = W n f_{av}.$$

Single Transpondor. Interrogator Beam Widths Not All the Same

<u>Maximum number r′ of interrogators to be considered.</u> Suppose we have a system where all interrogator beam widths are not all the same but are arranged as follows:

[10] J. R. Parsons, PPI display visibility of Morse-coded transpondor replies under heavy traffic conditions, NRL Letter Report Serial 15/47, 3 October 1947. See also H. H. Bailey, The probability of beacon response, p. 108 in Radiation Laboratory Series, Vol. III, Radar beacons, McGraw-Hill (1947).

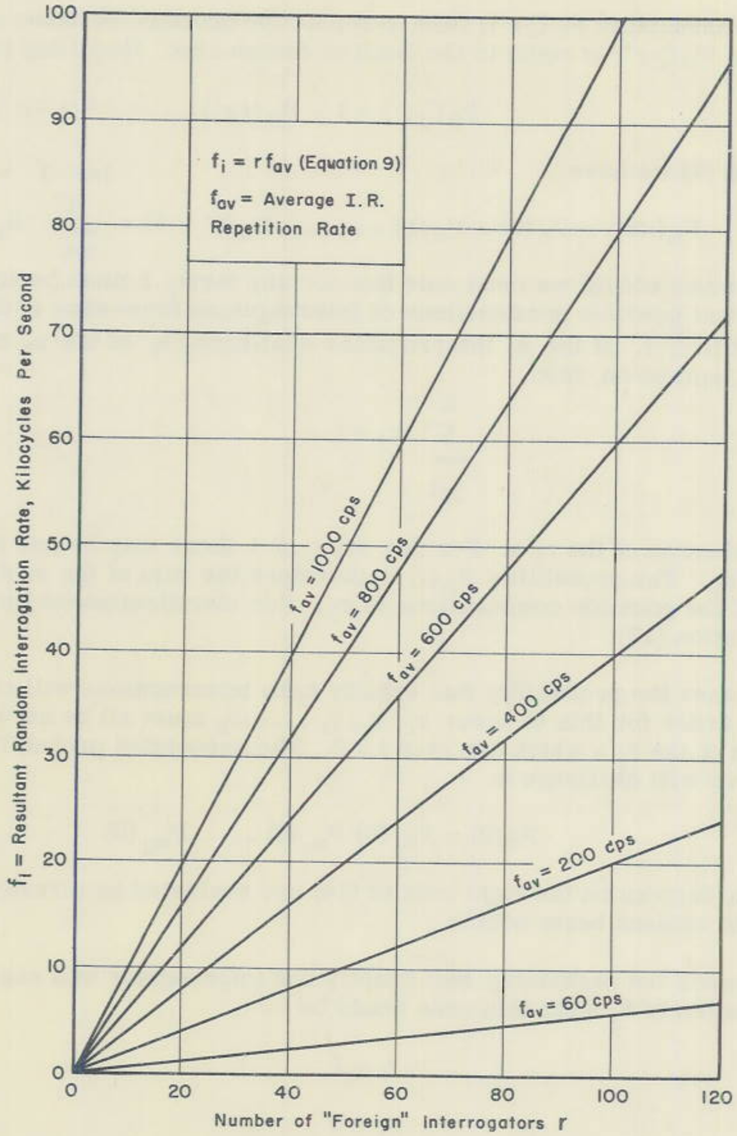

Fig. 6 - Interrogation rate as a function of r and average repetition rate

n_1	interrogators	....	beam width θ_1	....	probability	p_1
n_2	"	....	" " θ_2	....	"	p_2
n_3	"	....	" " θ_3	....	"	p_3
⋮		....	⋮			⋮
n_k	"	....	" " θ_k	....	"	p_k

n = Total number of "foreign" interrogators.

The problem is to find a number r′ of interrogators such that the associated probability $P_n(\geq r')$ = design risk. Essentially, it will be necessary to pick an arbitrary value for r′

and calculate the associated $P_n(\geq r')$; then to repeat the process for other values of r' until the resulting $P_n(\geq r')$ is equal to the desired design risk. Recalling equation (7), we may write

$$P_n(\geq r') = 1 - P_n(<r') \tag{11}$$

and from equation (4) we have

$$P_n(<r') = P_n(0) + P_n(1) + \ldots + P_n(r' - 1) = \sum_{i=0}^{i=r'-1} P_n(i) \tag{12}$$

To evaluate the terms of (12) we must note that for any term, i must be considered to be made up of different possible combinations of interrogators from each of the groups n_1, $n_2, \ldots n_k$. That is if r_1 of the n_1 interrogators challenge, r_2 of the n_2 challenge, r_3 of the n_3 challenge, and so on, then

$$\sum_{j=1}^{k} r_j = i \tag{13}$$

for a given combination of the r_j's. For any value of i there may be one or more possible combinations of r_j. The probability $P_n(i)$ is therefore the sum of the probabilities associated with each of the possible combinations of r_j. For clarification we proceed to evaluate the terms of equation (12):

$P_n(0)$ expresses the probability that exactly zero interrogators will challenge the transpondor. In order for this to occur $r_1, r_2, r_3 \ldots r_k$ must all be zero. This is the only combination of the r_j's which can give $i = 0$. The associated probability that exactly zero of each group will challenge is

$$P_n(0) = P_{n_1}(0)\, P_{n_2}(0) \ldots P_{n_k}(0) \tag{14}$$

where each of the factors on the right side of (14) are evaluated as already outlined for the case of single antenna beam widths.

$P_n(1)$ expresses the probability that exactly one interrogator will challenge. One possible combination of r_j's for this case would be

$$\begin{aligned} r_1 &= 1 \\ r_2 &= 0 \\ r_3 &= 0 \\ \vdots\ & \quad \vdots \\ r_k &= 0 \end{aligned}$$

The probability of this happening is

$$\Big[P_n(1)\Big]_1 = P_{n_1}(1)\, P_{n_2}(0)\, P_{n_3}(0) \ldots P_{n_k}(0)$$

where the subscript on the left hand term associates the probability with the first possible combination. A second possible combination of r_j's for this case would be

$$r_1 = 0$$
$$r_2 = 1$$
$$r_3 = 0$$
$$\vdots \quad \vdots$$
$$r_k = 0$$

and the associated probability for this combination is

$$\left[P_n(1)\right]_2 = P_{n_1}(0)\, P_{n_2}(1)\, P_{n_3}(0) \; \ldots \; P_{n_k}(0).$$

Since $P_n(1)$ is the sum of the probabilities for each possible combination, we may now write

$$P_n(1) = \sum_{c=1}^{c=k} \left[P_n(1)\right]_c = \sum_{c=1}^{c=m_1} \left[P_n(1)\right]_c \tag{15}$$

where the subscript c is associated with the possible combinations of r_j which give $i = 1$, and $m_1 = k$ is the number of possible combinations of r_j which give

$$\sum_{j=1}^{k} r_j = 1.$$

$P_n(2)$ expresses the probability that exactly two interrogators will challenge. One possible combination of r_j's for this case would be

$$r_1 = 2$$
$$r_2 = 0$$
$$r_3 = 0$$
$$\vdots \quad \vdots$$
$$r_k = 0$$

and the associated probability is written

$$\left[P_n(2)\right]_1 = P_{n_1}(2)\, P_{n_2}(0)\, P_{n_3}(0) \; \ldots \; P_{n_k}(0).$$

Determining the number of combinations of r_j which give

$$i = \sum_{j=1}^{k} r_j = 2$$

is more complicated than for the case $i = 1$. In general, however,

$$P_n(2) = \sum_{c=1}^{c=m_2} \left[P_n(2)\right]_c \tag{16}$$

where m_2 is the number of differerent possible combinations of the r_j's which will give

$$i = \sum_{j=1}^{k} r_j = 2 .$$

$P_n(i)$ the general term for (12), may now be written as

$$P_n(i) = \sum_{c=1}^{c=m_i} \left[P_n(i)\right]_c \tag{17}$$

where m_i is the number of different combinations of the r_j's which will give

$$\sum_{j=1}^{k} r_j = i ,$$

and c is associated with a particular combination. Equation (12) may now be written by substituting the value of the general expression for $P_n(i)$ given in (17):

$$P_n(<r') = \sum_{i=0}^{i=r'-1} \sum_{c=1}^{c=m_i} \left[P_n(i)\right]_c \tag{18}$$

and this value is substituted in (11) to determine the desired value of $P_n(\geq r')$.

Calculation of transpondor count down and fruit. Calculations are made exactly as for the case where all interrogator beams are the same except that r' is used instead of r.

A Number of Transpondors. All Interrogator Beam Widths the Same

Calculation of transpondor count down. Transpondor count down is independent of the number of transpondors in a region of operation and therefore the previous sections, pertaining to a single transpondor when all interrogator beam widths are the same, apply in this case.

Calculation of fruit. Our problem is to determine a tolerable fruit rate F_t', such that the probability, $P(\geq F_t')$, that "our" responsor will display fruit at a rate equal to or greater than F_t' is some small value. If $P(\geq F_t')$ is set low enough, for instance if $P(\geq F_t')$ is less than, or equal to, the design risk, then F_t' is the maximum displayed fruit rate that the system must tolerate for satisfactory operation. The logical way to select F_t' is to derive an equation which gives $P(\geq F')$ for any given value of F', and then select F_t' as that value of F' which corresponds to the probability $P(\geq F')$ = design risk.

Any value of fruit F' which might be obtained is the result of one or more possible combinations of A transpondors within "our" antenna beam being challenged by r "foreign" interrogators, where $A \leq B$ and $r \leq n$. Thus the probability $P(\geq F')$ which is calculated is a probability relating to various possible combinations of A and r. Suppose, for simplicity, we disregard count-down effects by assuming an "ideal" system with no count down. We may then calculate the probability $P(\geq F)$ relating to a given amount of fruit, F, from this "ideal" system. With beam widths and prf's fixed, this probability is a function of certain combinations of A and r, and is a probability relating to these combinations. If we now imagine the "ideal" system replaced by a practical system, the actual displayed fruit F'

[redacted]

for any combination of A and r is certain never to be greater than F and in most cases will be less. Since the combinations of A and r which have been considered are unchanged, the probability $P(\geq F)$ can be said to relate to the actual fruit as well, that is,

$$P(\geq F) = P(\geq F') \tag{19}$$

where F is obtained by the relationship

$$F = rAf_{av} \tag{20}$$

and where

$$F' < F \tag{21}$$

Summarizing, we avoid the difficulties encountered in finding $P(\geq F')$, which would necessarily include count-down effects, by ignoring these effects and deriving a means for finding $P(\geq F)$. Then, by selecting a value of "fruit" $F = F_t$ such that $P(\geq F_t)$ equals the design risk, we designate F_t as the maximum tolerance limit. By equations (19) and (21) we are assured that F_t is a safe maximum tolerance limit for design purposes. Further, since F_m, the maximum possible fruit under any conditions, is given by

$$F_m = nBf_{av}$$

and F_t is obtained from equation (20), where ordinarily r and A will be less than n and B respectively, we know that

$$F_t' < F_t < F_m \tag{22}$$

and we thereby avoid the over-design which would result from using F_m instead of F_t as the upper tolerance limit.

In calculating $P(\geq F)$ for any given value of F we recall equation (7). A similar relationship

$$P(\geq F) = 1 - P(< F) \tag{23}$$

fixes the relationship between the probability, $P(<F)$, that "our" responsor will obtain fruit at a rate less than F, and $P(\geq F)$. Thus if we calculate $P(<F)$ for any F, we find $P(\geq F)$ from equation (23). Recalling equation (4), we may write a similar expression for $P(<F)$:

$$P(<F) = P\left[F_1\right] + P\left[F_2\right] + \ldots + P\left[F_S\right] \tag{24}$$

where the terms on the right-hand side are to include the probabilities of obtaining every possible fruit rate from A transpondors and r interrogators, $A \leq B$ and $r \leq n$, the only restriction being that none of the fruit rates considered be equal to or greater than F. It should be noted that the subscripts refer to different possible combinations of A and r and not necessarily to different fruit frequencies. That is, there may be one or more combinations of A and r which can give the same value of fruit. The original assumptions of randomness imply that the position of "our" antenna and that of any "foreign" antenna, as well as the location of any transpondor are all independent of one another and therefore any term of equation (24) is expressed as the product

$$P\left[F_i\right] = \left[P_n(r_i)\right] \left[P_B(A_i)\right] \tag{25}$$

The factors of equation (25) are obtained from equations (2) or (3); that is,

$$P(F_i) = C^n_{r_i}\, p^{r_i}(1-p)^{n-r_i} \times C^B_{A_i}\, p^{A_i}(1-p)^{B-A_i} \tag{26}$$

 [redacted]

or

$$P(F_i) = \frac{e^{-np}[np]^{r_i}}{r_i!} \times \frac{e^{-Bp}[Bp]^{A_i}}{A_i!} \tag{27}$$

where (27) applies only in case the Poisson approximation is good, and where $p = \theta/360$, assuming θ equal to the "effective" beam width of all interrogator-responsors. Equation (24) may now be written

$$\begin{aligned} P(<F) = {} & P_n(0)\left[P_B(0) + P_B(1) + P_B(2) + \dots + P_B(B)\right] + \\ & P_n(1)\left\{P_B(0) + P_B(1) + P_B(2) + \dots + P_B\left[f(1)\right]\right\} + P_n(2)\left\{P_B(0) + \right. \\ & \left. P_B(1) + P_B(2) + \dots + P_B\left[f(2)\right]\right\} + \dots + P_n(r)\left\{P_B(0) + \right. \\ & \left. P_B(1) + P_B(2) + \dots + P_B\left[f(r)\right]\right\} + \dots + P_n(n)\left\{P_B(0) + \right. \\ & \left. P_B(1) + P_B(2) + \dots + P_B\left[f(n)\right]\right\} . \end{aligned} \tag{28}$$

Each term of (28) is the sum of the probabilities of a number of possible combinations of A and r, and therefore contains the probabilities of obtaining various possible fruit frequencies less than F. In the general term,

$$P_n(r)\left\{P_B(0) + P_B(1) + \dots + P_B\left[f(r)\right]\right\} \tag{29}$$

the last term of the series in the bracket is determined from the following considerations. If we solve for A in equation (20),

$$A = \frac{F}{rf_{av}}$$

then the product $P_n(r) \times P_B\left[\frac{F}{rf_{av}}\right]$ is a probability of obtaining a fruit rate exactly equal to F. However, the last term of (29) is a term of the series (28) which includes only probabilities associated with fruit rates less than F. We must therefore end the series of (29) with a value of A which is one less than the above product, that is,

$$f(r) = \left[\frac{F}{rf_{av}}\right] - 1 \tag{30}$$

and the general term (29) of equation (28) becomes

$$P_n(r)\left\{P_B(0) + P_B(1) + \dots + P_B\left[\frac{F}{rf_{av}} - 1\right]\right\} = P_n(r) \times P_B\left(<\frac{F}{rf_{av}}\right) . \tag{31}$$

Considering the first term of (28),

$$P_n(0)\left[P_B(0) + P_B(1) + P_B(2) + \dots + P_B(B)\right] = P_n(0) \times 1 = P_n(0) \times P_B(<\infty)$$

But since $\lim_{r \to 0}\left(\frac{F}{rf_{av}}\right) = \infty$, it is apparent that the general term (31) applies to all terms of (28) including the first, and we may now write (28) as follows:

[redacted]

$$P(<F) = P_n(0) \times P_B(<\infty) + P_n(1) \times P_B\left(<\frac{F}{f_{av}}\right) + P_n(2) \times P_B\left(<\frac{F}{2f_{av}}\right) + P_n(3) \times P_B\left(<\frac{F}{3f_{av}}\right) + \ldots + P_n(r) \times P_B\left(<\frac{F}{rf_{av}}\right) + \ldots + P_n(n) \times P_B\left(<\frac{F}{nf_{av}}\right) \tag{32}$$

or

$$P(<F) = P_n(0) + \sum_{r=1}^{n} P_n(r) \times P_B\left(<\frac{F}{rf_{av}}\right) \tag{33}$$

From (23)

$$P(\geq F) = 1 - \left[P_n(0) + \sum_{r=1}^{n} P_n(r) \times P_B\left(<\frac{F}{rf_{av}}\right)\right]. \tag{34}$$

The parameters involved in (34) are F, f_{av}, $p = \theta/360$, n and B. In plotting curves for (34) one could reduce the number of parameters by one by introducing the parameter

$$K = \frac{F}{f_{av}} \tag{35}$$

and plot $P(\geq F)$ for different values of K. Each graph would be a family of curves corresponding to various "effective" beam widths.

A Number of Transpondors. Interrogators Beam Widths Not All the Same

Calculation of transpondor count down. We again observe that transpondor count down is independent of the number of transpondors in a region of operation and therefore the previous derivations for the case of a single transpondor and interrogator beam widths not all the same applies in this case.

Calculation of fruit. We follow a similar argument as for the analogous situation for equal interrogator beam widths except that here we are concerned with combinations of A and r′ instead of A and r. Thus in writing the general term analogous to (31) for the equation analogous to (28), we find that the factor $P_n(r)$ is of the form of equation (17). Hence the final equation analogous to (34) would be

$$P(\geq F) = 1 - \left\{P_n(0) + \sum_{r'=1}^{n}\left[\sum_{c=1}^{c=m_{r'}} \left[P_n(r')\right]_c \times P_B'\left(\frac{< F}{r' f_{av}}\right)\right]\right\} \tag{36}$$

This expression is extremely more complicated than (34) owing to the double summation.

DERIVATIONS — A MORE COMPLEX IFF SYSTEM USING SEVERAL REPLY FREQUENCY CHANNELS

Single Transpondor

Suppose there are n interrogators operating on t channels, each interrogator being capable of triggering any transpondor. Assume, however, that each responsor is sensitive

[redacted]

only to a specific reply frequency channel which is determined by the particular interrogation channel used, and that the transpondor cannot reply to more than one challenge signal at a time. Such conditions will not affect the relationships involving the number r of interrogations or the count down experienced in the transpondor (equations 1 through 8 and Figures 1, 2, 3, 4 and 5). However, the effective interrogation rate, ${}_ef_i$, for calculating fruit response will be less than the value of f_i as given by equation (9) and will, in fact, be

$${}_ef_i = sf_{av} \tag{37}$$

where s is some number less than r. If it is true that any one interrogation channel may be used independently of any other, the law of independent trials again applies to calculate the probability that s of the r interrogations will occur on the interrogation channel which will produce a fruit reply on "our" responsor's reply channel. The probability p_c that one of the "foreign" interrogators will challenge on "our" channel is determined by the operating procedure in selecting the channels. If the interrogation channel selection is completely random, then $p_c = 1/t$. If the interrogation channels are assigned according to some definite plan, then p_c will be the ratio of the number of interrogators on "our" channel to the total number n. In any case, the probability that exactly s of the r interrogators will challenge correctly is

$$P_r(s) = C_s^r \, p_c^s \, (1-p_c)^{r-s} \tag{38}$$

and the probability that s or more of the r interrogators will challenge correctly is

$$P_r(\geq s) = 1 - \left[P_r(0) + P_r(1) + \ldots. + P_r(s-1) \right] \tag{39}$$

However, the Poisson approximation cannot be employed to determine s, since the necessary conditions (that r is large and p_c small) will in most cases not be met. The value of s for each particular example will be found by substituting the desired percentage for $P_r(\geq s)$ in equation (39) and carrying out the series to the required minimum number of terms to make the right-hand side a least equal to the left-hand side. Each term of the right-hand side is found by the formula of equation (38) where

$$C_s^r = \frac{r!}{s!(r-s)!} .$$

If $P_r(\geq s)$ is chosen small enough; then s rather than r becomes the maximum effective number of interrogations which need to be considered in calculating fruit. If $P_n(\geq r)$ has also been chosen small enough in determining r, then the maximum amount of fruit which we need consider is

$$F = Wsf_{av} . \tag{40}$$

If on the other hand, the transpondor has t separate and independent transmitting channels, permitting it to emit as many as t simultaneous replies, then both the count down and resulting fruit received on a given responsor will be different from what they were for a single interrogation channel. If p_c, the probability of an interrogation occurring on "our" channel, is used in determining s, then equation (9) for determining interrogation rate becomes

$${}_ef_i = sf_{av} \tag{41}$$

where ${}_ef_i$ is the effective interrogation rate on "our" channel. Fractional response, or count down, on our channel (which may be different than on other channels) is

[redacted]

$$W = \frac{1}{1 + e^{f_i \tau}} \tag{42}$$

Figure 6 may be used for this case if $_{e}f_i$ is substituted for f_i.

A Number of Transpondors

We follow a similar argument as for the analogous situation for a single reply frequency channel except that here we are concerned with combinations of A, r and s instead of A and r alone. Thus the equation analogous to (25) is a triple product

$$P\left[F_i\right] = \left[P_n(r_i)\right] \left[P_{r_i}(s_i)\right] \left[P_B(A_i)\right] \tag{43}$$

The general term analogous to (29) for the equation analogous to (28) in this case is

$$P_n(r) \times P_B(A) \left[P_r(0) + P_r(1) + \dots + P_r\left[f'(r)\right]\right] \tag{44}$$

where a similar derivation would give

$$f'(r) = \left[\frac{F}{Af_{av}} - 1\right] \tag{45}$$

Hence the final equation for $P(\geq F)$ analogous to equation (34) would in this case be

$$P(\geq F) = 1 - \left\{P_n(0) \times P_B(0) + \sum_{A=1}^{B} \sum_{r=1}^{n} \left[P_n(r) \times P_B(A) \times P_r\left(< \frac{F}{Af_{av}}\right)\right]\right\} \tag{46}$$

for the case where all interrogator beam widths are the same, and an even more complex expression

$$P(\geq F) = 1 - \left\{P_n(C) \times P_B(0) + \sum_{A=1}^{B} \sum_{r'=1}^{n} \left[\sum_{c=1}^{m_{r'}} \left[P_n(r')\right]_c \times \sum_{c=1}^{m_A} \left[P_B(A)\right]_c \times P_{r'}\left(< \frac{F}{Af_{av}}\right)\right]\right\} \tag{47}$$

for the case where all interrogator beams are not the same. The double summation in (46) and the multiple summation in (47) makes the task of finding $P(\geq F)$ for any value of F an extremely laborious one for any situation involving more than one reply frequency channel.

WORKING SUMMARY

Equations (1) through (10) can be readily used to work out problems involving transpondor count down and fruit from a single transpondor, assuming all interrogator antenna beams to be equal. For situations where all interrogator beams are not equal, and for problems on fruit from many transpondors, the subsequent relationships are not simple, and the extensive work to prepare curves or work examples is not done here.

We make use of the equations and curves to derive the maximum count down which must be tolerated for a given design risk as follows. We first find r, the number of "foreign" interrogators out of the total number n, such that the probability, $P_n(\geq r)$, that at least r will challenge a transpondor is equal to the given design risk. If this design risk is 1%, 5%, or 10%, then r can be read directly from Figure 1, Figure 2, or Figure 3, respectively. If some other value of design risk is assumed, the $P_n(<r)$ is found by using equations (7) and, finally, r is determined by carrying out the series of equations (4) or (5) (depending upon whether the Poisson approximation holds) until the right-hand side at least equals the left-hand side. Once r is known, the maximum count down which must be tolerated for satisfactory operation with the given design risk is obtained from equations (8) and (9), or, for convenience, by using equation (9) and Figure 5.

Use of equation (10) to find the maximum fruit which must be tolerated from a single transpondor for satisfactory operation with the given design risk, is of little practical importance, since "fruit" from a single transpondor is not likely to be excessive in most cases. Results from this equation could, however, be useful in observing what effects might be expected on fruit from many transpondors if the parameters n, θ, or f_{av} are changed.

ILLUSTRATIVE EXAMPLES

Example 1.

Given: An IFF system with a single transpondor operating in a region containing 200 "foreign" interrogators, with all interrogator antenna beam widths equal to 36°, and the average repetition rate equal to 200 cps.[11] The transpondor "dead" time is 125 microseconds.

Part 1. Find the count-down ratio (fractional response) such that the probability of having or exceeding this ratio is 1% or less.

From Figure 1 we select the graph corresponding to $\theta = 36°$. Since N = 200, the value of r can be read directly as

$$r = 31$$

From equation (9):

$$f_i = 6200 \text{ cps}$$

and from equation (8) or Figure 6:

$$W = 0.56$$

where W = fractional response such that the probability of having $W \leq 0.56$ is 1% or less. This count down may be referred to as the count-down tolerance limit for 1% design risk under the given conditions.

Part 2. Find the fruit rate on "our" responsor such that the probability of having or exceeding this rate is 1% or less. From equation (10):

$$F = Wrf_{av} = 0.56 \times 31 \times 200 = 3.47 \text{ kc.}$$

The value of F may be referred to as the fruit tolerance limit for a 1% design risk if a single transpondor is assumed working under the given conditions.

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[11] The value $\theta = 36°$ is given here as an effective IFF beam width which might be obtained from a system comparable to the present Mark V IFF system and was selected after consulting the references given in the text (footnotes 1 and 2).

Example 2.

What is the effect on the count-down tolerance limit and on the resulting fruit tolerance limit if the effective interrogator antenna beam width is reduced to $\theta = 18^{\circ}$ for the system given in Example 1?

From Figure 1, using the graph corresponding to $\theta = 18^{\circ}$,

$$r = 18$$

From equation (9):

$$f_i = 3600 \text{ cps}$$

and from equation (8) or Figure 6:

$$W = 0.69$$

Using equation (10):

$$F = 2.48 \text{ kc}$$

Example 3.

What is the effect on the count-down tolerance limit and on the resulting fruit tolerance limit if the effective interrogator antenna beam width remains $\theta = 36^{\circ}$ and the average repetition rate is reduced to $f_{av} = 100$ cps for the system given in Example 1?

The value of r in this case is unchanged;

$$r = 31$$

From equation (9):

$$f_i = 3100 \text{ cps}$$

From equation (8) or Figure 6:

$$W = 0.72$$

From equation (10):

$$F = 2.23 \text{ kc}$$

Example 4.

What is the combined effect on the count-down tolerance limit and on the resulting "fruit" tolerance limit for 1% design risk if the effective antenna beam width is reduced to 18° and the average repetition rate is reduced to 100 cps for the same given conditions?
From Figure (1):

$$r = 12$$

From equation (9):

$$f_i = 1200$$

From equation (8) or Figure 6:

$$W = 0.87$$

From equation (10):

$$F = 1.04 \text{ kc.}$$

* * *

[redacted]

ACKNOWLEDGMENT

Derivation of equations (1) through (7) and the associated ideas are due to L. S. Schwartz who initiated this study.

Many of the ideas in this work have been formed or clarified as a result of frequent discussions on the subject with Dr. C. E. Cleeton, Dr. A. J. Ruhlig, and Mr. C. V. Parker. The assistance thus obtained is greatly appreciated.

[redacted]

APPENDIX I

The Probability That Exactly r Out of n Independent Events Happen

Consider n independent events $A_1, A_2, A_3 \ldots\ldots A_n$, whose respective probabilities are $p_1, p_2, p_3 \ldots\ldots p_n$. The probability that exactly r of these events, beginning with A_1 happen is

$$p_1 p_2 p_3 \cdots\cdot p_r \left(1-p_{r+1}\right)\left(1-p_{r+2}\right) \cdots\cdot \left(1-p_n\right)$$

where the factors (1-p) are the probabilities that these events do not happen. The probability that exactly r of the n events, beginning with A_2, happen is

$$\left(1-p_1\right) p_2 p_3 p_4 \cdots\cdots p_r p_{r+1} \left(1-p_{r+2}\right)\left(1-p_{r+3}\right) \cdots\cdot \left(1-p_n\right).$$

The number which we are interested in, however, is not the probability that a certain selection of r of the n events happen, but the probability that exactly r, no matter how selected, of the n events happen. From the foregoing it follows that this probability, $P_n(r)$, is the sum of the probabilities of all possible specific selections of r events. This can be expressed as

$$P_n(r) = \Sigma\, p_1 p_2 p_3 \cdots p_r \left(1-p_{r+1}\right)\left(1-p_{r+2}\right) \cdots \left(1-p_n\right)$$

where the notation Σ denotes the sum of all possible combinations of r out of the n events forming a product of the type given in the equation.

APPENDIX II

The Probability That a Single Interrogator Searching Any Azimuth Sector in a Random Manner Will Challenge a Single Transpondor Located at Random

The challenging of the transpondor depends upon the fact that both of the following events occur:

(1) The transpondor lies in the searching sector of the interrogator.

(2) The transpondor, if it is in the searching sector, also lies within the interrogator beam width.

Let the interrogator beam width be θ degrees. Let the searching sector be ψ degrees, where ψ may have any value from θ to 360°. The probability that a transpondor randomly located will lie in the sector ψ is

$$\frac{\psi}{360} \, .$$

The probability that any transpondor within the search sector ψ also lies within the beam width θ is

$$\frac{\theta}{\psi}$$

Since (1) and (2) are independent events, the probability that both occur is

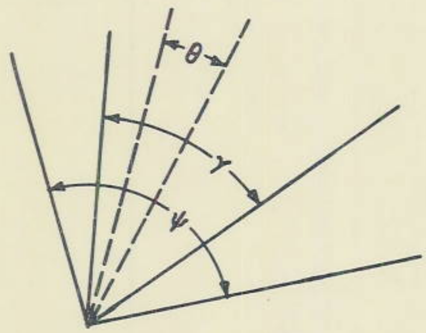
$$p = \frac{\theta}{\psi} \times \frac{\psi}{360} = \frac{\theta}{360}$$

which is the probability that a transpondor randomly located will be challenged by an interrogator of beam width θ searching any azimuth sector in a random manner.

APPENDIX III

The Probability That a Transpondor Located at Random Within an Azimuth Sector γ is Challenged by an Interrogator of Beam Width θ Searching a Sector ψ

θ and γ are included within ψ as shown in the diagram.



If a transpondor located within the azimuth arc γ is to be challenged by an interrogator, both of the following conditions must be fulfilled:

(1) The interrogator beam θ must lie within the sector γ.

(2) The transpondor within the sector γ must also lie inside the interrogator beam θ.

The probability that condition (1) is fulfilled is $p_1 = \frac{\gamma}{\psi}$. Assuming that condition (1) has been fulfilled, the probability that condition (2) is also fulfilled is $p_2 = \frac{\theta}{\gamma}$. Therefore the probability that both conditions are fulfilled, and hence that the transpondor is challenged, is

$$p = p_1 p_2 = \frac{\gamma}{\psi} \times \frac{\theta}{\gamma} = \frac{\theta}{\psi}.$$