[redacted] DECLASSIFIED NRL REPORT R-3368

UNCLASSIFIED

COPY NO. 78

FR-3368

A STUDY OF COMPUTERS AND RADAR FOR AIRCRAFT INTERCEPTION CONTROL

DECLASSIFIED by NRL Contract
Declassification Team
Date: 5 JAN 2017
Reviewer's name(s): [redacted]
Declassification authority: NAVY DECLASS GUIDE / NAVY DECLASS MANUAL, 11 DEC 2012
D8 SERIES

DECLASSIFIED: By authority of
DOD DIR 5200.10
Cite Authority Date
Entered by NRL Code

Transmission by Registered Guard Mail or U. S. registered mail is authorized in accordance with Article 76 (15) (e) and (f), U. S. Navy Regulations.

NAVAL RESEARCH LABORATORY

WASHINGTON, D.C.

DISTRIBUTION STATEMENT A APPLIES
Further distribution authorized by UNLIMITED only.

[redacted]

DECLASSIFIED

SECRET

NRL REPORT R-3368

COPY NO. 124

DECLASSIFIED

A STUDY OF COMPUTERS AND RADAR FOR AIRCRAFT INTERCEPTION CONTROL

W. S. Alderson,
P. A. Guarino and
A. A. Varela

October 13, 1948

Approved by:

Dr. R. C. Guthrie, Head, Search Radar Section
Mr. L. A. Gebhard, Superintendent, Radio Division II

NAVAL RESEARCH LABORATORY
CAPTAIN H. A. SCHADE, USN, DIRECTOR
WASHINGTON, D.C.

DECLASSIFIED

SECRET

[redacted]

DECLASSIFIED

DISTRIBUTION

	Copy No.
BuAer	
Attn: Code EL	1
Attn: Code EL-70	2
Attn: Code EL-74	3
Attn: Code EL-80	4-8
Attn: Code EL-90	9
Attn: Code AR-23	10
Attn: Code TD-4	11
BuShips	
Attn: Code 665	12
Attn: Code 910	13
Attn: Code 912	14
Attn: Code 915	15
Attn: Code 916	16-20
Attn: Code 930	21
Attn: Code 931	22
BuOrd	
Attn: Code Re4a	23
Attn: Code Re4c	24
Attn: Code Re4f	25
CNO	
Attn: OP-413-B2	26-30
Attn: OP-413-C6	31-35
Attn: OP-413-34G	36
Attn: OP-20	37
Attn: OP 34H8	38
Attn: Operations Evaluation Group, Mr. Everett	39
ONR	
Attn: Code 461	40-44
Attn: Code 467	45
Attn: Code 482	46-47
OinC, NRLFS, Boston	
Attn: Code 6110	48
Dir., USNEL	49-50
Cdr., USNOTS	
Attn: Reports Unit	51-52
SNLO, USNELO	53-55
Chief of Staff, USAF	
Attn: Maj. Sather, AFMEN	56
Attn: Maj. P. J. Schenk, AFOAC - E/P	57
Attn: Office of the Air Communications Officer, Miss L. Diamond	58

DECLASSIFIED

[redacted]

[redacted]

DECLASSIFIED

CONTENTS

DECLASSIFIED

[redacted]

ABSTRACT

The design difficulties encountered in attempting to incorporate a two-second scan period in a shipborne aircraft interception control radar have lead to an over-all study of the aircraft interception problem. As a result, the mathematical characteristics of three control-radar computers are developed and analyzed and a system of aircraft interception is suggested in which the shipborne interception control officer directs the interceptor close to the target and relays the relative position of the target in order that the center of the aircraft's radar search sector may be placed on the predicted position of the target. The probability that this search sector will include the target is calculated and favorable results are indicated for six-second scan periods. Some desirable aircraft radar characteristics are found, including the ability to search small solid sectors in a direction indicated by the interception control officer.

PROBLEM STATUS

This is an interim report on the problem; work is continuing.

AUTHORIZATION

NRL Problem R02-42R (BuShips Problem S1430)

A STUDY OF COMPUTERS AND RADAR FOR AIRCRAFT INTERCEPTION CONTROL

INTRODUCTION

As part of the over-all program of aircraft interception control, a study is being made of the radar requirements for "picket" ship fighter-direction. It was proposed to determine the characteristics of the most suitable height-finding radar system for use aboard ships of destroyer size or larger. This report summarizes the results of this study to date and discusses some of the tactical considerations as well as the radar requirements.

A request was made by the Bureau of Ships to study the possibilities of developing a ship-borne light-weight height-finding radar capable of providing range, bearing, and elevation information suitable for controlling aircraft interceptions.[1,2,3] Tentative specifications indicated an anticipated need for providing bearing and altitude data with an accuracy of 20 minutes of arc out to a range of 100 miles and up to an elevation angle of 25° with information rate no less than once every two seconds. It was also desired that this radar provide detection coverage out to 150 miles. In the course of this study it became apparent that there was need for studying the whole aircraft intercept problem to determine more accurately the minimum requirements for the intercept control radar.

HEIGHT-FINDING METHODS

Several methods of height determination should be considered for possible application to meet the requirements of this system.

The V-beam system of height-finding has already been applied to the GCI problem by the Air Forces in its AN/CPS-6 radar. This system uses two fan beams placed at an angle of 45° to each other and the delay between successive incidence of the beams on the target is a measure of its height.

---

[1] BuShips ltr to NRL, S67-5(2) (916B), Ser. R-916-1858 dtd. 13 June 1947 requesting assignment of problem S1430 (NRL No. 39R02-42R)

[2] BuShips ltr to NRL, S67-5(9) (910B), Ser. 2452 (910B) dtd. 14 July 1947. "Proposed Shipboard Combined Aircraft Detection and Tracking Radar" (Intermediate Range) - Military Characteristics for NRL Problem S1430.

[3] BuShips ltr to NRL S67-(26) (916B), Ser. C-916-10772 dtd. 26 September 1947, revising military characteristics of radar on problem S1430.

DECLASSIFIED [redacted]

A vertical scanning system for height-finding is already in use by the Navy in the SX radar. It consists of two radar systems, one for early warning and one for height-finding. Height information is obtained directly by means of the narrow beam scanning vertically.

A stacked-beam system, designated as the AN/SPS-2 and under development at this Laboratory, is also capable of accurate height-finding.[4] Such a system consists of several vertically narrow beams separately fed and tilted progressively upward so as to overlap and provide essentially solid coverage. By comparing the relative signal strengths on adjacent beams it is possible to determine the height of a target quite accurately.

GENERAL RADAR CONSIDERATIONS

Regardless of the type of system used, the choice of basic radar parameters is determined by the requirements of range, accuracy, resolution, data rate, and by the practical restrictions on antenna size. Without attempting to determine conclusively the optimum value of the parameters, the following discussion is submitted to indicate their approximate value and to bring out the significance of requirements of accuracy and data rate.

The large detection range, the desirable fine resolution, and the required azimuth accuracy dictate a narrow azimuth beam. The antenna horizontal aperture must then be large in terms of wavelength and because of the physical size limitations, a high frequency must be used. But to maintain detection range with increasing frequency, it is necessary to increase antenna gain approximately proportional to the frequency. The product of the vertical and horizontal beamwidths must then be equal to some constant times the wavelength and must decrease proportionately as the frequency is increased. If the horizontal beam width is limited because of scanning considerations, a further increase in frequency requires reduction of the vertical beam width. In the case of the V-beam or stacked-beam systems this means that the number of beams required to obtain the specified elevation coverage is proportional to the frequency chosen and will be excessive with too high a frequency. With a vertical scanning system either the time of scan must be increased with frequency or the sector scanned must be reduced. Hence, as a general proposition, it can be stated that the number of radar beams required is proportional to the frequency used and it is desirable to use the lowest frequency permitted by the accuracy and resolution requirements. It is also apparent that the number of beams required will be proportional to the data rate. Additional considerations such as power and pulse length limitations, atmospheric attenuation, propagation stability, and cloud echo clutter all favor a low frequency.

To establish an intelligent compromise on frequency and other radar parameters, it is necessary first to evaluate as accurately as possible the data accuracy and rate requirements. However, to present a picture of the character of a system that might be required, a sample set of parameters is given below:

$\theta_V = 2.1^0$	Rotational speed = 10 rpm
$\theta_H = 1.3^0$	F = 440 pps
$\tau = 3\mu s$	$\lambda = 10$ cm.
$P = 2 \times 10^6$ watts	R_{max} = 211 miles on 1-sq-meter target

---

[4] A. A. Varela, Proposed very long range radar with height-finding, NRL Report R-2759, 18 February 1946

DECLASSIFIED [redacted]

Scanning in azimuth only is assumed. Multiple beams are used for vertical coverage. The 10-rpm rotation giving a 6-second scan period is in wide divergence to the 2-second period originally specified. This matter will be discussed at length in this report.

Figure 1 shows the probability of a target being detected at or before the range indicated when the target is on a closing radial course with a velocity of 500 knots and at altitudes above 28,000 feet. The method used to obtain this curve was originated by the Operational Evaluation Group of the Chief of Naval Operations which has done some outstanding work in this field of radar design considerations.

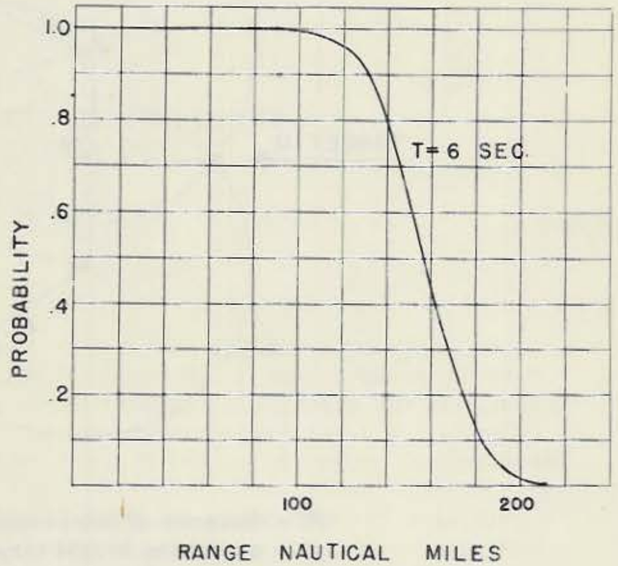

Fig. 1 - Probability of detection of target on radial course at speed of 500 knots

TACTICAL CONSIDERATIONS

The first thing considered in the tactical problem was the location of the picket ships. To be useful in this problem, they would be stationed far from the main body of the task force. It is assumed that the main body of the task force would carry much more elaborate control and search radars than possible for the picket ships. Therefore, the picket ships are required to furnish information on targets out of the line of sight of the main body radars. The picket ships have out-of-line-of-sight regions in between them which would permit diligent low flying targets to avoid detection by the picket ships altogether and escape detection until close to the main body of the task force. This leaves a hole in the defense which requires consideration in the over-all intercept control problem. Such a target may not even be dangerous when other factors are considered. However this hole cannot be eliminated by picket ship radar considerations except by increasing the number of picket ships.

A possible distribution of picket ships, P, and the main body of the task force, F, is shown with a target flying in on the weakest point of the defense (Figure 2). It is assumed that the target's speed is 520 knots and the interceptor's speed is 460 knots. The speed differential is used to allow for adverse wind conditions that are common at high altitudes. Other assumptions are that range of detection from the picket ships is 150 miles and that a 10-minute delay time is needed to allow for evaluation, take-off, and time lost in climbing. The interceptor is assumed to start from a deck at the main body of the task force.

In figure 2 following:

θ = angular displacement of pickets
PF = distance from picket to task force
PP = distance between pickets
DF = distance of detection from task force
IF = distance of interception from task force

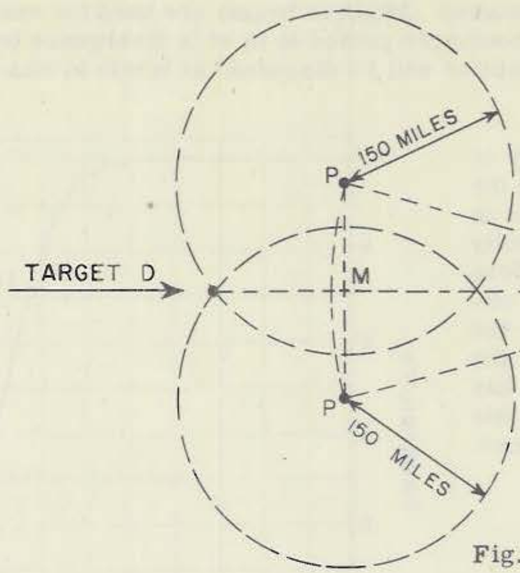

Fig. 2 - Tactical distribution

PI = distance of interception from picket
H = maximum height target can fly and avoid the line of sight regions of the pickets. Greatest restriction on altitude is at point, M, midway between the pickets. The antennas are assumed to be 66 feet high.

The following tabulation gives values of IF, PI, and H for the values of θ, PF, PP, and DF indicated and for a target speed of 520 knots and interceptor speed of 460 knots:

θ Deg.	PF Miles	PP Miles	DF Miles	IF Miles	PI Miles	H Feet
45	150	115	277	78	82	1480
45	200	152	313	97	117	2800
45	250	190	347	112	154	4800
30	150	77	288	83	72	540
30	200	102	333	105	103	1100
30	250	127	377	125	135	1900
30	300	154	417	146	165	3000

For θ equal to either 45 or 30 degrees, the range of the picket from the task force should be about 150 miles. These ranges of interception are adequate as determined by OEG.[5]

It should be noted from the table that the interception takes place well within the picket line and is actually closer to the task force than it is to the pickets. This fact raises some

[5] "Air defense by control of interceptor aircraft: a preliminary statement," Operations Evaluation Group Study No. 337, 5 Dec. 1947.

doubt as to the wisdom of using pickets rather than the task force itself for interception control. Special situations can be imagined where picket control might be necessary, for instance where an island or peninsula causes a radar blind sector for the task force, but for the ordinary case it appears that only severe disadvantages and unnecessary complexities will result from placing the control function in pickets. However, early warning of planes flying below the radar horizon of the task force is undoubtedly necessary and must be provided either by AEW or by pickets, but the radar required for this function can be extremely simple and might perhaps be carried by small submarines to give the advantage of less vulnerability than destroyers.

Since the picket-ship intercept-control radar would be required to furnish data on targets from outside the picket ring and on interceptors starting from the main body of the task force, one interception would require 180-degree azimuth coverage. More than one interception of targets coming in on either side of the picket ship would require 360-degree coverage. This coverage could be obtained in a number of ways.

One method would be to have a number of fast-scanning antennas scanning their respective sectors. Since the size of a single antenna is independent of the number used and since fast scanning is limited to small sectors, the weight and size of such a system would prohibit its use aboard a picket ship.

Another method would be to use a rotating antenna to furnish data during the least crucial part of interception and a fast scanning antenna during the final phase. This method has some good potentialities since it furnishes data at a fast rate when needed. Since most interceptions occur inside the picket ring, it is possible that one fast-scanning antenna aboard each picket ship would cover the area in which the final phase of most interceptions would take place.

Another method is simply a continuously rotating antenna and if it can furnish data of sufficient accuracy and rate it is the one that should be used.

It does not appear feasible to direct interceptions entirely from a ship. The interceptor weapons may be fixed guns, turret guns, fixed rockets, or guided missiles and sufficient information on the range and effectiveness of these weapons has not been obtained to permit an appraisal of the combat position accuracy required by the interceptor. However, apart from the position accuracy requirements, it does not seem possible to obtain in a control radar even of the fire control type, sufficient resolution to distinguish the targets in a close formation attack. Resolution of at least one tenth of a degree is required, but it is not practical to build large antennas for beamwidths much under one degree. Transfer of control of the closing phase to the aircraft radar and computer, in addition to being probably the only practical method, has the advantages of allowing greater flexibility in the introduction and use of weapons and provides the pilot independent action.

Concluding then that the interceptor must carry an AI radar which will direct the final phase of the attack, the problem reduces to determining whether a control radar giving the expected data accuracy with a continuously rotating antenna will be able to direct the interceptor so that there is high probability that the target will come into a practical cone of detection for the AI radar.

Because of the high relative velocities of the targets, the range for high probability of detection with AI radar is a very rapid function of scan rate.

It is desirable, therefore, that the cone of scan be continuously variable so that it may be very narrow, possibly only a beam width, when the target and interceptor are far apart and increased as this range decreases. The size of the cone should be as narrow as possible at all times while maintaining a high probability of including the target. With such a cone of scan it appears that reliable detection can be obtained at between 10 and 20 miles. This may be sufficient for weapon direction, but it is obvious that the AI radar antenna must be directed toward the target by the control radar.

COMPUTER CONSIDERATIONS

To determine the probability that a target plane can be brought into the detection cone of the AI radar by control radar data of assumed accuracy and rate, it is necessary to take account of the aircraft speed and maneuverability and the prediction accuracy of the computer. In the following analysis typical near-future aircraft performance is assumed and attention is directed primarily to the case of the target plane having constant course and speed since it appears very improbable that evasive action will be taken. Three different types of computers are considered.

The first computer will be called the Smoothed-Velocity Computer because it obtains a smoothed velocity without first smoothing the position data. It has a disadvantage of depending on the last data point to predict ahead and the error of this data point represents the minimum error obtainable. It is, however, a simpler computer than others.

The evaluation for the ratio of the root-mean-square error of the predicted coordinate value to the root-mean-square radar error in that coordinate is found to be (see Appendix I for full development):

$$\frac{\sigma_p}{\sigma} = \sqrt{1 + \frac{2t}{\ell m T} + \left(\frac{t}{\ell m T}\right)^2 2\ell}$$

where $\ell + m = n$, the number of data points spanned in calculating X_p, the predicted X-coordinate; T is the scan time, and t, the time of prediction from the last data point.

Figure 3 shows σ_p/σ when predicting ahead 720 seconds as a function of delay time for 6- and 2-second scan times. Delay time is the time used to gather data, and prediction ahead is actually 720 seconds minus delay time. The value of 720 seconds is the time it takes a 500-knot plane to fly 100 nautical miles.

Figure 4 is the same as Figure 3 except that it it based on predicting ahead 360 seconds. In both of these long-range predictions the difference between 6- and 2-second scan time is not critical.

Figure 5 gives the error ratio σ_p/σ when predicting ahead one scan period. For this case the difference between 6- and 2-second scan periods is more critical. One factor not showing on the graph is the radar error for 2-second scan time which is probably greater than that for a 6-second scan time. This will depend largely on the fluctuations of the echo signal, and sufficient data has not been obtained for a quantitative analysis. It is likely that the faster the scan rate the better the prediction, however, the object is to find a reasonable and practicable scan rate.

The second computer considered will be called the Smoothed-Position Computer. It finds the best fit to coordinate data and finds a smoothed value for both position and velocity.

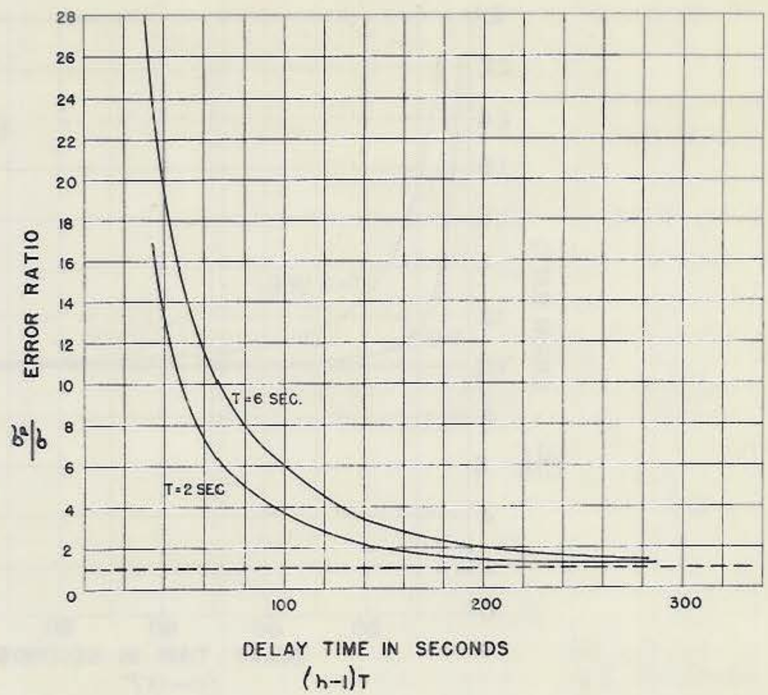

Fig. 3 - Smoothed velocity computer predicting ahead 720 seconds

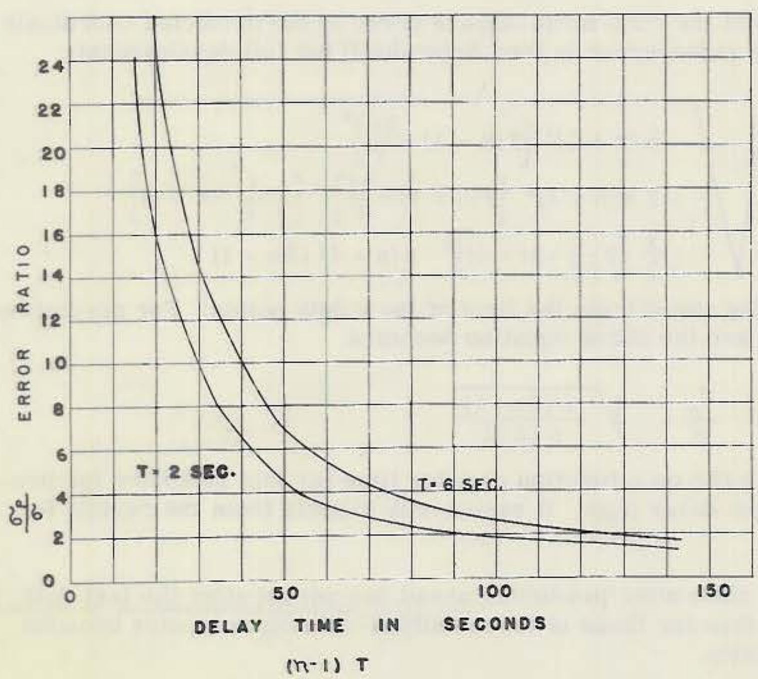

Fig. 4 - Smoothed velocity computer predicting ahead 360 seconds

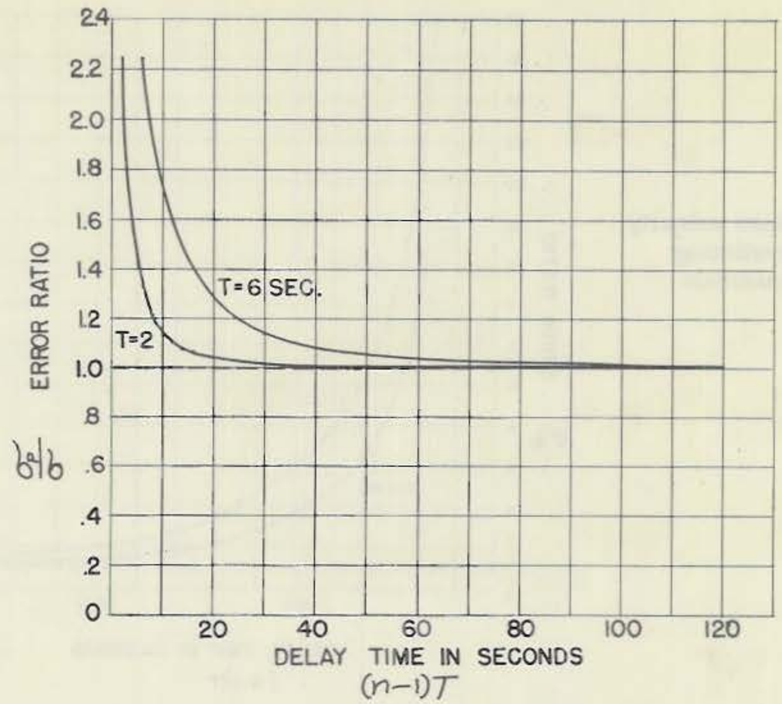

Fig. 5 - Smoothed velocity computer predicting ahead one scan period

The equation for the ratio of the root-mean-square error in the predicted coordinate value to the root-mean-square radar error is (see Appendix II for full development):

$$\frac{\sigma_p}{\sigma} = \frac{2}{n(n^2-1)} \sqrt{\begin{aligned} & n(n+1)^2 \left[2(n-1) - \frac{3t}{T}\right]^2 \\ & + 3n(n+1)^2 \left[2(n-1) - \frac{3t}{T}\right] \left[2\frac{t}{T} - (n-1)\right] \\ & + \frac{3}{2}\left[2\frac{t}{T} - (n-1)\right]^2 n(n+1)(2n+1) \end{aligned}}$$

where t is the time of predicting ahead from the first of the n data points. For predicting ahead one scan period, t = n T and the above equation becomes

$$\frac{\sigma_p}{\sigma} = \sqrt{\frac{2(2n+1)}{n(n-1)}}$$

Figure 6 shows the error ratio as a function of delay time for this computer for predicting ahead 360 seconds minus delay time. It varies only slightly from the results for the first computer.

Figure 7 shows the error ratio after predicting ahead one period after the last data point. The results are better than for those of the smoothed-velocity computer because the error ratio can go below unity.

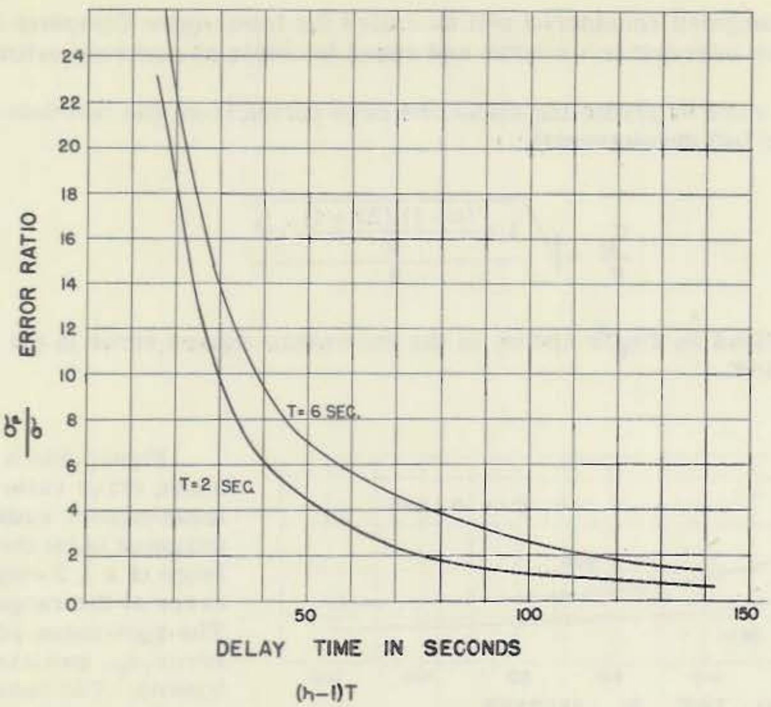

Fig. 6 - Smoothed position computer predicting ahead 360 seconds

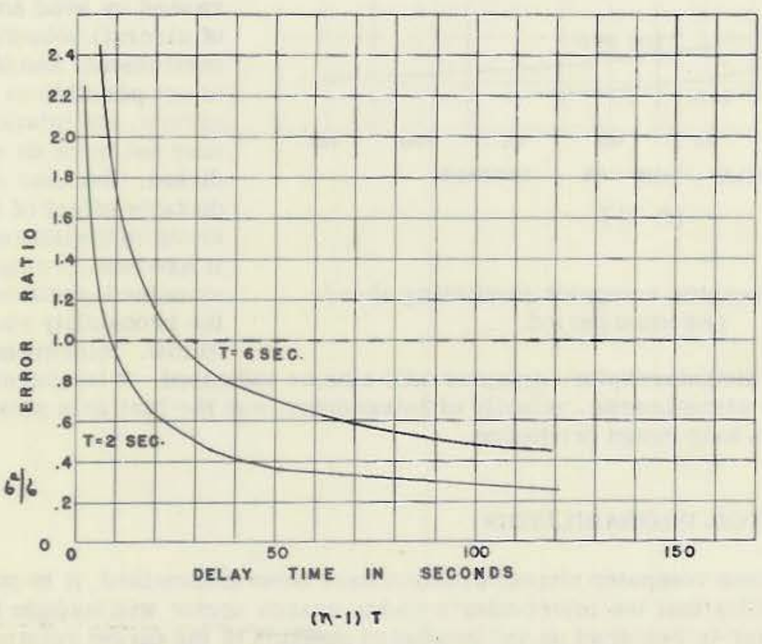

Fig. 7 - Smoothed position computer predicting ahead one scan period

The last computer considered will be called the Interceptor Computer because it uses knowledge of the interceptor's course and speed to obtain an accurate estimate of position.

The error ratio in predicting ahead one scan period from the last data point is (see Appendix III for full development):

$$\frac{\sigma_p}{\sigma} = \sqrt{\frac{1 + \frac{(n+1)\,(2n+1)}{6}\,m^2}{n}}$$

where m is defined as $T\sigma_V/\sigma$ and σ_V is the root-mean-square error in the average velocity for a scan period.

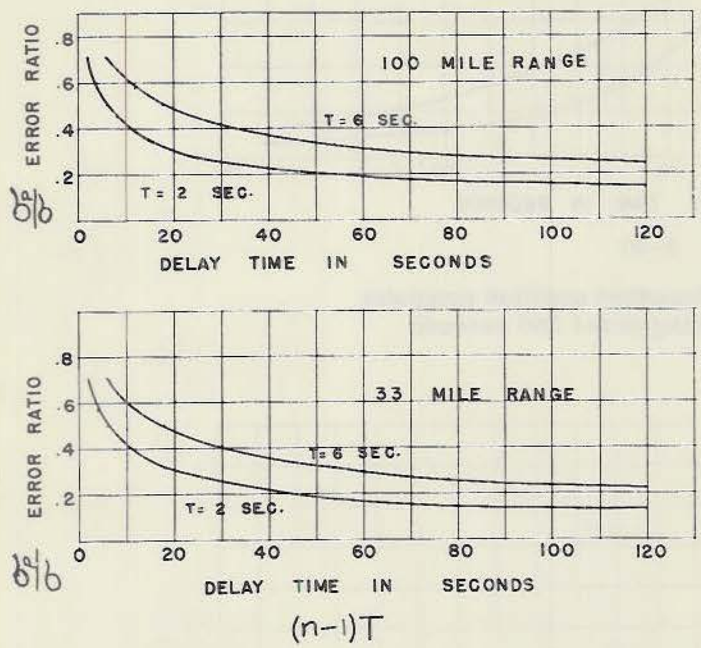

Fig. 8 - Interceptor computer predicting ahead one scan period

Figure 8 is a graph of the above error ratio. The root-mean-square radar error, σ, was assumed to be the error in distance of a 1/2-degree bearing error at the ranges considered. The root-mean-square velocity error, σ_V, was assumed to be 5 knots. The results, although even better than those of the smoothed-position computer, have been obtained by assuming that constant-velocity errors are eliminated. Such errors are caused by wind and calibration of aircraft velocity measuring instruments and if in practice it is not possible to eliminate these errors, the interceptor computer may not work as well as predicted. Because of the unpredictable effect of the constant error on the interceptor computer, it has been decided to use the smoothed-position computer in the probability studies which follow. Improvements which may arise by use of the interceptor computer will also be indicated. It is essentially a short range computer since course, velocity of interceptor, and the last data point are all that are required for long range prediction.

AI RADAR SECTOR PROBABILITIES

Now that some computer characteristics have been determined, it is possible to compute the probability that the interceptor's radar search sector will include the target when this search sector is centered on the predicted position of the target relative to the interceptor. To do this, present position of target and interceptor must be known. Present position is the most accurate just after the last data point has been taken and the least accurate just before the next data point is taken. The probability of the interceptor's search sector including the target is therefore lowest when prediction is one scan period ahead of the last data point. It is necessary to know the distribution of the radar errors in order to make this calculation. Since there is very little known about this distribution, a normal distribution is assumed. The radar root-mean-square error in range is assumed to be 200 yards and the bearing error to be 1/2 degree.

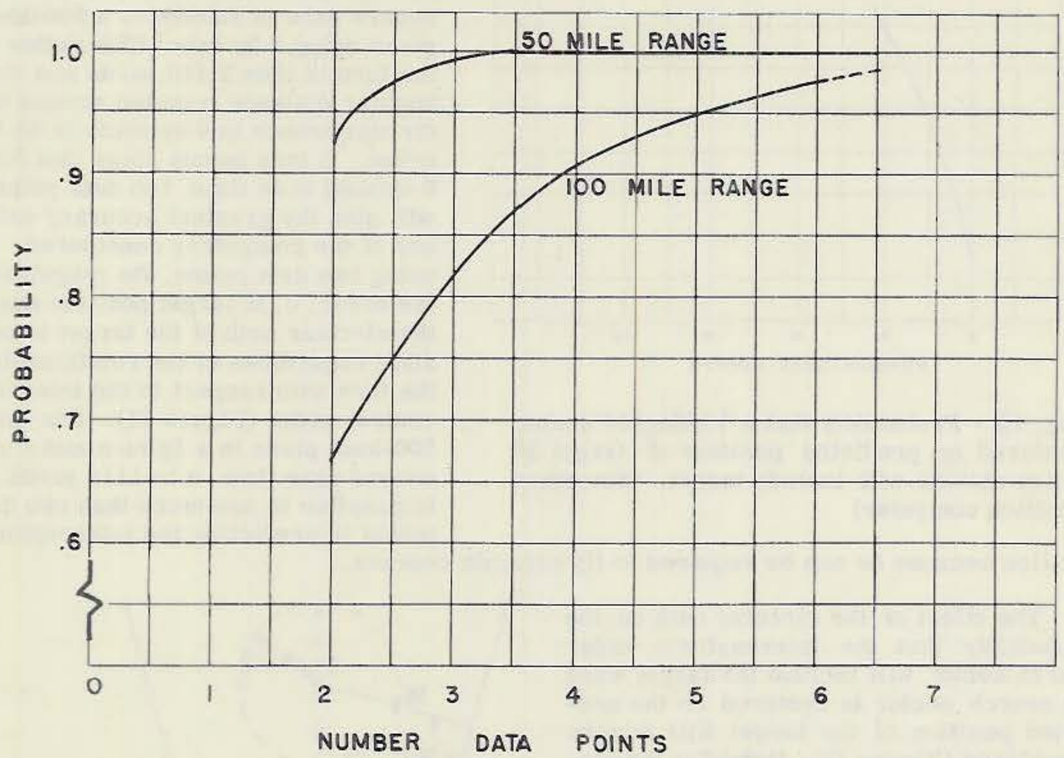

Fig. 9 - Probability that a 30-degree sector centered on predicted position of target 10 miles distant will include target (smoothed position computer)

Figure 9 shows the probability that a 30-degree sector will include the target when the aircraft are predicted to be 10 miles apart, as a function of the number of data points used. The two aircraft are assumed to be on the same radial line from the control radar since this is the worst case. The results are very encouraging in that only two data points are needed to have a probability of inclusion well over 0.9 when the range from the control radar is 50 miles or less. Figure 10 is the same except that it applies to a 10-degree sector and aircraft separation of 20 miles. Again at 50 miles, two data points are sufficient to bring the probability of inclusion close to 0.9. This indicates that by using this system of intercept control, scan periods of about 6 seconds would be adequate for intercept control radars.

If the interceptor computer is used instead of the smoothed-position computer to calculate the interceptor's position, the 100-mile curves would move close to the 50-mile curves.

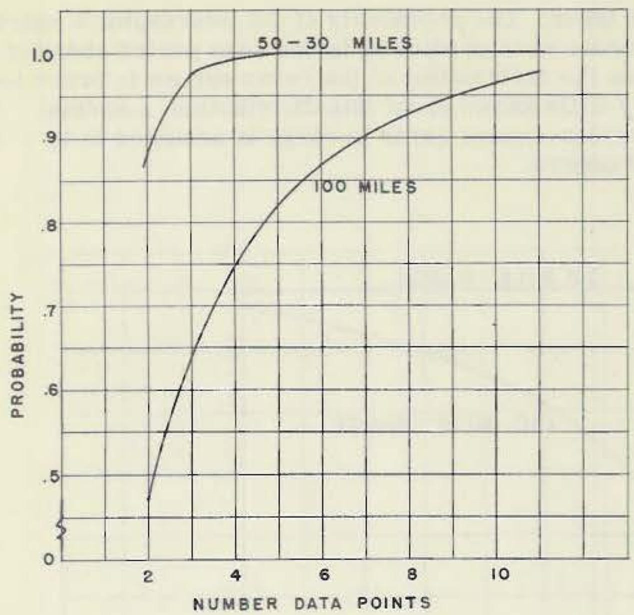

Fig. 10 - Probability that a 10-degree sector centered on predicted position of target 20 miles distant will include target. (Smoothed position computer)

CIRCULAR COURSE

The above results were obtained by assuming that the target flew on a straight course with constant velocity. Some thought should be given to what happens when the target is flying on the arc of a circle. To do this a rather severe case is chosen -- a 500-knot plane doing a 3g turn. The radius of the turn is then 2,410 yards and the angular distance traveled around the circumference in 6 seconds is 39.8 degrees. It then seems clear that for 6-second scan time, two data points will give the greatest accuracy using any of the computers considered. When using two data points, the magnitude of the error, e, in target position due to the circular path of the target is constant regardless of the relationship of the turn with respect to the intercept control radar (Figure 11). For the 500-knot plane in a 3g turn and a six-second scan time, e is 1114 yards. It is possible to use more than two data points in predicting the interceptor's position because he can be required to fly straight courses.

The effect of the circular path on the probability that the interceptor's radar search sector will include the target when the search sector is centered on the predicted position of the target will now be considered (Figure 12). Point T is the predicted position of the target and I is the

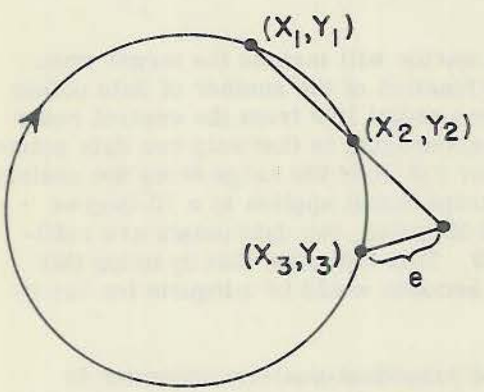

Fig. 11 - Changing course error

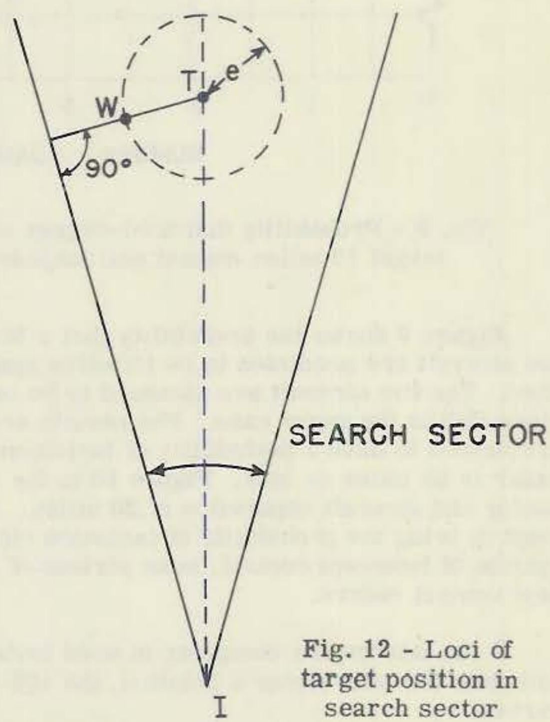

Fig. 12 - Loci of target position in search sector

predicted position of the interceptor. Actual positions of the target and interceptor are assumed to be on the same radial line from the intercept-control radar, which is the worst relative position because it gives the lowest value of probability of inclusion. The dotted circle represents the loci of all possible positions of the target if it was in a 3g turn instead of flying a straight course. The worst case is at W which comes the closest to placing the target outside the interceptor search sector.

Figure 13 shows the probability of the interceptor's radar search sector including the target for the case of a 500-knot target doing a 3g turn. The worst case was assumed both for relative position of target and interceptor and the orientation of the turn with respect to the intercept control radar.

The results indicate that, even under these severe conditions of turning, a 6-second scan time is sufficient for ranges out beyond 50 miles. If the intercept control radar is at the center of attack and the interception is beyond 50 miles, there is some time to spare and the interceptor can be instructed to approach the target from a more favorable direction and thus improve the probability of inclusion for the greater ranges.

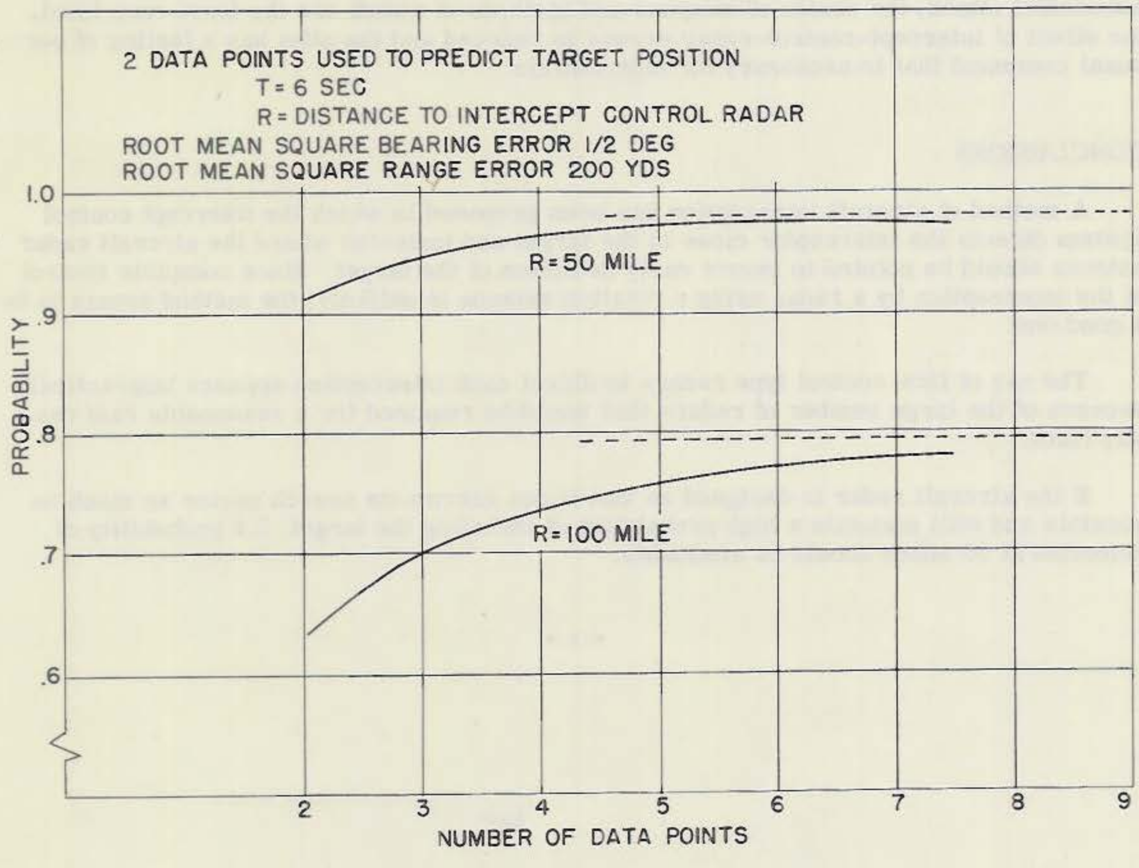

Fig. 13 - Probability that a 30-degree sector centered on predicted position of a 500-knot target doing a 3g turn will include the target if the target is predicted to be 10 miles from the interceptor. (Smoothed position computer)

AIRCRAFT RADAR

Assuming that there are two aircraft radars with 35 miles maximum range, one with 3-second scan time and the other with 1-second scan time, the ranges corresponding to 0.9 probability of detection would be 11 and 18 miles, respectively. If the two radars are similar except for the scan times, the maximum ranges may not be the same because of difference in scanning loss. Taking 35 miles as the maximum range of the 3-second scanning radar, the maximum range of the 1-second radar would be somewhere between 35 miles and $(1/3)^{1/8}$ 35, or 30.5 miles, depending on the appearance of the PPI scopes. Using a maximum range of 30.5 miles for the 1-second scanning radar, the range of 0.9 probability of detection would then be 15 miles which is still better than the 3-second radar's 11 miles. Therefore it appears that as the radar's search sector is narrowed, it is desirable to increase its scan rate to keep the number of pulses on the target the same for each scan. In order not to limit the direction of attack, the center of the aircraft radar's search sector should be variable.

The desirability of the interceptor detecting the target as far out as possible is apparent. As the interceptor's range of detection is increased the various phases of aircraft interception have less effect on others, target maneuvers have less effect on the chance of a successful attack, the choice of weapons and methods of attack are the least restricted, the effect of intercept-control-radar errors is reduced and the pilot has a feeling of personal command that is necessary for high morale.

CONCLUSIONS

A method of aircraft interception has been proposed in which the intercept control system directs the interceptor close to the target and indicates where the aircraft radar antenna should be pointed to insure early detection of the target. Since complete control of the interception by a radar using a rotating antenna is unlikely, the method seems to be a good one.

The use of fire-control type radars to direct each interception appears impractical because of the large number of radars that would be required for a reasonable raid density limit.

If the aircraft radar is designed so that it can narrow its search sector as much as possible and still maintain a high probability of including the target, 0.9 probability of detection at 20 miles should be attainable.

* * *

DECLASSIFIED

APPENDIX I

THE SMOOTHED-VELOCITY COMPUTER

This computer attempts to find the best value of velocity without smoothing the position data first. It is assumed that the plane is flying a straight course with constant velocity. The radar data has been broken down into rectangular coordinates and the coordinate points $X_1, X_2, X_3, \ldots, X_n$ are given at successive scans of the radar T seconds apart.

If "least squares" is taken as the criteria, the best value of velocity in the X direction, V_x, is the one that minimizes the sum:

$$S = \sum_{i=1}^{\ell} \left(V_{x_i} - V_x\right)^2, \text{ where } V_{x_i} = \frac{x_{i+m} - x_i}{mT}, \qquad (1)$$

m may be any integer such that $n/2 < m \leq n - 1$, $\ell + m = n$, and n is the number of data points used to obtain V_x. To minimize $\overline{S}$ differentiate (1) with respect to V_x and equate to zero.

$$\left.\frac{ds}{dV_x}\right]_{S=min} = -2 \sum_{i=1}^{\ell} \left(V_{x_i} - V_x\right) = 0. \qquad (2)$$

Therefore

$$V_x = \frac{1}{\ell} \sum_{i=1}^{\ell} V_{x_i} = V_{x\,avg.\,m\ell} \qquad (3)$$

$$V_{x\,avg.\,m\ell} = \frac{1}{\ell m T} \sum_{i=1}^{\ell} \left(x_{i+m} - x_i\right) \qquad \ell < m + 1 \qquad (4)$$

Since (4) is linear in the X's, it is possible to compute the root-mean-square error in velocity, σ_V, in terms of the root-mean-square error in radar data, σ, without assuming a distribution for the radar error as long as it is random.[6] It is also assumed that the

---

[6] For if $y = \sum_i a_i Z_i$, then $\sigma_y = \sqrt{\sum_i a_i^2 \sigma_{Z_i}^2}$ where σ_y is the root-mean-square error in y and σ_{Z_i} is the root-mean-square error in Z_i.

 DECLASSIFIED [redacted]

n data points are close enough together so that the root-mean-square error of each data point is the same. Then,

$$\sigma_V = \frac{\sigma}{T}\sqrt{\frac{2}{\ell m^2}} \tag{5}$$

Now ℓ and m should be chosen to minimize σ_V. This can be done by maximizing $\ell m^2 = (n-m)m^2$.

$$\left.\frac{d\,(n-m)m^2}{dm}\right]_{\ell m^2 = \max} = 2\,n\,m - 3\,m^2 = 0. \tag{6}$$

Therefore, $m = 2/3\,n$ and $\ell = 1/3\,n$ for minimum value of σ_V.

Predicted position is:

$$X_p = X_n + V_x\,t \tag{7}$$

where X_n is the last coordinate value and t is the time of predicting ahead from X_n.

Therefore,

$$X_p = X_n + \frac{t}{\ell m T}\sum_{i=1}^{\ell}\left(X_{(i+m)} - X_i\right) \tag{8}$$

The root-mean-square error in predicted coordinate, σ_p, can be calculated with the same assumptions as for σ_V.

$$\frac{\sigma_p}{\sigma} = \sqrt{\left(1+\frac{t}{\ell m T}\right)^2 + \left(\frac{t}{\ell m T}\right)^2 + \frac{(2\ell-1)\,t^2}{\ell^2 m^2 T^2}}$$

$$\frac{\sigma_p}{\sigma} = \sqrt{1 + \frac{2t}{\ell m T} + \left(\frac{t}{\ell m T}\right)^2 2\ell} \tag{9}$$

Given n and t, it is possible to find values of m and ℓ that minimize σ_p/σ. As may be expected, for large values of t the best values of ℓ and m are 1/3 n and 2/3 n respectively. When predicting ahead one scan period, t = T and (9) becomes

$$\frac{\sigma_p}{\sigma} = \sqrt{1 + \frac{2}{\ell m} + \frac{2}{\ell m^2}} \tag{10}$$

To minimize σ_p/σ for this case, minimize

$$\frac{m+1}{\ell m^2} = \frac{m+1}{(n-m)m^2}$$

DECLASSIFIED

$$\left.\frac{d\left[\frac{m+1}{(n-m)m^2}\right]}{dm}\right]_{\frac{m+1}{\ell m^2} = \min} = \frac{(n-m)\,m^2 - (m+1)\,(2mn - 3m^2)}{(n-m)^2\,m^4} = 0.$$

[redacted]

Therefore $(n-m)m = (m+1)(2n-3m)$, and

$$m = \frac{n-3 \pm \sqrt{(n-3)^2 + 16n}}{4} . \qquad (11)$$

Since m is restricted to integer values, the result is $m = \ell$; or $m = \ell + 1$ depending on whether n is even or odd.

Then in predicting ahead, the best values of m vary from 2/3 n for predicting ahead over long time periods to 1/2 n for predicting ahead one scan period. Likewise the best values of ℓ vary from 1/3 n to 1/2 n.

* * *

[redacted]

DECLASSIFIED

APPENDIX II

SMOOTHED-POSITION COMPUTER

This computer resolves position data on the basis of a straight line which best fits this data. After the radar data has been transformed into rectangular coordinates the solution, in one coordinate involves the determination of X and V_x such that the predicted coordinate value can be given by the equation

$$X_p = X + V_x t$$

The best values of X and V_x by the least squares criteria are those that minimize the sum

$$S = \sum_{i=1}^{n} \left[X + V_x (i-1) T - x_i \right]^2 , \tag{12}$$

where $x + V_x (i-1)T$ is the predicted position of the "i"th data point and x_i is this data point. Then $t = 0$ when the first data point is taken and X is the best estimate of the first data point.

To obtain the best values of X and V_x, differentiate (12) with respect to X and V_x, respectively, and equate to zero.

$$\left.\frac{ds}{dX}\right]_{S=\min} = 2 \sum_{i=1}^{n} \left[X + V_x (i-1) T - x_i \right] = 0 \tag{13}$$

$$\left.\frac{ds}{dV_x}\right]_{S=\min} = 2 \sum_{i=1}^{n} \left\{ \left[X + V_x (i-1) T - X_i \right] (i-1)T \right\} = 0 \tag{14}$$

Equation (13) becomes:

$$nX + V_x \frac{n(n-1)}{2} T = \sum_{i=1}^{n} X_i . \tag{15}$$

This can become

$$X = V_x \frac{n-1}{2} T = \frac{1}{n} \sum_{i=1}^{n} x_i . \tag{16}$$

The left-hand side of (16) is the best estimate of a point midway between X_1 and X_n. If the root-mean-square radar error is σ, the root mean square of the predicted midpoint is $\sigma / \sqrt{n}$.

DECLASSIFIED

[redacted]

Equation (14) becomes:

$$X T \frac{(n-1)n}{2} + V_x T^2 \sum_{i=1}^{n} (i-1)^2 = T \sum_{i=1}^{n} x_i (i-1),$$

$$X \frac{(n-1)n}{2} + V_x T \frac{n(n-1)(2n-1)}{6} = \sum_{i=1}^{n} x_i (i-1). \tag{17}$$

Solving (15) and (17) for X and V_x,

$$X = \frac{T \frac{n(n-1)(2n-1)}{6} \sum_{i=1}^{n} x_i - \frac{(n-1)n}{2} T \sum_{i=1}^{n} x_i (i-1)}{\frac{n^2 T (n-1)(2n-1)}{6} - \left[\frac{(n-1)n}{2}\right]^2 T}$$

$$X = \frac{2 \sum_{i=1}^{n} [2(n+1) - 3i] \; x_i}{n(n+1)} \tag{18}$$

Root-mean-square error in X is then:

$$\sigma_X = \frac{2\sigma}{n(n+1)} \sqrt{4n(n+1)^2 - 6(n+1)^2 n + \frac{3n(n+1)(2n+1)}{2}},$$

$$\sigma_X = \sigma \sqrt{\frac{2(2n-1)}{n(n+1)}} \tag{19}$$

$$V_x = \frac{n \sum_{i=1}^{n} x_i (i-1) - \frac{n(n-1)}{2} \sum_{i=1}^{n} x_i}{\frac{n^2(n^2-1)}{12} T}, \tag{20}$$

$$V_x = \frac{6 \sum_{i=1}^{n} [2i - (n+1)] \; x_i}{n(n^2-1) T}$$

Root-mean-square error in V_x is then:

$$\sigma_{V_x} = \frac{6\sigma}{n(n^2-1) T} \sqrt{\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)^2}{2} + n(n+1)^2}$$

$$\sigma_{V_x} = \frac{6\sigma}{T\sqrt{3n(n^2-1)}} \tag{21}$$

Predicted position is:

$$x_p = \frac{2\left\{(n-1)\sum_{i=1}^{n}\left[2(n+1)-3i\right]x_i + 3\frac{t}{T}\sum_{i=1}^{n}\left[2i-(n+1)\right]x_i\right\}}{n(n^2-1)}, \tag{22}$$

$$x_p = \frac{2\sum_{i=1}^{n}\left\{(n+1)\left[2(n-1)-3\frac{t}{T}\right]+3\left[2\frac{t}{T}-(n-1)\right]i\right\}x_i}{n(n^2-1)} \tag{23}$$

In equation (22), X_p is seen as simply the weighted sum of the X's plus another weighted sum of the X's multiplied by t.

Equation (23) is in a better form to compute root-mean-square error in predicted position.

$$\frac{\sigma_p}{\sigma} = \frac{2}{n(n^2-1)}\sqrt{\begin{array}{l} n(n+1)^2\left[2(n-1)-3\frac{t}{T}\right]^2 \\ +3\left[n(n+1)^2\right]\left[2(n-1)-\frac{3t}{T}\right]\left[2\frac{t}{T}-(n-1)\right] \\ +\frac{3}{2}\left[2\frac{t}{T}-(n-1)\right]^2 n\ (n+1)\ (2n+1) \end{array}}$$

For predicting ahead one scan period from the last data point, t = nT and equation (24) becomes

$$\frac{\sigma_p}{\sigma} = \sqrt{\frac{2\ (2n+1)}{n\,(n-1)}}\ .$$

* * *

APPENDIX III

THE INTERCEPTOR COMPUTER

Since we know or can know the course and speed of the interceptor with a fair degree of accuracy, it is desirable to use this information in determining its exact position. All that is necessary to determine (in one coordinate) is a computed value, X, for the first of the n data points used.

The best value of X is the one that minimizes the sum

$$S = \sum_{m=1}^{n} \left[x_m - \sum_{i=1}^{m-1} V_i T - X \right]^2 , \qquad (25)$$

where V_i is the average velocity from t_i to t_{i+1} and t_i is the time of data point i. T is the radar scan period. Differentiating (25) with respect to X and equating to zero,

$$\left.\frac{dS}{dX}\right]_{S = \min} = -2 \sum_{m=1}^{n} \left[x_m - \sum_{i=1}^{m-1} V_i T - X \right] = 0.$$

Therefore

$$nX = \sum_{m=1}^{n} \left(x_m - \sum_{i=1}^{m-1} V_i T \right) ,$$

$$= \sum_{m=1}^{n} x_m - \begin{bmatrix} V_1 T \\ +V_1 T + V_2 T \\ +V_1 T + V_2 T + V_3 T \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ +V_1 T + V_2 T + V_3 T + \ldots V_{n-1} T \end{bmatrix} .$$

$$nX = \sum_{m=1}^{n} x_m - T \sum_{i=1}^{n=1} V_i \, (n - i).$$

If the V_i are exact, the root mean square error in X is:

$$\sigma_X = \frac{\sigma}{\sqrt{n}}$$

If the V_i are in error in a random manner (no constant error), from equation (26),

$$\sigma_X = \frac{\sqrt{n\sigma^2 + T^2 \sum_{i=1}^{n-1} (n-i)^2 \sigma_V^2}}{n},$$

$$= \frac{\sqrt{n\sigma^2 + T^2 \frac{n(n-1)(2n-1)}{6} \sigma_V^2}}{n},$$

$$\frac{\sigma_X}{\sigma} = \frac{\sqrt{n + \frac{n(n-1)(2n-1)}{6} m^2}}{n},$$

where m is defined by $m = \frac{\sigma_V T}{\sigma}$.

Predicting ahead one scan period from the last of the n data points,

$$x_p = X + \sum_{i=1}^{n} V_i T,$$

$$= \frac{1}{n} \sum x_m - \frac{T}{n} \sum_{i=1}^{n-1} V_i (n-i) + \sum_{i=1}^{n} V_i T,$$

$$x_p = \frac{1}{n} \sum_{m=1}^{n} x_m + \frac{T}{n} \sum_{i=1}^{n} V_i i. \qquad (29)$$

If V_i is assumed to be equal to a constant V for all i (pilot attempts to fly a straight course at constant speed), equation (29) becomes

$$x_p = \frac{1}{n} \sum_{m=1}^{n} x_m + \frac{T(n+1)}{2} V. \qquad (30)$$

The first term on the right-hand side is the best estimate of the midpoint between the first and n'th data points. The time of prediction beyond this midpoint is $T(n+1)/2$.

Referring back to equation (29) it is possible to calculate the root-mean-square error of X_p.

$$\sigma_p = \frac{\sqrt{n\sigma^2 + T^2 \frac{n(n+1)(2n+1)}{6} \sigma_V^2}}{n},$$

$$\frac{\sigma_p}{\sigma} = \sqrt{\frac{1 + \frac{(n+1)\,(2n+1)}{6} \quad m^2}{n}} \qquad (31)$$

From a study of equation (30) and the results for the smoothed position computer in Appendix II, it is apparent that this method simply consists of finding the best midpoint value of the n data points used and predicting ahead from this point using the known velocity of the aircraft.

The general equation for predicting ahead t time units after the last data point is:

$$x_t = \frac{1}{n} \sum_{m=1}^{n} x_m + \left[\frac{n-1}{2} T + t \right] V \qquad (32)$$

* * *