REPORT DOCUMENTATION PAGE	Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY) 02-09-2021	2. REPORT TYPE Final Report	3. DATES COVERED (From - To) 10-Jun-2020 - 9-Mar-2021

4. TITLE AND SUBTITLE Final Report: Exploiting Complexity to Stabilize the Dynamics of Mechanical Systems	5a. CONTRACT NUMBER W911NF-20-1-0173
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER 611102
6. AUTHORS	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES Northwestern University Evanston Campus 1801 Maple Avenue Evanston, IL 60201 -3149	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSOR/MONITOR'S ACRONYM(S) ARO
	11. SPONSOR/MONITOR'S REPORT NUMBER(S) 76600-EG-II.3

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Adilson Motter
UU	UU	UU	UU		19b. TELEPHONE NUMBER 847-491-4611

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

RPPR Final Report
as of 09-Sep-2021

Agency Code: 21XD

Proposal Number: 76600EGII **Agreement Number: W911NF-20-1-0173**

INVESTIGATOR(S):

Name: Adilson E Motter
Email: motter@northwestern.edu
Phone Number: 8474914611
Principal: Y

Organization: **Northwestern University Evanston Campus**
Address: 1801 Maple Avenue, Evanston, IL 602013149
Country: USA

DUNS Number: 160079455 EIN: 362167817
Report Date: 09-Jun-2021 Date Received: 02-Sep-2021
Final Report for Period Beginning 10-Jun-2020 and Ending 09-Mar-2021
Title: Exploiting Complexity to Stabilize the Dynamics of Mechanical Systems
Begin Performance Period: 10-Jun-2020 **End Performance Period:** 09-Mar-2021
Report Term: 0-Other
Submitted By: Adilson Motter Email: motter@northwestern.edu
Phone: (847) 491-4611
Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: 1 **STEM Participants:** 1

Major Goals: This project builds on the previous discovery of scenarios in which unstable symmetric states of a system can be stabilized by breaking the given symmetry in the system itself. This effect has been termed asymmetry-induced symmetry (AIS). The previous literature on AIS was limited to synchronization phenomena in networks. In this proposal, we argued that this effect is expected for complex systems in general, including continuous systems described by PDEs and that are not naturally represented as networks.

The major goals of the project are: 1) To provide theoretical evidence for the occurrence of AIS in continuous systems by analyzing the transition between discrete (network) systems and continuous systems using a simple model amenable to mathematical analysis. 2) To demonstrate theoretically and computationally the occurrence of AIS in continuous systems of scientific relevance using Faraday wave instabilities as a representative example. 3) To implement experimental validation of these predictions to demonstrate in the lab that AIS can be exploited to prevent instabilities.

Accomplishments: Here I provide a brief description of how each of the three goals of the project was accomplished (additional details are included in a PDF uploaded below).

1) Analysis of the transition between discrete network systems and continuous systems.

This was achieved by theoretically analyzing parametrically driven arrays of coupled pendulums. The key insights were the realizations that i) bandgaps in the dispersion relation of the system can be used to inhibit the onset of symmetry-breaking instabilities and ii) such bandgaps can be created by introducing heterogeneities that break the symmetry of the system. Analysis of this model allowed us to both predict AIS in the pendulum array system and gain insight on how to design an experiment to observe it in continuous media. This research also led to the discovery that, for sufficiently large driving amplitude, an anharmonic response can be created by parametrically driving the system with a driving frequency inside a bandgap. This research was published in Phys. Rev. Research 3, 023106 (2021) and an animated summary of it is available here: https://bit.ly/3kwVKHC

2) Theoretical and computational demonstration of AIS in continuous systems exhibiting Faraday wave instabilities.

This was done by creating suitable bandgaps in the dispersion relation of a system consisting of water in a vertically vibrating container. We were able to show that the desired bandgaps can be created using a non-flat surface at the bottom of the container. The analysis was then completed in terms of Floquet-Fourier modes and validated by direct numerical simulations. These results constituted the theory/numerics part of the demonstration of AIS in Faraday instabilities presented in Nature Communications 12, 4486 (2021).

3) Experimental validation of predictions to demonstrate that AIS can be exploited to prevent instabilities.

This was achieved by implementing a Faraday wave experiment using 3D printed containers designed according to our theory. The results demonstrate that asymmetry in the container can suppress the onset of parametric instabilities (and thus of Faraday waves) for a wide region of the driving frequency vs. driving amplitude parameter space. The results were published in Nature Communications 12, 4486 (2021) and movies summarizing the results are available here: https://www.nature.com/articles/s41467-021-24459-0#Sec11

Training Opportunities: This research supported the training of a graduate student (Yuanzhao Zhang), who is now a Schmidt Science Fellow at Cornell University. It also contributed to the postdoctoral training of Zachary Nicolaou, who is now a WRF Postdoctoral Fellow and Acting Instructor at the University of Washington.

Results Dissemination: The research outcomes of this STIR project were published in the two peer-reviewed publications discussed in the other parts of the report. Both publications also include substantial supplemental information in the form of supplementary analysis and videos/animations illustrating the results. The core codes were published along with the papers and are available in ready-to-use form through our GitHub repository. This research has also been presented in various oral presentations, including the following talks by the PI:

1) Workshop "Disorder-Promoted Synchronization – Why this is important," July 22, 2021, U.S. Office of Naval Research.

2) Seminar, University of Chicago, July 14, 2021, Chicago, IL.

3) Theoretical Physics Colloquium, Carl von Ossietzky University of Oldenburg, June 24, 2021, Oldenburg, Germany.

4) Theory Seminar, Otto von Guericke University of Magdeburg, June 22, 2021, Magdeburg, Germany.

5) Physics Colloquium, University of Campinas, December 3, 2020, Campinas, São Paulo, Brazil.

Honors and Awards: Adilson E. Motter (PI):
2020 Fellow, Network Science Society.

Zachary G. Nicolaou (postdoctoral researcher):
2021 WRF Postdoctoral Fellow, University of Washington.

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: PD/PI
Participant: Adilson E. Motter
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Zachary G. Nicolaou
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: Yuanzhao Zhang
Person Months Worked: 3.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Staff Scientist (doctoral level)
Participant: Chao Duan
Person Months Worked: 1.00 **Funding Support:**
Project Contribution:
National Academy Member: N

ARTICLES:

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Nature Communications
Publication Identifier Type: DOI Publication Identifier: 10.1038/s41467-021-24459-0
Volume: 12 Issue: 1 First Page #: 4486
Date Submitted: 8/30/21 12:00AM Date Published: 7/1/21 5:00AM
Publication Location:
Article Title: Heterogeneity-stabilized homogeneous states in driven media
Authors: Zachary G. Nicolaou, Daniel J. Case, Ernest B. van der Wee, Michelle M. Driscoll, Adilson E. Motter
Keywords: Complex system, Stability, Control, Mechanical system
Abstract: Understanding the relationship between symmetry breaking, system properties, and instabilities has been a problem of longstanding scientific interest. Symmetry-breaking instabilities underlie the formation of important patterns in driven systems, but there are many instances in which such instabilities are undesirable. Using parametric resonance as a model process, here we show that a range of states that would be destabilized by symmetry-breaking instabilities can be preserved and stabilized by the introduction of suitable system asymmetry. Because symmetric states are spatially homogeneous and asymmetric systems are spatially heterogeneous, we refer to this effect as heterogeneity-stabilized homogeneity. We illustrate this effect theoretically using driven pendulum array models and demonstrate it experimentally using Faraday wave instabilities. Our results have potential implications for the mitigation of instabilities in engineered systems and the emergence of homogeneous states in
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Physical Review Research
Publication Identifier Type: DOI Publication Identifier: 10.1103/PhysRevResearch.3.023106
Volume: 3 Issue: 2 First Page #:
Date Submitted: 8/30/21 12:00AM Date Published: 5/1/21 10:00AM
Publication Location:
Article Title: Anharmonic classical time crystals: A coresonance pattern formation mechanism
Authors: Zachary G. Nicolaou, Adilson E. Motter
Keywords: Complex systems, Parametric instabilities, Mechanical systems
Abstract: Driven many-body systems have been shown to exhibit discrete time crystal phases characterized by broken discrete time-translational symmetry. This has been achieved generally through a subharmonic response, in which the system undergoes one oscillation every other driving period. Here, we demonstrate that classical time crystals do not need to resonate in a subharmonic fashion but instead can also exhibit a continuously tunable anharmonic response to driving, which we show can emerge through a coresonance between modes in different branches of the dispersion relation in a parametrically driven medium. This response, characterized by a typically incommensurate ratio between the resonant frequencies and the driving frequency, is demonstrated by introducing a time crystal model consisting of an array of coupled pendula with alternating lengths. Importantly, the coresonance mechanism is the result of a bifurcation involving a fixed point and an invariant torus, with no intermediate limit
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Partners

,

The portion of the research published in “Z.G. Nicolaou, D.J. Case, E.B. van der Wee, M.M. Driscoll, and A.E. Motte

I certify that the information in the report is complete and accurate:
Signature: Adilson E. Motter
Signature Date: 9/2/21 12:43AM

Summary of the publications resulting from this grant.

Heterogeneity-stabilized homogeneous states in driven media,
Z.G. Nicolaou, D.J. Case, E.B. van der Wee, M.M. Driscoll, and A.E. Motter,
Nature Communications ***12****, 4486 (2021).*

Summary: Understanding the relationship between symmetry breaking, system properties, and instabilities has been a problem of longstanding scientific interest. Symmetry-breaking instabilities underlie the formation of important patterns in driven systems, but there are many instances in which such instabilities are undesirable. Using parametric resonance as a model process, here we show that a range of states that would be destabilized by symmetry-breaking instabilities can be preserved and stabilized by the introduction of suitable *system* asymmetry. Because symmetric states are spatially homogeneous and asymmetric systems are spatially heterogeneous, we refer to this effect as heterogeneity-stabilized homogeneity. We illustrate this effect theoretically using driven pendulum array models and demonstrate it experimentally using Faraday wave instabilities. Our results have potential implications for the mitigation of instabilities in engineered systems and the emergence of homogeneous states in natural systems with inherent heterogeneities.

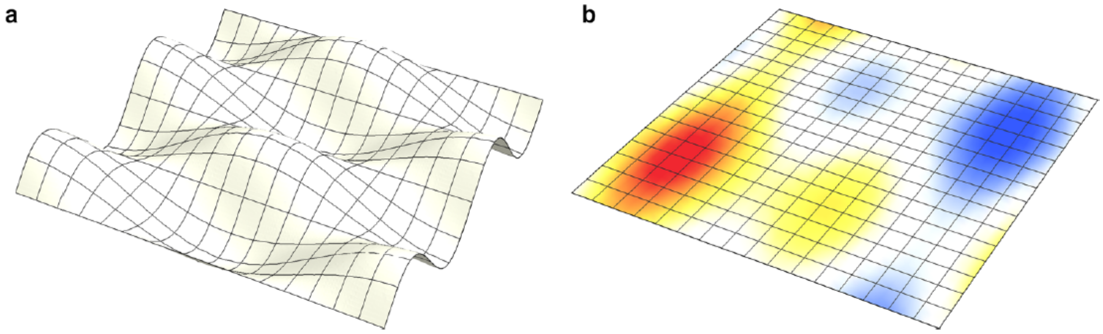

Fig. 1. Conceptual illustration of heterogeneity-stabilized homogeneous states. (**a**) A homogeneous membrane driven by a vertical vibration undergoes a symmetry-breaking instability, resulting in an inhomogeneous (undulated) state. (**b**) Introducing heterogeneity in the membrane (represented here by the color) stabilizes the homogeneous (planar) state. The boundary conditions are periodic.

Anharmonic classical time crystals: A coresonance pattern formation mechanism,
Z. G. Nicolaou and A. E. Motter,
Physical Review Research ***3****, 023106 (2021).*

Summary: Driven many-body systems have been shown to exhibit discrete time crystal phases characterized by broken discrete time-translational symmetry. This has been achieved generally through a subharmonic response, in which the system undergoes one oscillation every other driving period. Here, we demonstrate that classical time crystals do not need to resonate in a subharmonic fashion but instead can also exhibit a continuously tunable anharmonic response to driving, which we show can emerge through a coresonance between modes in different branches of the dispersion relation in a parametrically driven medium. This response, characterized by a typically incommensurate ratio between the resonant frequencies and the driving frequency, is demonstrated by introducing a time crystal model consisting of an array of coupled pendula with alternating lengths. Importantly, the coresonance mechanism is the result of a bifurcation involving a fixed point and an invariant torus, with no intermediate limit cycles. This bifurcation thus gives rise to a many-body symmetry-breaking phenomenon directly connecting the symmetry-unbroken phase with a previously uncharacterized phase of matter, which we call an anharmonic time crystal phase. The mechanism is shown to generalize to driven media with any number of coupled fields and is expected to give rise to anharmonic responses in a range of weakly damped pattern-forming systems, with potential applications to the study of nonequilibrium phases, frequency conversion, and acoustic cloaking.

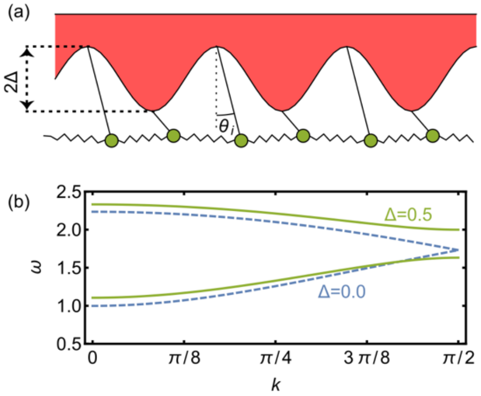

Fig. 2. Model system exhibiting an anharmonic response to periodic driving. (**a**) Array of coupled pendula of alternating lengths, which can oscillate about their pivots when driven by vertical vibrations through an anharmonic response incommensurate with the driving. (**b**) Dispersion relation between the angular frequency ω and wave number *k*, which governs wave propagation in the undriven pendulum array uniform array (blue dashed lines) and a symmetry-broken array (green solid lines).