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OPTIMUM CHARACTERISTICS FOR RADAR-REFLECTING MATERIALS

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OPTIMUM CHARACTERISTICS FOR RADAR-REFLECTING MATERIALS

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March 7, 1949

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ABSTRACT

A theoretical analysis is made to determine the optimum form for radar-reflecting materials. It is found that the best utilization of material for the frequency range 10^8 to 10^{10} cps is obtained from a netting made of wires of high conductivity having a radius of 3×10^{-5} cm and spaced 1.5 to 3 mm apart. The volume of material required to produce a given echo is found to vary inversely with the conductivity but is independent of the permeability. It is shown that, under "ideal" conditions, one ounce of aluminum material can be made to simulate the echoes from more than 2000 Flying Fortresses or Liberator bombers. Ideal conditions (never possible to achieve in actual operation) assume the netting to be in the form of a perfectly flat sheet oriented normal to the direction of radar energy.

PROBLEM STATUS

This is an interim report; work on this problem is continuing.

AUTHORIZATION

NRL Problem No. R06-26R.

OPTIMUM CHARACTERISTICS FOR RADAR-REFLECTING MATERIALS

INTRODUCTION

The problem of producing false radar echoes resolves itself into one of making the largest possible echo with the least possible material both in regard to mass and bulk. Recent developments in reflectors[1] show a trend toward the use of conducting mesh in the form of sheets, corners, and spheres. These meshes have been made of many materials. Initially, fine wires were woven at intervals in ordinary cotton fabric. Subsequently, netting was metalized by the Suchy process. A review of the work leads to speculation as to what extent wire diameter and conductivity may be decreased without destroying its effectiveness as a reflecting medium.

Preliminary experiments at the Naval Research Laboratory have indicated that a surprisingly small amount of material can produce large echoes. It was found that lines ruled in silver ink on paper as well as metallized condenser paper were excellent reflecting media. At the same time it was noted that certain materials, notably glass and polystyrene, were either commercially available in the form of fine fibers or could readily be put into that form. The combination of these two ideas leads to the concept of a new form for reflecting material. By metallizing such fibers or by use of a process which would render the fibers conducting, a "cobweb" or ball of "fluff" might be produced which would have a reflection efficiency comparable to the silver-ruled lines.

Such a material would have several outstanding advantages:

(1) Due to its extremely light weight, it would have a very slow rate of fall and remain in the air over extended periods of time.
(2) Since it would be non-resonant over a wide band of frequencies, it could be used effectively without advance knowledge of the characteristics of the enemy radar.
(3) Finally, there is the possibility that it might be loaded into a shell in very compact form and exploded into its final form at some point <u>in front of</u> the aircraft or ships being protected.

Before attempting to produce materials of this kind, it is desirable to provide for the development a theoretical foundation which will assist in establishing the specifications for the material and indicate the directions along which the research should proceed.

---

[1] Great Britain - Director of Misc. Weapons Dev., Technical History No. X, "Radar Reflectors," Secret (n.d.)

The present report consists of a mathematical treatment of the problem and gives the conclusions which may be drawn from that analysis.

HISTORICAL BACKGROUND

Since the time of Hertz it has been known that a series of parallel wires a short distance apart make a very effective reflector for electromagnetic waves, provided that the direction of polarization is parallel to the direction of the wires. A mesh is derived from a pair of such grids oriented so that the wires in one are at right angles to the wires in the other. Such an arrangement is insensitive to direction of polarization. Analysis of the reflection from the mesh, however, resolves itself into analysis of the grid, because the two components of the incident wave parallel to each of the grid directions can be treated separately.

Shortly after the experiments of Hertz, Thompson[2] and Lamb[3] published mathematical analyses of the reflection efficiency of a grid assuming infinite conductivity for the wires. Subsequently, the reflectivity of fine wire grids made of various metals was measured experimentally by Schaefer and Laugwitz.[4] They demonstrated that a drop in the reflectivity of the grid was experienced when wires of lower conductivity were used. An extension to the theory of grids to include the effect of conductivity was made by Gans.[5,6] His theoretical results did not check the Schaefer-Laugwitz experiments, but it was later shown by Schaefer[7] that the discrepancy was due to erroneous reporting of the earlier experimental work. A very extensive treatment of the grid reflection problem has been made by Ignatowsky.[8]

Since this general theory takes into account all possible current distributions around the circumference of the wire, it is considerably more complicated than necessary for the present work. A different approach to the problem has been made by Wessel.[9] He restricted himself to the first approximation of the Ignatowsky theory where a uniform current distribution on the wires is assumed. A feature of Wessel's work is the use of an integral equation method rather than a boundary value method to solve the problem. His results are of particular interest for the region where the spacing between wires approaches a wavelength in magnitude.

---

[2] J. J. Thompson, "Notes on recent researches in electricity and magnetism," Oxford University Press, (1893), pp 425 ff.

[3] H. Lamb, Proc. London Math. Soc., 29, 523-544, (1898).

[4] C. Schaefer and M. Laugwitz, Ann. d. Phys., Series 4, 23, 951-956, (1907).

[5] R. Gans, Ann d. Phys., Series 4, 61, 447-464, (1920).

[6] R. Gans, Ann d. Phys., Series 4, 66, 427-28, (1921).

[7] C. Schaefer, Ann. d. Phys., Series 4, 74, 275-284, (1924).

[8] W. von Ignatowsky, Ann d. Phys., Series 4, 44, 369-436, (1914).

[9] W. Wessel, Hochfreq. u. Elekakus., 54, 62-69, (1939).

A summary of the more recent work on the reflecting properties of gratings is given by Gooden [10] where curves of percent reflection as a function of a large range of wire diameters and spacings are presented. The data for the relatively large wire diameters is derived from Hayes,[11] and the data for the smaller wire diameters is derived from a report by the T.R.E. Mathematical Group.[12] The later work is based on the results of Wessel.

The curves presented by Gooden were used to calculate the volume of material required to give a certain percentage reflection. There was a marked decrease in volume as the spacing (i.e., wire size) was reduced, but as the smaller spacings were reached the curves tended to level off. These results confirmed earlier expectations and indicated that the region of very small wire diameters and spacings was of primary interest in this work. The Gans theory was applicable in this region and provided the basis for the present analysis.

DEVELOPMENT OF THE GANS THEORY

Assume that we have an infinite grid arranged as shown in Figure 1. The radius, a, of the grid wires is small in comparison with their spacing, which in turn is small with respect to a wavelength. A plane electromagnetic wave polarized in the direction of the z-axis approaches the grid along the positive x-axis. It is partly reflected and partly transmitted. The transmitted portion continues moving along the negative x-axis. Four different regions are recognized in the development of the theory. They are:

I. The region inside the wires themselves.

II. The region in the immediate vicinity of the grid (marked by two bounding planes parallel to the grid and a long distance from it in comparison to the spacing but a short distance from it with respect to a wavelength).

III. The region outside the bounding plane in the direction of the positive x-axis.

IV. The region outside the bounding plane in the direction of the negative x-axis.

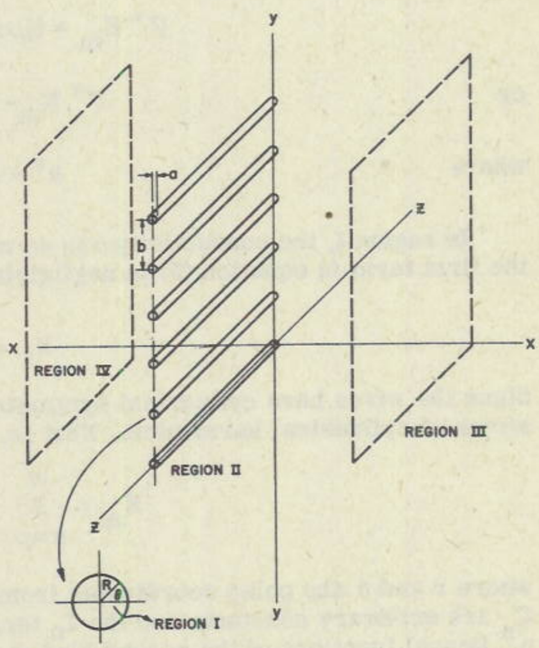

Fig. 1 - Graphic representation of assumed grid structure

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[10] J. S. Gooden, "Reflecting properties of metal gratings," Australia Council for Scientific and Industrial Research, Radiophysics Laboratory, Report No. R.P. 215, JEIA 5677, 31 July 1944, Confidential.

[11] W. D. Hayes, "Gratings and screens as microwave reflectors," Radiation Laboratory Report 54-20, 1 April 1943, Unclassified.

[12] Mathematical Group T.R.E. Swanage, "The elements of wave-propagation using the impedance concept," T.R.E. Report 261, 1 December 1941.

A solution to the problem is obtained in each of these areas, and the results are equated at the boundaries in order to determine the coefficients of the equations.

From Maxwell's equations it can be shown that the wave equation for a conducting region containing no charge takes the form

$$\nabla^2 E = \mu \frac{\partial}{\partial t} \left(\sigma E + \epsilon \frac{\partial E}{\partial t} \right). \tag{1}$$

If we assume a sinusoidal wave of frequency f cps, we have electric field intensity given by

$$E = E_m e^{j\omega t} \tag{2}$$

where

$$\omega = 2\pi f.$$

Substitution of equation (2) in equation (1) yields

$$\nabla^2 E_m = (j\mu\sigma\omega - \epsilon\mu\omega^2) E_m \tag{3}$$

or

$$\nabla^2 E_m + k^2 E_m = 0 \tag{4}$$

where

$$k^2 = (\epsilon\mu\omega^2 - j\mu\sigma\omega). \tag{5}$$

In region I, the conductivity σ is so much greater than the dielectric constant ϵ that the first term in equation (5) is negligible, hence

$$k_1 = \sqrt{-j\mu\sigma\omega}. \tag{6}$$

Since the wires have cylindrical symmetry, the solution to equation (4) for this region is in terms of cylindrical harmonics. That is,

$$E_m = \sum_{n=o}^{\infty} C_n J_n (k_1 r) \cos n\theta \tag{7}$$

where r and θ are polar coordinates from the axis of the wire (see Figure 1). The factors C_n are arbitrary constants and the J_n terms are Bessel functions of the first kind of order n. Bessel functions of the second kind do not appear in equation (7) because they become infinite for r = o.

In region III, the conductivity is zero and the permeability and dielectric constants are μ_o and ϵ_o. Equation (1) becomes

$$\nabla^2 E - \mu_o \epsilon_o \frac{\partial^2 E}{\partial t^2} = 0. \tag{8}$$

We can assume that the field intensity here varies in a sinusoidal manner and drops off exponentially as the origin is approached. Thus

$$E = E_o e^{j\omega t + \gamma x}. \tag{9}$$

Substitution of (9) into (8) yields

$$\gamma^2 + \omega^2 \epsilon_o \mu_o = 0$$

and

$$\gamma = \pm j \omega \sqrt{\epsilon_o \mu_o} = \pm \frac{j\omega}{c} = \pm j \frac{2\pi}{\lambda_o} \tag{10}$$

where c is the velocity of light and λ_o the wavelength in free space, and where the positive sign corresponds to the incident wave and the negative sign to the reflected wave. Consequently, the field intensity in region III can be written

$$E = E_m e^{j\omega t} = E_o e^{j\omega t} \left(e^{j \frac{2\pi x}{\lambda_o}} + A e^{-j \frac{2\pi x}{\lambda_o}}\right) \tag{11}$$

where A represents the fraction of the incident wave which is reflected. A similar situation exists in region IV except that here the transmitted wave exists alone. That is,

$$E = E_m e^{j\omega t} = E_o e^{j\omega t} \left(B e^{j \frac{2\pi x}{\lambda_o}}\right) \tag{12}$$

where B is the fraction of the incident wave transmitted through the grid.

The first two terms of the series expansion for e^x can be used to represent the function if x is sufficiently small. Thus an approximate expression for the coefficient E_m in equation (11) becomes

$$\begin{aligned} E_m &= E_o \left(1 + j \frac{2\pi x}{\lambda_o} + A - j A \frac{2\pi x}{\lambda_o}\right) \\ &= E_o \left(1 + A + j \frac{2\pi x}{\lambda_o} \; [1 - A]\right). \end{aligned} \tag{13}$$

Similarly an approximate expression for E_m in equation (12) is

$$E_m = E_o B \left(1 + j \frac{2\pi x}{\lambda_o}\right). \tag{14}$$

In considering region II, we can approximate the wave equation (4) by the Laplacian

$$\nabla^2 E_m = 0 \tag{15}$$

because k is a small number. The validity of this approximation has been discussed by Schaefer.[13] He compares the exact theory for dispersion by a cylinder with a solution based on the above. The results show that the approximate solution is appropriate for wires having good conductivity or for fibers having a high dielectric constant. Since the electric field intensity is independent of z, the Laplacian can be written in two dimensional form:

$$\frac{\partial^2 E_m}{\partial x^2} + \frac{\partial^2 E_m}{\partial y^2} = 0. \tag{16}$$

---

[13] Schaefer, op.cit.

We require solutions to equation (16) which will satisfy equation (7) on the surface of the wires and at the same time will go over into the form of equation (13) for large positive values of x or equation (14) for large negative values of x. Three functions which satisfy these requirements are

$$\left.\begin{aligned}
\phi_1 &= x - \frac{\pi a^2}{b}\,\frac{\sinh\frac{2\pi x}{b}}{\cosh\frac{2\pi x}{b} - \cos\frac{2\pi y}{b}}\\
\phi_2 &= \frac{1}{2}\ln\frac{1}{2}\left(\cosh\frac{2\pi x}{b} - \cos\frac{2\pi y}{b}\right)\\
\phi_3 &= x + \frac{\pi a^2}{b}\,\frac{\sinh\frac{2\pi x}{b}}{\cosh\frac{2\pi x}{b} - \cos\frac{2\pi y}{b}}\,.
\end{aligned}\right\} \tag{17}$$

It will be noticed that these functions are repetitive in character, i.e., they are the same at periodic values of y corresponding to the individual grid wires. For small values of u, $\sinh u \approx u$, $\cosh u \approx 1 + u^2/2$, and $\cos u \approx 1 - u^2/2$; hence, for small values of x and y equations (17) reduce to

$$\begin{aligned}
\phi_1 &= x - \frac{a^2 x}{x^2 + y^2} = x\left(1 - \frac{a^2}{r^2}\right) = \cos\theta\left(r - \frac{a^2}{r}\right)\\
\phi_2 &= \frac{1}{2}\ln\pi^2\left(\frac{x^2 + y^2}{b^2}\right) = \ln\frac{\pi r}{b}\\
\phi_3 &= x + \frac{a^2 x}{x^2 + y^2} = \cos\theta\left(r + \frac{a^2}{r}\right)
\end{aligned} \tag{18}$$

where r and θ are the polar coordinates from the axis of the wires as shown in Figure 1.

On the surface of the wires where r = a, equations (18) reduce to

$$\left.\begin{aligned}
\phi_1 &= 0 & \frac{\partial\phi_1}{\partial r} &= 2\cos\theta\\
\phi_2 &= \ln\frac{\pi a}{b} & \frac{\partial\phi_2}{\partial r} &= \frac{1}{a}\\
\phi_3 &= 2a\cos\theta & \frac{\partial\phi_3}{\partial r} &= 0.
\end{aligned}\right\} \tag{19}$$

These functions are valid on all the wires of the grid because, it will be noted, the equations (17) are periodic with respect to the spacing of the wires and take on the same values at each wire.

For very large positive values of u, $\sinh u \approx \cosh u \approx e^u/2$. Consequently for very large positive values of x equations (17) become

$$\left.\begin{aligned}
\phi_1 &= x - \frac{\pi a^2}{b}\\
\phi_2 &= \frac{1}{2}\ln\frac{1}{4}e^{\frac{2\pi x}{b}} = \frac{\pi x}{b} - \ln 2\\
\phi_3 &= x + \frac{\pi a^2}{b}
\end{aligned}\right\} \tag{20}$$

For large negative values of u, $\cosh u \approx -\sinh u \approx e^{-u}/2$. Consequently, for very large negative values of x, equations (17) become

$$\left.\begin{aligned} \phi_1 &= x + \frac{\pi a^2}{b} \\ \phi_2 &= \frac{1}{2} \ln \frac{1}{4} e^{\frac{-2\pi x}{b}} = -\frac{\pi x}{b} - \ln 2 \\ \phi_3 &= x - \frac{\pi a^2}{b} \end{aligned}\right\} \qquad (21)$$

At the boundary between the wires and the surrounding space the tangential components of both the electric and magnetic field intensity must be continuous. In terms of the present problem, the z components of electric field intensity and the θ components of magnetic field intensity must be the same on both sides of the boundary. Since the polarization was assumed to be parallel to the z-axis, $E_z = E_m$ and, at $r = a$,

$$E_m \text{ (outside)} = E_m \text{ (inside)}. \qquad (22)$$

The complete solution to the Laplacian is a linear combination of the functions ϕ_1, ϕ_2, ϕ_3, and a constant. Thus,

$$E_m = \alpha \phi_1 + \beta + \gamma \phi_2 + \delta \phi_3. \qquad (23)$$

At $r = a$, equation (23) must be equal to equation (7). From equations (19), (22), (23), and (7) we have at $r = a$

$$\gamma \ln \frac{\pi a}{b} + \delta\, 2a \cos\theta + \beta = C_0 J_0 (k_1 a) + C_1 J_1 (k_1 a) \cos\theta. \qquad (24)$$

Only the first two terms of equation (7) are used because there are no $\cos 2\theta$, $\cos 3\theta$, or higher terms on the left side of equation (24).

From Faraday's law,

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \qquad (25)$$

which, written in cylindrical coordinates, gives

$$\left(\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) = -\frac{\partial E_z}{\partial r} = -\mu \frac{\partial H_\theta}{\partial t}. \qquad (26)$$

The requirement for the continuity of the tangential magnetic field intensity and the above equation leads to

$$\frac{1}{\mu_0} \frac{\partial E_m \text{ (outside)}}{\partial r} = \frac{1}{\mu} \frac{\partial E_m \text{ (inside)}}{\partial r} \qquad (27)$$

for $r = a$. The combination of equation (27) with equations (19), (23), and (7) gives

$$\frac{1}{\mu_0} \left[2\alpha \cos\theta + \frac{\gamma}{a} \right] = \frac{1}{\mu} \left[C_0 k_1 J_0' (k_1 a) + C_1 k_1 J_1' (k_1 a) \cos\theta \right] . \qquad (28)$$

The coefficients of the cosine terms on each side of equations (24) and (28) must be equal as must the constant terms; hence,

$$\left.\begin{aligned} \beta + \gamma \ln \frac{\pi a}{b} &= C_0 J_0 (k_1 a) \\ 2a\delta &= C_1 J_1 (k_1 a) \\ \frac{\mu}{\mu_0} \frac{\gamma}{a} &= C_0 k_1 J_0' (k_1 a) \\ \frac{\mu}{\mu_0} 2\alpha &= C_1 k_1 J_1' (k_1 a) . \end{aligned}\right\} \tag{29}$$

The elimination of C_0 between the above equations gives

$$\frac{\beta + \gamma \ln \frac{\pi a}{b}}{\gamma} = \frac{\mu}{\mu_0 a k_1} \frac{J_0 (k_1 a)}{J_0' (k_1 a)} = \tau_0 . \tag{30}$$

Similarly, the elimination of C_1 leads to

$$\frac{\delta}{\alpha} = \frac{\mu}{\mu_0 k_1 a} \frac{J_1 (k_1 a)}{J_1' (k_1 a)} = \tau_1 . \tag{31}$$

From the definitions of τ_0 and τ_1, as given in equations (30) and (31), we have

$$\gamma = - \frac{\beta}{\ln \frac{\pi a}{b} - \tau_0} \quad ; \quad \delta = \alpha \tau_1 . \tag{32}$$

Substitution in equation (23) gives

$$E_m = \alpha \phi_1 + \beta - \frac{\beta}{\ln \frac{\pi a}{b} - \tau_0} \phi_2 + \alpha \tau_1 \phi_3 . \tag{33}$$

At the boundary between region II and region III equation (33) must be equal to equation (13). At this boundary the functions ϕ_1, ϕ_2, and ϕ_3 are at their upper limit of usefulness, and therefore equation (20) applies. So

$$\begin{aligned} \alpha \left(x - \frac{\pi a^2}{b}\right) + \beta - \frac{\beta}{\ln \frac{\pi a}{b} - \tau_0} \left(\frac{\pi x}{b} - \ln 2\right) + \alpha \tau_1 \left(x + \frac{\pi a^2}{b}\right) \\ = E_0 \left(1 + A + j \frac{2\pi x}{\lambda_0} [1 - A]\right) . \end{aligned} \tag{34}$$

Similarly, at the boundary between region II and region IV the application of equations (14), (21), and (33) leads to

$$\begin{aligned} \alpha \left(x + \frac{\pi a^2}{b}\right) + \beta - \frac{\beta}{\ln \frac{\pi a}{b} - \tau_0} \left(- \frac{\pi x}{b} - \ln 2\right) + \alpha \tau_1 \left(x - \frac{\pi a^2}{b}\right) \\ = E_0 B \left(1 + j \frac{2\pi x}{\lambda_0}\right) . \end{aligned} \tag{35}$$

The boundaries between region II and the exterior regions are indefinite. Thus, equations (34) and (35) are essentially valid for all values of x. Equating coefficients of like terms on each side of these equations leads to

$$\left.\begin{aligned}
E_o(1+A) &= \frac{-\alpha\pi a^2}{b}(1-\tau_1)+\beta+\beta\frac{\ln^2}{\ln\frac{\pi a}{b}-\tau_o}\\
j\frac{2\pi}{\lambda_o}E_o(1-A) &= \alpha(1+\tau_1)-\frac{\beta\frac{\pi}{b}}{\ln\frac{\pi a}{b}-\tau_o}\\
E_oB &= \frac{\alpha\pi a^2}{b}(1-\tau_1)+\beta+\beta\frac{\ln^2}{\ln\frac{\pi a}{b}-\tau_o}\\
j\frac{2\pi}{\lambda_o}E_oB &= \alpha(1+\tau_1)+\frac{\beta\frac{\pi}{b}}{\ln\frac{\pi a}{b}-\tau_o}\,.
\end{aligned}\right\}\qquad(36)$$

By addition and subtraction of equations (36) by pairs we obtain

$$\left.\begin{aligned}
E_o(1+A+B) &= 2\beta+2\beta\frac{\ln 2}{\ln\frac{\pi a}{b}-\tau_o}=2\beta\frac{\ln\frac{2\pi a}{b}-\tau_o}{\ln\frac{\pi a}{b}-\tau_o}\\
E_o(B-A-1) &= 2\alpha\frac{\pi a^2}{b}(1-\tau_1)\\
j\frac{2\pi}{\lambda_o}E_o(B-A+1) &= 2\alpha(1+\tau_1)\\
j\frac{2\pi}{\lambda_o}E_o(B+A-1) &= \frac{2\beta\frac{\pi}{b}}{\ln\frac{\pi a}{b}-\tau_o}\,.
\end{aligned}\right\}\qquad(37)$$

Equations (37) give the following ratios:

$$\left.\begin{aligned}
\frac{B-A-1}{B-A+1} &= j\frac{2\pi}{\lambda_o}\,\frac{2\alpha\frac{\pi a^2}{b}(1-\tau_1)}{2\alpha(1+\tau_1)}=j\frac{2\pi^2a^2}{\lambda_o b}\,\frac{(1-\tau_1)}{(1+\tau_1)}\\
\frac{B+A+1}{B+A-1} &= j\frac{2\pi}{\lambda_o}\,\frac{2\beta\ln\frac{2\pi a}{b}-\tau_o}{2\beta\frac{\pi}{b}}=-j\frac{2b}{\lambda_o}\left(\ln\frac{b}{2\pi a}+\tau_o\right).
\end{aligned}\right\}\qquad(38)$$

The coefficient of the right side of the first of equations (38) is essentially zero because the radius of the wires is assumed to be very small in comparison with the spacing and the wavelength. Hence,

$$B-A-1=0\quad\text{or}\quad B=A+1\,.\qquad(39)$$

From equation (39) and the second of equations (38) we have

$$\frac{2A + 2}{2A} = -j\Delta \tag{40}$$

where

$$\Delta = \frac{2b}{\lambda_o}\left(\ln\frac{b}{2\pi a} + \tau_o\right) = u + jv\,. \tag{41}$$

The factor A was defined as the ratio of the reflected electric field intensity E_r to the incident electric field intensity E_i. Solving equation (40) for A and substituting we have

$$A = \frac{E_r}{E_i} = \frac{-1}{1 + j\Delta} = \frac{-1}{1 - v + ju}\,. \tag{42}$$

Power is proportional to the square of the electric field intensity. Consequently, the reflection efficiency η, which is defined as the ratio of the reflected power to the incident power, is given by

$$\eta_o = \frac{P_r}{P_i} = \frac{|E_r|^2}{|E_i|^2} = \frac{1}{(1-v)^2 + u^2}\,. \tag{43}$$

The factors u and v are defined by equation (41). The defining equations for other terms required in the evaluation of u and v have been given earlier and are repeated here for convenience in reference:

$$\tau_o = \frac{\mu}{\mu_o k_1 a}\,\frac{J_0(k_1 a)}{J_0'(k_1 a)} \tag{30}$$

$$k_1 = \sqrt{-j\mu\sigma\omega} \tag{6}$$

Tables are available[14] which give the value of the function

$$\frac{2}{r}\,\frac{J_0(r\sqrt{i})}{J_1(r\sqrt{i})} = \frac{2}{r}\,\frac{b_0}{b_1}\,\underline{|\beta_0 - \beta_1} \tag{44}$$

for various values of r. The factor τ_o can be evaluated by use of these tables because the minus sign in $\sqrt{-i}$ simply changes the sign on the angles β_0 and β_1. Let

$$k_1 a = \sqrt{\mu\sigma\omega a^2}\,\sqrt{-i} = r\sqrt{-i}. \tag{45}$$

Then from equations (30), (44), and (45),

$$\begin{aligned}\tau_o &= \frac{\mu}{\mu_o r\sqrt{-i}}\,\frac{J_0(r\sqrt{-i})}{J_0'(r\sqrt{-i})} = \frac{-\mu}{2\mu_o}\,\frac{2}{re^{-j\frac{\pi}{4}}}\,\frac{(J_0(r\sqrt{-i})}{(J_1(r\sqrt{-i})}\\ &= \frac{-\mu}{2\mu_o}\,\frac{2}{r}\,\frac{b_0}{b_1}\,\underline{|0.5\llcorner - (\beta_0 - \beta_1)}\end{aligned} \tag{46}$$

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[14] E. Jahnke and F. Emde, "Tables of Functions with Formulae and Curves," Dover Publications, New York, (1943) pp. 266-267.

The symbol $\llcorner$ as used above conforms with the practice followed by the reference tables and signifies right angle units. Thus

$$0.5^{\llcorner} = 45^\circ = \frac{\pi}{4} \text{ radians.}$$

SIMPLIFICATION OF THE GANS FORMULA

From tne theory of Bessel functions we have the expansion,

$$\frac{x}{2}\,\frac{J_0(x)}{J_1(x)} = 1 - \frac{\left(\frac{x}{2}\right)^2}{2} - \frac{\left(\frac{x}{2}\right)^4}{12} - \frac{\left(\frac{x}{2}\right)^6}{48} - \frac{\left(\frac{x}{2}\right)^8}{180}\,, \tag{47}$$

which can be used to obtain an algebraic expression for τ_o. In the region where r is small, we have

$$\tau_o = \frac{-\mu}{\mu_o}\left[\frac{1}{r\sqrt{-i}}\;\frac{J_0(r\sqrt{-i})}{J_1(r\sqrt{-i})}\right] = \frac{-\mu}{\mu_o}\left[\frac{2i}{r^2}\;\frac{r\sqrt{-i}}{2}\;\frac{J_0(r\sqrt{-i})}{J_1(r\sqrt{-i})}\right]. \tag{48}$$

Substitution of equation (47) into equation (48) yields

$$\begin{aligned}\tau_o &= \frac{-\mu}{\mu_o}\,\frac{2i}{r^2}\left[1 + \frac{ir^2}{2x4} + \frac{r^4}{12x16} + \frac{ir^6}{48x64}\right]\\ &= \frac{-\mu}{\mu_o}\left[-\frac{1}{4} - \frac{r^6}{1536} + \ldots + \frac{2i}{r^2} + \frac{ir^2}{96}\right]\\ &\approx \frac{\mu}{\mu_o}\left[\frac{1}{4} - \frac{2i}{r^2}\right].\end{aligned} \tag{49}$$

Substitution of equation (49) into equation (41) gives

$$\Delta = \frac{2b}{\lambda_o}\left(\ln\frac{b}{2\pi a} + \frac{1}{4}\,\frac{\mu}{\mu_o} - \frac{2i}{r^2}\,\frac{\mu}{\mu_o}\right). \tag{50}$$

When r is sufficiently small the imaginary term is very much larger then the real terms in equation (50), so we have

$$\Delta \approx -i\frac{4b}{\lambda_o r^2}\,\frac{\mu}{\mu_o} = -i\,\frac{4b}{\lambda_o \sigma\omega a^2 \mu_o}\,. \tag{51}$$

A further simplification of equation (51) is possible by recognizing that $\lambda_o f = c = 3 \times 10^8$ meters per second, the velocity of light, and making the numerical substitution for $\mu_o = 4\pi \times 10^{-7}$ henry per meter. Then

$$\Delta \approx -\frac{ib}{60\pi^2\sigma a^2}. \tag{52}$$

From equation (43) we find that the reflection efficiency for normal incidence becomes

$$\eta_o = \frac{1}{\left(1 + \frac{b}{60\pi^2\sigma a^2}\right)^2}. \tag{53}$$

Under the conditions assumed it will be noted that the reflection efficiency is independent of the permeability μ of the wires making up the grid. Also the reflection efficiency is independent of frequency. The later result must be used with caution, however, because the magnitude of r is a function of frequency, and for certain frequencies r would no longer be very small.

CORRELATION OF GANS FORMULA WITH OTHERS

The Gans formula is limited in its application to those cases where $\lambda \gg b \gg a$. It is of interest, however, to consider situations where the spacing is nearly equal to a wavelength. A comparison of the Gans formula with others covering this region is desirable. The development of Wessel[15] is useful in this respect because it covers all spacings including multiples of a wavelength. However, our interest is confined to the region below one wavelength because all grids become transparent at multiples of a wavelength spacing. In a practical application it would be unwise to have such regions of zero response in the design band of frequencies.

If we assume that the conductivity of the wires in the grid is infinite, it can be seen from equation (45) that the variable r also becomes infinite. The function in equation (46) approaches zero for large values of r; hence, the factor τ_0 approaches zero in the limit. For infinite conductivity, equation (41) becomes

$$\Delta = \frac{2b}{\lambda_0} \ln \frac{b}{2\pi a} \quad . \tag{54}$$

Wessel defines the term $-ge^{j\delta}$ as the ratio of the reflected electric field intensity to the incident electric field intensity. Thus his term $-ge^{j\delta}$ corresponds to the term A used in this report. The reflection efficiency becomes

$$\eta_0 = |A|^2 = |ge^{j\delta}|^2 = \left|\frac{R\infty}{Z}\right|^2 \tag{55}$$

from Wessel's equation (59) where he defines Z as the impedance of the grid per unit length. It is important to note that this impedance is different from the grid impedance used in the transmission line analogy of T.R.E.[16] The factor R∞ is defined as the real portion of Z for infinite wavelength; however, it is constant for all wavelengths greater than the spacing of the grid. Hence, from equation (55) we have

$$\eta = \left|\frac{R\infty}{Z}\right|^2 = \left|\frac{R\infty}{R\infty + j\omega L}\right|^2 = \frac{1}{1+\left(\frac{\omega L}{R\infty}\right)^2} \tag{56}$$

for wavelengths greater than the grid spacing b (Wessel's symbol d). A comparison of equation (56) with equation (42) shows that

$$\Delta = \frac{\omega L}{R\infty} \quad . \tag{57}$$

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[15] Wessel, op. cit.

[16] T.R.E. Report 261, op. cit.

From Wessel's equations (56) and (57) we have

$$\Delta_{\lambda > b} = \frac{2\pi f\left[\frac{2}{c^2}\left\{\ln\frac{b}{2\pi a} + \sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{1}{\sqrt{1-\left(\frac{b}{n\lambda}\right)^2}} - 1\right)\right\}\right]}{\frac{2\pi}{cb}} \tag{58}$$

$$= \frac{2b}{\lambda_o}\left\{\ln\frac{b}{2\pi a} + \sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{1}{\sqrt{1-\left(\frac{b}{n\lambda}\right)^2}} - 1\right)\right\}.$$

Comparison of equation (58) with equation (54) shows that the effect of greater grid spacings appears as an additive series in the equation. Wessel also indicates the procedure for handling grids of finite conductivity, but the added complication is not warranted for our present purposes.

One other limitation on the Gans development is the assumption that the wave arrives at normal incidence. An analysis of oblique incidence has been made by T.R.E.[17] using the impedance concept. The results show that the shunt reactance of the grid is independent of the angle of incidence, when $\lambda \gg b \gg a$, i.e., in the region of applicability of the Gans formula. On the other hand, the surge impedance normal to the grid becomes $Z_o \sec\theta$, where Z_o is the surge impedance of free space and θ the angle of incidence.

A few fundamental ideas from the theory of reflection based on the impedance concept will assist in the utilization of these results. An obstruction such as a grid in space is analogous to an impedance shunted across a transmission line. The reflections produced by the grid are the same as those which would be produced by a transmission line terminated by a load consisting of the parallel combination of the surge impedance of space and the surface impedance of the grid. For infinitely conducting grids with a spacing of less than a wavelength,

$$\frac{E_i}{Y_o + (Y_o + jB_G)} = \frac{E_r}{Y_o - (Y_o + jB_G)} = \frac{E_t}{2Y_o}. \tag{59}$$

where E_i = incident wave, E_r = reflected wave, E_t = transmitted wave, Y_o = surge admittance of space, and B_G is the surface susceptance of the grid. Equation (59) reduces to

$$\frac{E_r}{E_i} = \frac{-jB_G}{2Y_o + jB_G} = -ge^{j\delta}. \tag{60}$$

From Wessel's equations (32) and (46) we have

$$ge^{j\delta} = \frac{2\pi}{cb}\,\frac{1}{Z} = \frac{2\pi}{cb}\,\frac{1}{R\infty + j\omega L} \tag{61}$$

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[17] Mathematical Group T.R.E. Malvern, "Surface impedance of an infinite parallel wire grid of oblique angles of incidence," T.R.E. Report 52/GGM, 26 November 1942.

where Z is the impedance of the grid in the Wessel sense. The combination of equations (60) and (61) leads to

$$\frac{j\,B_G}{2\,Y_o + j\,B_G} = \frac{2\pi}{cb}\,\frac{1}{R\infty + j\omega L} = \frac{\frac{1}{j\,X_G}}{\frac{2}{Z_o} + \frac{1}{j\,X_G}}$$

$$= \frac{Z_o}{Z_o + j\,2\,X_G} = \frac{1}{1 + \frac{2\,j\,X_G}{Z_o}} = \frac{2\pi}{cb\,R\infty}\left(\frac{1}{1 + \frac{j\,\omega L}{R\infty}}\right). \qquad (62)$$

Since $R\infty = 2\pi/cb$, equation (62) shows that

$$\frac{2\,X_G}{Z_o} = \frac{\omega L}{R\infty} \qquad (63)$$

or

$$X_G = \frac{1}{2}\,Z_o\Delta = Z_o\,\frac{b}{\lambda_o}\quad \ln\frac{b}{2\pi a} \qquad (64)$$

when the spacing is sufficiently small to be able to neglect the additive series in equation (58). Equation (64) is the grid reactance used by T.R.E. in their analysis.

For oblique incidence, the impedance of space used must be that normal to the grid. Consequently $Z_o \sec\theta$ must be substituted for Z_o in equation (62). As indicated above, the reactance of the grid is essentially constant. From equations (55), (61), and (62) we have for normal incidence

$$\eta_o = \left|\frac{Z_o}{Z_o + j\,2\,X_g}\right|^2 = \frac{Z_o^2}{Z_o^2 + 4\,X_g^2} \qquad (65)$$

and for oblique incidence

$$\eta_\theta = \left|\frac{Z_o \sec\theta}{Z_o \sec\theta + j\,2\,X_g}\right|^2 = \frac{Z_o^2 \sec^2\theta}{Z_o^2 \sec^2\theta + 4\,X_g^2}. \qquad (66)$$

From equation (65),

$$4\,X_g^2 = \frac{Z_o^2}{\eta_o} - Z_o^2. \qquad (67)$$

Substituting equation (67) in equation (66), we have

$$\eta_\theta = \frac{\sec^2\theta}{\sec^2\theta + \frac{1}{\eta_o} - 1} = \frac{\eta_o \sec^2\theta}{\eta_o \tan^2\theta + 1}. \qquad (68)$$

ECHO PRODUCED BY NETTING IN VARIOUS CONFIGURATIONS

There are two commonly accepted methods of expressing an area which would give an echo equivalent to that from a given target. These are (a) the effective area Ae based on an equivalent flat sheet[18] and (b) the cross sectional area σ of an equivalent isotropic reflecting surface.[19,20] Both concepts will be used in this report, but the major emphasis will be placed on (a) since it is better adapted to an analysis of reflectors of the conventional type.

Friis[21] has reported a simple transmission formula which is very useful in correlating (a) and (b) above. The effective area of any antenna is defined as the ratio, Pr/S, where Pr is the power available at the output terminals when the antenna is receiving a wave having an incident power density S. For a small uniform current element, i.e., the Hertz doublet, output power is equal to the induced voltage squared divided by four times the radiation resistance. That is,

$$Pr = \frac{E^2 dl^2}{4R}, \tag{69}$$

where E = electric field intensity, dl = length of current element, and R = radiation resistance = $80\pi^2 dl^2/\lambda^2$ ohms. The incident power density is equal to the electric field intensity squared, divided by the impedance of free space:

$$S = \frac{E^2}{120\pi}. \tag{70}$$

Hence, from the definition given above, the effective area of a Hertz doublet is

$$A_{hertz} = \frac{Pr}{S} = \frac{3\lambda^2}{8\pi}. \tag{71}$$

An isotropic antenna has 2/3 the gain or effective area of a Hertz doublet. That is,

$$A_{iso} = \frac{2}{3} \times \frac{3}{8}\frac{\lambda^2}{\pi} = \frac{\lambda^2}{4\pi}. \tag{72}$$

If we assume a transmitter of power P_t feeding an isotropic antenna, the power density at a receiving antenna a distance d away is

$$Sr = \frac{P_t}{4\pi d^2}. \tag{73}$$

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[18] S. D. Robertson, Bell System Tech. J. 26, 852-869, (1947).

[19] Martin Katzin, "Special report on radar cross section of ship targets," NRL report RA3A213A, 24 January 1944, Secret.

[20] K. A. Norton and A. C. Omberg, Proc. I.R.E., 35, 4-24, (1947).

[21] H. T. Friis, Proc. I.R.E., 34, 254-256, (1946).

From the definition of effective area of an antenna, the power picked up by a receiving antenna is

$$Pr = ArSr = \frac{P_t Ar}{4\pi d^2} \tag{74}$$

where Ar is the effective area of the receiving antenna. If the transmitting antenna is replaced by one having an effective area A_t, the power received will be increased by the ratio A_t/A_{iso}. The simple transmission equation follows directly:

$$\frac{Pr}{P_t} = \frac{Ar\,A_t}{4\pi d^2 A_{iso}} = \frac{Ar\,A_t}{d^2\,\lambda^2}. \tag{75}$$

Consider a large, flat, perfectly reflecting sheet oriented normal to the direction of incidence. The power density at the reflector is

$$Sr' = \frac{P_t \frac{A_t}{A_{iso}}}{4\pi d^2}. \tag{76}$$

The intercepted power Pr′ is equal to the product of the incident power density and the effective area Ae of the flat sheet. Since a reflection efficiency of 100% is assumed, the reradiated power P_t' is equal to the intercepted power. Thus,

$$Pr' = Sr'\,Ae = P_t'. \tag{77}$$

The returned power density at the transmitter is

$$Sr = \frac{P_t' \frac{Ae}{A_{iso}}}{4\pi d^2} = \frac{P_t \frac{A_t\,Ae^2}{A_{iso}^2}}{(4\pi d^2)^2}, \tag{78}$$

and the received power is

$$Pr = Sr\,A_t = \frac{P_t\,A_t^2\,Ae^2}{(4\pi d^2)^2\,\left(\frac{\lambda^2}{4\pi}\right)^2}$$

or (79)

$$\frac{Pr}{P_t} = \frac{A_t^2 Ae^2}{d^4 \lambda^4},$$

which is the radar equivalent of the Friis transmission formula. When a flat sheet is very large in comparison with a wavelength, its effective area Ae is equal to its physical area A. For that reason the concept of effective area is especially useful in evaluating the performance of reflectors made of flat surfaces.

On the other hand, consider the performance of a spherical reflector. A sphere has the property of reradiating equally in all directions the energy intercepted by it. It is an isotropic radiator. As before, we can say that the power density of the radiation incident on the sphere is given by equation (76). The power intercepted by the sphere, assuming it is large in comparison with its wavelength, is equal to the product of its cross sectional area, σ, times the incident power density. In this case, $\sigma = \pi a^2$, where a is the radius of the sphere, and is not to be confused with the definition of σ when discussing conductivity. If 100% reflection efficiency is assumed, this is also equal to the reradiated power P_t. So therefore

$$Pr' = Sr'\sigma = Sr'\pi a^2 = P_t' . \tag{80}$$

Since the sphere reradiates the power in an isotropic manner, the returned power density at the transmitter is

$$Sr = \frac{P_t'}{4\pi d^2} = \frac{P_t \frac{A_t}{A_{iso}} \sigma}{(4\pi d^2)^2} , \tag{81}$$

and the received power is

$$Pr = Sr\, A_t = \frac{P_t\, A_t^2 \sigma}{(4\pi d^2)^2 \left(\frac{\lambda^2}{4\pi}\right)} , \tag{82}$$

or

$$\frac{Pr}{P_t} = \frac{A_t^2\, \sigma\, \lambda^2}{d^4\, \lambda^4\, 4\pi} . \tag{83}$$

Comparison of equation (83) with equation (79) gives the relationship between cross sectional area and effective area. Thus,

$$\frac{\sigma\lambda^2}{4\pi} = Ae^2 \tag{84}$$

or

$$\sigma = \frac{4\pi Ae^2}{\lambda^2} . \tag{85}$$

Also, from equation (84) we find, on substitution for σ, the effective area of a spherical reflector. That is,

$$Ae = \frac{a\lambda}{2} . \tag{86}$$

Several important facts are illustrated by the above development. Targets which consist primarily of curved surfaces, such as aircraft, have a cross sectional area which is independent of wavelength, but their effective area is directly proportional to wavelength. Conversely, targets consisting of flat sheets or combinations thereof have effective areas independent of wavelength, but their cross sectional area is inversely proportional to wavelength squared.

Consider a flat sheet of netting having a physical area A and reflection efficiency η. The power intercepted would be

$$Pr' = Sr'\, A. \tag{87}$$

From the definition of reflection efficiency, the power reradiated would be

$$P_t' = \eta Pr' = \frac{P_t \frac{A_t}{A_{iso}} A}{4\pi d^2} \eta . \tag{88}$$

The returned power density at the transmitter becomes

$$Sr = \frac{P_t' \frac{A}{A_{iso}}}{4\pi d^2} = \frac{P_t \frac{A_t}{A_{iso}^2} A^2}{(4\pi d^2)^2} \eta , \tag{89}$$

since the directivity of the returned beam would be proportional to the physical area of the reflector. The received power becomes

$$Pr = Sr\, A_t$$

or

$$\frac{Pr}{P_t} = \frac{A_t^2 A^2}{d^4 \lambda^4} \eta . \tag{90}$$

Comparison of equation (90) with equation (79) leads to the expression for the physical area A of a semi-reflecting flat sheet in terms of its effective area Ae:

$$A = \frac{Ae}{\sqrt{\eta_o}} \tag{91}$$

In a similar manner the relationship between the physical area and the effective area of semi-reflecting material in spherical form can be determined. The power intercepted by the sphere is

$$Pr' = Sr' \pi a^2, \tag{92}$$

and the power reradiated is

$$P_t' = \eta_o Pr'. \tag{93}$$

Justification for using the reflection efficiency for normal incidence lies in the fact that the energy returned to the radar is from the portion of the sphere where normal incidence occurs. To avoid undue complication, we will neglect the energy transmitted through the surface of the sphere and reflected from the inside area on the opposite side of the sphere. The power density returned to the transmitter from the isotropic radiation of the sphere would be

$$Sr = \frac{P_t'}{4\pi d^2} = \frac{\frac{A_t}{A_{iso}} P_t \pi a^2}{(4\pi d^2)^2} \eta_o, \tag{94}$$

from which

$$\frac{Pr}{P_t} = \frac{A_t^2 \left(\frac{a^2 \lambda^2}{4}\right)}{d^4 \lambda^4} \eta_o. \tag{95}$$

Thus the effective area of the sphere becomes

$$Ae = \frac{a\lambda}{2} \sqrt{\eta_o} , \tag{96}$$

but the physical area is

$$A = 4\pi a^2. \tag{97}$$

Substitution of the radius a in (96) for the radius in (97) gives

$$A = \frac{16\pi A e^2}{\lambda^2 \eta_0}. \tag{98}$$

The angle of incidence on a dihedral oriented for maximum reflection is 45°. Hence the reflection efficiency from equation (68) becomes

$$\eta_{45} = \frac{\eta_0 \sec^2 45^\circ}{\eta_0 \tan^2 45^\circ + 1} = \frac{2\eta_0}{\eta_0 + 1}. \tag{99}$$

The incident wave undergoes two reflections before being returned. Thus the ratio of incident to reflected power is proportional to the reflection efficiency squared. The physical area of the material making up the dihedral is $\sqrt{2}$ times the intercept area. Consequently, in a manner similar to the above, we find that

$$A = \frac{\sqrt{2}\, Ae}{\eta_{45}} = \frac{Ae\,(\eta_0 + 1)}{\sqrt{2}\,\eta_0}. \tag{100}$$

A corner reflector viewed along its axis of symmetry has an angle of incidence of 54.74°. Hence, the reflection efficiency becomes

$$\eta_{54.74} = \frac{(1.731)^2 \eta_0}{\eta_0 (1.414)^2 + 1} = \frac{3\eta_0}{2\eta_0 + 1}. \tag{101}$$

According to Robertson,[22] a corner reflector 1.86a units on edge has an effective area of a^2 units. The physical area of the material making up such a reflector would be

$$A = \left(\frac{1.86a}{\sqrt{2}}\right)^2 \frac{3}{2} = 2.59\, a^2. \tag{102}$$

The returned beam would undergo a triple reflection; therefore the returned power would be proportional to the reflection efficiency cubed. Consequently, in a manner similar to that followed for a flat sheet, it develops that

$$A = \frac{2.59\, Ae}{\eta_{54.74}^{3/2}} = \frac{2.59\, Ae\,(2\eta_0 + 1)^{3/2}}{(3\eta_0)^{3/2}}. \tag{103}$$

VOLUME OF MATERIAL

The volume of each wire in a net of length l is $\pi a^2/l$. There are w/b wires in a width w. The volume of each cross wire at right angles to the first set is $\pi a^2 w$, and the number of wires in a length l is l/b. Thus, the total volume V of material making up a mesh l units by w units is

$$V = \pi a^2\, l \frac{w}{b} + \pi a^2\, w \frac{l}{b} = \frac{2\pi a^2}{b} A \tag{104}$$

when A is the physical area of the mesh.

[22] See footnote 18, page 15.

An expression for the volume of material required to produce A square meters of mesh having a reflection efficiency η_o can be found by combining equation (104) with equation (53):

$$\eta_o = \frac{1}{\left(1 + \frac{2\pi a^2 A}{60\pi^2 \sigma V a^2}\right)^2} = \frac{(30\pi\sigma V)^2}{(30\pi\sigma V + A)^2} \; . \qquad (105)$$

In order to simplify manipulations, let $\psi = 30\pi\sigma$. From equation (105) we have

$$\begin{aligned} \eta_o\,(\psi^2 V^2 + 2\psi A V + A^2) &= \psi^2 V^2 \\ \psi^2(\eta_o - 1)\,V^2 + 2\psi A \eta_o V + A^2 \eta_o &= 0. \end{aligned} \qquad (106)$$

Solving by the quadratic formula,

$$\begin{aligned} V &= \frac{-2\psi\eta_o A \pm \sqrt{(2\psi\eta_o A)^2 - 4A^2\eta_o\psi^2(\eta_o - 1)}}{2\psi^2(\eta_o - 1)} \\ &= \left[\frac{-\eta_o \pm \sqrt{\eta_o}}{\psi(\eta_o - 1)}\right] A . \end{aligned} \qquad (107)$$

The negative root of this expression has no physical significance. Since η_o is a number less than one, $\sqrt{\eta_o} > \eta_o$. Consequently, the negative sign on the radical is the only one which will give a positive value to the expression

$$V = \left[\frac{-\eta_o - \sqrt{\eta_o}}{\psi(\eta_o - 1)}\right] A = \frac{\eta_o + \sqrt{\eta_o}}{(1 - \eta_o)} \; \frac{A}{30\pi\sigma} \; . \qquad (108)$$

It will be noted that the volume of material required to produce a given area of netting with a certain reflection efficiency is independent of the dimensions. That is, in the region of the Gans formula, the same reflectivity can be obtained by using fine wires spaced close together or coarser wires spaced farther apart. Equation (108) also shows that the volume of material varies inversely as the conductivity, so that high conductivity materials should be used wherever possible.

An expression for the volume of material required to produce an effective reflecting area A_e when the netting is in the form of a flat sheet can be found by substituting equation (91) in equation (108):

$$\text{(flat sheet)}\; V = \frac{\eta_o + \sqrt{\eta_o}}{(1 - \eta_o)} \; \frac{1}{\sqrt{\eta_o}} \; \frac{A_e}{30\pi\sigma} \; . \qquad (109)$$

Similarly, the substitution of equation (98) in equation (108) gives the result for a spherical configuration:

$$\text{(sphere)}\; V = \frac{\eta_o + \sqrt{\eta_o}}{(1 - \eta_o)} \; \frac{16\pi}{\lambda^2 \eta_o} \; \frac{A_e^2}{30\pi\sigma} \; . \qquad (110)$$

Equations (100) and (108) give the results for a dihedral:

$$\text{(dihedral)}\; V = \frac{\eta_o + \sqrt{\eta_o}}{(1 - \eta_o)} \; \frac{(\eta_o + 1)}{\sqrt{2}\,\eta_o} \; \frac{A_e}{30\pi\sigma} \; . \qquad (111)$$

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Finally, from equations (103) and (108) we have

$$\text{(corner reflector)} \quad V = \frac{\eta_0 + \sqrt{\eta_0}}{(1-\eta_0)} \; \frac{2.59\,(2\,\eta_0 + 1)^{3/2}}{(3\,\eta_0)^{3/2}} \; \frac{A_e}{30\pi\sigma} \,. \qquad (112)$$

Rationalized MKS units have been used throughout the derivation, so the volume V is in cubic meters when A_e is in square meters and σ in mhos per meter cube. Tables give the resistivity of copper at 20°C as 1.77×10^{-6} ohm-cm. The conductivity is the reciprocal. Thus, $\sigma = 0.565 \times 10^{6}$ mho-cm.$^{-1}$ Converting to MKS units gives $\sigma = 5.65 \times 10^{7}$ mho-meter $^{-1}$. Similarly, for aluminum at 20°C $\sigma = 3.54 \times 10^{7}$ mho-meter^{-1}, and for silver at 18°C $\sigma = 6.15 \times 10^{7}$ mho-meter $^{-1}$.

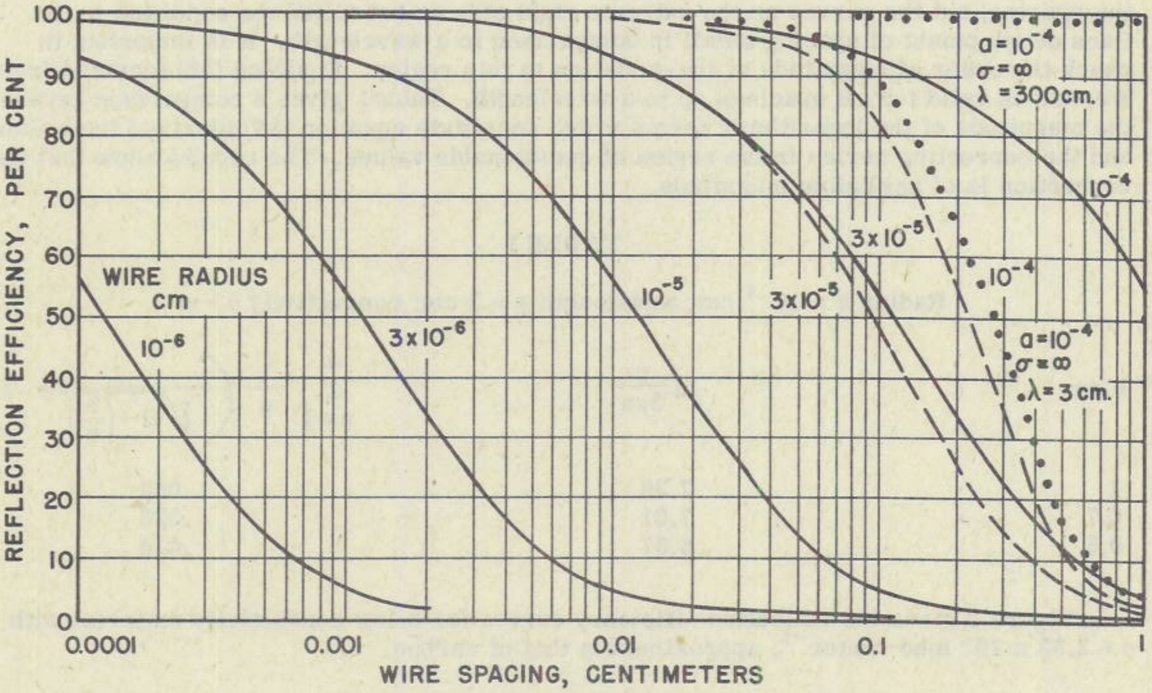

Fig. 2 - Reflection efficiency as a function of wire size and spacing for high conductivity metals (i.e. $\sigma = 5 \times 10^{7}$ mho-meter^{-1}). Solid curves are for $\lambda = 300$ cm and dashed curves are for $\lambda = 3$ cm. Dotted curves are for infinite-conductivity material

RESULTS

A series of curves have been plotted in Figure 2 showing the reflection efficiency of various size grid wires as a function of the spacing. A conductivity of 5×10^{7} mho-meters^{-1} has been assumed. It can be seen from the above that this figure is a good mean value to use to represent the good conducting materials. There may be some question about using the d-c conductivity for very high frequencies, but the work of Serin,[23]

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[23] B. Serin, Phys. Rev., 72, 1261-1262, (1947).

together with that of Pippard, Reuter, and Sondheimer,[24] indicates that this procedure is valid in the normal range of temperatures. The curves at the left of Figure 2 were calculated using equation (53). The curves on the right show how the situation changes when the approximate equation is no longer valid. The frequency limits used in the calculations were 10^8 cps (3 meters) and 10^{10} cps (3 cm). On the left side the curves are common to both of these frequencies, but on the right they separate and so the upper frequency curves have been dashed. These curves have been plotted by use of equations (41), (43), (45), and (46). Sample computations are seen in the Appendix. The change in the shape with increased frequency is due principally to the poorer utilization of the material because of skin effect. The curves for infinite conductivity (dotted) are based on equations (43) and (54), and the effect of the poorer utilization of the material is clearly demonstrated.

In all the curves of Figure 2, the radius of the wires is small in comparison with the spacing, but the curves on the extreme right side do not fulfill the condition in the Gans development of spacing small in comparison to a wavelength. It is important to check the order of magnitude of the deviation in this region. Equation (58) (derived from Wessel) is valid for all spacings up to a wavelength. Table1 gives a comparison between the magnitude of the logarithmic terms which constitute equation (54) (derived from Gans) and the correcting series in the region of questionable values. The results show that the correction is of negligible magnitude.

TABLE 1

Radius $a = 10^{-4}$ cm; wavelength $\lambda = 3$ cm; conductivity $\sigma = \infty$

b cm	$\ln \frac{b}{2\pi a}$	$\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{\sqrt{1-\left(\frac{b}{n}\right)^2}} - 1\right)$
1	7.36	.063
0.7	7.01	.028
0.5	6.67	.014

Figure 3 gives the reflection efficiency curves for a low conductivity material with $\sigma = 3.33 \times 10^5$ mho-meter^{-1}, approximately that of carbon.

Figure 4 is a plot of equations (109), (110), (111), and (112) for an assumed effective area of one square meter, a conductivity of $\sigma = 5 \times 10^7$ mho-meter^{-1}, and, in the case of equation (110), a wavelength of one meter.

Equation (109) has been rearranged in the following manner:

$$V = \frac{\eta_o + \sqrt{\eta_o}}{(1 - \eta_o)} \quad \frac{1}{\sqrt{\eta_o}} \quad \frac{A_e}{30\pi\sigma} \tag{109}$$

$$= \frac{\eta_o + \sqrt{\eta_o}}{(1 - \eta_o)} \quad \frac{1}{\sqrt{\eta_o}} \quad \frac{30\pi}{1} = \frac{\sigma V}{A_e} = (v\,\sigma) \tag{113}$$

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[24] A. B. Pippard, G.E.H. Reuter, and E. H. Sondheimer, Phys. Rev., 73, 920-921, (1948).

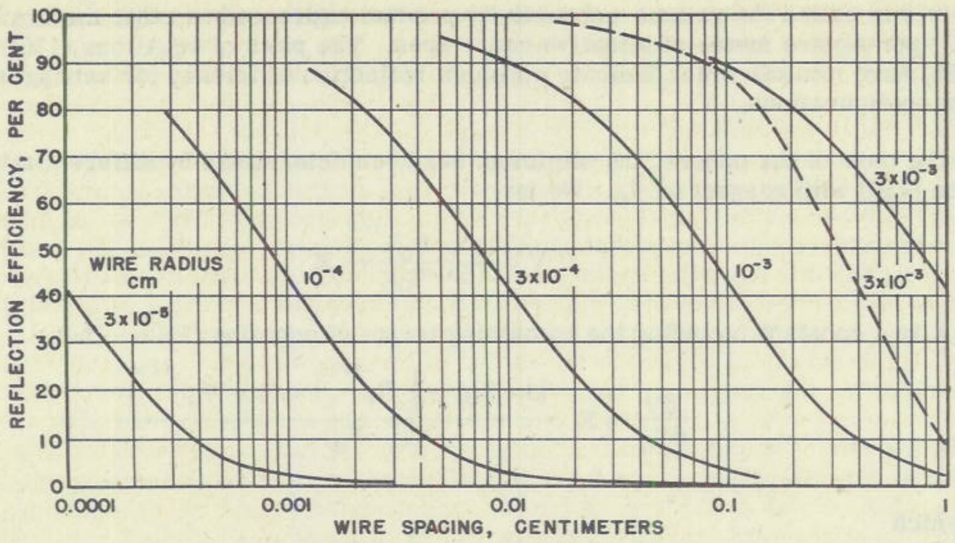

Fig. 3 - Reflection efficiency as a function of wire size and spacing for low conductivity materials (i.e., $\sigma = 3.33 \times 10^5$ mho-meter^{-1}). Solid curves are for λ = 300 cm and dashed curve is for λ = 3 cm

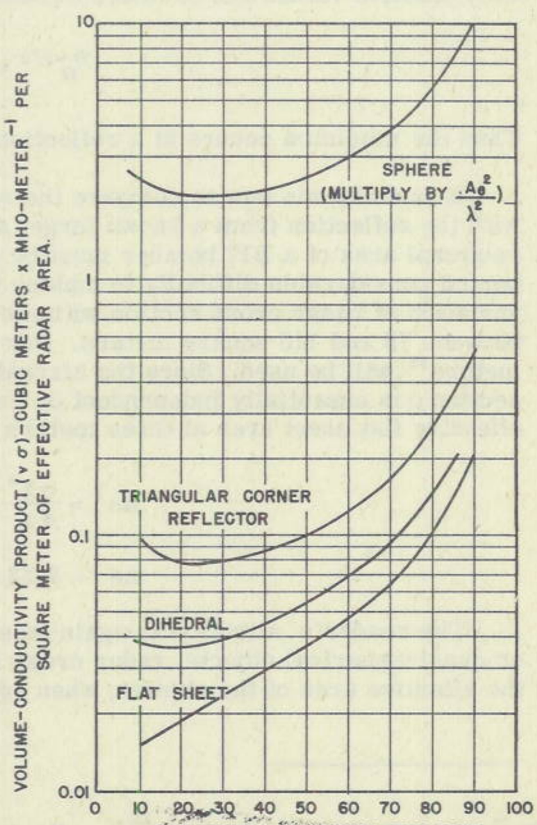

Fig. 4 - Volume-Conductivity Product Curves. To find the volume of material required to produce an echo area of A_e square meters, multiply by A_e and divide by σ, except in the case of the sphere. In the latter case, multiply by A_e/λ^2 and divide by σ.

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where $(v\sigma)$ is called the volume-conductivity product expressed in cubic meters X mho-meter^{-1} per square meter of effective radar area. The plots of equations (110), (111), and (112) have minima which indicate optimum reflection efficiency for netting in the various configurations.

In the case of the sphere, the minimum has been determined by differentiating equation (110) with respect to η_0. We let

$$V = \frac{1 + \eta_0^{-1/2}}{(1 - \eta_0)} K \tag{114}$$

where K is a constant including the remaining terms of equation (110). Then

$$\frac{dV}{d\eta_0} = K \frac{(1 - \eta_0)\left(-\frac{1}{2}\eta_0^{-3/2}\right) + (1 + \eta_0^{-1/2})}{(1 - \eta_0)} = 0 \tag{115}$$

from which

$$(\eta_0^{-1/2})^3 - 3(\eta_0^{-1/2}) - 2 = 0$$

$$(\eta_0^{-1/2} - 2)(\eta_0^{-1/2} + 1)(\eta_0^{-1/2} + 1) = 0. \tag{116}$$

Only positive values are of interest; hence

$$\eta_0^{-1/2} = 2 \text{ or } \eta_0 = \frac{1}{4}. \tag{117}$$

Thus the minimum occurs at a reflection efficiency of 25%.

It is desirable now to compare the reflection from a grid of metal such as aluminum with the reflection from a known target such as a B17 bomber. To do this the radar cross-sectional area of a B17 bomber must be determined. Independent investigators have reported considerable difficulty in making these measurements chiefly because of the extreme variation of radar cross section with the aspect of the aircraft. The spread of values is between 75 and 115 square meters. For the purposes of this report the value of 75 square meters[25] will be used. Since the aircraft surfaces are spherical in form, the radar cross section σ is essentially independent of frequency. From equation (85) the equivalent effective flat sheet area at three meters wavelength is

$$Ae^2 = \frac{\sigma\lambda^2}{4\pi} = \frac{75 \times 9}{4\pi} \tag{118}$$

$$Ae = 7.33 \text{ square meters.}$$

The reader's attention is again (see page 17) directed to the fact that, for spherical or quasi-spherical objects, radar cross section is independent of frequency even though the effective area of the objects, when equated to a flat sheet, is dependent thereon.

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[25] L. N. Ridenour, "Radar System Engineering," McGraw-Hill, New York (1947), p. 78.

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An assumption of reflection efficiency from the aluminum grid must be made. As can be seen from Figure 4, a flat sheet reflector has no minimum on its volume-conductivity product curve. However, there is a practical lower limit since for an extremely low value of volume-conductivity product, a flat sheet would become inordinately large. Thus a reflection efficiency of 25% yielding a volume-conductivity product of 0.021 is assumed.

The essential constants for aluminum are:

Conductivity at 20°C	$\sigma = 3.54 \times 10^7$ mho-meter^{-1}
Density	$d = 2.70$ grams/c.c.

Substitution of these values into equation (113) yields

$$V = (v\sigma)\frac{Ae}{\sigma} = \frac{0.021 \times 7.33}{3.54 \times 10^7} = 43.5 \times 10^{-10} \text{ cubic meters}$$

$$= 43.5 \times 10^{-4} \text{ c.c.}$$

$$\text{Weight} = dV = 2.70 \times 43.5 \times 10^{-4} = 11.74 \times 10^{-3} \text{ grams.}$$

This amount of aluminum in the form described would produce reflection equivalent to a B17 bomber. Since one ounce = 28.35 grams, reflection from one ounce of aluminum

$$= \frac{28.35}{11.74 \times 10^{-3}} = 2410 \text{ Flying Fortresses.}$$

CONCLUSIONS

As noted following the development of equation (108), the volume of material required to produce a given radar effective area becomes independent of the wire size and spacing. Consequently there is no advantage in striving for smaller wire sizes than those small enough to permit the use of the approximate Gans equation (53). From Figure 2, it can be seen that the transition from the exact to the approximate formula for good conductors occurs at a wire radius of 3×10^{-5} cm. The curves in Figure 4 indicate that the best utilization of material will occur if the reflection efficiency is made somewhere between 20 and 40%. Again from Figure 2, this reflection efficiency would give a wire spacing of from 1.5 to 3 mm for the 3×10^{-5} cm radius wires. Since equations (109) to (112) show that the volume of material required to simulate a given echo varies inversely as the conductivity, it is desirable to use materials of the highest possible conductivity.

ACKNOWLEDGMENT

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* * *

APPENDIX

Sample Computations of Reflection Efficiency Versus Wire Size and Spacing

For convenience in arriving at the relationship between reflection efficiency and the various parameters of a grid, sample computations are presented here using the exact and the approximate Gans formulae.

1. From Exact Gans Formula

We have the following equations from the body of the report:

$$k_1 a = \sqrt{\mu \sigma \omega a^2}\sqrt{-i} = r\sqrt{-i}; \qquad (45)$$

$$\tau_0 = -\frac{\mu}{2\mu_0}\,\frac{2}{r}\,\frac{b_0}{b_1}\,\underline{|0.5^{\llcorner} - (\beta_0 - \beta_1)}; \qquad (46)$$

$$\Delta = \frac{2b}{\lambda_0}\left(\ln\frac{b}{2\pi a} + \tau_0\right) = u + jv; \text{ and} \qquad (41)$$

$$\eta_0 = \frac{1}{(1-v)^2 + u^2}. \qquad (43)$$

Reflection from high conductivity metal will be assumed where

$$\sigma = 5 \times 10^7 \text{ mho-meter}^{-1}.$$

Radius of grid wire:

$$a = 10^{-7} \text{ meters.}$$

Also

$$f = 10^{10} \text{ cycles per second;}$$

$$\mu = 4\pi \times 10^{-7} \text{ henry/meter.}$$

Substitution of the above into equation (45) yields

$$r = \sqrt{4\pi \times 10^{-7} \times 5 \times 10^{7} \times 2\pi \times 10^{10} \times 10^{-14}}$$

$$= \sqrt{3.96 \times 10^{-2}} = 0.199.$$

By means of the Jahnke and Emde tables (reference 14) and the value of r, the functions of equation (46) are determined.

$$\tau_o = -103.01 \quad \underline{\big|\, 0.5^{\llcorner} + 0.5024^{\llcorner}}$$

$$= -51.505 \quad \underline{\big|\, 1.0024^{\llcorner}} = -51.505 \quad \underline{\big|\, 90.216}$$

$$= 0.208 - j51.504.$$

Assume a wire spacing:

$$b = 0.0001 \text{ meters.}$$

Then

$$\frac{2b}{\lambda_o} = \frac{2 \times 0.0001}{0.03} = 0.006667$$

$$\ln \frac{b}{2\pi a} = \ln \frac{0.0001}{2\pi \times 10^{-7}} = 5.069.$$

Substituting into equation (41) gives

$$\Delta = 0.006667\,(5.069 + 0.208 - j51.504)$$

$$= 0.006667\,(5.277 - j51.504)$$

$$= 0.03518 - j0.3434 = u + jv,$$

and

$$\eta_o = \frac{1}{(1 + 0.3434)^2 + (0.03518)^2} = 55.4\%.$$

2. From Approximate Gans Formula

This relation appears as equation (53) of the report where

$$\eta_o = \frac{1}{\left(1 + \frac{b}{60\pi^2 \sigma a^2}\right)^2}.$$

Using the same values for b, a, and σ as used in the exact formula, substitution is made into (53):

$$\eta_o = \frac{1}{\left(1 + \frac{10^{-4}}{60\pi^2 \times 5 \times 10^7 \times 10^{-14}}\right)^2}$$

$$= 0.558 \text{ or } 55.8\%.$$

Extension of these calculations leads to the curves of Figures 2 and 3 (see text).

* * *