**NRL REPORT 3503**

FR-3503

# FREQUENCY AND BANDWIDTH TOLERANCES FOR A TRANSPONDOR GO-NO-GO WAVEMETER


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**June 9, 1949**

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CONTENTS

## ABSTRACT

Consideration is given to the frequency and bandwidth tolerances of a wavemeter to be used as a part of a go-no-go transpondor transmitter tester in a pulse-operated interrogator-beacon system. If the bandwidth is too narrow, an excessive number of transmitters may be rejected while on the other hand, if the bandwidth is too wide, the instrument may not reject enough of the transmitters which should be rejected. It is shown that a statistical treatment of system frequency and bandwidth tolerances found in equipment specifications provides a method of designating suitable go-no-go wavemeter tolerances. The presentation includes an example which gives recommended wavemeter frequency and bandwidth tolerances for use with a typical interrogator-beacon system.

## PROBLEM STATUS

This is an interim report; work is continuing.

## AUTHORIZATION

NRL Problem R03-06R: BuShips Problem S1234X-S

# FREQUENCY AND BANDWIDTH TOLERANCES FOR A TRANSPONDOR GO-NO-GO WAVEMETER

## INTRODUCTION

A "go-no-go" tester for transpondors in a pulse-operated interrogator-beacon system must, among other things, test the frequency of the transpondor's transmitter. This portion of the "go-no-go" test requires a wavemeter which will reject all transmitters operating outside of certain frequency limits. The problem at hand is to determine these limits and to choose wavemeter tolerances so that the instrument will perform its function without rejecting an excessive number of transpondors which might otherwise have performed satisfactorily in the system. The problem is complicated by the fact that transpondor transmitter frequencies, responsor receiver frequencies and receiver bandwidths are not necessarily constant values from one equipment to another or from one instant to another in a given equipment. Further, the "go-no-go" wavemeter itself will resemble the responsor receivers in that its frequency and bandwidth will not be perfectly constant. A statistical treatment of various tolerances given in system specifications provides the relationships which lead to an appropriate set of frequency limits for the wavemeter to define. Further statistical consideration of frequency and bandwidth tolerances in the wavemeter itself leads to a suitable choice of its tolerances.

## RESULTS

Consideration is given to the percentage of transmitter rejects and the probability of a given reduction in maximum range on the reply path. The relationships derived are applied to a practical example of an interrogator-beacon system. The system characteristics are assumed to be:

*Transpondor transmitter frequency tolerance, ±3 Mc,*

*Responsor receiver frequency tolerance, ±2 Mc,*

*Responsor receiver bandwidth range, 8 Mc to 11 Mc,*

*Responsor receiver maximum bandwidth decrease factor, 1Mc.*

For these characteristics, it is shown that the percentage of transmitter rejects, which would otherwise have operated satisfactorily in the system, is less than 1 percent for any go-no-go wavemeter with a constant bandwidth greater than approximately 3 Mc centered exactly on the assigned frequency. As a definition is needed for "failure" on the reply path due to frequency or bandwidth variations in equipments which are passed by the go-no-go wavemeter, we arbitrarily define this failure as occurring when there is at least a 50 percent reduction in maximum range. The probability of such failure is certain to be less than 0.005 for any constant wavemeter bandwidth less than approximately 8 Mc centered exactly on the assigned frequency. In order to allow the greatest tolerance on wavemeter bandwidth and frequency variations, an average bandwidth of 5.5 Mc is chosen. The tolerance on this bandwidth is a function of transmitter frequency tolerance and wavemeter center frequency tolerance. The former tolerance is fixed for the system by transmitter specifications, hence the bandwidth tolerance is known for any given wavemeter center frequency tolerance. For example, if the wavemeter center frequency is accurate to within ± 0.1 Mc, then its bandwidth must be maintained at 5.5 Mc ± 2.49 Mc if the above maximum reject and probability values are not to be exceeded. If, on the other hand, the wavemeter center frequency is only accurate to, say, ± 1.0 Mc, then the bandwidth must be maintained at 5.5 Mc ± 1.50 Mc for the same results.

## STATISTICAL APPROACH

The problem is initially simplified by assuming that all receivers in the system have ideal rectangular pass-band characteristics. One can thus think of system failure on the reply path, due to frequency or bandwidth variations, as any situation in which a transpondor transmitter replies to an interrogator-responsor on a frequency outside of the responsor receiver's pass-band. For any system with given specifications, it is necessary to determine the probability of such a failure occurring. This probability can be determined by considering the transpondor transmitter frequency tolerance and the responsor receiver frequency and bandwidth toleranc s. Statistical combination of the responsor receiver frequency and bandwidth distributions gives the distribution of the upper and lower limits of receiver pass-bands in the system. The application of fundamental ideas in probability theory to the transmitter frequency distribution and the distribution of the receiver pass-band limits leads to a value for the probability of system failure.

The system failures thus far discussed are only those resulting from reasonable combinations of certain specified service conditions. That is, the calculated probability of failure relates only to a system under the influence of natural random forces and does not account for the unpredictable "human element." For instance, "extraordinary" causes such as unnecessary jarring, careless or unskilled detuning or poor maintenance are factors which would tend to increase the probability of failure above the ideal calculated value. A go-no-go wavemeter properly designed, while unnecessary for a perfectly maintained system, will reject transpondors outside of a certain bandwidth and thus eliminate the equipments subjected to these "extraordinary" causes which would tend to raise the probability of failure. An upper limit for the pass-band of such a wavemeter is selected on the basis of the largest tolerable probability of failure. The lower limit of such a wavemeter pass-band is determined by considering the percentage of transmitter rejects. For maximum allowable tolerance on wavemeter center frequency accuracy and bandwidth, an average wavemeter bandwidth is chosen half way between these limits. A statistical combination of wavemeter frequency distribution and wavemeter bandwidth distribution gives a resultant distribution of wavemeter pass-band limits. This distribution is fixed for any system with a fixed transpondor transmitter specification. Thus for any wavemeter frequency tolerance, one can obtain a corresponding bandwidth tolerance.

Considering a system with receivers having practical bandpass characteristics, it is no longer possible to regard system failure as definitely happening or not happening. Since the pass-band may be defined by the frequencies at which the receiver sensitivity is, say, down 6 db from its maximum value, we can only say from this that any received signal outside these limits will have an extreme range which is at least 50 percent below its maximum possible value. In practice, therefore, one does not determine a probability of failure, but rather a probability that the extreme range at which a transpondor will reply to a given interrogator will be reduced at least 50 percent below its maximum possible value. Let us consider the following example for clarification of the above statement. Assume a transponder transmitter with a given power output. If it transmits on a frequency corresponding to the point of maximum sensitivity of a responsor receiver, it can then be received satisfactorily at all ranges out to a certain extreme range, $R_M$. This is the maximum possible extreme range for this transmitter replying to the given responsor. If, on the other hand, the transmitter replies on any other frequency corresponding to a less sensitive point on the receiver's response curve, it can be received satisfactorily at all ranges out to a certain extreme range, $R_E$, which is less than $R_M$, the magnitude of the difference $R_M$-$R_E$ depending upon the difference in frequency. If the transmitter frequency lies outside of the receiver's bandwidth as specified by points 6 db down on its sensitivity curve, then

$$R_E \leqslant \tfrac{1}{2} R_M.$$

The practical consideration, then, is to determine the probability that

$$R_E \leqslant \tfrac{1}{2} R_M.$$

## A STATISTICAL VIEW OF SYSTEM "FAILURE"

### Necessary Assumption on Tolerances

A method of calculating suitable tolerance limits for transmitter or receiver frequency, or receiver bandwidth has been worked out by L. S. Schwartz;[1] the general idea is briefly as follows. Laboratory measurements were made on typical equipments to determine individual tolerances on equipment frequencies as certain parameters were varied over extreme ranges. Based upon the assumption that in actual operation the various parameters will vary randomly with respect to one another, it was shown that under the worst possible situations, and if the random parameters number at least four, the resultant distribution of equipment frequencies is closely approximated by the normal law. A method was given for calculating the standard deviation,

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[1] *Schwartz, L. S., Statistical Methods in the Design and Development of Electronic Systems, NRL Report No. R-3111, July 1947*

$\sigma$, of this normal distribution from data on the individual tolerances. For a normal distribution there is at least a 99.73 percent chance that the variable lies within ±3 $\sigma$ of the mean. This criterion was chosen as a basis for writing the tolerance limits in any specification. That is, from measured data on individual tolerances, one can calculate the standard deviation, $\sigma$, of the resultant frequency distribution and thus can set the overall tolerance limits at ±3 $\sigma$. Such a choice assures at least 99.73 percent of the time of operation the frequency of any equipment will be within ±3 $\sigma$ of the mean.

In the statistical treatment of this problem it is assumed that specifications for transmitter frequency tolerance, receiver frequency tolerance and receiver bandwidth have been written according to the statistical method outlined above. Thus, if the specification states that transmitters shall not deviate more than, say, ±3 Mc from the system frequency, then transmitter frequencies during operation will always be normally distributed about the system frequency and the standard deviation of this distribution will be 1 Mc. If receiver bandwidth specifications have been written according to a similar method, then specified bandwidth limits of, say, 8 to 11 Mc actually mean that receiver bandwidths during operation will be normally distributed about 9.5 Mc, and the standard deviation of this distribution is 0.5 Mc.

## Distribution of Transpondor Transmitter Frequencies

Let $F_o$ be the assigned frequency for a given interrogator-beacon system. That is, assume that transpondors are manufactured and maintained with the intention of setting the transmitter frequencies on $F_o$. Assume also that transpondor specifications state that transmitters must operate with a given maximum allowable frequency drift, for example ±3 Mc, due to all reasonable combinations of certain specified service conditions. Then, if an observation be made on any transmitter in the system at any instant, there is at least a 99.73 percent chance that its frequency, $F_t$, lies somewhere within the range $F_o$ ±3 Mc. If we denote the difference between a transmitter frequency at any instant and the assigned frequency by $f_t$, that is, let

$$f_t = F_t - F_o , \tag{1}$$

then an equivalent statement of the above transmitter specification is

$$-3 \text{ Mc} \leqslant f_t \leqslant 3 \text{ Mc}. \tag{2}$$

The distribution of a large number of observations of transmitter frequencies, $f_t$, between the limits ±3 Mc may be considered to be the resultant of at least four component frequency distributions caused by random variations of as many independent parameters. In general, these component distributions are unknown. It has been shown[2] that if they are assumed to be rectangular, the resultant distribution is pessimistic and closely approximates the normal law. On this basis we may regard $f_t$ as a random variable distributed about zero according to the relationship

[2] *Schwartz, op. cit. pp. 23-24.*

$$P(f_t)df = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{f^2}{2\sigma_t^2}} df, \qquad (3)$$

where $P(f_t)df$ is the probability that $f_t$ lies within the differential range $f \pm df/2$. The quantity $\sigma_t$ is the standard deviation of $f_t$ given by

$$\sigma_t^2 = \overline{[f_t - \overline{f_t}]^2} = \overline{f_t^2} . \qquad (4)$$

A plot of $P(f_t)$ is shown in Figure 1.

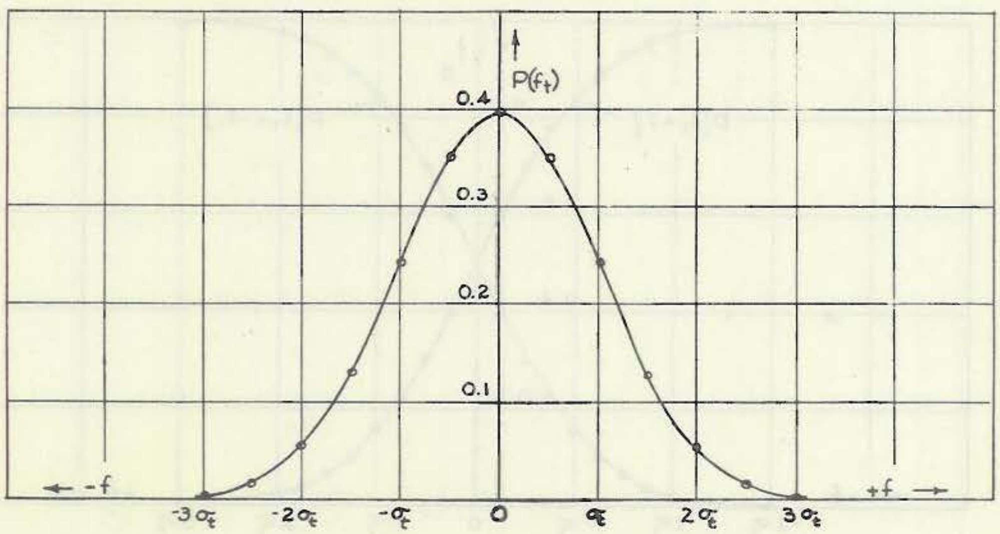


*Figure 1 - The distribution of transmitter frequency error, $f_t$, about the average, $\overline{f_t} = o$, during systems operation*

It may be seen from Figure 1 and equation (3) that the probability, $P[f_1 \leq f_t \leq f_2]$, that $f_t$ lies between the limits $f_1$ and $f_2$ is

$$P[f_1 \leq f_t \leq f_2] = \frac{1}{\sqrt{2\pi}\sigma_t} \int_{f_1}^{f_2} e^{-\frac{f^2}{2\sigma_t^2}} df. \qquad (5)$$

The value of this integral for any set of limits $\pm f_i$ is readily obtained from the probability integral tables. The probability that $f_t$ lies outside the limits $\pm f_i$ is, of course, $1 - P[-f_i \leq f_t \leq f_i]$, and the probability that $f_t$ exceeds $f_i$ in the positive direction is

$$P[f_t > f_i] = \tfrac{1}{2}\left[1 - \frac{1}{\sqrt{2\pi}\sigma_t}\int_{-f_i}^{f_i} e^{-\frac{f^2}{2\sigma_t^2}}\, df\right]. \tag{6}$$

The probability that $f_t$ exceeds $-f_i$ in the negative direction is also given by equation (6). That is, $P[f_t > f_i]$ and $P[f_t < -f_i]$ are symmetrical functions on either side of zero. A plot of these two functions is given in Figure 2.

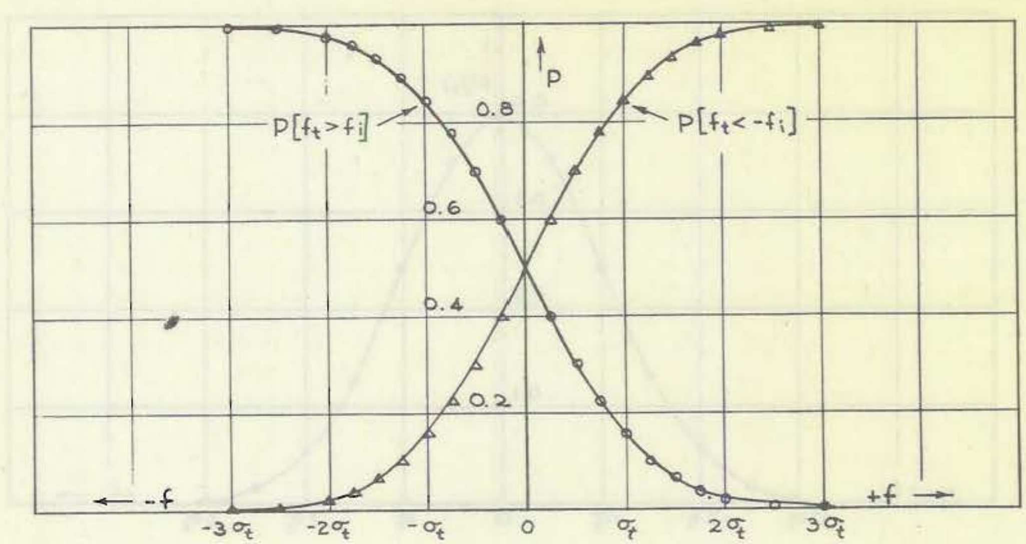


*Figure 2- The probability that $f_t$ exceeds $+f_i$ in the positive direction or $-f_i$ in the negative direction*

From probability tables, we find the probability that $f_t$ lies outside the limits $\pm 3\sigma_t$ is less than 0.0027. That is, there is at least a 99.73 percent chance that $f_t$ will be within $0 \pm 3\sigma_t$ for a normal distribution. Let us assume that the interrogator-beacon system being considered is manufactured and maintained such that any value $f_t$ has at least a 99.73 percent chance of lying within the range 0 ± 3 Mc. Then,

$$\left.\begin{aligned} 3\sigma_t &= 3 \text{ Mc} \\ \sigma_t &= 1 \text{ Mc.} \end{aligned}\right\} \tag{7}$$

Equations (5) and (6) will subsequently be used in determining the percentage of transpondor rejects and the probability of system failure. Before this can be done, however, similar statistical consideration of certain receiver characteristics is required.

Distribution of Responsor Receiver Frequencies

We assume that responsor receivers are manufactured and maintained with the intention of setting the receiver center frequency exactly on $F_o$. Assume, also, that the responsor specifications state that receivers must operate with a given maximum allowable frequency drift, for example, ±2 Mc, due to all reasonable combinations of certain specified service conditions. Then, if an observation be made on any receiver in the system at any instant, there is at least a 99.73 percent chance that its frequency, $F_r$, lies somewhere within the range $F_o$ ± 2 Mc. If we denote the difference between a receiver frequency at any instant and the assigned frequency by $f_r$, that is, let

$$f_r = F_r - F_o, \tag{8}$$

then an equivalent statement of the above receiver specification is

$$-2 \text{ Mc} \leq f_r \leq 2 \text{ Mc}. \tag{9}$$

Similar to the case of transmitter frequencies, $f_r$ may be regarded as a random variable distributed about zero according to the normal law:

$$P(f_r)df = \frac{1}{\sqrt{2\pi}\sigma_r} e^{-\frac{f^2}{2\sigma_r^2}} df, \tag{10}$$

where $P(f_r)df$ is the probability that $f_r$ lies within the differential range f ± df/2, and $\sigma_r$ is the standard deviation of this distribution given by

$$\sigma_r^2 = \overline{f_r^2}. \tag{11}$$

If we make an assumption for the variable $f_r$ similar to the one made for $f_t$, that is, assume any observation of receiver frequency has a 99.73 percent chance that $f_r$ is within the limits 0 ± 2 Mc, then

$$\left.\begin{aligned} 3\sigma_r &= 2 \\ \sigma_r &= 2/3 \end{aligned}\right\} . \tag{12}$$

A plot of equation (10) is similar to Figure 1, except the abscissa are measured in terms of $\sigma_r$. Receiver center frequency is the first characteristic for which a statistical consideration is required. The second statistical treatment applies to receiver bandwidth.

Distribution of Responsor Receiver Bandwidths

Let us assume that responsor receivers are manufactured and maintained with the intention of having over-all bandwidths, B, somewhere within certain specified limits set by the specifications. For example, suppose the specifications require the bandwidths of receivers at the time of manufacture or maintenance, and before being subjected to service conditions, must lie between 8 Mc and 11 Mc. Let us also assume that there are at least four independent random parameters which cause the distribution of B between these limits. Then B may be regarded as a random variable distributed about its average value $\overline{B}$, according to the normal law,

$$P(B)dB = \frac{1}{\sqrt{2\pi}\sigma_B} e^{-\frac{[B-\overline{B}]^2}{2\sigma_B^2}} dB. \qquad (13)$$

where P(B)dB is the probability that a receiver bandwidth lies within the differential range B ± dB/2. The mean bandwidth $\overline{B}$ for the example being considered is

$$\overline{B} = \frac{8 + 11}{2} = 9.5 \text{ Mc}, \qquad (14)$$

and $\sigma_B$ is the standard deviation for the distribution of B. If specifications are being met, it is reasonable to assume that any receiver bandwidth B has at least a 99.73 percent chance of being within the range 8 to 11 Mc before the receiver is subjected to service conditions. This is equivalent to saying that any value of B has at least a 99.73 percent chance of lying in the range 9.5 Mc ± 1.5 Mc. Hence,

$$\begin{aligned} 3\sigma_B &= 1.5 \text{ Mc}, \\ \sigma_B &= 0.5 \text{ Mc}. \end{aligned} \qquad (15)$$

This distribution of values of B will not necessarily remain unchanged after receivers are subjected to service conditions. Several independent random parameters will make B a variable with time for any one receiver. There are at least four of these parameters and hence $\Delta$ B may be regarded as a random variable ($\Delta$ B varies with time during service) distributed about zero according to the normal law:

$$P(\Delta B)\,dB = \frac{1}{\sqrt{2\pi}\,\sigma_\Delta} e^{-\frac{(\Delta B)^2}{2\sigma_\Delta^2}}\,dB, \tag{16}$$

where $P(\Delta B)dB$ is the probability that a bandwidth change lies in the differential range $\Delta B \pm dB/2$, and $\sigma_\Delta$ is the standard deviation of the distribution of $\Delta B$. Suppose the receiver specifications also impose a maximum allowable bandwidth change, $\Delta B_{Max}$. If specifications are being met, it is reasonable to assume that any bandwidth change, $\Delta B$, during operation has at least a 99.73 percent chance of being within the range $0 \pm \Delta B_{Max}$. For example, if specifications set $\Delta B_{Max}$ = 1 Mc, then it is reasonable to assume that any bandwidth change $\Delta$ B has at least a 99.73 percent chance of being within the range 0 ± 1 Mc. For this case

$$\left.\begin{aligned} 3\sigma_\Delta &= 1 \text{ Mc} \\ \sigma_\Delta &= 1/3 \text{ Mc} \end{aligned}\right\}. \tag{17}$$

The resultant distribution of receiver bandwidths at any instant during system operation arises from the component distributions of receiver bandwidth before service conditions are imposed, equation (13), and receiver bandwidth change during system operation, equation (16). Let us designate the observation of any receiver bandwidth during system operation by $B_R$. Then $B_R$ is a random variable distributed about an average value and is a resultant of all of the parameters which go to produce the two component distributions, i.e., distributions of B and $\Delta$ B. The distribution of $B_R$ is obtained by the superposition of these two component distributions. From the laws of probability, if two normal distributions are superimposed, the resultant distribution is also normal,

$$P(B_R)\,dB = \frac{1}{\sqrt{2\pi}\,\sigma_\beta} e^{-\frac{[B_R-\bar{B}]^2}{2\sigma_\beta^2}}\,dB, \tag{18}$$

where $P(B_R)dB$ is the probability that a receiver bandwidth at any instant during system operation lies within the differential range B ± dB/2. The standard deviation, $\sigma_\beta$, of the distribution of $B_R$ is related to the standard deviations of the component distributions by the relationship

$$\sigma_\beta^2 = \sigma_B^2 + \sigma_\Delta^2 \ . \tag{19}$$

Probability theory also relates the average values as follows:

$$\overline{B_R} = \overline{B} + \overline{\Delta B} \ . \tag{20}$$

For the example being considered, equations (19) and (20) give

$$\left.\begin{aligned} \sigma_\beta^2 &= (1/2)^2 + (1/3)^2 = 13/16 \\ \sigma_\beta &= 0.6 \text{ Mc} \\ \overline{B_R} &= 9.5 + 0 = 9.5 \text{ Mc.} \end{aligned}\right\} \tag{21}$$

It will presently be apparent, however, that we are concerned with values of $B_R/2$ instead of $B_R$. For any observation of a receiver bandwidth $B_R$, there is a corresponding value of half-bandwidth, $B_R/2$. The parameter $B_R/2$ is therefore a random variable distributed about its mean, $\overline{B_R}/2$, according to the normal law,

$$P\left[\frac{B_R}{2}\right] dB = \frac{1}{\sqrt{2\pi}\sigma_\alpha} e^{\frac{\left[\frac{B_R}{2} - \frac{\overline{B_R}}{2}\right]^2}{2\sigma_\alpha^2}} \ dB \tag{22}$$

where $P(B_R/2)dB$ is the probability that $B_R/2$ lies within the differential range $B_R/2 \pm dB/2$, and $\sigma_\alpha$ is the standard deviation for the distribution of $B_R/2$. For the example being considered

$$\left.\begin{aligned} \frac{\overline{B_R}}{2} &= \frac{9.5}{2} = 4.75 \text{ Mc} \\ \sigma_\alpha &= \frac{\sigma_\beta}{2} = 0.3 \text{ Mc} \end{aligned}\right\} \tag{23}$$

## Distribution of Responsor Receiver Bandwidth Limits

For simplicity in arriving at the ultimate relationships between tolerances and system failure it is necessary to assume that all receiver bandpass characteristics are rectangular and symmetrical about their mid-frequencies. That is, Figure 3, any signal falling inside the band limits is received, and any signal outside these limits is not received.

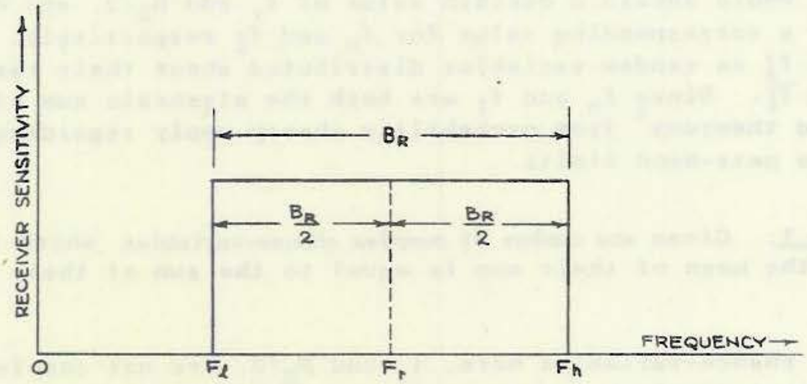


*Figure 3- Notation used in connection with the assumption of rectangular band-pass characteristics*

The lower frequency limit of a receiver pass-band is designated as $F_l$ and the upper limit is designated as $F_h$. Then the above assumption states that

$$F_h = F_r + \frac{B_R}{2}$$

and

$$F_l = F_r - \frac{B_R}{2} \quad . \qquad (24)$$

Let us define

$$\left.\begin{aligned} f_h &= F_h - F_o \\ f_l &= F_l - F_o \end{aligned}\right\} . \qquad (25)$$

Then from equations (8), (24) and (25):

$$\left.\begin{aligned} f_h &= F_r + \frac{B_R}{2} - F_o \\ f_h &= f_r + \frac{B_R}{2} \end{aligned}\right\} . \qquad (26)$$

$$\left.\begin{aligned} f_l &= F_r - \frac{B_R}{2} - F_o \\ f_l &= f_r - \frac{B_R}{2} \end{aligned}\right\} . \tag{27}$$

For an observation of any particular receiver at any instant during system operation one would obtain a certain value of $f_r$ and $B_R/2$, and hence from (26) and (27) a corresponding value for $f_h$ and $f_l$ respectively. We may regard $f_h$ and $f_l$ as random variables distributed about their respective means, $\overline{f_h}$ and $\overline{f_l}$. Since[3] $f_h$ and $f_l$ are both the algebraic sum of two random variables, two theorems from probability theory apply regarding the distribution of these pass-band limits.

Theorem 1: Given any number of complex chance-variables which need not be independent, the mean of their sum is equal to the sum of their individual means.

The two chance-variables here, $f_r$ and $B_R/2$, are not complex, but the theorem applies to real chance-variables by specialization. From theorem 1,

$$\left.\begin{aligned} \overline{f_h} &= \overline{f_r} + \overline{\frac{B_R}{2}} \\ \overline{f_l} &= \overline{f_r} - \overline{\frac{B_R}{2}} \end{aligned}\right\} . \tag{28}$$

For the example being considered:

$$\left.\begin{aligned} \overline{f_h} &= 0 + 4.75 = 4.75 \text{ Mc} \\ \overline{f_l} &= 0 - 4.75 = -4.75 \text{ Mc} \end{aligned}\right\} . \tag{29}$$

Theorem 2: If any number of complex chance-variables are independent, and if not more than one is of nonzero mean value, then the mean of the squared value of their sum is equal to the sum of the means of their individual squared values.

In order for this theorem to apply we must assume that the parameters which cause variations in receiver frequency are independent of those which cause variations in receiver bandwidth. From theorem 2:

$$\sigma_h^{\ 2} = \sigma_l^{\ 2} = \sigma_r^{\ 2} + \sigma_a^{\ 2} , \tag{30}$$

where $\sigma_h$ and $\sigma_l$ are the standard deviations for the distributions of $f_h$ and $f_l$, respectively. These distributions are also normal and are written

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[3] *Hoyt, R.S., Probability Theory and Telephone Transmission Engineering, The Bell System Technical Journal, Vol. 12, pp. 35-75, January 1933*

$$P(f_h)df = \frac{1}{\sqrt{2\pi}\sigma_h} e^{-\frac{[f-\overline{f_h}]^2}{2\sigma_h^2}} df, \quad (31)$$

and

$$P(f_l)df = \frac{1}{\sqrt{2\pi}\sigma_l} e^{-\frac{[f-\overline{f_l}]^2}{2\sigma_l^2}} df. \quad (32)$$

$P(f_h)df$ is the probability that a receiver pass-band upper limit will lie within the differential limits $f \pm df/2$, and $P(f_l)df$ is a similar probability related to a receiver pass-band lower limit. For the example being considered:

$$\sigma_h^2 = \sigma_l^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{3}{10}\right)^2 = \frac{481}{900} = .534$$

$$\sigma_h = \sigma_l = .73 \text{ Mc}$$

$$3\sigma_h = 3\sigma_l = 2.19 \text{ Mc}. \quad (33)$$

From equations (29), (31), (32), and (33) we may conclude that for an observation of the pass-band limits of any receiver operating in the system at any instant, there is at least a 99.73 percent chance that the upper limit $f_h$, will lie within the range 4.75 ± 2.19 Mc, and the lower limit, $f_l$, will lie within the range -4.75 ± 2.19 Mc. Figure 4 is a plot of equations (31) and (32), and illustrates the meaning of the above relationships graphically.

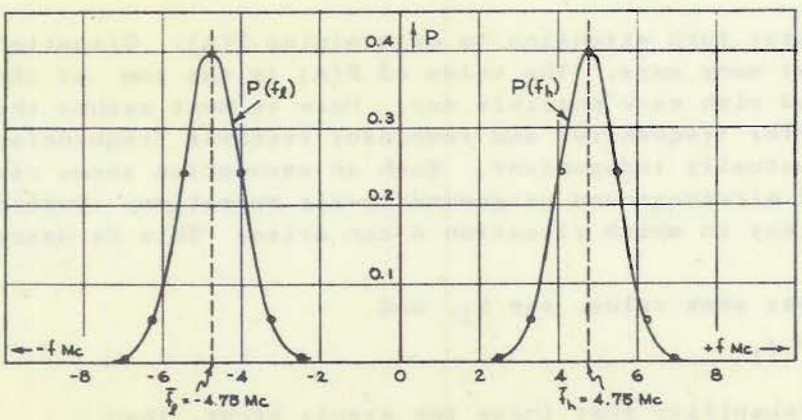


*Figure 4- Probability distributions of $f_h$ and $f_l$; the upper and lower limits, respectively, of responsor receiver pass-bands*

Definition of System "Failure"

With the initial assumption of rectangular receiver pass-bands, a definite failure in system operation due to frequency and bandwidth variations may be said to occur whenever a transpondor which is being interrogated emits a reply on a frequency which lies outside the pass-band of the receiver on the I-R which is doing the interrogating. This failure on the reply path will be observed whenever either of the following two situations occur:

$$\left.\begin{array}{ll}\text{Situation A:} & F_t > F_h \\ \text{Situation B:} & F_t < F_l\end{array}\right\}, \tag{34}$$

where $F_t$ is the frequency of transpondor transmitter and $F_h$ and $F_l$ are the upper and lower limits, respectively, of the responsor receiver pass-band. Recalling equations (1) and (25), situations A and B may also be described:

$$\left.\begin{array}{ll}\text{Situation A:} & f_t > f_h \\ \text{Situation B:} & f_t < f_l\end{array}\right\}. \tag{35}$$

Probability of System "Failure"

Let $P_F$ be the probability that system "failure" occurs on the reply path because of frequency or bandwidth variations. Let P(A) and P(B) be the probabilities that situation A and situation B occur, respectively. That is, P(A) is the probability that $f_t > f_h$ and P(B) is the probability that $f_t < f_l$. Since situation A and situation B are mutually exclusive,

$$P_F = P(A) + P(B). \tag{36}$$

Let us first turn attention to determining P(A). Situation (A) can arise in a great many ways. The value of P(A) is the sum of the probabilities associated with each possible way. Here we must assume that transpondor transmitter frequencies and responsor receiver frequencies and bandwidth are mutually independent. Such an assumption seems reasonable for the case of air-to-ground or ground-to-air operation. Consider the first possible way in which situation A can arise. This is described when

(1) $f_h$ has some value, say $f_1$, and

(2) $f_t > f_1$.

If $P_1$ is the probability that these two events occur, then

$$P_1 = [P(1)] \times [P(2)] \tag{37}$$

where P(1) is the probability that $f_h$ lies in the differential range $f_1 \pm df/2$; and where P(2) is the probability that $f_t > f_1$. From equations (6) and (31):

$$P_1 = \left[ \frac{1}{\sqrt{2\pi}\sigma_h} e^{-\frac{[f_1 - \bar{f}_h]^2}{2\sigma_h^2}} df \right] \times \tfrac{1}{2} \left[ 1 - \frac{1}{\sqrt{2\pi}\sigma_t} \int_{-f_1}^{f_1} e^{-\frac{f^2}{2\sigma_t^2}} df \right]. \qquad (38)$$

The second possible way in which situation A can arise is described when

(1) $f_h$ has some other value, say, $f_2$, and

(2) $f_t > f_2$.

If $P_2$ is the probability that these two events occur, then

$$P_2 = \left[ \frac{1}{\sqrt{2\pi}\sigma_h} e^{-\frac{[f_2 - \bar{f}_h]^2}{2\sigma_h^2}} df \right] \times \tfrac{1}{2} \left[ 1 - \frac{1}{\sqrt{2\pi}\sigma_t} \int_{-f_2}^{f_2} e^{-\frac{f^2}{2\sigma_t^2}} df \right]. \qquad (39)$$

In fact there is an infinite number of mutually exclusive ways in which situation A can arise, one for each possible value that can be assigned to $f_h$. The probability P(A) is therefore an infinite series

$$P(A) = P_1 + P_2 + \ldots = \sum_{i=1}^{\infty} P_i \qquad (40)$$

where $P_i$ is given by

$$P_i = \left[ \frac{1}{\sqrt{2\pi}\sigma_h} e^{-\frac{[f_i - \bar{f}_h]^2}{2\sigma_h^2}} df \right] \times \tfrac{1}{2} \left[ 1 - \frac{1}{\sqrt{2\pi}\sigma_t} \int_{-f_i}^{f_i} e^{-\frac{f^2}{2\sigma_t^2}} df \right]. \qquad (41)$$

Each of the factors of (41) can readily be obtained from the probability tables for any value of $f_i$.

Figure 5 is a plot of four different functions as follows:

Curve (A) Plots $P(f_h)$, equation (31)
Curve (B) Plots $P(f_l)$, equation (32)
Curve (C) Plots $P(f_t>f_i)$, equation (6)
Curve (D) Plots $P(f_t<-f_i)$, equation (6).

Figure 5 is, in fact, a superposition of Figure 2 and Figure 4 drawn to the same scale. It is clear from these curves that $P[f_l]$ and $P[f_h]$ are symmetrical and that $P[f_t<f_i]$ and $P[f_t>f_i]$ are symmetrical. Hence from (36)

$$P_F = 2\,P(A). \tag{42}$$

An upper limit for P(A) can be obtained by the approximate process of numerical integration. If the abscissa is divided into small intervals, say 0.1 Mc, then P(A) is approximately equal to the summation of $P_i$ equation (41), at each interval from $-\infty$ to $+\infty$. By inspection of Figure 5 it is apparent that:

(a) Since $P[f_t>f_i]$ is some number between 0.5 and 1.0 for all negative values of f, we are sure that the sum of all of the $P_i$ for $f_i<0$ is less than

$$1.0 \times \int_{-\infty}^{0} P(f_h)df.$$

(b) Since both factors of (41) decrease for all successive terms after the term for $f_i = 4.8$ Mc, we are sure that the sum of all of the $P_i$ for $f_i > 4.8$ Mc is less than

$$P[f_t>4.8] \times \int_{4.8}^{\infty} P(f_h)df.$$

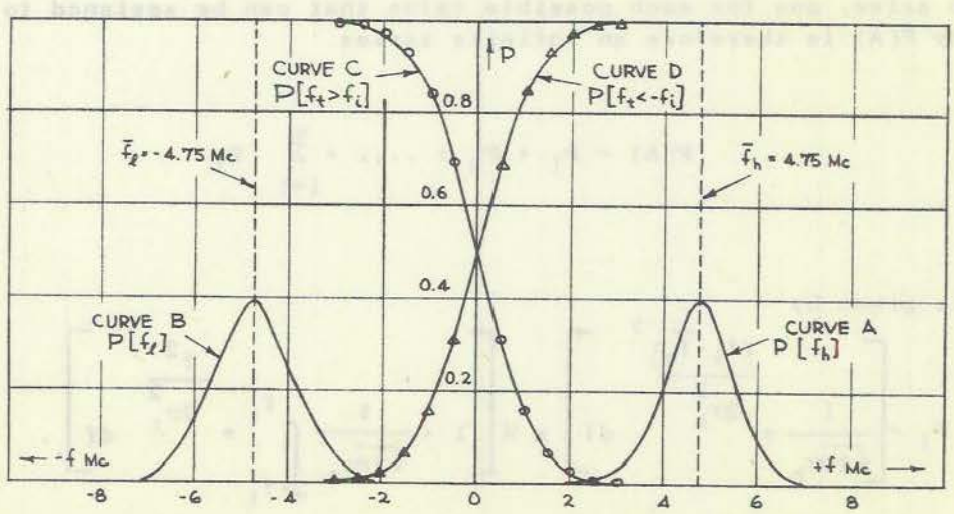


*Figure 5- Plots of $P[f_h]$, $P[f_l]$, $P[f_t>f_i]$, and $P[f_t<-f_i]$*

Thus a safe upper limit for P(A) can be set by summing the values for $P_i$ for each 0.1 Mc interval from 0 to 4.8 Mc and adding to this the products considered in (a) and (b) on p. 16. That is,

$$P(A) < \int_{-\infty}^{0} P(f_h)df + \sum_{i=0}^{4.8} P_i + \left[ P(f_t > 4.8) \times \int_{4.8}^{\infty} P(f_h)df \right] \quad (43)$$

where $P_i$ is determined from equation (41). In a practical situation the first and third terms of equation (43) are very small compared to the second term. The result obtained is dependent upon $\sigma_t$ and $\sigma_h$, which are, of course, determined by the assumptions made and equipment specifications. For the example being considered in which transmitter frequency tolerance is ±3 Mc, receiver frequency tolerance is ±2 Mc, and receiver bandwidths are distributed between 8 Mc and 11 Mc with an allowable decrease factor of 1 Mc, we get

$$P_F < 10^{-4} \quad (44)$$

which indicates that system failure due to frequency and bandwidth variations alone is extremely unlikely to occur. This assumes, of course, a single channel system; the possible failure due to adjacent channel interference is not included. This upper limit of $P_F$ is determined by equipment specifications and remains valid only upon the assumption that all equipments are meeting specifications on frequency and bandwidth tolerances. Furthermore, since all assumptions used in arriving at this value were pessimistic, we are assured that $P_F$ is, in fact, numerically smaller than $10^{-4}$.

## SUITABLE WAVEMETER BANDWIDTH AND FREQUENCY CHARACTERISTICS

### Distribution of Transpondor Transmitter Frequencies at the Time of the Go-No-Go Test

We are interested in a preservice go-no-go test on transpondor transmitter frequencies. We will assume that all transpondors in the system are meeting the frequency tolerance specifications (±3 Mc under service conditions). The distribution of $f_t$ given by equation (3) with standard deviation $\sigma_t = 1$ Mc is the distribution we would expect due to variations of a number of random parameters arising from reasonable combinations of service conditions. At the time of go-no-go test, however, the equipments may not necessarily be subjected to the full range of variation of several of these parameters, and we would hence expect the transmitter frequencies to be more closely concentrated about $F_o$. Let us assume for the present that we have a "perfect" wavemeter which can indicate absolutely the reply frequency as each transmitter is tested. The parameters which give rise to the frequency distribution about $F_o$ at the time of test are:

*(1) Variations in centering the transmitter on the frequency channel at the time of adjustment.*

*(2) Variations in wavemeter reading at the time of transmitter adjustment.*

*(3) Change in power supply voltages between the time of adjustment of transmitter and the time of the go-no-go test.*

*(4) Change in temperature and pressure between the time of adjustment of transmitter and the time of go-no-go test.*

*(5) Change in duty cycle between the time of adjustment of transmitter and the time of go-no-go test.*

*(6) Impedance mismatch.*

These parameters are random with respect to one another and thus the resultant distribution of transmitter frequencies at the time of go-no-go test may also be assumed to be normal about $F_o$. Let us designate a transmitter frequency at the time of go-no-go test and before the equipment is subjected to service conditions by $F_x$.

If we let

$$f_x = F_x - F_o, \tag{45}$$

then $f_x$ may be regarded as a variable which is normally distributed about zero. Let $\sigma_x$ be the standard deviation of this distribution. With transmitters which are meeting specifications regarding frequency tolerance, there is at least a 99.73 percent chance that $f_x$ will lie in the range $0 \pm 3\ \sigma_x$ Mc. This may also be stated by saying that there is at least a 99.73 percent chance that $f_x$ will be within a band X centered on zero, where

$$X = 6\sigma_x . \tag{46}$$

A value for $\sigma_x$ and X may be found by following the reasoning and assumptions of L. S. Schwartz.[4] Essentially, we take note of the fact that in addition to the six parameters leading to the distribution of $f_x$ at the time of go-no-go test, further variations in several parameters combine to give the ultimate distribution of $f_t$ with the specified limits ±3 Mc. These additional parameters are listed together with associated tolerance limits. The values used come from actual laboratory tests of equipments with specifications similar to those used in the example.

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[4] *Schwartz, op. cit. pp. 23-24*

| Parameter | Tolerance, $\pm b_i$ |
| --- | --- |
| Air temperature ± 50°C and air pressure variations. The worst possible frequency variation will result if these two parameters are not independent and if the separate tolerances add algebraically. For this situation the total tolerance is given as $b_1$. | $b_1 = \pm 0.75$ Mc. |
| Heater and $B^+$ voltages, ± 10 percent. The worst possible frequency variation will result if these two parameters are not independent and if the separate tolerances add algebraically. For this situation the total tolerance is given as $b_2$. | $b_2 = \pm 0.40$ Mc. |
| Duty cycle (dissipation), 0 to 1 percent. | $b_3 = \pm 0.60$ Mc. |
| Impedance mismatch | $b_4 = \pm 1.0$ Mc. |

The distribution law for each of these tolerances is unknown, but clearly the worst possible situation would result if they were all rectangular. For a rectangular distribution, the standard deviation[5] is given by

$$\sigma_i^2 = \frac{b_i^2}{3} . \qquad (47)$$

The parameters listed above are all random with respect to one another and also with respect to those which give the distribution of $f_x$. This latter distribution is related to the other component distributions by the equation.

$$\sigma_t^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_x^2 . \qquad (48)$$

For the example being considered, $\sigma_t^2 = 1$ Mc, and from equation (47), $\sigma_1^2 = 0.1875$, $\sigma_2^2 = 0.0533$, $\sigma_3^2 = 0.1200$, and $\sigma_4^2 = 0.3333$. Substituting these in (47) and solving for $\sigma_x$:

$$\sigma_x^2 = [0.1875 + 0.0533 + 0.1200 + 0.3333]$$

$$\sigma_x^2 = 0.3059$$

$$\sigma_x = 0.5531 ;$$

[5] *Schwartz op. cit. pp. 23-24*

from (46)

$$X = 6\sigma_x = 3.3186 \text{ Mc.}$$

The frequency band X = 3.3186 Mc is the band centered upon $F_o$ within which at least 99.73 percent of the transmitters will lie when tests are made on a system manufactured and maintained to meet the specifications given in the example.

## Selection of an Average Wavemeter Bandwidth

Let us initially assume an "ideal" go-no-go wavemeter with a constant center frequency equal to $F_o$ and a constant rectangular bandwidth W. The problem is to select a value for W. In this selection we must consider two factors:

(1) Probability of system failure due to frequency and bandwidth variations.

(2) Rejection of transmitters.

With respect to the first of these factors, two general groups of causes contribute to any system failure. The first may be thought of as "ordinary" causes, those variable parameters which have already been statistically accounted for in determining the frequency band X. If these were the only causes of frequency or bandwidth variations, then a system conforming to specifications would never require a go-no-go tester because the performance of equipments would be statistically predictable. The second group of causes of system failure may be thought of as "extraordinary" causes, those which are statistically unaccountable due to human failures such as mishandling of equipment, jarring, dropping, unskilled or careless detuning, etc. Because of the presence of these factors to a more or less degree depending on the personnel situation, some transmitters may be found to be outside the limits of the band X in excess of the statistically predicted value even in a properly designed system with equipment conforming to specifications. If these transmitters are not removed from service, the probability of system failure will have some higher value than that already found. Let us assume the wavemeter band W = X. (In the example being considered, let W = 3.3186 Mc.) We may conclude that:

(1) All transmitters outside of the band X for any cause whatsoever will be rejected. Less than 0.27 percent of the transmitters which have not been "mishandled" will be among the rejects.

(2) The probability of system "failure" remains certain to be some value $P_F$ such that $P_F < 10^{-4}$.

The go-no-go wavemeter effectively removes the statistically unpredictable "human factor" from transmitter maintenance as a contributing cause to system failure due to frequency and bandwidth variations.

If W is made less than X (in the example, let W<3.3186 Mc) then the percentage of rejects of transmitters which have not been "mishandled" increases above the 0.27 percent value, and at the same time the sure upper limit for $P_F$ will be reduced (in the example, below the value $10^{-4}$).

If W is made greater than X ( in the example, let W>3.3186 Mc), the number of transmitter rejects will be reduced. Obviously, the percentage of transmitters which have not been mishandled will also be reduced. The go-no-go wavemeter is in this case removing some of the mishandled transmitters (those outside of the band W) as contributing causes to $P_F$, but is not rejecting those which are inside of W and outside of X. Thus, for any W>X the sure upper limit for $P_F$ may be increased, the amount of increase depending upon the extent to which "extraordinary" causes are contributing to transmitter frequency variations. Since this cannot be statistically accounted for in system design, the exact amount of increase in the upper limit for $P_F$ cannot be determined. Let us consider a less desirable system, in which transmitter tolerances have been relaxed until the distribution band of transmitter frequencies at the time of go-no-go test takes on an increased value X' = W (where W>X). It seems reasonable to assume that such a system would have a new upper limit for $P_F$ which is in fact greater than the undetermined upper limit for $P_F$ in the original system. This new upper limit for $P_F$ can be determined statistically, and we can best express the effect of W on $P_F$ by determining this new sure upper limit for $P_F$ if transmitter tolerances are relaxed to the point where X' = W. Table 1 lists several values of W with corresponding values of percent rejects of transmitters not "mishandled" when tolerances remain unchanged, and the sure upper limit for $P_F$ if tolerances are relaxed so that X' = W. Figures 6 and 7 plot the information from Table 1. It appears from Figure 6 that transmitter rejects become appreciable (that is, greater than 0.5 percent) when W is approximately 3 Mc or less. For values of W greater than this, the only appreciable number of transmitter rejects will be those which are off frequency because of one or more of the "extraordinary" causes. From Figure 7 we may conclude that any W<8 Mc can be used with assurance that $P_F$ will not exceed 0.005. A suitable value for W from these considerations would be any value within the limits 3 Mc to 8 Mc. Since W and $F_W$ cannot be expected to be constant in practice, 5.5 Mc would seem the best choice for W to assure operation within the two extremes.

## Wavemeter Frequency and Bandwidth Tolerances

The calculation of W has thus far applied only to a fictitious perfect wavemeter centered exactly on $F_o$ with constant bandwidth. However, in practice the center frequency, $F_W$, and the bandwidth, W, cannot be expected to be constant. If the wavemeter is adjusted "on frequency" at regular intervals, then due to limitations in the accuracy with which this can be done and also to several randomly varying parameters in the instrument's environment, instantaneous observations of $F_W$ taken over a period of time which is long compared to the intervals between adjustments will show $F_W$ to be normally distributed about $F_o$. Let

$$f_W = F_W - F_o. \tag{49}$$

Table 1

| "Ideal" Wavemeter Bandwidth W (Megacycles) | Percent Rejects if Transpondor Tolerances Remain Unchanged | Sure Upper Limit for $P_F$ if Transmitter Tolerances are set so that $X' = W$ |
|---|---|---|
| 0.01 | 98.56 | - |
| 0.10 | 92.00 | - |
| 0.25 | 82.60 | - |
| 0.50 | 65.05 | - |
| 1.00 | 36.54 | - |
| 1.50 | 17.38 | - |
| 2.00 | 7.03 | - |
| 2.25 | 4.24 | - |
| 2.50 | 2.38 | - |
| 2.75 | 1.30 | - |
| 3.00 | 0.67 | - |
| 3.3186 | 0.27 | 0.00010 |
| 3.50 | 0.156 | - |
| 4.00 | 0.03 | 0.00016 |
| 5.00 | $6\times10^{-6}$ | 0.00045 |
| 6.00 | - | 0.00102 |
| 6.50 | - | 0.00159 |
| 7.00 | - | 0.00230 |
| 8.00 | - | 0.00454 |
| 9.00 | - | 0.00713 |

Then $f_W$ is normally distributed about zero according to the law

$$P(f_W)df = \frac{1}{\sqrt{2\pi}\,\sigma_w} e^{-\frac{f^2}{2\sigma_w^2}} df. \qquad (50)$$

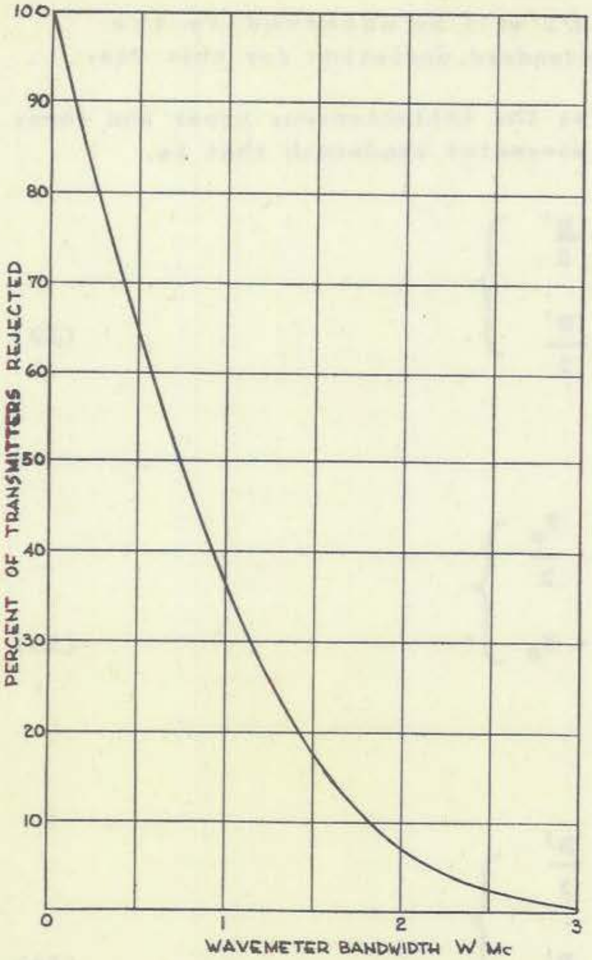


*Figure 6- Percent transmitter rejects for different wavemeter bandwidths, W, if the transmitter tolerances remain unchanged at ± 3 Mc*

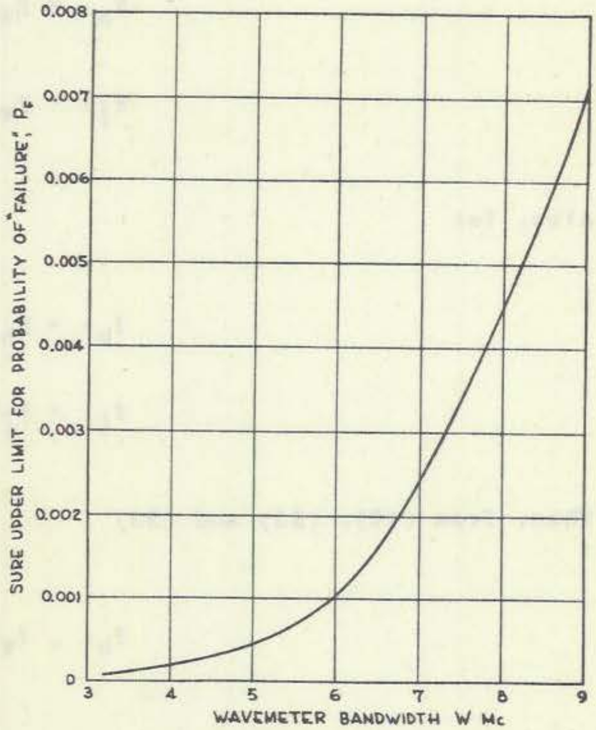


*Figure 7- Sure upper limit for probability of system "failure" for different wavemeter bandwidths, W, for the case where transmitter tolerances are relaxed until X = W*

$P(f_W)df$ is the probability that $f_W$ lies within the interval $f \pm df/2$ and $\sigma_w$ is the standard deviation for this distribution.

Similar to the analysis made for receiver bandwidths, the instantaneous wavemeter bandwidth, B , may be shown to be a random variable with a normal distribution about its average value B . Hence, the instantaneous value of half the wavemeter bandwidth, B /2, is a random variable normally distributed about $\overline{B}$ /2 according to the relationship

$$P\left[\frac{B'}{2}\right] dB = \frac{1}{\sqrt{2\pi}\sigma_{\alpha'}} e^{-\frac{\left[\frac{B}{2}-\frac{\overline{B}}{2}\right]^2}{2\sigma_{\alpha}^2}} dB, \qquad (51)$$

where $P(B'/2)dB$ is the brobability that $B'/2$ will be observed in the interval $B \pm dB'/2$ and where $\sigma_a$ is the standard deviation for this distribution.

Let us now designate $F_{h'}$ and $F_{l'}$ as the instantaneous upper and lower frequency limits, respectively, of the wavemeter bandwidth that is,

$$\left.\begin{aligned} F_{h'} &= F_W + \frac{B'}{2} \\ F_{l'} &= F_W - \frac{B'}{2} \end{aligned}\right\}. \tag{52}$$

Also, let

$$\left.\begin{aligned} f_{h'} &= F_{h'} - \frac{F_o}{2} \\ f_{l'} &= F_{l'} - F_o \end{aligned}\right\}. \tag{53}$$

Then, from (49), (52) and (53)

$$\left.\begin{aligned} f_{h'} &= f_W + \frac{B'}{2} \\ f_{l'} &= f_W - \frac{B'}{2} \end{aligned}\right\}. \tag{54}$$

Similar to the case of receiver bandwidth limits, it appears that $f_{h'}$ and $f_{l'}$ are random variables distributed normally about their respective mean values

$$\left.\begin{aligned} \overline{f_{h'}} &= \frac{\overline{B'}}{2} \\ \text{and} \qquad \overline{f_{l'}} &= -\frac{\overline{B'}}{2} \end{aligned}\right\}. \tag{55}$$

Also,

$$\left. P(f_{h'})df = \frac{1}{\sqrt{2\pi}\sigma_{hl}} e^{-\frac{[f - \overline{f_{h'}}]^2}{2\sigma_{hl}^2}} df \right\} \tag{56}$$

$$P(f_{l}')df = \frac{1}{\sqrt{2\pi}\sigma_h} e^{-\frac{[f - \overline{f_{l}'}]^2}{2\sigma_{hl}^2}} df \qquad (57)$$

where $P(f_h')df$ and $P(f_l')df$ are equal to the probability that $f_h'$ or $f_l'$ will be observed within the interval $f \pm df/2$, and $\sigma_{hl}$ is the standard deviation for these distributions given by:

$$\sigma_{hl}^2 = \sigma_w^2 + \sigma_{a'}^2 \quad . \qquad (58)$$

Figure 8 shows equations (56) and (57) with $\sigma_{hl}$ as determined in the following paragraphs.

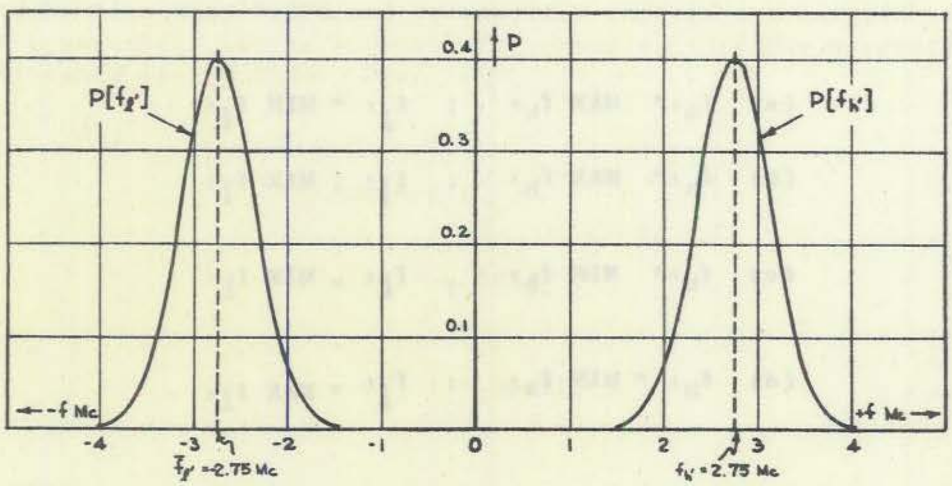


*Figure 8- Distribution of wavemeter upper and lower bandwidth limits.*

From this analysis we may conclude that:

(1) $f_h'$ will be within the interval $[\overline{B}'/2 \pm 3\sigma_{hl}]$ at least 99.73 percent of time of operation.

(2) $f_l'$ will be within the interval $[-\overline{B}'/2 \pm 3\sigma_{hl}]$ at least 99.73 percent of time of operation.

Let us define

$$\left.\begin{aligned} \text{MAX } f_{h'} &= \frac{\overline{B}'}{2} + 3\sigma_{hl} \\ \text{MIN } f_{h'} &= \frac{\overline{B}'}{2} - 3\sigma_{hl} \\ \text{MAX } f_{l'} &= -\frac{\overline{B}'}{2} + 3\sigma_{hl} \\ \text{MIN } f_{l'} &= -\frac{\overline{B}'}{2} - 3\sigma_{hl} \end{aligned}\right\} \tag{59}$$

Regardless of what the particular instantaneous values of $f_w$ and $B'$ may be, the wavemeter acceptance band may be specified by some $f_{h'}$ and $f_{l'}$, lying between the extreme values given by (59) for at least 99.73 percent the time of operation. Four extreme conditions can be indicated:

(a) $f_{h'} = \text{MAX } f_{h'}$ ; $f_{l'} = \text{MIN } f_{l'}$

(b) $f_{h'} = \text{MAX } f_{h'}$ ; $f_{l'} = \text{MAX } f_{l'}$

(c) $f_{h'} = \text{MIN } f_{h'}$ ; $f_{l'} = \text{MIN } f_{l'}$

(d) $f_{h'} = \text{MIN } f_{h'}$ ; $f_{l'} = \text{MAX } f_{l'}$

The greatest possible number of rejects will occur if the wavemeter operates under conditions (d). This is the same as for a fictitious perfect constant frequency wavemeter centered on $F_o$ with a constant bandwidth

$$B_d = 2\left[\frac{\overline{B}'}{2} - 3\sigma_{hl}\right]. \tag{60}$$

The greatest possible upper limit for $P_F$ will be obtained under condition (a). This is the same as for a fictitious perfect constant-frequency wavemeter center on $F_o$ with a constant bandwidth

$$B_a = 2\left[\frac{\overline{B}'}{2} + 3\sigma_{hl}\right]. \tag{61}$$

Let $\overline{B}' = 5.5$ Mc, $B_d = 3$ Mc and $B_a = 8$ Mc. Then from either of the equations (60) or (61):

$$\sigma_{hl} = \frac{2.5}{6} = 0.4166,$$

$$\sigma_{hl}^2 = 0.1736.$$

From equation (58), $\sigma_w$ and $\sigma_{\alpha'}$ must be selected so that

$$\sigma_w^2 + \sigma_{\alpha'}^2 = 0.1736. \qquad (62)$$

This equation specifies the tolerances for center frequency and bandwidth for a go-no-go wavemeter with average bandwidth 5.5 Mc in order that $P_F$ is certain to be less than 0.005 and transmitter rejects are certain to be less than 1 percent. Let us suppose, for example, that the wavemeter frequency tolerance is ± 0.1 Mc. Then

$$3\sigma_w = 0.1$$

$$\sigma_w = 0.0333$$

$$\sigma_w^2 = 0.00111,$$

and from (62),

$$\sigma_{\alpha'}^2 = 0.1725$$

$$\sigma_{\alpha'} = 0.415 \text{ Mc}$$

$$3\sigma_{\alpha'} = 1.245 \text{ Mc}.$$

This means that for this situation half the bandwidth must be 2.75 Mc ± 1.245 Mc, hence the bandwidth must be 5.5 Mc ∓ 2.49 Mc.

Let us now suppose the wavemeter frequency tolerance is relaxed to ± 1.0 Mc.

Then

$$3\sigma_w = 1.0$$

$$\sigma_w = 0.0333$$

$$\sigma_w^2 = 0.111.$$

From (62):

$$\sigma_{\alpha'}^2 = 0.1736 - 0.1111 = 0.0625$$

$$\sigma_{\alpha'} = 0.25 \text{ Mc.}$$

$$3\sigma_{\alpha'} = 0.75 \text{ Mc.}$$

This means that for this situation half the bandwidth must be 2.75 Mc ± 0.75 Mc, hence the bandwidth must be 5.5 Mc ± 1.5 Mc.

## PRACTICAL CONSIDERATIONS

### Practical Bandpass Characteristics and the Concept of System "Failure"

Suppose we assume that responsor receiver sensitivity vs frequency curves are not rectangular, but have sloping sides and are not necessarily flat. The bandwidth limits for a receiver with such a response are defined in the specifications as the frequencies at which the response is 6 db down from the maximum. For this case we can no longer state that a transmitter replying on a frequency outside the receiver's band definitely will not be received. Actually, if a transmitter replies on any frequency other than that for maximum receiver sensitivity, the result is a reduction in the extreme range for that transpondor from the maximum possible range. If a transmitter replies on a frequency exactly at one of the bandwidth limits, the extreme range for this transmitter is just 50 percent of the maximum possible range if the reply frequency had been the frequency of maximum receiver sensitivity. Thus, we can only state that any transmitter which replies on a frequency outside the receiver's pass-band will suffer a reduction of at least 50 percent in the extreme range at which it can be received. We no longer can refer to system "failure" as definitely occurring or not. Instead we must redefine $P_F$ as the probability that the extreme range for a transpondor replying to a given interrogator-responsor will be reduced to 50 percent of its maximum possible value or less because of frequency and bandwidth variations. $P_F$ with this new definition can still be used as a consideration in setting an upper limit for W.

The concept of failure or, more accurately, range reduction below 50 percent does not include any failures or range reduction due to interference of any kind or from any causes other than frequency and bandwidth variations.

Hence this analysis does not apply, for instance, in determining channel spacing in a multi-channel system or in providing a go-no-go wavemeter to prevent failure on the reply path due to cross-channel interference.

## Wavemeter Bandpass Characteristic and Transmitter Power Variations

The analysis thus far assumes that the go-no-go wavemeter accepts or rejects transmitters on the basis of frequency alone and independent of the transmitter power. In practice, this may not necessarily be so. Imagine, for instance, a wavemeter calibrated to give a "go" indication whenever a transmitter having at least a minimum acceptable power is operating at the wavemeter's most sensitive frequency (for convenience, the center of the wavemeter band). Consideration should be given to the slope of the edges of the wavemeter band. Suppose the band is defined by points which are 6 db below the point of maximum sensitivity and suppose also that the slope of the edges is gradual. In this case a number of transmitters operating inside the wavemeter band may be rejected because of insufficient power to give the "go" indication. At the same time, if there are a number of transmitters outside of the wavemeter band with excessive power, they will not be rejected if their power is sufficient to produce a "go" indication. The wavemeter under such conditions is not selecting or rejecting on the basis of frequency alone. With gradual sloping band edges the wavemeter might reject an excessive number of transmitters which would otherwise have operated satisfactorily in the system, or it might do just the opposite, that is, "pass" too many transmitters outside of its band. The performances will be dependent upon transmitter power distribution. This dependency can be minimized by making the slopes of the wavemeter pass-band edges as steep as possible.

Even with a pass-band having very steep edges, the wavemeter is bound to "pass" some of the transmitters just outside of its band which have a sufficient amount of excess power. According to the previous concept of system failure, a transmitter fails if it replies to a receiver outside of the receiver's pass-band, hence, some of these transmitters with the excess power may fail according to the above definition. Actually, these "off frequency" transmitters with high power might even have a greater maximum range than some low powered transmitters operating within the receiver's band and therefore not classed among the failures. The fact that the wavemeter "passes" a few higher powered transmitters outside of its band is less objectionable if at the same time one attaches this less objectionable meaning to the idea of system failure.

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Hence this analysis does not apply, for instance, in determining channel spacing in a multi-channel system or in providing a go-no-go wavemeter to prevent failure at the reply path due to cross-channel interference.

Wavemeter Bandpass Characteristic and Transmitter Power Variations

The analysis thus far assumes that the go-no-go wavemeter accepts or rejects transmitters on the basis of frequency alone and independent of the transmitter power. In practice, this may not necessarily be so. Imagine, for instance, a wavemeter calibrated to give a "go" indication whenever a transmitter having at least a minimum acceptable power is operating at the wavemeter's most sensitive frequency (for convenience, the center of the wavemeter band). Consideration should be given to the slope of the edges of the wavemeter band. Suppose the band is defined by points which are 6 db below the point of maximum sensitivity and suppose also that the slope of the edges is gradual. In this case a number of transmitters operating inside the wavemeter band may be rejected because of insufficient power to give the "go" indication. At the same time, if there are a number of transmitters outside of the wavemeter band with excessive power, they will not be rejected if their power is sufficient to produce a "go" indication. The wavemeter under such conditions is not selecting or rejecting on the basis of frequency alone. With gradual sloping band edges the wavemeter might reject an excessive number of transmitters which would otherwise have operated satisfactorily in the system, or it might do just the opposite, that is, "pass" too many transmitters outside of its band. The performance will be dependent upon transmitter power distribution. This dependency can be minimized by making the slopes of the wavemeter pass-band edges as steep as possible.

Even with a pass-band having very steep edges, the wavemeter is bound to "pass" some of the transmitters just outside of its band which have a sufficient amount of excess power. According to the previous concept of system failure, a transmitter fails if it applies to a receiver outside of the receiver's pass-band. Hence, some of these transmitters with the excess power may fail according to the above definition. Actually, these "off frequency" transmitters with high power might even have a greater maximum range than some low powered transmitters operating within the receiver's band and therefore not classed among the failures. The fact that the wavemeter "passes" a few higher powered transmitters outside of its band is less objectionable if at the same time one attaches this less objectionable meaning to the idea of system failure.

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