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UNCLASSIFIED

APPLICATION OF INTERSTAGE RC NETWORKS TO PROVIDE SELECTIVITY IN LOW-FREQUENCY AMPLIFIERS

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ABSTRACT

A selective 10-cps amplifier has been developed which is unique in that adequate gain and selectivity have been attained in a miniaturized assembly without recourse to the use of other than conventional resistance-capacitance coupling and loading networks. The resultant circuitry is similar to that of a typical RC-coupled audio amplifier with flat frequency response, but the component values are chosen so that the 1-db cutoff points at the low- and high-frequency limits are made to coincide at the desired operating frequency. By thus avoiding the need for relatively large filter components or complex selective-feedback circuit elements, and by employing miniaturization techniques insofar as the availability of suitable components permitted, the chassis area of the combined amplifier and associated power-supply has been reduced to approximately one-third that of the equipment which this unit was intended to supersede, and, at the same time, has provided greatly improved performance. The circuit design avoids the relatively sharp-peaked selectivity characteristic normally obtained with selective inverse-feedback networks, and more nearly provides the relatively broad-peaked curve of a band-pass amplifier without the limitation of selectivity by the unity-gain condition of the amplifier which would be normal for an inverse-feedback system.

Universal selectivity curves comparing the performance of the resistance-capacitance circuit with that of inductance-capacitance resonant circuits are given.

PROBLEM STATUS

This is a final report on that phase of the problem which was assigned to the Radio Techniques Branch. Work on the main problem is continuing.

AUTHORIZATION

NRL Problem C04-34D

APPLICATION OF INTERSTAGE RC NETWORKS TO PROVIDE SELECTIVITY IN LOW-FREQUENCY AMPLIFIERS

INTRODUCTION

In response to a request[1] made by the Chemistry Division, the Radio Techniques Branch, Radio Division II, has developed a small, compact, selective low-frequency amplifier to be used as part of a gas analyzer.[2] The Chemistry group had been working with such a device in the Laboratory, with sufficiently promising results to warrant the development of an improved and compact version that would be more suitable for the service intended. The original amplifier was bulky, somewhat unstable, and possessed too high a degree of peak selectivity due to the Wien-bridge (twin-T) feedback circuit employed. This rendered it very susceptible to shock excitation, an undesirable characteristic for use where vibration exists.

TECHNICAL REQUIREMENTS

It was stipulated that the input to this amplifier would be a signal having a frequency of about ten cycles per second. The a-c output of the amplifier was to be mechanically rectified synchronously with the input signal, and made to register on a recording voltmeter. Obviously, the suppression of signals from any other source would be necessary to insure correctness of the readings obtained. The amplifier therefore had to be sufficiently selective to attenuate hum components from the power supply and other possible sources, and to prevent very low frequency "flicker-effects" from being amplified. On the other hand, too great a degree of selectivity would cause the amplifier to be sensitive to shock excitation or variation in the frequency; in this respect, the behavior of the amplifier would then be similar to that of a tuned circuit of high Q. The specifications for the amplifier required a voltage amplification of approximately 1000, an input impedance of 1 megohm, and a 5000-ohm output impedance. The center or signal frequency was to be 10 cycles per second, with suitable attenuation of frequencies below 7 and above 14 cycles per second. The unit was to be complete with a self-contained power supply, including a special 6-volt d-c outlet for the filament of an exciter lamp, and two 110-volt a-c outlets for various circuits of the gas analyzer. Maximum possible miniaturization was to be given prime consideration in designing the amplifier, since the equipment was intended to be used where space would be at a premium.

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[1] *NRL Memo 806-10/48, dated April 27, 1948 to Code 1200 of NRL*

[2] *Elliot, M. A., "A positive filter type of infrared gas analyzer," NRL Report C-3449 (Unclassified), April 15, 1949*

BASIC DESIGN CONSIDERATIONS

Generally speaking, relatively large capacitors, reactors, and transformers are associated with low-frequency amplifier circuits, whereas the space limitations required that simple circuitry and small components be employed. Accordingly, it was decided to disregard the usual filters and selective-feedback circuitry, and to postulate the initial design of the interstage networks upon a modification of conventional theory[3] as applied to wide-band, flat-response, resistance-capacitance-coupled amplifiers. Table 1 shows the considerations leading to this decision.

In order that heat production be kept to a minimum, it was decided to operate the amplifier on a 108-volt plate supply, regulated by a miniature Type OB2 voltage-regulator tube. Since effective cathode bypassing for 10 cycles would require the use of very large capacitors, operation with unbypassed cathode resistors was indicated. The extent to which the unbypassed cathode resistor would affect the over-all selectivity was determined in the course of the design computations, as will be shown later. A triode output stage was decided on, to provide a relatively low output impedance which would be very tolerant of mismatched loading. Since this stage would not be required to furnish appreciable power, it was feasible to dispense with the output transformer and to use capacity coupling to the output load.

DEVELOPMENT OF THE INTERSTAGE NETWORKS

A postulated voltage amplification of approximately 5 for the low-impedance output stage required that the combined gain of the preceding stages be about 200, which could be obtained from two pentode stages with a voltage amplification of about 15 times each. To obtain the required gain with a low plate-supply voltage (108 V), the use of a rather high value of plate-load resistor was necessary in the first two stages. The value selected was 0.56 megohm as being approximately optimum for the Type 6AU6 miniature tubes chosen, under the particular conditions of operation selected (low values of electrode potential and an unbypassed cathode-bias resistor).

It is very desirable that the ratio of a-c to d-c load resistance be as near unity as possible in any amplifier. The ratio, however, in this case, was limited by the maximum permissible value of grid resistor. A 1.5-megohm resistor was chosen because it permitted the use of a standard value of commercially available capacitor to obtain the frequency characteristics desired for interstage coupling.

As was stated before, the calculations for this amplifier followed a treatment somewhat similar to that used in designing a more conventional audio amplifier in which the usual low- medium- and high-frequency ranges were involved, except that the 1-decibel low and high cutoff frequencies were used as the "medium" or center-frequency, which in this case was 10 cycles per second. Low-pass and high-pass R-C filter configurations of the general type shown in Figure 1, were used as coupling elements.

In the low-pass RC network configuration (series resistance, shunt capacitance), the ratio of output to input voltage for one section is:

$$x_L = \frac{E\ \text{out}}{E\ \text{in}} = \frac{1}{(1 + \omega^2 T^2)^{\frac{1}{2}}}$$

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[3] *Sturley, K. R., "Radio receiver design," Part II, Chapter 9, Section 9.3 London, Chapman & Hall, 1936*

TABLE 1
Relative Advantages and Disadvantages of Various Types of Selective Networks Applicable to 10-CPS Amplifier

Type of Circuit	Advantages	Disadvantages
(A) Resonant LC Circuits (Single anti-resonant or band-filter types)	(1) Cutoff not limited by amplifier gain: dependent only on frequency, effective Q, number of circuits, and leakage. (2) Associated amplifier not critical as regards phase shift.	(1) Requires large components for desired Q values. (2) Q limited by circuit values (such as grid-leak resistance). (3) Susceptible to opening of inductor windings. (4) Vulnerable to inductive pickup (hum, etc.). (5) Cutoff frequency is voltage dependent unless inductor iron is operated at proper flux density. (6) Requires more than one inductor if band-pass characteristic is desired. (7) May "ring" or oscillate excessively unless properly damped, in which case selectivity would be reduced.
(B) Bridge-Type Feedback Networks (Twin-T etc.)	(1) Simple RC circuitry. (2) Can be designed to lose little amplifier gain at peak response frequency. (3) Not so vulnerable to stray pickup as (A) above.	(1) Cutoff limited by amplifier gain, i.e., selectivity becomes zero at frequencies when E output/E input = 1. (2) Critical as regards phase-shifts in associated amplifier networks; demands large coupling and decoupling condenser values. (3) Higher amplifier gain required if feedback loop cannot include amplifier input and output circuits because of changing load conditions in those circuits. (4) Normally provides sharply peaked response with flaring skirts. (5) Band-pass characteristic difficult to obtain. (6) Requires at least 6 additional components. (7) Can ring or oscillate excessively owing to relatively high equivalent peak-Q. (8) Very vulnerable to amplifier overload.
(C) Interstage RC-Coupling Networks.	(1) Simple RC circuitry. (2) High- and low-pass characteristics separately and readily controlled. (3) Can provide triangular or band-pass selectivity characteristic, as desired, for same number of components. (4) Cutoff not limited by amplifier gain: dependent only on frequency and number of circuits. Will not "ring" appreciably if cutoff does not exceed 10 db per octave. (5) Uses components already in use as loading and coupling elements in the amplifier. (6) Takes advantage of amplifier phase shifts.	(1) Some loss of gain per stage as result of low- and high-pass crossover (e.g., 2 db per stage in amplifier described in this report).

and

$$\omega T_L = \left(\frac{1-x_L^2}{x_L^2}\right)^{\frac{1}{2}}. \quad (1)$$

Likewise, in the high-pass case (series capacitance, shunt resistance)

$$x_H = \frac{1}{\left(1+\frac{1}{\omega^2 T^2}\right)^{\frac{1}{2}}}$$

and

$$\omega T_H = \left(\frac{x_H^2}{1-x_H^2}\right)^{\frac{1}{2}} \quad (2)$$

Designating the frequency at which the low- and high-pass attenuation curves cross, as f_r, and selecting an attenuation of 1 db for each circuit at f_r (2 db total attenuation for a band-pass network consisting of one low and one high-pass circuit, we have:

$$2\pi f_r T_L = 0.51$$

$$T_L = R_L C_L = \frac{1}{12.3 f_r}$$

and

$$2\pi f_r T_H = 1.95$$

$$T_H = R_H C_H = \frac{1}{3.22 f_r}$$

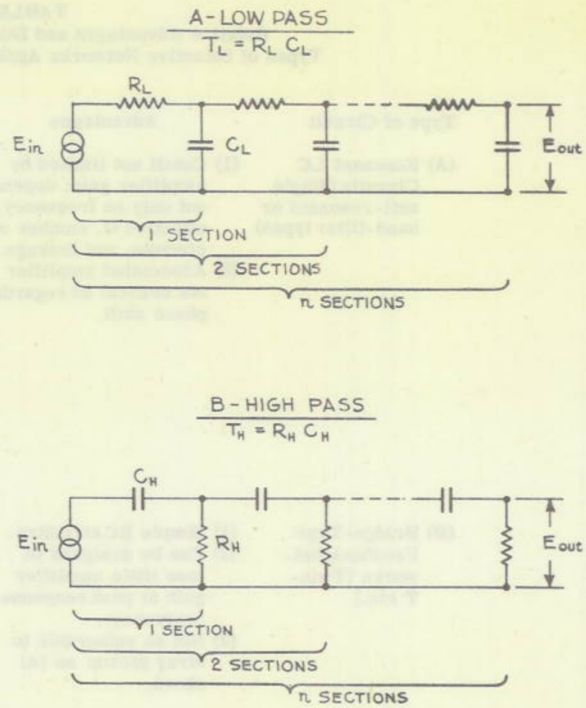

Fig. 1 - Low-pass and high-pass filter configurations

With two low-pass and two high-pass circuits cascaded to provide a greater degree of off-center selectivity, the value of x at f_r would be taken as 0.5 db, still producing a total attenuation of 2 db at f_r and providing almost the same peak bandwidth, so that

$$2\pi f_r T_L = 0.35$$

$$2\pi f_r T_H = 2.84$$

Similarly, three circuits of each type combined would be computed for an attenuation of 0.33 db at f_r, four circuits for 0.25 db, etc. Tables 2 and 3 show the variation in the value of the time constant (T) for filters consisting of one to ten RC sections in cascade. Figure 2 shows the band-pass transmission characteristic obtained by combining these low- and high-pass sections, as compared to single-section shunt LC resonant filters, plotted in log-log form. Figure 3 is a similar graph but plotted in db attenuation against a linear frequency scale. All the RC filter tabulations and curves are based on a total "insertion loss" or attenuation at f_r of 2 db.

Figure 4 shows the circuit of a typical resistance-capacitance coupled amplifier, in which C_o represents the output capacitance of V_1, C_w represents the wiring capacitance, C_g the input capacitance of V_2, and C_{gk} the grid-cathode capacitance of V_2.

Figure 5 represents the circuit equivalent to that in Figure 4, where R_p is the plate resistance of the tube, and C_s is the total value of the output capacitance V_1, the wiring capacitance, and the input capacitances of V_2.

TABLE 2
Value of Time-Constant T Producing 2-DB Total
Insertion Loss at Frequency f_r - Low-Pass Case

$T_L = R_L C_L$

No. Filter Sections in Cascade	Attenuation Per Section at f_r-(db)	$\omega_r T_L$ $\left(=2\pi f_r T_L\right)$	Ratio $\frac{T_L \text{ for n Sections}}{T_L \text{ for one Section}}$
1	1.0	0.51	1.0
2	0.5	0.35	0.69
3	0.33	0.29	0.57
4	0.25	0.25	0.50
5	0.2	0.22	0.44
10	0.1	0.16	0.31

TABLE 3
Value of Time Constant-T Producing 2-DB Total
Insertion Loss at Frequency f_r - High-Pass Case

$T_H = R_H C_H$

No. Filter Sections in Cascade	Attenuation Per Section at f_r-(db)	$\omega_r T_H$ $\left(=2\pi f_r T_H\right)$	Ratio $\frac{T_H \text{ for n Sections}}{T_H \text{ for one Section}}$
1	1.0	1.95	1.0
2	0.5	2.84	1.45
3	0.33	3.46	1.75
4	0.25	3.96	2.0
5	0.2	4.5	2.3
10	0.1	6.37	3.22

In the medium- or center-frequency range of the conventional audio amplifier, the series reactance of the coupling capacitor (C_c) is small compared to R_g and can be neglected; similarly, the reactance of the shunt capacitance (C_s) is high compared to R_L and can be omitted from the computation. The stray shunt capacity, C_k, across cathode resistor R_k is likewise negligible. The equivalent circuit for the amplifier at the medium frequencies then becomes as shown in Figure 6.

The amplification at the medium frequencies, neglecting the unbypassed cathode bias resistance (R_k), is

$$Am = \frac{\mu R'_l}{R_p + R'_l} \tag{3}$$

where $R'_l = R_l R_g / R_l + R_g$ = the effective plate-load resistance at the medium frequencies.

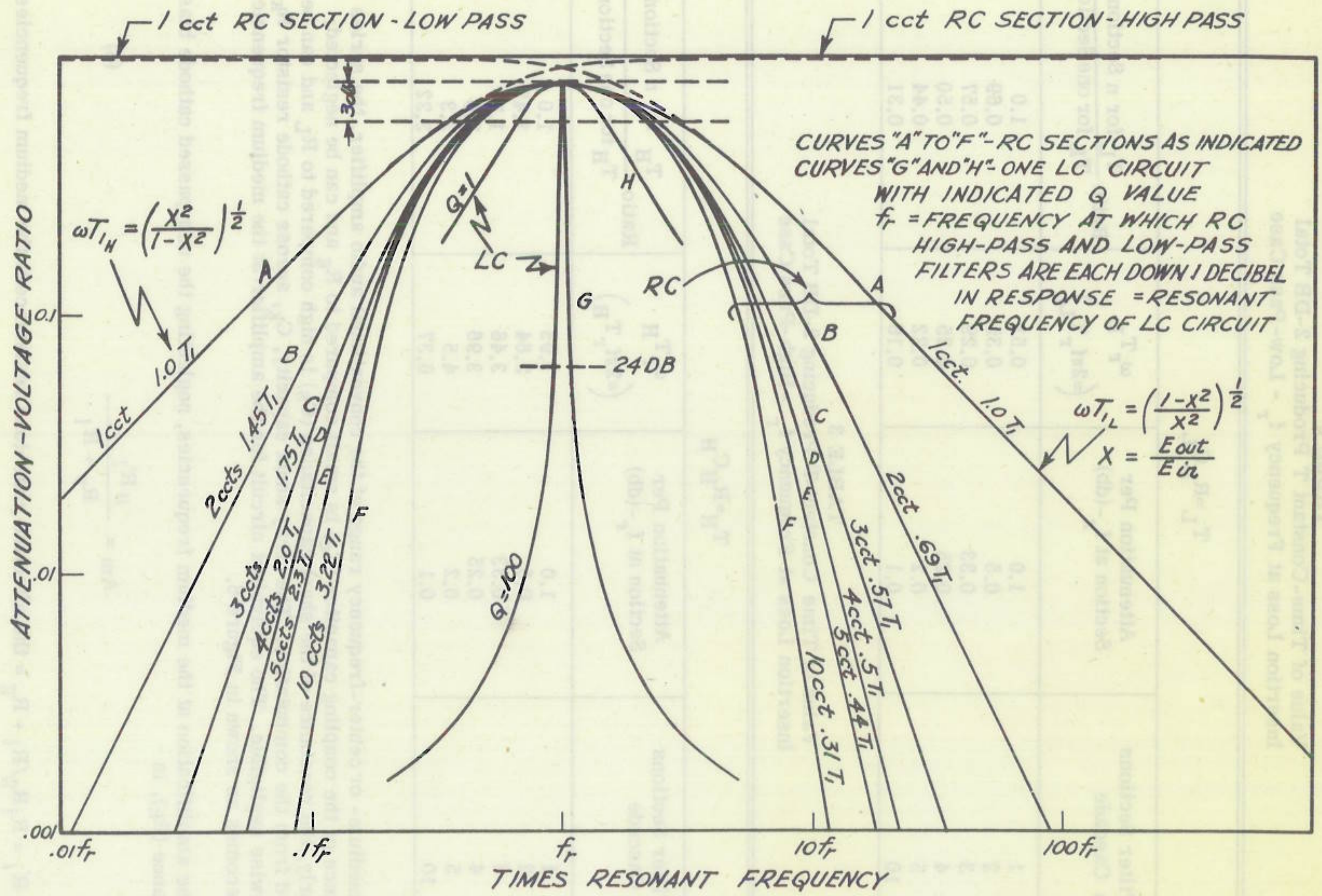

Fig. 2 - Pass-band characteristics of RC and LC filters

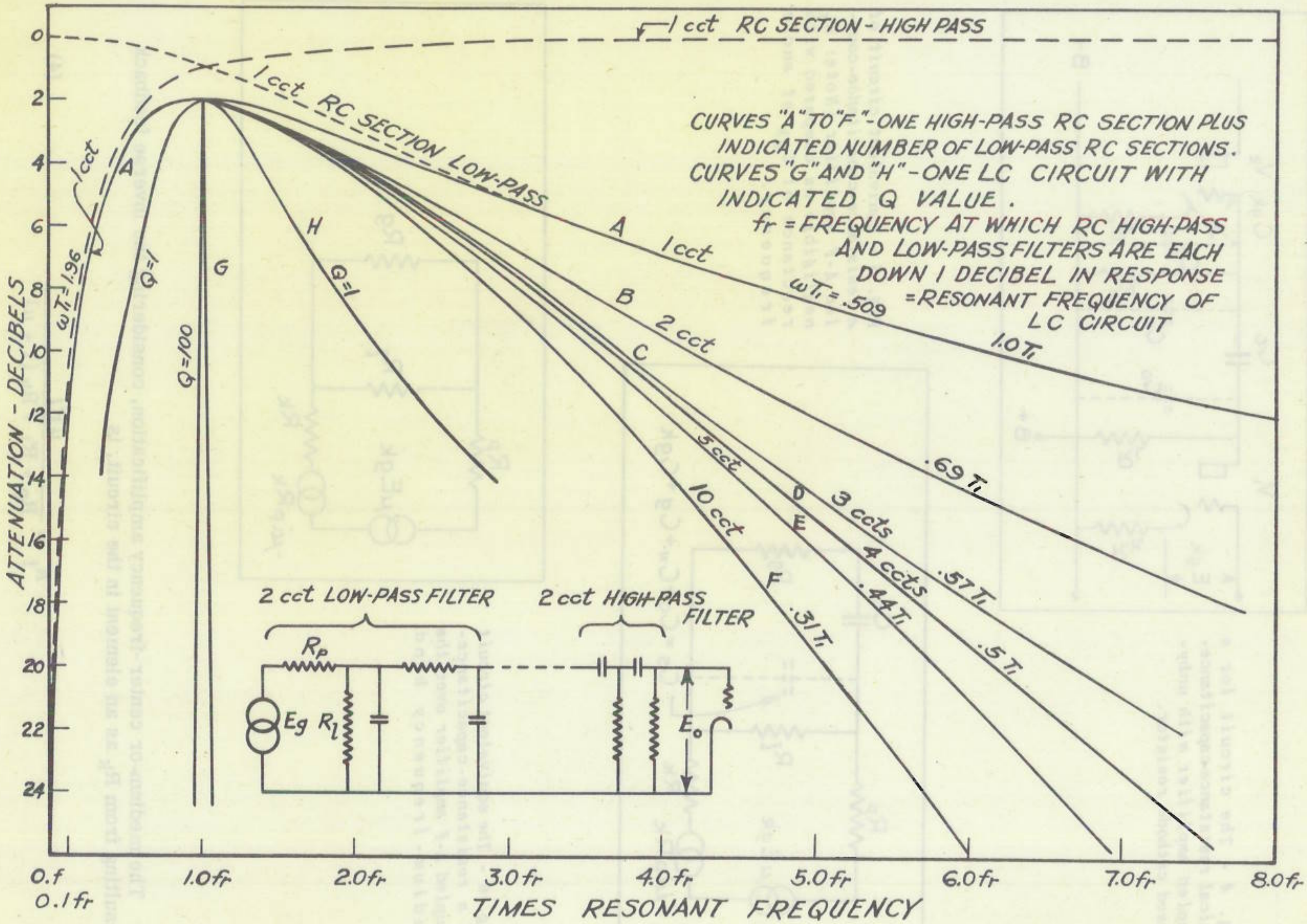

Fig. 3 - Pass-band characteristics of RC and LC filters

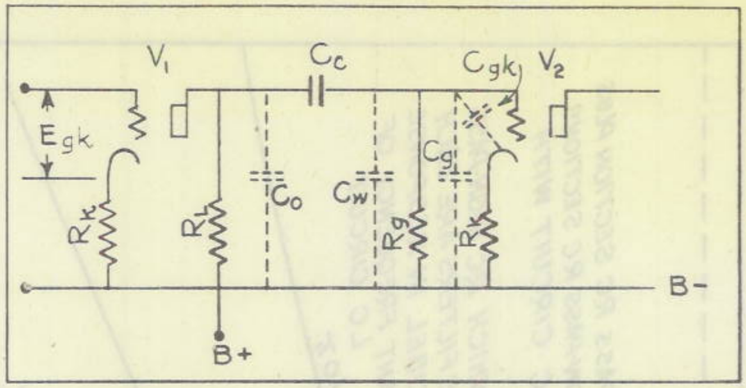

Fig. 4 - The circuit for a typical resistance-capacitance-coupled amplifier with unbypassed cathode resistor.

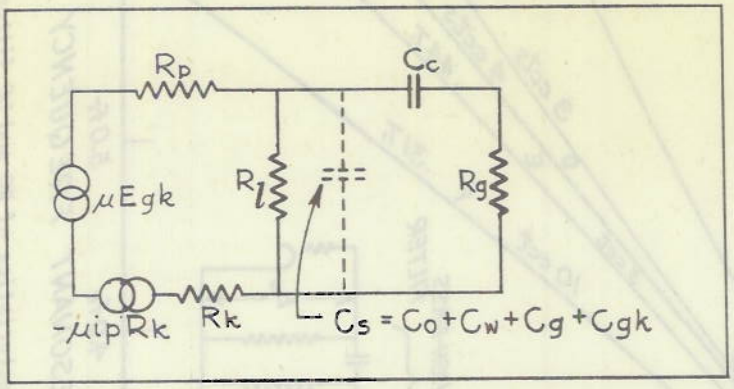

Fig. 5 - Equivalent circuit of a resistance-capacitance-coupled A-F amplifier (Note: R_k negligible as compared with reactance of C_{gk} at audio frequency.

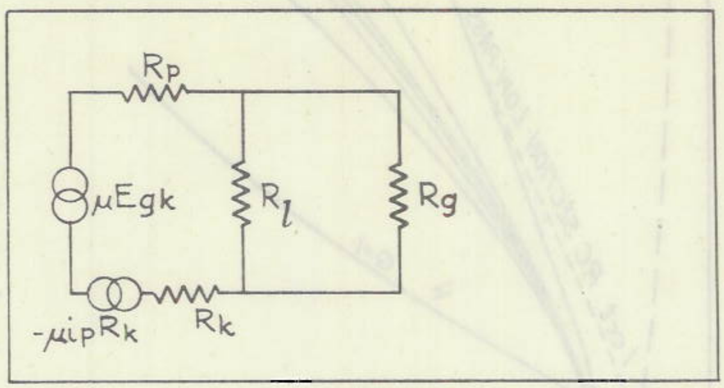

Fig. 6 - The equivalent circuit of a resistance-capacitance-coupled A-F amplifier over the medium-frequency band.

The medium-or center-frequency amplification, considering the inverse feedback resulting from R_k as an element in the circuit, is

$$A_f = \frac{\mu R'_l}{R_p + R'_l + R_k(1+\mu)} . \qquad (4)$$

The ratio of the amplification without feedback to that with feedback is

$$\frac{A_m}{A_f} = 1 + \frac{R_k(1+\mu)}{R_p + R'_l}$$

$$\cong 1 + \frac{\mu R_k}{R_p + R'_l} \qquad (5)$$

(for the pentode-type tube which usually has μ-values > 100). With $R_p \cong 2 \times 10^6$ ohms, $R'_l \cong 4 \times 10^5$ ohms, μ on the order of 1000, and $R_k \cong 6 \times 10^3$ ohms, $A_m/A_f \cong 3.5$, or a loss due to R_k of about 12 db. R'_l (or, more strictly, Z'_l) can vary between the limits of 5.6×10^5 and 0 ohms (at 0 and ∞ frequency, respectively), due to the actual presence of C_s in the circuit, and, disregarding phase variations, $A_m/A_f \cong 3.4$, at zero frequency and $A_m/A_f \cong 4$, at a very high frequency. The maximum possible variation in A_m/A_f is then about 1.4 db per stage; in other words, R_k unbypassed will not affect the over-all selectivity to any considerable extent.

The above consideration of the effect of R_k on selectivity is not strictly accurate; a more exact solution would, however, yield similar results. Such a solution can be obtained by using A_l and A_h (see following test) in place of A_m above, with appropriate modification of Equation (5).

The equivalent circuit for the amplifier in the low-frequency range is shown in Figure 7. The shunt capacitance (C_s) can still be neglected in these computations, whereas the reactance of the series capacitor (C_c), becomes comparable with the resistance of R_g.

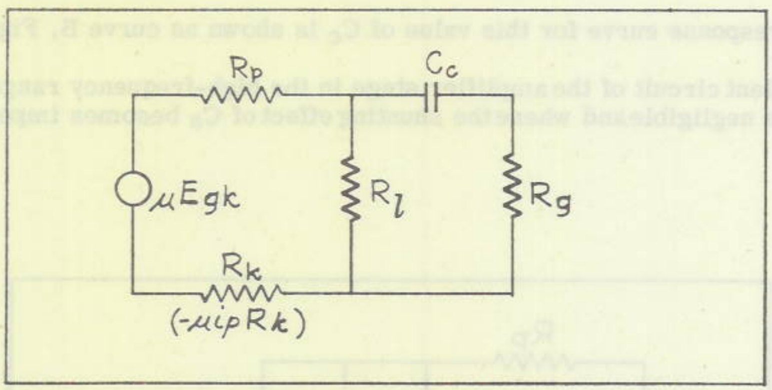

Fig. 7 - The equivalent circuit of a resistance-capacitance coupled A-F amplifier over the low frequency band

The relation of the gain at the low frequencies to that at the medium frequencies, neglecting R_k, is expressed by:

$$\frac{A_l}{A_m} = \frac{\dfrac{R_l R_p}{R_l + R_p} + R_g}{\dfrac{R_l R_p}{R_l + R_p} + R_g + \dfrac{1}{j\omega C_c}} = \frac{1}{1 - j\dfrac{X'}{R'}}$$

where

$$X' = \frac{1}{\omega C_c}, \quad \text{and} \quad R' = \frac{R_l R_p}{R_l + R_p} + R_g.$$

Hence

$$\frac{A_l}{A_m} = \frac{1}{\left[1 + \left(\frac{X'}{R'}\right)^2\right]^{\frac{1}{2}}}. \tag{6}$$

Basing the calculations for the low-frequency cut-off on an attenuation of 1 decibel (voltage ratio of 0.9) at 10 cycles per second, the value of the coupling capacitor is computed from Equation (6) for the condition that $A_l/A_m = 0.9$

whence

$$X' = 0.48\, R'.$$

It has been stated that 0.56 megohm was chosen for R_l, and 1.5 megohms was selected for R_g to permit the use of a commercially available value of C_c. With R_p assumed to be about 2 megohms, $R' = 1.94$ megohms, and $X' = 0.48R' = 930{,}000$ ohms, $C_c = 1/2X' = 0.017$ microfarad. A miniature tubular form of capacitor is available in a standard value of 0.018 microfarad, which was considered close enough for use as C_c. The reduction in voltage amplification at the low frequencies, in decibels, as referred to the medium or center frequency, is expressed by

$$\frac{A_m}{A_l} = 20 \log_{10} \left[1 + \left(\frac{X'}{R'}\right)^2\right]. \tag{7}$$

The computed response curve for this value of C_c is shown as curve B, Figure 9.

The equivalent circuit of the amplifier stage in the high-frequency range, where the reactance of C_c is negligible and where the shunting effect of C_s becomes important, is shown in Figure 8.

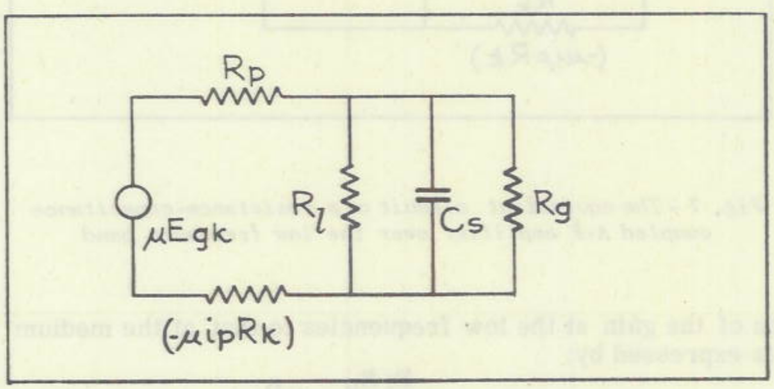

Fig. 8 - The equivalent circuit of a resistance-capacitance coupled A-F amplifier over the low frequency band

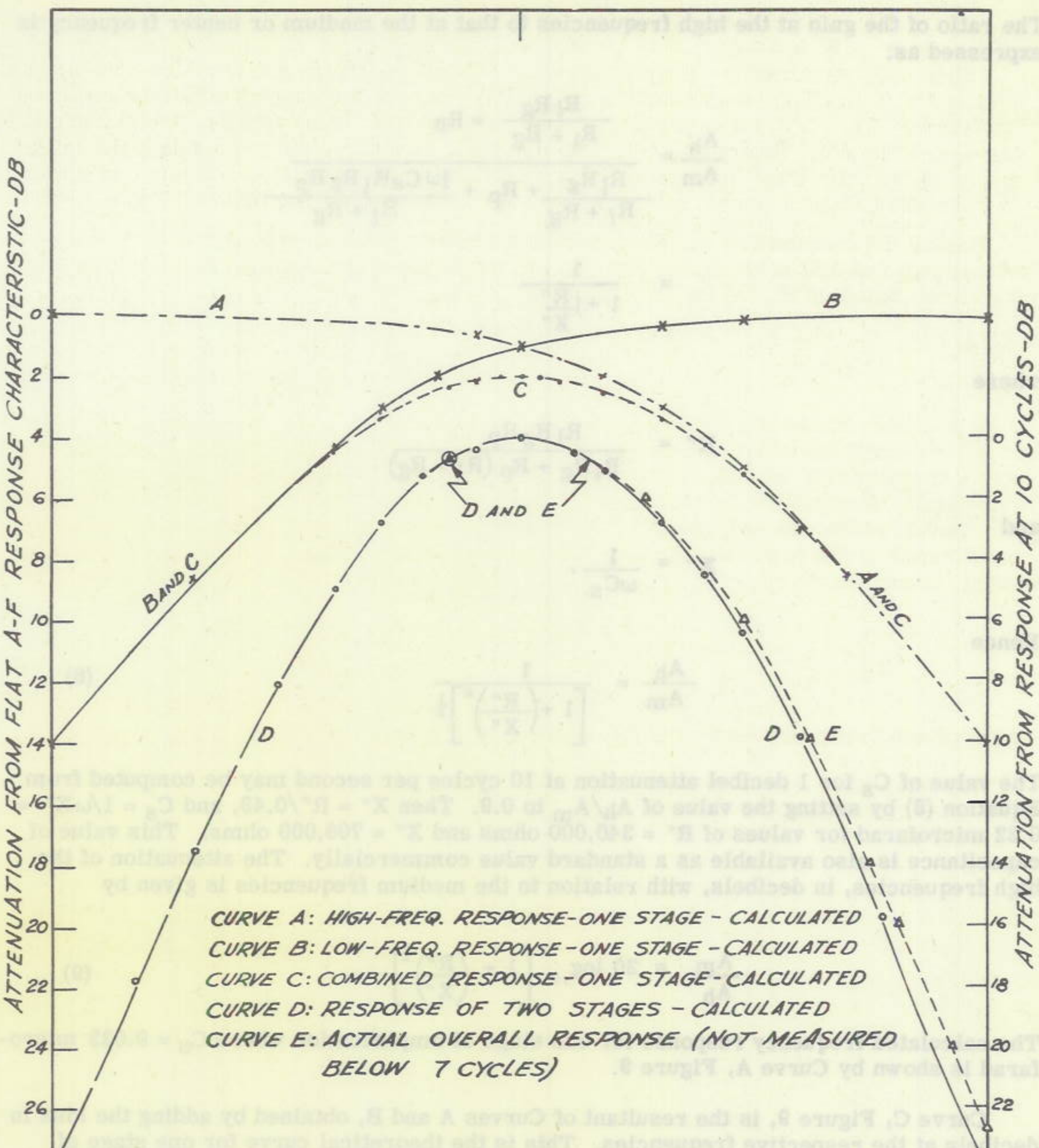

Fig. 9 - Frequency response characteristics of 10-cycle amplifier

The ratio of the gain at the high frequencies to that at the medium or center frequency is expressed as:

$$\frac{A_h}{A_m} = \frac{\dfrac{R_l R_g}{R_l + R_g} + R_p}{\dfrac{R_l R_g}{R_l + R_g} + R_p + \dfrac{j\omega C_S R_l R_g R_p}{R_l + R_g}}$$

$$= \frac{1}{1 + j\dfrac{R''}{X''}}$$

where

$$R'' = \frac{R_l R_g R_p}{R_l R_g + R_p (R_l + R_g)}$$

and

$$X'' = \frac{1}{\omega C_S}.$$

Hence

$$\frac{A_h}{A_m} = \frac{1}{\left[1 + \left(\dfrac{R''}{X''}\right)^2\right]^{\frac{1}{2}}} \tag{8}$$

The value of C_S for 1 decibel attenuation at 10 cycles per second may be computed from Equation (8) by setting the value of A_h/A_m to 0.9. Then $X'' = R''/0.48$, and $C_S = 1/\omega X''$ = 0.22 microfarad for values of R'' = 340,000 ohms and X'' = 706,000 ohms. This value of capacitance is also available as a standard value commercially. The attenuation of the high frequencies, in decibels, with relation to the medium frequencies is given by

$$\frac{A_m}{A_h} = 20 \log_{10} \left[1 + \left(\frac{R''}{X''}\right)^2\right]. \tag{9}$$

The calculated frequency response for one stage of amplification where C_S = 0.022 microfarad is shown by Curve A, Figure 9.

Curve C, Figure 9, is the resultant of Curves A and B, obtained by adding the loss in decibels at the respective frequencies. This is the theoretical curve for one stage of amplification, as calculated for the particular combination of C_c, C_S and the respective resistor values employed in the foregoing computations. Two identical cascaded stages would then increase the selectivity as shown by Curve D, Figure 9. Since the selectivity indicated by curve D appeared adequate, the triode-plate network of the final stage was designed to provide only the additional gain necessary for an over-all (three-stage) gain of approximately 1000 when working into an output impedance of 5000 ohms. Curve E of Figure 9 shows the actual over-all frequency response as measured through the range of 7 to 100 cycles per second, 7 cps being the low-frequency limit of the available generators.

MISCELLANEOUS DESIGN DETAILS

Figure 10 is a schematic diagram of the completed amplifier and power-supply circuits, and Figures 11, 12, and 13 are photographic views showing the physical arrangement of the components. With the relatively high impedance circuits employed in the first two stages, some difficulty was encountered initially with hum, apparently caused by capacitance coupling between the vacuum-tube grids and the ungrounded side of the heater circuit. The condition was remedied by "floating" both sides of the tube heaters, the ground connection being made through the slider contact of a 150-ohm wire-wound potentiometer which was shunted across the heater-supply winding. The slider contact was then adjusted for minimum hum with no signal at the input of the amplifier and left in that position.

The screen bypass capacitors are of the smallest value that could be used without objectionable loss of gain. A 2 x 0.1-microfarad bathtub-type capacitor was used to bypass the screens of both pentode amplifiers.

An RC plate isolation filter was found to be necessary in the first stage to prevent low-frequency oscillation of the amplifier due to common plate-supply coupling, and to supply needed additional filtering of the plate potential for the input circuit. Locating the plate isolation filter in the first amplifier stage, rather than the last, enabled its use for both purposes and afforded the added advantage of further reducing the hum component attributable to plate-supply ripple.

It will be noted in Figure 12 that the coupling and shunt capacitors used are not of the type mentioned in the discussion, but are a standard 0.01-microfarad mica type which is relatively large. These were hand-picked and parallel-connected to give the correct values. The miniature-type capacitors previously mentioned are manufactured by Sprague Electric Company, and carry the trade name "Prokar." The catalog number of the 0.018 μf condenser is 65P18392, and that of the 0.022 μf condenser is 65P22392. An order had been placed for these capacitors, but they were not received in time to have been included in the model.

CONCLUSIONS

It is concluded from the results of the above investigations and development that:

(a) It is practicable to use the frequency-selective characteristics inherent in RC coupling networks of the usual resistance-coupled amplifier to provide a desired selectivity.

(b) This technique is particularly useful for very low frequency audio applications and lends itself well to miniaturization of such equipment.

RECOMMENDATIONS

It is recommended that:

(a) In future applications, circuit constants, wherever possible, should be selected so that the two network capacitors C_s (shunt) and C_c (coupling) are of the same value, and that all stages have the same values of R_k (cathode resistor), R_l (plate load resistor), R_{sg} (screen-filter resistor) and R_g (grid-return resistor), except possibly the output stage.

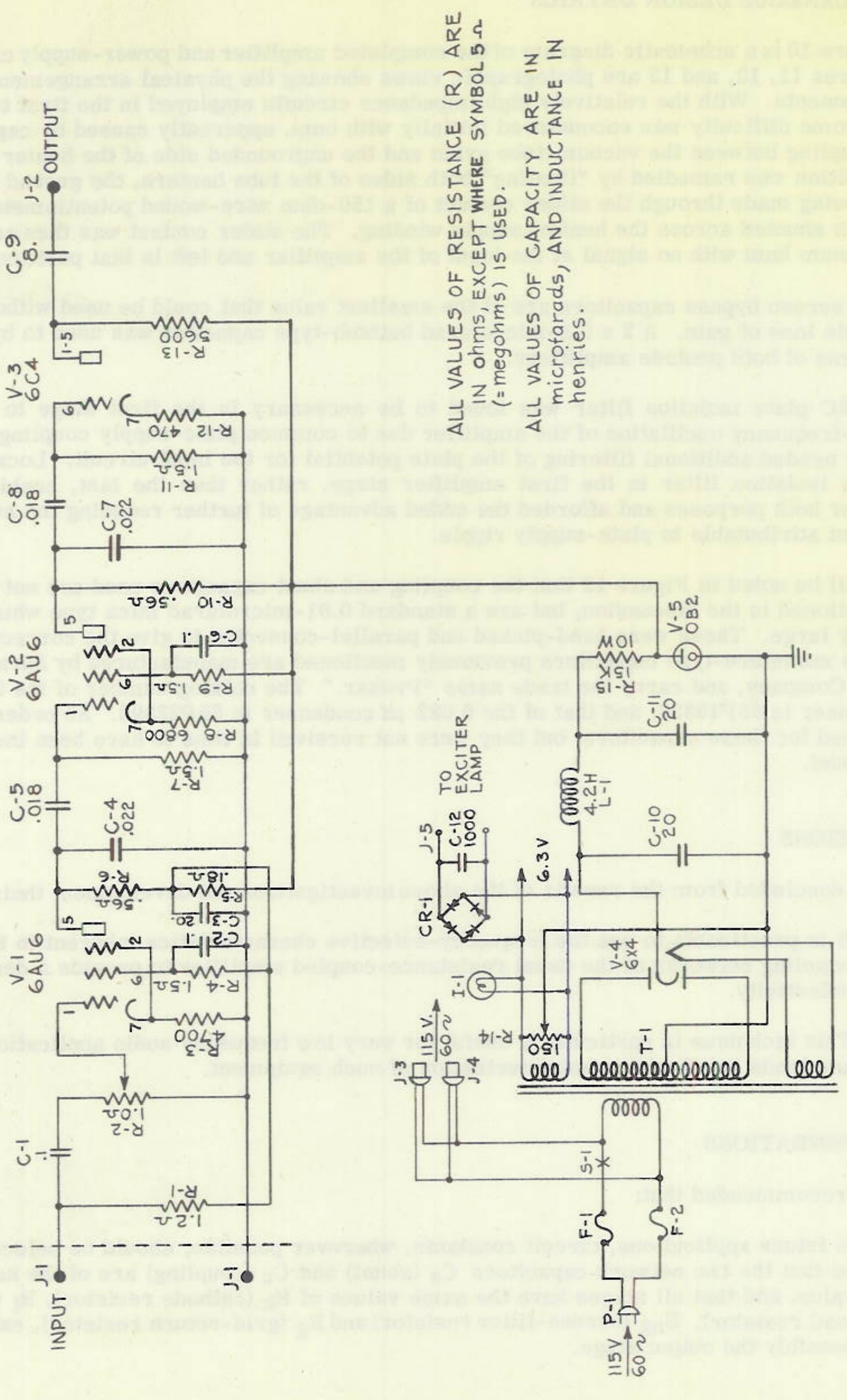

Fig. 10 - Schematic diagram of 10-cycle-per-second amplifier

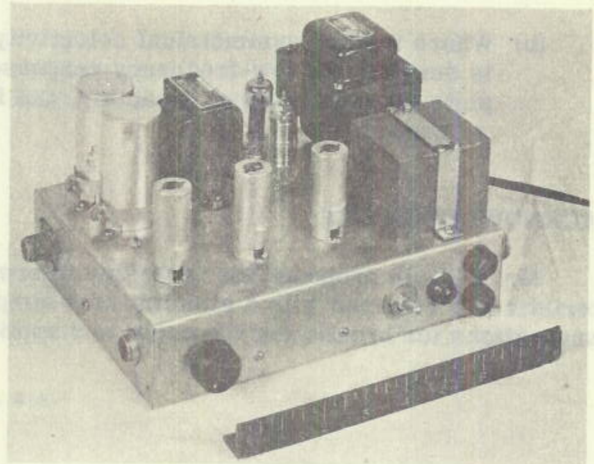

Fig. 11 - 10-CPS amplifier, left-front oblique view

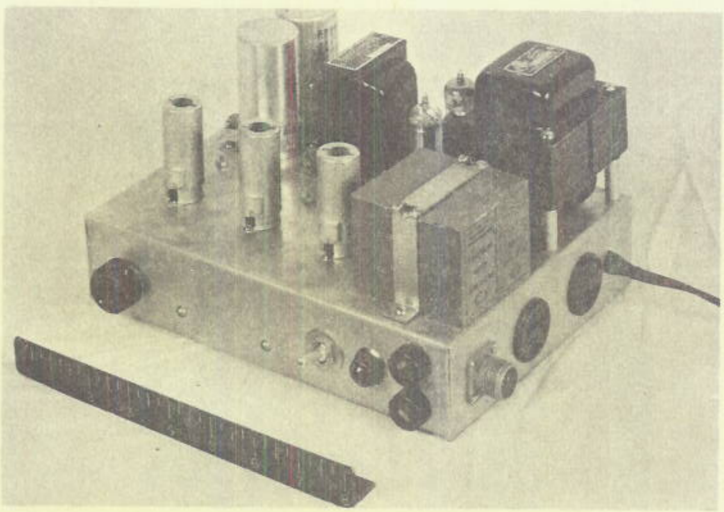

Fig. 12 - 10-CPS amplifier, right-front oblique view

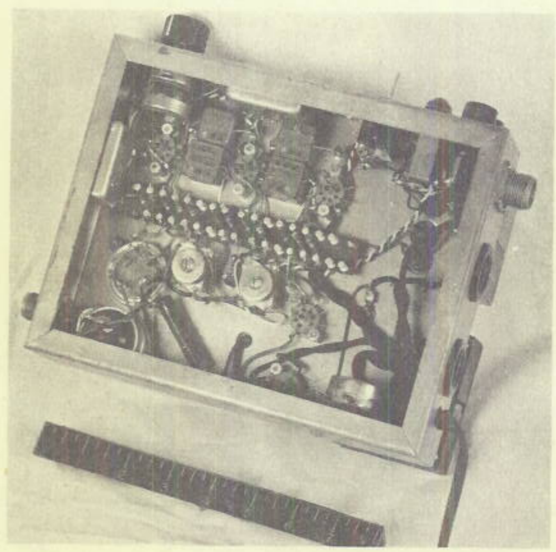

Fig. 13 - 10-CPS amplifier, bottom view of chassis

(b) Where a more symmetrical selectivity characteristic on a <u>linear</u> frequency scale is desired, the low-frequency response should be controlled by selection of the plate-filter (isolation) capacitor, and further shunt elements be added inter-stage, as needed.

ACKNOWLEDGMENT

Mr. E. Toth proposed the technique whereby the desired frequency-response characteristic was obtained with a simultaneous simplification of circuitry, and offered valuable suggestions for broadening the scope and application of this report.

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