UNCLASSIFIED

DECLASSIFIED

[redacted]

NRL REPORT 3507

FR-3507

FRAGMENT PENETRATION RESISTANCE LAWS IN THE THEORY OF AIRCRAFT VULNERABILITY

DECLASSIFIED: By authority of
NRL Classification Change Notice
No. 17-61 2 May 61
Sinda Jean Lambert 2028
Entered by NRL Code

DECLASSIFIED by NRL Contract
Declassification Team
Date: 12 JAN 2017
Reviewer's name(s): [redacted]
Declassification authority: NAVY DECLASS GUIDE/NAVY DECLASS MANUAL, 11 DEC 2012
D2 SERIES

[redacted]
Entered by NRL Code

DISTRIBUTION STATEMENT A APPLIES.
Further distribution authorized by UNLIMITED only.

NAVAL RESEARCH LABORATORY

WASHINGTON, D.C.

UNCLASSIFIED

[redacted] DECLASSIFIED

DECLASSIFIED

~~CONFIDENTIAL~~ UNCLASSIFIED

NRL REPORT 3507

FRAGMENT PENETRATION RESISTANCE LAWS IN THE THEORY OF AIRCRAFT VULNERABILITY

Robert E. Roberson, George R. Irwin and Waller George

UNCLASSIFIED

July 22, 1949

Mechanics Division-Dr. George R. Irwin, Superintendent (Acting)

NAVAL RESEARCH LABORATORY
CAPTAIN F. R. FURTH, USN, DIRECTOR
WASHINGTON, D.C.

UNCLASSIFIED ~~CONFIDENTIAL~~ DECLASSIFIED

DISTRIBUTION

BuOrd	
Attn: Code Re2b	1
Attn: Code Re2c	1
Attn: Code Re3	1
Attn: Code Re3a	1
Attn: Code Re9	2
Attn: Code Re9g	1
BuAer	
Attn: Code AR-AD-2	1
Attn: Code TD-4	2
BuShips	1
Attn: Code 424	1
CNO	
Attn: OP-57	2
Attn: OP-573	1
Attn: OP-36	1
ONR	
Attn: Code 432	1
CDR., USNOL	1
Attn: Mr. H. J. Plumley	1
Dir., USNEL	2
CDR., USNOTS	
Attn: Reports Section	2
Attn: Mr. L. T. Thompson	1
Attn: Dr. John Reinhardt	1
CO, USNPG, Dahlgren	
Attn: Dr. Lyddane	1
OCSigO	
Attn: Ch. Eng. & Tech. Div., SIGTM-S	1
CO, SCEL	
Attn: Dir. of Engineering	2
CG, AMC, Wright-Patterson AFB	6
Attn: Eng. Div., Electronics Subdiv., MCREEO-2	1
Deputy Chief of Staff, Materiel, Hqs., USAF	
Attn: Guided Missiles Group, AFOGM	1
Attn: Director Armament, AFMAR	1

DISTRIBUTION (CONT.)

Office of the Ch. of Ordnance	
Attn: Chief of Research and Development	2
Attn: ORDTM	1
CG, Aberdeen Proving Ground	
Attn: Ballistic Research Laboratory	1
Attn: Lt. Col. F. S. Allen, AMC Liaison Officer	1
Attn: Mr. R. O. Fleming	1
Attn: Mr. J. N. Sarmousakis	1
Attn: Mr. H. K. Weiss	1
Attn: Mr. T. S. Sterne	1
Attn: Mr. R. K. Kent	1
Attn: Mr. L. Simon	1
BAGR, CD, Wright-Patterson AFB	
Attn: CADO-D1	1
O-in-Ch., USNOEU, NBS	1
Attn: Ordnance Development Division	2
Attn: Div. of Research, OEG	1
Attn: Dr. A. V. Austin	1
Attn: Mr. Donald Burchan	1
RDB	6
Attn: Library	2
Attn: Navy Secretary	1
Attn: Dir. of Committee on Ord.	1
Attn: Guided Missiles Committee	1
Attn: Director	1
Naval Res. Sec., Science Div.	
Attn: Mr. J. H. Heald	2

[redacted]

[redacted]

CONTENTS

ABSTRACT

Estimates of the optimum velocity for guided missile warhead fragments made by Meyer, Morton and others of the Applied Physics Laboratory indicated values of the order of 10,000 fps. This report extends the theory on which this estimate has been made for the cases where damage to aircraft structures can reasonably be expected to follow the laws of fragment penetration into homogeneous targets such as armor plate.

In general upper bounds are found to exist on the optimum velocity if the penetration curve is steeper than a certain value or if it is discontinuous providing these characteristics exist at velocities less than or equal to $1.2a_o$ where a_o is the "Gurney" constant for the warhead explosive. An example of the first of these bounds is given in terms of previously unpublished data of George and Warner from firings of ½-inch steel spheres against 24S-T aluminum alloy plate. These data indicate in terms of the general extension of the theory developed in this report that an optimum velocity of about 6400 fps is to be expected. This is to be compared with values of approximately 9000 fps to be calculated from the Meyer-Morton treatment of the theory using a modified Poncelet type of penetration law.

PROBLEM STATUS

This is an interim report; work is continuing on the problem.

AUTHORIZATION

NRL Problem F05-01R (NR 435-010)

FRAGMENT PENETRATION RESISTANCE LAWS IN THE THEORY OF AIRCRAFT VULNERABILITY

INTRODUCTION

In designing warheads for guided missiles, the question of target vulnerability is of paramount importance. One must know, in particular, the mass and velocity of the warhead fragments which are most efficient in damaging a target.

A theory of aircraft vulnerability has been developed by Meyer-Morton[1] in which appears two basic assumptions. First, the theory assumes that the ability of fragments to damage complex aircraft and guided missile structures can be functionally related to the ability of the fragments to penetrate given thicknesses of homogeneous targets. The precise form of the assumption defines the simple target, the penetration of which is equivalent in some sense to an arbitrary degree of damage to the complex structures. Second, the theory assumes that the penetration of these homogeneous "equivalent" targets is functionally related to the fragment attack velocity, mass, and shape. This relation is commonly termed a "penetration law" when the striking velocities considered are the minimum values necessary to accomplish penetration. The constants in such penetration laws are governed, in part, by the properties of the target.

While it is clearly recognized that the first of the above assumptions is primary, it was thought interesting to study the sensitivity of the results of the Meyer-Morton theory to more general, physically reasonable, and experimentally verified forms of penetration law than were used by these authors. Therefore an integrated program of experimental and theoretical work was planned, and it was undertaken by the Naval Research Laboratory in cooperation with the Applied Physics Laboratory of Johns Hopkins University, the latter maintaining one or more staff workers at NRL continuously for this study since the summer of 1947. The NRL contribution is a portion of the early work sponsored by the Bureau of Ordnance on the general topic of penetration dynamics.

---

[1] *Meyer, Morton and Porter, "Aircraft Vulnerability as a Function of Fragment Penetration,"* *APL JHU Report TG-24, (Confidential) April 1, 1947*

In the Meyer-Morton theory, the explicit form of the first assumption was that the vulnerable area of a given aircraft or missile could be related to the thickness of a homogeneous target of the aluminum alloy, 24S-T. The second assumption took the form of an empirical penetration law of the deMarre type.[2,3]

The parameters were fitted to rather meager experimental fragment penetration data for 24S-T aluminum alloy over a range of striking velocities between 1,000 and 4,000 fps. The results of the theory indicated that optimum striking velocities were of the order of 10,000 fps and were independent of the fragment mass. It is obvious that the first of these results involved a very considerable extrapolation of the penetration law beyond the region over which it was verified.

The validity of the extrapolation of penetration data for attack velocities from 4,000 to 10,000 fps by means of any empirical law is subject to serious question in view of the existence of major changes in the physical mechanisms of the penetration resistance with increases in attack velocity above approximately 5,000 fps. The fact that only relatively small velocity changes are required to introduce major amounts of penetrator and/or plate shatter, restricts even the permissible extrapolation of physically reasonable penetration laws such as the modified Poncelet types which assume rigid penetrators.

For attack velocities above 5,000 fps, phenomena of plate and projectile shatter and impact flash with its associated pressure formation not only make the use of penetration laws questionable, but also probably seriously affect the first assumption of the theory.

It is important to note that conflicting factors are involved in using these assumptions beyond their valid ranges. Errors caused by the breakdown of one assumption may cancel those caused by the other. For example, shatter may appear to lead to lower optimum velocities, but associated impact flash and blast pressure effects may be sufficiently large in complex aircraft structures attacked by very high-velocity fragments to mitigate completely the fragment shatter characteristics associated with higher velocities.

The purpose of this report is to examine in greater detail the relationship between optimum velocity and the form of the penetration resistance law with a view of generalizing and extending one portion of the work of the APL group. First a discussion of the more realistic Poncelet type of penetration law is given. Nomographs are presented

---

2See a discussion in Cranz, C. "Lehrbuch der Ballistic." Berlin: Springer, 1926

3Beth, R. "Concrete Penetration." NDRC Report A-319 (OSRD Report 4856) *(U.S. Confidential), 1945* which yield the optimum velocity in terms of certain target parameters. This is followed by discussions of upper bounds on the optimum velocity introduced by certain characteristics of the penetration resistance laws which can be associated with the onset of fragment shatter, plate cavitation, and similar phenomena.

PRELIMINARY DISCUSSION

Although it is assumed that the reader is acquainted with the Meyer-Morton theory of vulnerability, the essential definitions and assumptions are repeated with some slight changes to prevent ambiguity.

Def. 1 - A target area is <u>vulnerable</u> to a fragment of a certain shape, mass, and velocity if such a fragment striking this area causes a specified degree of damage.

Def. 2 - Damage in a selected category is called a <u>kill</u>.

Def. 3 - Let each element of the target area vulnerable to fragments of a given character be projected as a sphere passing through the element of area and having a center at the burst. The totality of such projections, averaged over all possible orientations of burst with respect to the aircraft, is called the <u>vulnerable area</u> of the aircraft.*

Def. 4 - The <u>penetrance</u> of a fragment is the thickness of homogeneous plate, possessing given physical characteristics such as density and strength, required to allow the fragment just to penetrate for average orientation and given fragment velocity.

Notations used are:

m: fragment mass

V: fragment velocity

e: fragment penetrance

---

* *This is identical with the Meyer-Morton definition when the distance from the burst to the aircraft is large compared to the dimensions of the aircraft. Such is assumed to be the case. However, it must not be thought that Def. 3 is unique, since there are alternative definitions with intuitive appeal.*

A_f: average fragment area presented to the target

$A(m, V)$, $A(e)$ } vulnerable area with respect to fragments of mass m and velocity V, or of penetrance e

e_m: the solution of $A(e_m) = 0$ (Meyer-Morton use P_m)

e_o: a constant (Meyer-Morton use P_o)

C: mass of explosive charge of warhead

M: mass of metal in warhead

a_o: the "Gurney constant"

k: a constant for a given fragment (its form factor)

P, a: target penetration constants specified elsewhere in the text

$\overline{Q}$: the optimum value of any variable Q.

Other symbols are introduced as needed. Note that the principal deviation of this notation from that of Meyer-Morton is the replacement of their P by e. The symbol exp(x) rather than e^x is then used for the exponential function.

The assumptions of the theory are:

Assumption 1 - The air resistance to the fragment may be neglected. This has the effect of making burst velocity and fragment striking volocity interchangeable.

Assumption 2 - The fragmentation is controlled. All fragments from a given burst are of the same mass shape and initial velocity.

Assumption 3 - The initial velocity is given by the Gurney law

$$V = a_o\sqrt{\frac{C/M}{1 + ½\ C/M}}\ .$$

Assumption 4 - The number of kills from fragments of penetrance e obeys a Poisson distribution law.

Assumption 5 - The average presented area of a fragment is related to its mass by

$$M = k(A_f)^{3/2}$$

Note that this implies that the fragments are geometrically similar.

OPTIMUM VELOCITIES OF FRAGMENTS

The Efficiency Function

The efficiency function, ψ, chosen by Meyer-Morton is proportional to the number of kills per unit warhead mass. In view of their other results, this function takes the form (from Meyer, Morton, and Porter, op. cit., eq. 15)

$$\psi = \frac{1}{m}\left[\frac{1 - \exp\left\{-\frac{e - e_m}{e_o}\right\}}{1 + C/M}\right].$$

Using the Gurney law, a further reduction follows:

$$\psi = \frac{1}{m}\left(\frac{2\alpha_o^2 - V^2}{2\alpha_o^2 + V^2}\right)\left[1 - \exp\left\{-\frac{e - e_m}{e_o}\right\}\right]. \qquad (1)$$

The problem in warhead design is to choose the mass and velocity of the fragments in such a way that this efficiency is a maximum. Setting

$$\frac{\partial \psi}{\partial m} = \frac{\partial \psi}{\partial V} = 0 \, ,$$

as a necessary condition for maximization implies,

$$m\frac{\partial e}{\partial m} = \frac{4\,\alpha_o^4 - V^4}{8\,V\,\alpha_o^2}\,\frac{\partial e}{\partial V}\,. \qquad (2)$$

If this equation is now solved for V, the result is an optimum velocity, if one exists.

General Penetration Laws

Before Eq. (2) can be solved, however, a relation between e, m, A_f, and V must be known. Such a relation is a penetration law.* At the Naval Research Laboratory, the variables in terms of which the penetration law is written are

$$\left.\begin{aligned} X &= \frac{\rho\, e\, A_f}{m}\,, \\ Z &= \frac{\rho_f\, V^2}{2g}\,, \end{aligned}\right\}, \tag{3}$$

where ρ and ρ_f are target density and fragment density respectively, and g is the acceleration of gravity. The penetration law itself may be written as

$$Z = Z\,(X)$$

or

$$X = X\,(Z)$$

which from Eq. (3) and Assumption 5 yields,

$$e = m^{1/3}\, \phi\,(Z)\,, \tag{4}$$

where we define $\phi(Z) = \rho^{-1}\, k^{2/3}\, X\,(Z)$.

In this relation, the definition of the form factor k is taken as $k = m/(A_f)^{3/2}$. The form of the function $\phi\,(Z)$ is still unspecified.

In the general case, then, Eqs. (2) and (4) combine to give

$$V^4 + \frac{8g}{3\rho_f}\, a_o^2\, \frac{\phi(Z)}{\phi'(Z)} - 4a_o^4 = 0 \tag{5}$$

---

* *Strictly speaking this is for the average penetration since the law refers to penetration by fragments with average orientation.*

[redacted]

where the prime denotes differentiation with respect to the argument. When the form of the penetration law is specified, the solution of Eq. (5) for V gives an optimum fragment velocity.* It is noteworthy that the optimum velocity, given by a root of Eq. (5), is independent of the fragment mass. The shape of the fragment is implicit in $\phi(Z)$. It is measured by k, but since k does not appear in the ratio ϕ/ϕ', Eq. (5) is likewise independent of the shape. If ϕ is independent of m, Appendix I shows that a necessary and sufficient condition for Eq. (5) to be independent of shape is that ϕ have the form:

$$\phi = \phi_1(k)\ \phi_2(V).$$

It is not true that optimum mass is independent of optimum velocity. In deriving Eq. (5), the elimination between $\partial\psi/\partial m$ and $\partial\psi/\partial V$ lead to Eq. (2). A different combination of these two conditions might be expected to lead at once to an equation for optimum mass, $\overline{m}$, but such a relationship cannot be obtained directly. After finding the optimum velocity from Eq. (2), however, one may return to

$$\partial\psi/\partial m = 0,$$

which is of the form

$$\exp\left\{\frac{m^{1/3}\ \phi(\overline{Z})}{e_o} - \frac{e_m}{e_o}\right\} = 1 + \frac{1}{3}\ \frac{m^{1/3}}{e_o}\ \phi(\overline{Z}) .$$

It can be seen that, knowing e_m and e_o as the result of damage trials on aircraft, the optimum mass, $\overline{m}$, is determinable from this transcendental equation. If desirable, a nomograph can be easily constructed for giving $m^{1/3}$ as a function of e_o, e_m, and $\phi(\overline{Z})$.

The commonly observed (normal) form of the penetration law, when the maximum velocity observed is below approximately 4,000 fps, is shown in Figure 1 (A).**

The possibility and physical interpretation of multiple solutions for the optimum velocity is discussed in Appendix II.

***The orientation of axes in Figure 1 is standard for ballistic studies. In the discussion, however, Z is considered as the independent variable. Thus, contrary to custom, the axis of ordinates represents the independent variable.*

From practical considerations, the characteristics of the curve are:

$$\left.\begin{array}{l} \phi(Z) \geqslant 0, \\ \phi'(Z) > 0, \quad Z \geqslant 0 \\ \phi''(Z) < 0, \end{array}\right\} . \qquad (6)$$

However, for velocities greater than 5,000 fps, shatter, projectile deformation, and cavitation may profoundly alter the form of the penetration curve. Since such effects work to increase the striking velocity (and Z) required for a given depth of penetration, "abnormal" curves of the type shown in Figures 1(B) and 1(C) occur.

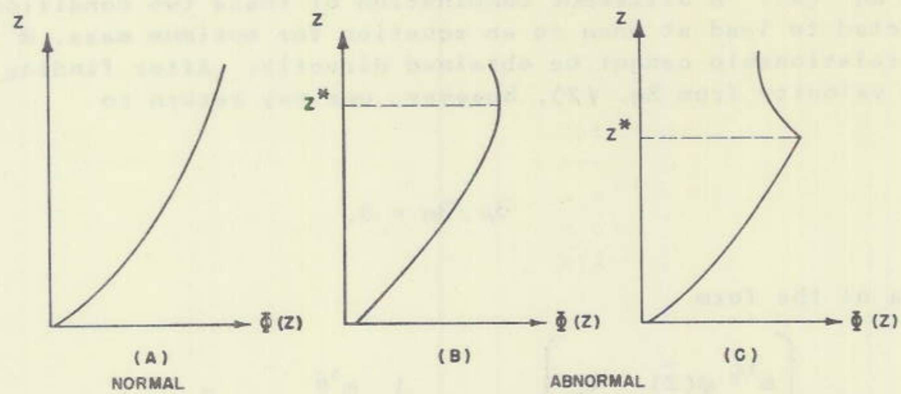


Figure 1 - General penetration curves.

In the case of projectile shatter such curves have been observed by the British[4] and by NDRC workers at Princeton University.[5] The existence of similar published results for fragment attack are not now known. Certainly the preliminary results of George and Warner[6] indicate the existence of analogous effects of fragment shatter, but further work is desirable. Target cavitation effects have been

---

4 *Milne, E.A., and Hinchcliffe, N. "On the Occurrence of Shatter." Proc. Ord. Board (British) Appendix to Proc. No. 20,002. Also issued as EBD Report No. 30 (British Secret, U.S. Confidential), November 1942*

5 *Emrich, R. J. and Mood, A. M. "The Shatter Velocity of Projectiles at Hypervelocities." OSRD No. 4357 (Confidential), November 1944.*

6 *George, W. and Warner, R., unpublished data on problem involving the firing of projectiles against armor plates, Naval Research Laboratory.*

observed both in America and Britain.[7] In addition to conditions of Eq. (6), the onset of fragment shatter, or of target cavitation at a certain $Z = Z^*$ may require that $\phi'(Z^*) = 0$, or possibly that $\phi'(Z^*)$ be discontinuous.

Statements can now be made about the value of the optimum velocity when the penetration curve is one of the three types discussed above and shown schematically in Figure 1. In Table 1 below are listed the formal requirements on $\phi(Z)$ and its derivatives which determine these classes of penetration curve.

TABLE 1

Normal Type "A" (Missile Unaltered During Penetration)	Abnormal Type "B" (Mild Missile Deformation)	Abnormal Type "C" (Major Missile Shatter or Deformation)
$\phi(Z) \geq 0,\ Z > 0.$	$\phi(Z) \geq 0,\ Z \geq 0.$	$\phi(Z) \geq 0,\ Z \geq 0.$
$\phi'(Z) > 0,\ Z > 0.$ $\phi''(Z) < 0,\ Z > 0.$	$\left.\begin{array}{l}\phi'(Z) > 0\\ \phi''(Z) < 0\end{array}\right\},\ 0<Z<Z^*.$	$\left.\begin{array}{l}\phi'(Z) > 0\\ \phi''(Z) < 0\end{array}\right\}\ 0<Z<Z^*.$
	$\left.\begin{array}{l}\phi'(Z) = 0\\ \phi''(Z) < 0\end{array}\right\},\ Z = Z^*.$	$\left.\begin{array}{l}\phi'(Z)\\ \phi''(Z)\end{array}\right\}$ Discontinuous for $Z = Z^*.$

Bounds on the Optimum Velocity

The results of this section will be stated as a series of theorems.

Theorem 1. If the Gurney law is obeyed, the optimum velocity is not greater than $\sqrt{2}\,\alpha_o$.
Proof: Trivial

[7] *Hill, R. "Cavitation Phenomena in Ductile Materials and the Dynamic Term in the Resistance to Penetration" (British) Armament Research Department (Fort Halstead, Kent). Theoretical Report No. 34/44 (British Secret, US. Confidential) 1944*

 [redacted]

Theorem 2. At the optimum velocity,

$$\frac{\phi'(\bar{Z})}{\phi(\bar{Z})} \geqslant \frac{2g}{3\rho_f\, \alpha_o^2} .$$

Proof: The optimum velocity must satisfy Eq. (5) so

$$\frac{\phi'(Z)}{\phi(\bar{Z})} = \frac{2g}{3\rho_f\, \alpha_o^2} \left[1 - \left(\frac{V}{\sqrt{2}\ \alpha_o} \right)^4 \right]^{-1} ,$$

$$\geqslant \frac{2\ g}{3\rho_f\ \alpha_o^2} .$$

Theorem 3. Provided conditions of Eq. (6) are satisfied and $\phi'(Z)$ is continuous, an optimum of efficiency with respect to velocity always exists.
Proof: From Eqs. (1) and (5)

$$\frac{\partial \psi}{\partial V} = \left[\frac{1}{\rho e_o k m^{2/3}} \left(4\alpha_o^4 - V^4 \right) \phi'(Z) \exp \left\{ - \frac{e - e_m}{e_o} \right\} - \frac{8\alpha_o^2\, V}{m} \left(1 - \exp \left\{ - \frac{e - e_m}{e_o} \right\} \right) \right] \left[2\alpha_o^2 + V^2 \right]^{-2} .$$

If V_m is the velocity corresponding to e_m,

$$\left. \frac{\partial \psi}{\partial V} \right|_{V_m} = \frac{4\ \alpha_o^4 - V^4}{\left(2\ \alpha_o^2 + V_m \right)^2} \ \frac{\left. \phi(Z) \right|_{V_m}}{\rho\ e_o\ k m^{2/3}} ,$$

[redacted]

since $$\phi(Z) \geqslant 0, \quad \partial\psi/\partial V\Big|_{V_m} \geqslant 0.$$

Similarly,

$$\frac{\partial \psi}{\partial V}\Bigg|_{V=\sqrt{2}\,\alpha_o} = \frac{1}{\sqrt{2}\,\alpha_o m}\left[1-\exp\left\{\frac{e-e_m}{e_o}\right\}\right]\Bigg|_{V=\sqrt{2}\,\alpha_o},$$

or $\frac{\partial \psi}{\partial V}\Big|_{V=\sqrt{2}\alpha_o} < 0.$ Therefore, $\partial\psi/\partial V = 0$ for at least one velocity in the velocity range $(V_m, \sqrt{2}\alpha_o)$. That ψ is a maximum at this point follows from the condition $\phi''(Z) < 0$.

Theorem 4. Let $\phi'(Z^*) = 0$ and V^* be the velocity corresponding to Z^*. If the performance curve satisfied conditions Eq. (6) for $V < V^*$, and if $\phi'(Z)$ is continuous, the optimum velocity is less than V^*.
Proof: An optimum velocity could not be as high as V^*, since at this point $\phi'(Z) = 0$, while Theorem 2 bounds ϕ' from below. But by Theorem 3, the optimum velocity exists. It must be concluded that the optimum velocity is less than V^*.

Theorem 5: If the performance curve satisfied conditions of Eq. (6) up to V^*, with $\phi'(Z^* - \epsilon) > 0$, $\phi'(Z^* + \epsilon) < 0$, ϵ, being an arbitrarily small positive number; the optimum velocity, $\bar{V}$, is equal to or less than V^*.

Proof: If $\frac{\partial \psi}{\partial V}\Big|_{V^* - \epsilon} < 0$, the argument of Theorem 3 applies with the result that $\bar{V} \leqslant V^*$. Suppose, however,

$$\frac{\partial \psi}{\partial V}\Bigg|_{V^* - \epsilon} > 0 \qquad \text{Since } \phi'(Z^* + \epsilon) < 0, \quad \frac{\partial \psi}{\partial V}\Bigg|_{V^* + \epsilon} < 0$$

and ψ has a cusped maximum at V^*. That is,

$$\bar{V} = V^*.$$

These theorems show that for penetration curves of the general "abnormal" types shown in Figure 1, the optimum velocity is never greater than V^*.

Discussion of Some Particular Penetration Laws

The penetration law used by Meyer-Morton is the deMarre formula, $X = a\,Z^{2/3}$, which is a special case of the more general formula,

$$X = a\,Z^{b}. \qquad (7)$$

Eq. (7) substituted into Eq. (5) gives an optimum velocity, $\overline{V}$, in terms of

$$\overline{V}^2 = \frac{2\alpha_o^2}{3b}\left[\sqrt{1 + 9\,b^2} - 1\right],$$

which is plotted in Figure 2. The function $\overline{V}/\alpha_o$ is not rapidly changing for b-values near 2/3, and therefore the Meyer-Morton estimate of $\overline{V} = 1.11$ will not be far in error for experimental curves fitted by b-values in this range. Figure 3 shows some experimental points obtained by firing 1/2-inch hard steel spheres at 24S-T. A least squares fit to Eq. (7) gives

$$X = 1.39\,Z^{(.594)},$$

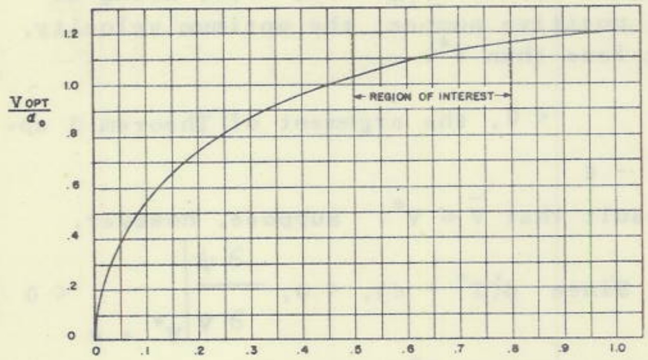


Figure 2 - Optimum velocities as a function of b in the empirical deMarre law of the type $X = aZ^b$.

which is the relation shown by the curve in Figure 3.

A penetration law based on sounder physical arguments is the so-called modified Poncelet law,

$$Z = \frac{\rho_f}{\rho}\,\frac{P}{\alpha}\left[\exp(\alpha X) - 1\right],$$

where P and α are physical constants of the target material. Solving this for X, one obtains,

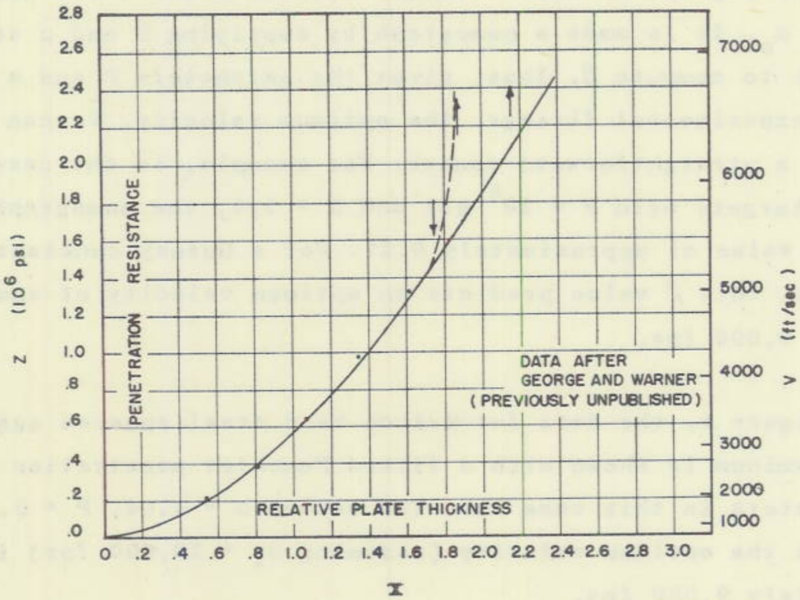


Figure 3 - Penetration resistance curve for ½-inch steel spheres against 24S-T aluminum, represented by $X = 1.39Z^{0.594}$ (Empirical deMarre type).

$$X(Z) = \frac{1}{\alpha} \ln \left[1 + \frac{\alpha \rho Z}{P \rho_f} \right] \qquad (8)$$

The substitution of Eq. (8) into Eq. (5)* and the introduction of $\beta = \rho\alpha/2gP$, $\xi = \beta V^2$ and $\gamma = 2\alpha_o^2$, leads to

$$(1 + \xi)\ln(1+\xi) = \frac{3}{2}\left(\beta\gamma - \frac{\xi^2}{\beta\gamma}\right),$$

---

*Note: $\frac{X}{X'} = \frac{\phi}{\phi'}$

which establishes $\bar{V}$ as a function of β and the Gurney constant a_0. Figure 4 is a graph of this relation between β and $\bar{V}$ for several values of a_0. It is made a nomograph by supplying P and a scales from which to compute β. Thus, given the parameters P and a determined by experimental firings, the optimum velocity, $\bar{V}$, can be obtained in a straightforward manner. For example, in the case of aluminum targets with $P = 10^6$ psi and $a = 1.4$, the nomograph indicates a β value of approximately 0.27. For a Gurney constant of 10,000 fps, this β value predicts an optimum velocity of somewhat less than 9,000 fps.

In Figure 5, the data for ½-inch hard steel spheres against 24 S-T aluminum is shown with a fitted Poncelet penetration law. The parameters in this case have the values $a = 1.04$, $P = 0.115$, from which the optimum velocity (assuming $a_0 = 10{,}000$ fps) is approximately 9,000 fps.

A further specification of the penetration resistance law correcting the modified Poncelet expression for fragment penetrator deformation has been given by Irwin and Roberson,[8] but in view of the large number of disposable constants in these relations, which at present are only vaguely known, it is considered superfluous to discuss this penetration resistance law in detail.

Thus far, the optimum velocities have been computed on the assumption that the penetration laws are of the stated forms for velocities as high as the optimum velocity, and the results obtained compare reasonably well with the optimum velocity calculated by Meyer-Morton.

---

[8] *Irwin, G. R. and Roberson, R. E., "Representation of Penetration Resistance," Memo from NRL to BuShips, Serial C-440-6/45* (Confidential), February 19, 1945

DECLASSIFIED

[redacted]

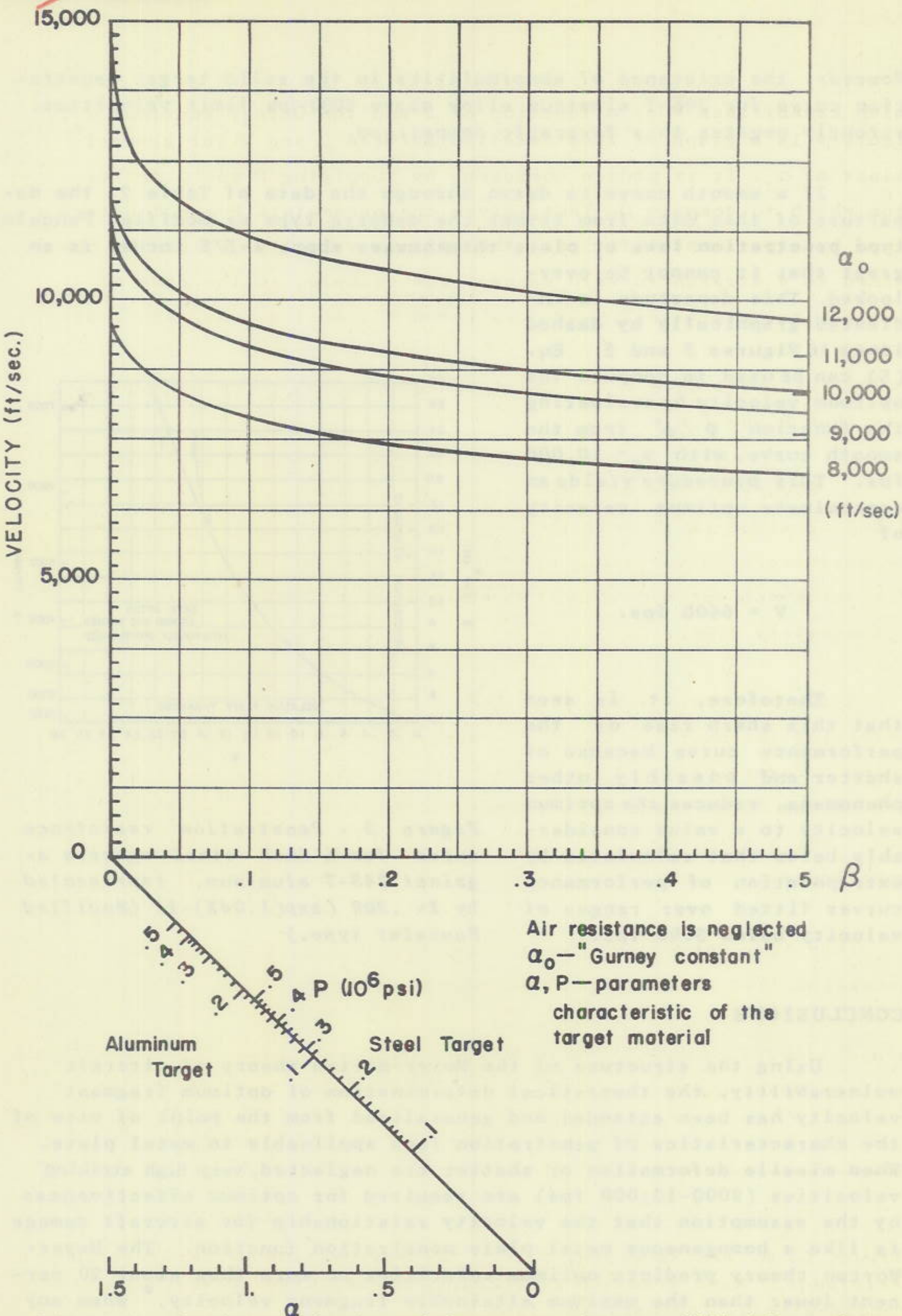


Figure 4 - Optimum velocity of fragments for a modified Poncelet penetration resistance law

DECLASSIFIED

[redacted]

However, the existence of abnormalities in the solid target penetration curve for 24S-T aluminum alloy above 6000-fps limit velocities, strongly negates this favorable comparison.

If a smooth curve is drawn through the data of Table 2, the departure of this data from either the deMarre type or modified Poncelet type penetration laws at plate thicknesses above 1-5/8 inches is so great that it cannot be overlooked. This departure is indicated graphically by dashed lines in Figures 3 and 5. Eq. (5) can be used to compute the optimum velocity by evaluating the function ϕ / ϕ' from the smooth curve, with $a_o = 10,000$ fps. This procedure yields an approximate optimum velocity of

$$\bar{V} = 6400 \text{ fps}.$$

Therefore, it is seen that this sharp rise of the performance curve because of shatter and possibly other phenomena, reduces the optimum velocity to a value considerably below that calculated by extrapolation of performance curves fitted over ranges of velocity below 5000 fps.

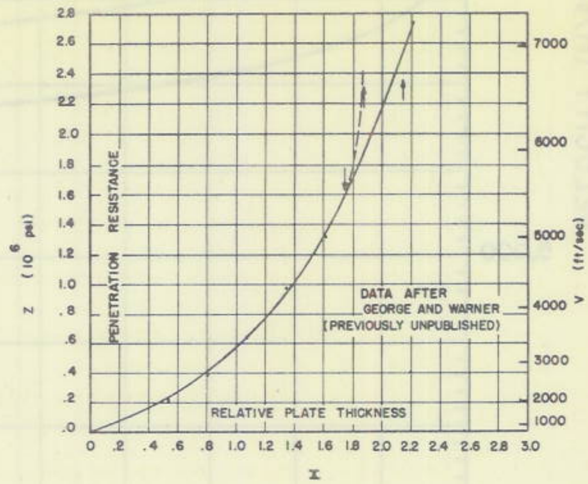


Figure 5 - Penetration resistance curve for ½ inch steel spheres against 24S-T aluminum, represented by Z= .308 (exp(1.04X)-1) (Modified Poncelet type.)

CONCLUSIONS

Using the structure of the Meyer-Morton theory of aircraft vulnerability, the theoretical determination of optimum fragment velocity has been extended and generalized from the point of view of the characteristics of penetration laws applicable to metal plate. When missile deformation or shatter are neglected, very high striking velocities (9000-10,000 fps) are required for optimum effectiveness by the assumption that the velocity relationship for aircraft damage is like a homogeneous metal plate penetration function. The Meyer-Morton theory predicts optimum velocities no more than about 20 percent lower than the maximum attainable fragment velocity,* when any

*Established by the Gurney relationship as $\sqrt{2a_o}$.

TABLE 2

½-inch Hard Steel Spheres Against 24S-T Aluminum
(Previously unpublished data by George & Warner)

e (in.)	X	Z (10^6 psi)
1/2	0.538	0.212 ± .020
3/4	0.807	0.381 ± .011
1	1.076	0.630 ± .019
1-1/4	1.345	0.972 ± .025
1-1/2	1.614	1.319 ± .041
1-5/8	1.749	<1.618 *
1-3/4	1.883	>2.249 *
2	2.152	>2.382 *

e: Plate thickness, inches
X: $= \rho e A_f/m$, dimensionless
Z: $= \rho_f V^2/2g$, surface density, lb. mass/sq. in.

*These data plotted as arrows indicating upper or lower limits to the penetration curve in Figures 3 and 5. The tips of arrows pointing down indicate upper limits — the tips of those pointing up indicate lower limits.

of a representative set of penetration laws is used in this theory. However, actual penetration curves depart widely from any of these commonly used penetration functions with onset of projectile deformation or shatter. This departure occurs at velocities much lower than the maximum attainable warhead fragment velocity. Reexamination of the aircraft damage theory showed that the velocity region where missile deformation or shatter grossly affects the experimental plate penetration curve lies closely above the theoretical optimum velocity for maximum aircraft vulnerability. If the results for steel spheres penetrating 24S-T aluminum are used as an aircraft damage-velocity relationship, the theoretical optimum velocity for warhead fragments lies in the range of 6000 to 7000 fps.

ACKNOWLEDGMENT

The over-all interpretation was considerably clarified by experimental work of Mr. Robert Warner and Mr. Bruce Johnston, both employees of Johns Hopkins University, Applied Physics Laboratory, working at the Naval Research Laboratory. Valuable assistance was provided by shop personnel, especially Mr. H. Whitman. Mr. Clifford Kingsbury has been helpful on matters pertaining to procurement of guns and powder. Lastly, Mr. David McGogney has been of considerable assistance in connection with some of the later experimental work.

* * *

APPENDIX I

Discussion of the Independence of $\bar{V}$ and m

The fact that Equation (2) yields a value of the optimum velocity, $\bar{V}$, independent of the mass and shape of the fragment, follows directly from the particular penetration law used in Equation (1). This result can be made somewhat more general by considering the conditions to be placed on Φ in the penetration law which lead to optimum velocity values independent of M and k separately or together.

The function Φ may be considered in its most general form as

$$\Phi = \Phi\,(m, V, k), \qquad (9)$$

with

$$e = m^{1/3}\,\Phi\,(m, V, k). \qquad (10)$$

The optimization requires that when $m = \bar{m}$ and $V = \bar{V}$,

$$m \frac{\partial e}{\partial m} = \frac{4\,\alpha_o^4 - V^4}{8V\alpha_o^2} \; \frac{\partial e}{\partial V}\,. \qquad (11)$$

Using Equation (10), Equation (11) becomes

$$m\left[\tfrac{1}{3}\, m^{-2/3}\,\Phi + m^{1/3} \frac{\partial \Phi}{\partial m}\right] = \frac{4\,\alpha_o^4 - V^4}{8V\alpha_o^2}\, m^{1/3} \frac{\partial \Phi}{\partial V}\,,$$

which reduces to

$$\tfrac{1}{3}\Phi + m \frac{\partial \Phi}{\partial m} = \frac{4\alpha_o^4 - V^4}{8V\alpha_o^2} \; \frac{\partial \Phi}{\partial V}\,, \qquad (12)$$

or

$$\frac{4\,\alpha_0^4 - V^4}{8V\alpha_0^2} = \frac{\partial\Phi/\partial V}{\frac{1}{3}\Phi + m\,\partial\Phi/\partial m} = F(V)\,. \qquad (13)$$

The left side of Equation (13) is independent of m and k; thus, the right side Equation (13) provides a criterion for the optimum to be independent of m and k. No general condition on Φ can be obtained which yields a Φ function independent of m and k. However, if one assumes Φ to be independent of m, i.e., $\partial\Phi/\partial m = 0$, one finds by inspection of Equation (13) that the optimum velocity will be independent of m. Further we note for this case that,

$$\frac{d\Phi}{\Phi} = \frac{F(V)}{3}\,dV \;;$$

whence

$$\ln \Phi = \tfrac{1}{3} \int F(V)\,dV + C(k)$$

$$\equiv I(V) + C(k)\;,$$

or

$$\Phi = \exp I(V)\, \exp C(k) = \Phi_1\,(V)\,\Phi_2\,(k)\,. \qquad (14)$$

Hence a necessary condition for

$$\Phi\;(m,\; V,\; k)$$

to yield optimum velocity values independently of k and m is that

$$\Phi\ (m,\ V,\ k) = \Phi_1\ (V)\ \Phi_2\ (k)\ .$$

This condition is also sufficient. The penetration law used in the text not only is such that $\partial\Phi/\partial m = 0$, but, in terms of Equation (14), Φ_2 = constant.

* * *

APPENDIX II

Multiple Solutions for Optimum Fragment Velocity

In this appendix, we are concerned with the possibility that Equation (5) has more than one physically meaningful solution (i.e. a real solution with V in the range $(0, \sqrt{2}\alpha_o)$). This, of course, would raise the question of whether the solution we have considered, which may be called the primary solution, is actually optimum.

In order to examine the solutions of this equation,

$$V^4 + \frac{8g}{3\rho_f}\alpha_o^2 \frac{\Phi(Z)}{\Phi'(Z)} - 4\alpha_o^4 = 0, \qquad (15)$$

and to relate them to various normal and abnormal penetration curves, let us write the equation in a nondimensional form by the introduction of the new quantities

$$\left.\begin{aligned} y &= (V/\sqrt{2}\,\alpha_o)^2 \\ \lambda &= 2g/3\rho_f\,\alpha_o^2 \end{aligned}\right\} . \qquad (16)$$

Then Equation (15) becomes

$$1 - y^2 = \lambda\Phi(y)/\Phi'(y) \equiv f(y). \qquad (17)$$

Since Z is proportional to y, the liberty has been taken of maintaining the symbol Φ for the penetration function, without regard for the scale change in its independent variable.

Equation (17) may have solutions of various kinds depending upon the nature of Φ. Consider Figures 6, 7, and 8 in which the various behaviors of Φ are shown in the (A) presentation of each chart, and the corresponding solutions of Equation 17 in

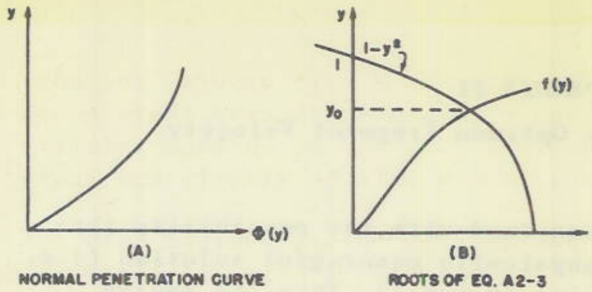


Figure 6 - Roots of equation (17) for normal presentation curve

the (B) presentation. The method for locating the roots of this equation is to plot $1-y^2$ as a function of y, and f(y) as a function of y. The latter depends only upon the nature of the penetration curve. The roots of Equation (17), of course, are the y-values corresponding to the intersections of these curves.

The arguments for establishing the behavior shown in these figures is as follows. With respect to the normal penetration curve as characterized in Table 1, the quantities $\phi(y)$ and $\phi(y)$ are always positive, whence

$$\frac{df}{dy} = 1 + \frac{\Phi\Phi''}{[\Phi']^2} > 0, \qquad (18)$$

and the f-curve is essentially like that shown in Figure 6B. There can be only one solution y_0 to Equation (17) in this case.

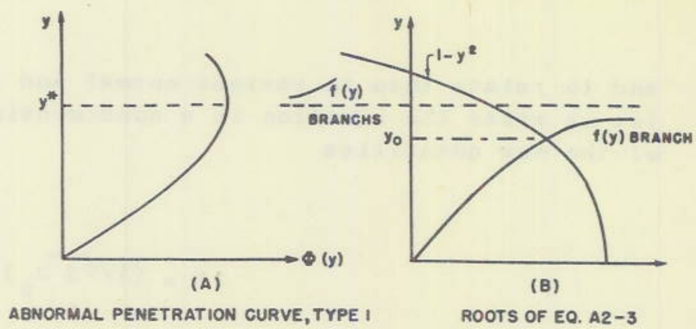


Figure 7 - Roots of equation (17) for abnormal penetration curve, type 1

For the abnormal but smooth penetration curve shown in Figure 7, f(y) has an infinity at some value of y, say y*. If y > y*, the function f(y) is negative, and any intersection of branch 2 of this curve with the curve $1-y^2$ must occur for y > 1, placing the intersection outside our range of physical interest.

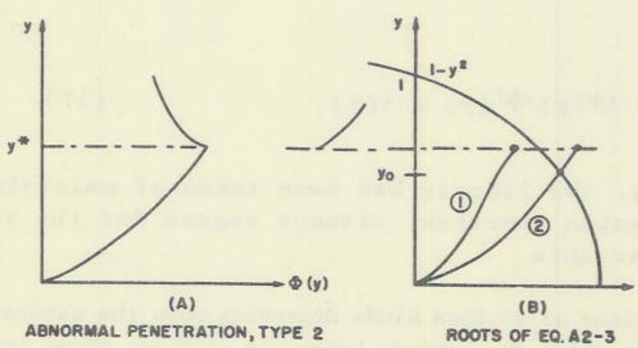
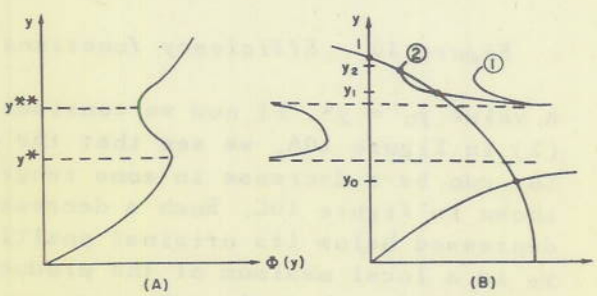


Figure 8 - Roots of equation (17) for abnormal penetration curve, type 2

The argument is similar for the case of abnormal penetration curves of Type 2, as shown in Figure 8. Provided the slope of the Φ-curve does not again become positive for y > y*, then the intersection of the second branch of the f(y)-curve with the $1-y^2$ curve must again occur for y > 1. Here, however, a phenomenon occurs which may be unexpected. If the value of f(y) does not become sufficiently large before the cusp at $y = y^*$ occurs, one may have a curve as shown by (1) of Figure 8B, in which case there would not be any real root at all of Equation (17) and no optimum velocity would exist. If curve (2) maintains, there will be exactly one root.

It thus appears that if the penetration curves exhibit only the gross characteristics shown above, no multiple solution of interest of Equation (17) exists, and one can say that any solution found is physically unique.

It might be noted that multiple solutions could conceivably appear in the case where a penetration curve has sufficiently large negative curvature or slope discontinuities equivalent to this. Such behavior cannot be ignored, since it can exist physically. For example, the penetration curve may be of the type shown in Figure 9 (or a broken curve of the same general nature). In this figure, it can be seen that the possibility of two roots y_0 and y_1 of Equation (17) is very real. Actually, there could be 0, 1, or 2 roots in addition to y_0. If $\Phi(y)$ is a piecewise smooth curve of this same general type, it is likely that there would be 0 or 1 root in addition to y_0.



A POSSIBLE PENETRATION CURVE ROOTS OF EQ. A2-3

Figure 9 - Roots of equation (17) for possible penetration curve

Let us consider the case of three roots, y_0, y_1, y_2, in some detail. It is clear from a consideration of $\partial\psi/\partial y$ that y_0, y_2 represent maxima and y_1 a minimum of the efficiency function ψ. It remains to see the relative magnitudes of

$$\psi_0 = \psi(y_0) \text{ and } \psi_2 = \psi(y_2).$$

In terms of y, the efficiency function ψ may be represented as

$$\psi \; \alpha \left(\frac{1-y}{1+y}\right)\left(1 - \exp\left[C_1 - C_2 \Phi(y)\right]\right). \qquad (19)$$

It is assumed that attention is confined to a certain value of m which remains fixed during the discussion. The two factors of this expression are graphed in Figure 10B for the penetration curve denoted by (1) in Figure 10A. One knows that the maximum of the product curve occurs at

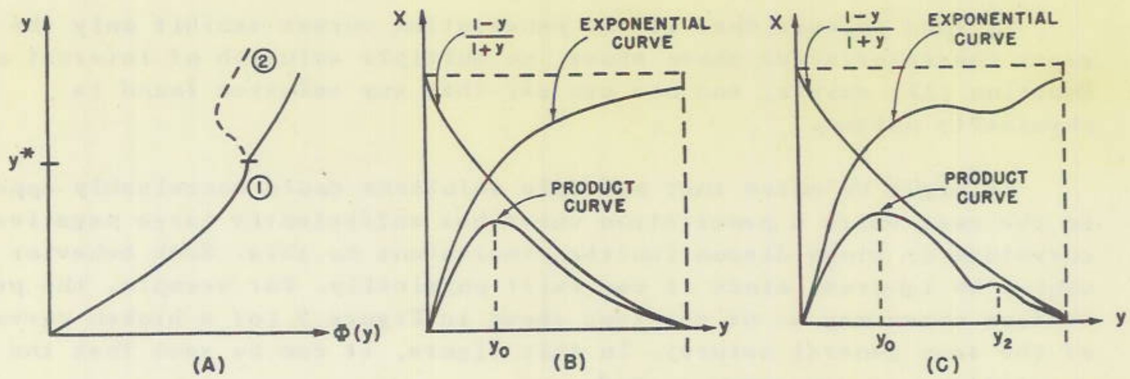


Figure 10 - Efficiency functions for two penetration curve types

a value $y_0 = y^*$. If now we consider the curve (2) formed by extending (1) in Figure 10A, we see that the only change in the exponential factor can be a decrease in some range of y-value beyond y_0. This is shown in Figure 10C. Such a decrease causes the product curve to be depressed below its original position for $y = y_0$. Thus even if some y_2 is a local maximum of the product curve (ψ-curve), this local maximum is below that given by y_0.

While counterexamples to the above behaviors might be constructed by sufficiently pathological penetration curves, the argument above indicates that for any such curves which seem to be physically reasonable, there exists only one solution to Equation A2-3 which is physically significant. Furthermore, one concludes that when two solutions exist mathematically, the one with the lower value is the significant solution.

* * *

  [redacted]