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# TRANSIENT ANALYSIS OF VOLTAGE-REGULATED AIRCRAFT D-C SYSTEMS

D. G. Scorgie

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Approved by:

N. W. Matthews, Head, Operational Equipment Branch
W. C. Hall, Superintendent, Electricity Division

NAVAL RESEARCH LABORATORY
CAPTAIN F. R. FURTH, USN, DIRECTOR
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## ABSTRACT

The stability of a d-c voltage-regulated aircraft system can be determined by the basic principles of feedback amplifier theory. The transfer function of a shunt-wound d-c generator has been defined and calculated, and a method given for accurately measuring the generator transfer function. There is a calculation of the terminal voltage transient resulting from removal of full load. For this purpose a simplified expression for the transfer function of a carbon pile voltage regulator is presented. The calculated terminal voltage transient is found to be in essential agreement with experiment.

## PROBLEM STATUS

This is an interim report; work on the problem is continuing.

## AUTHORIZATION

NRL Problem E03-04R (BuAer Reference, Engineering Division Aer-E-312-JWA, F36-1(1), 21 December 1945)
NR 423-003

# TRANSIENT ANALYSIS OF VOLTAGE-REGULATED AIRCRAFT D-C SYSTEMS

## INTRODUCTION

The analysis presented here is intended to provide a framework within which to investigate the factors affecting and methods of improving voltage regulation in aircraft d-c systems. In this report a system consisting of one shunt-wound generator and a carbon pile voltage regulator[1] has been analyzed. Basic principles of the theory of servomechanisms and feedback amplifiers have been utilized. The essential elements of the more general problem are present in the relatively simple system which has been dealt with here.

The great usefulness of servomechanism theory is due in part to the fact that a complex system may be broken into smaller components, and the characteristics of each analyzed and measured separately. This advantage is lost in analyzing by the usual approach[2] a system containing a carbon pile voltage regulator. The d-c reference voltage for the system is furnished by a critical balance of forces within the carbon pile regulator. If one allows this reference voltage to enter the study, the several transfer functions and the feedback loop within the regulator must be analyzed. This analysis is quite difficult, and cannot be checked empirically. Therefore the entire regulator is treated as a single component in a closed control loop.

The resulting analysis is not without disadvantages. The disadvantages, however, lie in the need for initial clarity and understanding rather than in the use of the results. In this report the inputs to the generator-regulator system are considered to be perfect equivalent-voltage generators introduced in the generator armature and field circuits because of external disturbances such as the removal of generator load. The variable, $E_0(t)$, corresponding to servomechanism output is defined as the contribution to the generator terminal voltage due to field resistance changes and other inputs to the field. To conform with standard terminology, this quantity is called the output. However, it is the algebraic sum of this output and the armature input which actually appears at the generator terminals and directly affects the voltage regulator. This sum is the error, $\epsilon_1(t)$, which is the variable to be controlled, and it becomes convenient later to define and utilize the error transfer function.

Both the generator and the regulator are highly nonlinear components, and the latter introduces the feature of a purposely variable parameter, namely carbon pile resistance.

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[1] Mills, R. L., "Dynamic Characteristics of Carbon Pile Voltage Regulators," NRL Report 3519 (Unclassified), September 10, 1949

[2] Whitely, A.L., "Theory of Servo Systems," J. Institute of Electrical Engineers," Pt. II, August 1946

However, linearizing assumptions have been made, and consequently it has been possible to derive the transfer functions of both regulator and generator. The utility of these approximations is illustrated by the close agreement between calculated and measured terminal voltage transients which are presented in this report.

By means of the frequency spectrum analysis technique discussed briefly in a later section, qualitative predictions of system performance can be made from an inspection of the transfer function of the control loop. The loop transfer function is easily determined from the derived transfer functions of the individual components. In turn, the effect of a single parameter change upon a component of the loop can be easily seen. The method, therefore, provides a swift, convenient tool with which to investigate methods of improving voltage regulation.

## THE REGULATOR TRANSFER FUNCTION

The system which will be analyzed consists of a shunt wound d-c generator and a carbon pile voltage regulator connected as shown in Figure 1. Each of these components is highly nonlinear. It was pointed out in an earlier report[3] that the carbon pile regulator remained linear for small changes from some arbitrary reference condition in the operating range. The transfer function of a stable linear component was defined there as the complex ratio (a function of frequency) of a sinusoidal output variation to the sinusoidal input which produced it.[4] In the case of the carbon pile regulator, the input quantity is a voltage disturbance at the operating coil terminals, and the output is a variation of the resistance of the carbon pile. Each is measured with respect to some arbitrary reference point taken as zero in the operating range. The transfer function of the regulator was derived as

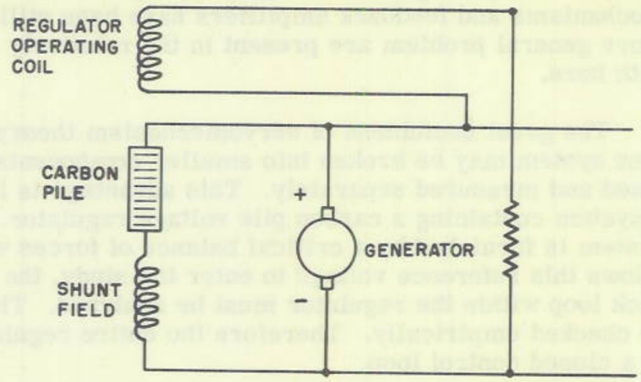


*Figure 1 - Regulated aircraft d-c generator*

$$T_r = \frac{UP}{R_cK - \omega^2 R_cM + j\omega(LK + U^2 - \omega^2 LM)}, \qquad (1)$$

where

U = a coupling coefficient equal to the rate of change of magnet force with operating coil current;

P = the carbon pile resistance change per unit change of carbon pile length;

$R_c$ = the resistance of the operating coil circuit;

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[3] *Mills, op. cit.*

[4] *An alternative definition is the ratio of the Laplace transforms of output to input, the input being an arbitrary function of time.*

K = the difference in rates of change of force with deflection between the magnet curve and the carbon pile curve;

M = the effective mass of the armature;

L = the incremental inductance of the operating coil.

The transfer function given in equation (1) is unwieldy, but can be simplified for use in analyzing the system under study. The simplification relies upon the fact that, for frequencies above a certain low band, the characteristics of the regulator contribute very little to the system performance. If the assumption is made that the top of this band is considerably below the frequency for which the regulator phase shift is $90^\circ$, the equation (1) can be approximated by

$$T_r = K_r \left( \frac{\alpha_r}{j\omega + \alpha_r} \right), \qquad (2)$$

where

$$K_r = \frac{UP}{KR_c},$$

and

$$\alpha_r = \frac{R_c K}{U^2 + KL}.$$

THE GENERATOR TRANSFER FUNCTION WITH NO LOAD

The generator no-load transfer function will first be defined assuming that the output of the regulator is the sole input to the generator. That is to say—the generator will be disturbed by a variation, R, of shunt field resistance.

Suppose for the moment that the generator is operating in its normal no-load condition, but with the regulator disconnected. If the field resistance is suddenly changed by a small step, $\Delta R$ ($\equiv R_2 - R_1$), the resulting terminal voltage transient is $E_O(t)$. The generator transfer function $T_g'$, is the ratio of the Laplace transforms of these time functions,[5]

$$T_g' = \frac{E_o}{R}(s).$$

In order to derive the transfer function, we assume that no-load saturation curve of the generator is a straight line with slope $K_S$ at the operating point. The following differential equation would be exact throughout the transient:

$$E_o(t) = L_f \frac{di(t)}{dt} + R_2 i(t) + I_1 \Delta R, \qquad (3)$$

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[5] *In this report the same symbols will be used for time functions and their Laplace transforms with the appropriate variable (t or s) indicated at all times.*

where

$i$ = the change of field current from the initial value

$I_1$ = the initial shunt field current.

$L_f$ = incremental self inductance of the field circuit.

But the saturation curve was assumed to be

$$E_t(t) = K_1 + K_S I_f(t), \tag{4}$$

so that

$$E_0(t) = K_S i(t). \tag{5}$$

From (3) and (5)

$$-I_1 \Delta R = \frac{L_f}{K_S}\frac{dE_0(t)}{dt} + \frac{E_0(t)}{K_S}\left(R_2 - K_S\right), \tag{6}$$

the Laplace transform of (6) is

$$-K_S \frac{I_1 \Delta R}{s} = E_0(s)\left[sL_f + R_2 - K_S\right], \tag{7}$$

or

$$E_0(s) = \frac{\Delta R}{s}\left[\left(\frac{K_S I_1}{R_2 - K_S}\right)\left(\frac{\frac{R_2 - K_S}{L_f}}{s + \frac{R_2 - K_S}{L_f}}\right)\right]. \tag{8}$$

If $\Delta R$ is very small relative to $R_1 - K_S$, equation (8) becomes

$$E_0(s) = \frac{\Delta R}{s}\left[\left(\frac{K_S I_1}{R_1 - K_S}\right)\left(\frac{\frac{(R_1 - K_S)}{L_f}}{s + \frac{(R_1 - K_S)}{L_f}}\right)\right] \tag{9}$$

This is the product of a driving function $\frac{\Delta R}{s}$ and the transfer function

$$-\left[\left(\frac{K_S I_1}{R_1 - K_S}\right)\left(\frac{\frac{(R_1 - K_S)}{L_f}}{s + \frac{(R_1 - K_S)}{L_f}}\right)\right]$$

which is characteristic of the generator. Replacing the symbol $\Delta R$ by R, the transfer function of the generator is

$$T'_g = \frac{E_o}{R}(s) = -K'_g\left(\frac{\alpha_g}{s+\alpha_g}\right), \tag{10}$$

where

$$\alpha_g = \frac{R_1 - K_s}{L_f},$$

and

$$K'_g = \frac{K_s I_1}{R_1 - K_s}.$$

It will be more convenient to consider the input to the generator as a voltage instead of a resistance. Rearranging equation (3), assuming

$$R \ll R_1,$$

$$E_o(t) - I_1 R = L_f \frac{di}{dt} + R_1 i(t). \tag{3}$$

Then evidently $-I_1 R$ is equivalent to a voltage, $\epsilon'$ impressed upon the field circuit of the generator. The second form of the transfer function is given by

$$T_g = \frac{E_o}{\epsilon'}(s) = +K_g\left(\frac{\alpha_g}{s+\alpha_g}\right), \tag{11}$$

where

$$K_g = \frac{K_s}{R_1 - K_s}.$$

$E_o$ also affects the shunt field directly. This constitutes a minor feedback loop within the generator, but in no way invalidates the over-all transfer function.

## ANALYTICAL SYSTEM RESPONSE

The system under study can be represented schematically by the block diagram of Figure 2. $E_i$ and $E'_i$ are voltages introduced into the generator armature and field circuit, respectively, due to some disturbance such as the removal of load. The other symbols are as given previously with the exception of $T_f$ which is the transfer function used to convert resistance change into an equivalent voltage in the field circuit. By definition

$$\epsilon_1(t) = E_i(t) + E_o(t), \tag{12}$$

and

$$\epsilon_2(t) = E'_i(t) + \epsilon'(t). \tag{13}$$

Of the seven variables indicated in Figure 2, only $\epsilon_1(t)$ is directly measurable while the system is operating. $\epsilon_1(t)$ is the total change of terminal voltage from the initial operating point, so it is this quantity with which we are chiefly concerned in voltage regulator problems. The Laplace transforms of equations (12) and (13) can be written

$$\epsilon_1(s) = E_i(s) + E_o(s) \text{ and} \quad (14)$$

$$\epsilon_2(s) = E_i'(s) + \epsilon'(s). \quad (15)$$

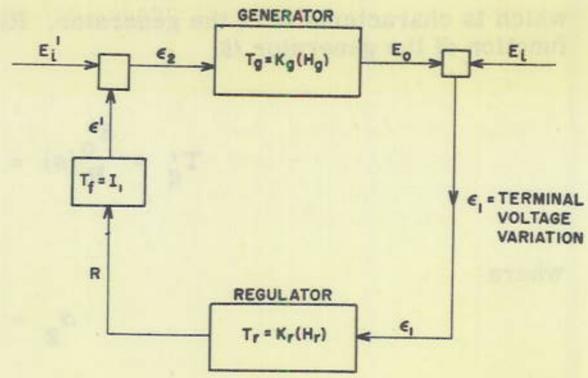


*Figure 2 - Block diagram of regulator-generator system*

Suppose for the moment that $E_i' = 0$. We get the additional equations

$$E_o(s) = \epsilon_1(s) \cdot T_r \cdot T_g \cdot T_f \quad (16)$$

and

$$T_r \cdot T_g \cdot T_f = \frac{E_o}{\epsilon_1}(s), \quad (17)$$

where $E_o/\epsilon_1(s)$ is known as the loop transfer function. From equations (14), (16), and (17) we get

$$\epsilon_1(s) = E_i(s) \left( \frac{1}{1 - \frac{E_o}{\epsilon_1}(s)} \right). \quad (18)$$

Therefore, the terminal voltage response can be determined from the input and the loop transfer function. Now suppose that $E_i = 0$ and $E_i'$ is not zero. We know that

$$E_o(s) = T_g \cdot \epsilon_2(s) \quad (19)$$

and

$$\epsilon'(s) = \epsilon_1 \cdot T_r \cdot T_f. \quad (20)$$

From equations (14), (15), (19) and (20)

$$\epsilon_1(s) = E_i'(s) \cdot \left( \frac{T_g}{1 - \frac{E_o}{\epsilon_1}(s)} \right). \quad (21)$$

The transfer function of the generator has been introduced, but nonetheless the modes of oscillation of the response will be very much the same for both inputs.

## INPUT DUE TO REMOVAL OF GENERATOR LOAD

It is desirable to check the validity of the analysis by both measuring and calculating a transient response to a known input. Unfortunately, the input disturbances which can be applied to the system are not readily controlled independent variables. The simplest common input is the removal of load from the generator. If a good circuit breaker is used, the load drops linearly to zero. Nevertheless the equivalent input is complicated and consists of several superimposed voltages as follows:

(a) The change of $IR_a$ voltage drop which is linear with time while current is decreasing. ($R_a$ = armature resistance.)

(b) A step function voltage which is introduced in the armature circuit as a result of armature circuit self-inductance and mutual inductance with the shunt field. An equal but opposite step function occurs as the load current reaches zero.

(c) A voltage due to the change of direct armature reaction and cross magnetizing of the pole face. This demagnetizing voltage is assumed to change linearly with load current and constitutes an input disturbance to the armature circuit.

(d) A step function voltage is introduced into the field circuit due to the mutual inductance between the armature and field windings. When the load current reaches zero an equal but opposite step function results.

These voltages can be considered as perfect generators in series with the armature and shunt field circuits. In Figure 3, these are represented by $G_a$ and $G_f$ respectively. Figure 3 also indicates the quantities $E_l$ and $E_l'$ which correspond to those of Figure 2. By definition the voltage of $G_f$ is $E_l'$. Assuming that the armature inductance and resistance are negligibly small with respect to the shunt field inductance and resistance, it is obvious that the voltage of $G_a$ is $E_l$. In Appendix 1 a discussion is presented showing that the input voltages (c) and (d) above lead to equal and opposite terminal voltage responses and hence will be neglected.

## MEASUREMENT OF GENERATOR TRANSFER FUNCTION

If the generator were truly a linear device, the measurement of its transfer function would be quite simple. It would only be necessary to introduce a step function change of resistance into the field circuit and measure the time constant of the resulting exponential rise of terminal voltage. Unfortunately, even small hysteresis in the magnetic circuit of the generator introduces large errors in the results obtained by such a test. In its place a method has been devised for introducing a sinusoidally varying resistance into the the generator field circuit and for measuring

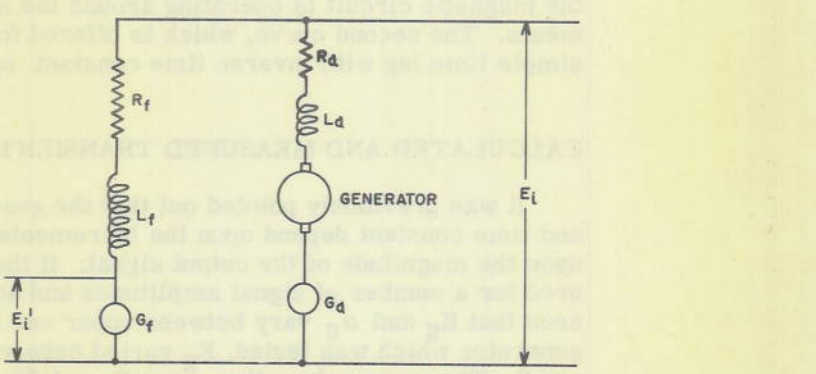


*Figure 3 - Voltages equivalent to removal of load*

the sinusoidal terminal voltage response. In a previous interim report [6] an electronic circuit was described for measuring the transfer function of a carbon pile regulator. This circuit is reproduced in Figure 4. The same circuit may be utilized in the analysis here, but in addition, a special carbon pile regulator is required. A regulator was obtained in which the armature compresses two carbon piles instead of one. The two piles are electrically isolated from each other, and one is used in the electronic circuit as shown in Figure 4. When a sinusoidal signal is introduced into the electronic circuit both piles respond. The second may be used to introduce the sinusoidal resistance variation into the field of the d-c generator, and the transfer function is thus easily measurable.

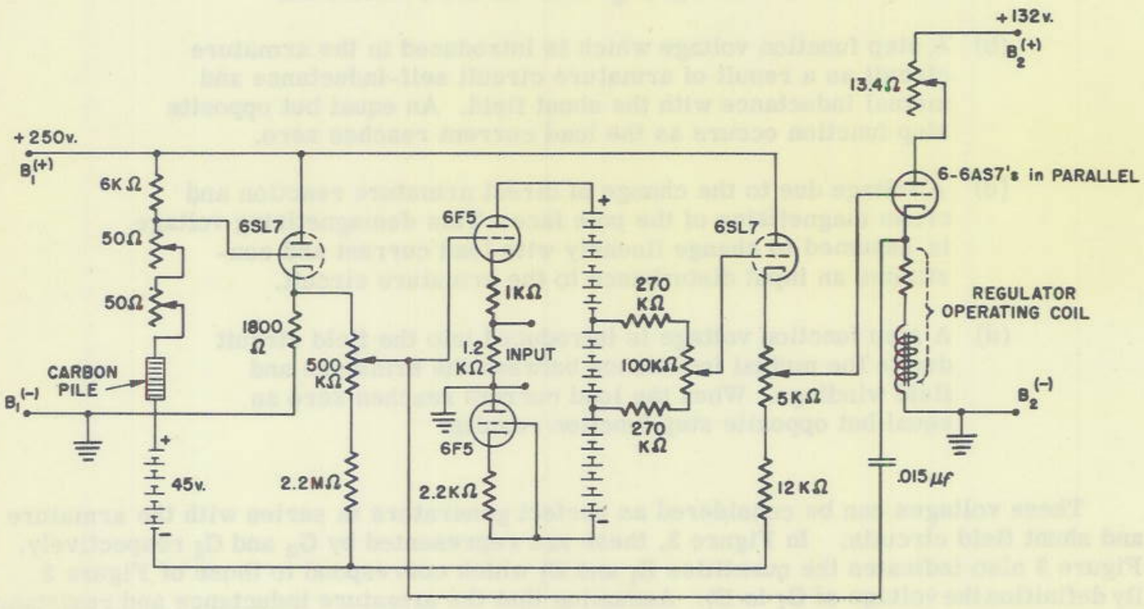


*Figure 4 - Schematic diagram of regulator test circuit*

A typical example is shown in Figure 5. The transfer function is measured with constant generator output variation at all frequencies in order to assure that the iron in the magnetic circuit is operating around the same minor hysteresis loop for all measurements. The second curve, which is offered for comparison, is the transfer function of a simple time lag with inverse time constant $\alpha_g = 5.7$ and amplification $K_g = 1.5$.

## CALCULATED AND MEASURED TRANSIENTS COMPARED

It was previously pointed out that the generator is nonlinear and that both amplification and time constant depend upon the incremental permeability of the magnetic circuit, hence upon the magnitude of the output signal. If the transfer function of the generator is measured for a number of signal amplitudes and at a number of initial operating points it is seen that $K_g$ and $\alpha_g$ vary between upper and lower limits. For an arbitrarily chosen generator which was tested, $K_g$ varied between 1.50 and 2.00, and $\alpha_g$ varied between 4 and 9. The mean values then were $K_g = 1.75$ and $\alpha_g = 6.5$. The same condition existed

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[6] Mills, op. cit.

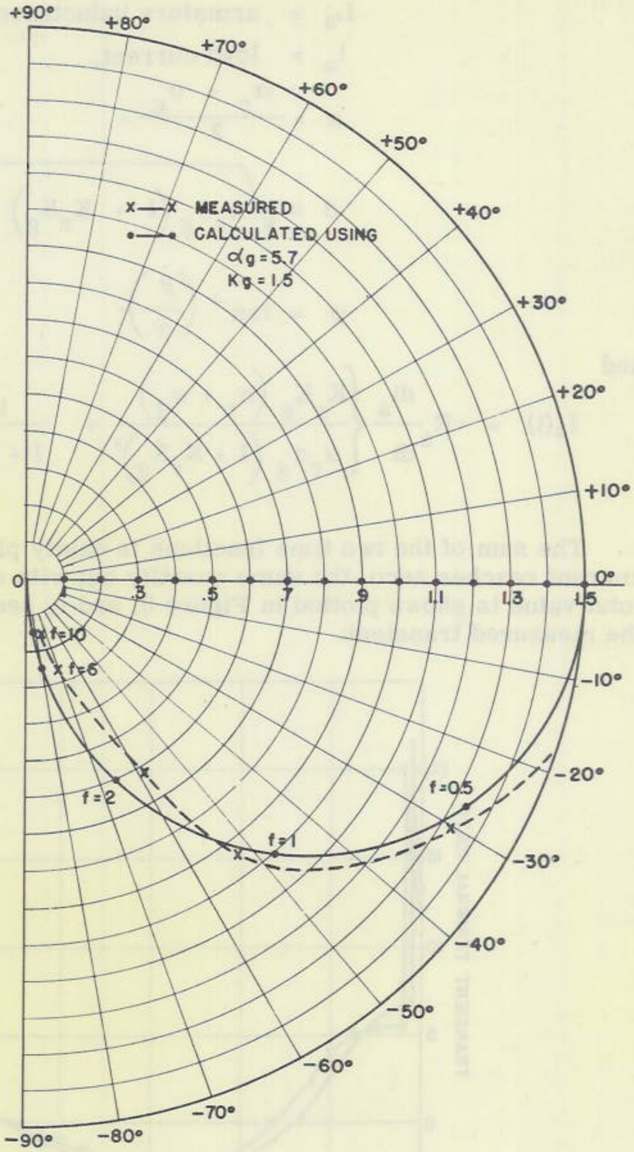


*Figure 5 - Generator transfer function*

when measuring the regulator transfer function, and an average yielded $K_r = 15$ and $\alpha_r = 65$.

Using these four constants, calculations have been made of the terminal voltage response to the removal of full load from the generator. It was assumed that a 75 amp load was reduced linearly to zero in 0.002 second. The mean inductance and resistance of the generator armature were measured as 0.000373 henry and 0.1 ohm respectively. Using the inputs (a) and (b) discussed above, and taking the inverse Laplace transform of equation 17 for both cases, the following two time functions are determined:

$$f_1(t) = \left[ -L_a + M\left(\frac{M + L_a}{M + L_f}\right)\right]\frac{di_a}{dt} \times$$

$$\left[\frac{1}{1 + K_rK_g} - K_rK_g\sqrt{\frac{\alpha_r\alpha_g}{\left(1 + K_rK_g\right)\left[\alpha_r\alpha_g\left(1 + K_rK_g\right) - \left(\frac{\alpha_r + \alpha_g}{2}\right)^2\right]}}\, e^{-\alpha t}\sin(\beta t + \psi)\right] \quad (22)$$

where M = Mutual inductance between the field and armature circuits, where the armature circuit is taken to include compensating and commutating windings,

$L_a$ = armature inductance,

$i_a$ = load current,

$$\alpha = \frac{\alpha_r + \alpha_g}{2},$$

$$\beta = \sqrt{\alpha_r \alpha_g \left(1 + K_r K_g\right) - \left(\frac{\alpha_r + \alpha_g}{2}\right)^2},$$

$$\psi = \tan^{-1}\left(\frac{\beta}{\alpha}\right);$$

and

$$f_2(t) = -R_a \frac{di_a}{dt} \left\{ \frac{K_r K_g \left(\alpha_r + \alpha_g\right)}{\alpha_r \alpha_g \left(1 + K_r K_g\right)^2} + \frac{t}{1 + K_r K_g} - \frac{K_r K_g}{\beta\left(1 + K_r K_g\right)} e^{-\alpha t} \sin(\beta t + 2\psi) \right\} \quad (23)$$

The sum of the two time functions is easily plotted. At time t = 0.002 when the load current reaches zero, the same quantity but with opposite sign is again introduced. The total value is shown plotted in Figure 6, and is seen to be substantially in agreement with the measured transient.

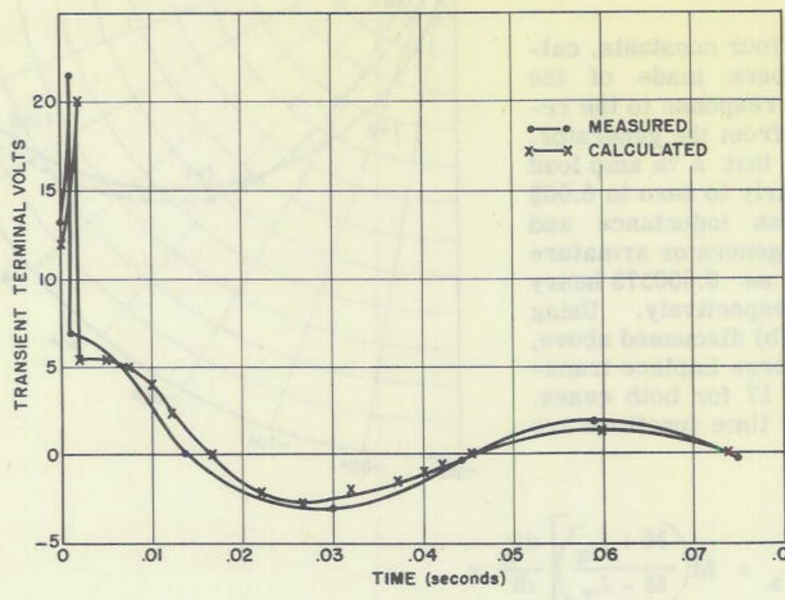


*Figure 6 - Generator response to removal of full load*

## FREQUENCY SPECTRUM ANALYSIS

The function $\epsilon_1/E_i(s)$, is called the error transfer function, and from it predictions of system performance can be made qualitatively without laborious calculations. The absolute magnitude of the ideal error transfer function would of course equal zero for all frequencies, but this is never even approximately realized in practice. Typically, the magnitude of the error transfer function is very small at zero frequency, then rises to a resonance peak. At higher frequencies it then drops asymptotically to 1.0. In general, the error response to a transient disturbance will take the form of a damped sinusoid of frequency near that of the error transfer function resonance peak. The oscillation will be damped to a lesser degree the larger the magnitude of the resonance peak, so every effort is made to keep this peak as small as possible. In addition, however, the error transient depends upon the predominant frequencies in the input disturbance. The common inputs such as step functions and ramp functions have frequency components which drop off at least as $1/\omega$ or faster. It is therefore better in general to have a resonance peak occur at higher frequencies. Special considerations must be taken into account individually. For instance, mechanical vibration of frequency near the resonance peak must be avoided.

From equation (18) it can be seen that $\epsilon_1 / E_i(s)$ can be expressed in terms of the loop transfer function alone. The loop transfer function in turn is easily determined from the characteristics of the individual components of the control loop. Once the relationship between loop and error transfer functions is established it is no longer necessary to plot the error transfer function in order to analyze the system.

We therefore plot the loop transfer function on the complex plane which indicates its magnitude and phase shift at each frequency (Figure 7). Using equation (18) a magnitude and phase shift (Figures 7 and 8 respectively) of the error transfer function can be determined corresponding to every point on the loop transfer function complex plane. Typical constant magnitude loci are shown to be circles concentric about the 1 + j0 point. The radii of these circles are given by

$$r = \frac{1}{\left|\frac{\epsilon_1}{E_i}\right|} .$$

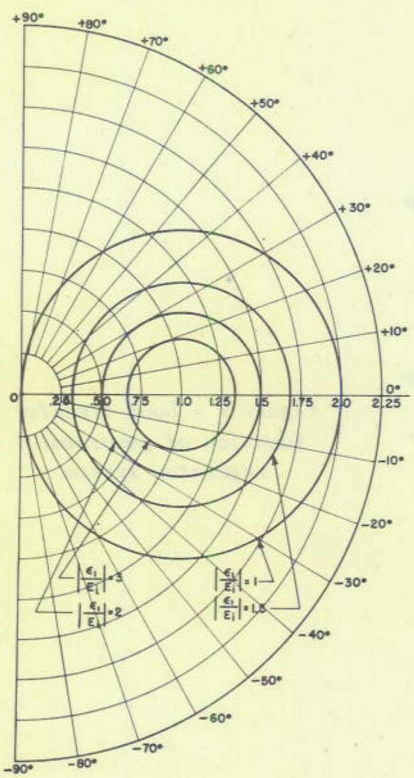


*Figure 7 .- Loci of constant magnitude of error transfer function on loop transfer function plane*

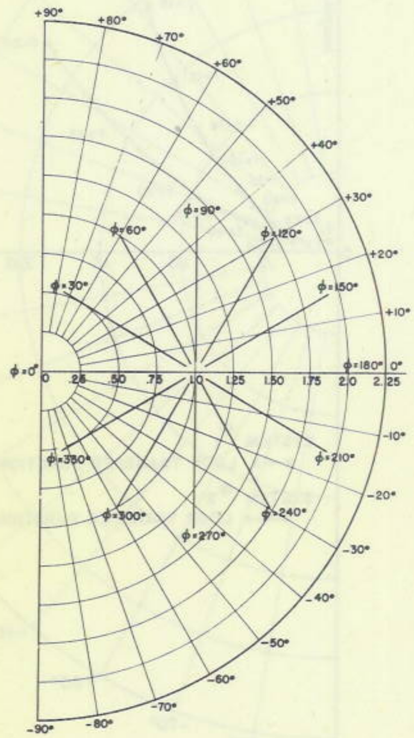


*Figure 8 - Loci of constant phase, φ, of error transfer function on loop transfer function plane*

If Nyquist's[7] stability criterion is satisfied then the peak magnitude of the error transfer function is determined solely by the point at which the loop transfer function approaches closest to the 1 + j0 point. As an example Figure 9 shows the loop transfer function of the generator and regulator which were used for the transient analysis (system #1). The frequency for which the curve most nearly approaches the critical point is 19 cycles. Noting the co-ordinates of this point and referring to Figure 7 this corresponds to an error transfer function magnitude of slightly less than 2.0. The transient response shown in Figure 6 is a damped sinusoid whose period is approximately .058 seconds. The reciprocal of the period is a frequency of about 17 cycles per second as would be expected. In Figure 9 and Figure 10 this system is compared with another which employed the same regulator with a different generator (system #2). In system #2 the peak magnitude of the error transfer function is considerably larger (2.87), but it occurs at a higher frequency and therefore is not excited to such a large degree by the input disturbance. The frequency at which the loop transfer function is closest to the 1 + j0 is about 34 cycles, whereas the corresponding transient response is a damped sinusoid of approximately 31 cycles per second.

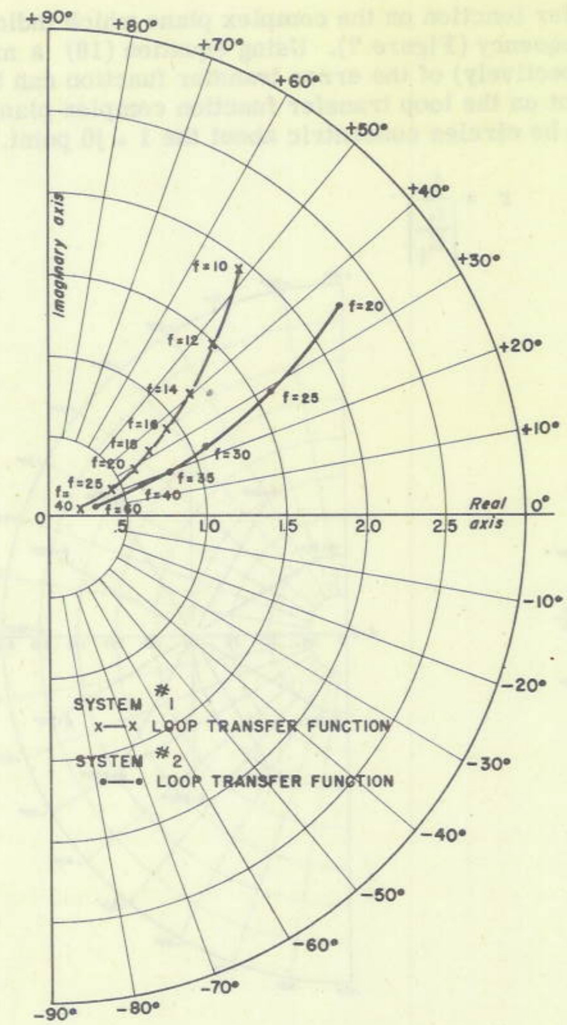


*Figure 9 - Loop transfer function of generator-regulator system*

[7] Nyquist, H., "Regeneration Theory," Bell Telephone System Monograph B-642, January 1932

## APPENDIX
### Neglected Input Disturbances

In the calculation of transient response to the removal of full generator load two voltage inputs were neglected. These were (c) and (d) under the general discussion of inputs. The first of these is a ramp function, $E_d = K't$, impressed upon the armature circuit, and the time response is the corresponding inverse Laplace transform of equation (18). The second input is a step function introduced in the shunt field of the generator due to mutual inductance between armature and field windings; the corresponding time function is the inverse Laplace transform of equation (21).

The two terminal voltage transients will be equal and opposite if equations (24) and (25) are satisfied:

$$\tan^{-1}\left(\frac{2\beta}{\alpha_r - \alpha_g}\right) + \tan^{-1}\left(\frac{2\beta}{\alpha_r + \alpha_g}\right) = 2\tan^{-1}\left(\frac{2\beta}{\alpha_r + \alpha_g}\right) \tag{24}$$

where

$$\beta = \sqrt{\alpha_r \alpha_g \left(1 + K_r K_g\right) - \left(\frac{\alpha_r + \alpha_g}{2}\right)^2} \; ;$$

$$D_a = K_g \alpha_g M, \tag{25}$$

where $D_a$ is the rate of change of terminal voltage with armature current due to demagnitization effects, and M is the mutual inductance between armature and shunt field circuits.

Every effort is made to satisfy the criterion of equation (24) in the manufacture of voltage regulators; that is $\alpha_r$ is made much larger than $\alpha_g$. In practical systems approximation, equation (24) is true to within about 4 degrees, which is negligible. Equation (25) can be shown to be true. Let us consider $\phi$ as a change from an initial reference value. Let e, $i_f$, and $i_a$ be the corresponding changes from the initial values of terminal voltage, shunt field current, and load current, respectively. From equation (5) we can write

$$K_s = \frac{\partial e}{\partial i_f}. \tag{26}$$

By definition

$$D_a = \frac{\partial e}{\partial i_a}. \tag{27}$$

Therefore,

$$D_a = K_p S \frac{\partial \phi}{\partial i_a}, \tag{28}$$

where $K_p$ = a constant of the generator and S is a constant generator speed.

But

$$\frac{\partial \phi}{\partial i_a} = \frac{M}{kN_f} = \frac{M}{L_f}\frac{\partial \phi}{\partial i_f} \tag{29}$$

where $N_f$ = the number of turns in the field winding

$k$ = a constant of proportionality between the field pole flux and the flux responsible for generated voltage.

From equations (28) and (29):

$$D_a = K_p S \frac{M}{L_f}\frac{\partial \phi}{\partial i_f} \tag{30}$$

Using (30) and the definitions of $K_g$ and $\alpha_g$ given with equations (10) and (11),

$$D_a = MK_g\alpha_g \frac{K_p S \frac{\partial \phi}{\partial i_f}}{K_S}. \tag{31}$$

From (26) and (31)

$$D_a = MK_g\alpha_g. \tag{25}$$

Therefore both criteria are satisfied and the two transients in question cancel each other.

* * *