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NRL REPORT 3561

FR-3561

FULL-SCALE SHOCK TESTS - USS NIAGARA APA87
PART 4 - THE CORRELATION OF MEASURES OF MAXIMUM VALUES OF KINEMATIC QUANTITIES

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FULL-SCALE SHOCK TESTS - USS NIAGARA APA87

PART 4 - THE CORRELATION OF MEASURES OF MAXIMUM VALUES OF KINEMATIC QUANTITIES

Sanford P. Thompson

November 2, 1949

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ABSTRACT

Two classes of instruments are commonly employed for the measurement of kinematic quantities resulting from the shock excitation of mechanical structures. The first class is composed of those instruments (for example, velocity meters) which give the instantaneous time dependence of the motion at the point of attachment. The second class is composed of peak-reading instruments (such as reed gages and putty accelerometers) capable of giving only, at best, representative values or rough measures of the maximum values of the actual kinematic quantities.

The data obtained from the underwater explosion trials held against the USS NIAGARA APA87 during the summer of 1948 are treated statistically to determine the degree to which rough measures obtained by the second class of instruments are truly representative of the motion as measured by those of the first class. In addition, the reproducibility of the experiments which composed these trials is also studied.

For instrument excitations of the type afforded by an underwater explosion near an APA, it is concluded that the multifrequency reed gage is capable of yielding an approximate value of the maximum velocity. It is also concluded, however, that under the same conditions both the reed gage and the putty accelerometer are incapable of responding correctly to the maxima of accelerations having significant durations. As regards the degree of reproducibility of experimental conditions which existed during the trials, it is concluded that the experiments were reproducible to within at least 91-95 percent.

PROBLEM STATUS

This is a report on a portion of NRL Problem No. F03-07R. Work is continuing.

AUTHORIZATION

F03-07R (BuShip SRD 509-48)
NR 433-070

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FULL-SCALE SHOCK TESTS - USS NIAGARA APA87
PART 4 - THE CORRELATION OF MEASURES OF MAXIMUM VALUES OF KINEMATIC QUANTITIES

INTRODUCTION

During the summer of 1948, the Naval Research Laboratory, in cooperation with other Naval agencies, participated in a series of experiments to measure the effects of non-contact underwater explosions upon a Naval vessel, the USS NIAGARA APA87. The work described here is a by-product of that investigation. The shock tests provided a desirable opportunity to determine how much objective reality may be attributed to the rough measures (of maximum values of kinematic quantities) calculated from reed-gage and putty-accelerometer data—that is, to what extent these rough measures may be regarded as being meaningfully representative of the actual[1] motions. The conclusion[2] reached, that these rough measures but poorly reflect the actual motions in an APA-type vessel, should serve to modify the high regard in which these quantities are sometimes held.

A very complete account of the background of the problem, the purposes of the trials, and the instrumentation employed will be found in a previous report.[3] A number of charges of HBX-1 were fixed at various depths and at various horizontal distances from the starboard side of an APA-type vessel. Table 1 describes the geometry of the target and the charge, which was in all cases in a plane normal to the axis of the vessel at frame 83.

Table 2 describes the location of the instrumentation aboard the vessel. Of special interest here are those British velocity meters, those putty accelerometers, and those reed gages installed by the Naval Research Laboratory.

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[1] *Inasmuch as the British velocity meter is known to be an exceptionally accurate shock and vibration instrument, little distinction will be maintained here between "true" measures and those calculated from velocity-meter records. Strictly speaking, of course it is the latter against which the rough measures are compared.*

[2] *Statistical techniques are employed in reaching this conclusion. Now, the application of statistics to practical problems, to a greater extent than seems true of other branches of mathematics, is always liable to the introduction of assumptions whose ramifications are often not intuitively obvious. It would therefore be strange if some of the writer's personal bias did not inadvertently creep into the text; this in spite of an honest effort to distinguish, in the writing of this report, between those things which are facts, those which are hypotheses, and those which are opinions or judgments.*

[3] *Cunningham, C.B., "Full-Scale Shock Tests - USS NIAGARA APA87, Part I - Techniques and Instrumentation," NRL Report F-3404*

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TABLE 1
APA Shot Identification[1]

BuShips No.	Firing Sequence	Distance[2] (ft)	Depth[3] (ft)	Charge Weight (lb)	Date Fired
1	1	325	70	250	5/21/48
2	2	325	70	600	5/27/48
3	3	325	70	1200	6/1/48
4	4	325	70	600	6/11/48
5	5	325	70	1200	6/18/48
6	6	185	70	600	6/23/48
7	7	185	17	600	6/23/48
8	8	185	70	600	7/1/48
9	9	325	17	600	7/2/48
10	10	185	70	250	7/7/48
11	11	220	70	1200	7/8/48
12	12	185	70	1200	7/12/48
13	13	185	70	640	7/19/43
13A	13A	185	70	640	7/20/48
14	14	155	70	1200	7/26/48
15	15	124	70	1200	7/27/48
16	16	110	70	1200	7/29/48
17	Not Fired - Cancelled				
18	17	131	53	250	7/30/48
19	18	170	110	1200	8/3/48
20	19	120	134	1200	8/4/48
21[4]		105	110	250	8/5/48
22	20	65	134	320	8/6/48
23	21	0	134	250	8/9/48
24	22	46	119	250	8/10/48
25	23	0	65	250	8/11/48

Notes:

[1] *Depth of water - 134 ft.*
[2] *Distance measured from starboard plating*
[3] *Depth measured from surface of water*
[4] *Charge 21 failed to detonate - no data*

These three instruments were thought by the designers of the experiments to be those most suitable, because of many considerations, for the field measurement of kinematic quantities under shock conditions; they represent the most practical instrumentation currently available, or likely to be available in the near future.

The first of these instruments yields a time record, and hence gives the instantaneous inertial velocity of the point of attachment. The other two instruments do not give direct measures of kinematic quantities (the displacement and its time derivatives). Rather, their data may be employed to calculate certain rough measures of maximum values of kinematic quantities.

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TABLE II
APA Test Position Descriptions

TEST POSITION NUMBER	DECK LEVEL	FRAME NUMBER	DISTANCE OFF CENTER LINE	HEIGHT ABOVE DECK LEVEL	BN EQUIPMENT NUMBERS			BRITISH VELOCITY METER NUMBERS			REED GAGE NUMBERS			PUTTY ACCELEROMETER NUMBERS					
					MOUNTING A	MOUNTING B	MOUNTING C	VERTICAL	TRANS.	LONG.	VERTICAL	TRANS.	LONG.	VERTICAL UP	VERTICAL DOWN	TRANS. PORT	TRANS. STBD	LONG. FWD	LONG. AFT
1	HOLD	89-90-91	24' TO STBD	ON DECK	23	5	36	113	—	—	104	13	—	L-61	—	L-71	—	—	—
2	HOLD	86-87-88	STBD FRAMING	4'	11-21	27	2	104	—	—	109	105	—	A-39	—	H-21	—	—	—
3	SECOND PLATFORM	83	15' TO STBD	4'	15	33	24	118	—	—	21	77	—	L-40	—	L-19	—	—	—
4	MAIN DECK	87	ON CL	ON DECK	7-39	34	3	117	—	—	61	33	—	L-60	—	L-121	—	—	—
5	MAIN DECK	83	9' TO STBD	ON DECK	19	38	13	126	—	—	25	80	—	L-101	—	L-26	—	—	—
6	NAVIGATION DECK	83	5' TO STBD	1.5'	16	39-29	25	123	—	—	65	36	—	L-38	—	L-69	—	—	—
7	NAVIGATION DECK	79-80-81	6' TO STBD	ON DECK	40	30	35-14	119	—	—	53	107	—	L-66	—	L-3	—	—	—
8	NAVIGATION DECK	79-80-81	10' TO STBD	4'	28	20	18	118	—	—	15	106	—	L-76	—	L-124	—	—	—
9	FIRST PLATFORM	63-64-65	ON CL	ON DECK	12	9	22	121	—	—	23	102	—	L-51	—	L-102	—	L-43	—
10	UPPER DECK	36-37-38	18' TO STBD	ON DECK	10	26	6	122	—	—	34	3	—	L-21	—	L-130	—	—	—
11	FWD MACH SPACE	FWD SIDE OF BOILER	STBD SIDE OF BOILER	BOILER FOUNDATION	—	—	—	111	110	116	38	82	59	L-12	L-116	L-72	L-14	L-103	L-106
12	FWD MACH SPACE	FWD SIDE OF BOILER	PORT SIDE OF BOILER	BOILER FOUNDATION	—	—	—	125	107	115	99	68	64	L-84	L-110	L-68	L-119	L-115	L-29
13	FWD MACH SPACE	AFTER SIDE OF BOILER	PORT SIDE OF BOILER	BOILER FOUNDATION	—	—	—	124	103	109	16	83	40	L-123	L-59	L-57	L-37	L-47	L-126
14	FWD MACH SPACE	AFTER SIDE OF BOILER	STBD SIDE OF BOILER	BOILER FOUNDATION	—	—	—	102	108	106	2	91	14	L-5	L-45	L-108	L-128	L-109	L-112
15	FWD MACH SPACE	TOP OF BOILER	CENTER OF STEAM DRUM	ON STEAM DRUM	—	—	—	—	—	—	27	—	—	—	—	—	—	—	—
16	AFT MACH SPACE	97	STBD FRAMING	4'	—	—	—	—	—	—	—	101	—	—	—	—	—	—	—
17	AFT MACH SPACE	95	18' TO STBD	ON DECK	—	—	—	—	—	—	22	—	—	—	—	—	—	—	—
18	HOLD	90	STBD FRAMING	4'	—	—	—	—	—	—	—	108	—	H-103	—	—	—	—	—
19	HOLD	86	24' TO STBD	ON DECK	—	—	—	—	—	—	50	—	—	—	—	—	—	—	—
20	NAVIGATION DECK	80	10' TO PORT	4'	—	—	—	—	—	—	58	103	—	—	—	—	—	—	—

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In calculating each rough measure, one is, in effect, making an assumption about the time variation of the displacement of the point of attachment. These assumptions are never exactly met in practice; if, however, the departure from ideality is not too great, one would expect each rough measure to be a reasonable approximation to the maximum value of the true kinematic quantity. Reed gages and putty accelerometers[4] are used for the measurement of kinematic quantities more from necessity than from choice. Unlike those requiring equipment for time records, these instruments are very dependable, compact, light, and quite inexpensive.

The tests on the NIAGARA provided one of the first opportunities to compare measures of the same kinematic quantity obtained by the use of different instruments subjected to simultaneous, identical excitations due to sizeable underwater explosions. It is the purpose of this report to define formally, by their method of calculation, certain rough measures of kinematic quantities; to make the use of these rough measures plausible by showing, for certain simple cases, their relation to the instrument excitation; and to intercompare statistically rough measures of the same kind.

A word of caution is necessary here. Because of the arbitrariness in the definition of the rough measures (or, what is equivalent, the assumptions made about the motion), there is no reason to expect a high correlation between a given rough measure and the true kinematic quantity for all types of excitation. On the contrary, one can easily conceive of excitations which would yield a very low correlation. The attempted correlation must be viewed as an empirical study; one must not conclude that the same degree of correlation will, or will not, necessarily hold in another vessel of different type. Only with this reservation may one apply the results to these instruments under different conditions of shipboard shock. or to the development of new instruments.

In conclusion, the reproducibility of the APA tests is considered. The discrepancies in the readings of a given instrument between supposedly identical shots are indicative of the degree of experimental control during the trials.

INSTRUMENTS[5]

Putty Accelerometer

The putty accelerometer (or putty gage) represents an attempt to construct a polarized, peak-bracketing accelerometer. By means of a guide, a stop, and a precompressed spring, each of eight masses is constrained to move, with reference to the case, only along a specific line, and then only if the inertial force of the mass overcompensates the precompression of the spring. Each mass is provided with a conical point which can indent a plane clay surface after a travel of a few thousandths of an inch.

The behavior of the instrument when the case is slowly accelerated (as, say, by a slowly accelerated centrifuge) is as follows. Until the acceleration of the case approaches a critical value, the mass remains in its original position. When, however, the case acceleration becomes sufficiently large, the force required to accelerate the mass exceeds the

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4 *No disparagement of the engineering usefulness of these instruments is intended. As will be mentioned subsequently, the reed gage is, in fact, particularly suited for certain engineering applications.*

5 *For details see Cunningham, op. cit.*

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precompression force of the spring, and the required additional force can be obtained only by a spring deflection. After the critical value of case acceleration has been reached, then, only a small increment is required to indent the clay (the diameter of the indentation providing a permanent record of the maximum deflection of the mass from its rest position). By suitable design, the eight spring-mass systems provide a graded series of critical accelerations. The peak value of a slowly changing acceleration of the case is known to lie between the critical acceleration of the last system which indented, and that of the first system which did not indent. Measurement of the indentation diameter provides, in addition, the possibility of interpolation within this interval.

The reading of a putty accelerometer is, then, a rough measure of the peak acceleration, which agrees with the actual peak acceleration for excitations which have low values of the third time derivative of the displacement. When the instrument is used under other than these conditions, however, other effects (due to the distribution of mass and elasticity within the spring, to the finite clearance between the clay and conical point, and to the possibility of resonance of the system with the excitation) make interpretation of the results difficult. These effects manifest themselves experimentally by a disordering of the indentation pattern. It was often observed that some systems indented, while, under the same excitation, those of lower critical acceleration did not. In this report, the only putty-accelerometer data employed are those for which the indentation pattern is ordered like the pattern obtained in a centrifuge.

Velocity Meter

The British velocity meter is a seismic instrument employing electromagnetic coupling. Its open-circuit voltage is directly proportional to the relative velocity between the case and the seismic element and, hence, is also proportional to the inertial velocity of the case for frequencies considerably in excess of its own natural frequency when the meter is acting as a one-degree-of-freedom mass-spring system.

This is the only kinematic instrument (apart from strain gages, which are not here considered) employed in the tests which yields a time history of its reading. From the test records not only may the velocity of the point of attachment be read, but the acceleration and the displacement may also be calculated. The British velocity meter has been found to be both an accurate and a dependable instrument; however, because of the finite natural frequency of the instrument, the errors in the calculated acceleration and displacement increase with the time. Since the values of kinematic quantities at a considerable time after the onset of the shock may be of interest, the question of these cumulative errors is treated in Appendix I.

In order to compare the numerical values of rough measures obtained from the putty accelerometers and reed gages with the velocity-meter records, it is necessary to obtain from each record a set of numbers, each member of the set giving a typical value to a different kinematic quantity. Since it is desired here to correlate results obtained by the use of peak-reading instruments with those obtained by the velocity meter, maximum values of quantities derived from the velocity-meter record are chosen. Since it was noted that the first peak of velocity (the greatest value of the velocity prior to the first sign reversal) was in most cases the maximum velocity and that the initial slope corresponded in most cases to the maximum acceleration, these quantities are used as the standard against which rough measures are judged.

Reed Gage

The reed gage is an assembly of cantilever springs each fixed at one end to the case of the instrument and each being tuned to a different fundamental frequency by a mass at its

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unsupported end. Two styli on the mass contact a sheet of waxed paper on a platen fixed to the case. When the case of the instrument is subjected to an excitation, the styli leave scratches on the paper indicating the maximum deflection of each mass relative to the case.

The reed gage owes its engineering importance to the following. Assuming that the paper introduces negligible damping, it can be shown (see Appendix II) that the deflection of any given reed relative to the case is the same as the relative deflection across the spring of any undamped single-degree-of-freedom system, of the same natural frequency which has been subjected to the same kinematic excitation. The reed gage therefore furnishes, in a form requiring no analysis whatever, the maximum deflection which would be experienced by a corresponding series of shock-mounted equipments subjected to the same kinematic excitation.

In addition to its engineering use, the reed gage is capable of furnishing rough measures of the velocity and acceleration of its point of attachment. The most simply calculated rough measure of velocity (and the one to be subsequently used) is the step of velocity excitation which would give the same reed deflection as was obtained by the actual excitation. A step of velocity might at first appear to be an unrealistic excitation upon which to base the definition of a rough measure. The procedure appears more plausible, however, when it is noted that the anvils of impact-type shock machines (which simulate, more or less, the gross features of shipboard shocks) are known[6] to possess motions which may be fairly well approximated by a velocity step.

By either a straightforward transient analysis or by simpler considerations of the conservation of mechanical energy, it may be shown that the magnitude of the step, V, is related to the maximum deflection, x, of the reed (assumed to be undamped) by

$$V_\omega = \omega_n x . \qquad (2)$$

The subscript, ω, on V serves to remind the reader that the value of V calculated by this formula from an actual reed deflection is dependent on the particular reed selected, because actual excitations are never of the assumed step-of-velocity type. The selection of the ω to be used in this report was somewhat arbitrary. The value finally selected was that corresponding to the 40-cps reed. Use of the 20-cps reed was rejected because of a dominant, and as yet unexplained, natural mode of the ship structure at approximately 18 cps, while progressively higher-frequency reeds gave deflections which could, because of their smallness, be read only with a progressively greater inaccuracy.

A rough measure of the acceleration is the value, for a high-frequency reed, of the quantity, N.[7] N is defined as the ratio, (the time-invariant acceleration required to produce the same deflection as that measured kinematically) / (the acceleration of gravity), that is

$$N_\omega = \frac{\omega^2}{g} x , \qquad (3)$$

the subscript, ω, indicating that N for an actual record is dependent on the reed selected. It is reasonable that one should employ a value corresponding to a high- rather than a

[6] *Walsh, J. P. and Blake, R. E., "The Equivalent Static Acceleration of Shock Motions," NRL Report F-3302, June, 1948.*

[7] *Walsh and Blake, op. cit.*

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low-frequency reed, since the period of the reed is then as small as possible as compared to the time required for an appreciable change in the excitation.

This statement is corroborated in Appendix III where it is shown that the N for a very-high-frequency reed corresponds to the asymptotic value of the gradually rising pulse of acceleration:

$$\ddot{Y}_c\,(t) \;=\; A\;(1-e^{-\alpha t}). \tag{4}$$

Such a function is, however, a very poor representation of an actual excitation. Correspondingly, one would suspect that the N for the highest-frequency reed, 920 cps, would correlate poorly with the other measures of acceleration, and this is the case as the subsequent analysis will show.

SELECTION OF DATA

Before describing the extent to which the previously defined rough measures of kinematic quantities correlate with each other and with the "true" measures, it is necessary to state which data obtained from the APA trials were employed and which were, for this report, rejected. (The remarks to be made apply both to the correlation of measures of kinematic quantities, and to the determination of the reproducibility of the experiment.) Those employed include all the usable data associated with test positions 1 through 10. Since it was desired to draw conclusions representative of "average" conditions aboard an APA-type vessel and since the use of data from test positions 11-15 inclusive would have given undue weight to machinery spaces, the remainder of the data was ignored.

Some of the data from test positions 1-10 were unusable and hence were also rejected. All cases of such rejection for the putty accelerometer fall into one of the following categories: (a) data whose indentation patterns were disordered (this exception was mentioned previously), (b) data for which no indentation occurred, (c) data unusable by reason of the "puffing" or swelling of the putty.[8] In addition, reed gage deflections less than 0.005 inch (the minimum detectable deflection being of this order) were ignored.

Enough data falls into these rejected categories that, for some comparisons, there remains such a small amount of usable data that the form of the probability distribution cannot possibly be inferred. In the following tabulations of results, therefore, the number of items of data is specified for each comparison. The procedure of using a random-error theory of correlation for all data seems reasonable since, in all cases where a sizeable amount of data was available, a random theory appeared justified.

CORRELATION OF MEASURES OF KINEMATIC QUANTITIES

It is desired now to compare actual kinematic quantities corresponding to the rough measures defined. The form of the most appropriate comparison is dependent both upon the objective relationship found empirically and upon the use to be made of the comparison. A typical scatter diagram, that for the correlation of first peak of velocity (velocity meter) with V_{40} (reed gage), together with the most probable linear relationship between them, is

---

[8] *Of a total of 450 putty accelerometer indentation patterns, 155 displayed no indentation, 16 displayed a "puffing" of the putty, and 69 displayed disordered patterns. The remaining 210 indented, unswollen, and ordered patterns provided the data used for this report.*

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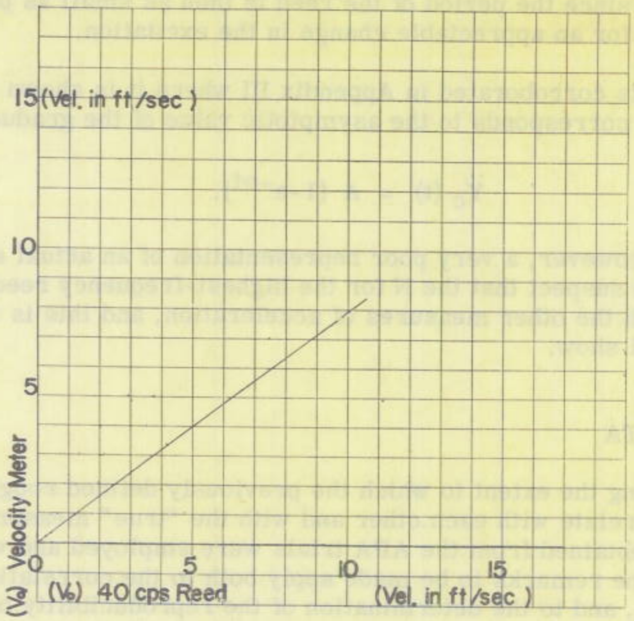

Figure 1

shown in Figure 1. Of all the forms of curves one might fit to these points, the straight line is as justified as any, and there is good empirical reason for taking the intercept as zero. Upon physical grounds any correct relationship obviously must pass through the origin. It is equally reasonable that the relationship be considered linear—all the instruments used are known to be linear, and hence for a series of shocks of graded intensity, there is no "preference" for either high or low readings. All the excitations are known, from inspection of the velocity-meter records, to be of the same form. That is, severe shocks did not afford excitations grossly different in form from those afforded by small shocks. For these reasons, the mathematical assumption is made that the most probable relationship between two measures, x and y, of the same quantity is of the form, $y = bx$.

Consider now the form the correlation for convenience should assume. The form chosen for this report is the one which answers the following questions. (1) Given a set of measures taken with instruments of type A, what are the most probable values of measures of the same kind which would be obtained with instruments of type B? (2) What uncertainty is to be associated with the calculated measures taken with type-B instruments?

These questions are answered in the following way. Given a set of measure values, y, taken with instruments B and a corresponding set, x, taken simultaneously at the same test positions with instruments A, one determines by least squares the most probable value of b in the relation, $y = bx$. The first question is now answered: given an arbitrary value for x, one may calculate the most probable value to be assigned to y, that is, the predicted mean value of a series of measurements of y, x being held constant at the arbitrary value.

Since we are concerned with the "reliability" of the measures we are employing, the second question is as important as the first. To answer it, one may now calculate the

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TABLE 3
Linear Comparison of Velocity-Meter Peak Velocity
with Reed-Gage Initial Velocity

y*	x†	Direction of Instrument Orientation	Slope of the Fitted Line (b)	Probable Error in y (PE_y)	Average Value of y ($\bar{y}$)	Number of Pairs (x, y) (N)
Peak Velocity (V.M.)	V_{40}	up	0.788	1.27	2.47	132

* *y= measure of velocity from velocity meter*
† *x= rough measure of velocity from 40-cps reed gage*

TABLE 4
Linear Comparisons of Measures of Acceleration
from Different Instruments

y*	x†	Direction	b	PE_y	$\bar{y}$	N
N_{920}	Acc (Putty)	up	2.2	663.2	1418.1	21
N_{920}	Acc (Putty)	port	1.2	607.6	1170.4	47
N920	Acc (Putty)	up and port	1.4	660.7	1246.9	68
V. M. Acceleration	N_{920}	up	0.027	30.52	34.50	37
V. M. Acceleration	Acc (Putty)	up	0.091	21.87	25.89	59

* *y= rough measure of acceleration from one instrument*
† *x= rough measure of acceleration from another instrument*

probable error of y about the most probable line. That is, not only may we predict a value for y corresponding to an arbitrary x, but we may also state the uncertainty in the prediction. A quantitative discussion of the statistical procedure, and the derivation of the fitting formula used are given in Appendix IV.

Correlation of Measures of Velocity

In Table 3 is displayed the correlation of the first peak of velocity with the rough measure from the 40-cps reed. Since the value of b is in the neighborhood of unity, it is possible that, for an APA, V_{40} and the first peak of velocity (as obtained from the velocity meter) are determined, on the average, by substantially the same quantity.

The probable error in the first peak of velocity, furthermore, is of the order of $\frac{1}{2}\bar{y}$ [9]. V_{40}, in a rough sense, is thus a valid measure of peak velocity. The correlation is sufficiently inaccurate that the utility of the relationship is doubtful. However, the correlation might be greater in vessels of more conventional design. The essentially sinusoidal response (at 18 cps) of all but the lowest decks of the APA is, of course, poorly approximated by a step of velocity.

[9] *The value of y is relevant to this discussion as an empirical standard against which the practical importance of the magnitude of the probable error (PE) of the y_i's may be judged.*

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Correlation of Measures of Acceleration

As regards the correlation of measures of acceleration (Table 4), N920 correlates with putty-gage acceleration to give values of b in the neighborhood of unity, and the probable error of N920 is of the order of half the mean of the N920's themselves.

However, the lack of correlation of velocity-meter acceleration with either N920 or putty-gage acceleration is striking. Not only are the values of b far different from unity, but the PE's are as large as the $\bar{y}$'s. One is forced to conclude that both N920 and putty-gage acceleration are very poor measures of velocity-meter acceleration. Because, as has been mentioned, one puts great credence in velocity-meter records,[10] he is further led to conclude that both of the peak-reading instruments give, on the average, very poor measures of the true initial acceleration (here, the significant[11] acceleration) in an APA-type vessel.

Suitability of Instruments

To sum up, the reed gage[12] has been shown to be, on the average, a very approximate substitute for a peak-velocity meter on an APA. In fairness, it should also be said that sufficient data would almost certainly show that its performance for positions low in the hull is considerably better than its average, or over-all, performance. The same instrument gives but a fair measure of the reading of a putty accelerometer, and fails completely to correlate with the initial acceleration obtained from a velocity meter. To the same degree, the putty accelerometer fails also to correlate with the initial acceleration obtained from a velocity meter. These are conclusions of fact.

Upon making the assumption (for whose correctness there is abundant evidence) that for all practical purposes the velocity meter yields accurate information, one may further conclude that the reed gage is capable of measuring, very approximately, the true peak of velocity. If in addition, then, one confines his inquiries about accelerations to those of significant duration, he may conclude that both the putty accelerometer and reed gage are probably generally unsuited for the measurement of acceleration. The utility of the putty accelerometer for shipboard shock trials is thus questionable; the reed gage, of course, has other and quite proper engineering uses. Stated conversely, one may conclude that the use of velocity meters is unavoidable if one expects to determine, with good accuracy, the significant maximum values of kinematic quantities. The peak-reading instruments employed in these tests are not, generally speaking, to be considered acceptable substitutes.

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[10] *It is to be noted, incidentally, that data-processing errors could not possibly account for the low values of b. By performing the experiment of repeatedly measuring the initial angle of slope of an actual record, one concludes that the measuring technique employed contains an implicit probable error of 5 minutes. The consequent probable error in velocity-meter acceleration depends on the quantity itself, being 0.4% on the average, and only 11% in the worst case.*

[11] *It is possible, of course, that the reed gage and putty accelerometer measure the peak acceleration. An extremely large acceleration of very short duration is undetectable in a velocity-meter record. The important point is that, in these tests, the maxima of the accelerations of significant duration were not the excitations which provided the putty-accelerometer and reed-gage records.*

[2] *It must be borne in mind that, when one speaks of the suitability of an instrument, the form of the rough measure chosen to typify the result of a measurement, and the technique used to process the actual data are both to be considered part of the instrument.*

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REPRODUCIBILITY OF THE APA TESTS

The shock trials against the NIAGARA represented the most extensive experience of our Navy with a series of shock and vibration experiments conducted under conditions closely approximating those normally encountered by surface vessels at sea. From the results of these trials will be drawn conclusions affecting the future design of vessels. In addition, the experience itself at least for a time, will constitute an important standard against which plans for future trials will be judged. For both purposes, it is of importance to question the validity of the experiment (i.e., the degree to which the experiments actually performed approximated the experiments believed to have been performed). An answer to this question will constitute not only a criterion of the validity of the APA data, but will also throw light on the suitability of the techniques employed to locate both the target and the charge.

One criterion of the validity of an experiment is its reproducibility. Fortunately, the test schedule provided four pairs of shots (2, 4; 3,5; 6,8; and 13,13A) in which both members of a given pair were obtained under conditions as much alike as practically possible. Unfortunately, for various reasons, not all possible data are available for each of the four pairs; enough, however, was provided that the reproducibility can be studied.

In this report, there is treated statistically a quantity, Q, indicative of reproducibility. This quantity is defined as the ratio: (algebraic difference of readings of the same instrument in the two experiments of a given pair) / (algebraic sum of the same readings). This quantity "should" be zero for each instrument.[13] The mean, M, and probable error, PE_Q, of the aggregate of quantities, Q, for a particular instrument category (say, for example, vertically oriented reed gages) is then calculated. The magnitude of the mean is indicative of the extent of unreproducibility; the magnitude of the dispersion is indicative of the uncertainty to be associated with the unreproducibility.

In Table 5 are displayed these quantities for the velocity meters in shot pairs 3,5 and 13,13A. In the case of the first pair, the uncertainty, PE_Q, associated with M is of the same order as M itself. There is therefore no real evidence that shot 5 was any more intense than shot 3. For the second pair, shots 13,13A, the mean of the Q's is over three times the probable error. Since an M of 0.035 is equivalent to a percentage difference of 7% in the "intensity" of the two shocks, one is justified in tentatively concluding that shot 13A was of the order of 5-9% more "intense" than 13—a degree of unreproducibility that is neither significant, nor, in the light of the experimental difficulties, surprising.

Less importance should be attributed to the reproducibility evidence from reed gages and putty accelerometers. Many of the Q's for reed gages are calculated from very small deflections, in whose measurement there is a good possibility of systematic error. The putty accelerometer Q's were calculated from indentation patterns which were interpolated by a rather arbitrary procedure. In the case of the reed gages (Table 6), it is only the 20-cps reed for shots 6 and 8 which evidences any detectable unreproducibility, the criterion (admittedly arbitrary) of detectability being $M \geq 2PE_Q$. Considering the order of magnitude of PE_Q's for other shots and in the light of the preceding remarks, this evidence seems inconclusive.

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[13] *Note that there is no question involved here of the absolute accuracy of the instruments. It is merely necessary to assume that all the instruments of a particular category (say, reed gages) are substantially alike, and that their response is, in a vague sense, linear with the "intensity" of the shock.*

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TABLE 5
Comparison of Velocity-Meter Peak Velocities for Supposedly Identical Shots

Shot Pairs* (a,b)	Direction of Instrument Orientation	Mean of the Q's† (M)	PE_Q	N
3,5	up	-0.12	0.098	9
13,13A	up	-0.035	0.010	5

*See Table 1 for details of the shot designations

$$†Q = Q(a,b) = \frac{\text{Peak velocity from shot } a - \text{peak velocity from shot } 6}{\text{Peak velocity from shot } a + \text{peak velocity from shot } 6}$$

TABLE 6
Comparison of Deflections from Reed Gages for Supposedly Identical Shots

Shot Pairs	Direction of Instrument Orientation	20-cps Reed			40-cps Reed		
		Mean of the Q's*(M)	PE_Q	N	Mean of the Q's(M)	PE_Q	N
2,4	all	-0.093	0.079	19	-0.11	0.17	12
	vert	-0.12	0.085	10	-0.036	0.17	8
	trans	-0.069	0.071	9	-0.25	0.14	4
3,5	all	-0.025	0.057	19	+0.095	0.18	15
	vert	-0.004	0.043	10	-0.027	0.065	8
	trans	-0.049	0.067	9	+0.24	0.22	7
6,8	all	-0.099	0.077	20	+0.013	0.20	19
	vert	-0.069	0.030	10	+0.002	0.10	10
	trans	-0.13	0.010	10	+0.026	0.15	9

$$*Q = Q(a,b) = \frac{\text{Deflection from shot } a - \text{deflection from shot } b}{\text{Deflection from shot } a + \text{deflection from shot } b}$$

TABLE 7
Comparison of Accelerations from Putty Accelerometers for Supposedly Identical Shots

Shot Pairs	Direction of Instrument Orientation	Mean of the Q's* (M)	PE_Q	N
2,4	all	+0.54	0.36	7
	up	+0.67	0.32	3
	port	+0.44	0.38	4
3,5	all	+0.058	0.31	9
	up	-0.53	0.26	3
	port	+0.18	0.19	6
6,8	all	-0.55	0.32	5†
	up	+0.016	---	2
	port	-0.88	---	2

$$*Q = Q(a,b) = \frac{\text{Acceleration from shot } a - \text{acceleration from shot } b}{\text{Acceleration from shot } a + \text{acceleration from shot } b}$$

†Includes one piece of data in forward direction.

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The values of Q and PE_Q for putty accelerometer data are presented in Table 7. Shots 2,4 (up) and 3,5 (up) give the only evidences (using the same criterion) of unreproducibility and these are so inconclusive that they may be justifiably ignored.

To sum up, it is concluded that the experimental procedures employed in the NIAGARA trials are reproducible to within 91-95%; and there is no clear evidence that the reproducibility may not actually be greater.

* * *

APPENDIX I
Correction of Velocity-Meter Records

It is well known that a seismic velocity meter gives valid information only for frequencies considerably in excess of its own natural frequency and that the low-frequency velocity components of a transient are recorded incorrectly by such a device. Although it is difficult to specify the conditions under which the velocity meter will not departfrom a specified accuracy, the converse problem is capable of solution. Given a record, it is possible to correct it for the error introduced by the finite natural frequency of the instrument.

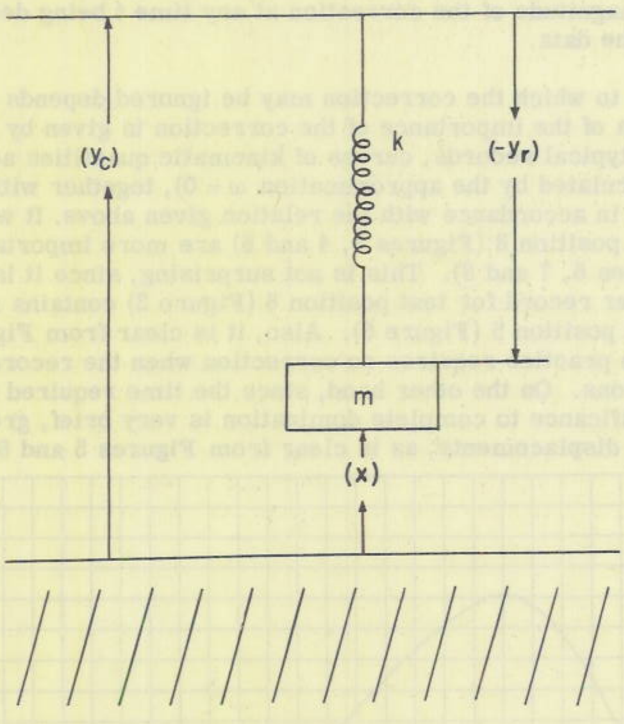

Figure 2

In Figure 2 let $y_c(t)$ be the inertial displacement of the case (whose first derivative it is desired to determine); y_r, the displacement of the seismic mass, m, relative to the case; and x, the inertial displacement of the bob, all taken positively upward. Then $x(t) = y_c(t) + y_r(t)$ from geometry, and $m\ddot{x}(t) = -ky_r(t)$ from the equation of motion, k being the spring stiffness.

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Combining these,

$$\ddot{y}_c(t) = -(\ddot{y}_r(t) + \omega^2 y_r(t)). \tag{5}$$

Selecting the zero of time as any convenient instant before the shock, one has the first integral of this expression,

$$\dot{y}_c(t) = -\left[\dot{y}_r(t) + \omega^2 \int_0^t y_r(p)\, dp\right], \tag{6}$$

since all kinematic quantities were zero before the shock. (The variable, p, has been introduced as the argument of y_r in order to distinguish the variable from the upper limit of the integral). For the same reason, one is justified in setting

$$y_r(p) = \int_0^p \dot{y}_r(q)\, dq, \tag{7}$$

which when substituted into the last equation gives:

$$\dot{y}_c(t) = -\left[\dot{y}_r(t) + \omega^2 \int_0^t dp \int_0^p \dot{y}_r(q)\, dq\right]. \tag{1}$$

The term in this equation containing the integral may be considered a correction to the data, $y_r(t)$, the magnitude of the correction at any time t being determined by the entire time history of the data.

The time up to which the correction may be ignored depends on the use to be made of the data. An idea of the importance of the correction is given by Figures 3 through 8, which show, for typical records, curves of kinematic quantities actually recorded (or, equivalently, calculated by the approximation $\omega = 0$), together with the correction (dashed line) to be added in accordance with the relation given above. It will be noted that the corrections for test position 8 (Figures 3, 4 and 5) are more important than those for test position 5 (Figures 6, 7 and 8). This is not surprising, since it is apparent that the original velocity-meter record for test position 8 (Figure 3) contains more low frequencies than that for test position 5 (Figure 6). Also, it is clear from Figures 4 and 7 that the velocity meter in practice requires no correction when the records are to be used to calculate accelerations. On the other hand, since the time required for the correction to rise from insignificance to complete domination is very brief, great caution must be used when calculating displacements, as is clear from Figures 5 and 8.

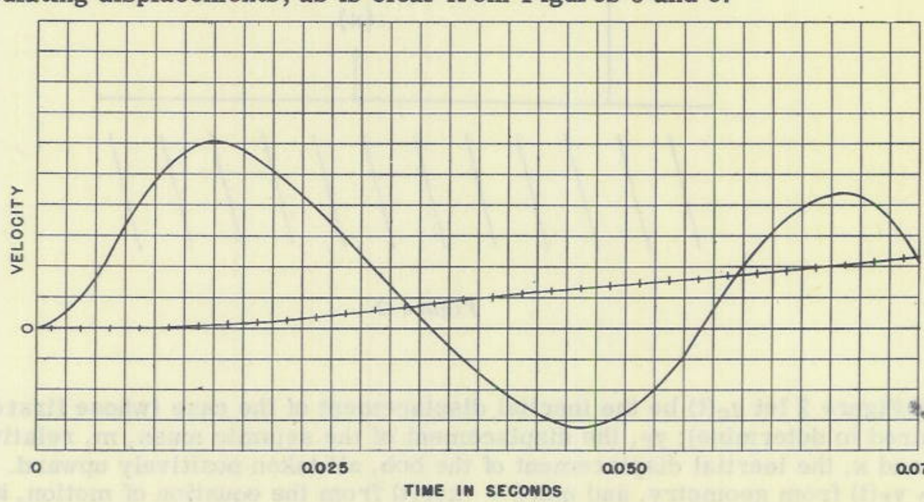

Figure 3

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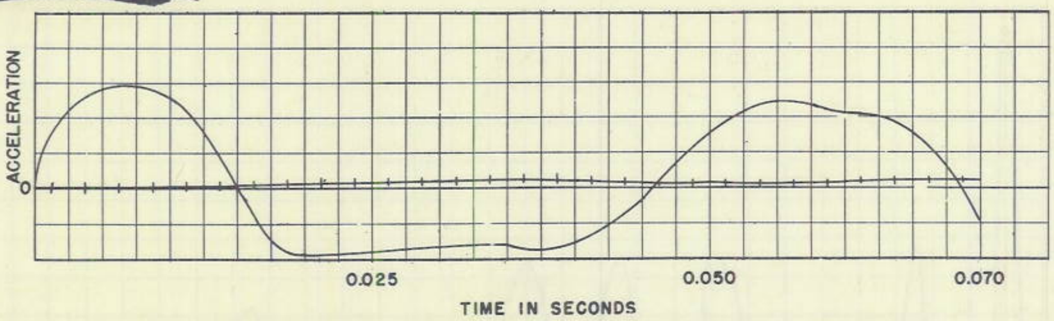

Figure 4

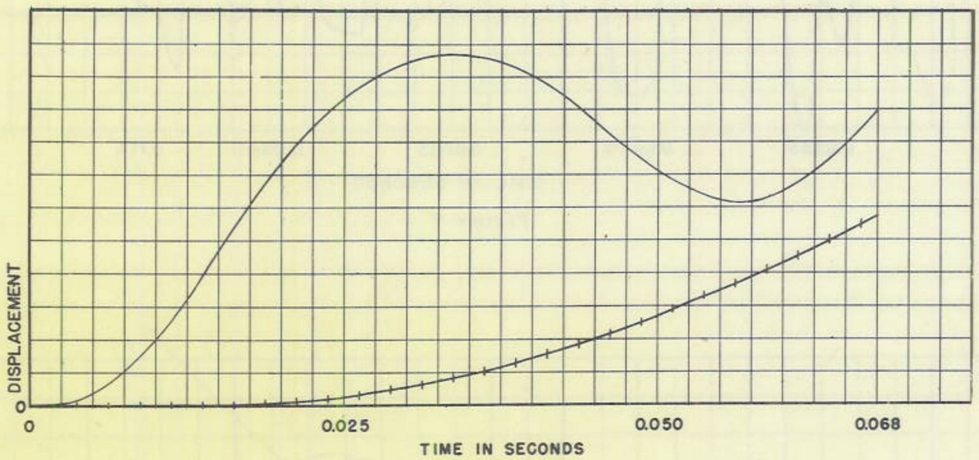

Figure 5

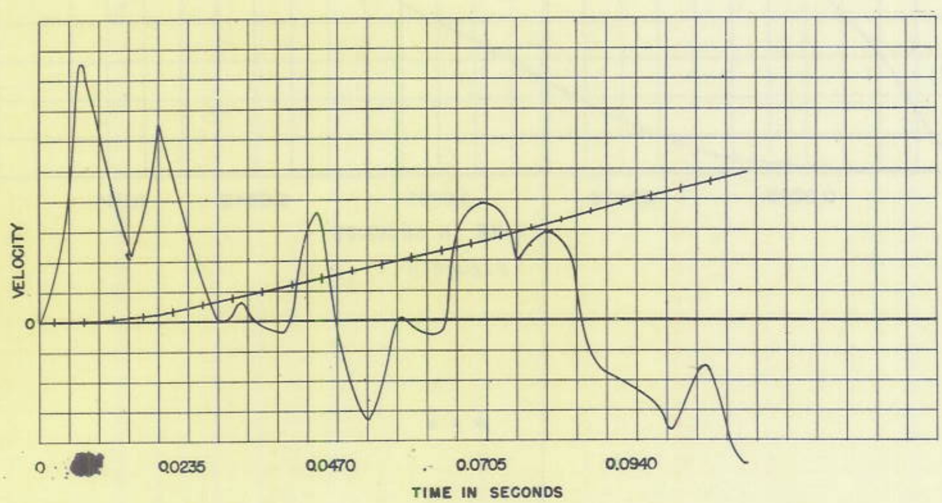

Figure 6

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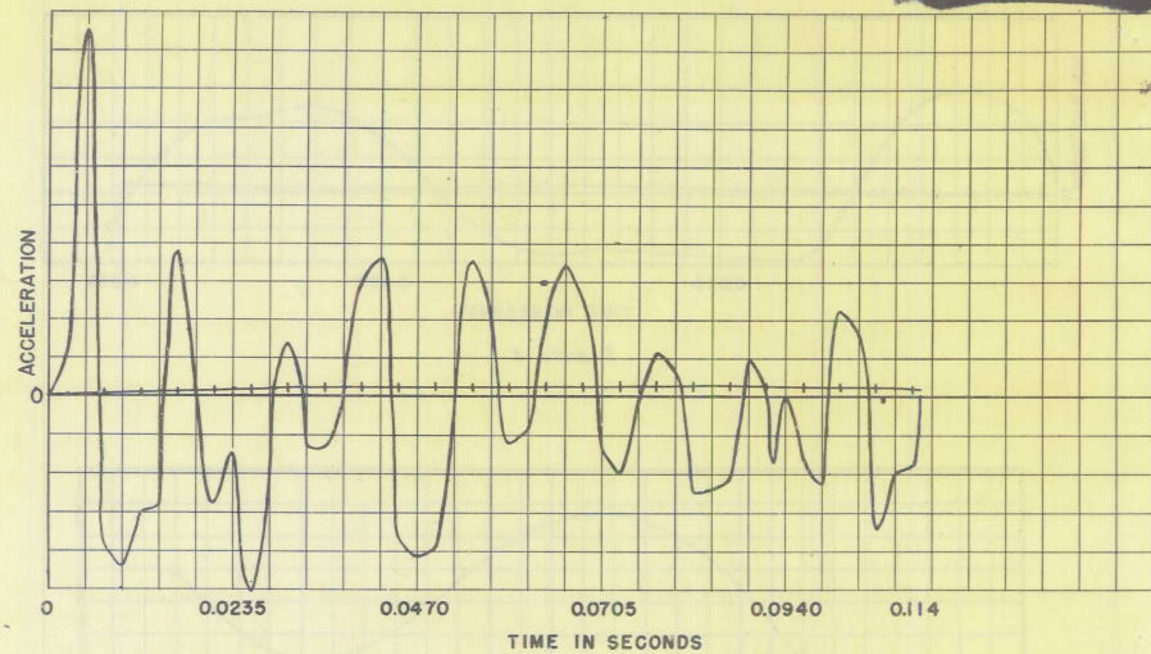

Figure 7

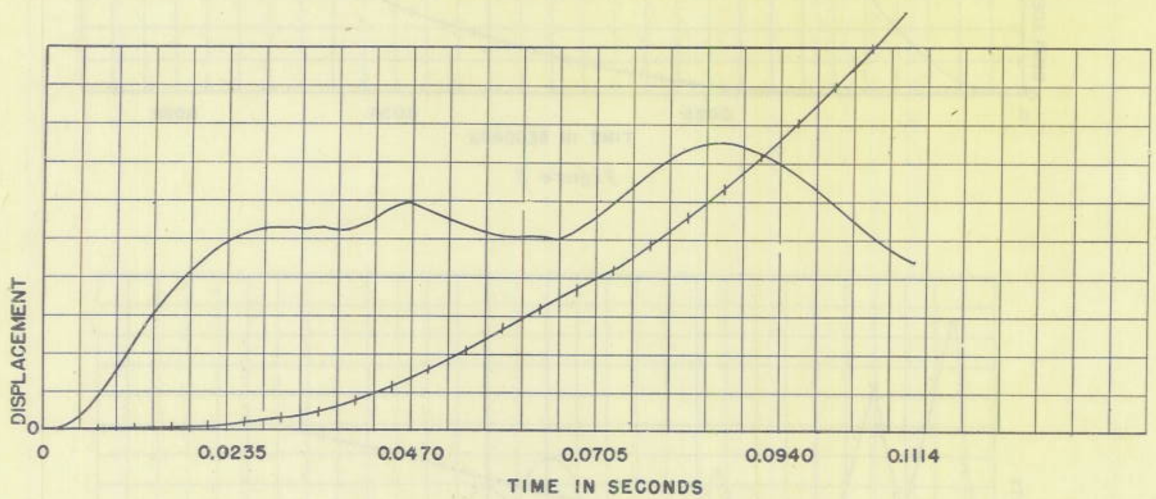

Figure 8

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APPENDIX II
The Motion of Undamped Single-Degree-of-Freedom Systems

Consider a single-degree-of-freedom system of spring stiffness, k, and mass, m, the end of the spring opposite the mass being subjected to an excitation. Using Figure 2 and the terminology of Appendix I, the analysis of this system is identical with that of the velocity meter considered there. Let us then repeat equation (5),

$$\ddot{y}_c(t) = -(\ddot{y}_r(t) + \omega^2 y_r(t)),$$

where $\omega^2 = k/m$, and consider the motion of the case, $y_c(t)$, to be the excitation, and the spring deflection, $y_r(t)$, to be the response. Since the spring is initially undeflected prior to the application of the excitation, it may be seen that the solution for $y_r(t)$ is dependent only upon the excitation and the parameter, ω.

The internal motion of any single-degree-of-freedom system is then specified completely by its natural frequency. The mass of the bob and the stiffness of its support enter the solution only in their ratio.

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APPENDIX III
The Response of a Reed Gage to a Gradually Rising Pulse of Acceleration

By means of the Laplace Transform,[14] the response of a single-degree-of-freedom system to a gradually rising pulse of acceleration will be calculated. It will be shown that, for a high-frequency reed, the value of N corresponds to the asymptote of the pulse. Using Figure 2 and the terminology of Appendix I,

$$\ddot{y}_C(t) = -\left[\ddot{y}_r(t) + \omega^2 \ y_r(t)\right]. \qquad (5)$$

Let $\ddot{y}_C(t) = A\,(1 - \epsilon^{-\alpha t})$ (see Figure 9).

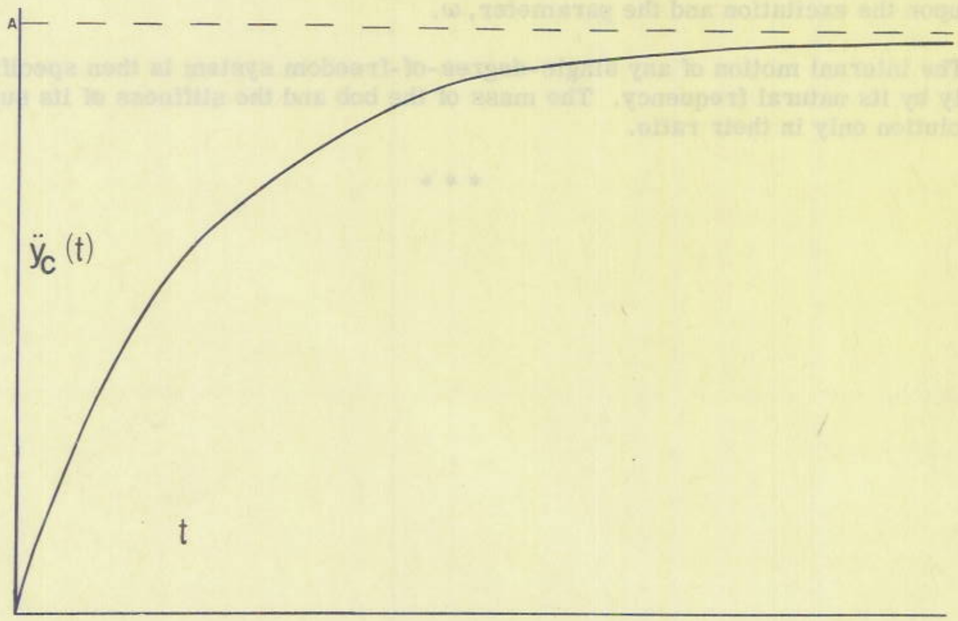

Figure 9

The initial conditions being

$$y_r = 0,\ t = 0+$$

and

$$\dot{y}_r = 0,\ t = 0+,$$

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[14] *For the theory of the Laplace Transform, see Garaner and Barnes, Transients in Linear Systems, 1, 1942, New York (Wiley).*

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the transformed equation becomes

$$Y_r(s) = -A\frac{\alpha}{s(s+\alpha)(s^2+\omega^2)}, \tag{8}$$

where $Y_r(s)$ is the Laplace Transform of y_r, $Y_r(s) = L\ (y_r(t))$. After a straightforward manipulation, one finds the inverse transform as

$$y_r(t) = L^{-1}(Y_r(s)) = A\left[\frac{\alpha(\alpha^2+\omega^2)^{\frac{1}{2}}}{\alpha^2\omega^2+\omega^4}\sin(\omega t+\phi) + \frac{\epsilon^{-\alpha t}}{\alpha^2+\omega^2} - \frac{1}{\omega^2}\right]. \tag{9}$$

If the acceleration is slow to rise, i.e., if $\omega >> \alpha$, this simplifies to

$$y_r(t) = A\left[\frac{\alpha}{\omega^3}\sin(\omega t+\phi) + \frac{1}{\omega^2}(\epsilon^{-\alpha t}-1)\right]. \tag{10}$$

The sine and the exponential function within the parentheses are both of magnitude between 0 and |1| so that, under the same assumption about the sizes of ω and α, the last relation further approximates to

$$y_r(t) = \frac{A}{\omega^2}(\epsilon^{-\alpha t} - 1), \tag{11}$$

the magnitude of whose maximum value is A/ω^2. The value of N corresponding to this deflection is

$$N = (y_r)_{max}\frac{\omega^2}{g} = \frac{A}{\omega^2}\frac{\omega^2}{g} = \frac{A}{g} \tag{12}$$

so that the N so calculated is the height of the excitation asymptote in g's.

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APPENDIX IV
The Statistical Comparison of Linearly Related Quantities

Material on the least-squares process of fitting empirical curves to data will be found in many sources, a particularly good place to begin being the summary of Margenau and Murphy.[15]

In order that the reader may be made familiar with the assumptions made, however, the fitting formula will also be derived here. First, it is assumed, on other than mathematical grounds, that the most probable relation between a set of quantities, y_i, and another set, x_i, is linear with zero intercept, that is,

$$y = bx. \tag{13}$$

This statement requires amplification. For reasons of convenience in computation, it is assumed that there are no errors in the x's – an assumption which is almost always made in applying least-squares theory and which, as is usually the case, obviously does not accord with the facts. The reason one comes to valid conclusions is that the essential question to be decided is the existence or nonexistence of a clearly defined relationship between x and y. Since the question in the end is decided by the gross values of statistical quantities, the final conclusions are not likely to be altered by small inaccuracies in the numerical results.

What, then, does one mean by "most probable relation" or, equivalently, what is the meaning of y in the assumed relation, $y = bx$? The y which appears in the relation is <u>not</u> any of the data, y_i; it is the <u>mean</u> of a set of numbers, y_i, which have been measured at a constant x. The relation, $y = bx$, is a prediction that if x is held constant at x_i the mean of a set of measured y's will be bx_i.

To put it another way, it is assumed that the y_i's, the actual data corresponding to a value of x equal to x_i, form a Gauss distribution about a mean value, $\overline{y}_i = bx_i$. Up to this point, one has assumed nothing about the manner in which the "spread" of the distribution of y's varies with x_i. The question of whether the probable error, calculated about bx_i, of one set of quantities, y_i (corresponding to x_i), is greater or less than the probable error, calculated about bx_k, of another set, y_k (corresponding to x_k), has not been raised.

It is next assumed that the dispersion about bx_i is independent of x_i, that is, that the probable error about the line, $y = bx$, is the same for any value of x. This assumption is usually (as in this case) made from necessity rather than from reason. Some assumption about the variation of probable error of y with varied x must be introduced in order to

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[15] *Margenau, H. and Murphy, G. M. <u>The Mathematics of Physics and Chemistry</u>, D. Van Nostrand Co., Inc., New York, 1943. See Ch. 13, Part 3, and the references there cited.*

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proceed any further; this assumption is the only one which allows the analysis to proceed a great deal further. Now, there is no particular reason to disbelieve the correctness of this assumption about the APA data, but the fact that there is no positive reason for belief, either, constitutes a warning to any who might credit the statistical results with an undeserved importance.

To proceed, one may now define a set of residuals, ρ_i, as

$$\rho_i = bx_i - y_i, \quad i = 1, 2, 3, \ldots, n \tag{14}$$

and form the sum of the squares of these residuals,

$$\sum_{i=1}^{n} \rho_i^2 = \sum_{i=1}^{n} (bx_i - y_i)^2. \tag{15}$$

The foregoing assumptions are then sufficient grounds for the application of the principle of least squares, which states that the most probable value of b is that one which makes $\sum_{i=1}^{n} \rho_i^2$ a minimum.

Then

$$\frac{d}{db} \sum_{i=1}^{n} \rho_i^2 = 0, \text{ or } b = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}. \tag{16}$$

Having determined b, one may calculate the probable error of the y_i,

$$PE\, y_i = 0.6745 \left(\frac{\sum_{i=1}^{n} \rho_i^2}{(n-1)} \right)^{\frac{1}{2}}, \tag{17}$$

the foregoing assumptions being sufficient to justify the application of this well-known formula.

It should be noted that the foregoing assumptions contain in them the implicit hypothesis that both b and PEy_i are quantities determined by the "population" (the aggregate of all possible x's and y's), not by the "sample" (the particular x's and y's measured). Their values, insofar as the assumptions are valid, relate to the objective situation under investigation; the experiments performed affect only the probable errors of b and PEy_i.

In this regard, it will now be clear that for our purposes there is no reason for evaluating the probable error of b. To do so would introduce no additional assumptions; on the other hand the result, in this case, would be of little value.

* * *