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COMPUTER FOR RADIATION PATTERNS OF HIGH-GAIN ANTENNAS

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ABSTRACT

An electromechanical analog computer has been developed jointly by NRL and the Massachusetts Institute of Technology to solve the following equation,

$$V(\psi) = \frac{1}{45} \sum_{\nu = -22}^{\nu = +22} A_{\nu} \exp \left[\Phi_{\nu} + \frac{\nu}{22} \psi \right].$$

This equation gives exactly the amplitude of the radiation pattern of a linear array of 45 equispaced elements fed with arbitrary amplitude, A_{ν} , and phase, Φ_{ν} . The independent variable is $\psi = 360 \cdot S/\lambda \cdot \sin \theta$, where S is the half-length of array, and θ is the aspect angle. The array pattern provides a good approximation to the pattern of a linear aperture with illumination of arbitrary amplitude and phase. Further extension to a plane aperture may be made by auxiliary integrations, and thence, by calculations based on optical techniques, to focusing antennas of the reflecting or refracting type. Linear arrays of fewer than 45 elements can also be analyzed.

The computer has been built, tested and put into operation for calculating radiation patterns of high-gain antennas. The accuracy of the apparatus has been evaluated by direct measurement of errors, and by comparison of test patterns obtained from it with the same patterns computed by standard techniques. The machine error is absolute, resulting primarily from statistical sources, and has a maximum value of 0.25 percent of full-scale output in amplitude. On a logarithmic power scale, this will introduce a random error of the order of 1 db at the 34 db level. Operation is manual; the usual problem takes an experienced operator one to four hours for solution.

PROBLEM STATUS

This report concludes the work on the problem. Unless otherwise notified by the Bureau, the Laboratory will consider this problem closed one month from the mailing date of this report.

AUTHORIZATION

NRL Problem R09-48R
NR 509-480

GLOSSARY OF LETTER NOTATION*

Letter	Page of 1st Appearance	Definition
a	5	Contact: on ρ ; provides phase information.
A	1	Relative amplitude of component of the field at a point on the aperture.
A	16	Amplitude bar.
A_i	10	Rack bar.
A_{ν}	2	Normalized relative amplitude of excitation of ν^{th} element.
b	5	Contact: on r ; provides amplitude information.
n	4	Number of interelement spaces in half array.
P_o	11	Gear-locking knob.
r	5	75K resistor = amplitude potentiometer.
R	5	2000K isolating resistor, permits averaging outputs e_b .
S	2	Half-width of line aperture.
V	1	Complex radiation-vector component.
V	7	Normalized alternating output voltage of computer, or normalized space factor of array.
V_o	5	Direct output voltage of computer network.
x	1	Horizontal coordinate of point on plane aperture.
y	1	Vertical coordinate of point on plane aperture.
θ	1	Latitudinal direction angle of V ; space aspect angle of line aperture or linear array.
λ	2	Free-space wavelength of radiation.
ν	2	Ordinal number of an element in a linear array.
ρ	5	5 K resistor = phase potentiometer.
σ	5	Manual reversing toggle switch.
ϕ	1	Longitudinal direction angle of V .
Φ	1	Relative phase of the field at a point on the aperture.
Φ_ν	2	Relative phase of excitation of ν^{th} element.
Ψ	1	Relative phase due to the difference between the length of path from a point on the aperture to a point P with arbitrary direction angles, and a point P_o with $\theta = 0$. Angle of equivalent advance in phase of excitation at the end of a line aperture due to aspect angle θ ; angle of rotation of gear #45.

* Not all the symbols in the text are listed here, as a number are well defined in the places where they appear. In a number of cases, symbols appear at points remote from where they are defined in the text.

COMPUTER FOR RADIATION PATTERNS OF HIGH-GAIN ANTENNAS

INTRODUCTION

The design and study of narrow-beam and shaped-beam antennas may be based on either computation or experimentation, or, more generally, a combination of the two. Either approach is by successive approximations and hence is tedious. Therefore any method of facilitating some of the operations involved should contribute to a better understanding of directional antennas.

The laboratory evaluation of antenna performance has been greatly simplified by the mechanization of repetitive processes. Systems for the automatic recording of patterns and phase fronts developed recently ^{1,2,3,4,5} have been effective tools in antenna design. A similar mechanization of antenna computing processes should be equally fruitful. Investigation has shown that utilization of any of the existing general-purpose automatic computers is both costly and inconvenient. Hence, the need is for mechanized computation specifically designed for the solution of antenna problems.

An important computational problem encountered in antenna design is the determination of the radiation pattern of an antenna. A rigorous formulation of the problem, based on Maxwell's equations, is possible. However, the mathematical operations involved are difficult to perform, and a knowledge of the radiating system is required that in practice cannot be provided. Several simplifications of the exact formulation of the problem can be made to render the problem computable, and to base it on data physically available. The particular simplification selected will depend on the size and nature of the radiating system, the frequency at which it operates, the type of pattern it produces, and the accuracy with which the predicted pattern should conform to the actual one.

One method, effective in evaluating the radiation pattern of a high gain antenna, is to approximate the electromagnetic field of the antenna over a plane aperture. It is then possible, subject to some simplifying approximations, to compute the radiation field at point P. This field computed from the aperture distribution is that of the antenna.⁶ If all the reflecting and refracting surfaces of the antenna are sufficiently far from the primary sources and if the curvature of these surfaces is small, optical techniques⁷ may be used to predict the amplitude, phase, and polarization over a chosen plane. If the field vanishes over all but a finite part of the plane (the aperture⁸ of the antenna), and if the phase is nearly constant, then any polarization component, $V(\theta, \phi)$, of the radiation vector in the direction (θ, ϕ) , see Figure 1, is given by

$$V(\theta, \phi) = \iint A(x, y) e^{j[\Phi(x, y) + \psi(\theta, \phi, x, y)]} ds \quad (1)$$

where the integration is extended over the aperture. The functions $A(x,y)$ and $\Phi(x,y)$ are the relative amplitude and phase of the corresponding polarization component of the field at a point (x,y) on the aperture. The function $\psi(\theta, \phi, x, y)$ represents the phase difference arising from different path lengths. For P in the Fraunhofer region of the radiating system, ψ is given by

$$\psi(\theta, \phi, x, y) = \frac{2\pi}{\lambda} \sin \theta (x \cos \phi + y \sin \phi). \quad (2)$$

In any direction for which $\phi = 0$ (and similarly for $\phi = \pi/2$), the integration of Equation (1) with respect to y is independent of ϕ , and may be performed by standard techniques. This integration gives $A(x)$ and $\Phi(x)$, the amplitude and phase of the field on an equivalent line aperture. Equation (1) then becomes

$$V(\theta, 0) = \int A(x) e^{j[\Phi(x) + \psi(\theta, x)]} dx \quad (3)$$

where the integration is now extended over the line aperture. This integral may be approximated by a finite sum and

$$V(\theta, 0) \doteq \sum_{\nu=-n}^n A_{\nu} e^{j(\Phi_{\nu} + \frac{\nu}{n}\psi)} \cdot \frac{1}{2n+1}. \quad (4)$$

Here $\psi = (2\pi S/\lambda) \sin \theta$ where $2S$ is the width of the aperture measured along the x -axis, and the total number of equal intervals is $2n$. Experience has shown that for the more common types of antennas, Equation (4) is a better approximation than would be obtained by using weighted sums or unequal subintervals.

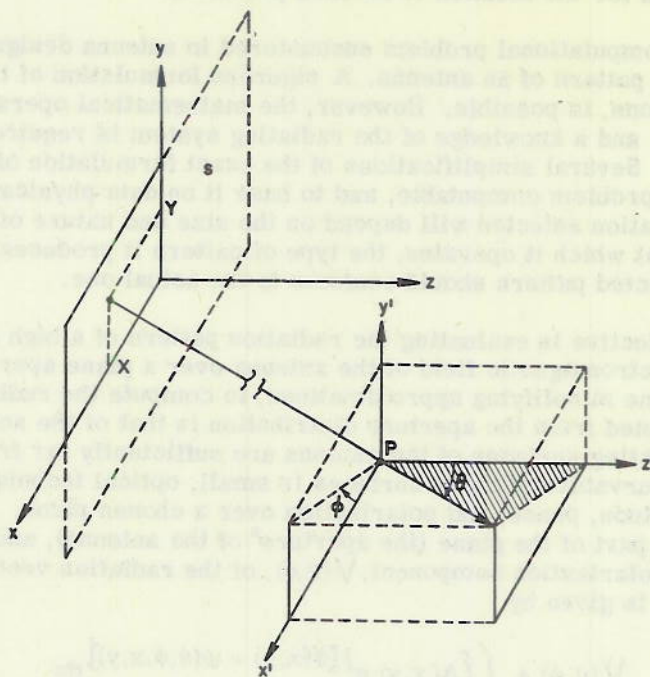


Figure 1 - Coordinate systems used in equation (1) et seq.

Although Equation (4) was derived as an approximate expression for the radiation vector of a high-gain antenna, it has other interpretations. It is an exact expression for the array factor of a linear array with equispaced elements. Also, it is exact for many constant- ϕ cuts of the array factor of a uniformly-spaced planar array. It is an approximation to the radiation vector due to known current distributions on plane surfaces or along a line source.

The requirements to be met by a machine which is to handle the problem from this point; i.e., to be an analog of a linear array of point sources, or, in terms of the optical approach, to compute the Fourier transform of the line-aperture illumination, are implicit in Equation (4). It is to be emphasized, first, that the independent variable, θ , is contained in the parameter ψ . Although for any given antenna, operated at any particular frequency, the fundamental parameter is the space aspect angle θ , it would be at least permissible, if not desirable, to have the output of a pattern computer as a function of $\psi = (2\pi S/\lambda) \sin \theta$ (or $360 \cdot S/\lambda \cdot \sin \theta$ in degrees). This would give more general results, as the pattern would then be independent of frequency or of aperture width. On the other hand, it would then be desirable to have some means, such as an auxiliary mechanism, or a chart⁹ (see Figure 2) to introduce the scaling factor; i.e., to convert the independent variable to θ for any particular value of the S/λ ratio.

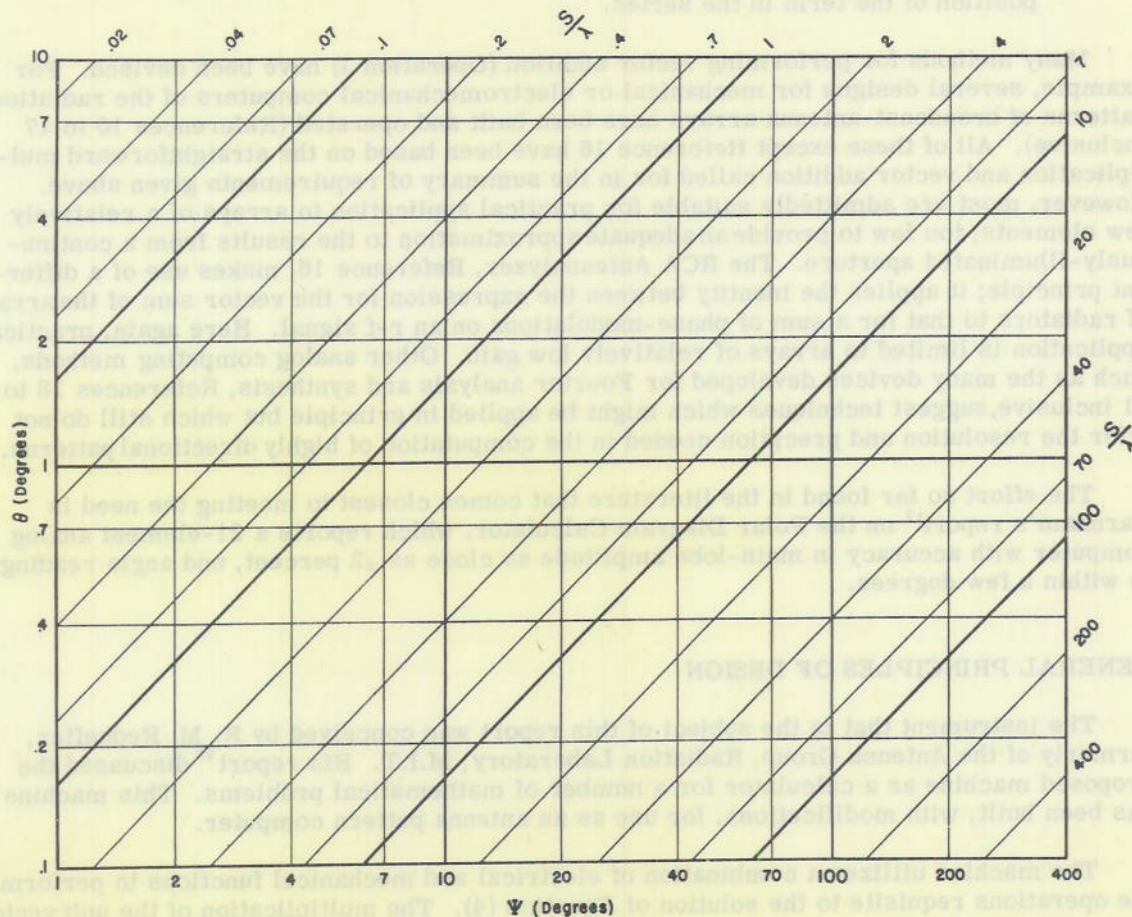


Figure 2 - Conversion Chart for $\psi = 360 \cdot \frac{S}{\lambda} \cdot \sin \theta$

Next, it is noted that the signal to be given by the analog machine is a complex quantity, $V(\psi)$, proportional to the amplitude of the radiation vector. Since the more usual need, as far as antenna design is concerned, is for a measure of radiation intensity or relative power; and since, furthermore, the machine is also to be used to perform the integration of Equation (1) with respect to y , providing $A(x)$ and $\Phi(x)$ for the equivalent line aperture, the complex output should be in polar form.

In more detail, the machine must perform the following operations:

- (1) It must sum a series of complex quantities;
- (2) For each of the terms in the sum it must combine a given magnitude with the corresponding angle; this is essentially a multiplying process, as indicated by writing the complex quantity in rectangular form:

$$A_{\nu} \left[\cos \left(\Phi_{\nu} + \frac{\nu}{n} \psi \right) + j \sin \left(\Phi_{\nu} + \frac{\nu}{n} \psi \right) \right];$$

- (3) Each of the angles must be the sum of two angles, one a fixed angle, Φ_{ν} , the other an angle, $(\nu/n) \psi$, proportional to the independent variable and to the position of the term in the series.

Many methods for performing vector addition (Operation 1) have been devised. For example, several designs for mechanical or electromechanical computers of the radiation patterns of broadcast-antenna arrays have been built and operated (References 10 to 17 inclusive). All of these except Reference 16 have been based on the straightforward multiplication and vector addition called for in the summary of requirements given above. However, most are admittedly suitable for practical application to arrays of a relatively few elements; too few to provide an adequate approximation to the results from a continuously-illuminated aperture. The RCA Antennalyzer, Reference 16, makes use of a different principle; it applies the identity between the expression for the vector sum of the array of radiators to that for a sum of phase-modulations on an r-f signal. Here again, practical application is limited to arrays of relatively low gain. Other analog computing methods, such as the many devices developed for Fourier analysis and synthesis, References 18 to 21 inclusive, suggest techniques which might be applied in principle but which still do not offer the resolution and precision needed in the computation of highly directional patterns.

The effort so far found in the literature that comes closest to meeting the need is Garnham's report¹⁷ on the Polar Diagram Calculator, which reports a 21-element analog computer with accuracy in main-lobe amplitude as close as ± 2 percent, and angle readings to within a few degrees.

GENERAL PRINCIPLES OF DESIGN

The instrument that is the subject of this report was conceived by R. M. Redheffer, formerly of the Antenna Group, Radiation Laboratory, M.I.T. His report²² discussed the proposed machine as a calculator for a number of mathematical problems. This machine has been built, with modifications, for use as an antenna pattern computer.

The machine utilizes a combination of electrical and mechanical functions to perform the operations requisite to the solution of Equation (4). The multiplication of the unit vectors, $e^{j(\Phi_{\nu} + \frac{\nu}{n} \psi)}$, by the corresponding amplitudes, A_{ν} , and the subsequent summation, are performed electrically, using direct current in linear potentiometers. This is shown

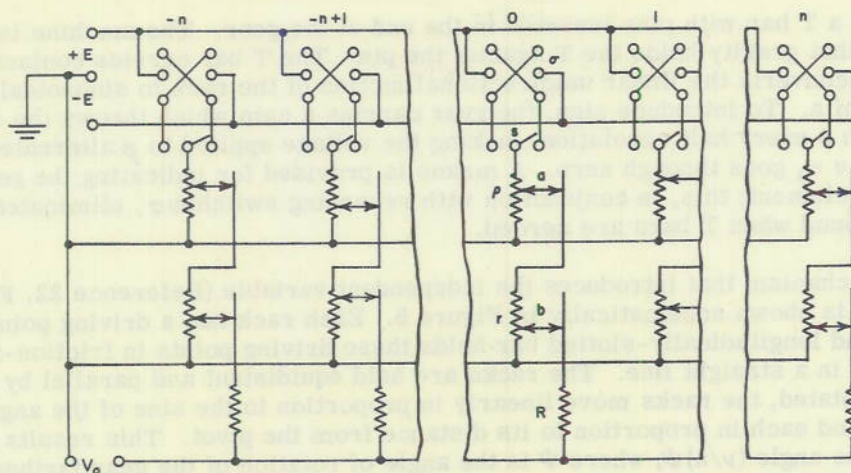


Figure 3 - Circuit for obtaining the sum of the products:

$$V_o = \frac{E}{2n+1} \sum_{v=-n}^n A_v B_v$$

schematically in Figure 3 (corresponding to Reference 22, Figure 2c). Symbols +E and -E represent stabilized direct voltages, adjusted to be as nearly equal as possible. Switches σ and s , potentiometers p , and sliding contacts a constitute the electrical components needed to insert the phase information. Potentiometers r and sliding contacts b introduce the amplitude function. Resistors R provide isolation, so that V_o is the arithmetic average of the voltages e_b . To minimize loading error, $R \gg r \gg p$, and the maximum settings of the sliders are restricted to roughly 0.7 of the full resistance of their respective potentiometers.²³ The amplitude function is readily set in manually by pushing the sliders b . Since linear potentiometers are used, the insertion of the phase function and control of the continuous phase shift require mechanical linkages.

The insertion of the phase information is accomplished by means of the mechanism shown in Figure 4 (Reference 22, Figure 7). A rack drives a many-toothed gear which in

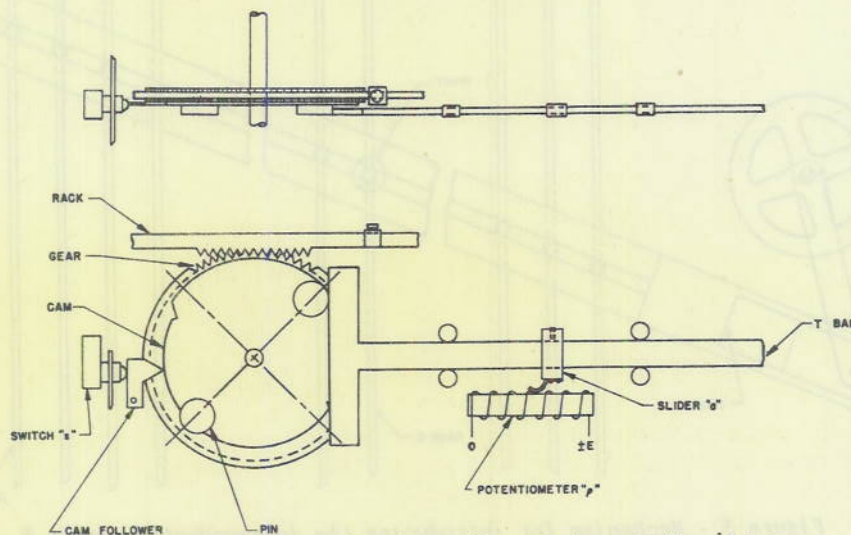


Figure 4 - Mechanism for insertion of phase information:

$$\left. \begin{matrix} \sin \\ \cos \end{matrix} \right\} \left(\Phi_v + \frac{v}{n} \Psi \right)$$

turn pushes a T bar with pins inserted in the end of the gear. The machine is operated at a slant, so that gravity holds the T against the pin. The T bar carries contact a . This mechanism converts the linear unidirectional motion of the rack to sinusoidal reciprocating motion of a . To introduce sign, the gear carries a cam which throws the reversing microswitch s every half revolution, making the voltage applied to p alternately $+E$ and $-E$ each time e_a goes through zero. A means is provided for indicating the polarity of e_a for each element; this, in conjunction with reversing switches σ , eliminates the 180° ambiguity found when T bars are zeroed.

The mechanism that introduces the independent variable (Reference 22, Figure 9 and Figure 11b) is shown schematically in Figure 5. Each rack has a driving point on its top. A pivoted and longitudinally-slotted bar holds these driving points in friction-free but snug confinement in a straight line. The racks are held equidistant and parallel by guides. When the bar is rotated, the racks move linearly in proportion to the sine of the angle of rotation of the bar, and each in proportion to its distance from the pivot. This results in the introduction of the angle $(\nu/n)\Psi$, where Ψ is the angle of rotation of the gear farthest from the pivot. The slotted bar is pivoted at the center of the array, and driven by a crank-and-eccentric mechanism at one end.

The mechanism of Figure 4 introduces the phase information as either the sine or cosine of the total phase angle, depending on which end of the T-bar travel is taken as zero reference. The gears may be locked and rotated as a unit. The maximum value of V_0 that occurs as the locked gears are rotated is proportional to the magnitude of the complex number, $V(\theta, 0)$, in Equation (4). The angle through which the gears are rotated to maximize V_0 represents the angle of this complex number. The value of V_0 for no rotation corresponds to either the real part of V or the imaginary part, depending upon how the phase data was originally inserted in the machine.

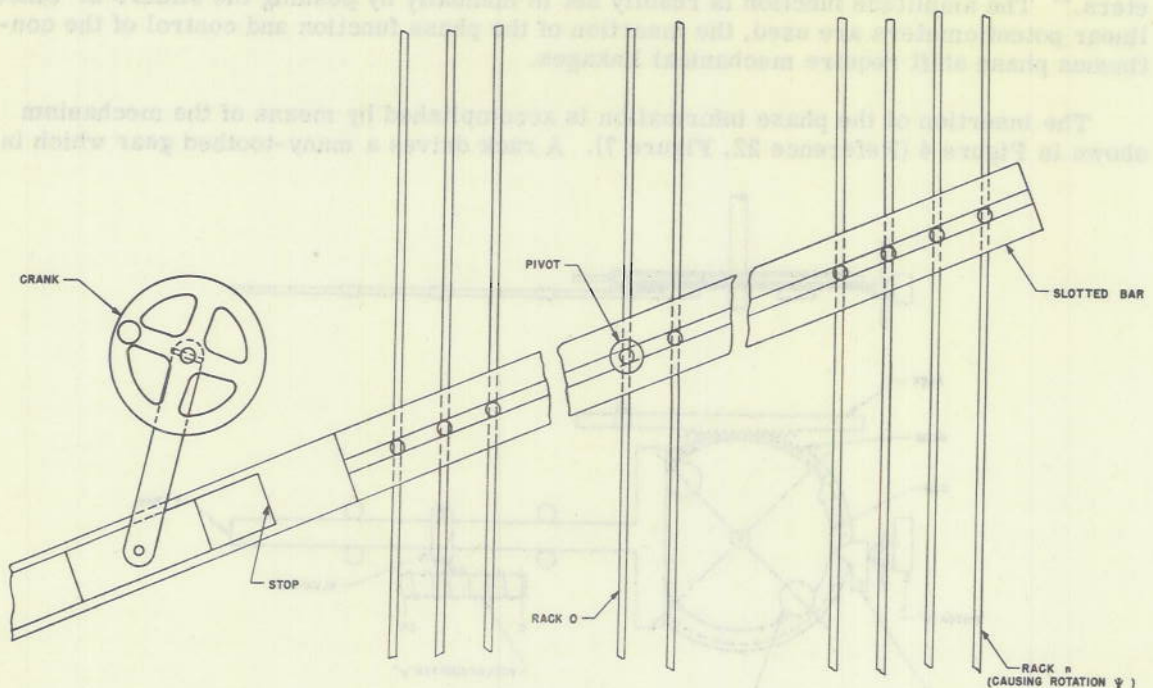


Figure 5 - Mechanism for introducing the independent variable Ψ

Due to requirements of accuracy, the gears have large diameter and small teeth. Limitations of space require that the racks be no longer than about one quarter of the gear circumference. Since, for the complete solution of a problem, the first gear must be moved half a revolution and the n th gear many revolutions, a mechanism is provided which lifts simultaneously all racks out of mesh with their gears. With the gears locked, the out-of-mesh racks may be returned to their initial positions so that successive forward sweeps of the in-mesh racks permit the advancement of Ψ to any desired value.

The setting in of the initial phase function, Φ_v , also makes use of the gear-locking device. The slotted bar is removed from the machine, permitting each of the racks to be set independently of the others. With all gears initially at zero, each is driven by its rack to the position corresponding to Φ_v . The gears are then locked, and the unmeshed racks are lined up in the replaced slotted bar.

DETAILS OF DESIGN AND CONSTRUCTION

The basic number of elements ($2n + 1$) is 45. This number is expected to provide adequate resolution for most approximations to the integral (Equation (2)). Under exceptional conditions, such as when the amplitude distribution varies widely and with relatively great frequency across the aperture, the effective number of elements may be expanded to 90, and again, if required, to 180.

In order to simplify accurate reading, automatic recording if desired, and normalization or reference to a particular level, the output direct voltage (V_o) is converted to an alternating voltage in an electronic unit, the Output Indicator. This unit includes means for normalization, and for reading converted and normalized output V .

A block diagram of the over-all assembly is shown in Figure 6. The computer is divided into three main components, each physically independent of the other two except for cable interconnections. The first section is primarily electronic, and itself has three units: the Power Supply, regulated source of $+E$ and $-E$; the Output Indicator, mentioned above; and the Polarity Indicator, referred to on page 6. The second component is the principal mechanical assembly, and contains all of the elements involved in controlling phase information except the bank of manual reversing switches σ which is included in the Polarity Indicator. The connection from the electronic assembly to this Phasing Unit is by means of three 45-conductor shielded cables. The final component contains potentiometers r , sliders b , and summing resistors R (Figure 3). This Amplitude-Function Unit is connected to the Phasing Unit by a fourth 45-conductor cable, and to the Output Indicator by a single shielded lead.

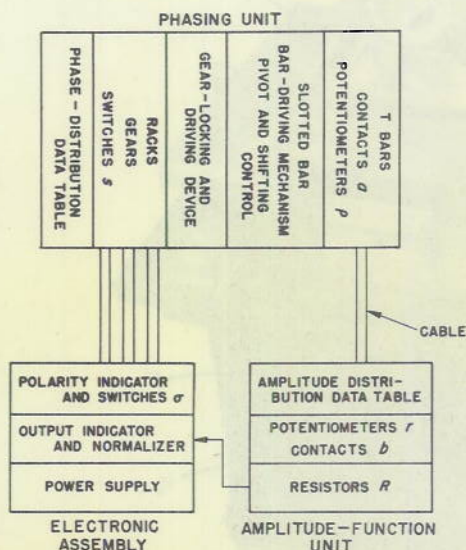


Figure 6 - Block diagram of over-all assembly

A set of photographs grouped as Figure 7 shows views of the computer. The Power Supply, item H, is shown schematically in Figure 8. This circuit has excellent regulation, having an output of 10 ± 0.05 v for load variation from 0 to 100 ma, but

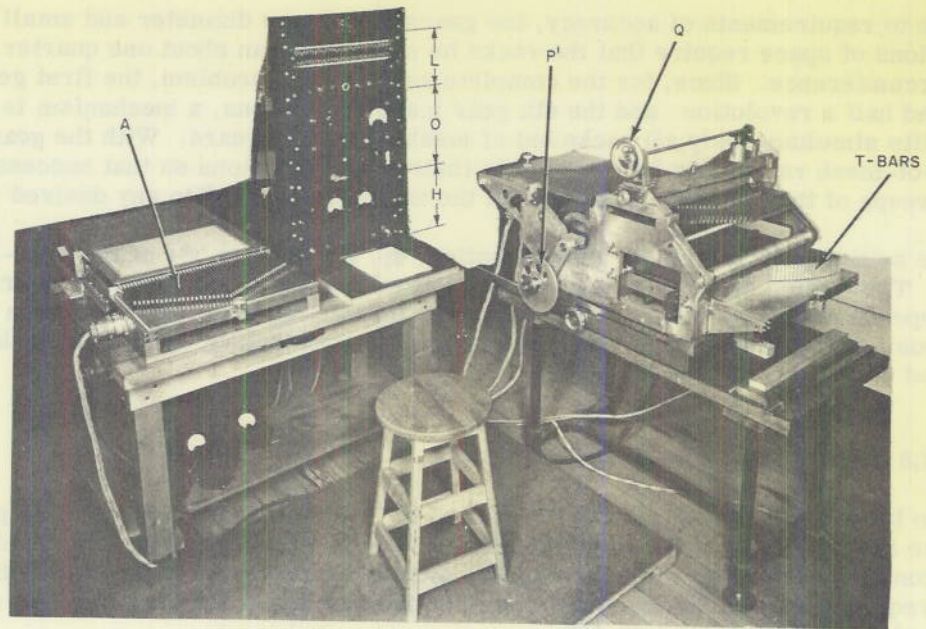


Figure 7a - A view of the antenna pattern computer

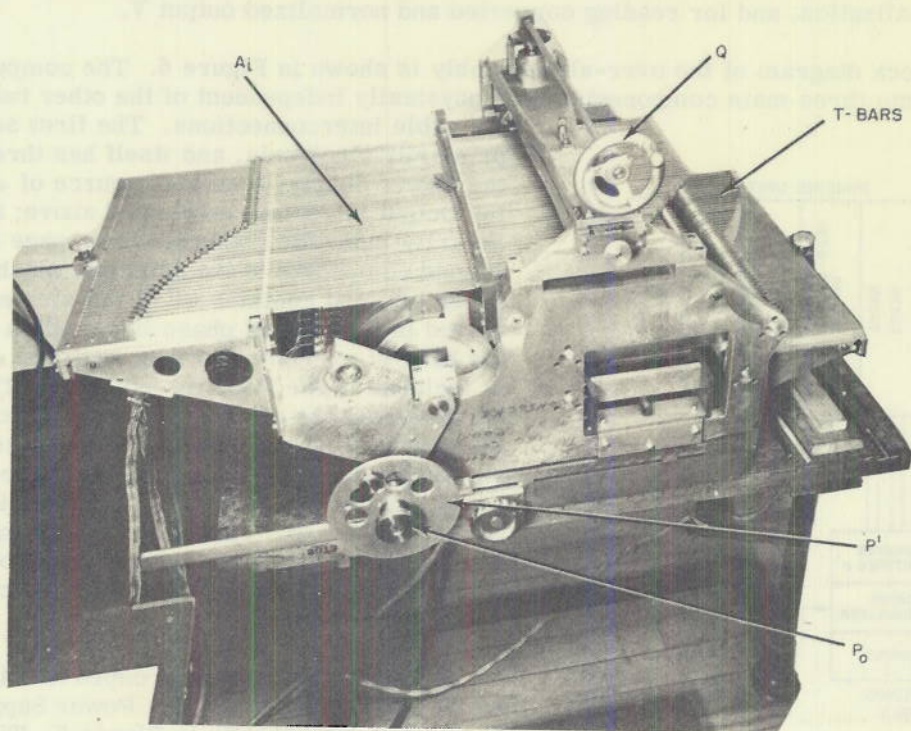


Figure 7b - A view of the phasing unit in the upright position

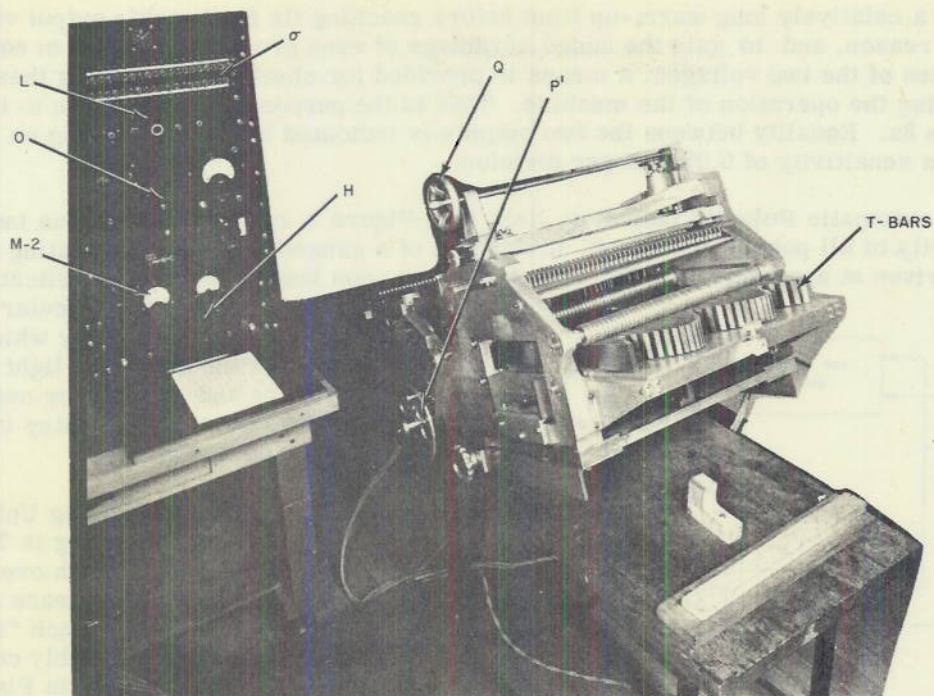


Figure 7c - A view of the phasing unit in operating position

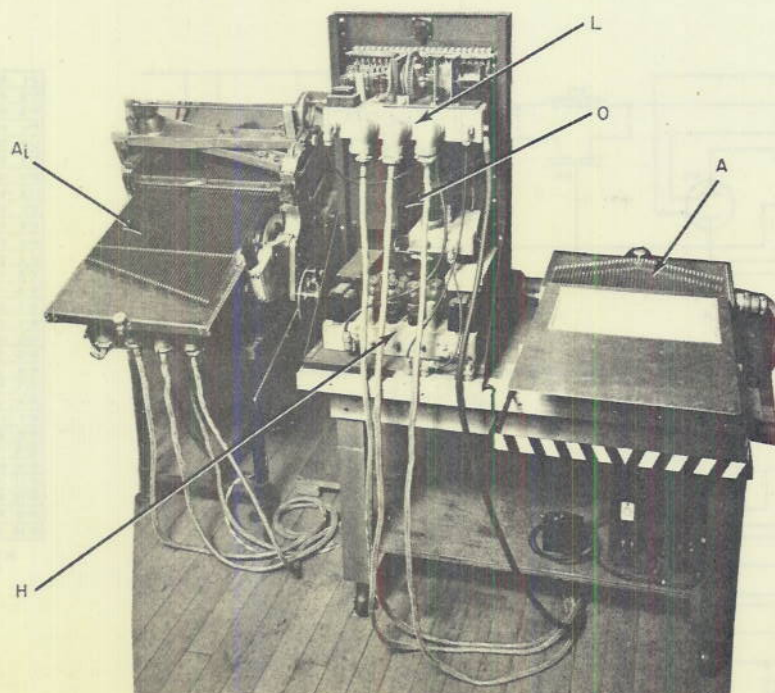


Figure 7d - Another view of the antenna pattern computer

requires a relatively long warm-up time before reaching its final stable output voltage. For this reason, and to gain the added advantage of even greater precision in equating the magnitudes of the two voltages, a means is provided for checking and setting these at any time during the operation of the machine. This is the purpose of the network to the right in Figure 8a. Equality between the two outputs is indicated by a zero reading on M-2, at a maximum sensitivity of 0.05 volt per division.

The automatic Polarity Indicator, item L in Figure 7, can give continuous indication of the polarity of all potentiometers ρ . It consists of a ganged two-section rotating switch, motor-driven at approximately 500 rpm; a bank of neon lamps, each lamp indicating the

polarity of a particular potentiometer; and circuitry which provides that the lamp will light for positive and stay off for negative polarity. The circuitry is diagrammed in Figure 9.

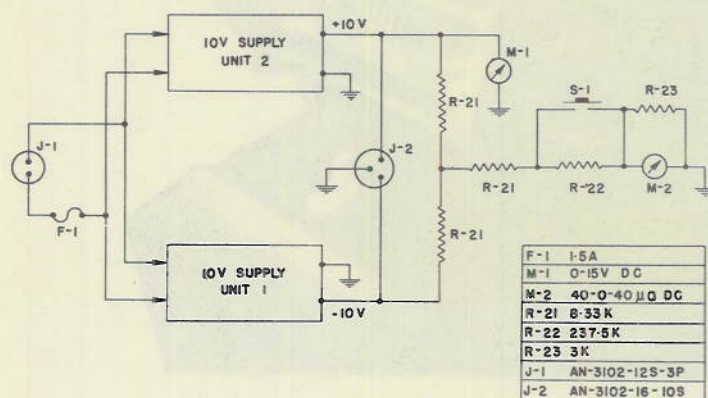


Figure 8a - Power supply assembly

On the Phasing Unit, the interelement spacing is $7/16$ ", giving a total width over 44 spaces of $19 1/4$ ". The gears are 11" P.D., 48 pitch. Each "rack" is actually an assembly consisting of a flat strip (A₁ in Figure 7) to one edge of which is fastened a length of 48-pitch rack and to the other edge, the driving pin. The

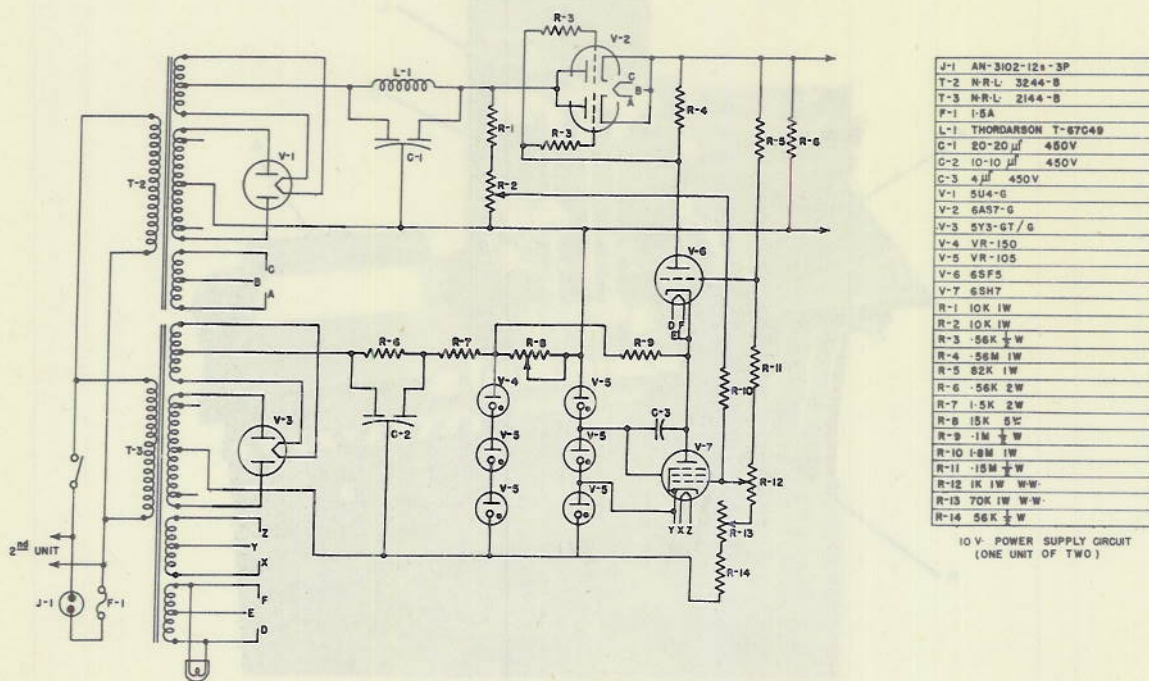


Figure 8b - Power supply schematic

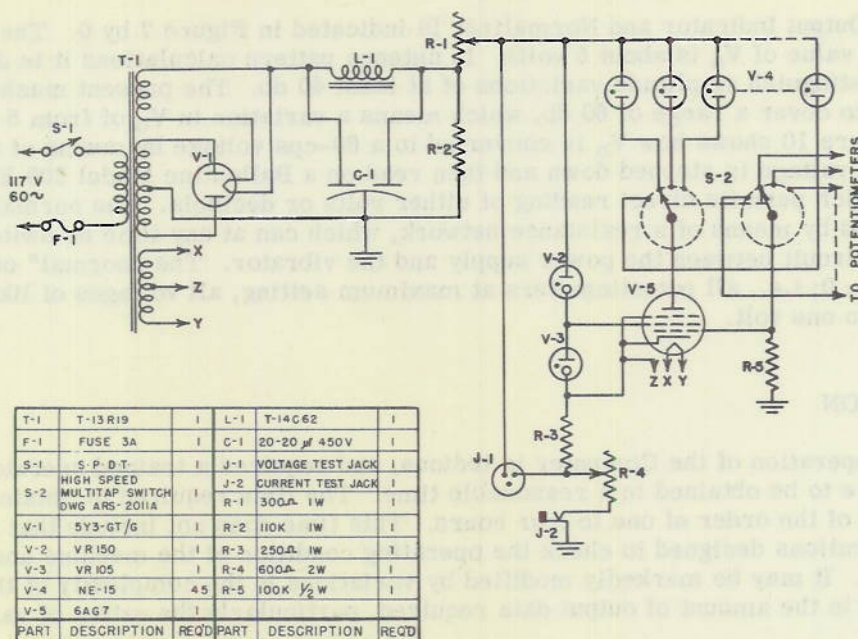


Figure 9 - Polarity indicator schematic

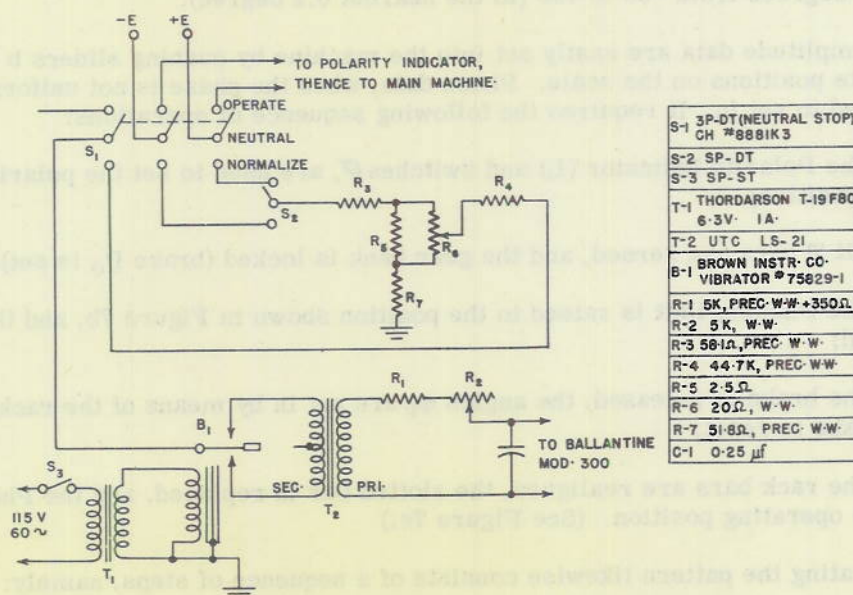


Figure 10 - Output indicator and normalizer schematic

gear-locking device (P' in Figure 7) is a modification of the original design (Reference 22, Figure 12). The revised method, suggested by W. J. E. Edwards, consists of a draw bar carrying idler gears in mesh with the main gears. When locking is desired, the stack is pulled up by knob P₀, which effectively converts the assembly to a single pinion, keyed to the shaft by the stops at either end. More certain braking and rapid release are accomplished by the insertion of discs of molded brake lining between the steel gears.

The Output Indicator and Normalizer is indicated in Figure 7 by 0. The maximum or "normal" value of V_0 is about 5 volts. In antenna pattern calculations it is desirable to be able to distinguish amplitude variations of at least 40 db. The present machine is therefore designed to cover a range of 60 db, which means a variation in V_0 of from 5 volts to 0.005 volt. Figure 10 shows how V_0 is converted to a 60-cps voltage by means of a driven vibrator. This voltage is stepped down and then read on a Ballantine Model 300 Electronic Voltmeter, which permits direct reading of either volts or decibels. The normalization is accomplished by means of a resistance network, which can at any time be switched into the over-all circuit between the power supply and the vibrator. The "normal" output (for $A = 1, \psi = \Phi = 0$; i.e., all potentiometers at maximum setting, all voltages of like polarity) is adjusted to one volt.

OPERATION

The operation of the Computer is tedious, and requires a trained operator if accurate results are to be obtained in a reasonable time. The time required to obtain data for one pattern is of the order of one to four hours. This time does not include that required for extra operations designed to check the operating condition of the machine and the accuracy of results. It may be markedly modified by variations in the complexity of the input functions, and in the amount of output data required, particularly the extent of variation of ψ .

The input data, A_ν and Φ_ν , are generally provided as 45 values of each. A_ν is the relative amplitude from 0 to 1 (to the nearest 0.002). The corresponding relative phase angles, Φ_ν , are in degrees from -90 to +90 (to the nearest 0.2 degree).

The amplitude data are easily set into the machine by pushing sliders b (bars A) to the appropriate positions on the scale. Phase data, when the phase is not uniform, are more complicated to set in. It requires the following sequence of operations:

- (1) The Polarity Indicator (L) and switches σ , are used to set the polarities of all elements positive;
- (2) All T bars are zeroed, and the gear bank is locked (brake P_0 is set);
- (3) The Phasing Unit is raised to the position shown in Figure 7b, and the slotted bar is removed;
- (4) The brake is released, the angles Φ_ν are set in by means of the rack bars (A_i), and the brake is reset;
- (5) The rack bars are realigned, the slotted bar is replaced, and the Phasing Unit is lowered to operating position. (See Figure 7c.)

Generating the pattern likewise consists of a sequence of steps, namely:

- (1') The brake is released and ψ is advanced the desired amount by crank Q; ψ is given in degrees (up to 360; beyond that, revolutions must be counted) by a scale attached to gear #22;
- (2') The brake is locked, the racks are unmeshed by the bar-raising mechanism, the power supply is balanced and V is maximized (to $|V|$) and read;
- (3') The gear bank is returned to its position before maximization, the racks are re-meshed, the brake is released, and ψ is again advanced;

(4) ψ is thus advanced in steps until the stop is reached at 60° ; (in many patterns readings at 60° intervals are sufficient); the brake is then locked, the racks are unmeshed, the crank is run backwards until the stop is again hit, the racks are remeshed, and the sequence is repeated until the desired range of ψ has been examined. Although the pattern has a half period in ψ of 3960° , the region of interest for most patterns is within the first quarter of this displacement.

(5) When Φ_ν is not constant or when A_ν and Φ_ν are not both symmetrical about the center element, the pattern will, in general, be unsymmetrical about $\psi = 0$. In this case, upon completion of the readings for positive ψ , the machine is driven backwards in 60° steps (without taking readings) to $\psi = 0$, then operation is continued in this direction to obtain the pattern for negative ψ .

If the input amplitude distribution is symmetrical, and if the phase distribution has odd symmetry about the phase of the center element taken as reference, the sum (Equation (3)) will be pure real. That is, if $A_\nu = A_{-\nu}$, $\Phi_\nu = -\Phi_{-\nu}$ and $\Phi_0 = 0$. (this last being at the discretion of the operator by his liberty to add any constant angle to the whole bank), then

$$V_0(\psi) = \sum_{\nu=-n}^n A_\nu \cos \left[\Phi_\nu + \frac{\nu}{n} \psi \right]. \quad (5)$$

This special case allows for simpler operating procedure, by elimination of steps (2) and (3).

Several check tests are made, generally at the beginning and end of each day's operations, to assure that the machine is operating properly within error tolerances. These tests involve checks on the zero settings and linearity of the potentiometers, phase settings and proper operation of the reversing switches, normalization of the Output Indicator, and continuity throughout.

EVALUATION

In evaluating the performance of the present machine, the sources of error have been itemized and individually examined. In many cases, it has been possible to isolate an error and estimate its magnitude either by direct measurement or by critical observation of the progress of a test pattern. Three test patterns were computed on the machine and compared with the same patterns determined by other methods. No error was found large enough to impair the usefulness of the machine as an extremely accurate means of computing directive antenna patterns.

The possible sources of error are enumerated in Table 1. These sources are of two general types: A- those that are inherent in the construction of the machine, and must be accepted as part of the current design; B- those that are, to greater or lesser extent, under the control of the operator. In operation, the errors due to type-B sources must be periodically investigated and, where found excessive, must be reduced by suitable maintenance procedures to a magnitude less than that of the expected error due to other sources. Some of these type-B sources result in errors of such gross magnitude when they do occur that their presence will be observed by the operator, operation will be stopped, and the run will be repeated after the source of error has been eliminated.

The fundamental limitation on accuracy is imposed by the type-A group of "inherent" sources of error. To investigate these, the one most readily isolated was attacked first: namely, (3) potentiometer setting error. The procedure used was to observe the output due to each element of the array set at nominally identical amplitude and phase levels, then

TABLE 1
Sources of Error

No.	Title	Description and Causes	Type	Error
(1)	Tooth-meshing	Cumulative error that can occur due to the imperfection of the process of meshing rack bars.	A	Insignificant
(2)	Microswitch	Statistical error due to the failure of microswitches to reverse polarity at exactly the minimum-potential position of their respective T-bar contacts.	B	Insignificant
(3)	Potentiometer setting	Statistical error having several possible sources, as follows: a - Nonlinearity of potentiometer windings; b - Error in zero adjustment of potentiometer sliders; c - Imperfections in the phase-potentiometer driving mechanism, such as eccentricity of driving-point path, irregularity of rack-and-gear couplings, inequalities in interelement spacings; d - Errors in positions of maximum limit stops for amplitude sliders.	A A B A A	2.5 mv, or 0.25% of maximum output, due primarily to parts (a) and (d).
(4)	Circuit	Error inherent in the electrical circuit of the computer, due to loading of each potentiometer by a combination of the other potentiometers and the summing resistors.	A	Insignificant
(5)	Electrical Misc.	Other electrical errors, such as pickup in the indicator, circuit noises from various causes, etc.	B	Insignificant
(6)	Mechanical Misc.	Other mechanical errors, such as slipping of the brake, sticking or slipping of a microswitch cam follower, etc.	B	Possibly significant, but detectable by check tests.
(7)	Operator	<u>Small errors</u> , such as in setting up initial distributions, setting or reading angle ψ , normalizing and balancing power supplies, reading output; <u>gross errors</u> , such as failure to set the brake before raising the rack bar, incorrect setting of initial polarities.	B	Variable; can be made insignificant.

to determine the deviation from the mean of these readings. Operator error was eliminated insofar as setting the potentiometers was concerned by setting the bars against fixed stops; operator error in reading the output was minimized by care and checking. For the phase potentiometers, the T bars were set against a precision straight edge; this eliminated error (3c) as well as (1), (2), (6), and (7). For the amplitude potentiometers, the bars were raised to their maximum-limit stops, so any error (3d) in the positions of these stops is included. This error is discussed in more detail below. Since each pair of contacts was raised to essentially the same level, error (4) was not a factor, except for the second-order effect of variations in the loading due to differences in the values of total resistances of potentiometer cards and summing resistors. Error (3b) is also a second-order factor, as explained subsequently; likewise is error (5).

The mean normalized output reading per element for the level selected was 13.8 mv, roughly two thirds of the 22.2-mv nominal maximum. The maximum deviation from this mean was 0.7 mv, and the total deviation was 2.5 mv. The total is the sum of the deviations without regard to sign, since in the general case the polarities of the elements may be of either sign. Thus, this maximum deviation figure gives directly the maximum error to be expected from this source, and provides the order of magnitude to be used in determining whether a source of error is significant. This figure is here used to review assumptions about other sources of error, and to consider a more detailed breakdown of its own sources.

To appraise the contribution of (3d) amplitude-slider maximum-limit-stop error, note that, with phase contacts set at maximum, the output per inch of amplitude potentiometer is approximately 2.2 mv, which is comparable to the observed 2.5-mv maximum error. Hence, a setting error of the order of $1/45 = 0.022$ inch per element would produce a significant part of the 2.5 mv. Construction specifications give a tolerance of $\pm 1/64$ inch in the 10-inch travel from zero to maximum setting (stop to stop) of the slider, and $\pm 3/64$ inch in the 14-3/8-inch nominal length of potentiometer winding. This corresponds to a tolerance in potentiometer setting of 0.05 inch, which is somewhat higher than the 0.022-inch limit of significance. Hence it may be expected that significant error is present from this (3d) source, and is included in the 2.5-mv over-all figure.

Another "inherent" source of error is (3c), which involves mechanical errors in the Phasing Unit. In considering magnitudes of errors due to this source, it is best to relate them to motions measured in numbers of teeth. Since the gear and rack teeth have a diametral pitch of 48, and the maximum travel of a phase contact is 4-1/2 inches, the maximum output error due to a one-tooth error in rack position is 0.323 mv. Hence a total mechanical error equivalent to an error in rack position (or gear rotation near zero setting) of several teeth would be significant. The figure under consideration is the total for the 45 elements. If this total were the sum of widely distributed errors too small to be detectable in individual elements, the net effect would be significant only when many T bars are near zero position simultaneously. This is at most an infrequent condition, therefore such errors need not be given too serious individual consideration, and they are lumped with electrical errors under source (3). In any case, small errors of this type would be very difficult to evaluate by direct measurement. For these reasons, they were eliminated in the above described tests; it was left for test-pattern results to show whether or not they are of significant magnitude. The remaining "inherent" errors, (1) and (4), were also left for test-pattern operation to evaluate.

With regard to the adjustable sources, error (2) was set to be within $\pm 1^\circ$ of rotation of any gear. This allows a tolerance of ± 1.5 teeth, which, as in the case of errors (3c), could result in important error only at, or near, any point at which a number of phase elements is going through zero simultaneously. But again, since such a circumstance will

prevail very rarely, and would at worst give an erroneous reading at a low point in a pattern, this source will receive little further consideration.

Error (3b), adjusted to $1-1/2 \pm 1$ turns on each card, and error (5), reduced to < 1 mv by suitable grounding, shielding, cleaning of contacts, etc., are both well within the 2.5-mv figure of significance. Error (6) was minimized by cleaning and adjustment; this source results either in gross errors which obviously nullify a run, or in secondary effects, such as increased tooth-meshing error, which will be evaluated separately; hence, no further consideration need be given to this source. Error (7) cannot be predicted accurately; it will vary with the caliber and experience of the operator, and will be quite sensitive to the speed of operation and the distractions imposed on the operator.

The remaining investigation of sources (1), (3c) and (4) made use of a well-known problem to provide a standard test pattern. This is the case of the uniform array with constant feeding coefficients (uniform amplitude and phase). A 45-element array may be computed on the machine and may also be calculated "manually"; its normalized space factor, or relative amplitude is (see Appendix A):

$$V = \frac{\sin 45 \frac{\psi}{44}}{45 \sin \frac{\psi}{44}} \quad (6)$$

This pattern is called "Test Pattern #1." The calculated pattern is plotted in magnitude* in Figure 11. Only the first 40 db (1000 to 10 mv) are shown, although the next decade down was calculated and is given by the machine. Side lobe #12 through most of side lobe #18 is also omitted. The positions of the nulls are shown by short vertical lines astride the horizontal axis.

The first application of Test Pattern #1 in the investigation of errors was in the determination of the error remaining after the elimination of any possible cumulative errors, and the removal, as far as possible, of mechanical errors (except (3c)), such as those due to sources (1), (2), or (6); in other words, Test Pattern #1 was used to obtain the total error due to essentially electrical sources: (3), (4), and (5). To accomplish this, advantage was taken of the fact that, when the machine is operated with a linear distribution of input phase, there are several values of ψ at which the T bars are arranged in groups having equal travel from zero position. These values of ψ have been called "check points." In the course of running a pattern, they provide the opportunity to determine the presence of any accumulated tooth-meshing error. Conversely, the use of "check points" made it possible to set the T bars to the positions they should have for these several selected points in Test Pattern #1, and to compare the resulting Computer readings with the values obtained by calculation. This comparison is made in Table 2.

The deviations from Computer values at all of these test points were well within the predicted maximum of 2.5 mv which could be due to potentiometer error alone (sources (3a) and (3d)). But these test readings included possible error from sources (4), (5), and (7). Hence it is reasonable to assume that the total of errors from electrical sources will be within the 2.5-mv limit. Furthermore mechanical errors (2) and (3c) are heavily accentuated at the 1980° point, at which 22 of the 45 T bars are at zero position; the 990° point

* Although a slight modification would enable the machine to give polarity, this is not significant in most work for which the Computer will be used. Hence, voltage magnitude, analogous to relative field strength, has been plotted in all patterns discussed in this report.

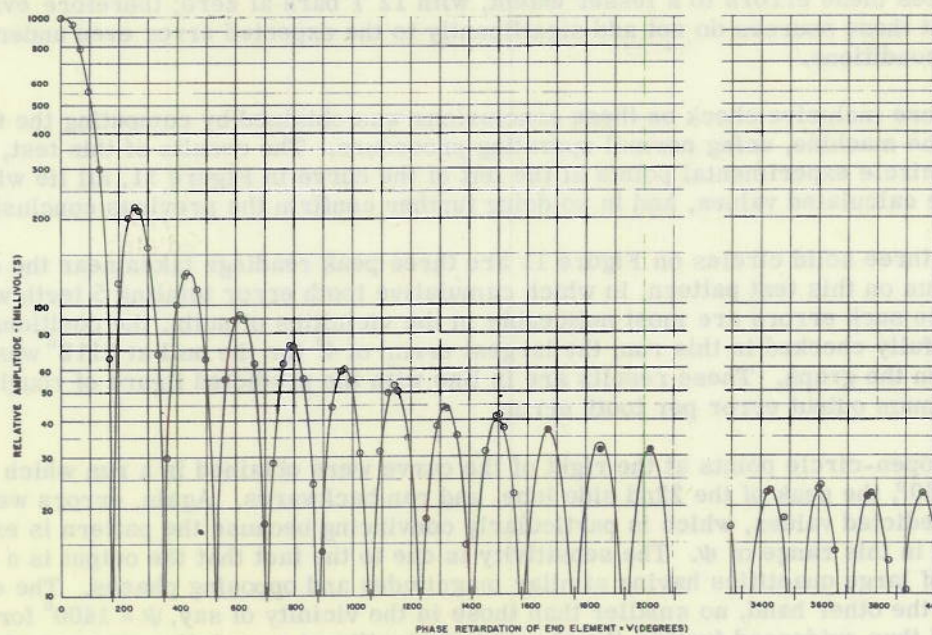


Figure 11 - Test pattern no. 1 - Uniform amplitude, uniform phase

TABLE 2
Total Electrical Error as Given by Readings Near Check Points

Ψ (degrees)		V (millivolts)		Error (mv)
Check Point	Reading Point	Computer Reading	Calculated	
990	990	44.0 to 45.0*	43.6	1.4
990	964.9	60.0	59.4	0.6
1980	1980	21.6 to 23.0†	22.2	0.8
1980	2024	30.8	30.3	0.0
3960	3960	21.5	22.2	0.7

* This variation obtained by juggling polarities of "zero" elements.

† This variation obtained by juggling polarities of pairs of elements having opposite sign and maximum amplitude.

emphasizes these errors to a lesser extent, with 12 T bars at zero; therefore evidence is given that these sources do not add significantly to the expected error even under these unusual conditions.

A more inclusive check on these conclusions was obtained by computing the test pattern on the machine, using normal operating procedure. The results of this test, shown as open-circle experimental points at the left of the curve in Figure 11, all lie within 2.5 mv of the calculated values, and in so doing further confirm the previous conclusions.

The three solid circles on Figure 11 are three peak readings taken near the end of an earlier run on this test pattern, in which cumulative tooth error totaling 5 teeth was present. Since such errors are most noticeable in the vicinities of nulls, the positions of nulls was carefully checked in this run; the largest error of 4° for the null at 2112° was too small to show on the graph. These results are in line with the predicted figure of roughly 0.32 mv maximum output error per tooth error.

The open-circle points at the right of the curve were obtained in a run which started at $\psi = 3960^\circ$, the peak of the 22nd side lobe, and ran backwards. Again, errors were well within predicted values, which is particularly convincing because the pattern is especially sensitive in this range of ψ . The sensitivity is due to the fact that the output is a sum of a number of large quantities having similar magnitudes and opposing phases. The errors were, on the other hand, no smaller than those in the vicinity of say, $\psi = 1400^\circ$ for the other runs, and thus evidenced further the absence of significant cumulative sources of error.

As further indication that error is absolute, neither cumulative nor a function of output level, and also to include other possible sources of error, two other test patterns were computed, plotted and run. The first of these, called "Test Pattern #2," is the pattern of an array having a uniform phase distribution and a sinusoidal distribution of amplitude ($A_\nu = \sin \nu \delta$). With $n = 22$ and δ selected to be 40° , the pattern is given by:[†]

$$V = \frac{1}{2} \left[V_1 (\psi - 880^\circ) - V_1 (\psi + 880^\circ) \right] \quad (7)$$

where $V_1 (\psi)$ is Test Pattern #1. This pattern is plotted in Figure 12, from $\psi = 0$ to 1200° . The total pattern is a split-lobe pattern, with the two main lobes at 880° on either side of the normal. Thus, starting the run at $\psi = 0$, fairly low-level side lobes are obtained first. This, then, provides the further evidence that significant cumulative errors do not occur when the machine is run in the simpler manner (the output is in the form of Equation (5)), as may be seen from the fact that points near the peak of the main lobe show no significant error beyond the absolute 2.5 mv.

This #2 pattern provides further information in the fact that the sinusoidal distribution of amplitude necessitates setting in such a distribution manually by preliminary setting of the amplitude sliders. Thus, certain possible sources of error which were minimized or absent in running Test Pattern #1 were effective in the #2 test run: (4) circuit error was accentuated, especially so because of the extreme variation in the amplitude-potentiometer settings; and (7) operator error in setting the distribution was introduced. The machine results again were all within 2.5 mv of the computed values, indicating that these additional sources of error were not significant. This is true in spite of the fact that the amplitude distribution is far more sharply varying than the usual problem would require. A plot of this amplitude distribution is shown in Figure 13.

[†] See Appendix A.

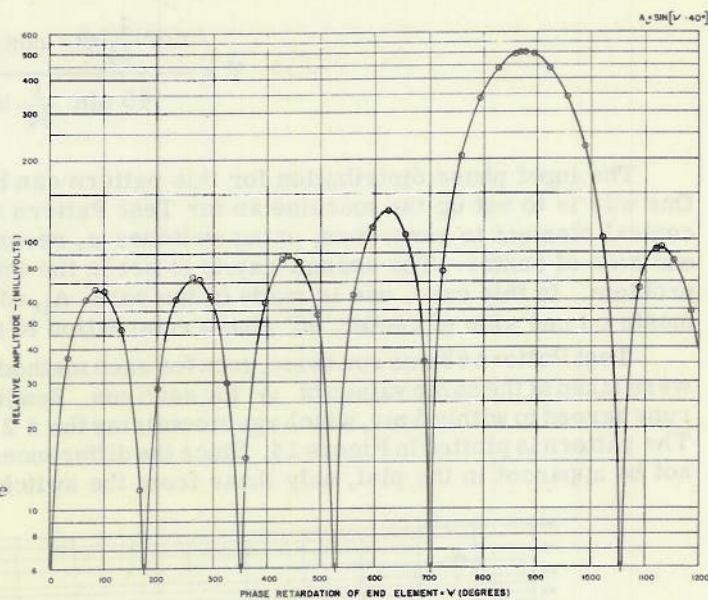


Figure 12 - Test pattern no. 2 -
Sinusoidal amplitude, uniform phase

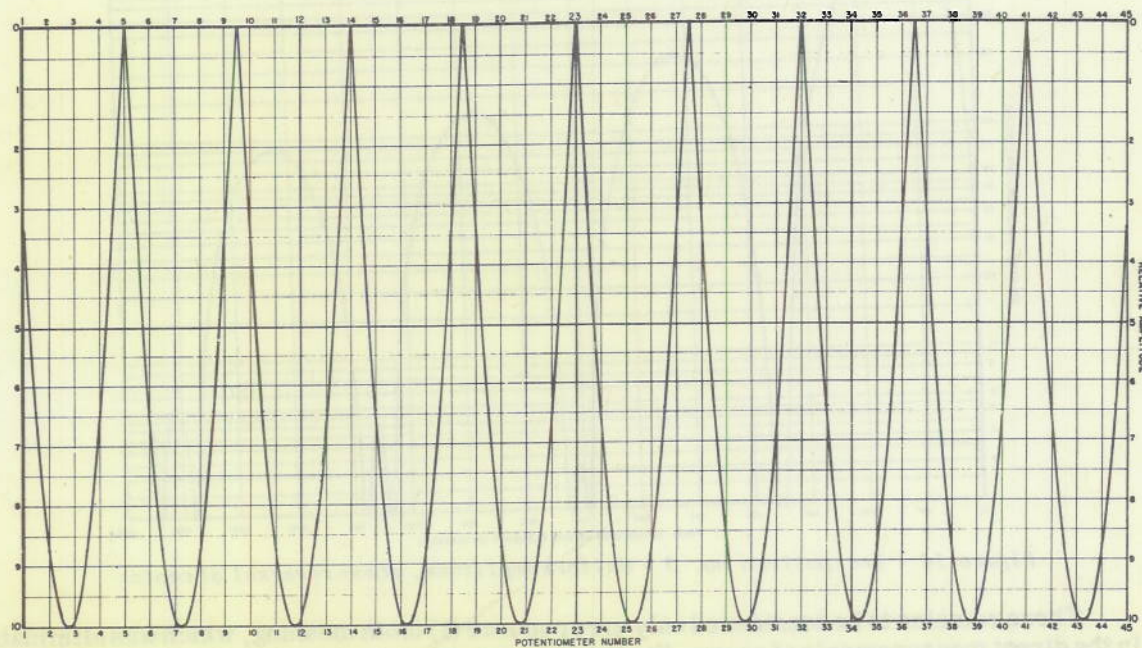


Figure 13 - Input amplitude distribution, test pattern no. 2

Test Pattern #3 provided some additional information needed to evaluate the procedure of setting in the initial phase distribution. This pattern is that due to the array having uniform amplitude distribution, but with a phase reversal at the center (zero amplitude on the center element, maximum amplitude on others; all to the left of center in phase opposition to all to the right). The pattern is given by:§

§ See Appendix A.

$$V = \frac{\cos \frac{1}{44} \psi - \cos \frac{45}{44} \psi}{45 \sin \frac{1}{44} \psi} \quad (8)$$

The input phase distribution for this pattern can be set into the machine in two ways. One way is to set up the machine as for Test Pattern #1, but setting the amplitude of the central element to zero; then, using switches σ , reverse the polarity of all elements on one side of center. The second way is to set in the initial distribution as for a general problem. In this case, use is made of the bars A_i , adding 90° phase advance to all elements on one side of center, 90° phase retardation to all on the other side.

Test Pattern #3 was run twice, once for each method of setting in the phase reversal. Data were taken at the same values of ψ for each run. Readings at corresponding points of the two runs agreed to within 5 mv, which again confirms the ± 2.5 -mv figure for maximum error. The pattern is plotted in Figure 14. Since the differences between the two sets of readings would not be apparent in the plot, only those from the switch-reversing method are shown.

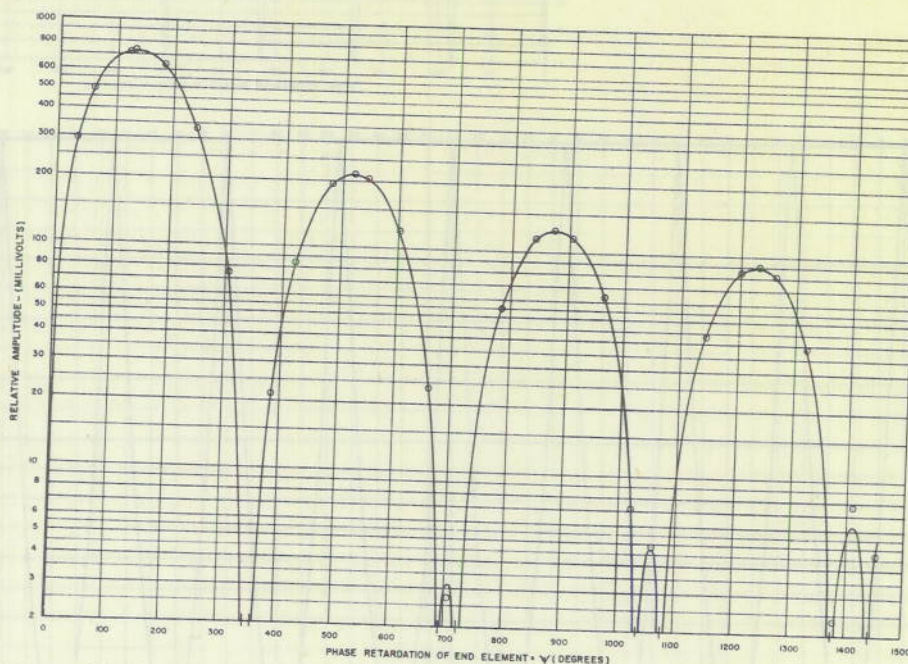


Figure 14 - Test pattern no. 3 - Uniform amplitude, phase reversal at center

There remains to be considered only error source (1) tooth meshing, which was eliminated in the direct measurements of error. Preliminary trial test-pattern runs had shown, by visual inspection at the check points, large cumulative tooth errors. Subsequent refinements in operating technique plus some further modification of the machine had made possible the reduction of this error to the small statistical total due to the possible gain or loss of one tooth in any element at each remeshing. This error was investigated first by noting, in the course of running the three test patterns, any accumulated tooth-meshing error at a check point near the end of the run. In all cases, there was no tooth error. However, all of these test patterns are of the special case of symmetrical input distribution which permits the simpler operating procedure used for a non-complex output.

A more severe test results from operation of the machine for the general case, in which the pattern may be complex and in which it is desired to obtain magnitude, and perhaps phase,

by maximizing the output. As has been previously described, this process involves considerable manipulation of the machine at each point of reading, including forward and backward rotation of the full gear bank by means of the brake mechanism, determination of a null position by this means, then rotation through 90° to obtain the magnitude reading, and finally a return to the original position and remeshing. This procedure very much increases the possibility of tooth error, due to both the increase in number of meshings, and to the introduction of the essentially new source of brake slipping under (6). The added source of meshing error may be attributed in part to an additional source of operator error due to the need for setting to precisely the correct angle before remeshing. Error in a new factor must also now be considered: namely, the value of the phase angle, which is subject to the same sources of error as is the value of the output.

Test Pattern #1 was used to evaluate operation of the machine in this manner. A constant phase of 90° was impressed on the phase potentiometers at the output, making the resulting output the imaginary component of the pattern. To compute the pattern, it was then necessary to rotate the full gear bank 90° before each reading, to maximize the output, since this pattern is purely real. After the reading was recorded, the gear bank was returned to its $\phi = 90^\circ$ position. Two such computations were made for the range $\psi = 0$ to 990° . Readings taken in steps of 30° in no case differed by more than 2.5 mv from the theoretical value. The total accumulated tooth error for one run was 4, for the other, 2 teeth. This is sufficient evidence that, when the machine is operated with proper care and dexterity, errors due to source (1) will remain negligible in general-case operation.

A final consideration is the accuracy to which the phase angle of the complex output can be obtained. This was also given by the last-described tests, since at each point the output before rotation of the gear bank, i.e., the imaginary component, should be zero. The rotation required to reduce the reading to a null was in each case less than 0.5° , which figure may therefore be taken as the limit of error in phase-angle readings.

CONCLUSIONS

A study of the machine under normal conditions indicates that it performs well the function for which it was designed, namely the computation of the radiation pattern due to a linear aperture. The extent of its usefulness in antenna theory and design does not appear to be limited by its operational accuracy or complexity. In the hands of a trained and alert operator, the machine has exhibited an accuracy greater than had been expected. The maximum error observed, 2.5 mv, when compared with the normal maximum output of 1000 mv, is negligible from the antenna-design point of view.

Since the machine is an exact analogue of a linear array and an excellent approximation to a line source, there are no theoretical errors limiting its usefulness in analyzing such antennas. However, for antennas that contain reflectors or lenses, there are significant errors present in approximating their performance by that of an equivalent line source. A comparison of predicted patterns with those actually measured will provide valuable information on the validity of the aperture-field approximation for various types of antennas.

The time required for the computation of a single pattern is moderate. Patterns vary widely among themselves in both complexity and extent, and any estimate of the time required for the generation of a pattern is subject to an equal variation. The majority of the patterns that will be presented to the machine for computation, however, should require not more than two to four hours for evaluation. Since the preparation of data for insertion in the machine will usually require at least a few hours and in many instances far more,

there seems to be little need to increase the speed of operation of the machine. If, on the contrary, experience should indicate that a reduction in operation time is desirable, some of the functions now performed manually by the operator can be mechanized.

Automatic recording of the output could be accomplished by the use of a drum or table carrying a standardized record paper, and servo-driven to advance in proportion to the aspect angle θ . Such a mechanism would require means of converting the machine motion, which is proportional to the angle ψ , to θ motion; this conversion means has been designed in principle. Marks on the record, positioned in proportion to the relative magnitude of the computer output, would have to be made by a mechanism under control of the operator, due to the necessity of maximizing output before reading. A number of such mechanisms exist. Several other functions might be mechanized, and some of the electronic adjustments might well be made automatic. Of the latter, the most valuable would be self-equalization of the power-supply outputs. However, all of these together would not greatly increase the operating speed.

It is in the synthesis of patterns that a marked increase in speed would be most desirable. In problems of this sort, one seeks by the method of successive approximation, an aperture illumination or set of feeding coefficients to produce the prescribed pattern. It does not appear likely that the present machine by itself can be used successfully for such work. Machines with the speed necessary for synthesis problems can be designed, but their speed is achieved at the expense of accuracy. The present machine with its high resolution and extreme accuracy could be used profitably in conjunction with a high-speed calculator; the latter to determine rapidly a rough approximation to a prescribed pattern and the former for final precise adjustment. Several possible high-speed machines have been proposed and considered (see Appendix B). The feature common to all is continuous cathode-ray presentation of the pattern with simultaneous adjustment in amplitude and phase of the feeding coefficients.

Concerning use of the present machine, more important than the speed of operation is the question of "down time"; that is, the proportion of total machine time that is taken by repair and maintenance, as against that chargeable to operation. Up to date an excessive amount of "down time" has been experienced. Most of this, however, has been due to known mechanical causes which would be largely eliminated by further minor modifications in the Gear-Locking Device and the Amplitude-Function Unit.

How much further effort will be put into this development depends on the demand for the services of this type of computer. As little as one problem per month would warrant improvement of the present machine at least to the extent of correcting the most marked deficiencies mentioned above (amplitude potentiometers, brake, power-supply balance). An appreciable demand for design data, involving trial-and-error pattern synthesis, would warrant serious consideration of the development of a high-speed machine.

ACKNOWLEDGMENTS

Acknowledgments for those involved in the earlier development of the Computer are given in the Redheffer report.²² In particular, Mr. H. Kylin was responsible for designing the principal mechanical components and for supervising their construction.

Credit is due many members of the Antenna Research Branch for help and guidance in the completion of the project and the writing of this report. Mr. O. A. Tyson provided invaluable guidance in the design of electronic components and of mechanical modifications. In machine design, in the selection of evaluation procedures, and in the preparation of the report, Dr. J. E. Eaton provided theoretical guidance. Dr. M. L. Kales assisted in the selection and derivation of test patterns. As the regular operator, Mr. H. F. Cogan obtained most of the data used in the evaluation.

* * *

APPENDIX A Derivation of Test-Pattern Formulas

Formulas for the calculation of test patterns were derived from equation (4):

$$V = \frac{1}{2n+1} \sum_{\nu=-n}^n A_{\nu} e^{j(\Phi_{\nu} + \frac{\nu}{n} \psi)}$$

1. The pattern for the array with constant feeding coefficients ($A = 1$, $\Phi = 0$) is given by:

$$V_1 = \frac{1}{2n+1} \sum_{\nu=-n}^n e^{j \frac{\nu}{n} \psi} = \frac{1}{2n+1} \left[1 + 2 \operatorname{Re} \sum_{\nu=1}^n e^{j \frac{\nu}{n} \psi} \right]$$

Using the expansion:

$$\sum_{\nu=1}^n Z^{\nu} = Z \left[\frac{1 - Z^n}{1 - Z} \right], \quad (a)$$

$$V_1 = \frac{1}{2n+1} \cdot \frac{\sin \frac{2n+1}{2n} \psi}{\sin \frac{1}{2n} \psi}$$

For $n = 22$, this becomes:

$$V_1 = \frac{\sin 45 \frac{\psi}{44}}{45 \sin \frac{\psi}{44}} \quad (6)$$

The nulls in this pattern occur at $\psi = \hat{\psi} = 44/45 \cdot 180^\circ k$, ($k = 1, 2, 3, \dots$). By differentiating V_1 with respect to $\psi/44$ and setting $= 0$, the peaks are found to occur at $\psi = \hat{\psi}$ given by: $45 \tan(\hat{\psi}/44) = \tan 45(\hat{\psi}/44)$.

2. The pattern for the array fed with sinusoidal distribution of amplitude and no additional phase variation ($A = |\sin \nu \delta|$, $\Phi = \pm 90^\circ$) may be expressed as:

$$V_2 = \frac{1}{2n+1} \sum_{\nu=-n}^n (\sin \nu \delta) e^{j \frac{\nu}{n} \psi}$$

$$V_2 = \frac{1}{2n+1} \cdot \frac{j}{2} \left\{ \sum_{\nu=-n}^n e^{j \nu \left(\frac{\psi - n\delta}{n} \right)} - \sum_{\nu=-n}^n e^{j \nu \left(\frac{\psi + n\delta}{n} \right)} \right\}.$$

$$\sum_{\nu=-n}^n e^{j \nu \left(\frac{\psi + n\delta}{n} \right)}$$

is, except for the addition to ψ of the constant angle $n\delta$, the same as the sum giving the pattern for the constant amplitude case. Hence the pattern for the sinusoidal distribution of amplitude is, in magnitude, equal to one-half the difference between two constant-amplitude patterns separated by an angle $\psi_s = 2n\delta$. That is:

$$V_2 = \frac{j}{2} \left\{ V_1(\psi - n\delta) - V_1(\psi + n\delta) \right\}.$$

For $n = 22$ and $\delta = 40^\circ$, and neglecting phase of the radiation:

$$V_2 = \frac{1}{2} \left[V_1(\psi - 880^\circ) - V_1(\psi + 880^\circ) \right]. \quad (7)$$

3. The pattern for the array fed with constant amplitude but with a phase reversal at the center and no other phase variations ($A_\nu = A_{-\nu} = 1$, except $A_0 = 0$; $\Phi_{-\nu} = -90^\circ$, $\Phi_\nu = 90^\circ$) is given by:

$$V_3 = \frac{1}{2n+1} \left\{ \sum_{\nu=-1}^n e^{j(-90^\circ + \frac{\nu}{n}\psi)} + \sum_{\nu=1}^n e^{j(90^\circ + \frac{\nu}{n}\psi)} \right\}$$

$$V_3 = \frac{1}{2n+1} \cdot 2 \operatorname{Re} \sum_{\nu=1}^n e^{j(90^\circ + \frac{\nu}{n}\psi)}$$

$$V_s = \frac{1}{2n+1} \cdot 2 \operatorname{Im} \sum_{\nu=1}^n e^{j \frac{\nu}{n} \psi}.$$

Again using expansion (a):

$$V_s = j \frac{\cos \frac{1}{2n} \psi - \cos \frac{2n+1}{2n} \psi}{(2n+1) \sin \frac{1}{2n} \psi}.$$

For $n = 22$, and neglecting phase, this becomes:

$$V_s = \frac{\cos \frac{\psi}{44} - \cos 45 \frac{\psi}{44}}{45 \sin \frac{\psi}{44}}. \quad (8)$$

For this function, $\hat{\psi}$ is given by: $(\hat{\psi}/44) = |360k - 45(\hat{\psi}/44)|$, ($k = 0, 1, 2, 3, \dots$), and $\hat{\psi}$ is given by:

$$45 \left[\cos \hat{\psi} - \cos \frac{23}{22} \hat{\psi} \right] = 2 - \left[\cos \hat{\psi} + \cos \frac{23}{22} \hat{\psi} \right].$$

* * *

APPENDIX B

Proposed Designs for High-Speed Computer

Several possible high-speed machines have been proposed by members of the Antenna Research Branch. One such is electro-mechanical in nature, like the present machine, and is based on the same principles. The differences in execution, however, would permit continuous display of the pattern, or an expanded portion thereof, on a cathode-ray-tube screen, while adjustments in the array feeding coefficients were made. This type of operation would be accomplished in an a-c machine, using, in place of the pair of potentiometers to represent each element, a variable auto transformer for adjusting relative amplitude, and a synchro with adjustable stator position for adjusting relative phase. The array of synchro rotors would be geared into a train in such ratios that driving of the assembly would produce variation in their outputs equivalent to the variations in contribution of the corresponding elements when the analogous antenna array is rotated. The outputs of these rotors would then be summed and applied to a C-R tube deflection system; the C-R tube would be swept in synchronism with the synchro drive.

Another possible design is primarily electronic. It requires the use of a radio-frequency supply. The individual element would be represented by an electronic-circuit, the output of which is manually controllable in magnitude and phase. In addition, each successive "element" would have a frequency-multiplying circuit, the array providing successive harmonics of the supply frequency. "Rotation" of the array would consist of shifting the phase of the input common to all but a reference (fundamental-frequency) circuit, or "center element." The output of each element would be at fundamental frequency from a generator triggered by the frequency multiplier. The rectified sum would again be applied to C-R deflection, swept at audio frequency in synchronism with the phase shift of the r-f input to the multipliers. This system appears complicated, but has advantages over the electro-mechanical system in lightness, compactness, ease of servicing, and the much greater number of elements (hence better approximation to the continuous aperture) that would be practical. There is also the possibility of an extension of the technique to the 3-dimensional pattern of the plane array.

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