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OPTIMIZED MAINTENANCE SCHEDULING IN SUPPORT OF AT-SEA DETERRENCE

by

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The United States counters adversarial threats with submarine-launched fleet ballistic missiles and strategic weapon systems developed, produced, and supported through their lifetime of service by Strategic Systems Programs (SSP). Much of the service support of this fleet occurs at strategic weapons facilities, where analysts balance competing requirements to schedule the modification, maintenance, onloading, offloading, and delivery of missiles and their components. The schedules are created with project management tools that limit productivity of the skilled analysts and in turn the operational effectiveness of the U.S. Navy and its ability to maintain nuclear deterrence. The current use of these tools involves manually organizing data and decisions across spreadsheets, tediously balancing the fleet and manufacturing requirements, inventory, and workload. To help support this process, we wrote out a statement of the basic decision problem addressed by SSP and formulated it as an integer linear programming problem. This model helps complete the data processing and manage the requirements and inventory to produce a feasible, improved schedule that meets the demands of the strategic weapons facilities. This schedule is produced in a manner that requires less manual data manipulation by the analyst, provides the analysts with more visibility for the future, and provides the ability to tackle what-if scenarios. It can be extended to help keep the facilities’ workloads leveled.

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OPTIMIZED MAINTENANCE SCHEDULING IN SUPPORT OF AT-SEA DETERRENCE

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ABSTRACT

The United States counters adversarial threats with submarine-launched fleet ballistic missiles and strategic weapon systems developed, produced, and supported through their lifetime of service by Strategic Systems Programs (SSP). Much of the service support of this fleet occurs at strategic weapons facilities, where analysts balance competing requirements to schedule the modification, maintenance, onloading, offloading, and delivery of missiles and their components. The schedules are created with project management tools that limit productivity of the skilled analysts and in turn the operational effectiveness of the U.S. Navy and its ability to maintain nuclear deterrence. The current use of these tools involves manually organizing data and decisions across spreadsheets, tediously balancing the fleet and manufacturing requirements, inventory, and workload. To help support this process, we wrote out a statement of the basic decision problem addressed by SSP and formulated it as an integer linear programming problem. This model helps complete the data processing and manage the requirements and inventory to produce a feasible, improved schedule that meets the demands of the strategic weapons facilities. This schedule is produced in a manner that requires less manual data manipulation by the analyst, provides the analysts with more visibility for the future, and provides the ability to tackle what-if scenarios. It can be extended to help keep the facilities' workloads leveled.

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List of Acronyms and Abbreviations

DOD	Department of Defense
MILP	mixed-integer linear program
NCE1	needs component exchange 1
NCE2	needs component exchange 2
NCE3	needs component exchange 3
NPI	needs process inspection
NPS	Naval Postgraduate School
NRAE	needs RA exchange
PCE1	processing subcomponent exchange 1
PCE2	processing subcomponent exchange 2
PCE3	processing subcomponent exchange 3
PPI	processing process inspection
PRAE	processing RA exchange
RCPSP	resource constrained project scheduling problem
RCPSPBM	resource constrained project scheduling problem with bounded multitasking
RFI	ready for issue
SSP	Strategic Systems Programs
SWFs	strategic weapons facilities
SWFLANT	Strategic Weapons Facility, Atlantic

SWFPAC	Strategic Weapons Facility, Pacific
USN	United States Navy

Executive Summary

The United States demonstrates its commitment to deter and counter adversarial threats with a safe, secure, and effective nuclear deterrence strategy; Strategic Systems Programs (SSP) maintains the U.S. Navy's submarine-launched fleet ballistic missiles and strategic weapon system, securing reliability for a critical mission in deterrence. SSP's strategic weapons facilities (SWFs) conduct the hands-on service support for the fleet of ballistic missiles. Expert analysts at these facilities conduct critical planning for and execution of tracking, maintaining, upgrading, and loading of ballistic missiles and their components into submarines. The analysts' current planning and scheduling tools severely limit their ability to create schedules in a time-efficient manner and adequately understand how future scenarios will affect long term operations. Arming these analysts with a capable planning tool broadens SSP's assurance in maintaining this critical fleet.

SSP requests NPS's assistance in creating a more efficient, standardized tool for the SWFs. To do this, we built a mixed integer linear programming (MILP) model, SSPM, that automates the balancing of schedules, inventories, and rules and optimizes the schedule based on the SWFs' priorities. To test the model, case studies of different sizes, created with likeness data, are used as inputs. The smaller cases verify and validate the model and display advanced behavior over time. The largest instance stresses the efficiency of model development and machine performance, and as future work, the current model needs to be modified to reduce its size in order to solve these bigger problems. Overall, the results from the cases show that SSPM reduces the time to produce a schedule from thirty days to less than one day, requires less manual data manipulation due to automation, provides greater visibility for the analysts, and prioritizes the time that components spend at-sea. These improvements in scheduling allow analysts more time to do more thorough analyses and the means to address important operational questions in support of the ballistic missile submarines. This work helps contribute to a safe, secure, and effective strategic deterrent.

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CHAPTER 1:
Introduction

1.1 Background

The United States counters adversarial threats with submarine-launched fleet ballistic missiles and strategic weapon systems developed, produced, and supported through their lifetime of service by Strategic Systems Programs (SSP). Much of the service support of this fleet occurs at strategic weapons facilities, where analysts balance competing requirements for the modification, maintenance, onloading, offloading, and delivery of missiles and their components with project management tools that limit productivity of the skilled analysts and in turn the operational effectiveness of the U.S. Navy and its ability to maintain nuclear deterrence. Foundational to this nuclear deterrence strategy is what is known as America's nuclear triad, which includes the ballistic missile submarines, as well as intercontinental ballistic missiles and long-range bombers (Vergun 2020). General Mark A. Milley, Chairman of the Joint Chiefs of Staff, emphasized its importance, saying, "The nuclear Triad has kept the peace since nuclear weapons were introduced and has sustained the test of time" (United States Department of Defense 2020). Though each piece of the triad plays a vital role in the U.S.'s nuclear deterrence efforts and maintaining peace, this thesis focuses on the submarine leg of the triad.

Acting as the stealthiest and most survivable part of the triad are the fleet ballistic missile submarines, also known specifically as "boomers" or, more generally, as boats. Boomers complete critical missions to maintain nuclear deterrence and display the power and lethality of the U.S., and specifically the United States Navy (USN). They are built for extended sea patrols and the distribution of nuclear warheads (United States Department of Defense 2022). The independently targeted nuclear warheads, along with many other crucial components, are contained in ballistic missiles that are able to be launched from boomers (United States Department of Defense 2020).

1.2 Mission and Facilities

SSP is a workforce dedicated to this ballistic missile mission, along with carrying out the Polaris Sales Agreement with the United Kingdom and creating and advancing hypersonic weapons (Strategic Systems Programs 2023). The Headquarters of SSP is located Washington DC, where the major missions are directed and organizational decisions are made, but there are locations around the U.S. and one in the United Kingdom that support SSP's strategic goals.

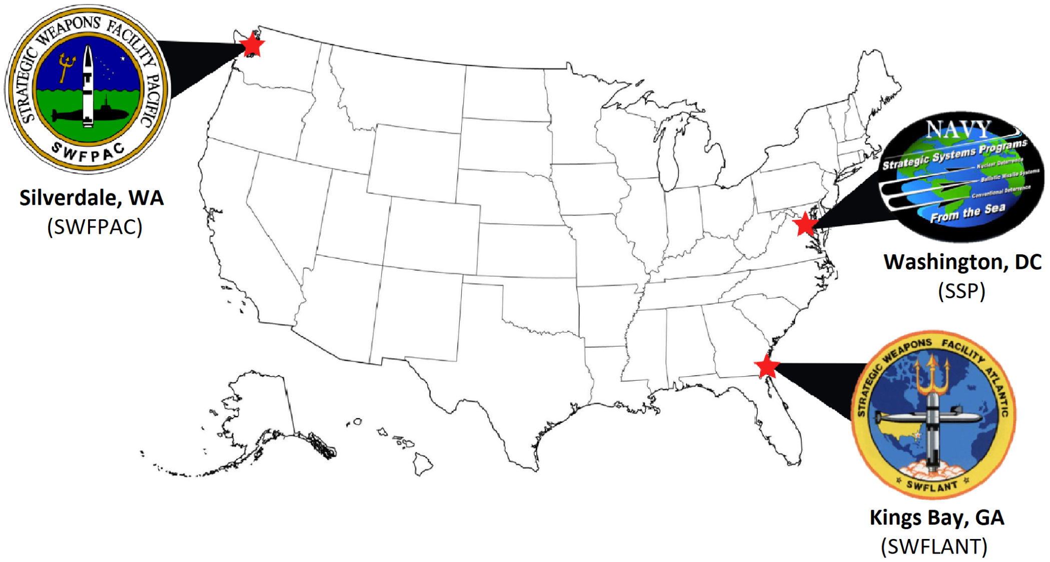

Figure 1.1. The locations of the three main sponsor facilities.

The two primary facilities that provide the service support for the ballistic missile mission of SSP are Strategic Weapons Facility, Atlantic (SWFLANT) and Strategic Weapons Facility, Pacific (SWFPAC), located in Kings Bay, Georgia and Silverdale, Washington, as shown in Figure 1.1. All ballistic missile submarines pull into port at either SWFLANT or SWFPAC for regular servicing of their missiles. The strategic weapons facilities (SWFs) handle the constant maintenance and upkeep of missiles that they remove from and load onto the submarines. They also control any modification or processing required while the missiles are on shore. These adjustments can include changes to the missiles configuration before it is delivered to a boat. A big portion of this processing for the components involves replacing one or more of the component's subcomponents to increase the overall service life left on that component, a process referred to as a *subcomponent exchange*. Other types of processing that can be scheduled are *RA exchanges*, to replace an attachment to a component, and *process inspections*, which do not modify anything but are required to prepare components for issue to a boat. At both of the SWFs, there is a generalized flow for a component on shore as it moves through facilities to get one of these types of processing. After a component is taken off of a boat, it is placed in storage and marked as *in need of processing*. This storage can be broken down by what type of processing it needs: process inspection, subcomponent exchange of a specific type, or RA exchange. It then moves to new location to get processed. Finally, the component gets moved into storage where it is ready to be issued back onto a boat or be sent back to complete more processing, or if it is a recalled component, it is put in a separate storage to be sent away after receiving a RA exchange. To facilitate the timely and necessary modification, maintenance, and delivery of missiles and their important components at the shore facilities, skilled analysts working at the SWFs are tasked with creating a detailed schedule of operations that can be carried out for both the missiles and their important components. The two types of schedules, one at the missile and one at the component level, directly support the overall mission of nuclear deterrence by giving the ballistic missile submarines the capability of operating with effective, deadly weaponry at all times. However, this scheduling is not a simple effort.

1.3 Planning Shore Operations

Analysts create schedules that balance restrictions and competing priorities. Restrictions come from multiple sources that include, but are not limited to, fleet operations, personnel management, material handling, and inventory management. When considered in combination, these restrictions limit what is acceptable as a usable schedule and complicate the scheduling problem. The fleet schedule displays when the submarines will be porting at the SWFs for maintenance and work and is determined before it gets passed on to SSP and the SWFs. Due to its inflexibility, it exists as a strict input to the analysts' schedule but, on rare circumstances, can be changed by the fleet to meet fleet mission needs. For the missiles' components' schedules, the missile schedule that specifies which missiles will be pulled off and put on a submarine when it is in port acts as another input for the component specific analysts. In addition, the component recall schedule indicates which specific components must be returned to the manufacturer's facility by a certain time. The contracted workforce at the SWFs has a schedule of work that they can handle and a set of resources available per day or week. The times to complete processing of missiles and their components are determined by the available workforce. All of these aspects restrict how many jobs the analysts can fit in a schedule. Due to the sensitivity of the missiles and their components, there are also processing and manufacturing rules that reduce the processing availability. Other higher level details that affect the scheduling are fluctuations in nuclear treaty adherence with other nations, large platform retirement and fielding, such as the eventual replacement of the Ohio-class submarines by the Columbia-class boomers, and adjustments in missile makeup requirements passed down from the fleet. The last major facet that goes into this scheduling is the inventory of missiles, their components, the subcomponents of the main components, and the resources required to hold components while on shore, referred to as containers. The supply of many of the parts and resources, especially containers, is limited, making balancing and tracking the existing inventory crucial for a usable schedule at the SWFs. With the inventories, it is also necessary to track the service lives of the subcomponents, components, and missiles. The service lives are the amount of time left before the expiration of that element, which can be increased with certain processing specified by the analysts' schedules. Competing priorities include maximizing at-sea component life, minimizing cost, and leveling the workload in the facilities. While all are important, the analysts' schedules at the SWFs directly control the components and their life onboard a submarine, making this priority the easiest to impact initially and therefore, the top priority for this work.

A standard component schedule at both of the SWFs provides comprehensive information about what components on shore should be processed at what time and in which type of processing, as well as which components should be (or must be) pulled off of and put on boats while those boats are in port. The processing decisions are in preparation for specific events, such as boats pulling into port, and the schedule specifies how far in advance the processing is completed. In addition, while creating this schedule, inventories for all missiles and components must be tracked, and all of the previously mentioned rules, schedules, and requirements must be taken into consideration and followed. While abiding by the requirements, analysts have some choices when creating a schedule. Within the bounds of their problem, the choices include which of the available components to put on which boats and what work to schedule to best prepare those components for a service life at sea. A big portion of this preparation includes subcomponent exchanges and process inspections.

The schedules the analysts at the SWFs build are twofold in their purpose. First of all, the workforce needs a detailed schedule to execute for their operations. For this, the analysts provide a day to day or week to week schedule of work that states which specific components need to be on-loaded and off-loaded for a boat in port and what processing should be done on shore to prepare and maintain the components. The tactical schedule is short term and used for real work, focused on only a few months out at a time. The second purpose is to answer longer-term questions passed down from a higher level such as SSP to the SWFs. These questions, known as "what-ifs," can range from "What if the processing facility is shut down for two weeks later this year?" to "What if a whole class of submarines is decommissioned two years earlier than originally planned?" Once the questions are asked, it is up to the analysts to perform what-if analysis by adjusting their schedules to determine how the possible changes will affect their facilities and workforce and if they can feasibly handle them. This can involve looking many years into the future to see how a decision made today at the higher level would have an impact on the submarines and future operations at the SWFs. If they are not able to complete this analysis themselves, the analysts may also rely on contracted support for help. This schedule is thorough, but is more long term, spanning years into the future to answer operational planning questions.

The tools currently utilized by analysts are multiple, independent spreadsheets with varying purposes. The functions of the different sheets include managing inventory of the many components and availability of resources as well as tracking the workforce schedule, boat schedules, and service lives of components and subcomponents. Because of the disconnectedness of the spreadsheets, the scheduling takes place in a manual, time consuming, and heuristic manner, requiring by-hand data manipulation with the analysts going between the different spreadsheets to build the schedule one part at a time. It incorporates the expert analyst's judgement in an ad-hoc method. A feasible, but not necessarily the best or optimal, schedule that meets requirements and schedules for the following three months can take analysts more than 30 days to complete in this current manner. In addition, when what-if questions requiring thorough analysis are passed to the SWFs from SSP, the analysts must either manually adjust their current schedule to test possible scenarios, which is extremely time consuming, or depend on contracted analysts, limiting the scope of government visibility and general visibility for the future.

We have developed, in coordination with SSP, SWFPAC, and SWFLANT, a statement of the basic decision problem and formulated it as an integer linear programming problem. This model helps complete the data processing and manage the requirements and inventory to produce a feasible, improved schedule that meets the demands of the strategic weapons facilities. This schedule is produced in a manner that requires less manual data manipulation by the analyst, provides the analysts with more visibility for the future, and provides the ability to tackle what-if scenarios. Not only that, but with an optimization tool, all possible schedules can be compared to choose the most valuable one, not just a feasible schedule.

For this thesis, the focus will be on the scheduling specific to the important components that are a part of the ballistic missiles. It continues with Chapter 2 discussing previous project scheduling and inventory problems. Chapter 3 defines the model created to solve this problem, and Chapter 4 introduces the instances implemented in the model and resulting output. Lastly, Chapter 5 wraps up the work completed and outlines possible future work for this problem.

CHAPTER 2:
Literature Review

The United States Navy has a long history of facing project scheduling and inventory challenges, and continues to do so today, as seen by the problem previously explained that SSP and the analysts at the SWFs are encountering. Other organizations and industries, such as medicine, finance, and other branches of the military, also deal with but do not always fully grasp the intricacies of both project scheduling and inventory management. Silver et al. (1998) describes the importance of these topics and the benefits that can be gained from completing them carefully and with tools such as optimization models. The following review addresses some of the history of project management, as well as project and inventory problems past and present and how optimization can be used to solve them.

2.1 Modern Project Management History

The Polaris Program, started in 1955 under the Fleet Ballistic Missile Program, is one of the Navy's most well-known and successful projects both in terms of weapons development and project management. It began in order to keep up with Soviet Union's missile development (Engwall 2012). With the start of this project came the establishment of the Special Projects Office (SPO), known today as SSP. They were charged with creating and testing the first-ever submarine-launched ballistic missile system with limited time to do so (Strategic Systems Programs 2023). This led to the development of the Project Evaluation and Review Technique (PERT), a project management tool used to evaluate schedules by identifying the necessary tasks, order in which they need to be completed, and time to complete each task. PERT assists in proper scheduling, which for this problem, helped save time in the successful and on-time completion of the Polaris missile program (Hitchcock and Bliss 1964).

PERT went on to be used in projects both within the military and a variety of industries. Its central idea of sequencing tasks to manage limited resources became the basis for the project scheduling problem and project management (Moder et al. 1983). Shortly after the development of PERT, Morgan R. Walker and James E. Kelley established the critical path method (CPM), another tool to assist in complicated project management and scheduling (Kelley 1961). As suggested by its name, CPM focuses on identifying the critical path or paths. The path is the sequence of tasks that are critical or time restrictive in terms of time to complete, which usually sets the overall duration of the project. If any of the critical path tasks are delayed, then the project will also be delayed, so the uncertainty in the completion of tasks must be considered as well. CPM inherently involves optimization, by ordering the tasks in the most time efficient while still feasible manner. Both PERT and CPM laid the foundation for concepts used in project scheduling today, including some ideas that can be applied when solving these problems with optimization.

2.2 Project Scheduling and Inventory Modeling Using Optimization

Project management can best be described as the planning, scheduling and controlling of tasks while balancing resources in order to accomplish goals associated with time, cost, or efficiency. Following the 1950s and 1960s, project management gained momentum because of the previously mentioned Polaris project, as well as the Apollo moon project at NASA and other Department of Defense construction programs (Demeulemeester and Herroelen 2002). The popularity led to a whole class of problems being termed Scheduling Problems, to categorize both old and new problems, and soon after, sub-classes such as the resource constrained project scheduling problem (RCPSP), time-cost trade-off problem (TCTP), and resource leveling problem (RLP) (Ke et al. 2015). Most relevant to this work are RCPSPs, where the tasks have a start time, deadline, and penalty for delays and are being completed by a workforce that is restricted. Cavalcante et al. (2013) further explored this sub-class with an IT dispatcher problem including multitasking or tasks occurring in parallel. This extension is referred to as the resource constrained project scheduling problem with bounded multitasking (RCPSPBM). In this problem, customers report IT issues which leads to the creation of a ticket with a certain severity and a specific deadline to be resolved. The tickets are then assigned by a dispatcher using a decision-support system to one of the analysts in a way that will hopefully allow the ticket to be handled before the deadline. The multitasking comes in with analysts handling more than one ticket at a time. The system utilized by the dispatchers does not always lead to an acceptable solution, so the authors proposed a mixed-integer linear program (MILP) of the RCPSPBM as a way to optimize the planning and see if there was a more efficient way to schedule these tickets. The MILP uses an objective function focused on minimizing penalties in planning and uses constraints to enforce rules of the planning. The situation Cavalcante et al. (2013) faced is just one example of a project scheduling problem that utilized an optimization technique to help answer an operational question.

Even more recently, Ahmed (2022) addressed a scheduling problem for the Iraq residential gate project with the aim of using an optimization technique to complete the project within the time constraint and maximize the profit. The two main aims were to schedule the tasks to have the overall project completed by the deadline and to complete the fourth activity specifically within ninety days without having to accelerate it. For this project, the author applied a method called goal programming, used to solve multi-objective decision-making problems. A linear goal programming model was built to create a schedule that achieved both of the two aims while maximizing profit.

Project scheduling and the challenges that come along with it exist in the Navy in many different forms as well. In 2009, Brown et al. addressed a problem looking to determine what interdicting actions can be scheduled to maximally delay an enemy in their creation of nuclear weapons. The enemy here has the ability to adjust their schedule based on any such actions in order to minimize the time, and the interdiction resources available constrain the one looking to delay the weapons project. To optimize this unique scheduling problem, a max-min model was built that incorporated a CPM sub-model (Brown et al. 2009).

A more common Navy scheduling problem that has been researched in many different technical papers revolves around scheduling maintenance for surface ships in shipyards in a way that allows them to still meet fleet operational requirements. In each of these papers, the analysts define the problem with a set of procedural rules, resource constraints and planning objectives, which allows them to apply an optimization technique, MILP, in order to find the best schedule out of all possible schedules. Brown (1992) first introduced the application of a MILP to optimally solve the shipyard scheduling problem in an efficient way that would better utilize the shipyards while also saving the Navy money. It works to schedule the docking of ships in a manner that evens out the workload and is cost efficient. Brown's original model has been extended many times to represent different operational and logistic considerations. One extension by Hilliard et al. (2020) took into consideration options such as enhancing the drydocks' capabilities and double docking, docking two ships in the same drydock at the same time, to improve the scheduling possibilities. More recently, O'Malley and Lin (2021) created a MILP for this problem focusing on minimizing the workload fluctuation on shore by shifting maintenance job's starting date within an allowable window. All of these versions of the same problem use a MILP with constraints to enforce the rules of the scheduling while the objective function works towards the most efficient schedule in terms of time and cost.

For most problems, project scheduling cannot be discussed without including some aspect of inventory management. Oftentimes building schedules for short-term, long-term, and ongoing projects includes managing inventory in one way or another, whether it be resources or the parts being altered throughout the schedule. Problems including inventory management can be handled in a similar manner to project scheduling ones. In 2016, Salmeron and Craparo (2016) completed a project deliverable in order to assist Naval Supply Systems Command (NAVSUP) with their inventory of over 430,000 items used to support Navy, Marine Corps, Joint and Allied Forces. They created the Site Demand-Based Level Inventory Optimization Model (SIOM) to aid NAVSUP in constructing a schedule of when and how much to order to meet uncertain demand and use the budget efficiently. Ersoz (2016) reformulated SIOM, a MILP, into SIOMsQ, which resulted in a model that solved the same problem still handling the immense amount of inventory but in a faster manner with improved solution quality. In the model name, s represents the order point and Q represents the order quantity for this inventory and scheduling problem. Two years later, Deibler (2018-09) developed the s and Q Unit Identification (sQUID) model from SIOMsQ in order to help the Expeditionary Support Unit (ESU), who is responsible for the logistical support of U.S. Navy Explosive Ordnance Disposal units deployed and in home-ports. This problem, while smaller in inventory items, has greater operational importance and is a great example of how an optimization tool can assist in mission critical problems. It allowed the decision-makers to track inventory and make choices about the restock times and quantities.

2.2.1 Contribution

The above review represents just a few of the inventory and project scheduling problems both within and outside the Navy that analysts worked on over the previous decades. Inventory and project management ideas allow analysts to define these time-consuming and complicated problems, and optimization tools allow those same analysts to solve them in less time, with better results, and with less manual data manipulation involved. This thesis intends to contribute to Navy project management body of knowledge through applying project scheduling and inventory modeling with optimization to the SSP scheduling problem. To optimize this problem, SSP requires a MILP to schedule the component processing in order to meet fleet requirements while tracking inventory, service lives, and processing capacity limitations. In the next two chapters, the MILP model built will demonstrate this capability.

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CHAPTER 3:
An Integer Programming Formulation of the SSP Planning Problem

3.1 The Method

The issues facing the analysts at SWFLANT and SWFPAC can be characterized as an inventory management and project scheduling problem. The concepts behind these types of problems, as well as the MILP optimization technique, were applied to building a MILP model, $\mathcal{SSPM}$, that both generates a detailed schedule for the life-cycle support of missiles' components and maintains inventory, all while abiding by the different fleet and processing rules and regulations.

3.2 Modeling Considerations

Various assumptions were made in the completion of this model. Shore facilities that complete necessary work on components, as well as storage facilities and boats, are modeled as locations. Once a component remains at a location long enough to complete that specified work, the work is acknowledged as complete, and the component is permitted to move to the next location. For locations that do not represent processing facilities, the component is permitted to move after one time period unless specifically restricted otherwise. In addition, when the amount of work that is required to move or modify a component is completed, credit for that work must be taken in that time period of completion. When a boat pulls into port, the model allows components to be taken off of it in that same time period it pulled in, as well as the following time period, and it allows components to be put on the boat in the two time periods following a boat pulling in. With the way variables and parameters are defined in this model, good service life is represented by more service life or a higher amount of life left on a component. Components that come off boats need some sort of processing before they can be moved either on to another boat or recalled back to the manufacturer.

3.3 The Optimization Model

Sets and Indices

$b \in B$	main components
$u \in U$	subcomponents that are a part of main components
$l \in \mathcal{L}$	all locations
$l \in Bo \subset \mathcal{L}$	boat locations
$l \in \mathcal{L}_{report} \subset \mathcal{L}$	reportable locations
$l' \in ALL_l$	acceptable location moves (l') when starting from location l
$l \in start\_exch$	all possible starting locations for subcomponent exchanges
$l' \in EXCH_l$	location moves (l') from location l that signify a subcomponent exchange
$l \in SUBEXCH_u$	the starting location (l) for a u specific subcomponent exchange
$t \in T$	time periods
$t \in T_{report} \subset T$	reportable times
$r \in R$	types of resources
$c \in C$	types of containers that hold components when on shore
$v \in V$	versions of components

Parameter Definitions

μ_{ct}	penalty on amount of additional containers of type c required for successful schedule fulfillment at time t.
slr	reward for components on shore having good service lives.
uwp	penalty for doing unnecessary work.
scr	reward for subcomponents having good service lives.
$reset_u$	full service life for new subcomponent u.
$res_{ll'r}$	resource amount of type r required for l to l' move or modification.
$avail_{rt}$	amount of resource type r available for time period t.
$d_{ll'}$	number of time periods it takes to move and/or modify from l to l'.
e_{lt}	1 if event can occur at location l at time t; 0 otherwise. [0,1 indicator]
f_{blt}	1 if component b at location l at time t must be moved due to regulation; 0 otherwise. [0,1 indicator]
req_{ltv}	required number of components of version v needed in location l by time t to support on-loading of components.
ver_b	component version of component b.
$h_{bll't}$	1 if component b must make a specific move from l to l' by time t. [0,1 indicator]
$contreq_b$	type of container required for component b.
w_init_c	initial inventory for type c containers.
i_init_{bl}	1 if components b is in location l for the initial inventory; 0 otherwise. [0,1 indicator]
svc_init_{bu}	initial service life for subcomponent u on component b.
sn_b	serial number for component b.
$ratype_b$	attachment type on component b.

Derived Parameters

ρ_{bt}	reward for service life of component b at time t; equal to $e^{-0.01\cdot(t+1)}$.
max_life	maximum possible life of a component; equal to the minimum of the $reset_u$ values.
$\overline{reset}$	maximum value for OS variable; equal to $1.8 + (0.1 \cdot max\_life)$.

Variables

I_{blt}	binary, 1 if component b is in location l at time t.
$X_{bll't}$	binary, 1 if component b is currently getting worked on in order to transition from location l to location l' at time t.
$Z_{bll't}$	binary, 1 if component b has finished work, making the transition from location l to location l', at time t.
SVC_{but}	non-negative value for the service life of subcomponent u on component b at time t.
EXP_{bt}	non-negative value for the overall service life of component b at time t.
OS_{bt}	non-negative value indicating the objective function representation of service life of component b at time t if the component is in a reportable location.
SSL_{bt}	non-negative value expressing the service life of component b at time t, for those components not in a reportable location but instead on shore.
W_{ct}	real value indicating the number of containers of type c available at time t or if negative, the number of containers needed of type c at time t.
E_{ct}	non-negative value indicating the additional amount of containers of type c needed in time period t to fulfill requirements.

Formulation

$(\mathcal{SSPM})$

$$\begin{aligned} \max \sum_{b\in B,t\in T} \rho_{bt}\cdot OS_{bt} + \sum_{\substack{b\in B,u\in U,\\ t\in T}} scr\cdot SVC_{but} \quad &+ \sum_{b\in B,t\in T} slr\cdot SSL_{bt} - \sum_{\substack{b\in B,l\in\mathcal{L},\\ l'\in ALL_l,t\in T}} uwp\cdot X_{bll't} \\ &- \sum_{c\in C,t\in T} \mu_{ct}\cdot E_{ct} \end{aligned} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{l\in\mathcal{L}} I_{blt} = 1 \qquad \forall\ b\in B, t\in T \tag{3.2}$$

$$\sum_{b\in B:ver_b=v} I_{blt} \geq 2\cdot req_{ltv} \qquad \forall\ l\in L, t\in T, v\in V \tag{3.3}$$

$$\begin{aligned} I_{blt} = I_{bl(t-1)} &+ \sum_{l'\in\mathcal{L}:l\in ALL_{l'}} Z_{bl'l(t-1)} \\ &- \sum_{l'\in ALL_l} Z_{bll'(t-1)} \end{aligned} \qquad \forall\ b\in B, l\in\mathcal{L}, t\in T, t>1 \tag{3.4}$$

$$\sum_{b\in B} I_{blt} \leq 1 \qquad \forall\ l\in Bo, t\in T \tag{3.5}$$

$$I_{bl1} = i\_init_{bl} \qquad \forall\ b\in B, l\in\mathcal{L} \tag{3.6}$$

$$\sum_{l'\in ALL_l} X_{bll't} \leq I_{blt} \qquad \forall\ b\in B, l\in\mathcal{L}, t\in T \tag{3.7}$$

$$\sum_{b\in B,l\in L,l'\in ALL_l} X_{bll't}\cdot res_{ll'r} \leq avail_{rt} \qquad \forall\ r\in R, t\in T \tag{3.8}$$

$$d_{ll'}\cdot Z_{bll't} \leq \sum_{\substack{t'\in T:\\ t-d_{ll'}+1\leq t'\leq t}} X_{bll't'} \qquad \forall\ b\in B, l\in\mathcal{L}, l'\in ALL_l, t\in T \tag{3.9}$$

$$Z_{bll't} \leq e_{ll't} \qquad \forall\ b\in B, l\in\mathcal{L}, l'\in ALL_l, t\in T \tag{3.10}$$

$$f_{blt} \leq \sum_{l'\in ALL_l} Z_{bll't} \qquad \forall\ b\in B, l\in Bo, t\in T \tag{3.11}$$

$$h_{bll't} \leq \sum_{t'\in T:t'\leq t} Z_{bll't'} \qquad \forall\ b\in B, l\in\mathcal{L}, l'\in ALL_l, t\in T \tag{3.12}$$

$$\begin{aligned} SVC_{but} \leq SVC_{bu(t-1)} &- 1 \\ &+ reset_u \sum_{\substack{l\in start\_exch,\\ l'\in EXCH_l}} Z_{bll't} \end{aligned} \qquad \forall\ b\in B, u\in U, t\in T, t>1 \tag{3.13}$$

$$SVC_{bu1} = svc\_init_{bu} \qquad \forall\ b\in B, u\in U \tag{3.14}$$

$$SVC_{but} \leq reset_u \qquad \forall\ b \in B, u \in U, t \in T \tag{3.15}$$

$$EXP_{bt} \leq SVC_{but} \qquad \forall\ b \in B, u \in U, t \in T \tag{3.16}$$

$$OS_{bt} \leq EXP_{bt} \qquad \forall\ b \in B, t \in T \tag{3.17}$$

$$OS_{bt} \leq 1.8 + (0.1 \cdot EXP_{bt}) \qquad \forall\ b \in B, t \in T \tag{3.18}$$

$$OS_{bt} \leq \overline{reset} \cdot (1 - \sum_{l \in \mathcal{L}: l \notin \mathcal{L}_{report}} I_{blt}) \qquad \forall\ b \in B, t \in T \tag{3.19}$$

$$SSL_{bt} \leq EXP_{bt} \qquad \forall\ b \in B, t \in T \tag{3.20}$$

$$SSL_{bt} \leq max\_life \cdot (1 - \sum_{l \in \mathcal{L}_{report}} I_{blt}) \qquad \forall\ b \in B, t \in T \tag{3.21}$$

$$\begin{aligned} W_{ct} = W_{c(t-1)} &- \sum_{\substack{b \in B: contreq_b = c, \\ l \in Bo, l' \in ALL_l}} Z_{bll'(t-1)} \\ &+ \sum_{\substack{b \in B: contreq_b = c, \\ l' = RFI, l \in Bo}} Z_{bl'l(t-1)} \qquad \forall\ c \in C, t \in T, t > 1 \end{aligned} \tag{3.22}$$

$$W_{c1} = w\_init_c \qquad \forall\ c \in C \tag{3.23}$$

$$W_{ct} + E_{ct} \geq 0 \qquad \forall\ c \in C, t \in T \tag{3.24}$$

$$I_{blt} \in \{0, 1\} \qquad \forall\ b \in B, l \in \mathcal{L}, t \in T \tag{3.25}$$

$$X_{bll't} \in \{0, 1\} \qquad \forall\ b \in B, l \in \mathcal{L}, l' \in ALL_l, t \in T \tag{3.26}$$

$$Z_{bll't} \in \{0, 1\} \qquad \forall\ b \in B, l \in \mathcal{L}, l' \in ALL_l, t \in T \tag{3.27}$$

$$SVC_{but} \geq 0 \qquad \forall\ b \in B, u \in U, t \in T \tag{3.28}$$

$$EXP_{bt} \geq 0 \qquad \forall\ b \in B, t \in T \tag{3.29}$$

$$OS_{bt} \geq 0 \qquad \forall\ b \in B, t \in T \tag{3.30}$$

$$SSL_{bt} \geq 0 \qquad \forall\ b \in B, t \in T \tag{3.31}$$

$$W_{ct} \in \mathbb{R} \qquad \forall\ c \in C, t \in T \tag{3.32}$$

$$E_{ct} \geq 0 \qquad \forall\ c \in C, t \in T \tag{3.33}$$

3.4 Description of the Model

The objective function 3.1 maximizes the reward for having a larger amount of available service life on each component both on boats and on shore as well as the service lives of the subcomponents, while minimizing the penalty associated with needing extra containers and doing unnecessary work. The first five constraints focus on the component inventory. Constraint 3.2 manages the inventory of every component throughout the model run, ensuring each component can only be in one location. If the required number of components, req_{ltv}, is greater than zero, constraint 3.3 requires a certain number of components of an explicit version type to be in a set location at a specific time, and two times the specified number are required to account for spares. Constraint 3.4 tracks the inventory of the components in every time period other than the first one, updating the time index as time passes and updating the location index if work was completed to move a component, for each component. Constraint 3.5 ensures that no more than one component can be in a specific position on a boat at a given time. Constraint 3.6 sets the inventory in the first time period to the current inventory when the model is run.

The next group of constraints transitions into managing active work and completed work. Constraint 3.7 makes sure work in a specific location can only be done on a component in a time period if that component is actually in that location in that time period. Constraint 3.8 ensures that no more than the available amount of resources for each time period can be utilized for work for every type of resource. Constraint 3.9 ensures that work done on a component can only be marked as completed if enough work, based on requirements, was done in the time periods leading up to that specific time period. The $d_{ll'}$ parameter represents the transit time requirement for each l to l' move or the amount of time periods it takes to complete the work to make that specific move. Constraint 3.10 makes sure that work can only be completed to move a component from one location to another if that move is feasible during the given time period for each set of moves and component. If f_{blt} is one, constraint 3.11 forces that component b that was in a boat location to be moved off of the boat in that time period t. If $h_{bll't}$ is one, constraint 3.12 forces that component b to be moved to a specific location l' by the time t. This could occur in the event of a component getting recalled.

Constraints 3.13 through 3.21 center on the service life of components and subcomponents, as well as their representations in the objective function. Constraint 3.13 tracks the service life of each subcomponent on each component. This life is reduced by one for each passing time period, and if a subcomponent exchange of that type is completed, enough life is added to make sure the service life for that specific subcomponent is reset to its maximum life.

Constraint 3.14 sets the initial service life of each subcomponent on each component in the first time period to its current service life when the model is run. Constraint 3.15 prevents the subcomponent service life from being bigger than its maximum value. Constraint 3.16 ensures that the overall service life for a component is based on the minimum service life left on that component's subcomponents, or in other words, a components overall expiration date is set by which of its subcomponents will expire first. Constraint 3.17 prevents the objective function service life representation from being greater than the overall service life for each component. Constraint 3.18 restricts the objective function service life variable in an equation where the slope allows less reward per unit of service life to be given to good service lives in time periods that are further out. Constraint 3.19 forces the objective function service life variable to zero for any component that is not in a reportable location and limits it to the maximum service life for OS if in a reportable location. The compilation of constraints 3.17 through 3.19 can be seen in Figure 3.1. Constraint 3.20 ensures the non-reportable location component service lives are not greater than the overall service life of that component in that time period. Constraint 3.21 forces the objective function shore service life variable to zero for components that are in reportable locations.

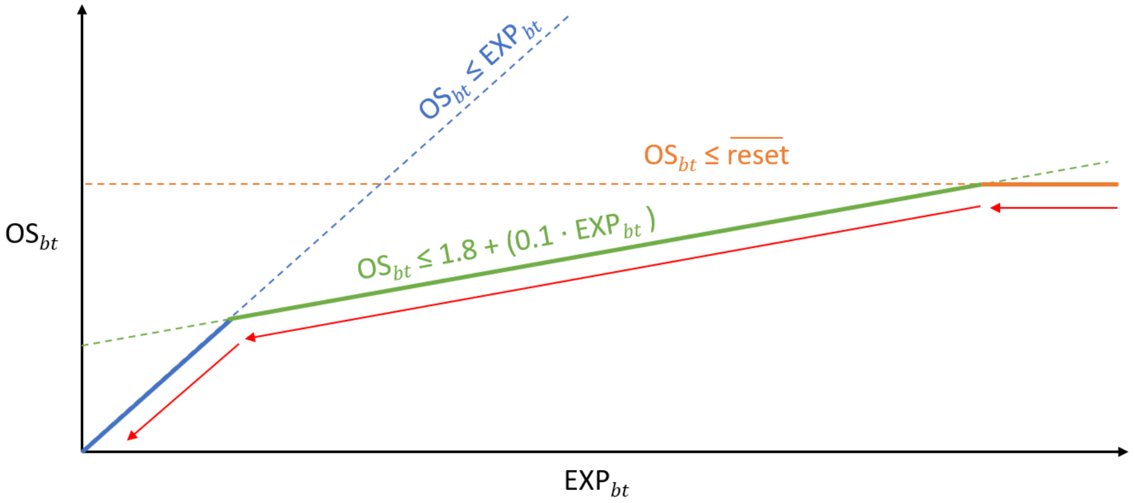

Figure 3.1. In terms of service lives, EXP_{bt} is the exact overall service life of a component b. The three constraints shown here constrain the objective function representation of the service life, OS_{bt}, so that the value decreases as the service life becomes smaller, as represented by the red arrow.

The next group of constraints focuses on container inventory. Constraint 3.22 tracks the inventory of the containers by container type, updating the time index as time passes, reducing the inventory when a component is moved from a boat to the shore, and increasing the inventory when a component is moved from shore to a boat, freeing up a container space. Constraint 3.23 sets the initial inventory in the first time period for each type of container to the current inventory when the model is initially run. Constraint 3.24 forces the additional container variable to be positive if the true inventory for that type of container becomes negative during a time period. The last nine constraints define the domain of each decision variable. Constraints 3.25, 3.26, and 3.27 restrict the binary variables I_{blt}, $X_{bll't}$, and $Z_{bll't}$ respectively. Constraints 3.28 through 3.31 and 3.33 force SVC_{but}, EXP_{bt}, OS_{bt}, SSL_{bt}, and E_{ct} to take on non-negative values, and constraint 3.32 sets the domain of W_{ct} to all real numbers.

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CHAPTER 4:
Data, Analysis, and Results

This chapter describes a collection of case studies and analyses applying the model, $\mathcal{SSPM}$, to four SSP planning scenarios. We report computational results for these cases that demonstrate how the model balances the rules, requirements, and inventories handled by the SWFs, as well as how it optimizes the scheduling of components.

4.1 Description of Cases

All the data used in the case studies is purely likeness data, developed in discussions with the analysts from both SWFs to replicate the general magnitude, complexity and generalized utility to meet their needs. Each case defines a time period as one week.

4.1.1 Shore Facilities

We consider the same generic shore facilities for each of the first three case. The initial storage locations are labeled needs process inspection (NPI), needs component exchange 1 (NCE1), needs component exchange 2 (NCE2), needs component exchange 3 (NCE3), and needs RA exchange (NRAE). The processing locations are labeled specifically as processing process inspection (PPI), processing subcomponent exchange 1 (PCE1), processing subcomponent exchange 2 (PCE2), processing subcomponent exchange 3 (PCE3), and processing RA exchange (PRAE), and the after-processing storage location is called ready for issue (RFI). Figure 4.1 shows a visualization of those locations common across the main three cases. Note that the double-headed arrows in the storage needs processing box represent a component being able to transition from any one of those needs processing locations to any other one, representing the type of processing scheduled being adjusted by the analyst.

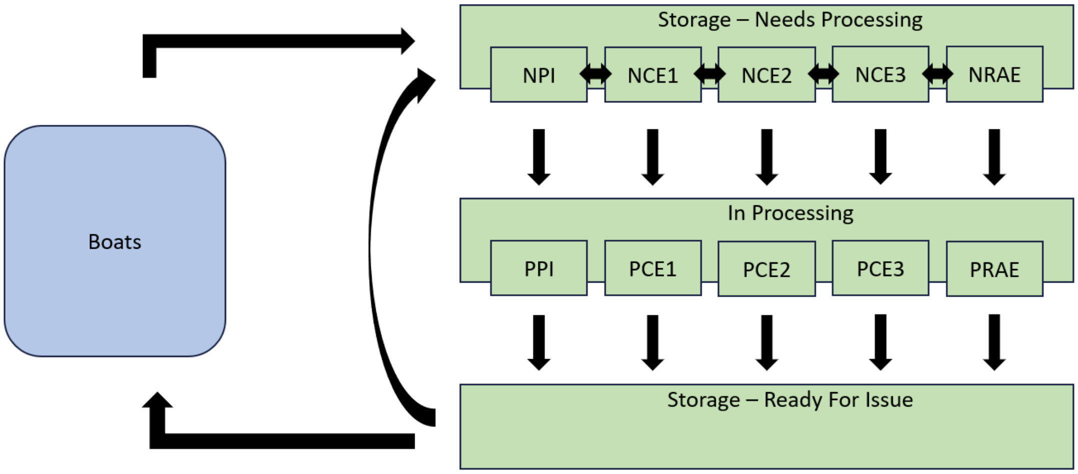

Figure 4.1. Locations used for the main three instances. The blue represents sea locations, the green represents shore locations, and the black arrows indicate the allowable location to location transitions.

4.1.2 Boats and Components

The cases vary in level of scheduling complexity in terms of number of boats and components to track and process. All components have three subcomponents, each with their own service life. The service life of a brand new subcomponent 1, or its maximum service life, is 225, subcomponent 2's is 250, and subcomponent 3's is 250 time periods. The processing facilities have the capacity to conduct three subcomponent exchanges, three process inspections, and two RA exchanges per time period. It takes two time periods to complete a process inspection, four for all subcomponent exchanges, and three for a RA exchange. Every other location to location move takes one time period. The differences of the cases can be seen outlined in Table 4.1, highlighting the varying size of each case.

Table 4.1. The cases vary in size and complexity.

case	size	boats	components	recalls	time periods	constraints
1	small	1	3	0	20	7,500
2	medium	4	15	0	40	135,000
3	large	8	1,200	0	75	>1,000,000
4	medium	4	15	1	40	135,000

Case 1, the smallest case built, has a starting inventory with one component on the boat, one in NPI, and one in RFI. The one boat comes into port to be serviced at time period ten. This very small case verifies the constraints are doing what we built them to do by enforcing the rules and restrictions, and it validates the model with a simple process.

The starting inventory for case 2, the medium size instance, has a component in each of the eight boat locations and the other seven components on shore, which can be seen in Figure 4.2. During the forty time periods in this instance, boat 1 comes into port at time period 32, boat 2 at time period 18, boat 3 at time period 10, and boat 4 at 25. The medium case was built to continue to verify the model, but also validate it. The constraints that were not be able to be seen in the small instance due to the small number of components could be verified in the medium instance. This includes constraints that restrict the number of components being processing, because only two components were ever off the boat at the same time in the small case. Additionally, because there are two components per boat, the medium case verified that the components on a boat were able to come off in either the time period it ported or the next. Due to the fact that this case contained more than one boat and twelve more components, its behavior could be used to validate the model, testing that its building a schedule more applicable to real life than the small case. The model has more options for components to process and components to put on a boat, as each boat pulls into port. The decisions it makes when scheduling component moves and processing can help display that it is deciding like an analyst would.

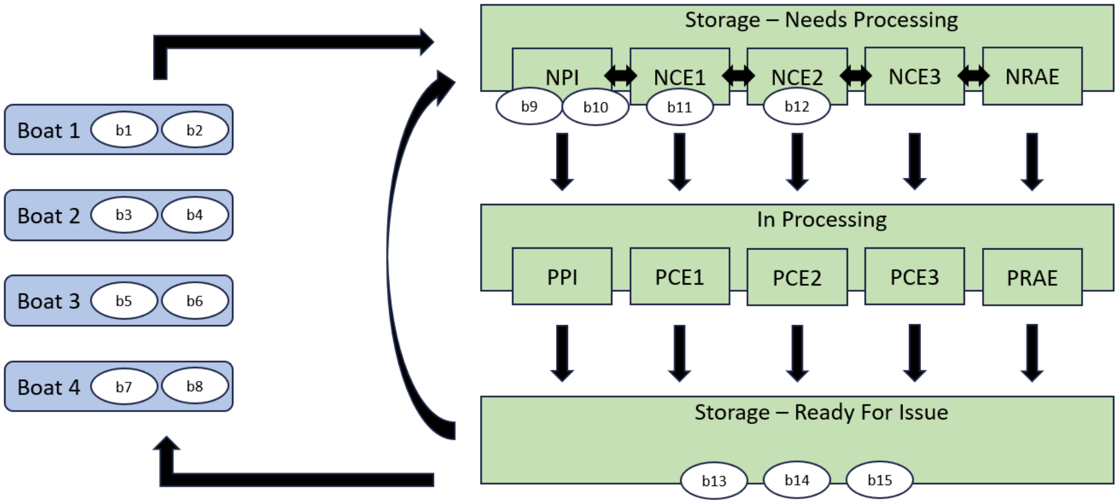

Figure 4.2. The starting locations for the fifteen components, represented by white ovals, are shown in the location model for the medium case.

In the large instance or case 3, each of the eight boats has twenty missiles each with five component positions on each missile for a total of 100 component locations on each boat. For the inventory of components in the first time period, every position on a boat is filled with a component, and the other 400 components are in shore locations. This case models a scenario representative of the size of the problem the analysts are handling.

Output data that can be gathered for the instances includes inventory data for every time period for both components and containers, service lives of subcomponents and components for every time period, and also work that is being done or was completed for each time period.

4.1.3 Running the Model

The model, $\mathcal{SSPM}$, was written in Python language, utilizing the Pyomo package. It was implemented using the solver Gurobi Optimizer version 10.0.1 on a personal computer with 64 GB of memory, Intel Core i7 at 3.60GHz CPU model, and 8 physical cores and 16 logical processors.

4.2 Case 1 Results

For case 1, it took less than three seconds to run through the $\mathcal{SSPM}$ model with a result of 2.6 percent optimization gap. The results of the component inventory and work completed can be seen in Figure 4.3. For the most part, this simple chart shows the movement of the three different components throughout the time periods run in the model. Movement in the outputted schedule verifies the constraints are restricting the problem correctly. For example, only one component is on the boat a time. One output of the model that needs to be improved is modeling the need for a spare component on shore. In the code, we specified that because one component is being replaced on boat 1 in time period 10, two need to be in RFI to support this event by time period 8, one as replacement and one as a spare. However, the model does not force the spare to remain in that location, so in time period 9, the spare moves to get other processing, which is not ideal. Additional modification of the model's parameters is necessary to ensure this meets the planner's intent.

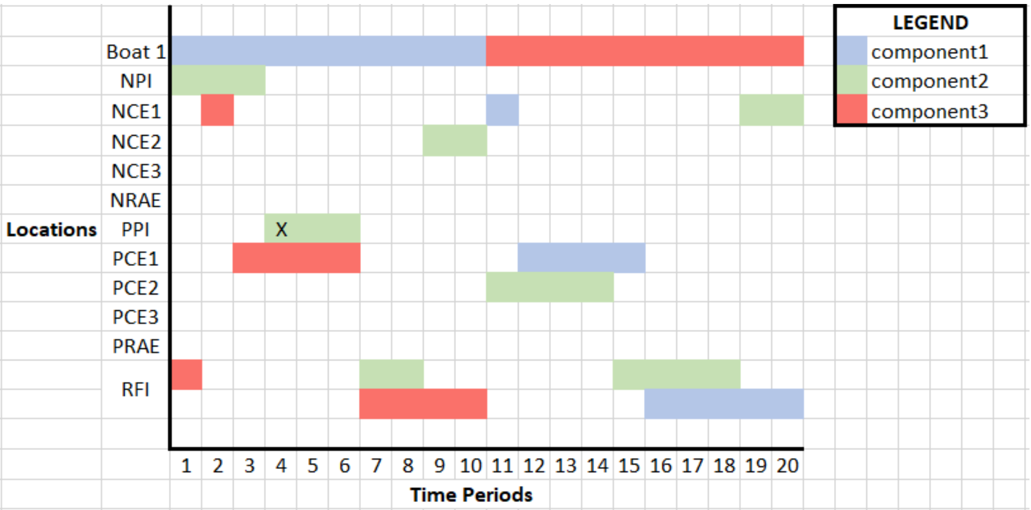

Figure 4.3. The optimal schedule for three components over twenty time periods created by running the case 1 through the MILP model. The X represents time periods the component is in a processing location but not yet being processed.

The results from this small case show the model is making decisions to optimize the schedule, based on the objective. It is easy to spot the point at which the boat came into port by the change of components in the boat location. This component replacement was chosen in a way that maximizes the model's objective function, in terms of putting the component with the best service life on a boat. In Figure 4.4, at time period 10, it is clear that component 3 in green has a higher service life than component 2 in red, so the model chose it to replace component 1 on the boat.

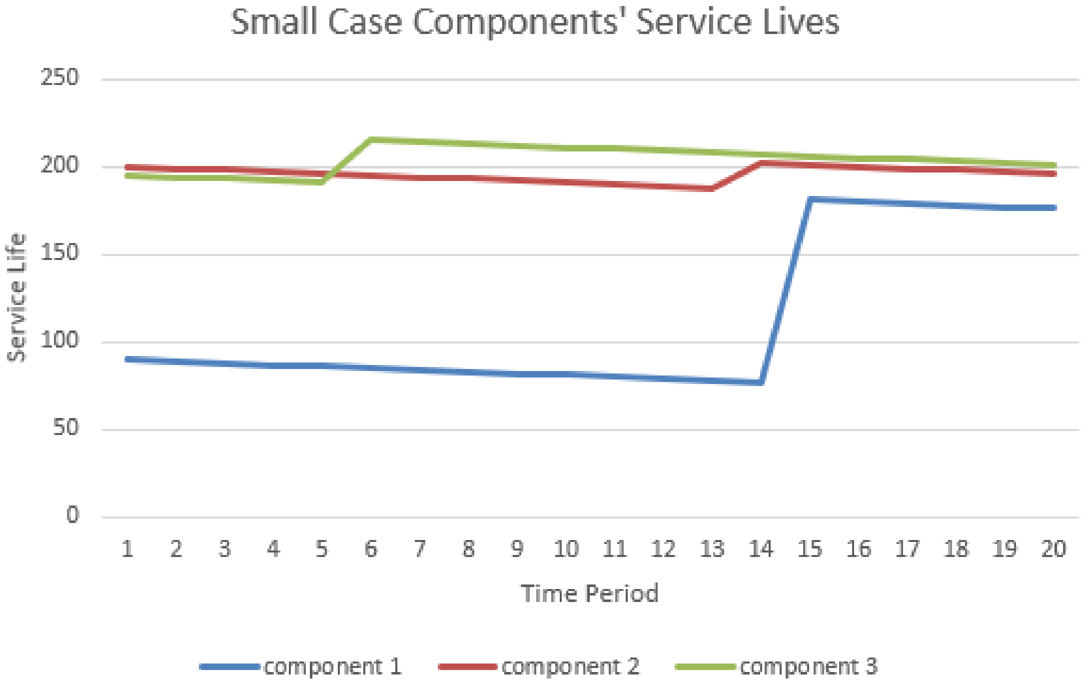

Figure 4.4. The service lives of the 3 components in the small case study over 20 time periods.

The results validate the model in the way it chooses which processes to have each component go through; the model produces a schedule that reflects decisions an analyst would make. For example, this can be seen in component 1's specific schedule. Once component 1 is taken off the boat in time period 10, it moves to storage and then to get processed through a subcomponent exchange 1. As seen in Figure 4.5, the lowest subcomponent service life, and therefore the one setting the overall life for component 1, is subcomponent 1. This is recognized by the model, so it exchanges subcomponent 1, so that the overall service life of the component is increased, now being restrained by subcomponent 2.

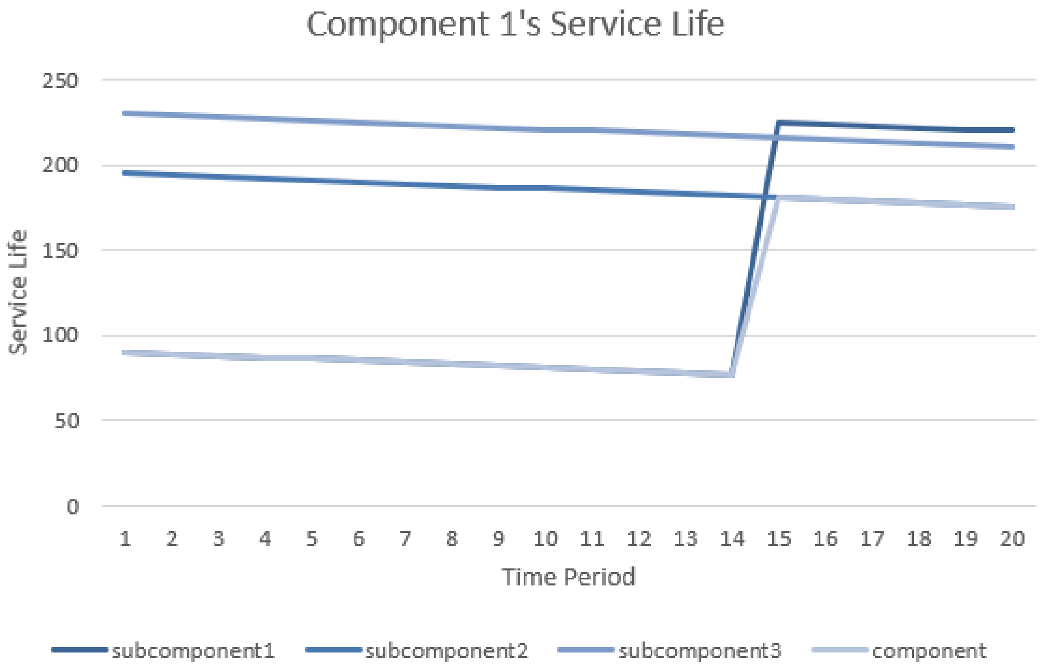

Figure 4.5. The service lives of the 3 subcomponents on component 1, as well as the overall component 1 service life, determined by the lowest subcomponent service life, over 20 time periods in the small case.

4.3 Case 2 Results

The scheduling results for the main medium case, case 2, can be seen in Figure 4.6. This case took just over five minutes to run with a result of 7.4 percent optimization gap. With more components and more time periods, the case study shows some advanced aspects of the model. Analysis of the medium case further verifies that the model manages the inventory and process completion as required. For inventory, each component is accounted throughout all 40 time periods and only make allowable moves from location to location. In terms of processing, the model correctly restricts the schedule to only three subcomponent exchanges being processed at one time. For example, in time period 5 in Figure 4.6, there are four components in the subcomponent exchange processing locations, but as denoted by the X, component 9 has not begun its processing yet. It is being held in that location until it begins its processing in time period 6, after component 11 and 12 have finished their subcomponent exchanges. Another point of verification can be seen when the components are being taken off boats 1 and 2 when they are in port. The model specifies that components can be taken off in the period the boat comes into port or the time period after, 32 or 33 for boat 1 and 18 and 19 for boat 2. The model decides to take component 2 off in time period 32 and component 1 off in time period 33, and a similar decision was made for components 3 and 4 on boat 2. This decision to wait one time period for the later components was made because the model was waiting for a different component to finish its processing, so that it could take the components place on the boat. This displays the model adhering to the constraints.

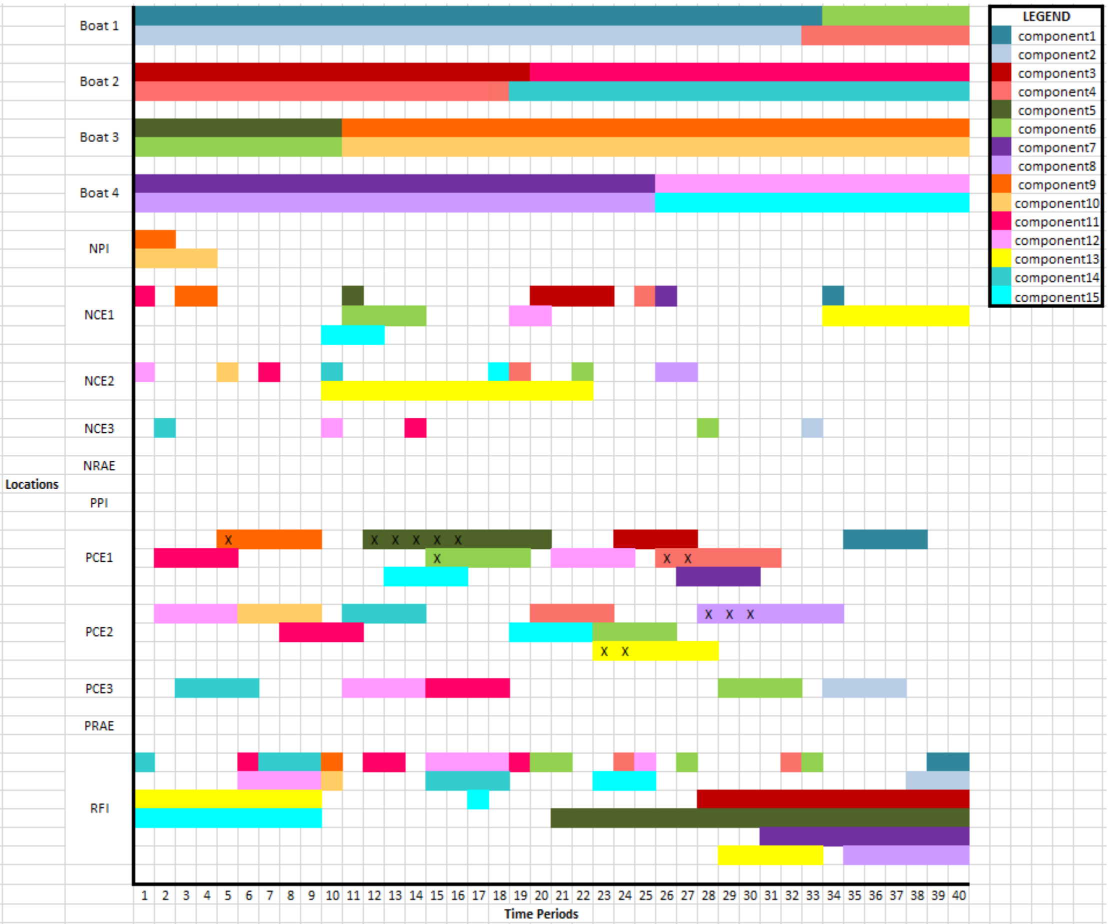

Figure 4.6. The optimal schedule for fifteen components over the forty time periods created by running case 2 through the MILP model. The X's in the colored boxes are time periods the component is in a processing location but not yet being processed.

The subcomponent exchanges are what give the components more life, helping to increase the objective function. Because no RA exchanges were required in this case and RA exchanges do not benefit the objective function, location PRAE was never utilized. In addition, because the components had enough time to process a subcomponent exchange and no process inspections were specifically required, PPI was not used. These decisions are what we expected based on the objective function and constraints enforced for this case study.

The main part of the model's objective function, in support of one of the SWFs priorities, is to maximize the service life of components while at sea. Figure 4.7 shows how the schedule supports this aspect of the objective. Each time one of the boats comes into port, its components are scheduled to be replaced by components from shore that significantly increase the boats average service life for components on board, as seen by the jump in that boat's line. As a result, the average life of components across all boats, shown by the red line, climbs throughout the 40 time periods.

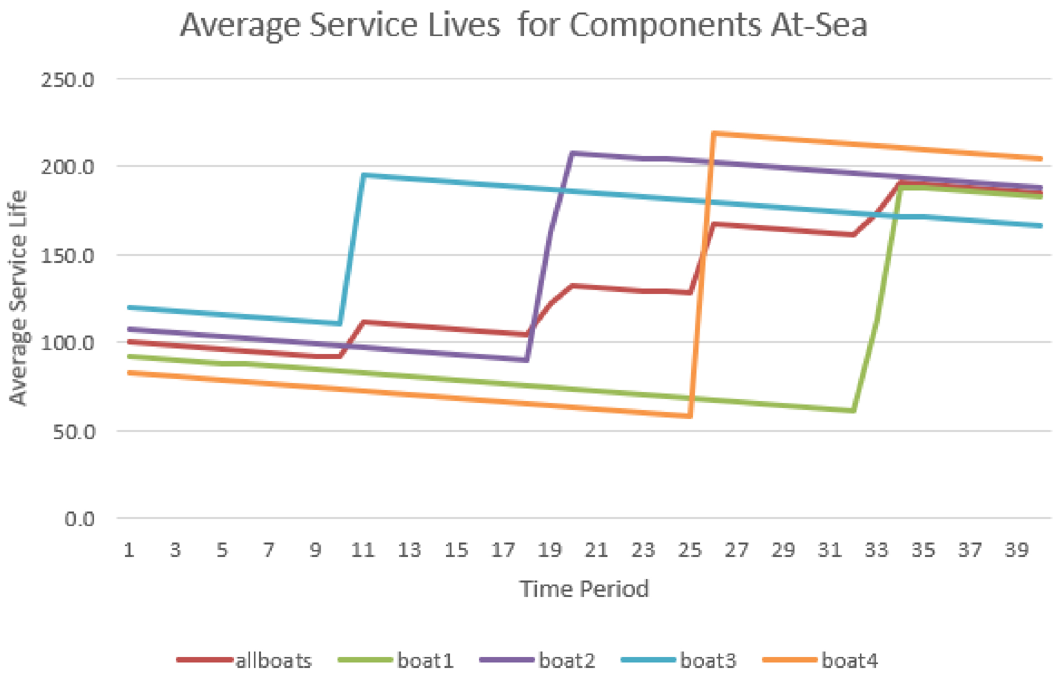

Figure 4.7. The average service lives of the components on the 4 boats and the average service life of all components on boats, over 40 time periods in case 2.

For another visual, Figure 4.8 displays the schedule for just one component, component 4, from the case 2. It follows the component as it comes off boat 2, goes through two rounds of processing, and gets onloaded to boat 1.

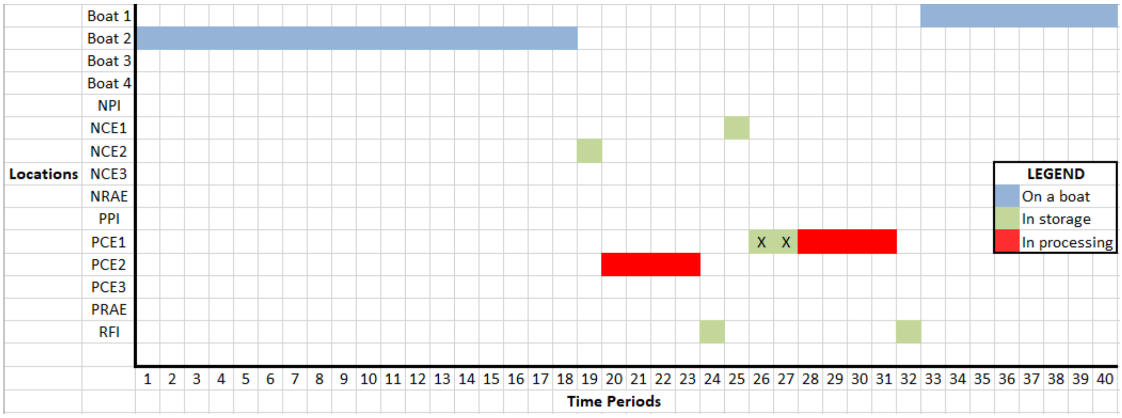

Figure 4.8. The optimal schedule, color coded by location, for component 4 over 40 time periods created by running case 2 through the MILP model. The X's represent the component in a processing location but not yet being processed.

The model $\mathcal{SSPM}$ selects an ordering and timing of processes that follows logic applied by the analysts at the SWFs. In Figure 4.9, it is clear that subcomponent 2 has the lowest service life, so the first processing component 4 receives is a subcomponent 2 exchange, finishing in time period 23. The next lowest is subcomponent 1, leading to a subcomponent 1 exchange that ends in time period 31. These two processes allow component 4's overall service life to be brought up to 168 by the time it is put on boat 1.

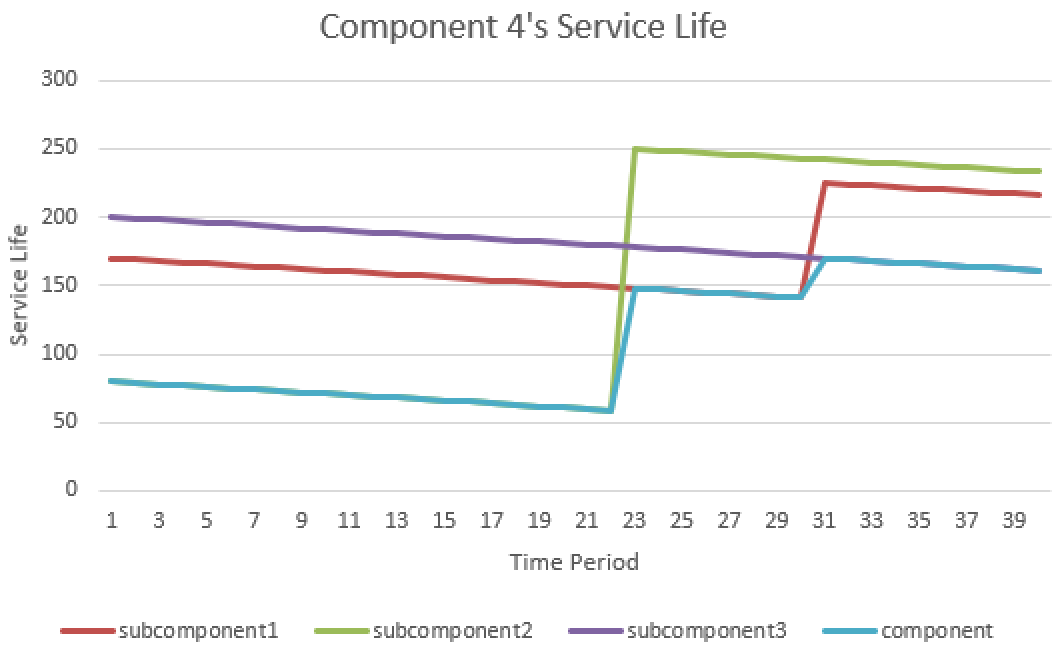

Figure 4.9. The service lives of the 3 subcomponents on component 4, as well as the overall component 4 service life determined by the lowest subcomponent service life, over 40 time periods in case 2.

This is just one of the 15 components in the medium case study. Similar decisions were made for the other components' schedules to maximize the model's objective. Also, SWFs' policies such as every component must receive some sort of processing before being put back on a boat were successfully adopted in the medium case study, helping to validate the $\mathcal{SSPM}$ model.

4.4 Case 3 Results

The current large instance cannot be solved on a computer with 64 GB of memory due to the magnitude of the case. The interaction of activities at the facilities through shared consumption of limited resources, requirement for back-up components, and narrow windows for component loading poses challenges for decomposing this problem into smaller, more manageable problems. Case 3 is a demonstration of the complexity and size of like-data scenarios in terms of number of boats, components, and time periods that require scheduling. This is not the largest scenario for a case, for there is no natural end to the sequence of required tasks for the analysts at the SWFs to plan. This large case points more to the complexity from the depth in terms of boats and components, but not as much to the breadth of solving the scheduling for the full service life of many of these components. Once the challenges posed by this case are addressed, another, longer term case, should be examined.

4.5 Case 4 Description and Results

A variation of the medium sized instance was created with a component recall occurring during the time periods. The instance is almost identical to the main medium instance, with the addition of two locations, sendoff and recall. The starting locations for this instance are in Figure 4.10, and this figure also displays the feasible moves for a component being recalled, from PRAE to sendoff to be held in storage until it is sent to recall. The move between sendoff and recall only takes one time period of work to complete for this case.

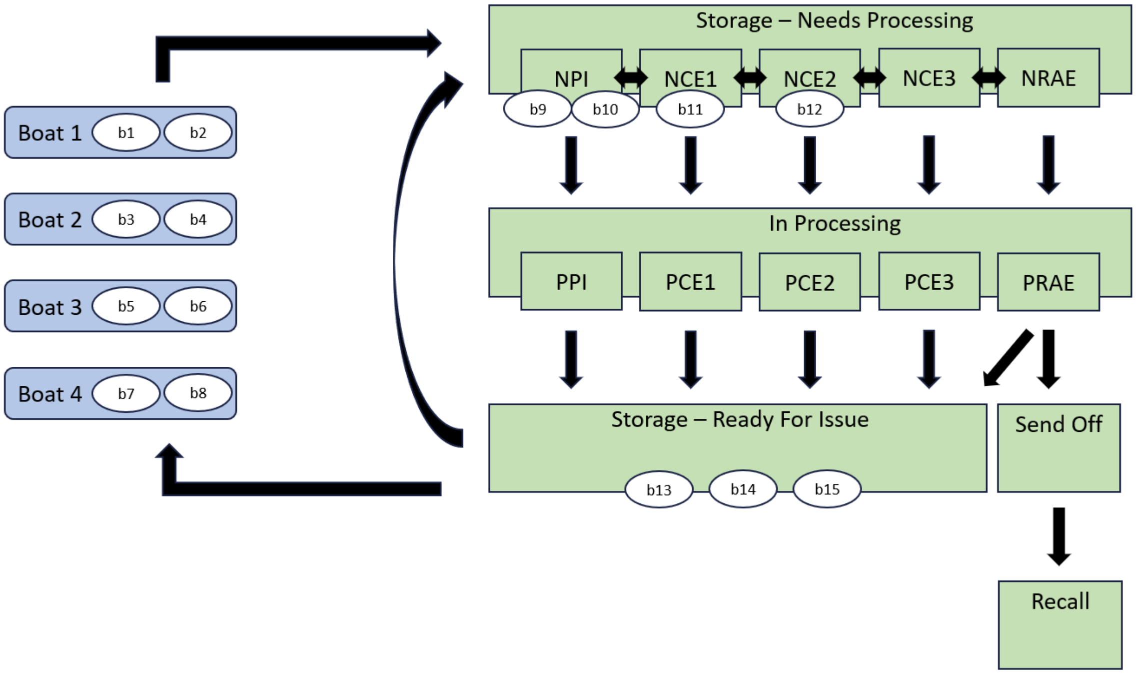

Figure 4.10. The starting locations for the fifteen components are shown in the location model for the medium case with additional locations for the component recall.

Case 4 took under four minutes to run with a result of 7.9 percent optimization gap. In this case, component 6 is being recalled. To enforce this, the parameter $h_{bll't}$ is initially set to one for component 6, sendoff to recall move, and time period 25. This makes sure that component 6 is in the recall location by time period 26. As seen in the schedule in Figure 4.11, the results from this case verify that the constraints are enforcing that component recall. Because of the added component 6 recall, this schedule varies from the schedule for the other medium case, case 2. The model made different decisions in scheduling processing, because in case 2, component 6 was placed on boat 1. In addition, this recall is a form of validation, as the component comes off a boat, gets processed through a RA exchange, stored, and then sent away by the required time period, meeting the policy held by the SWFs.

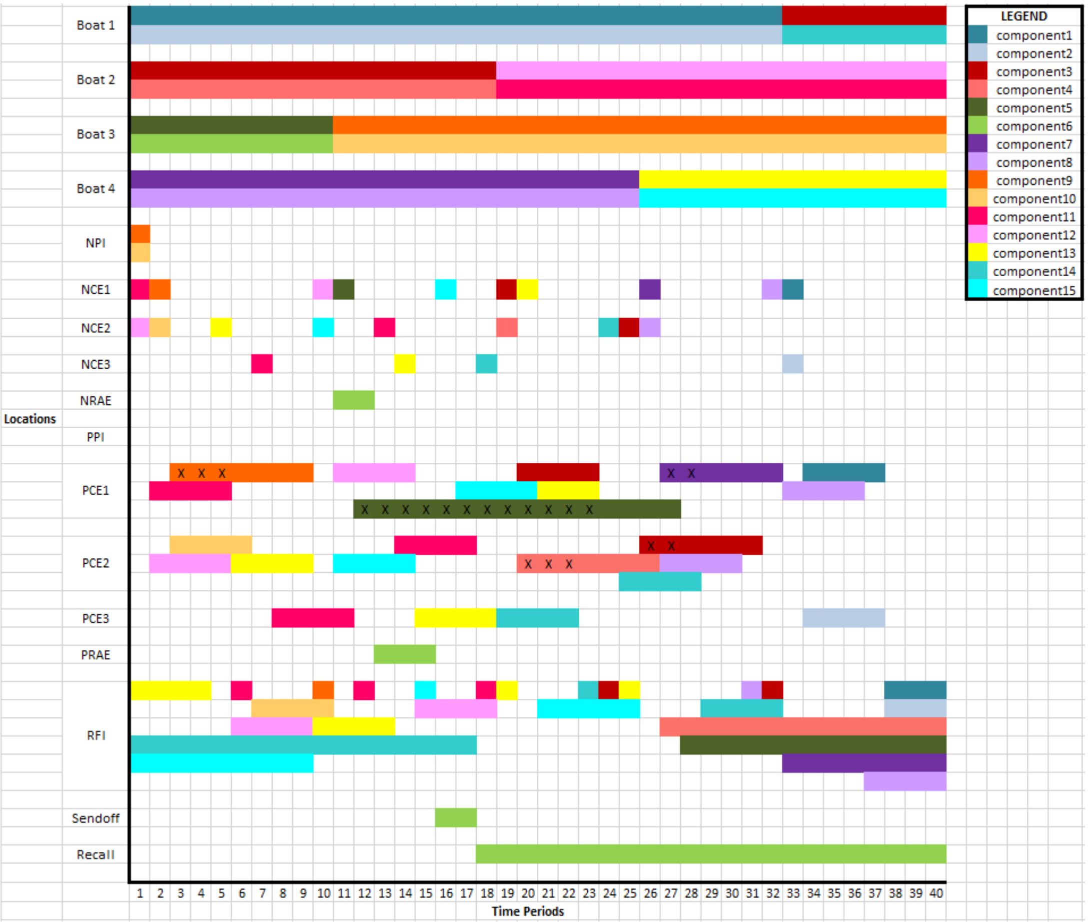

Figure 4.11. The optimal schedule for fifteen components over the forty time periods created by running the medium case with a component recall, case 4, through the MILP model. The X's in the colored boxes are time periods the component is in a processing location but not yet being processed.

In the small, medium, and medium with recall case, there is room for improvement in the modeling. Specifically in case 4, component 5 is held in PCE1 for twelve time periods before it is actually processed. Because of this extended time period, it would make more sense for it to be held in NCE1 until it is going to be processed. In addition, many components receive multiple types of subcomponent exchanges back to back. Currently, the model forces the components to storage where they are assigned a type of processing, processed, RFI, back to storage to be assigned a new type of processing, and repeat. While this is valid, it would be reasonable to add locations where components are automatically assigned more than one type of processing. In Figure 4.11, component 8 receives a subcomponent 2 exchange followed by a subcomponent 1 exchange. Instead, one location could be created in the model to get both of these processes complete.

Overall, the above results from the small and medium cases verified that the constraints built into the $\mathcal{SSPM}$ model are performing as they should, managing inventory and processing. Other than the minor improvements mentioned, they also displayed valid output that matches processing regulations and rules taken into consideration by the analysts. By diving into the service lives, the model also shows to be scheduling in a way that optimizes the life of components. The difficulty with the large case due to the size of the model implementation shows the complexity of this problem and moving forward, could possibly be used to develop a local search heuristic that can provide feasible solutions in reduced time.

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CHAPTER 5:

Conclusions and Future Work

SSP requested assistance from Naval Postgraduate School (NPS) in their scheduling operations at the SWFs. They recognize the challenges the analysts face creating both short term tactical and long term operational schedules at both the missile and component level while balancing different input schedules, rules, and inventories. Their current project management tools, which are independent spreadsheets, make it difficult to handle the range of demands in a time efficient manner. To help with this deficiency, this work walks through the creation of a MILP, $\mathcal{SSPM}$, that automates the schedule development and tracking of inventories over a desired time period. It builds the manufacturing and processing rules and regulations considered by the analysts into constraints that the model is forced to abide by. In addition, it optimizes the scheduling process and increases the analysts' visibility for the future. The model optimizes by maximizing the service life of the components and subcomponents and minimizing the additional resources required and unnecessary work scheduled. The objective makes it so the outputted schedule from the model efficiently uses the limited resources and the component's lifetime.

In conclusion, this thesis supports the difficult work underway by the scheduling analysts at SWFLANT, SWFPAC, and SSP to help increase their efficiency, productivity, and visibility in supporting the servicing ballistic missiles. The model created is superior to the manual method of building schedules because it is thorough in accounting for rules and keeping tracking of inventory. It goes further than building a usable schedule by optimizing the components' service lives. In addition, having a standardized tool that is tailorable by location will produce a new level of transparency between SSP and both SWFs. When utilizing the $\mathcal{SSPM}$ model, the analysts will be able to get a schedule in less time than the 30 days the manual method takes. With this time saved, they will then be able to apply their expert knowledge to deeper analyses on the schedules and what-if questions, and this more efficient tool will give SSP the ability to ask more of these questions to the SWFs. Also, the tool, with its increased capability for visibility, offers Department of Defense (DOD) personnel means to join in conversation with contractors on inventory and workforce matters that they did not have before.

5.1 Future Research

Several challenges remain to provide SSP with a model that will serve as a minimal viable product. The challenges, in order of priority to the analysts at SSP are: 1) accelerate the model to handle like-sized instances; 2) conduct a validation meeting with the SWFs to discuss model emergent behavior.

5.1.1 Improving Efficiency of the Formulation

Further development of solution acceleration techniques are necessary. $\mathcal{SSPM}$ creates a large number of constraints when offered like-data scenarios in terms of number of boats and components, more than acceptable for a computer available to the end-user. To improve the model, the formulation should be modified to occupy less memory at run-time, so we accommodate the cases with more components and longer time horizons.

Case 3 identified complexity in terms of boats and components, but it may not be the largest scenario encountered by a user, for there is no definite end to the scheduling for the analysts at the SWFs to complete. Resource constraints and component configuration requirements may require event preparation far in advance of the supported event. The end effects are significant; it is easy to see how tasks are completed in a fiscal year that do not return value in the same fiscal year but instead are benefiting future operations. Extending the scenario in the dimension of time does not reduce complexity as in other limited task problems due to the continuous nature of the mission. Combined with end-effects concerns, future work should leverage successes in model acceleration to pose new, larger cases for investigation of model efficiency.

This thesis, and the defined model $\mathcal{SSPM}$, may offer a logical foundation and feasibility definition for a reformulation or local search heuristic algorithm that accelerates solution development.

5.1.2 Model Validation

Several interesting model behaviors emerged from the case studies tested, and although they were built using likeness data, adjustments can be made to more closely model the analysts' schedules to validate the model. One important aspect to modify in the model's constraints is the requirement for the primary and spare components for a boat event.

Currently, the model requires the primary and spare to be in RFI together before an event for only one time period. This could have implications in making the schedule infeasible for the SWFs. Another interesting behavior of the model is scheduling a component to be moved into the processing location many time period before it is actually processed. Discussions need to be had in collaboration with the analysts to investigate these behaviors and determine the schedules acceptability in order to properly revise the model. Additionally, once the accelerated model can be utilized, operational opportunities should be examined. A validation meeting will allow the expert analysts to validate the model's capabilities, and from there, the model should be tested using real data within their native environment. Future validation should also include discussion of the objective function. The current objective function focuses on the service lives of components, but other terms can be added to account for things such as workload leveling. The objective can be explored further in terms of the penalties and rewards to appropriately balance considerations. It could even by adjusted to address the specific needs or challenges faced by one of the SWFs.

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