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STOCHASTIC ANALYSIS OF GROUND-BASED ANTI-SHIP MISSILE SYSTEM LOGISTICS

by

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June 2023

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The ground-based anti-ship missile (GBASM) capability is the Marine Corps' top modernization priority and a critical element of its Expeditionary Advanced Base Operations concept and Force Design 2030 initiative. While experimentation and limited study regarding firing, targeting, and the tactics, techniques, and procedures (TTPs) of this nascent capability is ongoing, no research has been conducted on the logistics associated with the weapon system. This thesis uses stochastic modeling and simulation methods to explore firing dynamics between GBASM launchers supported by a supply depot and maritime targets. The models are parameterized to account for varying factors such as number of blue shooters, size of the ammunition depot, and salvo size. The primary output of the research are insights in the form of policies that attempt to maximize blue probability of win in response to a strategic adversary, which can inform live-force experimentation and tactics development.

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STOCHASTIC ANALYSIS OF GROUND-BASED ANTI-SHIP MISSILE SYSTEM LOGISTICS

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ABSTRACT

The ground-based anti-ship missile (GBASM) capability is the Marine Corps' top modernization priority and a critical element of its Expeditionary Advanced Base Operations concept and Force Design 2030 initiative. While experimentation and limited study regarding firing, targeting, and the tactics, techniques, and procedures (TTPs) of this nascent capability is ongoing, no research has been conducted on the logistics associated with the weapon system. This thesis uses stochastic modeling and simulation methods to explore firing dynamics between GBASM launchers supported by a supply depot and maritime targets. The models are parameterized to account for varying factors such as number of blue shooters, size of the ammunition depot, and salvo size. The primary output of the research are insights in the form of policies that attempt to maximize blue probability of win in response to a strategic adversary, which can inform live-force experimentation and tactics development.

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List of Acronyms and Abbreviations

BDA battle damage assessment

CLF Combat Logistics Force

CTMC continuous time Markov chain

DMO Distributed Maritime Operations

DTMC discrete time Markov chain

EAB expeditionary advanced base

EABO Expeditionary Advanced Base Operations

GBASM Ground-based Anti-ship Missile

iid independent and identically distributed

MOE measure of effectiveness

MLR Marine Littoral Regiment

MMSL Medium Missile Battery

NPS Naval Postgraduate School

NSM naval strike missile

OMFTS Operational Maneuver from the Sea

TTPs tactics, techniques, and procedures

USMC United States Marine Corps

USN U.S. Navy

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Executive Summary

The Ground Based Anti-Ship Missile (GBASM) weapon system is the Marine Corps' current top modernization priority and is the cornerstone of its newest operational concept, Expeditionary Advanced Based Operations (EABO). EABO aims to support the Navy's sea control and sea denial missions, particularly in the Pacific; the GBASM provides the Marine Corps a long-range fires capability to contribute to those missions, allowing small units of Marines deployed with the weapon system to control vital sea lines of communication. To date, GBASM research has solely focused on weapon effectiveness and developing firing tactics, techniques, and procedures (TTPs); no research has been conducted regarding the logistical considerations of the system. This thesis aims to fill that gap by modeling the weapon system and its supply depot in order to gain insights into battle dynamics when logistics is considered.

We develop three models in this study: a discrete time Markov chain (DTMC), a Monte Carlo simulation, and an analytical model. The primary model used for the analysis is the Monte Carlo simulation. The DTMC is used to verify the results of the simulation and the analytical model complements and reinforces the findings of the simulation.

We model an artillery duel between GBASM launchers ("Blue") supported by a supply depot and an enemy maritime target ("Red"). The duel begins with all Blue shooters firing at the Red target with a probability of kill α. If still alive, the Red target returns fire at both the shooters, with a probability of kill β, and at the depot, with a probability of kill γ. The duel continues until one of three victory conditions are met: 1) Blue destroys Red, 2) Red destroys all Blue shooters, or 3) Red destroys the Blue depot. The measure of effectiveness for the model is the probability of winning.

There are several underlying assumptions and limitations to the model. We assume that Blue always fires first, which is consistent with the GBASM concept of employment, which stresses concealment of the weapon until it is ready to fire. Red is assumed to have an unlimited supply of missiles; in the base model, we make the same assumption for Blue for simplicity, but this is relaxed in the extended model. We assume that all Blue shooters fire on each salvo; unit tactics and fire control are not modeled. A key limitation of the model is that time and distance are not considered, which means that Blue does not need to traverse to the supply depot to reload. We believe this assumption does not affect the main conclusions, as we model the engagement over several waves, and the batteries get reloaded after each wave. Additionally, the study does not use actually weapon kill probabilities as that would elevate the classification of the study.

We run the base model, described above, to understand trends in the Blue and Red's behavior. We then run two model extensions, one in which we limit Blue's ammunition to the number of missiles a GBASM platoon would have in reality and one in which we considered a duel against two Red targets.

Several key findings emerge from the research. First, Red's optimal strategy is characterized by a threshold policy. Below some threshold of the number of Blue shooters which Red faces, it is more advantageous to Red to fire at the shooters; beyond that threshold, it is more advantageous to target the Blue depot. This phenomenon was observed in several different iterations of the simulations and is also reinforced by the analytical model. This leads to two important insights for GBASM units: 1) blue shooter survivability increases with the number of shooters deployed and 2) supply depot signature management is crucial, given that it will likely be a tempting target of opportunity for the enemy.

Another key finding of the research concerns the size of Blue's supply depot. We run several simulations while varying the number of missiles available to Blue. We find that after some threshold, the probability of Blue winning remains the same, indicating that there is a right-sized depot depending on how many Blue shooters are present. The insight for GBASM units is then that logistics planners should carefully balance the number of Blue shooters deployed to a site with the efficacy of adding more ammunition that may not be needed.

This research provides GBASM operators and logisticians key insights into battle dynamics that may be applied to the ongoing development of weapon TTPs. It is important to note that the specific numerical findings of the research are heavily dependent on the model assumptions as well as parameter values and should not be considered predictions; instead, the key output of the research are the general trends and insights observed.

Acknowledgments

I am extremely grateful to my friends and family for their support throughout this course of study. They all know by now my tendency to put far too much pressure on myself, especially academically, and I may have crumbled under the self-imposed pressure if not for their encouraging words and unwavering support.

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I want to thank each and every professor, as well, for their support throughout the program; the OR Department is filled with world-class academics who could have worked in any university or company of their choosing, but who remain dedicated to the education of military servicemen and women and advancing national security research.

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CHAPTER 1:
Introduction

1.1 Purpose

The purpose of this thesis is to develop stochastic models to represent the tactical-level operation and logistics of the Marine Corps' nascent weapon system, the Ground-based Anti-ship Missile (GBASM), in order to gain insights that can inform live-force experimentation and tactics development. We develop both analytical models as well as Monte Carlo simulations which represent an artillery duel between GBASM launchers and enemy maritime targets. Our research builds on previous research done on this topic by adding a supply depot in support of the GBASM, to gain insight into how battle dynamics change when logistics factors are considered.

1.2 Background

1.2.1 Operational Context

The Navy and Marine Corps have developed new operational concepts with an eye towards a potential near-peer conflict with China, namely Distributed Maritime Operations (DMO) and Expeditionary Advanced Base Operations (EABO). The DMO concept calls for naval forces which operate in small-sized units in a distributed manner in order to complicate adversarial targeting. The EABO concept complements DMO by focusing on deploying platoon-reinforced units throughout the theater, quickly moving from island to island, performing missions which contribute to U.S. Navy sea denial and sea control efforts in the area of operations (AO). Specifically, the Commandant of the Marine Corps has called for the Marine Corps to provide ground-based, long-range, anti-surface fires in order to "attack the adversary's sea lines of communication while defending our own" (Berger (2019)).

The Marine Corps has fielded a new weapon system to fulfill the need for a GBASM capability to engage maritime targets in support of a naval campaign. The material solution developed for this capability is the Navy-Marine Corps Expeditionary Ship Interdiction

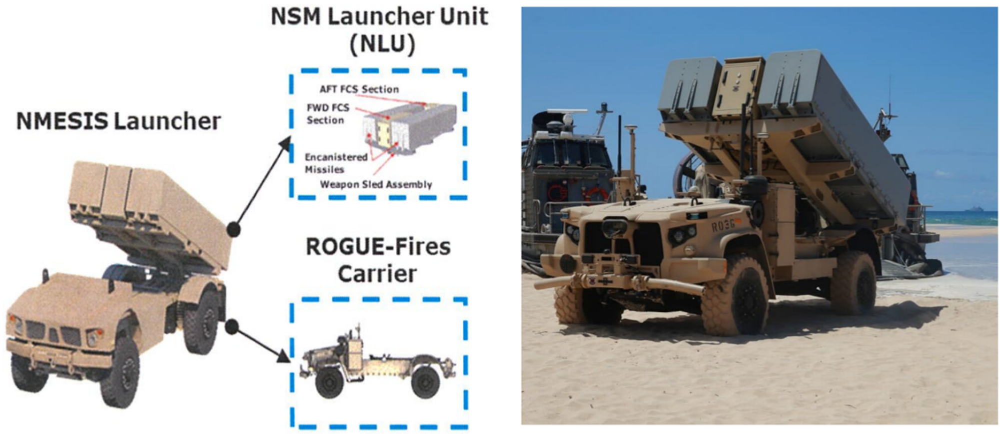

Figure 1.1. Source: Navy-Marine Corps Expeditionary Ship Interdiction System Concept of Employment, Headquarters, U.S. Marine Corps, Combat Development and Integration, dated August 5, 2021. GBASM system components. The term "GBASM" in this thesis will refer to a single missile launcher, as shown on the right.

System (NMESIS), which consists of the Remotely Operated Ground Unit Expeditionary Fires (ROGUE Fires) unit which is an unmanned variant of the Joint Light Tactical Vehicle (JLTV) equipped with a naval strike missile (NSM). For the purposes of this thesis, we refer to the weapon system as the GBASM, which will represent, specifically, one missile launcher.

1.2.2 Employment and Logistics

The GBASM will be operated by the Marine Corps' nascent Marine Littoral Regiment (MLR). The MLR can deploy a GBASM unit to an expeditionary advanced base (EAB), where launchers are emplaced in forward firing positions, with an assembly area, command and control elements, and a resupply point in the rear. The MLR has an organic Littoral Logistics Battalion which provide logistical support to the EABs, managing cache sites and connecting to higher headquarters logistics support. As this is a new unit supporting a new capability, there are no established tactical level logistics tactics, techniques, and procedures (TTPs); the unit, which is still in its initial operational capability phase, is currently experimenting in training exercises in order to develop these TTPs. This study will aid in providing insights that can inform the development of these TTPs.

1.3 Research Questions

This thesis will refer to a GBASM launcher as a "shooter." Friendly forces are referred to as "Blue" forces and adversary maritime targets as "Red" forces. This study aims to answers the following research questions:

1. What is the optimal strategy for Blue to defeat a single Red target? Two Red targets?
2. What is the optimal strategy for Red to defeat Blue? Should Red target the Blue shooters or the depot?
3. What is the optimal size of Blue's supply depot?

1.4 Scope and Methodology

This thesis examines the tactical level operation and logistics of the GBASM, focusing on platoon-sized elements operating at an EAB with a single supply depot. These elements will engage in artillery duels with either one or two adversary maritime targets. These duels will be modeled using a discrete time Markov chain (DTMC), an analytical model, and Monte Carlo simulation. The DTMC is used to confirm the results of the Monte Carlo simulation; the Monte Carlo simulation and analytical model complement each other as two approaches to solve the problem. The main focus of the analysis is exploring the firing dynamics between Blue launchers, a Blue depot, and Red targets using the Monte Carlo simulation and conducting sensitivity analyses to gain insights into how model parameters effect model outcomes. The primary measure of effectiveness (MOE) is probability of Blue win; in some cases, the complementary probability of Red win is used to compare results.

1.5 Organization of the Study

Chapter 2 is a literature review of previous operations research work done on combat models, salvo models, and operational level logistics.

Chapter 3 summarizes the models exploRed, parameters, and assumptions.

Chapter 4 discusses research results.

Chapter 5 provides a summary and recommendations for future work.

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CHAPTER 2:
Literature Review

The model discussed in this thesis falls under the umbrella of combat models, with a focus on interactions between artillery fire and maritime targets as well logistical resupply of ammunition. This literature review discusses previous work related to study the dynamics of artillery warfare and combat logistical models.

No discussion of combat models is complete without referencing the work done by Lanchester to model warfare. Lanchester (1916) developed simple and intuitive equations by which mathematicians could model warfare; in particular, his linear law equations are used to model artillery duels. Brackney (1959) applied the Lanchester equations to a "mixed" warfare model, in which one force is modeled using aim-fire tactics and the opposing force is modeled as using area-fire tactics. Deitchman (1962) expanded on the mixed warfare model, applying it to guerilla warfare under the premise that aim-fire modeling applied to guerrilla forces ambushing conventional forces, whereas the conventional forces returned fire using area-fire, given the assumption that they only knew the general area where the guerilla forces were located. Kress and Szechtman (2009) extended Deitchman's work to modern guerilla warfare, capturing the effects of "soft" capabilities such as intelligence and populace support in order to shed light on the complicated dynamics of counterinsurgency operations.

The mixed/guerilla warfare model would also be applicable to artillery duels involving land-based artillery firing on a sea-based enemy, where the land-based artillery fire "directly" at Red targets and where Red targets return area-fire, given the assumption that they only have general understanding of where the Blue artillery is located. In fact, Mahon (2007) considered this very idea and subsequently developed the Littoral Combat Model. The Littoral Combat Model applies both the Lanchester linear equation as well as Hughes' salvo model to model interactions between sea-based ships and land-based anti-ship cruise missile batteries. Hughes Jr (1995) developed a deterministic salvo model to model interactions between two sea-based forces in which offensive striking power is degraded by the defensive and staying power of the opposing force; the resultant maxim "fire effectively first" (to have the advantage) continues to be relevant today.

The Hughes model is best used to model attrition at sea, while Lanchester's model applies to attrition on land. Mahon combined attributes of both to develop his Littoral Combat Model, which assumed that land forces (truck-mounted missile launchers akin to the GBASM) with few defensive capabilities were vulnerable to area fire; in contrast, the sea-based force with active defenses vulnerable to the aim-fire of the land-based anti-ship cruise missiles. Mahon showed that 1) "fire effectively first" still applied and 2) the sea-based force can overcome the staying power of a land-based force with direct-fire weapons and effective targeting.

While the aforementioned models all yield important and significant insights into the nature of artillery warfare, all are deterministic and involve no stochasticity. While deterministic models are lauded for their simplicity and interpretability, stochastic models incorporate randomness, allowing for the analysis to incorporate outcome variability and engagement duration. There is a large body of work associated with stochastic duels which model the interaction between multiple agents firing at one another. Early examples which are foundational to this body of work include the work of Ancker, Gaffarian, and Kress. Ancker (1979) developed for the first time the general solution to the one-on-one duel by modeling the interaction as a continuous time Markov chain (CTMC); Gafarian and Ancker (1984) extended this work to the two-on-one duel, providing the state equations, win probabilities, mean value function and variance function. Of interesting note, the authors compared their model solution to the results obtained using Lanchester equations and found neither the Lanchester nor the Stochastic Lanchester results to be satisfactory approximations to their general model. Kress (1987) further extended the model, deriving state equations and win probabilities for a general case of a many-on-one duel; he also examined the relationship between the relative effectiveness of both sides of the duel under certain parameters. Kress (1991) also expanded upon the two-on-one case, including dynamic aspects of the interaction, such as combined arms teamwork, as well as tactical considerations such as when to advance on the enemy.

Several Naval Postgraduate School (NPS) theses develop more recent stochastic duel models. Shim (2017) developed a CTMCs to model artillery warfare in order to study shoot-and-scoot tactics, concluding that moving frequently reduces risk to friendly forces, but also limits friendly forces' ability to inflict damage on the enemy. Lim (2022) incorporates surveillance and counter-battery radar aspects to both DTMC and CTMC artillery duel models, concluding that more frequent scooting enhances the win probability of a force with superior detection capabilities and more lethal weapons. Of note, stochastic analyses of artillery warfare are not limited to anaytical Markovian methods. Kadrmas (2021) used agent-based simulation to examine United States Marine Corps (USMC) artillery warfare in the context of a peer conflict and recommended changes to TTPs to improve mission success probability against a Russian artillery force.

Previous studies relating specifically to the GBASM system are also concentrated at NPS, where students and faculty have used both simulation and Markovian analysis to study various aspects of GBASM employment. Sampson (2021) used agent-based simulation to examine the effect of the GBASM munitions mix and range on survivability and lethality while Moecher (2022) used it to examine the effect of a sea-based guard force on survivability. This thesis is most closely related to Barlow (2022), which developed a DTMC to model duels between the GBASM and an enemy surface ship. Barlow's models are parameterized to examine the effects of varying factors such as Blue or Red lethality, number of Blue and Red platforms in the duel, and number of missiles shot per platform.

This literature review is incomplete without a discussion of combat logistics research. As described in the previous chapter, the two-missile capacity of the GBASM system introduces a unique logistical challenge; the associated support infrastructure and methodology must be agile and adaptive not only to the EABO environment, but also to a system that requires more frequent resupply than conventional artillery. As this study aims to provide insights to the nascent MLR which is currently experimenting with resupply TTPs in training exercises, a review of operations research related to tactical logistics is appropriate. The study of battlefield logistics, however, almost exclusively focuses on the operational and strategic levels of war; the optimization of tactical-level logistics, and in particular, "last-tactical mile" logistics, are mostly limited to best practices and TTPs developed through user-level experimentation in military training. A comprehensive review of operations research in the field of battlefield logistics is beyond the scope of this thesis, but some examples are provided for context. Early examples include Chase (1973) and Taylor (1982), who both adapted the Lanchester equations to include logistics considerations by modeling the attrition coefficients as functions of supply, such as food, ammunition or fuel. By setting these coefficients to zero, they could simulate a force running out of some category of supply; thus, they were able to gain insight into how battlefield logistical decisions affected the outcome of the battle.

More recently, linear programming and queuing models have been used to model strategic and operational logistics problems, such as navy ship resupply. Pilnick (1989) used a queueing model to optimize a replenishment schedule which compared favorably to the more arduously locally-produced schedule. Brown and Carlyle (2008) developed an integer linear program to model the US Navy's Combat Logistics Force (CLF), which was used to examine how CLF composition and employment affected the Navy's ability to resupply its fleets worldwide.

Other examples of logistical models demonstrate how Operations Research has been used to inform nascent operating concepts, particularly in the Marine Corps. Several NPS theses examined the logistics considerations of the Marine Corps' Operational Maneuver from the Sea (OMFTS) concept of the late 1990's; results showed the limitations of this concept from a logistical standpoint. Beddoes (1997) developed an analytical model to determine max standoff distance of sea-based logistical vessels from shore and determined that the fleet at the time was unable to satisfy the demands of the ground force in the given OMFTS scenario. Hagan (1998) also examined sustainment requirements and standoff distances for several landing force scenarios, but focused on air delivery in an OMFTS environment; his analysis showed that logistical requirements outpaced available aircraft in several of the scenarios simulated. Lastly, Frey (2000) used discrete event simulation to analyze sustainment requirements of forces ashore and noted that aircraft attrition made the scenario logistically infeasible.

More recent work at NPS has focused on examining all aspects of the emergent EABO concept, but work focusing on the logistical aspects are currently limited to the operational level. Several theses are addressing the so-called "wicked" problem of satisfying logistical demands at multiple expeditionary locations over extremely long distances under a persistent missile threat. Sentinella (2021) and Mirsch (2022) continued development of a mixed-integer linear program developed by the Marine Corps which optimizes the sea connector fleet and develops sea distribution plans in the EABO environment. Similarly, Johnson (2022) developed a mixed-integer linear program to optimize sustainment deliver in a EABO environment, but introduced stochasticity into the model to account for uncertain demand location and quantity.

One instance of a tactical-level logistics model is found in Lenhardt (2001), who developed a vehicle-routing linear program to evaluate novel concepts involving the task organization of Marine Corps tactical-level logistics units. Using the vehicle-routing model as well as discrete event simulation to introduce stochasticity, he was able to show that the proposed logistics task organization was sufficient to meet the sustainment needs of the supported units.

We did not find any meaningful instance of research conducted on tactical-level logistics using Markovian methods. Additionally, to date, no research has been conducted that incorporates logistics considerations to the operational employment of the GBASM. Modeling these aspects is one of the main contributions of this thesis.

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CHAPTER 3: Models

3.1 Scenario

This thesis focuses on a tactical-level artillery duel between Blue and Red forces. We imagine m Blue shooters from a GBASM battery deployed to an EAB with a supply depot some distance away from the shooters' positions. The shooters come into contact with an enemy maritime target and a salvo of fire begins with Blue firing first. Some number of m shooters engage the Red target; the Red target may engage either the shooters, the depot, or both. Victory conditions may vary by model, but in general they are: 1) Blue destroys the Red target, 2) Red destroys all the Blue shooters, 3) Red destroys the supply depot, or 4) Blue runs out of missiles (sometimes referred to as "Blue winchester").

Various methods are used to model the engagement, including DTMCs, Monte Carlo Simulation, and analytical models. This thesis will explore these models with the aim of gaining insight into how changing model parameters affect the primary MOE, Blue probability of winning.

3.2 Model Parameters and Assumptions

The following parameters apply to each model:

Parameter	Description
m	number of Blue shooters deployed to an EAB
α	single-shot kill probability of a Blue shooter against a Red target
β	single-shot kill probability of a Red shooter against a Blue shooter
γ	single-shot kill probability of a Red shooter against a Blue supply depot
d	Red salvo size against the shooters
k	Red salvo size against the depot

Table 3.1. Model parameters.

The following assumptions apply to each model:

- We assume a full combat environment (vice competitive or "gray zone" environment) in which the rules of engagement allow for GBASM engagement with enemy maritime targets.
- All shots fired by Blue or Red are independent.
- All Blue shooters are homogeneous and have the exact same capabilities and vulnerabilities (training, leadership, maintenance, etc.).
- All Red shooters are homogeneous and have the exact same capabilities and vulnerabilities (training, leadership, maintenance, etc.).
- Time and distance are not considered. This leads to the following additional assumptions: all enemy targets are within range of the Blue shooters, all Blue shooters and the Blue depot are within range of the Red target, and Blue shooters are instantaneously resupplied (they do not need to traverse to the supply point in order to reload). We believe this assumption does not affect the main conclusions, as we model the engagement over several waves, and the batteries get reloaded after each wave.
- Blue fires first on each engagement. This assumption is derived from the GBASM concept of employment, which specifies that launchers are thoroughly concealed when emplaced and only reveal themselves upon firing at enemy targets.
- Red has an unlimited supply of missiles. In some simulations, Blue is also assumed to have an infinite depot; this will be specifically stated in the model explanation.
- Red fires both at the Blue shooters and at the Blue depot on each salvo, unless otherwise stated.
- Red does not have battle damage assessment (BDA) capabilities. No matter how many Blue shooters are alive, Red will continue to fire d sized salvos at the shooters.
- Red wins an engagement if Red destroys the Blue depot.
- All Blue shooters fire on each engagement; tactics such as firing by section are not considered.
- While not explicitly modeled, we assume that it will take multiple hits to destroy a Red target. This is reflected in a low value of the α parameter used.
- We assume 100% annihilation as the victory condition for both Blue and Red; that is, neither will disengage from the fight until either side is completely defeated.

3.3 Methodology

We consider three methodologies in our analysis: 1) Monte Carlo Simulation, 2) DTMC modeling, and 3) an analytic model. Monte Carlo simulation and DTMCs complement each other well, but have their respective advantages and disadvantages. Monte Carlo simulation lends itself well to quickly changing battle assumptions, allowing for analyses of different aspects of battle dynamics, as well as changing basic parameters, which is conducive to conducting sensitivity analyses. However, Monte Carlo simulations are more computationally costly and slower than DTMC; additionally, the results are considered approximations. DTMCs, in contrast, are not computationally costly, extremely fast to compute, and produce exact results; however, changing model assumptions and parameters requires new transition probability matrices for each change, making them more cumbersome for this type of analysis. Monte Carlo simulation was the most appropriate for our use, given the different types of scenarios that we explored which have different underlying assumptions and our emphasis on sensitivity analyses. A single DTMC is presented for the base model in order to verify the results of our base simulation. Lastly, an analytical model is presented which complements and reinforces the findings of the simulation.

3.4 Models

3.4.1 Discrete Time Markov Chain Model (DTMC)

We first consider a DTMC model of a battle between m Blue shooters, supported by a supply depot, and a single Red target. Blue fires first; if Blue destroys Red, Blue wins and the engagement is over. If Blue does not destroy Red, Red may then fire back, firing at the shooters and the supply depot. If Red destroys the depot, Red wins and the engagement is over. If Red destroys a shooter, the engagement continues unless that shooter was the last remaining Blue shooter. If not, another salvo begins and Blue fires back. The engagement continues until one of three victory conditions are met: 1) Blue destroys the Red target, 2) Red destroys all the Blue shooters, or 3) Red destroys the supply depot. Of note, in this model, Blue has an infinite depot.

Two additional parameters, d and k, are introduced, denoting the salvo size Red fires against the shooters and against the depot, respectively. Of note, for k, we assume that if Red lands two hits, for example, Red destroys two different Blue shooters.

Model State Space

This model has state space $X = \{x, B, R\}$, where $x = \{m, m-1, ...1\}$ is the number of Blue shooters alive, B indicates that Blue has won (that is, the Red target is destroyed) and R indicates that Red has won (that is, all Blue shooters destroyed or the depot has been destroyed).

For example, if $m = 3$, the associated state space is:

- *3* - three Blue shooters alive, supply depot intact, Red target alive
- *2* - two Blue shooters alive, supply depot intact, Red target alive
- *1* - one Blue shooter alive, supply depot intact, Red target alive
- B - Blue Victory (i.e., Red target destroyed) - *absorbing state*
- R - Red Victory (i.e., either all Blue shooters have been destroyed or the supply depot has been destroyed) - *absorbing state*

Transition Probability Matrix ($m = 3$)

The transition probability matrix will necessarily change for each starting m number of Blue shooters involved in an engagement. An example transitional probability matrix for is:

$$\begin{array}{c} \\ 3 \\ 2 \\ 1 \\ B \\ R \end{array} \begin{array}{c} \begin{array}{ccccc} 3 & 2 & 1 & B & R \end{array} \\ \begin{bmatrix} (1-\gamma)^k(1-\alpha)^3(1-\beta)^d & (1-\gamma)^k(1-\alpha)^3 d\beta(1-\beta)^{d-1} & (1-\gamma)^k(1-\alpha)^3\binom{d}{2}\beta^2(1-\beta)^{d-2} & 1-(1-\alpha)^3 & 1\text{-}\sum \\ 0 & (1-\gamma)^k(1-\alpha)^2(1-\beta)^d & (1-\gamma)^k(1-\alpha)^2 d\beta(1-\beta)^{d-1} & 1-(1-\alpha)^2 & 1\text{-}\sum \\ 0 & 0 & (1-\gamma)^k(1-\alpha)(1+\beta)^d & 1-(1-\alpha) & 1\text{-}\sum \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad (3.1)$$

where $1 - \sum$ refers to the compliment of the sum of all the previous entries of the same row.

3.4.2 Monte Carlo Simulation

Base Model

Various Monte Carlo Simulations were conducted to explore the battle dynamics between Blue shooters and Red targets. As with the DTMC, m Blue shooters, supported by a supply depot some distance away, engage a single Red target (a model extension considers multiple

Red targets). Blue shoots first and if still alive, Red fires back, both at the shooters and the depot. Red fires d-sized salvos against the shooters (regardless of how many Blue shooters are alive) and k against the depot. The engagement continues until one of four victory conditions are met: 1) Blue destroys the Red target, 2) Red destroys all the Blue shooters, 3) Red destroys the supply depot, or 4) Red destroys the shooters and depot simultaneously.

Recall, the model parameters are as follows:

Parameter	**Description**
m	number of Blue shooters deployed to an EAB
α	single-shot kill probability of a Blue shooter against a Red target
β	single-shot kill probability of a Red shooter against a Blue shooter
γ	single-shot kill probability of a Red shooter against a Blue supply depot
d	Red salvo size against the shooters
k	Red salvo size against the depot

Table 3.2. Model parameters.

Each simulation is run for 10,000 iterations in order to produce a sufficiently narrow confidence interval such that a single value for the results is presented. Blue and Red shots are modeled as generated random variables, as shown in Table 3.3. Of note, generating Red and Blue hits using binomial random variates carries with it the additional assumption that if more than one hit lands, different targets are destroyed. For example, if Red lands two hits, we assume that two different Blue shooters are destroyed, viec both hits landing on the same shooter.

Engagements continue via loop until victory conditions are met. Various MOEs such as number of missiles fired and battle outcomes are measured.

Parameter	Random Variate Distribution
Number of Blue hits on Red	$\sim Binomial(x, \alpha)$
Number of Blue shooters hit by Red	$\sim Binomial(d, \beta)$
Depot destroyed	$\sim Binomial(k, \gamma)$

Table 3.3. Random variate generation for Monte Carlo simulations. Recall, x denotes the number of live Blue shooters (out of the m shooters Blue started with) and d denotes the salvo size fired against the shooters and k denotes the salvo size fired against the depot.

We begin with the unlimited ammunition assumption for simplicity; these simulations are fast, straightforward to implement, and provide an approximation to the trends we expect to observe in limited depot simulations. Given the assumption that both Red and Blue have unlimited ammunition, the Blue probability of winning is in fact a joint probability: $P(Blue\ Win\ \&\ x\ missiles\ fired\ by\ Blue)$.

The following is a summary of the base model extensions.

Limited Depot

After exploring the results of the base model, we run two extensions in order to make the simulations more representative of realistic Blue/Red engagements. An inherent assumption of EABO are long, contested supply lines; as such, the assumption of a continuing supply of ammunition is unrealistic to our scenario. To this end, we limit Blue's depot, simulating a finite amount of missiles available at the EAB where the launchers are emplaced. We introduce a new parameter, h, for Blue depot size. Red's depot remains infinite; as such, the results can be interpreted as "worst-case scenario." Given the new scenario, the Blue probability of winning is now a conditional probability; for example, $P_{15}(Blue\ Win)$, which represents the Blue probability of winning given it has $h = 15$ missiles available to it in its depot. We introduce a new victory condition: Red win by "Blue winchester," or Blue running out of ammunition.

Again, various simulations are ran in order to examine the sensitivity of battle outcomes to various parameters.

Multiple Red Targets

Lastly, we run an additional model extension to consider Blue engaging multiple red targets. Each previous model examined the battle dynamics between some number of m Blue shooters and a single Red target. We can imagine a scenario in which Red understands that Blue is trying to control access to a sea line of communication using the GBASM, and would therefore send multiple Red ships in order to increase the probability of one surviving the GBASM barrier. Therefore, we introduce the parameter r for the number of Red targets which much be destroyed by Blue.

Naturally, the introduction of multiple Red targets increases the complexity of the battle by implying tactical decisions that need to be made by both sides. Examples of these decisions include: should Blue engage the Red targets simultaneously or sequentially, how many Blue shooters should engage the Red targets, should unengaged Red targets engage Blue, etc. This study examines the optimal policies of both Blue and Red while scoping the complexity of the battle. Blue may target Red either (1) sequentially, meaning Blue must destroy the Red targets one at a time, and (2) simultaneously, meaning Blue may destroyed multiple Red targets in one salvo. Red may target (1) only the Blue shooters, (2) only the depot, and (3) both the shooters and the depot. We further scope by only considering two Red targets ($r = 2$), as greater numbers of Red targets add an additional level of complexity to the model.

We also experiment with blue salvo size in this model. All other models assume that all live Blue shooters fire on each salvo. While this may increase the probability of win, it can also lead to quickly and unnecessarily depleting the Blue missile depot. In actuality, we can envision a GBASM unit commander coordinating fires in order to use available missiles more efficiently. To this end, we experiment with limiting the number of Blue shooters which can shoot on a given salvo. We assume that Red can see all Blue shooters, whether firing or not, and require that Red destroy all the Blue shooters in order to win. We introduce the parameter j, or blue salvo size, and explore the battle outcomes under these circumstances.

3.4.3 Analytical Model

An analytical model is developed to describe the dynamics between a Red target and Blue shooters with the aim of determining optimal strategies for both. The parameters associated with the analytical model are as follows:

Parameter	Description
X_i	number of Blue shooters alive in period i for $i = 1, 2...$
α	single-shot kill probability of a Blue shooter against a Red target
β	single-shot kill probability of a Red shooter against a Blue target
γ	single-shot kill probability of a Red shooter against a supply depot

The battle dynamics for this analysis are:

- Blue shoots first.
- Red wins if it destroys all of Blue's shooters or its depot.
- All shots are independent and identically distributed (iid).

We begin by comparing two simple policies that Red can take: 1) only fire at the batteries and 2) only fire at the depot. After determining Blue's optimal strategies in response to these two policies, we then consider an extension in which Red can target multiple Blue shooters per round.

Red targets depot only

The number of Blue shooters remains constant, so Blue maximizes its kill probability by setting the number of shooters to $x = X_1$ for all periods. Let

$$c = P(\text{Red survives by only shooting at the depot}).$$

The i.i.d. assumption implies that c equals the probability that Blue doesn't kill Red (i.e., $(1-\alpha)^x$) times the probability that Red destroys the depot (γ) plus the probability that Red doesn't destroy the depot times the probability that Red survives by only shooting at the depot (i.e., $(1-\gamma)c$). Putting everything together results in,

$$c = (1-\alpha)^x(\gamma + (1-\gamma)c). \tag{3.2}$$

Arguments of this kind arise frequently when dealing with models that regenerate. In our case, if each side fails to destroy any of the enemy targets', then the engagement restarts on the same conditions.

Eq. (3.2) is a linear equation in c, leading to,

$$c = \frac{\gamma(1-\alpha)^x}{1-(1-\gamma)(1-\alpha)^x}. \tag{3.3}$$

The interpretation is that Red's survival probability is the odds ratio of Red killing Blue to both sides missing their shots.

Red targets a single shooter per round

From the earlier discussion, we know Blue sets its number of shooters to $x = X_1$. Then, Red's survival probability is,

$$E\left[\exp\left(-\log\left(\frac{1}{1-\alpha}\right)(x \times \tau_1 + (x-1) \times \tau_2 + \cdots + 1 \times \tau_x)\right)\right], \tag{3.4}$$

where $\tau_1, \tau_2, \ldots, \tau_x$ are i.i.d. Geometric(β).

Using the fact that $\tau_1, \tau_2, \ldots, \tau_x$ are i.i.d., we can split the expectation in (3.4) into the product of x expectations, where each expectation follows Eq. (3.3). Therefore, (3.4) equals,

$$\prod_{k=1}^{x} E\left[\exp\left(-\log\left(\frac{1}{1-\alpha}\right) k \times \tau_{x-k+1}\right)\right] = \prod_{k=1}^{x} \frac{\beta}{1-(1-\beta)(1-\alpha)^k}. \tag{3.5}$$

Equation 3.5 shows that, for a given k, $\beta/(1-(1-\beta)(1-\alpha)^k)$ is the odds ratio that Red destroys a Blue shooter versus that both sides remain intact when Blue has k live shooters. Then, (3.5) is the odds ratio that Red destroys all of Blue's x shooters (one at a time), to no side destroying the other when Blue has $k = 1, \ldots, x$ shooters.

Blue optimal strategy

From the analysis above, we can determine Blue's optimal policy when Red can target only one shooter or the depot in each round.

All Blue shooters but one are deemed "safe" in any given round, as Red can only target a single shooter or the depot in that round. Therefore, Blue is better off by activating all its shooters in order to diminish Red's survival probability. Facing this optimal Blue strategy, it then behooves Red to target the depot if the number of Blue shooters exceeds the intersection point of the survival functions. Red will never switch between targeting the shooters or the depot, since either decision remains optimal if it is optimal for the first round. Hence, the only equilibrium point is where Blue activates all its shooters. This is true regardless of whether Red knows Blue's total number of shooters.

Model Extension: Red targets d shooters per round

After arriving at the above analysis, we next explore an extension of the model in which Red can target multiple shooters, denoted by parameter d, in each round.

If Blue starts with x shooters, it will remain there for Geometric $(1-(1-\beta)^d)$ periods; the parameter being one minus the probability that all of Red's shots fail. Red's survival probability is then,

$$\prod_{k=1}^{x} P(\text{getting to state } X=k)\frac{1-(1-\beta)^d}{1-(1-\beta)^d(1-\alpha)^k}.$$

The probability of a sample path getting to state $X=k$ can be computed recursively:

$$P(\text{getting to state } X=k)=\sum_{\ell=1}^{d} P(\text{getting to state } X=k+\ell)\frac{P(\text{Binomial}(d,\beta)=\ell)}{1-P(\text{Binomial}(d,\beta)=0)}, \tag{3.6}$$

for $0 \le k \le x-d$. It's a similar formula for the top boundary states. The probability in the l.h.s. of (3.6) is roughly constant. Figure (insert figure reference) shows the results behavior.

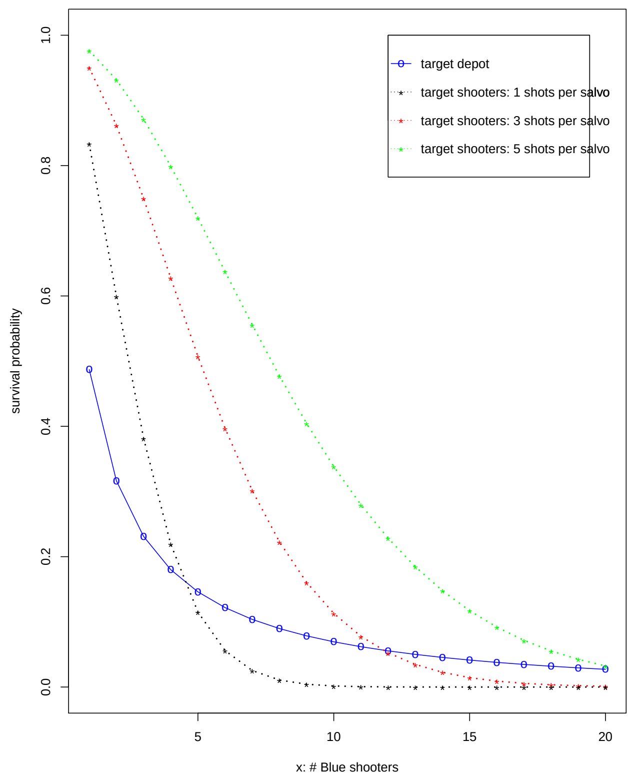

Figure 3.1. Per the analytical model, we conclude that Red's survival probability is optimized by targeting the shooter(s) until some threshold, after which it is more optimal to target the depot. The threshold occurs when the survival functions intersect.

We can conclude that when Red only targets d shooters, the game follows the same structure as when it can target only one shooter.

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CHAPTER 4:
Results

In this chapter, we present the results of the models introduced in Chapter 3. Unless otherwise specified, all models are ran using the following base case parameter values:

Parameter	Description	Value
α	single-shot kill probability of a Blue shooter against a Red target	0.05
β	single-shot kill probability of a Red shooter against a Blue shooter	0.1
γ	single-shot kill probability of a Red shooter against a Blue supply depot	0.04
d	Red salvo size against the shooters	1
k	Red salvo size against the depot	1

Table 4.1. The following parameter values were used to run all models unless otherwise stated.

In Section 4.1, we cover the results of the DTMC. In section 4.2, we cover the results of the Monte Carlo Simulation. We first present the results of the base model with the unlimited ammunition assumption for simplicity in order to explore the battle dynamics and gain insight to the baseline trends. We then incrementally inject more realism into the simulation, first by dropping the unlimited ammunition assumption, then adding a second Red target, and finally by limited Blue's salvo size.

4.1 Discrete Time Markov Chain Results

The nature of our study lends itself well to Monte Carlo simulations, in which individual parameters can be changed easily to conduct sensitivity analyses to battle characteristics. However, it is prudent to verify simulation results by analytical means. To that end, we solve a DTMC using its transition matrix to verify our simulation results. Given the parameter values shown in 4.1, we run both our DTMC model and our initial Monte Carlo simulation model at $m = 4$ and $d = 2$. Both models yield a Blue probability of win of 0.75, confirming the validity of our simulation.

4.2 Monte Carlo Simulation Results

4.2.1 Base Model Simulation Results

We first examine the results of the base model by conducting sensitivity analyses to model parameters.

Effects of Varying Base Model Parameters

We begin by exploring the effect of varying Blue's lethality by changing the α parameter. As expected, Figure 4.1 shows that Blue's probability of win increases as Blue lethality increases. More interestingly, we see a flattening of the curve beyond some threshold of m, the number of Blue shooters, where no matter how many more Blue shooters are present, Red probability of winning remains the same. As concluded by the analytical model discussed in Chapter 3, this is threshold beyond Red is better off targeting the depot instead of the shooters; after this threshold, the probability of Red win converges on its ability to destroy the depot. That is, for large m, the model simplifies to the depot only scenario discussed in the analytic model.

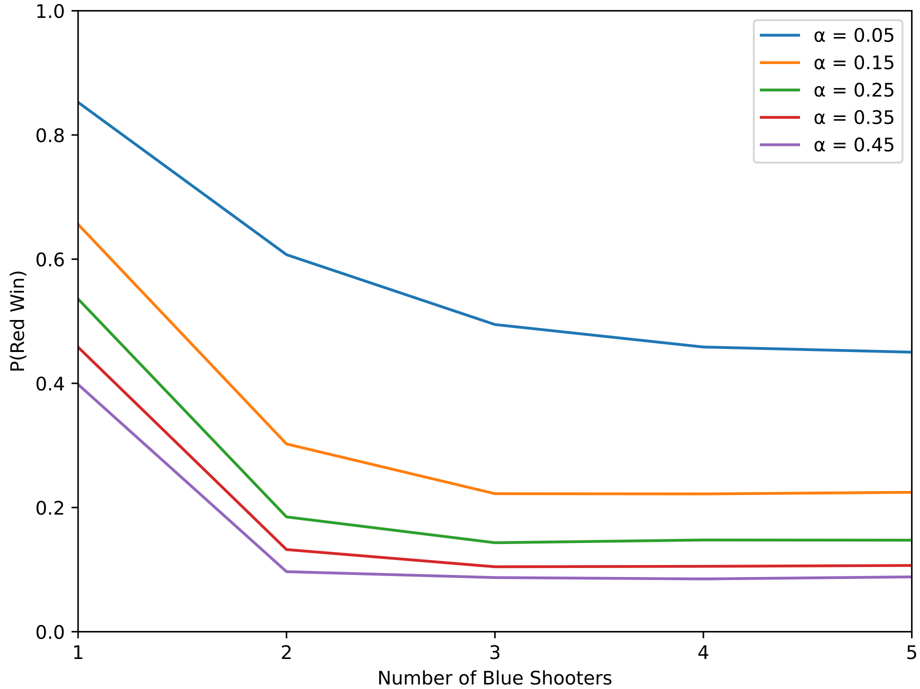


Figure 4.1. As the number of Blue shooters present increases, Red is better off targeting the depot vice the shooters; this results in a flattening of the curve at some threshold of m, beyond which it will destroy the depot prior to destroying all the Blue shooters.

We next examine the effect of varying the γ parameter. Again, Figure 4.2 shows that Red's probability of win increases as its lethality against the depot increases. More interestingly, we again observe an elbow in the curve representing a threshold beyond which the model favors Red targeting the depot vice the shooters. After the threshold, the probability of Red win levels out, indicating that Red will destroy the depot before destroying all the shooters. Before the threshold, probability of win is agnostic to lethality against the depot, as Red will destroy the Blue shooter(s) prior to destroying the depot.

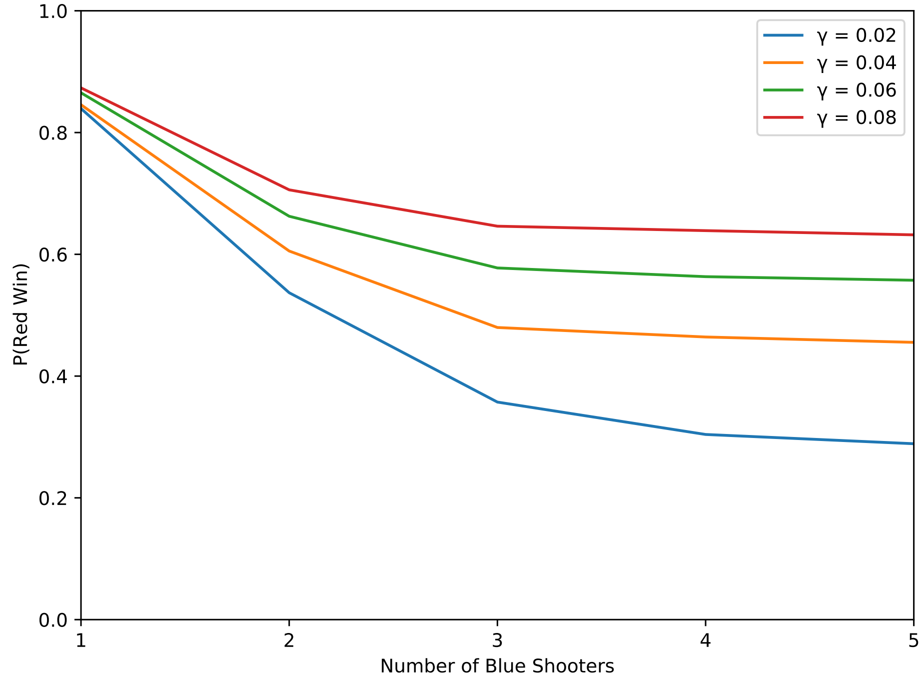


Figure 4.2. As the number of Blue shooters increase, Red lethality against the depot has a greater effect on the battle's outcome as Red is better off destroying the depot vice the shooters in order to win.

Given that an adversary may shoot more than a single missile in a salvo, we next explored the effect of d, Red salvo size against Blue's shooters, on the outcome of the battle. Figure 4.3 shows that for low numbers of Blue shooters, probability of Red win unsurprisingly increases with increased salvo size against the shooters. The probability of Red win levels off at $m = 5$, however, the threshold beyond which it is more advantageous for Red to target the depot and the outcome becomes agnostic to salvo size fired against the shooters.

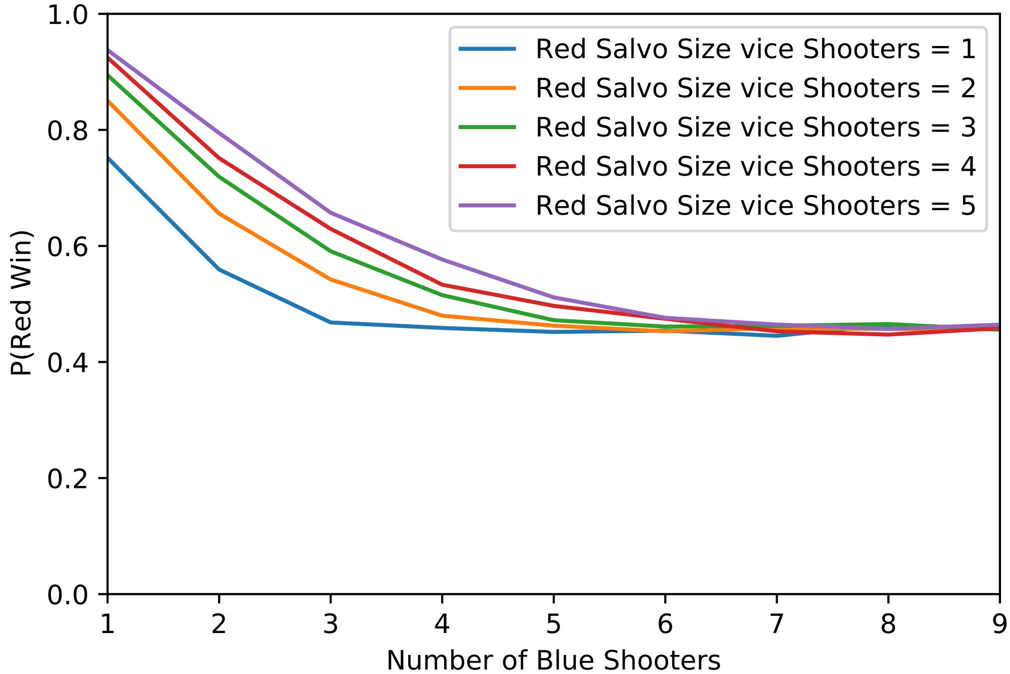


Figure 4.3. As the number of Blue shooters increases, the outcome of the battle is not effected by salvo size targeting the shooters as it is more advantageous for Red to target the depot instead.

Over all, in each case, Red probability of win remains constant after some threshold m, indicating that the model favors Red targeting the depot vice the shooters after said threshold.

Red Salvo Size

Next, we explore the effect of salvo size on probability of Red win. By varying parameters d, salvo size fired at Blue shooters, and k, salvo size fired at the Blue depot, we observe clear efficient frontiers in the resultant contour plots. Figure 4.4a, for example, shows the relationship between d and k when Red engages a single Blue shooter ($m = 1$); given the starting parameters, Red maximizes its probability of win by firing 7-missile salvos at the shooter and it is slightly more efficient for Red to target the shooter. Beyond ($m = 1$), it becomes more and more efficient for Red to target the depot vice the shooters. For large m, Red's win probability is agnostic of how many missiles it uses to target the shooters and only depends on how many missiles are fired at the depot. This further reinforces the notion that when Red faces a higher number of shooters, it achieves the highest win probability by targeting the depot instead of the shooters.

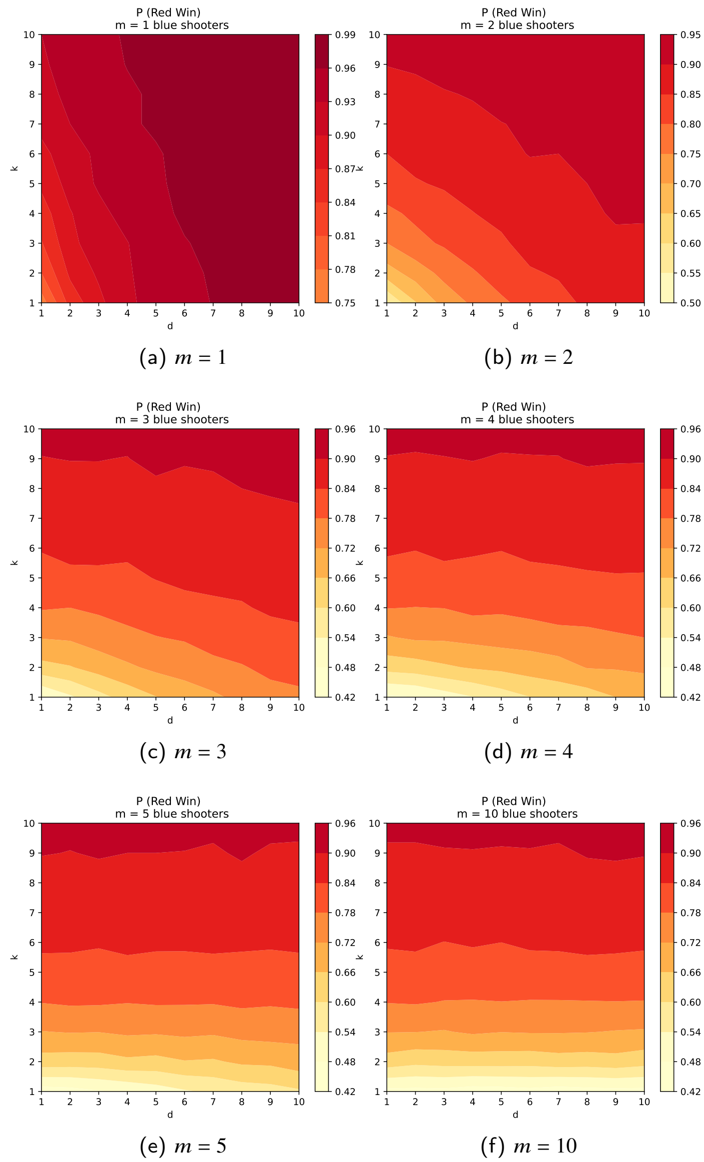


Figure 4.4. Contour plots reflecting Red probability of winning as a function of d, salvo size fired at Blue shooters and k, salvo size fired at the depot. We observe clear efficient frontiers in each case. At low values of m, it is more efficient for Red to fired at the shooters; at high values of m, the outcome of the battle is agnostics to the salvo size fired against the shooters and it is most efficient for Red to fire at the depot. Starting parameters: $\alpha = 0.05$, $\beta = 0.1$, and $\gamma = 0.04$.

Blue Number of Missiles fired

We next explore the battle dynamics through the lens of the number of missiles shot by the Blue shooter(s). Recall that in these simulations, Blue's depot is infinite; by plotting histograms of battle results according to number of missiles fired, we can begin to understand the effect of the number of missiles available to Blue on the battle outcome. Figure 4.5 shows histograms of battle outcomes for $m = 1, 2, 3,$ and 4 Blue shooters against a single Red target; results are binned based on number of missiles fired by Blue during the engagement, regardless of outcome. Each possible outcome is reflected in the histogram: 1) Blue wins, 2) Red wins by destroying all Blue shooters, 3) Red wins by destroying Blue's depot, and 4) Red wins by destroying all shooters and the depot simultaneously. We observe the maximum number of missiles Blue fires in order to defeat Red in the tails of each histogram; this implies the possibility that there is a "right-sized" depot according to how many Blue shooters are present. In the case of one Blue shooter, for example, Blue only ever fires 13 or fewer missiles. This idea is explored further in the next model, in which we limit Blue's depot size.

Of note, in Figure 4.5b-d, we see incremental dives in Blue win's probability. These dives reflect battles in which Red destroys one of the say, two shooters, and therefore, only one Blue shooter was left to fire (thus, the dives occur at multiples of the available number of shooters). As expected, Blue win probability decreases dramatically when it initially loses one or more shooters.

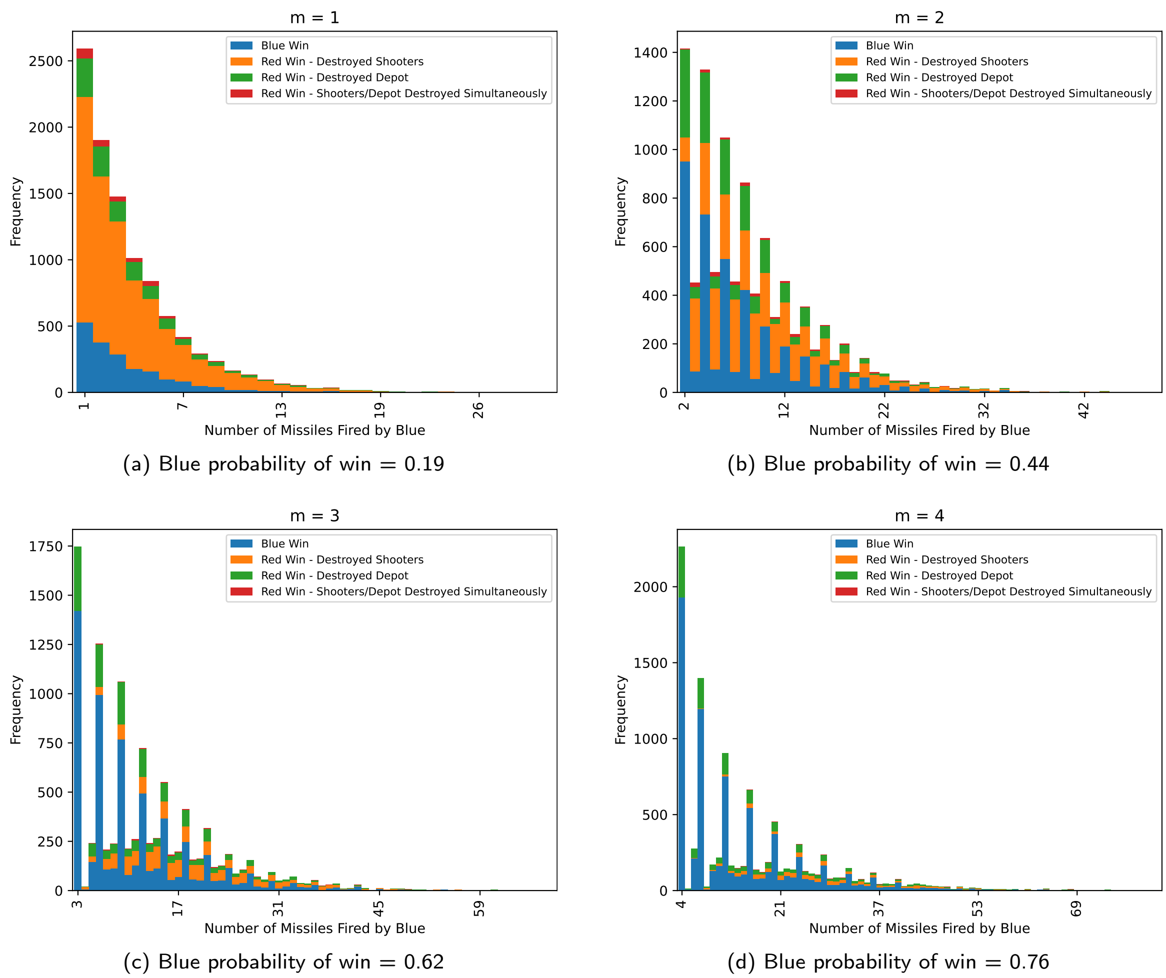
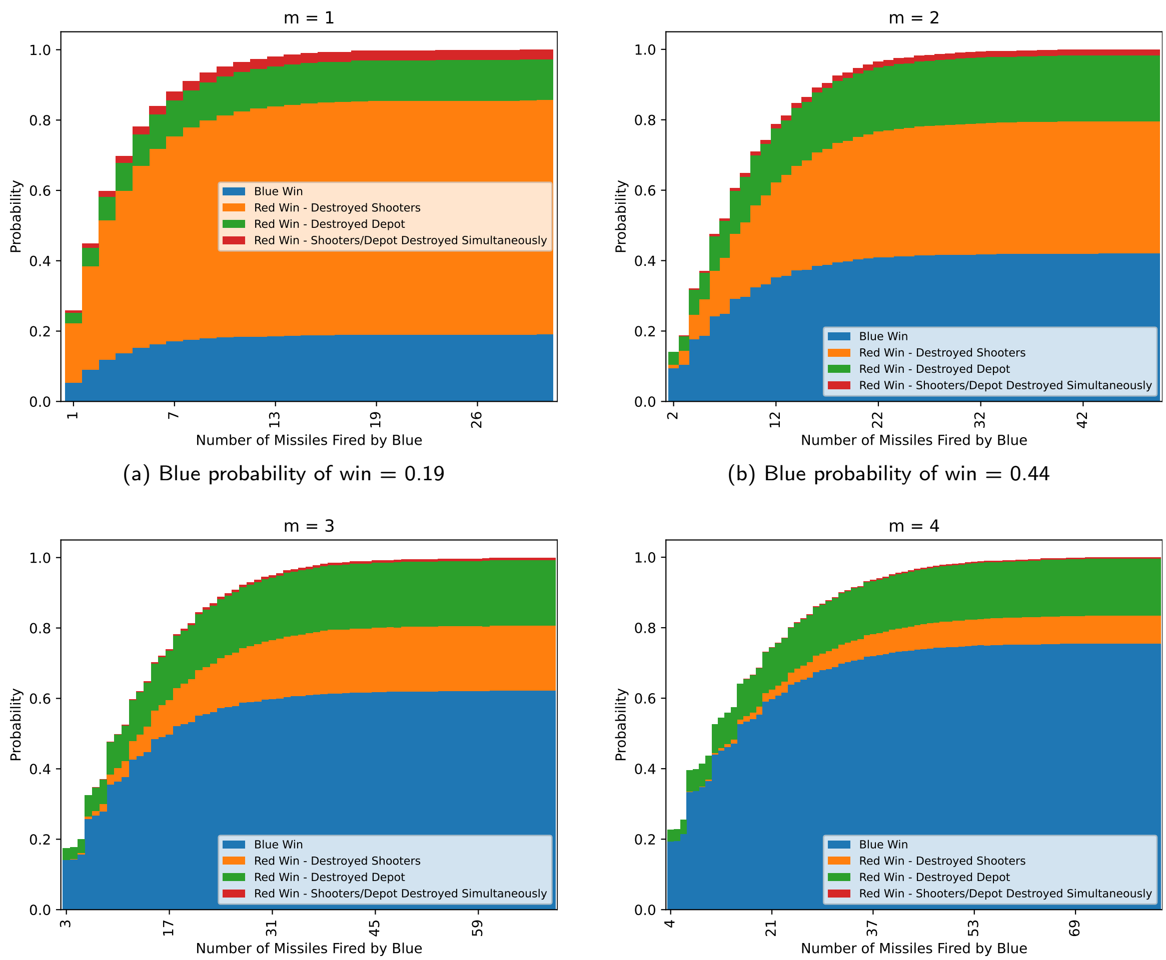


Figure 4.5. Outcome of battles based on number of missiles fired by Blue. Incremental dives in Blue probability of win reflect battles when Red destroys one or more Blue targets during the engagement.

By taking the cumulative sum of these histograms, we form cumulative distribution functions (CDF) to describe the outcome of a battle, given some number of missiles shot by Blue. In each case, we see a flattening of the curve such that providing more missiles to Blue's depot for resupply will not change the outcome of the battle, further reinforcing the "right sized" depot concept. For example, in Figure 4.6a, Blue's win probability increases until it reaches seven missiles, after which its win percentage does not change. We expect this flattening of the CDF plots to hold in generality, for other baseline parameter configurations.



(a) Blue probability of win = 0.19

(b) Blue probability of win = 0.44

(c) Blue probability of win = 0.62

(d) Blue probability of win = 0.76

Figure 4.6. Outcome of battles based on number of missiles fired by Blue. In each case, we observe a flattening of the curve such that adding additional missiles to the Blue depot will not change the outcome of the battle.

4.2.2 Limited Depot Simulation Results

While in previous simulations, both Blue and Red have an infinite number of missiles in their depot with which to defeat the adversary, in this model, we limit the Blue depot to size h. Sensitivity to the model parameters is explored.

We first explore the relationship between depot size and m, the number of Blue shooters present. In Figure 4.7, we see an increase in Blue win probability with a larger depot until some threshold for each m value, when there are no longer enough shooters to take advantage of the larger depot. For a single Blue shooter ($m = 1$), for example, we see no advantage to providing more than 20 missiles to the depot; the battle will win end prior to Blue being able to fire more than 20 missiles. This against reinforces the idea that there is a right-size depot that is dependent on the number of shooters deployed to an EAB, such that providing more missiles will not affect the battle outcome. These results can be used to plan for the size of a Blue depot; logistics planners should carefully balance the number of shooters deployed to an EAB with the efficacy of adding more ammunition that may not be needed when planning for ammunition posturing. It is important to recall that simulation results are very much dependent on the underlying assumptions of the model, the number of Red targets the shooters are engaged with (here, a single Red target), and the parameter values used in the simulation; therefore, the optimum depot size shown in the figures should not be interpreted as specific recommendations for logistics planners, but as insights to be gleaned from.

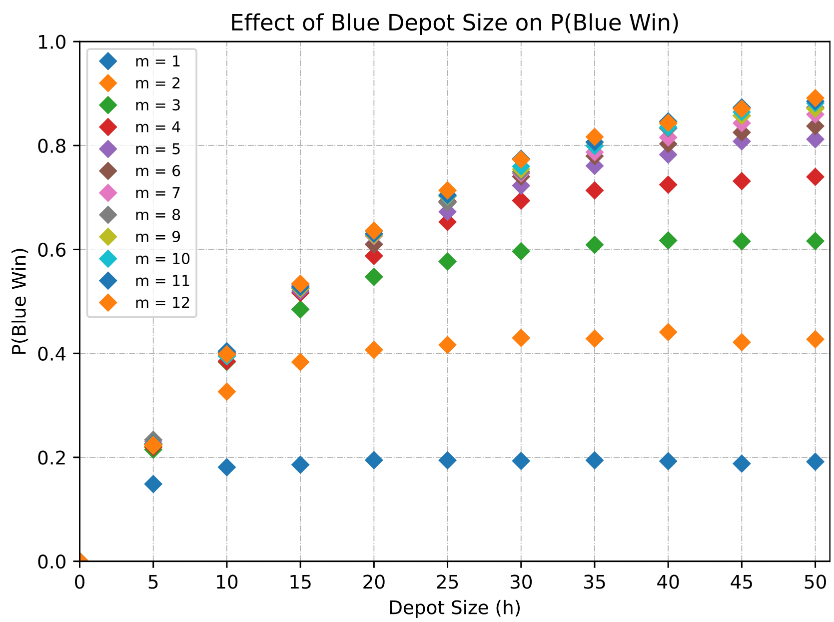


Figure 4.7. A larger depot increases the probability of Blue winning up to a certain threshold. Results are shown for varying values of the number of Blue shooters, m.

We next explore the relationship between depot size and α, blue probability of kill. Of note, these figures were simulated for three Blue shooters (a GBASM section). Figure 4.8 shows a steady increase in Blue probability of win as depot size increases to a point. This effect is lessened, however, for higher values at α, indicating that depot size is less important when Blue is much more lethal than Red, as it will take fewer and fewer missiles to defeat Red.

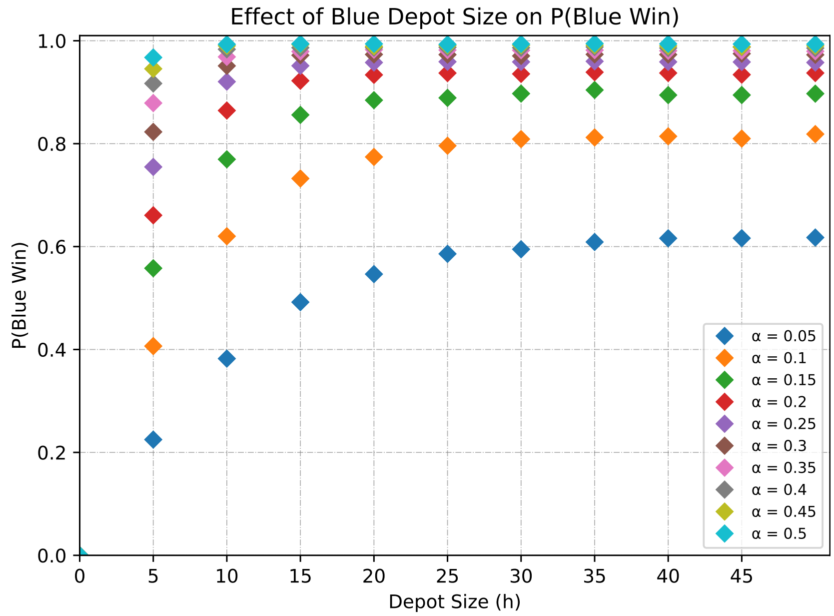
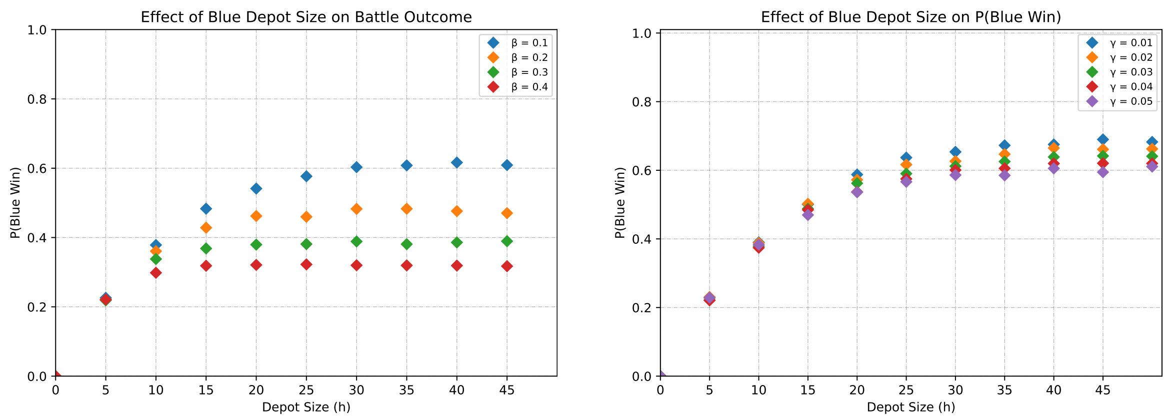


Figure 4.8. A larger depot increases the probability of Blue winning up to a certain threshold; this effect is lessened, however, when α increases. That is, as Blue becomes more lethal, the depot size becomes less important. Results are shown for $m = 3$, representing a GBASM section.

Lastly, we explore the effect of depot size by red probabilities of kill, β and γ. As seen in Figure 4.9, the concept of the right-sized depot is reinforced; that is, after a certain threshold of h, blue probability of win does not change.



(a) (b)

Figure 4.9: Depot size has little effect on battle outcomes when Red probabilities of kill are varied as depot size does not affect Red's ability to destroy blue. Results are shown for $m = 3$, representing a GBASM section.

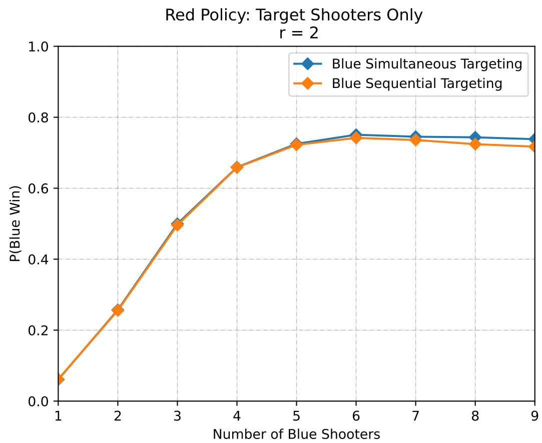
4.2.3 Two Red Targets Simulation Results

Recall that in all previous simulations, m Blue shooters engage a single Red target. In this section, we examine battle dynamics between m Blue shooters and multiple Red targets, denoted by the parameter r. For this study, we limit the number of Red shooters to two as greater numbers of Red targets increase the complexity of the simulation and the operational assumptions that need to be made. We limit the depot size to 54, the number of missiles available to a single GBASM platoon.

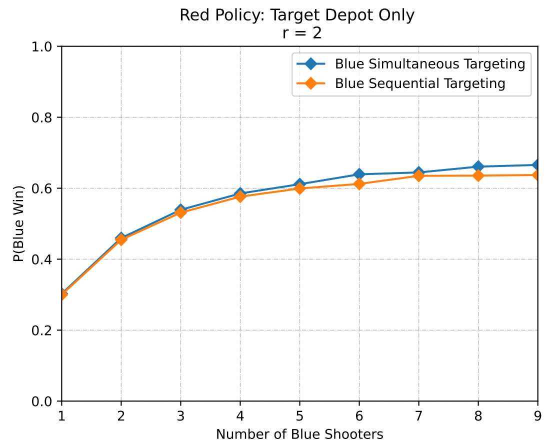
As previously discussed, the introduction of multiple red targets complicates the model as it raises tactical questions such as should the red targets be targets sequentially or simultaneously, how many Blue shooters should engage the Red targets, should unengaged Red targets engage Blue, etc. We scope the study to consider the difference between Blue targeting policies of engaging the Red target sequentially versus simultaneously. We run the simulation against three Red targeting policies: 1) target only the shooters, 2) target only the depot, and 3) targets both the depot and shooters. Lastly, we briefly explore the effects of limiting the blue salvo size.

Figure 4.10 shows simulation results exploring the blue targeting policy for $r = 2$. Each separate figure represents a different Red targeting policy; the lines on the figures represent Blue targeting policy. Most notably, we observe a negligible difference between the two Blue policies. A difference may emerge when Blue faces greater numbers of Red targets which is an opportunity for future study. We do see a slight advantage to simultaneous targeting when Blue greatly outnumbers Red (that is, high m) which matches our intuition. Of note, in Figure 4.10a, we observe a slight dip in Blue probability of win as m increases. This can be attributed to the fact that every Blue shooter fires a missile on each salvo in our simulation; this dip represents Blue running out of missiles prior to being able to defeat both Red shooters.

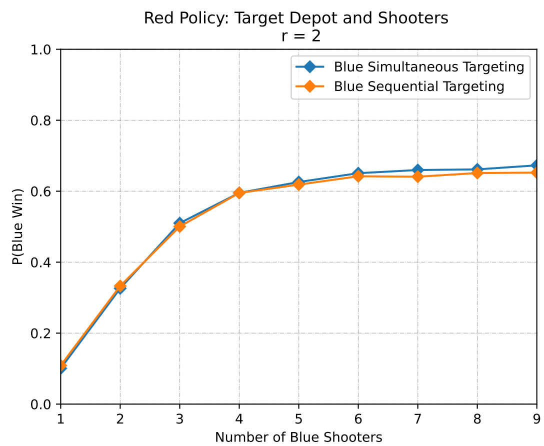
From Red's point of view, we observe the same trend to previous simulations in which beyond some threshold of m, it is more advantageous to target the depot instead of the shooters. Blue's probability of win is minimized when Red targets the shooters, but only until $m = 3$; beyond that, Blue's probability of win is lowest when Red targets the depot.



(a)



(b)

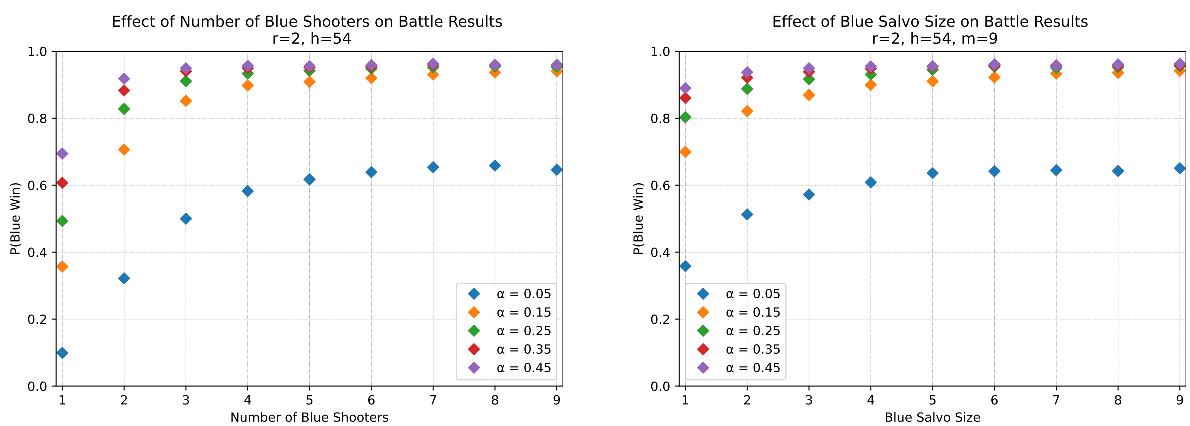


(c)

Figure 4.10. When Blue faces two Red targets, it must make a tactical decision to attempt to defeat them simultaneously or sequentially. Here, we observe that there is a negligible difference between Blue shooting strategies at low m; at high m, we see a slight advantage to the simultaneous targeting strategy.

Finally, we briefly explore the effect of blue salvo size (j) on battle outcomes against two red units which are targeted sequentially. Recall that in previous simulations, each Blue shooter fires on each salvo; therefore, a large Blue unit depletes its depot quickly, and ostensibly, unnecessarily because more than one shooter may succeed in destroying the red unit. We explore this dynamic by limiting the number of Blue shooters that fire on a given salvo. For example, if $j = 3$ and $m = 9$, only three of nine shooters fire on any given salvo. In order to win, Red must still defeat all nine shooters (or the depot). We hope to find the optimum number of shooters to fire against two red targets. We run the simulation using a blue sequential targeting policy and a red targeting policy of firing against both the depot and shooters. For comparison, we present the same simulation where all live Blue shooters available fire on every salvo.

As seen in Figure 4.11, we only see a difference in battle outcome at low values of m. When only j of m shooters fire (Figure 4.11b), we see an increase in probability of Blue winning when compared to Figure 4.11b at low values of m. This is because at $m = 1$, for example, Red only has to defeat a single Blue shooter in order to win; therefore, the probability of Blue winning is low (Figure 4.11a). However, at $j = 1$, only one Blue shooter is firing at each salvo, but the victory conditions require that Red defeat all nine shooters to win; so probability of Blue winning is higher. At high values of m and j, the battle outcome is identical; beyond a certain threshold, the simulation is leveling off to where the battle results are only sensitive to Red's ability to defeat the depot, which will always occur prior to it being able to destroy all nine shooters, given the parameters used. This reinforces the same trend observed throughout this study in which beyond a certain threshold of m Blue shooters, the optimal strategy for Red is to target the depot.



(a) All m shooters fire on each salvo.

(b) J of m shooters fire on each salvo.

Figure 4.11. Probability of Blue winning is higher at $j = 1$ vice $m = 1$ because Red must defeat nine Blue shooters to win vice just one; however, battle results are identical after $m = 5$ and $j = 5$, where both simulations favor Red winning by destroying the depot.

The main takeaway of this batch of simulation is that Blue should be judicious in choosing the salvo size when the Red units are targeted sequentially. We believe this issue will remain valid for more than two Red targets.

CHAPTER 5:
Conclusions

The goal of this research is to gain insights into tactics and policies related to the operation and logistics of the GBASM that maximize probably of win against adversary maritime targets. To this end, we formulate analytical models and run various Monte Carlo simulations that allow us to examine the effect of various parameters on battle outcomes. This chapter provides a summary of the results, explains the limitations of the work, and discusses potential future work.

5.1 Results

The first set of results concern the engagement of Blue shooters against a single Red target. Against a single Red target, we observe a common trend across several instances of the simulation (sensitivity of changing Blue probability of kill, Red probability of kill, and salvo size): beyond a certain threshold of the number of Blue shooters, it is more advantageous for Red to attack the ammunition depot vice the Blue shooters. That is, until the threshold is reached, Red probability of win is maximized by targeting the shooters; beyond it, Red probability of win is maximized by only targeting the depot. Notably, the analytical model reached the same conclusion. This suggests two important insights: 1) Blue shooter survivability increases with the number of shooters deployed, and 2) supply depot signature management is crucial as it is a tempting target of opportunity for Red.

We next examine the effect of depot size on battle outcomes. We observe that Blue probability of win levels off after some threshold of missiles available in the depot, depending on how many shooters are engaged. This suggests that there is a right-sized depot according to how many launchers are deployed to an EAB. We provide figures that can serve as planning aids, but planners should keep in mind that they are sensitive to model parameters and assumptions.

We briefly examine battle outcomes against two Red targets vice one. Engaging multiple targets raises questions of tactical decisions, which complicates simulation. We scope our examination to study of whether or not Blue should engage the Red targets simultaneously or sequentially. Against two Red targets, we do not observe a significant difference between the Blue targeting policies explored.

5.2 Limitations

Readers of this work should keep in mind several limitations of the study prior to making tactical and operational decisions based on the results. First, model parameters do not reflect specific knowledge of the weapon system or adversary capabilities. The results are sensitive to these parameters and should not be interpreted as predictions but instead as general insights on how the weapon or enemy may behave in a combat situation. Additionally, the model does not reflect a GBASM unit working in sections and coordinating fires. In actuality, a unit is expected to communicate and coordinate fires in an engagement to maximize effectiveness; therefore, the simulation is a reflection of the lower bound for combat performance and the results may be considered a conservative estimate of the probability of win for Blue. The model does not consider time or distance; all targets are assumed to be within range of the weapon system at some arbitrary distance away. Additionally, the distance from launcher to the depot is not considered and immediate weapon reload is assumed. Lastly, the model does not consider intangibles such as the training, leadership, or command and control of the sections or the platoon.

5.3 Future Work

The limitations listed above are ripe for potential follow on work to this thesis. First, the results could be refined with specific knowledge of the weapon system and adversary capabilities, though this would most certainly raise the classification of the work. Similarly, the results could be further refined by incorporating time and distance factors, such as accounting for weapon travel time to the supply depot for reload or varying distances to the Red targets. There are many opportunities for expanding the multiple red targets model and determining blue's optimal policy when facing more than one target. Additionally, one could consider how either side or both having BDA capabilities affects the results. Further work could also consider the effects of different tactics, such a scoot-and-shoot for the Blue shooters or rapid relocation of the supply depot.

Lastly, this work solely focuses on tactical-level employment and logistics of the GBASM weapon system, in a full combat scenario. Previous operations research work on the weapon system likewise solely focuses on tactical-level employment in a full combat scenario. Given our current posture in the competition zone vice conflict zone, future work could explore the operational effects of the weapon system on sea control and sea denial. Additionally, future GBASM study could consider the effect of contested logistics and in particular, the country's limited NSM stockpile, on weapon system deployment. Further study on how many missiles will be needed in various scenarios can inform ongoing efforts to determine the required stockpile size in a near-peer conflict.

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APPENDIX: Monte Carlo Simulation Code

The following can be considered pseudo code used for the base Monte Carlo Simulations.

A.1 Infinite Depot Simulation

```
blue_wins = 0
game_over = False
while game_over == False:
    a = ss.binom.rvs(m,alpha)
    b = ss.binom.rvs(d,beta)
    c = ss.uniform.rvs()
    if a > 0:
        blue_wins += 1
        game_over = True
    else:
        m -= b
        if (m <= 0 or c < gamma):
            game_over = True
```

A.2 Limited Depot Simulation

```
blue_wins = 0
missiles_available = h
game_over = False
while game_over == False:
    if missiles_available == 0:
        break
    a = ss.binom.rvs(min(m, missiles_available),alpha)
    b = ss.binom.rvs(d,beta)
    c = ss.uniform.rvs()
    missiles_available -= min(live_blue, missiles_available)
    if a > 0:
        blue_wins += 1
```

```
    game_over = True
else:
    m -= b
    if (m <= 0 or c < gamma):
        game_over = True
```

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