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STOCHASTIC DUELS FOR EVALUATING GROUND-BASED ANTI-SHIP SYSTEMS

by

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The Ground-Based Anti-Ship Missile (GBASM) is a critical capability at the center of the Marine Corps' concept of Expeditionary Advanced Base Operations and Force Design 2030 initiatives. This research formulates stochastic models for the evaluation of the efficiency and effectiveness of varying GBASM battery configurations in the context of a duel with a surface ship. The models produced are discrete-time Markov chains that model duels between a Blue GBASM battery and a Red surface ship. The models are parameterized to account for varying factors including lethality of Blue and Red, and evaluate salvos based on the number of GBASM delivery platforms and the number of missiles shot per platform. The primary output of this research is a modeling framework that allows an analyst to robustly analyze GBASM systems. Insights from the models reinforce Wayne Hughes' mantra of "fire effectively first," and highlight the importance of Blue being able to mass fires into an effective salvo.

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STOCHASTIC DUELS FOR EVALUATING GROUND-BASED ANTI-SHIP SYSTEMS

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ABSTRACT

The Ground-Based Anti-Ship Missile (GBASM) is a critical capability at the center of the Marine Corps' concept of Expeditionary Advanced Base Operations and Force Design 2030 initiatives. This research formulates stochastic models for the evaluation of the efficiency and effectiveness of varying GBASM battery configurations in the context of a duel with a surface ship. The models produced are discrete-time Markov chains that model duels between a Blue GBASM battery and a Red surface ship. The models are parameterized to account for varying factors including lethality of Blue and Red, and evaluate salvos based on the number of GBASM delivery platforms and the number of missiles shot per platform. The primary output of this research is a modeling framework that allows an analyst to robustly analyze GBASM systems. Insights from the models reinforce Wayne Hughes' mantra of "fire effectively first," and highlight the importance of Blue being able to mass fires into an effective salvo.

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List of Acronyms and Abbreviations

A2AD	anti-aerial/access denial
ASuW	anti-surface warfare
DMO	distributed maritime operations
DOD	Department of Defense
EABO	expeditionary advanced base operations
GBASM	ground-based anti-ship missile
INF	Intermediate-Range Nuclear Forces Treaty
LOCE	littoral operations in a contested environment
MLR	Marine Littoral Regiment
NMESIS	Navy-Marine Expeditionary Ship Interdiction System
NPS	Naval Postgraduate School
PLAN	Chinese People's Liberation Army Navy
ROGUE-Fires	remotely operated ground unit for expeditionary fires

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Executive Summary

The United States Marine Corps is building capability in the area of Anti-Surface Warfare, specifically in acquiring ground-based anti-ship missiles (GBASM) and their associated firing platforms. Our research provides a method for analyzing the force structure associated with this new capability. Our approach models tactical-level duels between GBASM batteries and an enemy surface vessel using discrete-time Markov models. The models have sufficient complexity to address critical force design questions, and key characteristics of the duel are parameterized to allow for a robust sensitivity analysis.

In naval missile combat, it is important to determine the desired salvo size, S, to give the salvo a high enough probability of killing the enemy ship. What is unique about the GBASM concept is the ability to spread this salvo across several platforms and in a manner more tailorable to a specific tactical scenario than if the salvo was delivered from a surface ship. In this manner, if we have a salvo of size S, and we spread that salvo across K platforms, then each platform fires N missiles in a given salvo such that $K \times N = S$. With this formulation we are able to analyze the tradeoffs in the number of platforms and the number of missiles fired per platform in terms of the lethality and survivability of these configurations. This provides the foundation for a cost-benefit analysis.

We model the scenario where a GBASM Battery comes into contact with enemy surface vessels. We begin with the simple scenario and build in complexity. We pit GBASM launchers against one enemy surface vessel in a duel. The GBASM side is referred to as Blue and the surface vessel as Red. We initially assume that both sides have adequate missile supply and the duration of the exchange is limited such that we can treat the supply as infinite. The GBASM fires in units of salvos, which each comprises at least one missile. Following a salvo by Blue, the Red surface vessel has the opportunity to return fire.

In the setting described we assume that Blue has the first-shot advantage. We believe the assumption of a first-shot advantage is not unreasonable given the asymmetric situation the introduction of the GBASM creates in the littorals. The GBASM is mobile and has the potential to move into hard to detect places and only come out when ready to fire. The goal of the GBASM is to remain undetected by a Red vessel until it successfully targets the Red

ship. Once the Red ship is targeted, the GBASM system fires and moves to a new location. Without perfect information regarding the movements of the GBASM, the Red ship is at a continuous disadvantage.

Further, the model captures the ability of Red to conduct defensive measures against Blue's salvo. These defensive countermeasures are accounted for with the parameter λ which is the average number of incoming Blue missiles Red can intercept according to a Poisson distribution. Modeling Red's ability to conduct countermeasures in this way accounts for the diminishing capability of Red to conduct countermeasures as Blue's salvo size increases. Similarly, we account for the reduced ability of Red to target Blue's distributed launchers. Red's ability to kill Blue's distributed platforms is captured with the parameter ϕ which represents the average number of Blue platforms Red can kill in return fire according to a Poisson distribution. This again accounts for limited effectiveness of Red to target and kill Blue as the number of Blue platforms increases.

In our analysis of the model we have come across several key findings. First, it is most important to determine the desired salvo size S that provides a sufficiently high probability of killing the enemy ship. This is not simply a matter of "more is always better" as there is a point of diminishing returns in salvo size. As one might expect, we also conclude that an increased number of platforms K improves the survivability and thus the lethality of the GBASM battery. However, the magnitude of the improvement is sensitive to other parameters and is generally small when the salvo size is sufficiently large.

The primary output of the research is the models created and the ability to them to conduct further analysis. None of the parameter values used anywhere in this thesis are informed by the capabilities of specific GBASM systems or potential enenemy surface ships. Thus, our results should be taken as generalizations from exploring likely areas of the parameter space. The models provide the ability to conduct specific analysis informed by the capability of the particular systems in question.

CHAPTER 1:
Introduction

1.1 Background

In 2019, the Commandant of the Marine Corps released the *38th Commandant's Planning Guidance*. The document signaled large changes to the combat formations of the United States Marine Corps and placed an emphasis on the traditional role of the Marine Corps in sea-control and sea-denial. Central to this vision is the development of a ground-based anti-ship missile (GBASM) capability (Berger 2019). This specific capability development falls under the larger umbrella of expeditionary advanced base operations (EABO), which seeks to place a persistent Marine Corps presence inside the enemy's weapon engagement zone, to secure objectives necessary to the conduct of a naval campaign (Berger 2019; Headquarters Marine Corps 2021). The larger effort is done with a regional focus on the South-Western Pacific and the South China Sea. This thesis focuses on the effectiveness of various GBASM systems in the context of tactical-level duels between GBASM batteries and enemy surface ships. We provide an analytical basis for evaluating various GBASM launchers and battery make-up under varying conditions.

1.1.1 Naval Missile Combat

Modern naval surface combat is characterized by the use of missiles. Lacking a modeling paradigm to match this reality, Wayne Hughes developed a mathematical representation of this type of combat with salvo equations in 1995 (Hughes 1995). The implications of this modeling effort and other work have led to suggestions for the changes to the make-up of the United States fleet. Hughes and others have made the call for a larger number of smaller and less exquisite naval vessels that collectively make up sufficient firepower to place enemy vessels at risk, particularly in littoral zones (Hughes 2000). The degree to which these arguments have had a significant impact on the make-up of the surface fleet is minimal. The Chinese People's Liberation Army Navy (PLAN) has reviewed Hughes' work (Xu Xiaoming and Wei 2010) and arguably has acted in a manner his work would suggest. The PLAN has acquired the Type 22 Missile Boat to serve the exact purpose of a small vessel capable of collectively massing fires in littoral waters.

1.1.2 Land-Based

In addition to the Type 22, the PLAN has developed a robust land-based anti-aerial/access denial (A2AD) capability that in its current composition severely restricts the use of U.S. surface vessels in key contested littoral waters. This led to the question of the feasibility of U.S. land-based Anti-Surface Warfare (ASuW) capability. In 2013, the Rand Corporation completed a study for the U.S. Army exploring the use of Land-Based Anti-Ship Missiles in the Western Pacific. The study gave a strong endorsement for the operational and subsequent strategic benefits of developing such a capability (Headquarters Marine Corps 2021). Though this study was conducted for the Army, the Army has not taken serious action in developing this capability. In 2018, the United States ended its adherence to the Intermediate-Range Nuclear Forces Treaty (INF). This development allowed military planners to look at previously banned cruise missiles being fired from the land due to previous range restrictions of the INF.

1.2 Research Focus

As the Marine Corps makes large institutional changes that are largely centered around the capability of the GBASM, it is important to provide analytical rigor to inform the force design decisions being made. This research will be a part of the larger body of efforts work that will contribute to the institutional decisions being made regarding this capability.

The overarching question is how the Marine Corps should build its ASuW capability around the GBASM weapon system. This relates both to the procurement of systems and the concepts for tactical and operational employment.

As has been previously summarized, there has been work conducted to establish the need for developing this capability and the potential operational and strategic impacts. Thus our research will not attempt to duplicate these efforts, and will instead have a more bottom-up approach of investigating tactics. From those tactical insights, we seek to inform decisions regarding force design and future employment.

1.3 Operational Setting

The concept of the Marine Corps employment of GBASMs falls into a larger strategic vision of a return to amphibious nature and specifically sea denial. Though sea denial consists of a mission set that has use cases globally, regional emphasis is placed on countering PLAN aggression in the South China Sea. The employment of the GBASM is not conducted in isolation, but through a new operational concept, EABO. As defined by the Tentative Manual for Expeditionary Advance Base Operations. "EABO is a form of expeditionary warfare that involves the employment of mobile, low-signature, persistent, and relatively easy to maintain and sustain naval expeditionary forces from a series of austere, temporary locations ashore or inshore within a contested or potentially contested maritime area in order to conduct sea denial, support sea control, or enable fleet sustainment" (Headquarters Marine Corps 2021).

The development of EABO has necessitated the creation of a new organizational structure, the Marine Littoral Regiment (MLR) (Berger 2019, 2020). The MLR will be purpose-built to perform the functions outlined in the above definition of sea-denial, support sea control, or enable fleet sustainment.

The proposed system for GBASM employment is known as Navy-Marine Expeditionary Ship Interdiction System (NMESIS). As defined in the Tentative Manual for EABO, NMESIS is an anti-ship missile system that combines Remotely Operated Ground Unit for Expeditionary Fires (ROGUE-Fires) vehicles with the Naval Strike Missile (NSM). The unit of employment for the NMESIS system is the NMESIS Battery. There is set to be one NMESIS Battery in an MLR.

There is institutional momentum in developing the capability as summarized above and outlined in more detail in the Tentative Manual for EABO. However, many of these concepts and capabilities are proposals and long-term projects. Thus research still has the ability to shape certain decisions. For this reason, the models described in our work do not seek to exactly replicate the proposed structure of the NMESIS Battery in a MLR. Rather we develope a more generic framework that we can modify easily to compare this proposed framework to other alternatives.

1.4 Literature Review

A study of modern naval combat should begin with a review of the work of Wayne Hughes. Hughes' work on salvo equations made use of the ideas of Lanchester's equations (Lanchester 1916) with the idea of attrition models with difference equations. Though Hughes saw the nature of naval combat with missiles different from both previous work done with Lanchester equations and from previous forms of naval combat (Hughes 2018, 1995). The key difference Hughes noted was the more catastrophic effects of a missile hitting its target. The difference was captured with his difference equations having larger discrete impacts from missiles in a salvo verse continuous effects of fire (Hughes 1995). In the equations, he also accounted for the ability of the targeted ship to intercept incoming missiles with countermeasures. He modeled this as a fixed number of missiles that could be intercepted in any given incoming salvo (Hughes 1995). Hughes made other useful extensions on the equations to include the importance of scouting and targeting (Hughes 1995). With the salvo equations at the center of his analysis, he drew conclusions about tactics and the makeup of modern navies. He concluded that in missile combat one should "fire first effectively" (Hughes 2018). This conclusion led him to the further insight that the platform of delivery was not as critical as the ability to mass fire in a salvo. Thus a large number of small ships with the ability to mass fires in a salvo can defeat larger and more exquisite ships (Hughes 2018).

Hughes' work is known by other navies in the world, including the PLAN. In the review of Hughes' work done at the University of Naval Engineering at Wuhan, they come to similar conclusions regarding the importance of massing an effective salvo and also on the viability of this being done with smaller ships (Xu Xiaoming and Wei 2010). The PLAN has acted on this concept in developing the Type 22 missile boat which is the type of small, less exquisite ship for which Hughes advocated.

Proponents of concepts like Distributed Maritime Operations (DMO) and EABO in the Marine Corps and Navy often invoke Hughes. Thus, his work is relevant both in the context of developing specific tactical-level models and in the larger context of concepts of operational employment.

In the specific context of salvo models, Armstrong (2005) has proposed stochastic versions of Hughes salvo equations (Armstrong 2005). The stochastic approach adds some richness to the salvo equations as they provide for variability in results. Where Armstrong's work most directly impacted our research is in his analysis of effective salvo sizes, particularly in the impact of the countermeasures of a ship (Armstrong 2007). While we do not adopt his methodology exactly, we do model the number of missiles a ship can intercept as a stochastic random variable with diminishing probability as the salvo gets larger.

Our models fall under the larger umbrella of combat models (Washburn and Kress 2009). The field includes the well-known Lanchester equations (Lanchester 1916), stochastic duels (Williams and Ancker Jr 1963), and the salvo equations discussed above (Hughes 1995; Armstrong 2005).

While our models are salvo models and most similar to those by Hughes and Armstrong there are key differences. The main difference is in the tactical scenario. In our case, we do not have salvos being exchanged between ships, but rather between a land-based GBASM and a ship. The GBASM being on land makes the duel inherently asymmetric. The GBASM system has the ability to "shoot-and-scoot" much like in an artillery engagement. There is also less reason to model staying power for the GBASM launchers as a hit from any modern missile would result in a kill every time. Thus rather than staying power on a more continuous scale, we have discrete state changes. These discrete state changes lend themselves to Markov models.

Stochastic Markov models have been used in many contexts for combat modeling. These include diverse applications of investigating force ratios in ground combat with stochastic Markov Lanchester equations (Kress and Talmor 1999), and evaluating shoot-and-scoot tactics of artillery (Shim and Atkinson 2018).

Our research is driven by current Marine Corps initiatives aimed at large organizational changes. These changes are meant to modernize the force, return to maritime operations, and with a specific focus on potential conflict in the Western Pacific. The changes called for were outlined in "The Commandant's Planning Guidance" and "Force Design 2030" (Berger 2019, 2020). The idea of using land-based missiles in a regional conflict in the Western-Pacific was explored in a Rand study. The study demonstrated the viability of controlling key maritime choke points with current available anti-ship missiles used as a deterrent to conflict or to make the cost of conflict very high to the adversary (Kelly et al. 2013). While the study was conducted for the United States Army, the concepts were later studied and adopted by the Marine Corps (Berger 2019). The goals of the Marine Corps are a part of joint initiatives with the other maritime services the Navy and Coast Guard. The joint service approach is focused on developing operational concepts of DMO, EABO, and littoral operations in a contested environment (LOCE) (Office of the Secretary of the Navy 2020). The Marine Corps is specifically responsible for EABO (Berger 2019). The development of a GBASM missile capability is cited as a top service priority in line with supporting EABO concepts (Berger 2020).

The development of the GBASM capability is among the top initiatives in the Marine Corps. As such, there are many parallel efforts for analyzing this problem. Other organizations in the Marine Corps are looking at this problem throughout the various analysis organizations of the Marine Corps and Department of Defense (DOD). Various efforts exist throughout Headquarters Marine Corps. There have been several theses written at Naval Postgraduate School (NPS) at various classification levels. The primary approach in other projects is data farming using an agent-based simulation model. The studies have generally utilized a single tactical scenario to conduct analysis and explore different design parameters related to tactical encounters of GBASM systems and enemy surface combatants through data farming techniques. The NPS Operations Research department has also devoted wargaming resources to this topic.

1.5 Modeling Focus

The goal of this research is to develop a series of analytical models that aid in the ongoing force design efforts. There are several criteria to consider including fire effectiveness, survivability, and supportability.

This project differs from those previously mentioned in the methods employed. Rather than a detailed agent-based simulation that hinges on many assumptions, we develop transparent analytical models that require only a few key assumptions. The focus of the research is on tactical-level stochastic duels between Blue GBASM launchers and Red ships. The goal is to use Markov models of tactical duels between GBASM launchers and enemy combatant ships. The aim is to better understand tradeoffs between factors such as the number of GBASM launchers, the number of missiles per platform, the probability of kill for salvos, and the implications of these variables at a basic tactical-level. For example, our models will be able to answer questions like: Is it better to have one platform that can shoot a salvo, or is it better to spread one salvo across multiple GBASM launchers? After gaining an understanding of tactical employment we discuss recommendations that are relevant to force design efforts.

We parameterize key inputs to the model giving us the ability to explore the possible design space and conduct sensitivity analysis. All models are computaionally implemented in Python Van Rossum and Drake (2009). This implementation provides a way to easily analyze many scenarios and explore the parameter space.

1.6 Outline

This thesis is organized into four chapters. Chapter 2 develops the models. Chapter 3 covers the initial results. Chapter 4 gives conclusions and recommendations.

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CHAPTER 2:
Discrete-Time Markov Duels

2.1 Setting

In the introduction, we described the operating environment in a more macro sense. Here we narrow in to more specifics to provide for the tactical setting of our modeling effort.

We model the scenario where a GBASM Battery comes into contact with enemy surface vessels. We begin with the simple scenario and build in complexity. We pit GBASM launchers against one enemy surface vessel in a duel. The GBASM side is referred to as Blue and the surface vessel as Red. We initially assume that both sides have adequate missile supply and the duration of the exchange is limited such that we can treat the supply as infinite. The GBASM fires in units of salvos, which each comprises at least one missile. Following a salvo by Blue, the Red surface vessel has the opportunity to return fire.

In the setting described we assume that Blue has the first-shot advantage. We believe the assumption of a first-shot advantage is not unreasonable given the asymmetric situation the introduction of the GBASM creates in the littorals. The GBASM is mobile and has the potential to move into hard to detect places and only come out when ready to fire. The goal of the GBASM is to remain undetected by a Red vessel until it successfully targets the Red ship. Once the Red ship is targeted, the GBASM system fires and moves to a new location. Without perfect information regarding the movements of the GBASM, the Red ship is at a continuous disadvantage.

We use Discrete-Time Markov Chain to model the duel. The nature of missile exchanges lends itself to discrete changes in the state of a model. Unlike a Lanchester model, where attrition is continuous over time due to continuous fire (Lanchester 1916), in combat, with missiles, a hit causes a more immediate and discrete change to the state of the combatants (Hughes 1995). As we demonstrate, the transitions between states in the model can be modeled with sufficient richness to describe the scenario.

2.2 Base Duel

In the base model, we start with a duel between a single GBASM platform and a Red surface vessel. The model follows the following order of events. Blue fires a salvo at Red, if Red is killed by Blue (with the probability of p_b), the duel is over. If Red is not killed, then Red has a chance to kill Blue before Blue successfully makes it to a new location. Red fires one salvo of return fire at Blue, which kills Blue (with probability p_r). If Red kills Blue the duel is over. Otherwise, the duel continues with a new exchange of salvos as described above.

Note that the actual size of a Blue GBASM salvo is not yet modeled here. Rather we abstract this into a single parameter p_b, the probability that Blue kills the Red ship in a single salvo.

2.2.1 Model States and Parameters

We begin by defining the state space of the model.

Model States:

D - Dueling – A transient state that represents the situation where both sides are alive.
BV - Blue Victory – Absorbing state.
RV - Red Victory – Absorbing state.

Model Parameters:

p_b – Probability Blue kills Red in a single salvo.
p_r – Probability Red kills Blue in a single salvo.

The ongoing duel state, D, is the only transient state in the model. Once the duel transitions from D to BV or RV the duel terminates. The duel starting in state D remains in that state if Blue fails to kill Red (with probability $q_b = 1 - p_b$) and Red fails to kill Blue with probability ($q_r = 1 - p_r$). We denote the transition probability from state D back to state D as $P_{D,D}$. Thus we have the probability of returning to state D as

$$P_{D,D} = q_b \cdot q_r. \tag{2.1}$$

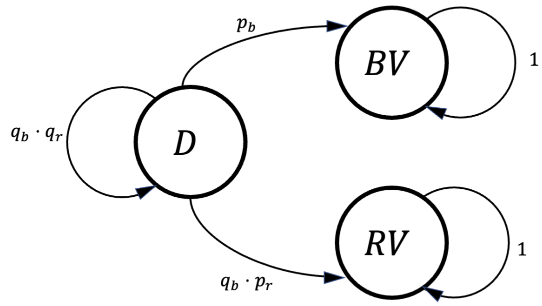

Figure 2.1. Transitions in the base model

Given the first shooter advantage, Blue wins if it hits with its salvo and Red does not have an opportunity to fire back. Thus, the transition probability of moving from the dueling state to Blue victory, $P_{D,BV}$, is given by

$$P_{D,BV} = p_b. \tag{2.2}$$

In order for Red to win the duel in any given round, two things must occur. First Blue must fail to kill Red with its salvo and then Red must kill Blue. The transition probability of moving from the dueling state to Red victory, $P_{D,RV}$, is given by

$$P_{D,RV} = q_b \cdot p_r. \tag{2.3}$$

The transitions are shown in Figure 2.1 and the transition matrix is shown below.

$$\begin{array}{c} \\ D \\ BV \\ RV \end{array} \begin{array}{c} \begin{array}{ccc} D & BV & RV \end{array} \\ \begin{bmatrix} q_b \cdot q_r & p_b & p_r \cdot q_b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \tag{2.4}$$

From this we can calculate the overall proability of Blue victory $p(BV)$ as:

$$p(BV) = \frac{p_b}{1 - q_b \cdot q_r}. \tag{2.5}$$

2.3 Blue has Limited Salvo Capacity

One obvious extension to the base model is to limit the salvo capacity of Blue. That is to account for the fact that as a ground-based asset there will likely be a limited number of salvos at Blue's disposal. We can make this addition by simply adding a parameter C, representing the starting number of salvos of Blue, and then by replacing our state D with the states representing the number of salvos remaining. We can also add an additional absorbing state Winchester or W. This state represents the event that Blue runs out of missiles and neither side has been killed. Even though W effectively represents a Blue loss and Red win, decisions makers may be interested in examining the likelihood of Winchester versus Blue being killed. To show an example we start with $C = 3$.

2.3.1 Model States and Parameters

Model States:

3 - Dueling – Represents ongoing trading of missile salvos where Blue has three salvos remaining.

2 - Dueling – Represents ongoing trading of missile salvos where Blue has two salvos remaining.

1 - Dueling – Represents ongoing trading of missile salvos where Blue has one salvo1 remaining.

BV - Blue Victory – Absorbing state.

RV - Red Victory – Absorbing state.

W - Winchester – Absorbing state.

The parameters p_b and p_r remain the same. As there is not just one dueling state, the model must transition between the different dueling states. For example, the model can move from dueling states 3 to 2 in the event both Blue and Red fail to kill the other in exchange. Thus we have the transition probability $P_{3,2}$ as:

$$P_{3,2} = q_b \cdot q_r. \tag{2.6}$$

On the other hand, the model cannot stay in dueling state 3. As state 3 represents the remaining salvo capacity of Blue and reaming in the state would imply a salvo was not fired. Thus we have $P_{3,3} = 0$. Note the model can also not transition from state 3 to 1 as this would imply that Blue fired two salvos in an exchange, making $P_{3,1} = 0$.

The behavior of the model transitioning into the states BV and RV from the dueling states remains the same as the base model. However, we have the additional Winchester state W. This absorbing state can only be entered from state 1 when Blue has one remaining salvo. The conditions for entering this state are that both Blue and Red fail to kill each other. Thus we have:

$$P_{1,W} = q_b \cdot q_r. \tag{2.7}$$

The transition matrix is shown below.

$$\begin{array}{c} \\ 3 \\ 2 \\ 1 \\ BV \\ RV \\ W \end{array} \begin{array}{c} \begin{array}{cccccc} 3 & 2 & 1 & BV & RV & W \end{array} \\ \begin{bmatrix} 0 & q_b \cdot q_r & 0 & p_b & q_b \cdot p_r & 0 \\ 0 & 0 & q_b \cdot q_r & p_b & q_b \cdot p_r & 0 \\ 0 & 0 & 0 & p_b & q_b \cdot p_r & q_b \cdot q_r \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{2.8}$$

2.4 Duel with Multiple GBASM Launchers

The next extension of the model is to consider a scenario where we have multiple GBASM launchers in a duel with one Red ship. As stated previously, the baseline idea of what a GBASM battery would look like is based on the structure of the current Marine Corps HIMARS battery (Manual for EABO). This includes situations where multiple launchers fire at the same Red ship. We again start with the simplifying assumption that the salvo capacity of Blue is large enough to be treated as infinite. Before we begin with the specifics of the model we briefly discuss the tactical considerations when there are multiple launchers.

2.4.1 Note on Tactics

We model two tactics of the GBASM battery. Specifically, two tactics: (1) all GBASMS conduct shoot-and-scoot together simultaneously and (2) a leap-frog tactic where GBASMs fire one at a time in sequence. We might choose tactic (1) in the situation where we expect that there is added benefit to a large volume of fire incoming at one time to overwhelm Red's defensive measures. Tactic (2) is adequate in a situation where a more constant rate of fire with a lower simultaneous volume of fire would be beneficial to Blue.

Under current model assumptions, the two tactics generate identical win probability for Blue. We currently assume that probabilities of kill between individual Blue salvo shot by multiple launchers are independent and identical. Likewise, we model that Red kills each GBASM system with individual and independent probability p_r. The key thing noted is that we always give Blue the first shooter advantage and then Red can fire back, regardless of whether this is done in a large-massed salvo or smaller ones as in the leap-frog case. Thus there is no difference between the two tactics.

Later, in Section 3.5.1, we discuss how defensive measures from Red may distinguish these tactics and how assumptions on the targeting of Blue change depending on chosen tactics.

2.4.2 Model with Multiple GBASM Launchers

For now, we model the following tactical situation: Assume there are K launchers at the beginning of the duel and the current number remaining in the duel is denoted by k. Note that all remaining active k launchers each fire one missile simultaneously having their fire act as one collective salvo of k missiles. Where the missile from each GBASM kills Red with probability p_s, where we define p_s as the single-shot probability of kill for one Blue missile. Thus the probability of kill from the fire of all k launchers is the complement of all the launchers missing:

$$1 - q_s^k.$$

In previous models we did not model the effects of the individual missiles this way, because we aggregated the entire probability of kill for a salvo as p_b. In p_b we may also have been able to aggregate the effects of any Red countermeasures. Here, when considering individual missiles we do not account for Red countermeasures of any kind; rather, p_s is the probability that a missile is on target and kills Red or not. Also note that in this model we only allow each GBASM platform to a single missile, a limitation we will later remove.

We have also made the assumption that Red has an independent probability p_r of killing each of the k launchers independently. Thus the number of GBASM launchers killed after Red fires follows a binomial distribution. If we denote X as the random variable representing the number of GBASM killed after Red fire, we can say:

$$P(X = x) = \binom{k}{x} p_r^x \cdot q_r^{k-x}.$$

2.4.3 Model States and Parameters

When we account for the fact that there are K launchers to begin with we will denote the current number of launchers as k. Then our ongoing duel states will be denoted as Bk to note the number of Blue launchers still in the duel. Thus in the case of an ongoing duel between a Red ship and two Blue launchers the model would be in state $B2$.

To illustrate this example we begin with the total number of GBASM launchers $K = 2$.

Model States:
$B2$ - The duel is ongoing with two GBASM launchers.
$B1$ - The duel is ongoing with one GBASM launcher.
BV - Blue Victory
RV - Red Victory

The model parameters remain unchanged. Below is the transition matrix.

$$
\begin{array}{c c}
 & \begin{array}{cccc} B2 & B1 & BV & RV \end{array} \\
\begin{array}{c} B2 \\ B1 \\ BV \\ RV \end{array} &
\begin{bmatrix}
{q_s}^2 \cdot {q_r}^2 & 2p_r \cdot q_r \cdot {q_s}^2 & 1 - q_s^2 & {q_s}^2 \cdot p_r^2 \\
0 & q_s \cdot q_r & p_s & q_s \cdot p_r \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
\tag{2.9}
$$

2.5 Duel with Capacity and Multiple GBASM Launchers

With capacity and multiple launchers modeled separately, it is a natural extension to model them together. Here, we show a model where we have capacity $C = 2$ salvos per GBASM system and a total of $K = 2$ launchers.

2.5.1 Model States and Parameters

Model States:

$(B2, 2)$ - The duel is ongoing with two launchers each having two missiles remaining.

$(B2, 1)$ - The duel is ongoing with two GBASM launchers each having one missile remaining.

$(B1, 1)$ - The duel is ongoing with one GBASM system having one missile remaining.

BV - Blue Victory

RV - Red Victory

W - Blue is Winchester

Note we cannot have a situation with one GBASM having two salvos. Below is the transition matrix.

$$
\begin{array}{c|cccccc}
 & (B2,2) & (B2,1) & (B1,1) & BV & RV & W \\
(B2,2) & 0 & {q_s}^2 \cdot {q_r}^2 & 2q_r \cdot p_r \cdot {p_s}^2 & 1-q_s^2 & {p_r}^2 \cdot q_s^2 & 0 \\
(B2,1) & 0 & 0 & 0 & 1-q_s^2 & {p_r}^2 \cdot q_s^2 & q_s^2 \cdot (1-p_r^2) \\
(B1,1) & 0 & 0 & 0 & p_s & p_r \cdot q_s & q_s \cdot q_r \\
BV & 0 & 0 & 0 & 1 & 0 & 0 \\
RV & 0 & 0 & 0 & 0 & 1 & 0 \\
W & 0 & 0 & 0 & 0 & 0 & 1
\end{array} \tag{2.10}
$$

2.6 Modeling Salvo Makeup

Given the basic structure of the duel, we can examine the outcomes of duels based on either side's probabilities of kill, the size, and the salvo capacity of the Blue force. We have not as of yet discussed or modeled the make-up of a Blue salvo. For example, how many missiles are fired across how many launchers? Until now we have left it abstract and wrapped the details into a single parameter p_b. One of the main goals that Hughes sought to achieve with his models was in choosing the appropriate salvo size for missile combat (Hughes, 1995). With some expansion, we can do the same.

Rather than just take p_b as a given value, we can compute it based on how the GBASM battery is constituted. We do this by defining some additional parameters. Assume that the GBASM battery fires a single homogeneous type of missile and that missile has a single shot probability of kill against the Red ship of p_s. That is if only one of these missiles is shot at Red it results in a kill with probability p_s. There are K launchers and each launches N missiles in a salvo. Note that K is the starting number of launchers and we annotate the active number as k (meaning in a duel our salvo size is equal to $k \cdot N$). Then, if we assume that each of these $k \cdot N$ missiles has an independent probability of killing Red, we can define our single salvo probability of kill, p_b, as a function of k, N, and p_s as such:

$$
p_b(N, k, p_s) = 1 - (1 - p_s)^{Nk}. \tag{2.11}
$$

It is also be useful to refer to the complement as:

$$q_b(N, k, p_s) = 1 - p_b(N, k, p_s) = (1 - p_s)^{Nk}. \tag{2.12}$$

By more explicitly modeling the make-up of a salvo and how the Blue side achieves a certain p_b we can conduct a much richer analysis. We can now compare missile types as reflected by their lethality in p_s, the salvo size S, and the best way to achieve that salvo size through a number of launchers K and the number of missiles fired per platform N. This parameter space provides for robust analysis of the duel.

For now, we can continue to assume that every active GBASM has a probability p_r of being killed by the Red ship after every GBASM salvo is delivered. That is p_r is fixed and independent of k (the number of active Blue launchers). In this way, the number of Blue launchers killed by Red in a single salvo follows a binomial distribution, we examine the impact of this assumption later on.

2.6.1 Model State Space

All models described have the same state space that is defined by the number of active launchers and the Red ship. We start with K launchers at the beginning of the duel and have transient states $K, K-1, K-2,...,0$. The absorbing states are RV signifying Red victory and (BV, k) for Blue victory, and k representing the remaining number of GBASM launchers.

The transitions in the transient states are as follows. If we are in state k we can transition to any transient state in the range $k,\ k-1, \ldots,\ 1$. This is done only when Blue fails to kill Red with its salvo, where that probability is defined in Equation 2.12 as $q_b(N, k, p_s)$. Then Red will kill some number of Blue launchers X where $X < k$, and we know that $X \sim Binom(k, p_r)$. Thus we have the probability that Red kills some number of launchers as:

$$P(X = x) = \binom{k}{x} p_r^x q_r^{k-x}.$$

The probability of transition from k to any transient state $k - x$ is given by:

$$P_{k,k-x} = q_b(N, k, p_s) \cdot \binom{k}{x} p_r^x q_r^{k-x}. \tag{2.13}$$

Similarly, we transition to the absorbing state RV for Red victory when Blue fails to kill Red again as defined by $q_b(N, k, p_s)$ and then that the number of Blue launchers that Red kills is $X = k$. Thus we have the probability that we transition from state k to the absorbing state RV is:

$$P_{k,RV} = q_b(N, k, p_s) \cdot p_r^k. \tag{2.14}$$

Then we have the probability that Blue will kill Red which is the probability the model transitions from k to (BV, k) which is defined in Equation 2.11.

$$P_{k,(BV,k)} = 1 - (1 - p_s)^{Nk}$$

The transition matrix for the model is shown below.

$$\begin{array}{c} \\ K \\ K-1 \\ \vdots \\ BV,K \\ BV,K-1 \\ RV \end{array} \begin{array}{c} \begin{array}{ccccccc} K & K-1 & \dots & BV,K & BV,K-1 & \dots & RV \end{array} \\ \begin{bmatrix} q_b(N,K,p_s) \cdot q_r^K & q_b(N,k,p_s) \cdot \binom{K}{1} q_r^{K-1} p_r & \dots & p_b(N,K,p_s) & 0 & \dots & p_b(N,K,p_s) \cdot p_r^K \\ 0 & q_b(N,K-1,p_s) \cdot q_r^{K-1} & \dots & 0 & p_b(N,K-1,p_s) & \dots & p_b(N,K-1,p_s)] \cdot p_r^{K-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 1 \end{bmatrix} \end{array} \tag{2.15}$$

2.7 Modeling Salvo Makeup with Capacity

If we combine all previous models we have a model that explicitly models the makeup of salvos based on the parameters N and K and we also add in the capacity constraint C.

As before we have K launchers each of which fires N missiles per salvo, making our initial salvo size $K \cdot N$. If we add in the initial capacity C, we define our transient state space in the duel based on the current values of K and C represented respectively by k and c respectively.

2.7.1 Model State Space

We have the usual absorbing states of BV and RV for Blue and Red victory respectively, as well as the Winchester state W as we are modeling salvo capacity. However, we expand the Winchester state to include the number of launchers remaining. Thus the Winchester absorbing states are $(W, K), (W, K - 1), ..., (W, 1)$. The states that change from previous models are the ongoing duel, transient states. The transient states are defined by the current number of launchers starting at K and the remaining capacity starting at C. These are denoted as (K, C) to indicate the starting state with K launchers and a capacity of C, $(K, C - 1)$ indicates K launchers that have fired one salvo.

2.7.2 Model Transitions

Beginning in the state (k, c) we discuss the various transitions possible. First, Blue achieves victory in the event that the salvo of $k \cdot N$ missiles scores a hit. This would be the transition probability $P_{(k,c),(BV,k)}$; recalling Equation 2.11 this is done with probability:

$$P_{(k,c),(BV,k)} = p_b(N, K, p_s) = 1 - (1 - p_s)^{Nk}. \tag{2.16}$$

Red could also achieve a victory from the state (k, c) in the event the Blue salvo fails to kill Red with the probability defined in Equation 2.12, $q_b(N, k, p_s)$, and Red kills $x = k$ launchers with probability p_r^k from Equation 2.13. Thus the transition probability from (k, c) to RV is given by:

$$P_{(k,c),RV} = q_b(N, k, p_s) \cdot p_r^k = (1 - p_s)^{Nk} \cdot p_r^k. \tag{2.17}$$

In the case that Red is not killed and Blue is not fully eliminated we move from Blue having a capacity of c to having capacity $c - 1$. We then move to a state defined by the number of launchers killed, which we have stated is defined by a binomial distribution. Say the number of launchers killed is x according to Equation 2.13. Then we move from sate (k, c) to $(k - x, c - 1)$ with probability:

$$P_{(k,c),(k-x,c-1)} = q_b(N, k, p_s)\binom{k}{x} p_r^x \cdot q_r^{k-x} \tag{2.18}$$

The above equations can generalize for transitions from all dueling states. The case not covered is entering the Winchester absorbing states. This can only be done from a state where the current capacity $c = 1$. We show this from the state $(k, 1)$. Note, the Winchester states are defined also by the number of remaining launchers as well. Thus from the state $(k, 1)$ we transfer to the state $(W, k - n)$ if Blue fails to kill Red and Red kills n launchers. Thus similar to Equation 2.18 we have:

$$P_{(k,1),(W,k-n)} = q_b(N, k, p_s)\binom{k}{n} p_r^n \cdot q_r^{k-n}. \tag{2.19}$$

2.8 Sensitivity to Assumptions

In our modeling efforts thus far we rely on several assumptions about how the duel would unfold tactically. These assumptions are necessary as there is no historical examples to examine of GBASM fire against surface ships. Here, we will re-examine some of those assumptions and the sensitivity of our model to those assumptions.

2.8.1 Impact of Red Countermeasures

We have stated that in any salvo of size S each missile has an independent and identical chance of killing Red. This assumption is valid in the sense that we have homogeneous missiles using the same targeting information and therefore we would not expect one to be more lethal than another. However, this assumption does fail to capture the added effects of firing numerous missiles near-simultaneously in a salvo when considering the ability of Red to intercept incoming missiles.

To illustrate the point let us propose two scenarios. In scenario A, we have $K = 3$ launchers each firing $N = 1$ missiles per salvo. The salvo is depicted in Figure 2.2. Here, we assume that the three launchers can coordinate well enough to have the missiles come at Red near simultaneously.

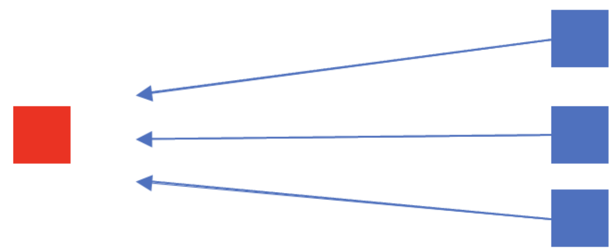

Figure 2.2. Scenario A. Blue salvo with $K = 3$ launchers and $N = 1$ missiles per system.

In Scenario B, we have $K = 1$ GBASM system firing $N = 3$ missiles per salvo. However, we assume that there is some time delay in Blue's ability to fire the three missiles due to reloading and thus the missiles do not arrive at the Red target simultaneously. This situation is shown in Figure 2.3.

Figure 2.3. Scenario B. Blue salvo with $K = 1$ GBASM system and $N = 3$ missiles with time delay.

For the moment, we will not account for any advantage in offensive ability that Red may gain in Scenario B and will instead focus on comparing the difference in the two Blue salvos.

In terms of the models, we have presented thus far that the lethality of Blue's fire is equivalent in both scenarios presented. If we say each of our missiles has a single shot probability of kill of p_s, then in Scenario A our salvo probability of kill is the probability that at least one missile kills Red, or the complement on no missiles killing Red. In this case:

$$1 - (1 - p_s)^3$$

.

Scenario B is no different. The probability of Blue killing Red is again the complement of none of the three missiles killing Red.

With this set of assumptions then all that matters in terms of Blue's lethality is how many missiles Blue can fire at Red before Red can return offensive fire. This is the behavior we observed when investigating the differences between simultaneous shoot-and-scoot tactics verse dispersed shoot-and-scoot.

Red's anti-missile countermeasures must be more explicitly accounted for in the model to capture this behavior. Modern surface warfare ships have various anti-missile countermeasures from surface to air anti-missile missiles to close-in weapons systems. Hughes accounted for these defensive measures in his salvo equations and let the number of missiles being intercepted be deterministic and constant (Hughes 1995). Let us use Hughes' logic in our two previous scenarios, and let assume that Red can reliably intercept two missiles in any given salvo where the incoming missiles are near-simultaneous. In Scenario A three missiles arrive at Red simultaneously, which means that Red would intercept two of these missiles. The other missile would then kill Red with probability p_s. Now consider Scenario B, where from the perspective of Red, the time delay between missiles makes it appear as the salvo from Blue is actually three distinct, time-delayed salvos of one missile each. Thus Red would have the defense capacity to intercept each missile as it came leaving the probability of Blue killing Red as 0.

From before, with our base assumptions, we said the two scenarios were equivalent in terms of Blue's lethality. However, when considering the countermeasures of Red we can see that there is in fact benefit in Blue firing a larger simultaneous salvo.

2.8.2 Stochastic Modeling of Red Defensive Measures

In our original models, we implicitly assume that there are no Red countermeasures to Blue's fire. Thus our value of p_s is the probability that the missile is on target and produces a kill. This does not change as we discuss modeling Red countermeasures.

Above we mentioned Hughes modeled the countermeasures of the Red ship with a deterministic constant: the number of missiles that could be intercepted from every incoming salvo (Hughes 1995). Here, we replicate this in a stochastic context. We include the number of incoming missiles that Red can intercept as a random variable I. The number of missiles intercepted can be represented as a Poisson random variable, $I \sim Pois(\lambda_I)$ where λ_I is the average number of incoming missiles that the Red ship can intercept against a simultaneous salvo. We previously defined the single salvo probability of kill for Blue firing at Red as p_b which is a function of N, K, and p_s. Adding in the defensive measures of Red we can define p_b a function of N, k, p_s, and λ_I.

$$p_b(p_s, k, N, \lambda_I) = \sum_{i=0}^{kN-1} \left(\frac{\lambda_I^i e^{-\lambda_I}}{i!} \right) \left(1 - (1 - p_s)^{kN-i} \right) \tag{2.20}$$

Figure 2.4 shows how the probability of kill for a salvo, p_b, when we do not account for Red defensive measures, compared to modeling the defensive measures of Red for several values of λ_I the average number of missiles the Red ship will intercept from a salvo. In each case shown below we have $p_s = 0.5$ and the Salvo Size on the x-axis is effectively $k \cdot N$.

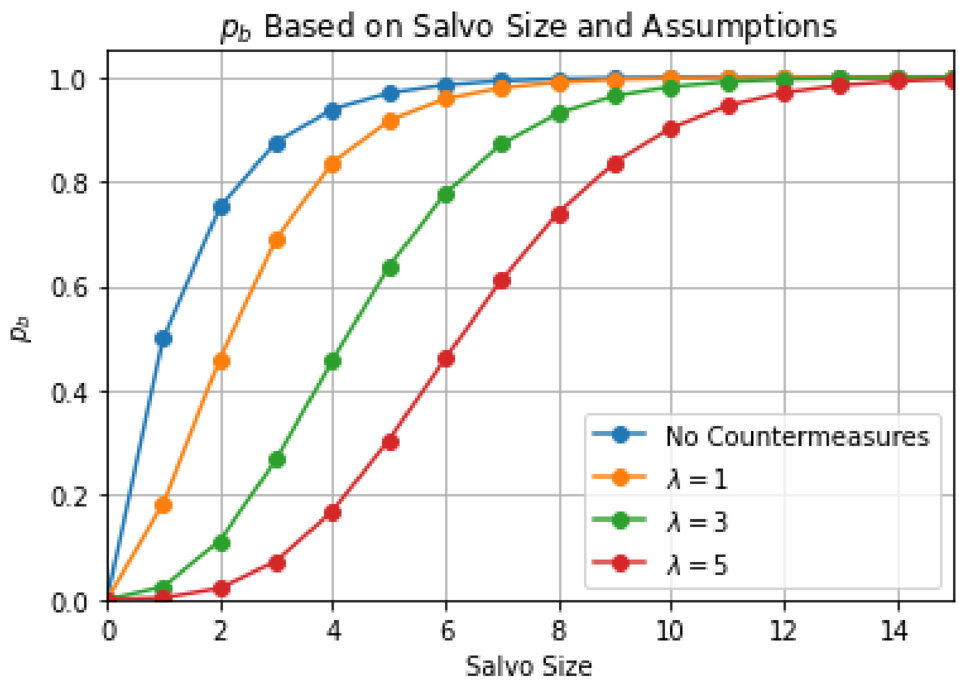

Figure 2.4. Comparing p_b with assumption of no Red countermeasures to different values of λ_I. Note that in all cases $p_s = 0.5$, and the salvo size is the total $k \cdot N$.

We can see that in all cases p_b grows monotonically as the salvo size grows. In the base case of no countermeasures, we can see that p_b has logarithmic growth. When we model the defensive measures, the growth of p_b is more like a sigmoid, where increases are initially small with salvo size until salvo size becomes greater than λ_I, at which point there is logarithmic growth with respect to salvo size.

2.8.3 Implication for Tactics

Modeling Red defensive countermeasures this way has impacts on the choice in tactics we introduced earlier. Before we said there was no difference between a leap-frog and simultaneous shoot-and-scoot tactics. When we model the defense of Red we can see this is no longer the case. Assuming Red has missile countermeasures, Blue needs to mass fires in one simultaneous salvo in order to achieve a high p_b. Thus Blue must organize its batteries to fire the sufficient salvo size. Any alteration of synchronizing all launchers to fire simultaneously would only be in the case where Blue has so many launchers this would cause the Blue salvo to be so large as to waste ammunition. There are no known advantages to having fire be continuous as would be the case of suppressing fire for maneuver in ground combat, here we are only trying to overwhelm the defenses of the Red ship in a single salvo.

2.8.4 Effects of Distributed GBASM Fire

In our previous models, Red has the ability to kill any GBASM launcher with an independent and identical probability p_r. This means that the number of GBASM launchers killed by a Red salvo follows a binomial distribution. Let k be the number of GBASM launchers and X is the number of launchers killed. We have $X \sim B(k, p_r)$ with the probability mass function:

$$P(X = x) = \binom{k}{x} p_r^x \cdot (1 - p_r)^{k-x}.$$

Modeled this way there is no limitation on Red's capabilities based on the number of Blue targets it must engage. However, there is likely some upper limit to the number of targets Red can engage effectively. In order for Red to destroy Blue launchers in a duel, they must detect the launchers, be able to allocate fires to the target, and then hit the target before it moves. This must all be done with a limited number of sensors and shooter launchers. For our purposes here we can continue with the observation that due to the necessarily limited capacity of Red assets, as the number of GBASM launchers K increases, there will be diminishing capacity for Red to identify and prosecute the targets. Thus we propose to model the number of Blue launchers killed by Red in a single exchange with a Poisson distribution. We will parameterize the lethality of Red as ϕ, as the average number of launchers Red can kill in a single salvo. Let us continue to say X is the number of Blue launchers killed and that k is the number of launchers Blue has. The probability that Red kills some number of Blue launchers x is:

$$P(X = x) = \begin{cases} \frac{\phi^x e^{-\phi}}{x!} & x < k \\ 1 - \sum_{i=0}^{k-1} \frac{\phi^i e^{-\phi}}{i!} & x = k \end{cases}. \tag{2.21}$$

Several transition probabilities change from our previous Markov models when this Red targeting is included. Recall that the state of an ongoing duel is denoted by an ordered pair of the number of active GBASM launchers k and the remaining number of salvos Blue has c written as (k, c). The altered transition probabilities are as follows. First, when Red and Blue miss we transition from (k, c) to $(k, c-1)$ with probability:

$$(1 - p_b(n, k, p_s, \lambda)) \cdot P(X = 0).$$

where $p_b(n, k, p_s, \lambda)$ was given in Equation 2.20 and $P(X = 0)$ is from Equation 2.21.

When Blue misses and Red kills $x < k$ launchers we transition from (k, c) to $(k - x, c - 1)$ with probability:

$$(1 - p_b(n, k, p_s, \lambda)) \cdot P(X = x).$$

Then in the case where Blue again fails to kill and Red kills all k of Blue's launchers (k, c) to RV with probability:

$$(1 - p_b(n, k, p_s, \lambda)) \cdot p(X = k).$$

2.9 Notes on Computational Implementation of Models

All of the models described in this chapter were implemented in Python primarily using the packages Pandas Wes McKinney (2010), NumPy Harris et al. (2020), and Math. Pandas was used for creating the transition matrices of the models used; while this could have been done using NumPy alone, the indexing used was more convenient in Pandas DataFrames. NumPy was then used for the matrix algebra that is discussed in Chapter 3. All models are implemented in the most generalized manner possible. This is important for analysis as in a less generalized format such as a spreadsheet it would not be as easy to perform sensitivity analysis on certain parameters. For example, the choice of Blue capacity C can greatly change the state space of the model. As the number of transient states is determined by the number of launchers K and the capacity C, the number of transient states being $(C - 1) \cdot (K - 1) + 1$. When we consider even how small differences in C can change the number of states it is useful to be able to quickly be able to build the model state space and use logic to determine the transition probabilities.

CHAPTER 3:
Model Analysis and Results

In this chapter, we will summarize results and insights gained from the models discussed in Chapter 2. We will progress through the results in the same manner as in Chapter 2, beginning with the Base Duel and working to our more complex models.

3.1 Absorption Probabilities

Before we begin to review results, we will describe our methodology for calculating absorption probabilities of the Markov Models used. The transition matrices presented in Chapter 2 are in a modified canonical form of Markov Chains. They have been of the form

$$\begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{3.1}$$

where if the entire state space is of m dimensions, and s of those states are absorbing, where $\mathbf{I}$ is an $s \times s$ identity matrix representing the transition probabilities for the absorption states, and $\mathbf{R}$ is an $m - s \times s$ matrix representing the probabilities of entering the absorbing states from the various transient states. $\mathbf{Q}$ is an $m - s \times m - s$ of the transition probabilities between the transient states.

Note the standard canonical form is:

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix}. \tag{3.2}$$

In this form, it can be shown that we can calculate absorption probabilities from any given transient state to an absorption by calculating.

$$(\mathbf{I}_{m-s\times m-s} - \mathbf{Q})^{-1}\mathbf{R} \tag{3.3}$$

The result of the above matrix algebra is an $m - s \times s$ matrix of absorption probabilities from particular transient states to specific absorbing states (Lin 2020). For all of our models and subsequent analysis, this process is automated in Python with the implementation discussed in Section 2.9.

3.1.1 Expected Duration of Duel

It will also be useful to calculate the expected duration of the duel until absorption. Again, utilizing the modified canonical form in Equation 3.1 we can calculate the expected duration of the duel from each transient state. Note that we will have $m - s$ transient states. If we denote these transient states as $t_i \; \forall \, i \; \in \{1, 2, ..., m - s\}$, then we can calculate the expected duration from any transient state t_i as:

$$\begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_{m-s} \end{pmatrix} = (\mathbf{I} - \mathbf{Q})^{-1} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \tag{3.4}$$

Note that in most circumstances we will only be interested in the duration of our duel from the starting state, which is the transient state t_1.

This calculation is also automated in the Python implementation introduced in Section 2.9.

3.2 Base Duel

We begin with the base duel model described in Section 2.2. Beginning with a numerical example where we have $p_b = p_r = 0.5$. Based on the transition matrix in Equation 2.4, we have the vector:

$$\begin{array}{c} \\ D \\ BV \\ RV \end{array} \begin{array}{c} \begin{array}{ccc} D & BV & RV \end{array} \\ \begin{bmatrix} 0.25 & 0.5 & 0.25 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \tag{3.5}$$

Based on our methods described in Section 3.1 we end up with absorption probability matrix:

$$\begin{matrix} BV & RV \\ [0.67 & 0.33] \end{matrix} \cdot \tag{3.6}$$

What we can see from this example where probabilities of kill for a salvo are equal is that there is a significant advantage gained from Blue shooting first. As we move on to further examples we will see this trend continues. Consequently, we will also continue to see that the most influential parameter in our models is p_b.

In further numerical examples it is useful to compare different values of p_b and p_r. In Figure 3.1 we have several curves that each represent various values of p_b for a single value of p_r. In the case where $p_r = 1$ we have a situation where Blue fires a salvo at Red, if they hit the duel is over with Blue victory. If the Blue salvo does not hit, they are killed with certainty by Red. This is obviously an extreme case where Red is very lethal. However, as an example, it gives some intuition for how the model behaves. In this case, we can see that the outcome of the duel is essentially the outcome of a single Bernoulli trial. This shows that in the worst-case scenario where Red can kill Blue with certainty in an exchange, Blue's lower limit to the probability of victory is p_b. In cases where $p_r < 1$ there is the chance that the duel re-enters the dueling state and subsequent salvos are exchanged. This results in more chances for Blue to win and the probability of Blue victory, or $P(BV)$, where $P(BV) > p_b$. In general, for the base model and all subsequent models we can say that:

$$P(BV) \geq p_b.$$

This simple finding echoes the fundamental lesson of Hughes' work on missile combat. Which is to "fire effectively first." As we analyze the more complex versions of the model we will continue to come back to this fundamental point. We will demonstrate that there are multiple ways to achieve a larger p_b, but regardless of how that is accomplished, achieving a high p_b is the most important factor in Blue victory.

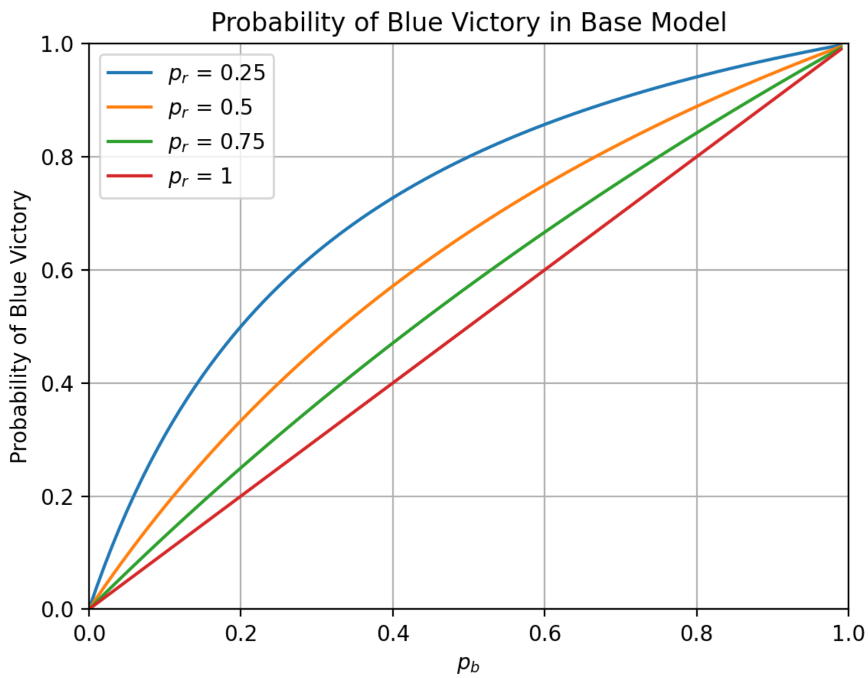

Figure 3.1. Probabilities of GBASM victory in base model for several values of p_r across a range of $p_s \in (0, 1)$.

3.3 Multiple GBASM Launchers

In Section 2.4 we discuss adding more than one GBASM platform. Here, we examine the results of the duel by examining the case where the starting number of GBASM launchers is $K = 2$. We again start with the case where $p_s = p_r = 0.5$. Using thee values in the transition matrix 2.9 we have:

$$\begin{array}{c} \\ B2 \\ B1 \\ BV \\ RV \end{array} \begin{array}{c} \begin{array}{cccc} B2 & B1 & BV & RV \end{array} \\ \begin{bmatrix} 0.0625 & 0.125 & 0.75 & 0.0625 \\ 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}. \tag{3.7}$$

We then have the absorption probability matrix:

$$
\begin{array}{c} \\ B2 \\ B1 \end{array}
\begin{array}{c} \begin{array}{cc} BV & RV \end{array} \\ \begin{bmatrix} 0.889 & 0.111 \\ 0.667 & 0.333 \end{bmatrix} \end{array}. \tag{3.8}
$$

The absorption probabilities in the $B1$ row vector are identical to the absorption matrix 3.6 in our base case where we only had one GBASM system. The $B2$ row vector shows the improvement in having the second GBASM platform firing an additional salvo.

Figure 3.2 is similar to Figure 3.1. In figure 3.1 we varied p_s for several values of p_r where we have one GBASM system. Likewise, in Figure 3.2 we examine the probabilities of Blue victory for several values on p_r when Blue begins with $K = 2$ GBASM launchers.

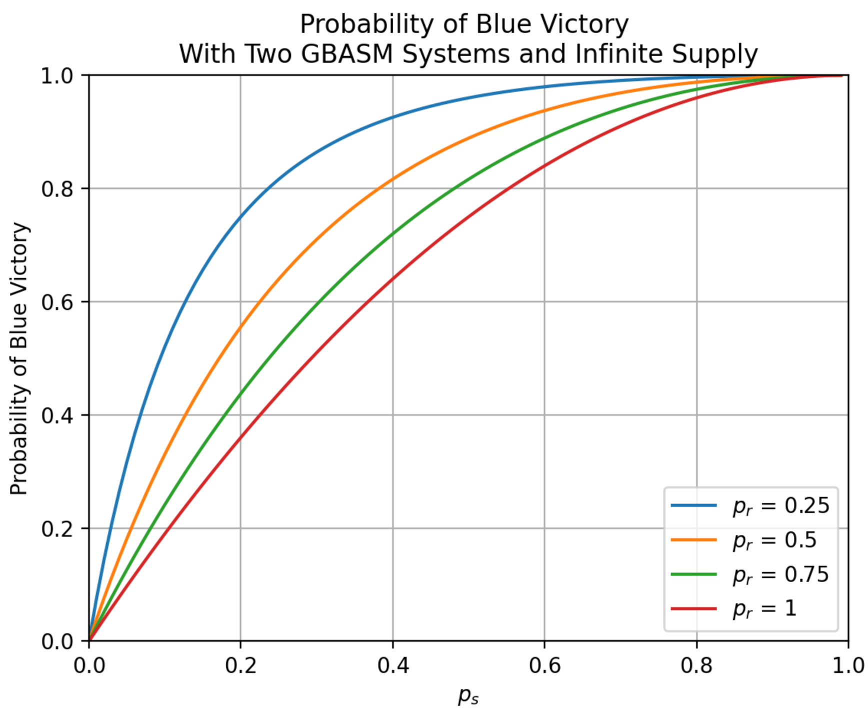

Figure 3.2. Probabilities of GBASM victory in the case where there are $K = 2$ GBASM launchers to begin the duel for various levels of p_r across a range of $p_s \in (0, 1)$.

Again, we can use the case where $p_r = 1$ set the lower bound on Blue's victory probability. In this scenario, we see that the two launchers will each fire a salvo with independent probabilities p_s of killing the Red ship. If both the GBASMs miss then they will both be killed by Red with absolute certainty. Thus given the first shooter advantage of Blue we see that the probability of Blue victory is the outcome of a Bernoulli trial. Specifically that $P(BV) = 1 - (1 - p_s)^2$. We once again see that the case where $p_r = 1$ provides a floor for $P(BV)$. Thus for the two GBASM case we have:

$$P(BV) \geq 1 - (1 - p_s)^2.$$

For a general number of GBASM k we have:

$$P(BV) \geq 1 - q_s^k.$$

We can make the additional observation that the marginal impact of having additional launchers is higher when p_r is smaller.

3.3.1 With Limited GBASM Salvo Capacity

Here, we will update the model to include the salvo capacity as we did in Section 2.5. For our numerical example shown in Figure 3.3, we will again use the number of launchers $K = 2$ and then add the number of salvos available to each GBASM system to be $N = 2$.

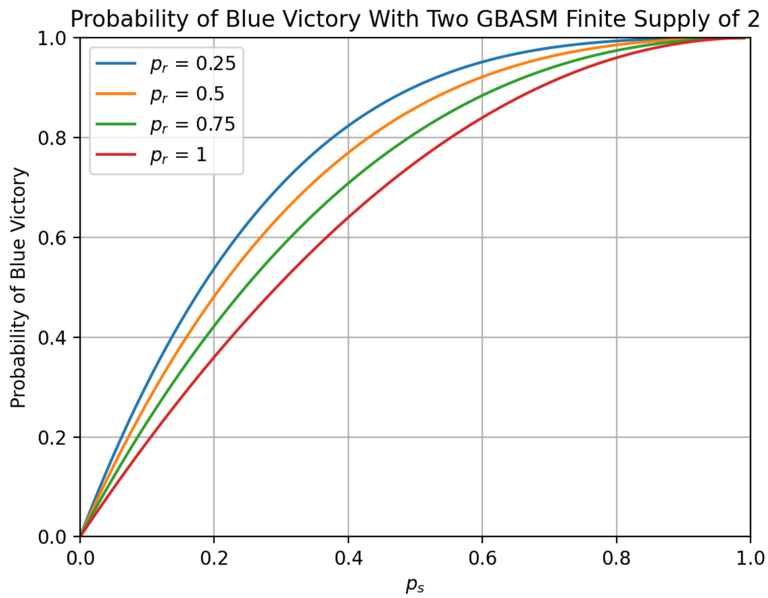

Figure 3.3. Probabilities of GBASM victory in the case where there are $K = 2$ GBASM launchers with finite supply of two missiles to begin the duel for various levels of p_r across a range of $p_s \in (0, 1)$.

In this example, we see the same floor created by the case where $p_r = 1$. This is due to the fact that regardless of the salvo capacity of Blue if Blue misses after the first salvo Red will kill all launchers with probability $p_r = 1$. Thus we still have that:

$$P(BV) \geq 1 - q_s^K.$$

However, the outcomes do change when Red is less lethal. When p_r is smaller in previous models we saw that Blue would not be killed as easily by Red, and Blue did not run out of missiles. In the case with an infinite capacity if p_r is very small the duel can continue for many rounds, but when we model the capacity of $C = 2$ we can only have two salvo exchanges. Thus the advantages of having more rounds of salvo exchanges from a low p_r is capped by C. This is shown in Figure 3.4 where we see that the largest marginal payoff for an increased salvo capacity happens on the lower end of Red lethality. We can also see that there is a point for all levels of p_r where little is gained from additional salvo capacity.

Salvo capacity will be discussed again in the context of the full model in Section 3.6.

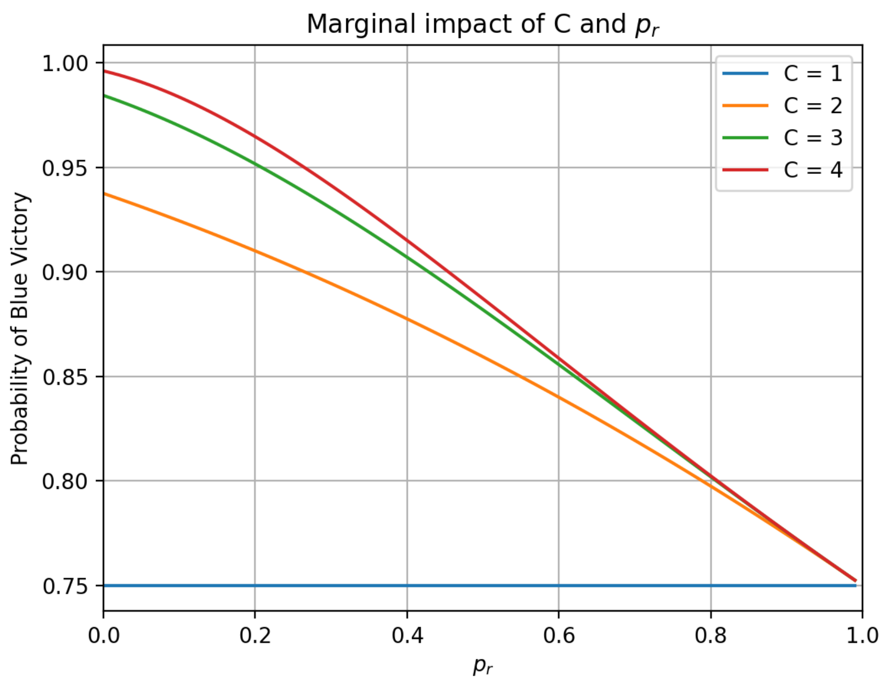

Figure 3.4. Impacts of having more salvo capacity, C, over different values of p_r. Note, $p_s = 0.5$

3.4 Salvo Makeup

The previous examples examine how p_b is the key input in providing a lower bound on the probability of Blue victory. With the model discussed in Section 2.6 we can evaluate how different GBASM configurations generate p_b and how configurations impact performance. Recall Equation 2.11which is the overall probability of kill of a single salvo given the number of launchers k and the number of missiles fired per platform N.

$$p_b(N, k, p_s) = 1 - (1 - p_s)^{Nk}.$$

3.4.1 Example Case

We choose an initial salvo size S and vary the number of GBASM launchers K and the number of missiles fired per platform N together to maintain that value S. Here, we set salvo size $S = 6$ and we examine all configurations of K and N such that $S = N \times K = 6$.

Those configurations are shown below.

- $N = 1, K = 6$
- $N = 2, K = 3$
- $N = 3, K = 2$
- $N = 6, K = 1$

We can then set values for our other parameters. Starting with our single-shot probability of kill for a single Blue missile, p_s which then determines the initial single salvo probability of kill.

$$p_b(S, p_s) = 1 - (1 - p_s)^S. \tag{3.9}$$

In this case we have have $S = 6$ and we set p_s to a low value, $p_s = 0.1$. Thus for our initial salvo before any attrition of launchers we have:

$$p_b(6, 0.1) = 1 - (1 - 0.1)^6 \approx 0.4686.$$

We set $p_r = 0.25$. Thus we have $p_s = 0.1$ and $p_r = 0.25$. Figure 3.5 shows the resulting probabilities of Blue victory for the various configurations listed above where $S = 6$.

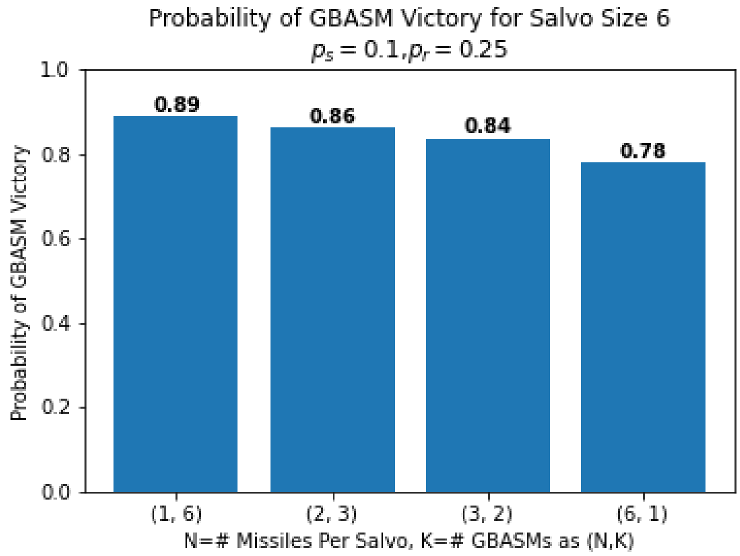

Figure 3.5. Probabilities of GBASM victory with a fixed salvo size while varying the number of GBASM firing launchers.

In this first example, the probability of GBASM victory is monotonically increasing with the number of launchers. This is due to the more survivable nature of having the salvo spread over more than one platform. Note that when $K = 1$ the duel is over as soon as one platform is hit. This is obviously not the case when K is larger. When K is larger there is a greater chance the duel will last for more rounds, increasing the chances that Blue will kill the Red ship.

3.4.2 Change in Single Shot Probability of Kill

We can also see how our results change as we vary the single-shot probability of kill for individual missiles, p_s, which in turn changes the salvo probability of kill p_b. Figure 3.6 shows the various configurations of N and K for $S = 6$, and $p_r = 0.25$ while also varying three levels of p_s.

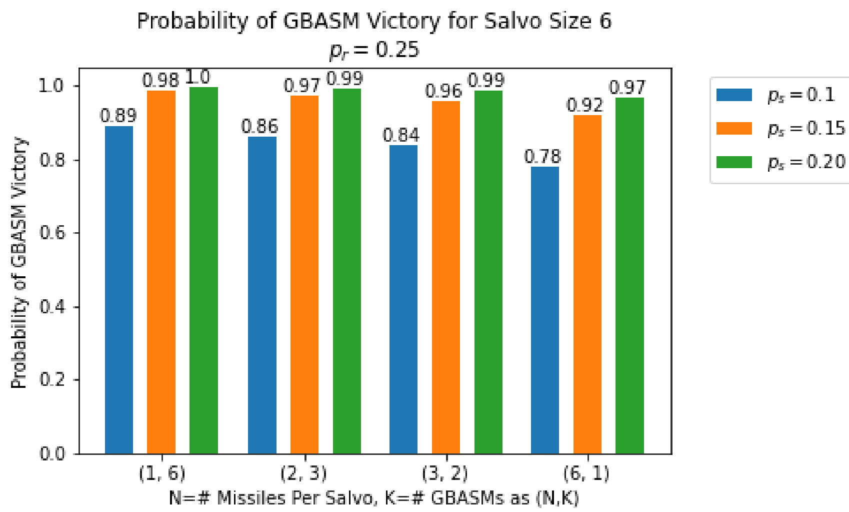

Figure 3.6. Probability of Blue victory for a fixed salvo size with differing configurations of N and K varying the lethality of Blue missiles.

We can see from Figure 3.6 that the probability of Blue victory is also monotonically increasing with p_s. What we do see is that as p_s and subsequently $p_b(N, K, p_s)$ increase, the added marginal benefits of having a larger K, Blue launchers, is smaller. This is further confirmation from our base model that a large p_b is the most important factor in determining the outcome of the duel.

What this also suggests is that while we can always improve the probability of Blue victory by adding more launchers, there is a point where the marginal benefit is likely not worth the additional cost.

3.4.3 Change in Red Lethality

Here, we continue to use our base case to examine the impacts of various levels of p_r. We return to having a single value of $p_s = 0.1$. Recall from before that with $S = 6$ and $p_s = 0.1$:

$$p_b(6, 0.1) = 1 - (1 - 0.1)^6 \approx 0.4686.$$

We then vary p_r between 0.25, 0.5, and 0.75. We can see the results of varying these values of p_r from our example case in Figure 3.7. We can see that the probability of Blue victory decreases as p_r increases, and as Red becomes more lethal, the marginal benefit of more launchers is again smaller. Recall in the Section 3.4.2 we found that more Blue launchers were most beneficial when p_b was small. Thus we can say that both a large p_b or a large p_r tend to dominate and be more important than any particular configuration of N and K. This again supports that achieving a large p_b is the most crucial part of winning the duel.

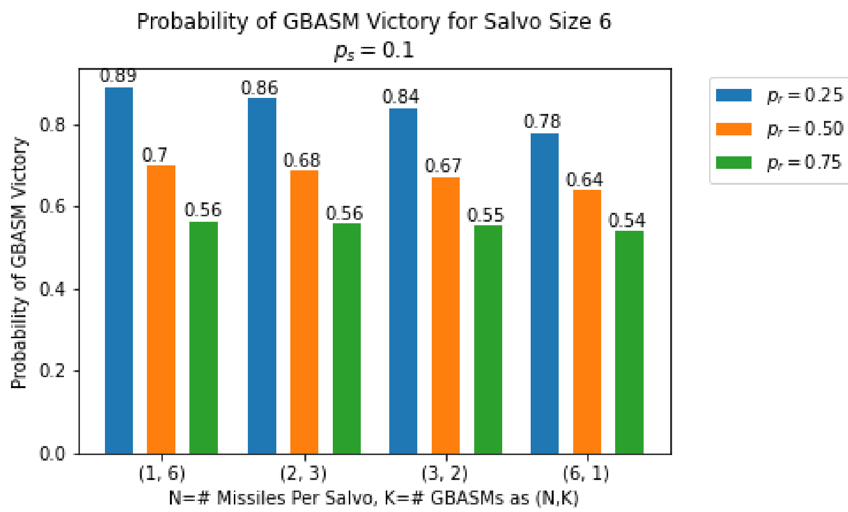

Figure 3.7. Probability of Blue victory for a fixed salvo size with differing configurations of N and K and varying Red lethality.

3.5 Determining Values of N and K

One of the decisions our research is trying to address is the choice of N and K in a salvo. We have already addressed some of the basic considerations. Here, we examine the tradeoffs.

As we have said, a high p_b is the best way for Blue to win any duel, thus we would want both N and K to be as large as possible if all we are concerned with is winning the duel. This obviously is not possible in reality, so it is worthwhile to examine the tradeoffs that exist.

We continue to use the model described in Section 2.6. We conduct our analysis based primarily on whether or not Blue wins the duel. Recall that from the transition matrix in Equation 2.15 there are multiple absorption states for Blue victory. Specifically, any state for Blue victory is denoted as $(BV,\ k)$ where k is the remaining number of Blue launchers. Investigating the surviving number of launchers k is certainly worthwhile as it gives indications of survivability and the sustainability of the fires in follow on engagements. However, for now, the main measure of effectiveness will be the probability of Blue victory in the duel.

Table 3.1 shows a numerical comparison of various combinations of N and K with fixed values of probability of kill of a single Blue missile (p_s), the probability Red kills each Blue platform (p_r), and the salvo capacity of each Blue platform (C). The range of the number of missiles N per salvo is from 1 to 4. This range is used as it seems in the realm of possibility for a launcher system but was not done with any current platform in mind. Further, the range from 1 to 4 missiles is wide enough to show the impacts of the choice of N. It should be noted again that we assume that all N missiles fired across all k launchers are fired near-simultaneously enough to appear as one salvo to the Red ship. The number of launchers K is varied across the range 1 to 6. This choice is made to reflect the current size of an artillery battery. The ranges were chosen for N and K are used throughout our analysis but should not be taken as a recommendation of any kind. The probabilities of Blue victory are calculated as discussed in Section 3.1.

Table 3.1. Probability of Blue Victory — $p_s = 0.6$, $p_r = 0.8$, $C = 10$

Number of Missiles **N**	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	0.6522	0.879	0.9579	0.9854	0.9949	0.9982
2	0.8678	0.9825	0.9977	0.9997	1.0	1.0
3	0.9481	0.9973	0.9999	1.0	1.0	1.0
4	0.9794	0.9996	1.0	1.0	1.0	1.0

We observe several things in the results shown in Table 3.1. First, we make the obvious observation that as total salvo size S increases Blue's probability of victory increase regardless of the particular combination of N and K. Next, when we look at different combinations for a particular salvo size S, we see that the probability of Blue victory increases as the number of launchers K increases. For example, in the cases where $S = 4$ from the combinations of

$N = 4,\ K = 1$ (0.9794); $N = 2,\ K = 2$ (0.9825); and $N = 1,\ K = 4$ (0.9854), we see that the highest chance of Blue victory occurs when $N = 1,\ K = 4$. We can also see that while the probability of Blue victory increases with K it is not a very large increase. This is the same behavior identified in Section 3.4.1.

Note that the numerical results here are sensitive to changes in parameters p_s and p_r. Specifically, we see an increase in the importance of a large K when the lethality of Red and Blue are both reduced. Observe the results in Table 3.2 where we reduce the lethality of Red and Blue with the parameters $p_r = 0.4$ and $p_s = 0.2$.

Table 3.2. Probability of Blue Victory — Reduced Lethality of Blue and Red — $p_s = 0.2,\ p_r = 0.4,\ C = 10$

Number of Missiles **N**	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	0.5492	0.7967	0.9084	0.9587	0.9814	0.9916
2	0.7368	0.9307	0.9818	0.9952	0.9987	0.9997
3	0.8264	0.9699	0.9948	0.9991	0.9998	1.0
4	0.8781	0.9852	0.9982	0.9998	1.0	1.0

As in the case displayed in Table 3.1 when we had $p_r = 0.8$ and $p_s = 0.6$, we again observe in Table 3.2 that the probability of Blue victory increases for a given salvo size S when we have a larger K. We also observe in this less-lethal scenario that the impact of a larger K is enhanced. Again looking at the case when $S = 4$ we see an 8% improvement from the case of $K = 1$ to $K = 4$. Compare this to our previous case shown in Table 3.1 where the improvement was less than 1%. In this lower lethality scenario, we see that adding more missiles to S regardless of N and K does not necessarily increase Blue's probability of victory. Observe the case for $S = 3$ when we have $K = 3$ launchers shooting $N = 1$ missile, the probability of Blue victory is larger in this case than for $S = 4$ where we have a single platform $K = 1$ shooting $N = 4$ missiles. We can make the specific observation that when we have less lethal configurations for Red and Blue, the duel is expected to last longer and that Blue gains an advantage from more distributed fires and thus a larger initial number of launchers K.

Also, note that we have held $C = 10$. This was done deliberately as $C = 10$ is a large enough salvo capacity for Blue that it really has no impact on the results discussed so far. We examine the implications of a smaller C later in Section 3.6.

3.5.1 Duel Outcomes with Red Countermeasures

Recall in Section 2.8.2 we introduced Red defensive countermeasures with the addition of the parameter λ where λ is the average number of Blue missiles Red can intercept in a given Blue salvo, modeled according to a Poisson distribution. We begin with cases where the salvo capacity of Blue is large enough so it is generally not a limiting factor. For now we have $C = 10$. Recall that C is the salvo capacity of each launcher, so the total number of missiles would be $N \cdot C$. We also fix values for $p_s = 0.6$, $p_b = 0.8$ and $\lambda = 2$ and then vary the configurations of the number of launchers K and the number of missiles per platform N. We do this for K ranging from 1 to 6 and N from 1 to 4. The resulting probabilities of Blue victory in the duel are shown in Table 3.3.

Table 3.3. Probability of Blue Victory with Red Countermeasures — $p_s = 0.6,\ p_r = 0.8,\ C = 10,\ \lambda = 2$

Number of Missiles N	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	0.0995	0.308	0.5513	0.7497	0.8764	0.9444
2	0.3228	0.7582	0.9465	0.9909	0.9986	0.9998
3	0.5717	0.9476	0.9965	0.9998	1.0	1.0
4	0.7635	0.991	0.9998	1.0	1.0	1.0

There are several things to note in the results displayed in Table 3.3. First observe the case where we have $N = 1$ and $K = 1$, which means a salvo size of $S = 1$. In the results that did not account for Red's countermeasures shown in Table 3.1 we had the probability of Blue victory of 0.6522. This is much larger than the value for $S = 1$ in Table 3.3 of

0.0995. Note that we do not reach this previous probability of Blue victory of 0.6522 again until the cases where $S = 4$, for the cases of $N = 4,\ K = 1$ (0.7635); $N = 2,\ K = 2$ (0.7582); and $N = 1,\ K = 4$ (0.7497). This shows how drastically the inclusion of Red's countermeasures impacts Blue's decision for appropriate salvo size. The choice for $\lambda = 2$ means that on average Red can intercept 2 incoming Blue missiles with a decreasing probability of intercepting more missiles according to a Poisson distribution.

There is one surprising outcome we observe in Table 3.3. In previous models before the inclusion of Red countermeasures, we observe that the probability of Blue victory is monotonically increasing with the number of Blue launchers K when we have a fixed salvo size S. However, this does not hold true here. Again observe the cases where we have $S = 4$, which is true when $N = 4,\ K = 1$, $N = 2,\ K = 2$ and $N = 1,\ K = 4$. Note that the resulting probability of kill with a single salvo p_b is equivalent in these three cases. According to the previously identified behavior, the probability of Blue victory should be higher when K is larger, but here we see the opposite is the case. The probability of Blue victory is actually highest when we only have a single Blue platform shooting 4 missiles per salvo. This is counter to the intuition that Blue's fires are more survivable and thus more lethal over the course of the duel.

This is a surprising and unexpected phenomenon but does have an explanation. When Red has an independent probability of killing any of the K GBASM launchers, in the case where $K = 4$, there is a larger probability that on the second salvo Blue will be shooting a smaller salvo than when $K = 2$ or $K = 1$. What we see with Red countermeasures when the salvo gets smaller it becomes ineffective. So, while a larger K may mean Blue will last more rounds, Blue may last more rounds just to be wasting missiles with small salvos.

This observation does not necessarily mean that Blue should go for the smaller configuration of K. It may suggest that it is good if Blue is able to dynamically adjust N as it takes casualties or find a breakpoint in the duel when Blue takes a certain number of casualties and can no longer return effective salvos. Something like: "if we lose two launchers, then we go into hiding," rather than continuing the duel. For now, we will not go into situations of alternate stopping conditions or dynamically updating salvo size as K changes in the duel. We will instead address the issue of independent chances of Red killing Blue launchers.

3.5.2 Duel Outcomes with Limits on Red Lethality

In analyzing our models we observe some unexpected results with regards to the make-up of a salvo in terms of N and K. Specifically, we would have expected to see a larger benefit from the distributed fires that come with a larger number of GBASM launchers K. However, this has not consistently been the case in our analysis. First, in Section 3.4.3 we observed that the benefits of a larger K are diminished when Blue and Red are lethal with high probabilities of kill. In Section 3.5.1, we observe the unexpected behavior that a larger K may actually lead to a smaller probability of Blue victory.

These observations lead us to the possibility that the way Red's targeting of Blue is currently modeled is not representative of behavior we would expect in reality. Thus we alter our model as described in Chapter 2.8.4. Recall that instead of treating Red's chances of killing all K launchers as independent and identical, we propose to add a parameter ϕ which represents the average number in a Poisson distribution of launchers that Red can kill in a given salvo exchange. This accounts for the limited capacity of Red to locate and prosecute distributed targets. Table 3.4 shows the probabilities of Blue victory with the same parameters as before in Table 3.3 but changing Red's process of killing of Blue launchers to the behavior described above with $\phi = 2$.

Table 3.4. Probability of Blue Victory — Limited Red Targeting — $p_s = 0.6,\ C = 10,\ \lambda = 2, \phi = 2$

Number of Missiles **N**	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	0.0927	0.3262	0.6113	0.8276	0.9404	0.9835
2	0.3061	0.7732	0.9611	0.9957	0.9996	1.0
3	0.5526	0.9515	0.9976	0.9999	1.0	1.0
4	0.7492	0.9917	0.9999	1.0	1.0	1.0

In the results here we see that as K increases the probability of Blue victory increases when we consider the same salvo size S. Consider when we have the salvo size $S = 4$. The difference when we have $K = 1$ platform shooting $N = 4$ missiles versus when we have $K = 4$ launchers shooting $N = 1$ missile we have an almost 8% increase in the Blue probability of kill. Though observe that with these parameters it is still more important to have a larger S than the make-up between N and K. We see in Table 3.4 that in any two cases where we have different salvo sizes S, the case where S is larger also corresponds to a larger probability of Blue victory. This continues to support the conclusion that the primary concern of Blue should be to determine a salvo size and then explore the tradeoffs in N and K to achieve that salvo size $S = N \times K$.

3.6 Capacity and Duration

So far, we have left the salvo capacity of Blue fixed at $C = 10$, which is a large enough capacity to be a non-constraining factor in the duels we have explored. Here, we explore what the impact of smaller salvo capacities is.

As a reminder from Section 2.7, the capacity C is the number of salvos that each GBASM platform can fire at a given number N missiles per salvo. Thus if we are looking for the total number of missiles on hand for Blue it would be $C \times N \times K$.

In Table 3.5, we can see the same probabilities of Blue victory as in Table 3.4 with the addition of the expected duration of the duel calculated as described in Section 3.1.1.

Table 3.5. Probability of Blue Victory —
$p_s = 0.6,\ C = 10,\ \lambda = 2, \phi = 2$

Number of Missiles **N**	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	0.0927	0.3262	0.6113	0.8276	0.9404	0.9835
2	0.3061	0.7732	0.9611	0.9957	0.9996	1.0
3	0.5526	0.9515	0.9976	0.9999	1.0	1.0
4	0.7492	0.9917	0.9999	1.0	1.0	1.0

Expected Duel Duration

Number of Missiles **N**	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	1.142	1.3567	1.42	1.3171	1.1788	1.0835
2	1.1086	1.1263	1.0502	1.0116	1.0021	1.0003
3	1.07	1.0288	1.0036	1.0003	1.0	1.0
4	1.0393	1.0052	1.0002	1.0	1.0	1.0

Note the bottom table is the expected duration of the duel regardless if the duel ends in Blue victory or not. What stands out in the values of the expected duration is that in no cases here is the duel expected to last much beyond one round. As a reminder, one round is Blue firing a salvo (and if it hits the round and duel are over), and Red has a chance to fire back. Thus we would expect when Blue has lethal combinations of fires the duration would be short. The same is true when Red has lethal fires. That is indeed what we observe here. Thus we can also see that Blue having a Capacity of $C = 10$ is likely not necessary to have enough salvos to complete the duel. To examine this we will keep the same model parameters and then change the capacity to $C = 2$. The results are shown in Table 3.6.

Table 3.6. Probability of Blue Victory —
$p_s = 0.6,\ C = 2,\ \lambda = 2, \phi = 2$

Number of Missiles **N**	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	0.0913	0.319	0.597	0.8121	0.9295	0.9781
2	0.3031	0.7689	0.9594	0.9954	0.9996	1.0
3	0.5502	0.9506	0.9976	0.9999	1.0	1.0
4	0.7481	0.9916	0.9999	1.0	1.0	1.0

Expected Duel Duration

Number of Missiles **N**	Number of Launchers **K**					
	1	**2**	**3**	**4**	**5**	**6**
1	1.1243	1.2939	1.3272	1.2393	1.1357	1.0661
2	1.098	1.1133	1.0455	1.0107	1.002	1.0003
3	1.0654	1.0273	1.0035	1.0003	1.0	1.0
4	1.0378	1.0051	1.0002	1.0	1.0	1.0

The decrease to $C = 2$ means that there is a maximum duration of two rounds for any duel. Recall that the model has a Winchester state when Blue runs out of missiles and neither side has been fully attired. When comparing these results to our previous results we do not see significant changes in the probability of Blue victory or in expected duel duration. In most combinations of N and K, we do see a slight decrease in the probability of Blue victory, but in all cases, the change is less than 0.01. In the cases where we see a decrease in the probability of Blue victory we also see a slight decrease in expected duration. This would indicate that more duels ending when Blue goes Winchester after two salvos rather than Red winning more engagements.

These small changes in expected duration and probability of Blue victory indicate that there is limited utility in extending Blue capacity when considering a single duel. As we have observed it is important to have a salvo with a high probability of killing Red in a single salvo rather than plan for an extended duel with a large salvo capacity C.

3.7 Cost-Benefit Analysis of N and K

We have found that it is important for Blue to fire a large enough Salvo S to achieve a large probability of kill in a single salvo. Our models further let us explore how to best make this salvo, where we spread out the salvo across K launchers firing N missiles each. We did find in most circumstances that there is a survivability of fires benefit to having the salvo spread out over a larger number of launchers. However, in most cases the benefit is not that significant.

In our discussion of results we speak in terms of the duel outcomes. However, cost of the various systems is also a factor in acquisition and force design decisions. While we do not speculate on the specific costs of the systems under consideration, we can make several inferences. In general, we can assume that it is more expensive to have more launchers in a GBASM battery. Additional launchers will mean additional requirements, in manpower, maintenance, and fuel like any other piece of military equipment. Given that we have found only minor benefits to highly distributed fires these additional costs should be carefully considered before defaulting to a battery with more launchers. We should also note that if there is a smaller amount of launchers, they must have the capability to fire all the required S missiles near-simultaneously to achieve the effects of firing as a salvo. Further note, this is all in the context of a duel with a single Red ship.efStochastic Modeling of Red Defensive Measures

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CHAPTER 4: Conclusions

The stated goal of this research is to provide analytical models to analyze the Marine Corps' development of a GBASM capability. To that end, we present a series of models of varying complexity to address key considerations associated with the use of this capability in a tactical scenario that can inform force design decisions. Our models address issues of the lethality of fires in terms of salvos and individual missiles, survivability of fires, the make-up of a GBASM battery, Red's counter-fire and probability of raid annihilation, and the impacts of distributed GBASM fires on Red's lethality. This is all done through the use of relatively simple, easy-to-understand, and transparent stochastic models.

4.1 Recommendations

The primary focus of the research is to provide an analytical baseline to provide specific force design recommendations. More careful consideration is needed of how current systems and systems in development will affect the choices of certain model parameters and assumptions. In developing the models, we examined numerical results to help understand the behavior of the models and their sensitivity to the various assumptions and parameters. From this process, we are able to provide the robust analytical baseline presented here and provide some general force design recommendations.

4.1.1 Limitations

It should be noted that no parameter space explored is done with specific knowledge of systems under consideration. Thus no results displayed in Chapter 3 or here should be viewed as such.

Another limitation of the model that should be noted is that all duels are between a GBASM battery and a single Red surface ship. This does not specifically address two things. First, a duel may be more dynamic between more combatants and systems on either side. Second, the ending conditions of Blue are not specifically analyzed for setting conditions for continuing to follow on duels. However, in terms of the second, the model does provide the ability to analyze these conditions as for both Blue victory and Winchester absorption states we have an absorption state to indicate the number of remaining GBASM launchers which would be an indicator of ability to carry out follow on duels.

4.1.2 Fire Effectively First

When we consider all the various aspects of the duels between GBASM and surface ships the maxim to "fire first effectively," holds as the paramount consideration Hughes (2018). In terms of our models, this amounts to selecting a combination of a missile with a p_s, a large enough salvo size S, when matched up against a ship with the ability to intercept missiles at a rate λ to achieve a high single salvo probability of kill p_b. Other factors such as the survivability of fires are important, but when Blue has the ability to fire first we see the best way to have to remain survivable is to eliminate the threat in a single salvo.

That being said, the sum of our analysis is not simply fire as many as possible missiles to win. We also demonstrated the ability to analyze the appropriate salvo size S in terms of marginal effectiveness of added missiles in Section 2.8.2. We observe that the probability of kill from a salvo asymptotically approaches 100% as S gets large and that the probability of Blue victory increases monotonically up to this point, there is also a point where the marginal utility of adding more missiles decreases.

4.1.3 Survivability and Distributed Fires

A key consideration in the EABO concept is the idea of distributed fires to achieve survivability. In our analysis, we find that within the context of a single duel this notion is sensitive to the lethality of Red and to certain model assumptions. We observe in Section 3.5.1. That without the ability to dynamically update Blue salvo size that distributed fires across more Blue launchers has an adverse impact on Blue's probability of victory. We also must note that this finding was under specific conditions regarding both the lethality and method of modeling Red fires and should not be generalized. Under other conditions, we see an increase in the number of Blue launchers K and thus the distribution of fires has a positive impact on Blue's suitability and thus on their overall probability of victory. Though the degree to which this distribution of fires leads to a higher probability of Blue victory is sensitive to the conditions of the model, particularly the lethality of Blue and Red salvos. So, while we demonstrate a positive impact from distributed fires, it is subordinate in importance to the ability of Blue to mass an effectively large salvo, whether the missiles in that salvo come from distributed launchers or not.

We do find that when we consider Red has a limited ability to locate and target Blue launchers the impact of distributed fires is greater. This is discussed in Section 3.5.2.

4.1.4 Tactics

As we have shown that the ability to fire first effectively is paramount in our considerations, this is also the primary driver of tactics that should be considered. The ability to fire first comes through the employment of scouting techniques to acquire Red targets and to remain in hiding until Blue has to fire. Once the duel starts the question is whether or not Blue should move all of its launchers at once and shoot in mass or employ a shoot when ready tactic. When considering that Blue fire is most effective in the form of massed salvos, it is apparent that the movements and fires will be most effective when synchronized.

4.2 Future Work

There are several categories we can place future work in, there are model expansions, analysis using the model, and work related to GBASM and EABO concepts not directly related to the models discussed in this thesis.

One obvious extension of the model is to consider duels that involve more than one Red ship. Salvo models from Hughes (1995) and Armstrong (2005) decrements combat power as ships take damage short of a kill. It is possible this level of detail could be beneficial, particularly when modeling duels with larger more resilient Red ships.

As the primary output of this research is modeling and not analysis, there is significant future work in terms of analysis. First, this would include using the model in its current form and to evaluate possible duels by analyzing current systems and using the parameters from that analysis. The model provides a rich parameter space, but all results shown in this thesis are done without knowledge of specific systems that may engage in the duel. Thus it is necessary to evaluate the value of the parameters used for making recommendations. Also, we did not evaluate all the different absorption probability states for Blue Victory and Winchester states. This goes beyond the analysis of the probability of winning the duel that is done here and will give an idea of the long-term survivability and supportability of the capability in sustained combat operations.

Additional modeling is needed to connect this tactical-level model of duels to operational level employment. This effort would evaluate the operational effectiveness of the GBASM capability. This would likely be in the form of an optimization model that would look to maximize the operational effectiveness of employing GBASM batteries in a specific geographic location. This would more directly answer questions regarding the range of missiles, and the number of forces required to achieve desired results. There is also the issue of supporting GBASM batteries and EABO sites more generally with logistic support.

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