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PENETRATION OF GAMMA RADIATION THROUGH THICK TARGETS

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ABSTRACT

An experimental and theoretical investigation has been made of the modifications in intensity and spectral distribution of a monochromatic beam of gamma rays due to scattering and absorption in thick shields of lead and aluminum. The intensity is calculated by considering the multiple scattering as taking place in a succession of steps. The spatial distribution is deduced from the equilibrium between quanta in a given group and those added to and removed from this group by scattering and photoelectric absorption.

PROBLEM STATUS

This is an interim report on this problem; work is continuing.

AUTHORIZATION

NRL Problem H13-01R

NR 463-010

PENETRATION OF GAMMA RADIATION THROUGH THICK TARGETS

INTRODUCTION

The penetration of gamma radiation through thick targets results in a modification of the intensity and spectral distribution by multiple scattering[1,2,3] and direct absorption taking place in the medium. The generation of soft secondary radiation by multiple scattering leads to equilibrium between the primary and secondary radiation. When equilibrium obtains, the ratio of primary to secondary intensity approaches a constant and the spectrum is independent of the depth of penetration. The region in which equilibrium exists should therefore be characterized by an exponential attenuation of the transmitted radiation. That this is the case is shown experimentally.

The intensity transmitted through a thick barrier, due to a flux of radiation incident normally upon a surface, is calculated by assuming that the multiple scattering takes place in a series of steps. The spatial distribution of radiation is deduced by assuming that equilibrium exists between quanta in a given energy group and those added to or removed from this group by scattering and absorption. An experimental method of observing these effects is described and theoretical calculations are compared with the experimental results.

THEORY

The multiple scattering occurring in a medium can be described by dividing the quanta into groups determined by the number of scatterings, i.e., the k^{th} group of quanta have been scattered k times. If it is now assumed, as in footnote 3, that all quanta of the k^{th} group have the mean energy of the group, then the energy of the k^{th} group is

$$E_k = \frac{\sigma_s}{\sigma} E_{k-1} \qquad (1)$$

where σ_s/σ is evaluated at the energy E_{k-1}. The ratio σ_s/σ is the fraction of incident energy given off in the form of a scattered quantum. σ_s/σ is calculated by integrating the Klein-Nishina formula for intensity over a sphere and dividing by the product, σ I, of the total cross section and the incident intensity. The succession of energy values assumed by quanta of 1.25 Mev initial energy[4] is given in Table I.

To calculate the spatial distribution of radiation in a plane target of infinite area, upon a surface of which a flux of quanta F is incident, let $F_k = F_k^+(\rho) + F_k^-(\rho)$. Here $F_k(\rho)$ is the number of quanta of a group k crossing unit area per sec. at a depth ρ in the target. The quantity F_k^+/F_k is the fraction of group k quanta making angles θ between zero and $\pi/2$ relative to

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[1] Hirschfelder, J. O., Magee, J. L. and Hull, M. H., The Penetration of Gamma-Radiation Through Thick Layers. Phys. Rev. 73, 852-862 (1948).

[2] Bethe, H. A., Fano, U. and Karr, P. R., Penetration and Diffusion of Hard X-Rays Through Thick Barriers. Phys. Rev. 76, 538-540 (1949).

[3] Faust, W. R., Multiple Compton Scattering II. Phys. Rev. 77, 227 (1950).

[4] Mean Energy of the two Co^{60} gamma rays. Lind, D. A., Brown, J. R. and DuMond, J. W. M., Precision Wave-Length Measurements of the 1.1 and 1.3-Mev Lines of Co^{60} with the Two-Meter Focusing Curved-Crystal Spectrometer, Phys. Rev. 76, 1838-1845 (1949) give the energies of these lines as 1.1715 and 1.3316 Mev.

the normal of a plane parallel to the target surface. Similarly F_k^-/F_k is the fraction with $\frac{\pi}{2} < \theta \leqslant \pi$.

TABLE I
GROUP ENERGIES AND ABSORPTION COEFFICIENTS

Energy (Mev)	σ_s/σ	Aluminum density = 2.69 gm/cm^3					Lead density = 11.15 gm/cm^3				
		μ^+ (cm^{-1})	μ^- (cm^{-1})	μ_{c1} (cm^{-1})	μ_{ph} (cm^{-1})	μ (cm^{-1})	μ^+ (cm^{-1})	μ^- (cm^{-1})	μ_{c1} (cm^{-1})	μ_{ph} (cm^{-1})	μ (cm^{-1})
1.25	0.533	0.10922	0.03758	0.14680	0.00005	0.14685	0.36808	0.12662	0.49470	0.1352	0.6299
0.666	0.619	0.140779	0.05776	0.198539	0.000095	0.198634	0.4744	0.19266	0.66906	0.4825	1.1516
0.413	0.681	0.16495	0.078168	0.243116	0.00028	0.243395	0.55586	0.26343	0.81929	1.482	2.3013
0.281	0.735	0.18358	0.09782	0.28139	0.00269	0.28408	0.618633	0.32964	0.94827	4.770	5.766
0.208	0.792	0.19712	0.11461	0.311728	0.00768	0.319408					
0.165	0.815	0.20640	0.13658	0.34299	0.01559	0.35858					
0.135		0.21355	0.14153	0.355081	0.03223	0.387311					
0.113		0.21956	0.15385	0.37342	0.05889	0.43231					

The total cross section can also be decomposed in a similar manner, i.e., $\sigma_k = \sigma_k^+ + \sigma_k^-$ where σ_k^+/σ_k is the fraction of quanta scattered in the directions with $0 < \theta \leqslant \pi/2$, and σ_k^-/σ_k is the fraction scattered into $\pi/2 < \theta \leqslant \pi$. The quantity σ_k^+ is obtained by integration of the differential cross section between zero and $\pi/2$ while the integration is between $\pi/2$ and π for σ_k^-.

It will be assumed that equilibrium exists between the quanta in a given group and those added to and removed from this group by scattering and absorption. If N is the electron density in the target then the contribution to $F_k^+(\rho)$ from the region between ρ and $\rho + \delta\rho$ is

$$N\sigma_{k-1}F_{k-1}^+\delta\rho\,\frac{\sigma_{k-1}^+}{\sigma_{k-1}} + N\sigma_{k-1}F_{k-1}^-\delta\rho\,\frac{\sigma_{k-1}^-}{\sigma_{k-1}} \tag{2}$$

This formulation is not entirely correct since it assumes, for example, that the first term represents outward scattered quanta. Actually quanta traversing the region between ρ and $\rho + \delta\rho$ in a solid angle $\delta\Omega$ at angle θ will be scattered so that some will also be scattered inward.

In spite of this objection (2) will be used in the following calculations. The forward component of group k emerging from the region between ρ and $\rho + \delta\rho$ is then

$$F_k^+(\rho + \delta\rho) = F_k^+(\rho) - N(\sigma_k + \varphi_k)F_k^+\,\delta\rho + N\left[\sigma_{k-1}^+F_{k-1}^+(\rho) + \sigma_{k-1}^-\,F_{k-1}^-(\rho)\right]\delta\rho. \tag{3}$$

Here $N(\sigma_k + \varphi_k)F_k^+\ \delta\rho$ represents the quanta removed from the k^{th} group by scattering and absorption; φ_k is the photoelectric cross section per electron for the k^{th} group. If terms of order $(\delta\rho)^2$ are neglected, then

$$\frac{\partial F_k^+}{\partial\rho} + \mu_k F_k^+ = \mu_{k-1}^+F_{k-1}^+ + \mu_{k-1}^-F_{k-1}^- \tag{4}$$

where $\mu_k = N(\sigma_k + \varphi_k)$, etc. In a similar manner it can be shown that

$$-\frac{\partial F_k^-}{\partial\rho} + \mu_k F_k^- = \mu_{k-1}^-F_{k-1}^+ + \mu_{k-1}^+F_{k-1}^- \tag{5}$$

In the case of a plane parallel beam incident upon a target of thickness Z, the boundary conditions are $F_0^+(o) = F$ (no. of incident quanta per cm^2 per sec.), $F_k^-(Z) = 0$. Particular solutions of equations (4) and (5) satisfying these boundary conditions are:

$$F_k^+(\rho) = e^{-\mu_k \rho} \int_0^\rho e^{\mu_k \xi} \left\{ \mu_{k-1}^+ F_{k-1}^+ + \mu_{k-1}^- F_{k-1}^- \right\} d\xi \tag{6}$$

$$F_k^-(\rho) = e^{\mu_k \rho} \int_\rho^Z e^{-\mu_k \xi} \left\{ \mu_{k-1}^- F_{k-1}^+ + \mu_{k-1}^+ F_{k-1}^- \right\} d\xi \tag{7}$$

An obvious solution of (4) and (5) for the zero group is $F_0^+ = F e^{-\mu_0 \rho}$, $F_0^- = 0$. Equations (6) and (7) can be solved successively, and numerical results are given in Tables II and III for lead and aluminum respectively. Table IV lists the solutions (6) and (7) for lead and aluminum.

TABLE II

NUMERICAL SOLUTIONS FOR F_k^+ FOR LEAD

Z (cm)	F_0^+	F_1^+	F_2^+
0	1	0	0
1	0.534	0.0839	0.0255
2	0.2845	0.131	0.030
3	0.1563	0.0881	0.0228
5	0.0429	0.0398	0.00785
7	0.01225	0.00835	0.00251
9	0.00345	0.00243	0.000705
11	0.001	0.000705	0.000204

Table I lists the total absorption coefficients as well as the partial scattering coefficients $\mu +$ and $\mu -$ for both lead and aluminum. The photoelectric absorption was calculated from the exact theoretical values of Hulme[5] while the scattering cross section was calculated from the Klein-Nishima formula.

The radiation emerging from the target may be observed on a Geiger Counter. If the effective area of the counter is A and its efficiency for the k^{th} group is E_k, then it will register counts at a rate

$$C = A \sum_{k=0}^{k=\alpha} \varepsilon_k F_k^+(Z) \tag{8}$$

where $F_k^+(Z)$ is evaluated at the far side of the target. Generally the detector ceases to record at some energy E_ρ so that the above expression converges and may be written

$$C = A \sum_{k=o}^{k=\rho} \varepsilon_k F_k^+(Z) \tag{9}$$

where ρ is the group number corresponding to the counter cut-off energy.

DESCRIPTION OF EXPERIMENT

The experimental apparatus was constructed as shown schematically in Figure 1. Lead and aluminum sheets approximately 3 ft by 3 ft in area were used as target materials. A Geiger Counter, of much smaller area than the target, was placed in the center of and directly behind the target. The behavior, then, of the target was essentially that of an infinite plane. By placing a 183-rd, Co^{60} source, 12 feet in front of the target, a constant and approximately normal flux was incident upon the front surface of the target.

Counters used were constructed of 1-inch copper tubing with 1/32-inch walls and were filled to a pressure of 29 cm with a 5 to 1 ratio of argon to ether mixture. The measured efficiency of

[5] Hulme, H. R. and Jaeger, J. C., On the Production of Electron Pairs. Proc. Roy. Soc. 153, 443-447 (1935). Values of μ_{ph}, outside the range of values given by this reference were obtained by extrapolation, using the formulas given in Heilter, Quantum Theory of Radiation, 2 ed. pp. 123-6. Oxford (1944).

TABLE III

NUMERICAL SOLUTIONS, F_k^+, FOR ALUMINUM

Z (cm)	F_0^+	F_1^+	F_2^+	F_3^+	F_4^+	F_5^+	F_6^+
0	1.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.86344	0.09196	0.006896	0.000575	0.000097		
3		0.19537	0.04213	0.00796	0.00134	0.00024	0.000013
5	0.47986	0.229437	0.078132	0.022689	0.00456	0.00098	0.000225
7		0.229437	0.10276	0.03909	0.0118	0.00301	0.00090
10	0.2307270	0.19634	0.116147	0.057816	0.0219	0.00754	0.00268
15	0.110495	0.12588	0.07195	0.065055	0.0345	0.01698	0.0085
20	0.053024	0.07214	0.067787	0.052971	0.0337	0.0178	0.0105
25	0.025444	0.038976	0.040987	0.036323	0.0258	0.0167	0.0104
30	0.012209	0.020307	0.023398	0.022417	0.0151	0.0117	0.0092
35	0.005859	0.010364	0.012675	0.012822	0.0104	0.0077	0.00477
40	0.002811	0.005183	0.006646	0.0061037	0.00582	0.00449	0.00343
45	0.01349	0.002569	0.003494	0.003777	0.00313	0.00257	0.00201
50	0.0006473	0.0012628	0.001719	0.0023535	0.00196	0.00151	0.00120
55	0.00031067	0.0006174	0.0008513	0.0009951	0.00105	0.00087	0.00063
60	0.00014908	0.0003004	0.0004235	0.0004976	0.0005	0.00047	

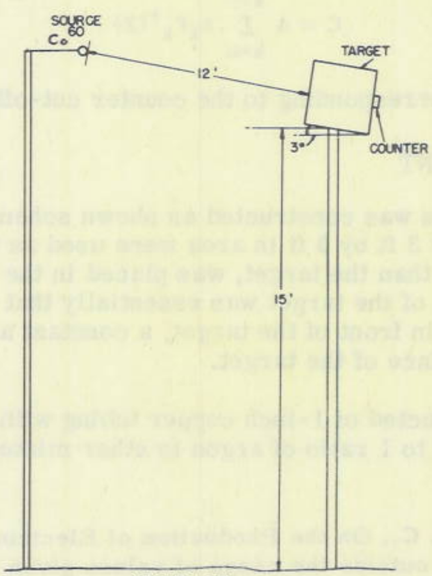

Fig. 1: Schematic Arrangement of experimental apparatus.

TABLE IV

Solutions of Equations (5)
Evaluated at Target Thickness Z cm

I Lead

(a) $F_o^+ = e^{-\mu_o Z}$

(b) $F_1^+ = 0.7055\ (e^{-\mu_o Z} - e^{-\mu_1 Z})$

(c) $F_2^+ = 0.20448\ e^{-\mu_o Z} - e^{-\mu_1 Z}\ (0.291126) + 0.082677\ e^{-\mu_2 Z}$
$+ 0.082677\ e^{-(\mu_o + \mu_1 + \mu_2)Z}$

II Aluminum

(a) $F_o^+ = e^{-\mu_o Z}$

(b) $F_1^+ = 2.10938\ (e^{-\mu_o Z} - e^{-\mu_1 Z})$

(c) $F_2^+ = 3.126687\ e^{-\mu_o Z} - 6.63433\ e^{-\mu_1 Z} + 3.49307\ e^{-\mu_2 Z}$
$+ 0.0142123\ e^{-(\mu_o + \mu_1 + \mu_2)Z}$

(d) $F_3^+ = 3.890\ 11\ e^{-\mu_o Z} - 13.01782\ e^{-\mu_1 Z} + 14.160110\ e^{-\mu_2 Z}$
$- 5.030710\ e^{-\mu_3 Z} - 0.0076909\ e^{-(\mu_o + \mu_1 + \mu_2)Z} + 0.04254\ e^{-(\mu_o + \mu_2 + \mu_3)Z}$
$+ 0.0043294\ e^{-(\mu_o + \mu_1 + \mu_3)Z} - 0.0408459\ e^{-(\mu_1 + \mu_2 + \mu_3)Z}$

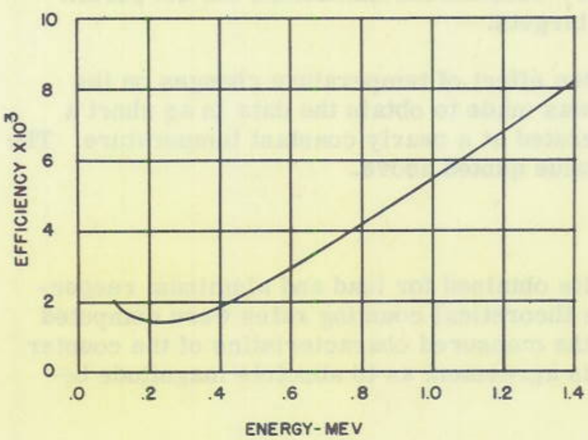

Fig. 2: Efficiency of Gamma-ray counter.

this counter agreed within experimental error with the results of Bradt[6]. Bradt's curve of counter efficiency, reproduced in Figure 2, was therefore used in calculation of the counting rate. The effective area of this counter was 4.35 sq inches.

In order to minimize the effect of scattering off the ground, the target was placed 15 ft above the ground. To determine this distance a counter was mounted behind a lead shield 20 cm thick and a source placed 3 feet in front of the shield. This apparatus was then raised off the ground as a unit and the counting rate observed, as a function of the height. It was found that at distances greater than 13 feet off the ground the counting rate became constant though still slightly greater than background. This residual counting rate was ascribed to air scattering.[7] The curves representing these results are shown in Figure 3.

The experiment proper consisted simply of observing the counting rate obtained through various thicknesses of lead and aluminum. Shielding material was added in the case of lead until the counting rate decreased to a constant value (above background). This residual counting rate was ascribed to air scattering, an assumption demonstrated to be correct by placing a one-inch lead shield directly behind the counter, in which case the counting rate (above background) was

[6]Bradt, H. et al, Empfindlichkeit von Zählrohren mit Blei-, Messing-, und Aluminiumkathode für γ-Strahlung im Energieintervall 0,1 MeV bis 3 MeV. Helv. Phys. acta. 19: 77-90, 1946.

[7]Cowan, C. L., The Absorption of Gamma-Radiation. Phys. Rev. 74, 1841-1845 (1948).

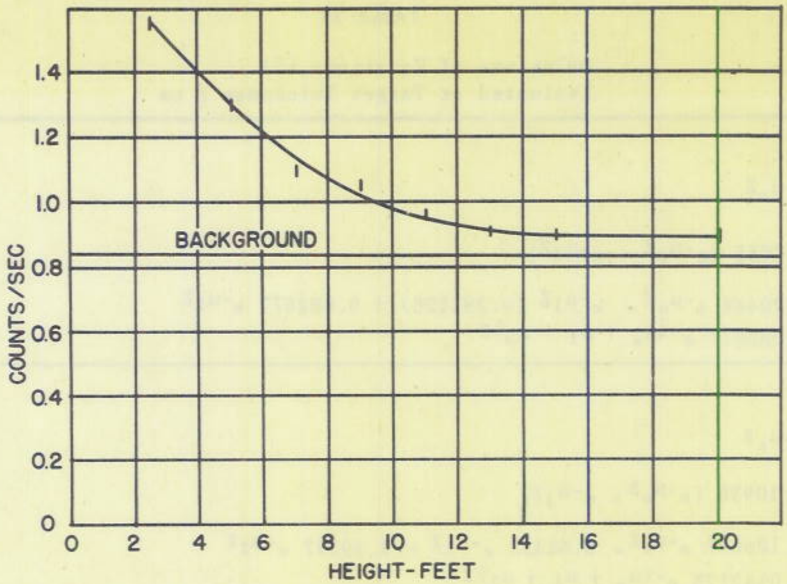

Fig. 3: Counting rate of Test apparatus as a function of height above ground. Data was used to estimate the scattering from the ground.

attenuated by a factor of approximately 30 times. Estimates of the mean energy of the air scattering, made from the known thickness of the shield, indicated that it was about 0.6 Mev, which is consistent with multiple scattering in the air. Mechanical limitations did not permit the air scattering to be observed with the aluminum targets.

The maximum variation in counting rate due to the effect of temperature changes on the counter efficiency did not exceed 1.5%. An attempt was made to obtain the data in as short a period as possible so that the counting apparatus operated at a nearly constant temperature. The error due to this effect was never greater than the value quoted above.

DISCUSSION OF RESULTS

Figures 4 and 5 compare the experimental results obtained for lead and aluminum respectively with the theoretical counting rate curves. The theoretical counting rates were computed from equation 9 with the aid of Tables II and III and the measured characteristics of the counter and source. As can be seen from the curves, there is agreement as to absolute magnitude between theory and experiment.

In Figure 4, the observed counting rates above background are represented by the short vertical lines, the length of which indicates the probable error. The observed counting rate decreased to a constant value of 0.232 counts per second beyond 12 cm of lead. This residual counting rate was shown to be due to air scattering from the source and must be subtracted from the raw data. The data corrected for air scattering is represented by the lower group of vertical lines. Theoretical results are represented by the solid line which passes close to the experimental points.

For thickness greater than 5 cm of lead the attenuation of the transmitted radiation is exponential in character. This indicates that the ratio of primary to total scattered intensity is a constant and that the spectral distribution is independent of the depth of penetration in this region. The results of Table IV well illustrate this equilibrium.

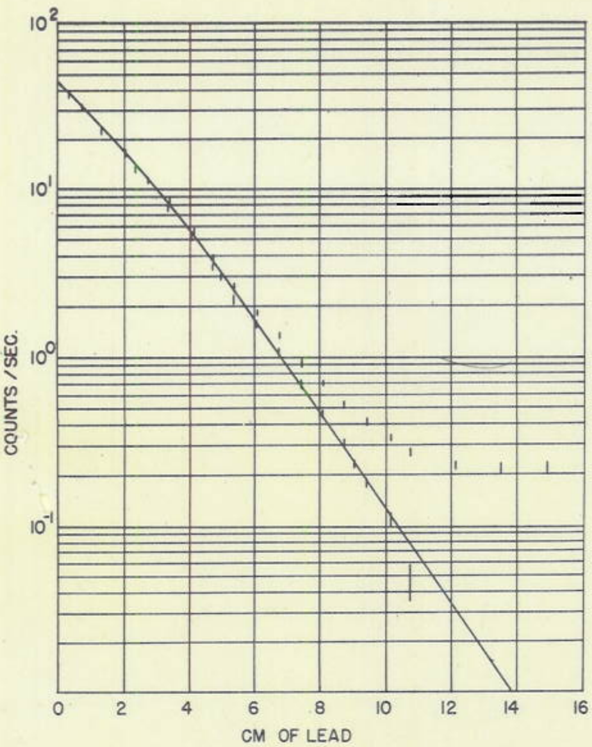

Fig. 4: Transmission of 1.25 Mev gamma rays through lead. The solid curve was calculated from equation 9 with the aid of Table II and the characteristics of source and counter. Raw experimental data is represented by the upper group of short vertical lines. The experimental data corrected for air scattering is given by the lower series of vertical lines.

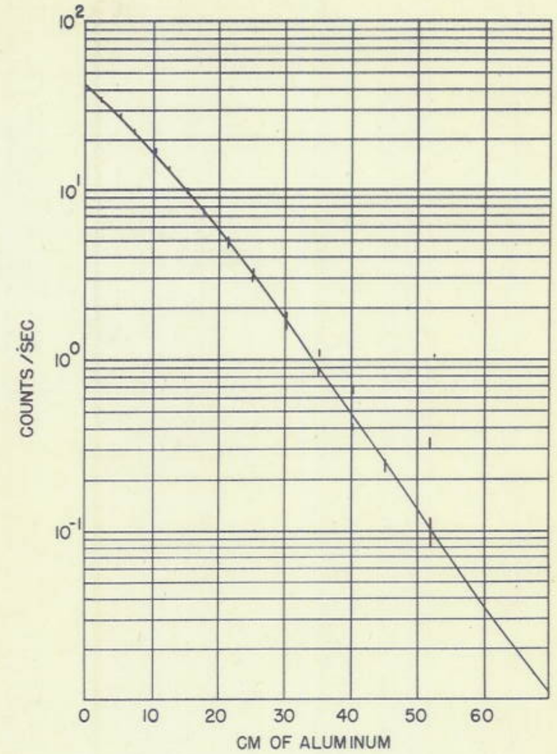

Fig. 5: Transmission of 1.25 Mev gamma rays through Aluminum. The solid curve was calculated from equation 9 with the aid of Table III and the characteristics of source and counter. Raw experimental data is represented by the upper group of vertical lines. The experimental data corrected for air scattering is given by the lower series of vertical lines.

Essentially the same results were obtained for the aluminum shielding experiments as shown in Figure 5. Generally the agreement of theory and experiment has been good (within 5%) so that these methods are probably reliable for all energies below 1.25 Mev. For initial energies much greater than 1 Mev, where pair production is appreciable, the theoretical results should be modified to include absorption due to this effect.

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