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THE OUTPUT CURRENT OF A NONLINEAR DEVICE

Bernard Salzberg

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Dr. J. M. Miller, Superintendent, Radio Division I

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CONTENTS

ABSTRACT

A voltage consisting of a cosine term superposed on a steady term is applied to the input of a nonlinear device. The output current of the device is zero for negative voltages, and proportional to the nth power of the voltage when it is positive, the exponent n being 1, 3/2, or 2. The resultant current may be zero, may be continuous, or may be discontinuous, depending on the relative values of the steady term and the peak value of the cosine term. In general, the current may be represented by a Fourier series, the coefficients of the various terms of which are functions of n and the magnitudes of the two components of the voltage. For n = 1 and n = 2, these coefficients are expressible in terms of trigonometric functions. For n = 3/2, the coefficients are given in terms of the complete elliptic integrals K and E. The root-mean-square value of the current, for n = 1, 3/2, and 2, is given in terms of trigonometric functions. Graphs of coefficients of the leading Fourier terms, and of the root-mean-square current, are presented for both continuous and discontinuous current, for n = 1, 3/2, and 2.

PROBLEM STATUS

The material presented here is incidental to the main problem, work on which continues.

AUTHORIZATION

NRL Problem P01-13R
NR 481-130

LIST OF SYMBOLS

i - instantaneous output current

v - instantaneous input voltage

G - constant of proportionality in the current-voltage law of the nonlinear device

n - exponent in the current-voltage law of the nonlinear device

V - peak value of alternating component of the voltage

V_o - value of the steady component of the voltage

$r = (V_o/V)$

θ - product of angular frequency and time

α - one-half of the total angle of conduction, when the current is discontinuous

$\hat{I}$ - maximum value of the current

I - root-mean-square value of the current

I_o - value of the steady term in the Fourier series for the current

I_m - magnitude of coefficient of the mth term in the Fourier series for the current

$K(k)$ - complete elliptic integral of the first kind

$E(k)$ - complete elliptic integral of the second kind

THE OUTPUT CURRENT OF A NONLINEAR DEVICE

INTRODUCTION

Analyses of the behavior of low-frequency circuits which utilize nonlinear electronic devices as oscillators, amplifiers, or harmonic-multipliers are usually based on results of the following situation: The output current of the nonlinear device is assumed to be zero when the input voltage is negative, and proportional to the nth power of the input voltage when it is positive. The exponent n is 1, 3/2, or 2, these values having been found typical of many vacuum tubes. The input voltage consists of an alternating term which is a simple harmonic function of time, superposed upon a steady polarizing term. The resultant output current may be zero, may consist of discontinuous pulses, or may be a continuous function of time, depending upon the relative values of the polarizing voltage and the amplitude of the alternating voltage. In general, the output current may be represented by a Fourier series, the coefficients of the various terms of which are functions of the exponent n, the polarizing voltage, and the amplitude of the alternating voltage. These coefficients are essential to analyses of the behavior of circuits utilizing nonlinear devices.[1,2,3,4] For n = 1 and n = 2, explicit expressions have been derived for the coefficients for the important case of discontinuous output current. For n = 3/2, coefficients of the leading Fourier terms have been determined by graphical methods for the case of discontinuous output current; explicit expressions for these coefficients, occasionally required in analysis, do not appear to have been worked out.

In this paper explicit expressions are derived for the coefficients of the Fourier terms when n = 1, 3/2, and 2, for the case of continuous output current, as well as for the case of discontinuous output current. When n = 1 and n = 2, the coefficients are expressible in terms of trigonometric functions. When n = 3/2, the coefficients are given in terms of the complete elliptic integrals K(k) and E(k). Explicit expressions are also derived for the corresponding values of root-mean-square output current; these also may be required occasionally. Graphs of coefficients of the leading Fourier terms and of the root-mean-square current are provided for both continuous and discontinuous output current, for n = 1, 3/2, and 2.

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[1] *Terman, F. E., and Ferns, J. H., "The Calculation of Class C Amplifier and Harmonic Generator Performance of Screen-Grid and Similar Tubes," Proc. I.R.E., vol. 22, No. 3, pp. 359-373, March 1934*

[2] *Terman, F. E., and Roake, W. C., "Calculation and Design of Class C Amplifiers," Proc. I.R.E., vol. 24, No. 4, pp. 620-632, April 1936*

[3] *Wagener, W. G., "Simplified Methods for Computing Performance of Transmitting Tubes," Proc. I.R.E., vol. 25, No. 1, pp. 47-77, January 1937*

[4] *Terman, F. E., "Analysis and Design of Harmonic Generators," Trans. A.I.E.E., vol. 57, pp. 640-644, November 1938*

ANALYSIS

The output current of the nonlinear device, expressed as a function of the input voltage, is

$$\begin{aligned} i &= 0 \quad \text{for } v \leqslant 0 \\ i &= Gv^n \quad \text{for } v \geqslant 0 \quad . \end{aligned} \tag{1}$$

The input voltage is

$$v = V \cos\theta + V_O \ , \tag{2}$$

where θ is the product of angular frequency and time. The voltage is symmetrical about θ, i.e., it has the same value for $-\theta$ as it has for θ, and it has a period 2π in the typical range $-\pi \leqslant \theta \leqslant \pi$. The current, correspondingly, is also symmetrical about θ, and it also has a period 2π in the typical range $-\pi \leqslant \theta \leqslant \pi$. The form of the current in any other similar range $(2m-1)\pi \leqslant \theta \leqslant (2m+1)\pi$ will be identical with that in the typical range; therefore only the latter need be considered.

When $V_O \leqslant -V$, the voltage is negative for all values of θ; the current, by equation (1), is therefore zero for all values of θ. This is a case of no interest. When $V_O \geqslant V$, the voltage is positive for all values of θ; the current is therefore continuous for all values of θ. In this case it is useful to write equation (2) as

$$v = V(\cos\theta + r) \ , \tag{2a}$$

where $r = (V_O/V)$ is a positive parameter having a value greater than or equal to unity. The corresponding form of equation (1) is

$$i = GV^n (\cos\theta + r)^n \quad . \tag{1a}$$

When $-V \leqslant V_O \leqslant V$, the voltage is positive for values of θ in the range $-\alpha \leqslant \theta \leqslant \alpha$, where α is defined by $\cos\alpha = -(V_O/V)$, and is negative for all other values of θ. The current is therefore discontinuous, being finite in the range $-\alpha \leqslant \theta \leqslant \alpha$, and zero for all other values of θ. For this case it is convenient to write equation (2) as

$$v = V(\cos\theta - \cos\alpha) \ , \tag{2b}$$

where the angle α has a value between 0 and π. The total angle during which the output current is finite, or the total "angle of conduction," is 2α. A useful form of equation (1) for this case is

$$\begin{aligned} i &= 0 \quad \text{for} \quad -\pi \leqslant \theta \leqslant -\alpha \ \text{ and } \ \alpha \leqslant \theta \leqslant \pi \\ i &= GV^n (\cos\theta - \cos\alpha)^n \quad \text{for } -\alpha \leqslant \theta \leqslant \alpha \quad . \end{aligned} \tag{1b}$$

The maximum value of current occurs when $\theta = 0$, so that

$$\hat{I} = G(V + V_O)^n = GV^n (1 + r)^n = GV^n (1 - \cos\alpha)^n \quad . \tag{3}$$

The mean-square value of the current is obtained by integrating the square of the current over the portion of a period during which it is finite, and dividing by 2π. Using equation (1a) this gives, for the case in which the current is continuous for all values of θ,

$$I^2 = (G^2 V^{2n}/\pi) \int_0^{\pi} (\cos\theta + r)^{2n} \, d\theta \quad . \tag{4a}$$

The mean-square value of the current, for the case in which the current is discontinuous, is

$$I^2 = (G^2V^{2n}/\pi)\int_0^{\alpha}(\cos\theta - \cos\alpha)^{2n}\,d\theta\,. \tag{4b}$$

In general the current may be represented by a Fourier series of the form

$$i = I_o + I_1\cos\theta + I_2\cos 2\theta + \text{----} + I_m\cos m\theta + \text{----}\,, \tag{5}$$

where the coefficients are given by

$$I_o = (G/\pi)\int v^n(\theta)\,d\theta \tag{6}$$

$$I_m = (2G/\pi)\int v^n(\theta)\cos m\theta\,d\theta. \tag{7}$$

In equations (6) and (7) $m = 1, 2, 3, \ldots.$, and the integration limits are 0 and π for the case of continuous current, and 0 and α for the case of discontinuous current.

Continuous Current, n = 1

The maximum value of current is, from equation (3),

$$I = GV(1 + r). \tag{3a.1}$$

The root-mean-square value of the current is obtained by putting $n = 1$ in equation (4a), carrying out the indicated integration, and taking the square-root of the result.

$$I = GV(1/2 + r^2)^{1/2}. \tag{4a.1}$$

The coefficients of the various terms in the Fourier series for the current are obtained by using equation (2a) in equations (6) and (7), with the limits 0 and π. The results are

$$I_o = GV_o = GVr \tag{6a.1}$$

$$\left\{\begin{array}{l} I_1 = GV \\ I_m = 0 \text{ for } m = 2, 3, 4, \ldots \end{array}\right. \tag{7a.1}$$

Continuous Current, n = 2

The maximum value of current is

$$\hat{I} = GV^2(1 + r)^2. \tag{3a.2}$$

The root-mean-square value of the current is

$$I = GV^2(3/8 + 3r^2 + r^4)^{1/2}. \tag{4a.2}$$

The coefficients of the terms in the Fourier series for the current are

$$I_o = GV^2(1/2 + r^2) \tag{6a.2}$$

$$\left\{\begin{array}{l} I_1 = 2GV^2r \\ I_2 = (1/2)GV^2 \\ I_m = 0 \text{ for } m = 3, 4, 5, \ldots \end{array}\right. \tag{7a.2}$$

Continuous Current, n = 3/2

The maximum value of current is

$$\hat{I} = GV^{3/2} \, (1 + r)^{3/2}. \tag{3a.3}$$

The root-mean-square value of the current is

$$I = GV^{3/2} \; r^{1/2} \; (r^2 + 3/2)^{1/2}. \tag{4a.3}$$

The coefficients of the terms in the Fourier series for the current are given by

$$I_O = (GV^{3/2}/\pi) \int_O^{\pi} (\cos\theta + r)^{3/2} \, d\theta$$

$$I_m = (2GV^{3/2}/\pi) \int_O^{\pi} (\cos\theta + r)^{3/2} \cos m\theta \, d\theta.$$

The integrals which appear in these expressions cannot be evaluated in terms of the elementary functions. However, they can be evaluated in terms of the complete elliptic integrals K and E, for which tables are available.[5,6]

The evaluation is carried out as follows.[7] Introduce the variable $x^2 = (\cos\theta + r)$; the integral which appears in the expression for I_O is transformed to

$$2R_4 = 2 \int_{x_1}^{x_O} x^4 P^{-1/2} \, dx,$$

where the limits of integration are given by

$$x_O^2 = (r + 1)$$

$$x_1^2 = (r - 1)$$

and

$$P = (1 - r^2) + 2rx^2 - x^4 = -(x^2 - x_O^2)(x^2 - x_1^2).$$

Let p = 1, 2, 3, ...; we note that

$$\frac{d}{dx}(x^p P^{1/2}) = \left[p x^{p-1} P + (1/2) x^p \frac{dP}{dx} \right] P^{-1/2}.$$

Integration of this expression between the limits x_O and x_1 yields a recurrence relation between definite integrals of the type R_4, given above; this relation is

$$(p + 2) R_{p+3} = 2r\,(p + 1) R_{p+1} - (r^2 - 1)\, p R_{p-1}, \tag{8a}$$

in which

$$R_p = \int_{x_1}^{x_O} x^p P^{-1/2} \, dx. \tag{9a}$$

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[5] *Peirce, B. O., "A Short Table of Integrals," Ginn (New York), 1929*

[6] *Dwight, H. B., "Tables of Integrals and other Mathematical Data," Macmillan (New York), 1947*

[7] *For those interested in following through the details of the evaluation, a useful reference is J. Pierpont, "Functions of a Complex Variable," Ginn (New York), 1914*

By setting $p = 1$ in equation (8a), it will be seen that the definite integral R_4 can be expressed in terms of R_2 and R_O. Consequently I_O, which is proportional to R_4, can be written as

$$I_O = (2GV^{3/2}/3\pi)\,[4rR_2 - (r^2 - 1)\,R_O]\,.$$

The elliptic integrals R_2 and R_O can be converted to standard form by means of the transformation $x^2 = x_O^2 - (x_O^2 - x_1^2)\sin^2\phi$, the results being

$$\begin{cases} R_2 = (r+1)^{1/2}E(k) \\ R_O = (r+1)^{-1/2}K(k), \end{cases} \tag{10a}$$

in which K and E are the complete elliptic integrals of the first and second kinds, respectively, and their modulus is given by $k^2 = 2/(r+1)$.

The zeroth-order Fourier coefficient is therefore

$$I_O = (2GV^{3/2}/3\pi)\,(r+1)^{1/2}\,[4rE - (r-1)\,K]. \tag{6a.3}$$

The mth-order Fourier coefficients are obtained by means of the same transformations as those used above. The factor $\cos m\,\theta$ which appears in the integral for these coefficients can be expressed[8] as a sum of terms each of which is proportional to a positive integral power of $\cos\theta$. Since $\cos^n\theta = (x^2 - r)^n$, any given coefficient can be written as a sum of terms each of which is proportional to a definite integral of the type R_{2p}. By means of the recurrence relation (8a), the coefficient can be reduced to a form which is linear in R_2 and R_O. Then, by using the conversions to standard form (10a), the mth-order coefficient can be expressed as

$$I_m = A_m\,E(k) + B_m\,K(k).$$

The results of carrying out this procedure for the leading mth-order coefficients are

$$\begin{cases} I_1 = (4GV^{3/2}/5\pi)\,(r+1)^{1/2}\,[(r^2+3)\,E - r(r-1)\,K] \\ I_2 = (4GV^{3/2}/35\pi)\,(r+1)^{1/2}[\,4r(2-r^2)\,E + (r-1)\,(4r^2-5)\,K] \\ I_3 = (4GV^{3/2}/315\pi)\,(r+1)^{1/2}[(32r^4 - 57r^2 + 21)\,E - r(r-1)\,(32r^2-33)\,K]. \end{cases} \tag{7a.3}$$

Explicit expressions for still higher order coefficients can be obtained, when required, in exactly the same way.

Discontinuous Current, n = 1

The maximum value of current is, from equation (3),

$$\hat{I} = GV\,(1 - \cos\alpha). \tag{3b.1}$$

The root-mean-square value of the current is obtained by putting $n = 1$ in equation (4b), performing the indicated integration, and taking the square-root of the result:

$$I = GV\pi^{-1/2}\,[\,\alpha + (\alpha/2)\cos 2\alpha - (3/4)\sin 2\alpha\,]^{1/2}. \tag{4b.1}$$

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[8] Dwight, reference 6.

The Fourier coefficients for the current are obtained by using equation (2b) in equations (6) and (7), with the limits 0 and α. The results are

$$I_o = (GV/\pi)\,(\sin\alpha - \alpha\cdot\cos\alpha) \qquad (6b.1)$$

$$I_m = (GV/\pi\, m)\left[\frac{\sin(m-1)\alpha}{(m-1)} - \frac{\sin(m+1)\alpha}{(m+1)}\right]. \qquad (7b.1)$$

Discontinuous Current, n = 2

The maximum value of current is

$$\hat{I} = GV^2\,(1 - \cos\alpha)^2. \qquad (3b.2)$$

The root-mean-square value of the current is

$$I = GV^2\pi^{-1/2}\left[(9\alpha/4) - (5/3)\sin 2\alpha - (25/96)\sin 4\alpha + 2\alpha\cos\alpha + (\alpha/8)\cos 4\alpha\right]^{1/2}. \qquad (4b.2)$$

The coefficients of the terms in the Fourier series for the current are

$$I_o = (GV^2/\pi)\;\left[\alpha + (\alpha/2)\cos 2\alpha - (3/4)\sin 2\alpha\right] \qquad (6b.2)$$

$$I_m = (GV^2/\pi)\left[\frac{m+2}{m+1}\cdot\frac{\sin m\alpha}{m} + \frac{1}{m(m+1)}\,\frac{\sin(m+2)\alpha}{(m+2)} + \right.$$

$$\left.\frac{m-1}{m}\cdot\frac{\sin(m-2)\alpha}{(m-2)} - 2\cdot\frac{\sin(m-1)\alpha}{(m-1)}\cdot\cos\alpha\right]. \qquad (7b.2)$$

Discontinuous Current, n = 3/2

The maximum value of current is

$$\hat{I} = GV^{3/2}\,(1 - \cos\alpha)^{3/2}. \qquad (3b.3)$$

The root-mean-square value of the current is

$$I = \left[GV^{3/2}/2(6\pi)^{1/2}\right]\left[27\sin\alpha + 11\sin 3\alpha - 54\alpha\cos\alpha - 6\alpha\cos 3\alpha\right]^{1/2} \qquad (4b.3)$$

The coefficients of the terms in the Fourier series for the current are given by

$$I_o = (GV^{3/2}/\pi)\int_o^{\alpha}(\cos\theta - \cos\alpha)^{3/2}\,d\theta$$

$$I_m = (2GV^{3/2}/\pi)\int_o^{\alpha}(\cos\theta - \cos\alpha)^{3/2}\cos m\theta\,d\theta.$$

The integrals which appear in these expressions cannot be evaluated in terms of the elementary functions. They can be expressed, however, in terms of the complete elliptic integrals K and E, in very much the same way as was done in the corresponding case of continuous current. Let $x^2 = (\cos\theta - r)$, where $r = (V_O/V) = \cos\alpha$, the range of r now being $-1 \leqslant r \leqslant 1$. The integral which appears in the expression for I_O is transformed to

$$2R_4 = 2\int_O^{x_O} x^4 P^{-1/2}\,dx ,$$

where the upper limit of integration is given by $x_O^2 = (1 - r) = 2\sin^2(\alpha/2)$, and

$$P = (1 - r^2) - 2rx^2 - x^4 .$$

Let p = 1, 2, 3,....; we note that

$$\frac{d}{dx}\,(x^p P^{1/2}) = \left[px^{p-1}\,P + (1/2)\,x^p\,\frac{dP}{dx}\right] P^{-1/2} .$$

Integration of this expression between the limits x_O and 0 yields the following recurrence relation between definite integrals of the type R_4, given above:

$$(p + 2)\,R_{p+3} = (1 - r^2)\,pR_{p-1} - 2r\,(p + 1)\,R_{p+1}, \qquad (8b)$$

in which

$$R_p = \int_O^{x_O} x^p P^{-1/2}\,dx . \qquad (9b)$$

The definite integral R_4 can be expressed in terms of R_O and R_2 by setting p = 1 in equation (8b). Thus I_O, which is proportional to R_4, can be written as

$$I_O = (2GV^{3/2}/3\pi)\,[\,(1 - r^2)\,R_O - 4r\,R_2\,] .$$

The elliptic integrals R_O and R_2 can be converted to standard form by use of the transformation $x^2 = x_O^2 \cos^2\phi$; the results are

$$\begin{cases} R_O = 2^{-1/2}\,K(k) \\ R_2 = 2^{1/2}\,E(k) - 2^{1/2}\,(1-k^2)\,K(k), \end{cases} \qquad (10b)$$

in which K and E represent, as usual, the complete elliptic integrals of the first and second kinds, respectively, and their modulus is given by $k^2 = (1 - r)/2 = \sin^2(\alpha/2)$.

The zeroth-order Fourier coefficient is

$$I_O = (2^{1/2}\cdot 4GV^{3/2}/3\pi)\,[\,(1 - k^2)\,(2 - 3k^2)\,K - 2(1 - 2k^2)\,E\,] \;. \qquad (6b.3)$$

The mth-order Fourier coefficients are obtained by means of the same transformations as those used above. As in the corresponding case of continuous current, the factor cos m θ which appears in the integral for these coefficients can be expressed as a sum of terms each of which is proportional to a positive integral power of cos θ. Since $\cos^n\theta = (x^2 + r)^n$, any given coefficient can be written as a sum of terms each of which is proportional to a definite integral of the type R_{2p}. By use of the recurrence relation (8b), the coefficient can be reduced to a form which is linear in R_O and R_2. Then, by using the conversions to standard form (10b), the mth-order coefficient can be expressed in a form which is linear in E(k) and K(k). The results of carrying out this procedure for the leading coefficients are

$$\begin{cases} I_1 = (2^{1/2}\cdot 8GV^{3/2}/5\pi)\left\{2\,[(1-k^2)^2 + k^2]\,E - (2-k^2)\,(1-k^2)\,K\right\} \\ I_2 = (2^{1/2}\cdot 8GV^{3/2}/35\pi)\,[\,(1-k^2)\,(2+5k^2-8k^4)\,K - 2(1-2k^2)\,(1+4k^2-4k^4)\,E\,] \\ I_3 = (2^{1/2}\cdot 8GV^{3/2}/315\pi)\,[\,2(128k^8 - 256k^6 + 135k^4 - 7k^2 - 1)\,E \\ \quad + (1-k^2)\,(128k^6 - 144k^4 + 15k^2 + 2)\,K\,] . \end{cases} \qquad (7b.3)$$

Explicit expressions for higher order coefficients can be obtained, when required, by following the procedure outlined above.

GRAPHS

Results of the foregoing analysis are plotted in Figures 1 through 14. Features of the discontinuous current case, for $n = 1$, $3/2$, and 2, are shown in Figures 1 through 7, the dependent variables for these figures being in order $I/\hat{I}$, $I_0/\hat{I}$, $I_1/\hat{I}$, $I_2/\hat{I}$, I/I_0, I_1/I_0, and I_2/I_0. In each of these figures the independent variable is 2α, the total angle of conduction, results being shown for the entire possible range (0-360 degrees) of this variable.

In Figures 8 through 14 the same dependent variables are plotted as functions of the variable $r = (V_0/V)$, the ratio of steady component of the applied voltage to peak value of the alternating component. In each of these figures the independent variable ranges from the value $r = -1$ to $r = 9$. The portion of the range $r \geqslant 1$ corresponds to the case of continuous current, and the remaining portion, $-1 \leqslant r \leqslant 1$, constitutes another version of the case of discontinuous current.

ACKNOWLEDGMENT

It is a pleasure to express my thanks to Mr. Walter Weber, formerly of this Laboratory, for having carefully checked the calculations and for carrying out computations needed for the figures.

* * *

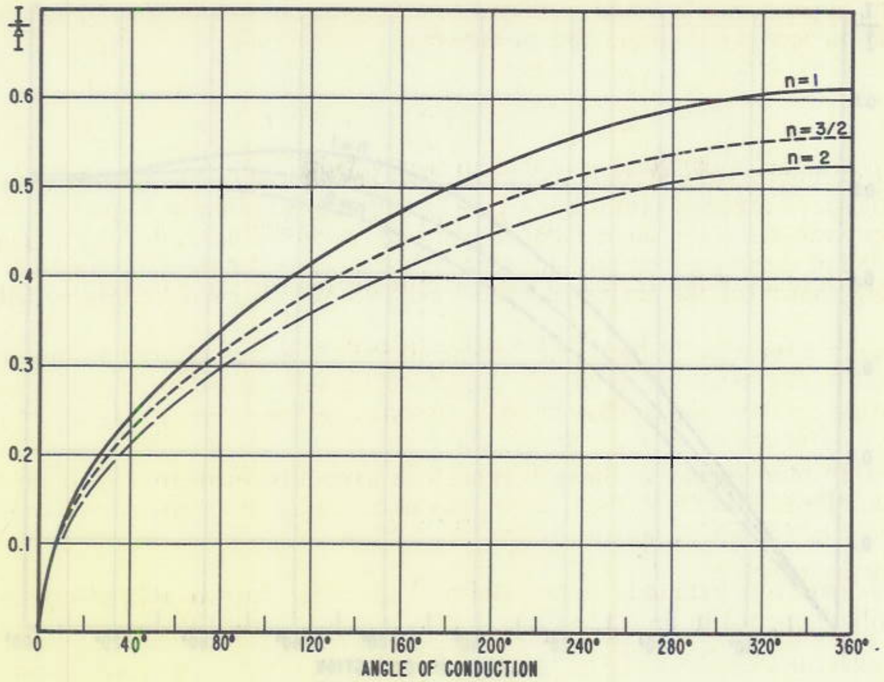

Figure 1 - The ratio of the rms value of output current to the maximum value, as a function of the total angle of conduction, for n = 1, 3/2, and 2

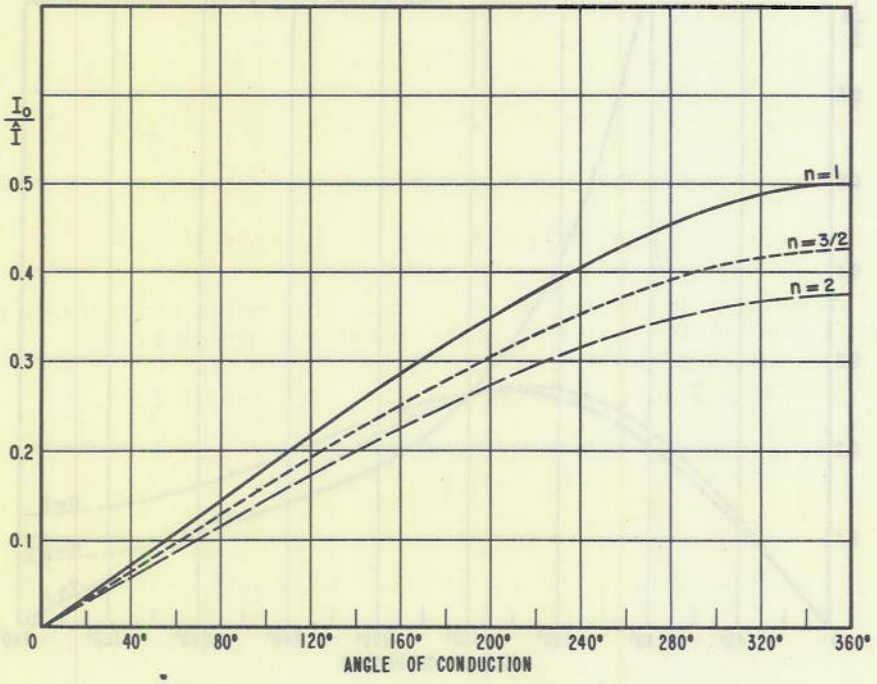

Figure 2 - The ratio of the d-c component of output current to the maximum value, as a function of the total angle of conduction, for n = 1, 3/2, and 2

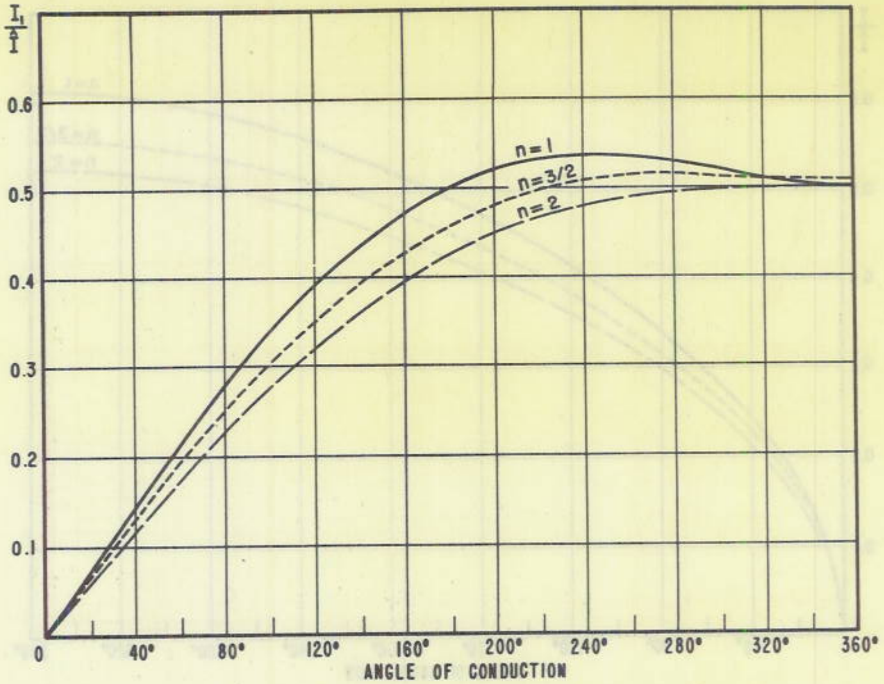

Figure 3 - The ratio of the fundamental component of output current to the maximum value, as a function of the total angle of conduction, for n = 1, 3/2, and 2

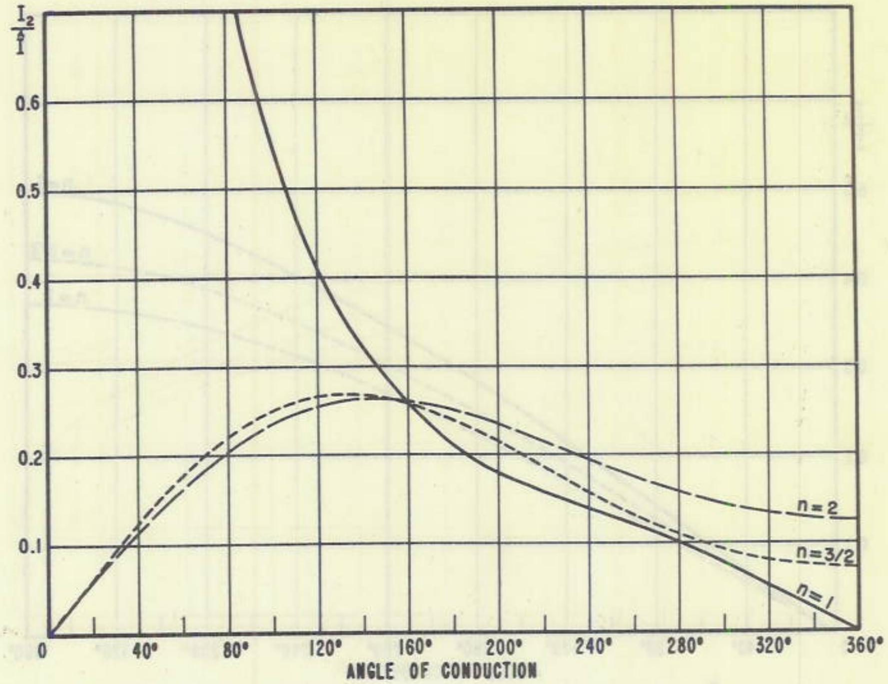

Figure 4 - The ratio of the second harmonic component of output current to the maximum value, as a function of the total angle of conduction, for n = 1, 3/2, and 2

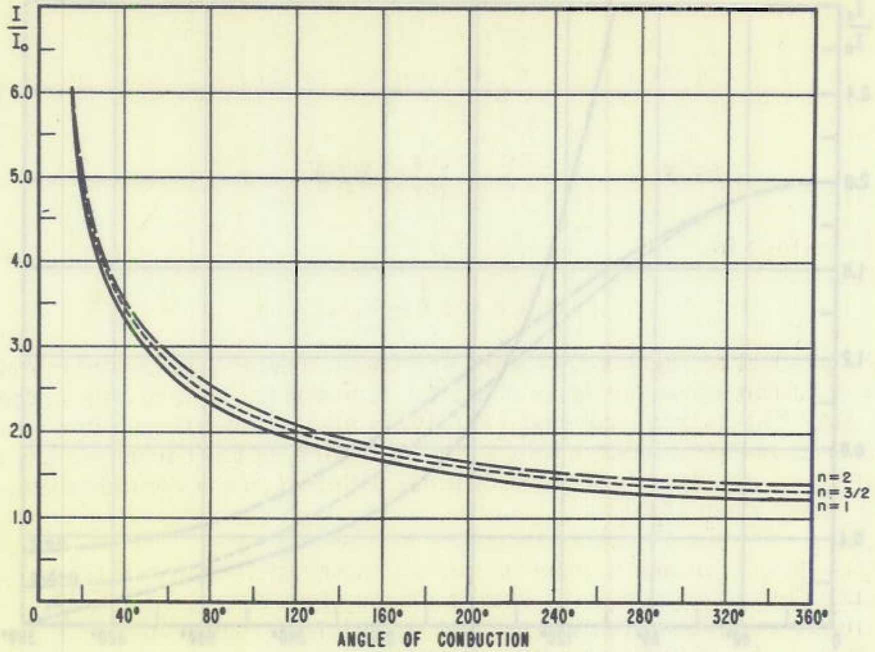

Figure 5 - The ratio of the rms value of output current to the d-c component, as a function of the total angle of conduction, for n = 1, 3/2, and 2

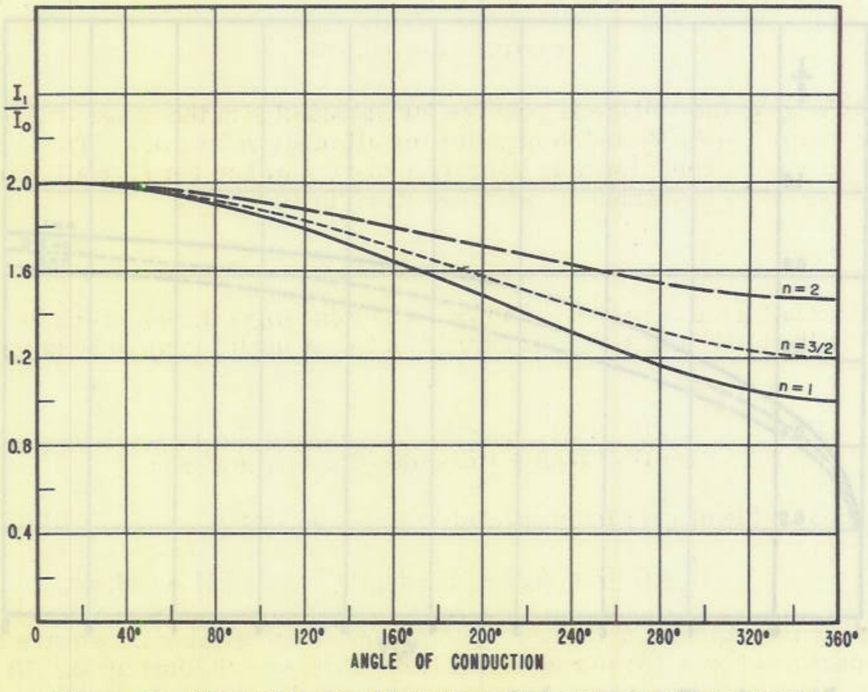

Figure 6 - The ratio of the fundamental component of output current to the d-c component, as a function of the total angle of conduction, for n = 1, 3/2, and 2

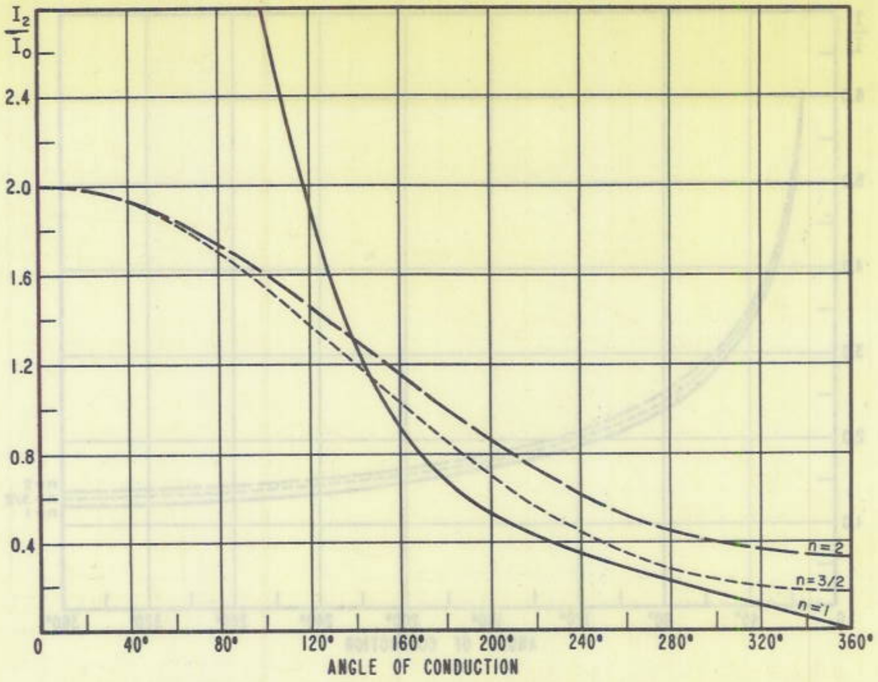

Figure 7 - The ratio of the second harmonic component of output current to the d-c component, as a function of the total angle of conduction, for n = 1, 3/2, and 2

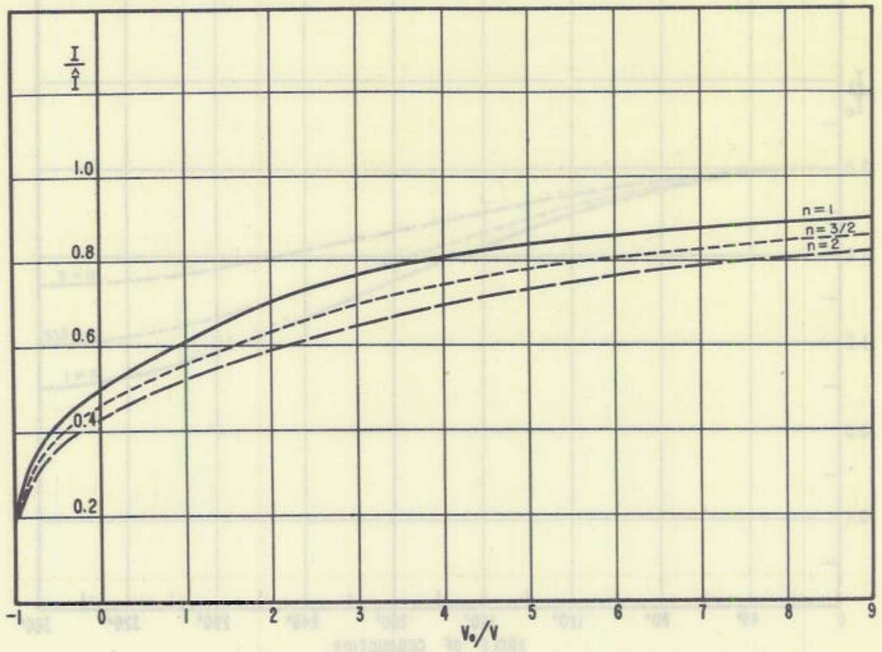

Figure 8 - The ratio of the rms value of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for n = 1, 3/2, and 2

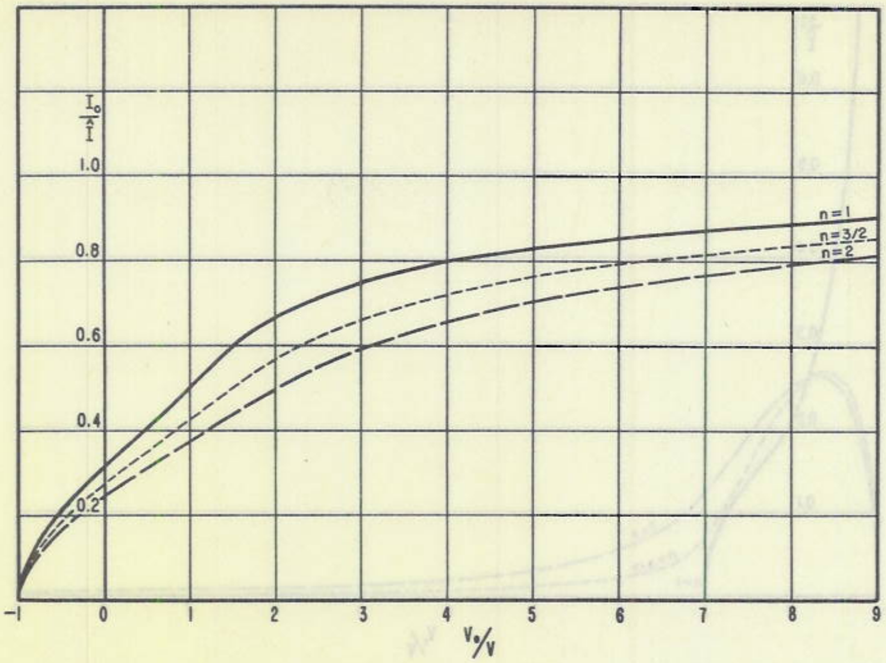

Figure 9 - The ratio of the d-c component of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for n = 1, 3/2, and 2

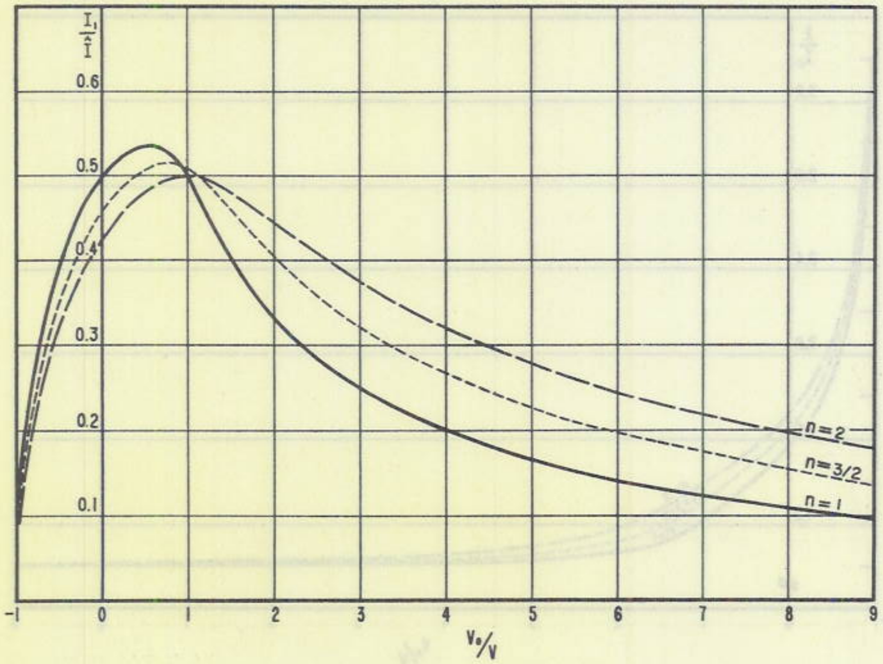

Figure 10 - The ratio of the fundamental component of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for n = 1, 3/2, and 2

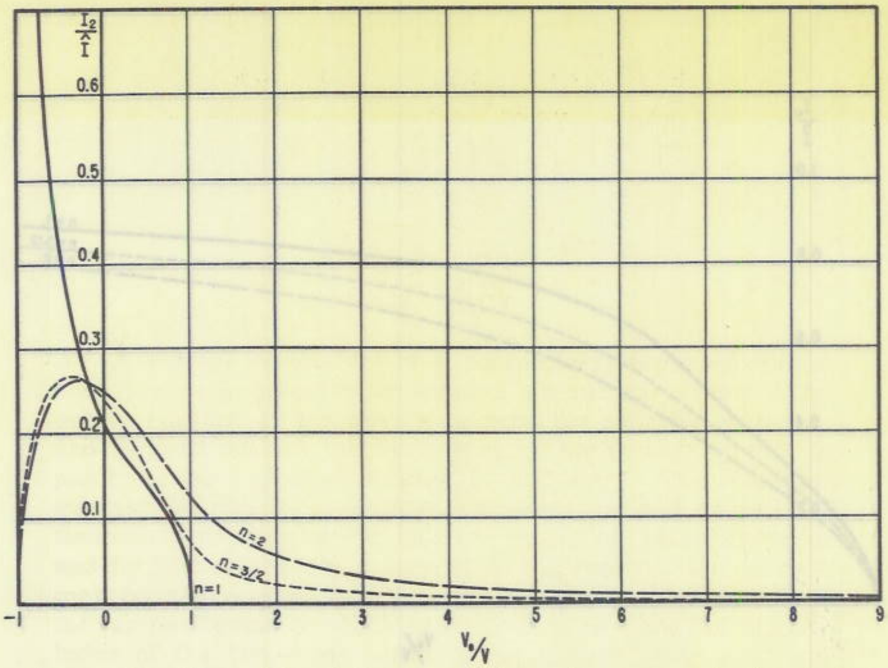

Figure 11 - The ratio of the second harmonic component of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for n = 1, 3/2, and 2

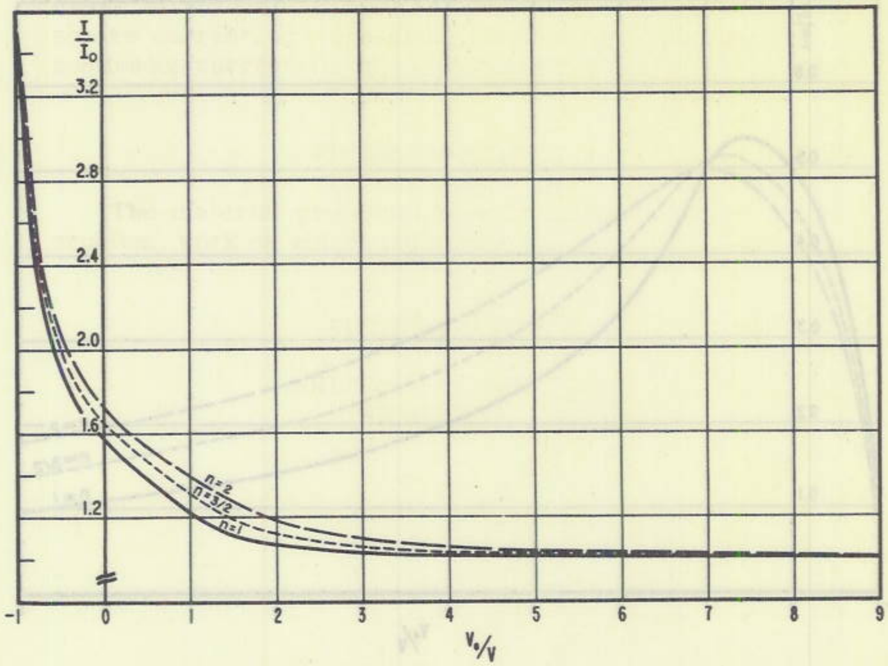

Figure 12 - The ratio of the rms value of output current to the d-c component, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for n = 1, 3/2, and 2

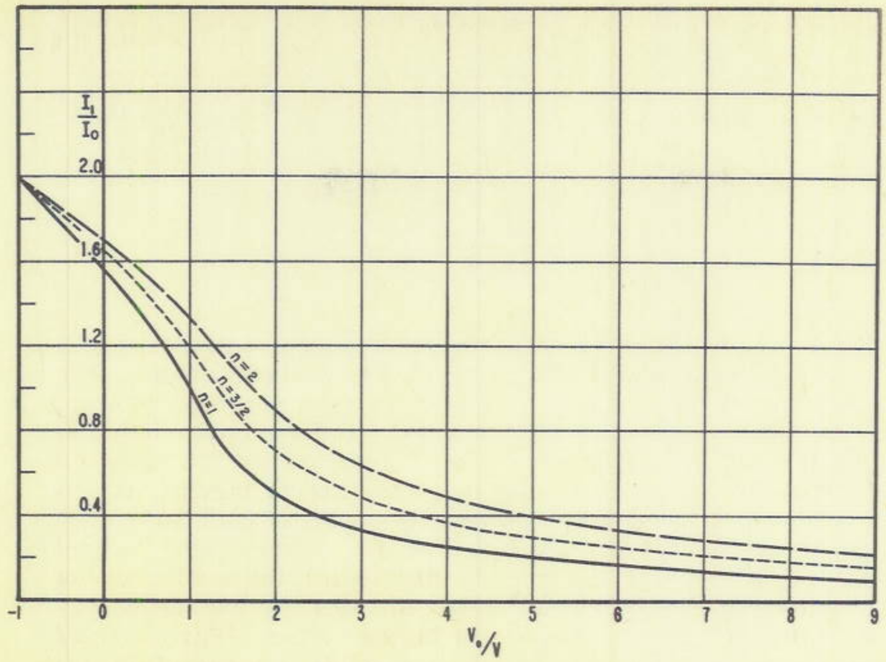

Figure 13 - The ratio of the fundamental component of output current to d-c component, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for n = 1, 3/2, and 2

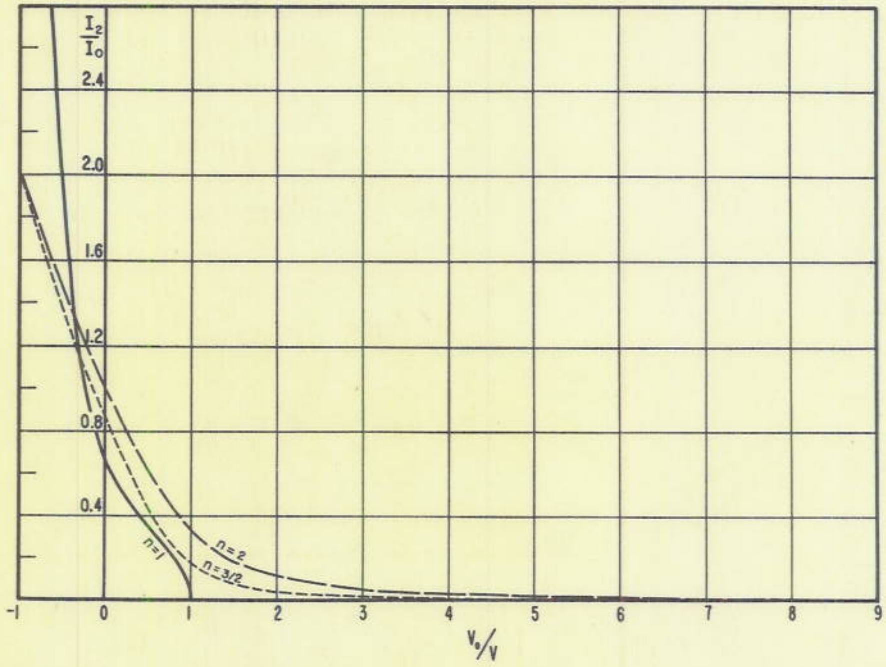

Figure 14 - The ratio of the second harmonic component of output current to the d-c component, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for n = 1, 3/2, and 2

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