

**EFFECTS OF TERMINAL VOLTAGE, LOAD CURRENT
AND MINIMUM ROTOR SPEED ON THE WEIGHT OF
D-C AIRCRAFT GENERATORS**



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ABSTRACT

As one step toward determining the weights of the various electrical system components for an over-all system weight study, empirical equations were developed to express the relation between total generator weight and the power output for minimum rotor speed. The ranges over which these three variables were investigated were: terminal voltage from 30 to 120 volts, load current from 100 to 400 amperes, and minimum rotor speed from 3000 to 8000 rpm. The empirical equations are limited by the assumptions made to derive them; but the method of derivation can be used to obtain substitute equations if different assumptions are required.

PROBLEM STATUS

This report is a final report on one phase of NRL Problem No. E01-08R and concludes the weight study on d-c aircraft generators. Three separate phases of this problem are continuing; (1) an investigation of the factors affecting the weight of aircraft transformers, (2) an investigation of the relations between current and temperature rise in bundled cables, and (3) simulation of electrical loads for aircraft electrical systems.

AUTHORIZATION

NRL Problem No. E01-08R
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EFFECTS OF TERMINAL VOLTAGE, LOAD CURRENT AND MINIMUM ROTOR SPEED ON THE WEIGHT OF D-C AIRCRAFT GENERATORS

INTRODUCTION

The weight of aircraft electrical power systems is of major importance because unnecessary weight carried in an airplane reduces the payload.

In order to make a weight study of aircraft electrical power systems it is necessary to obtain the weight of the electrical components. The generators which furnish the primary electrical power are often the heaviest single units in the system.

Many factors affect the weight of d-c generators, among them environmental conditions encountered, type of cooling used, duty requirements expected, length of life required, and method of mounting. The above factors have such a profound influence that the terminal output ratings of these generators do not necessarily establish their weight. For this reason, the authors in the technical literature usually restrict their discussion on the subject of weight of d-c generators to the active portion of the armature, which does not have all of the above limitations.

However, for aircraft use the operating conditions are sufficiently defined so that assumptions can be made and approximate weight curves obtained.

The methods used for obtaining the final weight curves are included. The range of investigation included in this report was limited to existing generators so that experimental evidence could be obtained to check the assumptions. However, the investigation can be extended to include higher values of speed and terminal voltage. The curves given include larger power output ratings than have been built, but have an upper limit of armature current owing to commutation limitations.

GENERAL DESCRIPTION AND OPERATING CONDITIONS OF D-C AIRCRAFT GENERATORS

The operating requirements for aircraft generators are more severe than those for most conventional generators. The generators are normally driven by the aircraft engine through a suitable gear box, and thus must operate over a wide speed range as the engine speed is changed. This range is frequently a three to one speed change.

In order to reduce their weight the generators are ventilated by forced convection. Openings are designed into the leading edge of the wing or in the forward surface of the airplane in such a manner that air can be admitted to the generator through special cooling tubes. Specifications govern the location of the openings and the design of the cooling tubes in order to provide the required pressure difference between the opening and the generator compartment, as well as to provide for the proper flow of cooling air.

Since the smaller generators are attached rigidly to the gear box or gear pad which is built into the aircraft engine, extreme vibration is encountered, especially on reciprocating type of engines.

FACTORS AFFECTING GENERATOR WEIGHT

Performance Requirements

The weight of d-c generators depends on the performance requirements. It is not possible for the designer to arrive at a generator design of minimum weight until the performance of the generator is specified. Thus, it is logical to begin a weight study by the application of basic performance equations.

The generated voltage of a d-c generator is expressed as

$$E_g = K\phi n \quad (1)$$

where E_g is the generated voltage, ϕ is the total air-gap flux in lines, n is the rotor speed in rpm and K is a constant depending on the particular generator. The constant K is further defined as

$$K = \frac{Z}{p(60)(10^8)} \quad (2)$$

where Z is the total number of conductors in the armature, and p is the number of current paths through the armature. By defining ϕ as the number of poles, P , times the flux per pole, ϕ_p ; and substituting (2) in (1),

$$E_g = \frac{P\phi_p Zn}{p(60)(10^8)} \quad (3)$$

The power developed in the rotor is obtained by multiplying both sides of (3) by the total armature current, I_a .

$$W_a = E_g I_a = \frac{P\phi_p Zn I_a}{p(60)(10^8)} \quad (4)$$

By defining the average flux density over the armature surface as

$$B_{ave} = \frac{P\phi_p}{\pi D_a L_a} \quad (5)$$

where D_a = diameter of the active armature (air-gap diameter in inches)
 L_a = length of armature stack in inches,

and by defining the ampere conductors per inch of armature periphery as q ,

$$q = \frac{Z I_a}{p \pi D_a} \quad (6)$$

the value of $D_a^2 L_a$ can be solved by substituting (5) and (6) in (4).

$$D_a^2 L_a = \frac{W_a(60)(10^8)}{n B_{ave} q \pi^2} \quad (7)$$

* A complete list of symbols is given in Appendix I, pages 23 to 26.

Equation (7), therefore, is a measure of the volume of the active armature in terms of the power developed in the armature, the rotor speed of rotation, the average flux density of the air gap, and the ampere conductors of armature periphery. The ampere conductors per inch of armature periphery, q , is sometimes called the specific loading.

Equations similar to (7) appear in many d-c machine design textbooks such as Still,¹ Kuhlmann,² and Spreadbury.³ Kuhlmann and Spreadbury have attempted to establish values of B_{ave} and q for typical machines.

A careful study of (7) clearly illustrates that the weight of at least the active armature depends on the performance requirements. For example, the maximum speed will affect the structural requirements, which, in turn, will affect the mass density. The type of materials selected for lamination will affect B_{ave} . The type of cooling used will affect the permissible value of q . The efficiency required will also affect B_{ave} and q . The voltage developed will affect the space factors of the windings and will, therefore, affect B_{ave} and q , even though W_a is not changed.

Terminal Voltage

In (7) no voltage term appears. Thus, for a given value of W_a it would appear that the voltage of the generator will not affect the volume of the rotor. As voltage increases, however, the insulation thickness must increase. Also, the number of conductors per inch of armature periphery will increase. Both of the above factors will reduce the ratio of copper area to slot area, thereby decreasing the value of q for a given current density in the copper. Therefore, as the voltage is increased, the rotor volume will increase in size for constant values of power developed, rotor speed and average air-gap flux density.

As the rated voltage of the generator is increased, there is a range of values over which the commutator weight is essentially inversely proportional to voltage. The actual commutator diameter, the speed of rotation, the reactance voltages, and the volts per commutator bar are factors which impose limits on the inverse relation between commutator weight and terminal voltage. If the actual diameter of the commutator becomes too large the centrifugal forces will be such as to require a more rigid design. An increase in speed of rotation will give the same result. Improper brush setting or lack of commutating windings will increase the reactance voltage, and there is a definite maximum allowable voltage between commutator bars. Consider a specific generator which has commutating and compensating windings, and a commutator relatively small in diameter. Also assume a 30-volt terminal rating and a 9-KW output. The above unit falls in the range of values described in which commutator weight is inversely proportional to voltage. Therefore, as the voltage is increased above 30 volts it is expected that the increased armature weight will tend to be compensated for by a decreased commutator weight until structural limitations or reactance voltages impose a limit.

¹ Alfred Still, *Elements of Electrical Design* (New York and London, McGraw-Hill Book Co., Inc., 1932, 2nd Edition)

² John H. Kuhlmann, *Design of Electrical Apparatus* (New York, John Wiley & Sons, Inc., London: Chapman & Hall, Limited, 1940, 2nd Edition)

³ F. G. Spreadbury, *Aircraft Electrical Engineering* (London, Sir Isaac Pitman & Sons, Limited, 1943)

Type of Cooling

Equation (7) does not furnish a complete means of predicting the total generator weight. The type of cooling employed will determine the necessary surface area for ventilation and the openings through the generator for the coolant. It will also influence some of the values of equation (7), since the magnitudes of B_{ave} and q can be influenced by the rate at which heat is carried away. The use of forced-convection cooling will permit smaller openings and smaller surfaces. Thus, the type of cooling has a direct influence on the total generator weight.

Structural Requirements

Aircraft generators are frequently flange-mounted directly on the engine drive-pad in a cantilever fashion. Thus, the size of the yoke may be dictated by structural requirements rather than by flux-density requirements, and one end bell may also be heavier because of the necessity of supporting the entire generator weight while subject to vibration and to the gravitational changes encountered in aircraft operation.

Speed Range

Both the maximum speed and the minimum speed of the rotor will have a definite effect upon the generator weight. The maximum speed will determine certain structural requirements. The minimum speed will impose a limit on the magnetic circuit. The lower the required minimum rated speed, the greater will be the flux requirement to maintain rated terminal voltage and load current.

METHOD OF DETERMINING WEIGHT DATA

Possible Simplification of Factors

The miscellaneous factors discussed above tend to make a study of weight in d-c aircraft generators very complex. However, some of the factors will be the same for all aircraft generators. For example, if only blast-cooled generators are considered, the type of cooling can be standardized for a study. Thus, one of the variables can be eliminated. In like manner, all aircraft generators are expected to be subjected to vibration; thus, the same general rules will need to be observed for all generators with respect to the structural requirements which vibration imposes.

Design Study Proposed

One way to obtain realistic weight data for aircraft generators is to design generator for specific ratings and at the same time carefully control the flux densities, current densities, temperatures, and the operational speed range. For example, in order to determine the effect of terminal voltage on the generator weight, several designs should be made each with a different terminal voltage, for the same power output, rotor speed, flux densities, current densities, and cooling-air temperature rise. Also, for a given voltage, several designs should be made, each with a different power output, but with all other variables held constant.

Basic Generator Selected

In order to establish the flux densities, current densities, speed range, etc., a specific aircraft generator was selected as a basic generator. The restrictions placed on the basic generator were that it should be a 30-volt, 9-KW, high-speed generator which was being used on Naval aircraft. Within the above limitations, the final choice of the basic generator was determined by availability. The determination of some of the constants resulted in the destruction of a unit. Thus, a generator type was selected of which identical units were available.

Design Method Explained

The design of d-c generators is somewhat empirical and the designer normally has an extensive background of design data and design experience to draw from in order to feel confident of his design values. It was necessary to design the suggested line of machines with limited design experience and data. Therefore, an approach was sought which would tend to minimize the errors due to the above limitations.

The design method used involved the determination of the constants of the basic generator. From the constants, the output of the basic generator was expressed in equation form. For each new design the values of B_{ave} , current densities, and temperature rise of cooling air were retained. The dimensional changes to retain the above values were then calculated, and the output equations determined for the designed generator. From the calculated dimensions, the change in volume for each component part was computed. The weight change for each part was then computed based on the mass density observed on the basic generator combined with the calculated volume change, or a standard mass density for the material combined with the calculated volume change, whichever was more logical to use. The changes in weight were then tabulated and the net weight change was combined with the weight of the basic generator to establish the weight of the designed generator. In this manner, only the changes were considered and a large amount of detail work was omitted.

ANALYSIS OF BASIC GENERATOR

Performance Equations

Predicting the performance of a d-c generator from design constants suggests that it should be possible to measure the constants of an existing generator and predict its performance. Errors in the method of prediction can then be detected by experimental test, and the method corrected prior to using it to predict the performance of a generator which is being designed.

VanValkenburg⁴ has developed an equation expressing the no-load air-gap flux of a d-c generator in terms of four constants, A, B, C, and D as

$$\phi = A \tanh(BN_f I_f + C) + D \quad (8)$$

where $N_f I_f$ is the shunt-field ampere turns per pair of poles.

⁴ VanValkenburg, E. S., "Steady-State Analysis of Aircraft D-C Generators" (NRL Report No. E-3130)

VanValkenburg and Matthews⁵ suggest that for approximate analysis (8) can be simplified to a pure hyperbolic tangent function. However, it is desired to retain the constant D which represents the residual flux. Therefore, (8) will be simplified by omitting only the constant C, or

$$\phi = A \tanh BN_f I_f + D \quad (9)$$

Substituting (9) in (1):

$$E_g = Kn(A \tanh BN_f I_f + D) \quad (10)$$

Equation (10) is the equation of the no-load terminal voltage of a separately excited d-c generator in terms of the generator constants, the rotor speed and the shunt-field excitation. Evaluation of the constants should permit expressing the no-load saturation curve of the basic generator in mathematical form.

Although the rated speed range of the generator was 4550-8000 rpm, the lower limit of speed, or the minimum speed when experimentally measured was found to be 4510 rpm.

The minimum speed of a blast-cooled aircraft d-c generator is defined as the lowest speed at which the generator will furnish rated load-current at rated terminal-voltage under stabilized temperature conditions, with a definite resistance in series with the shunt, field, and under definite conditions of flow of cooling air.

In Figure 1 is shown the measured value of the no-load saturation curve for the basic generator at 4510 rpm. The measured value is the mean value of the increasing and decreasing voltage values obtained by plotting the increasing and decreasing excitation-readings, calculating the mean value from the plotted curve, and replotting the mean value. The calculated curve of Figure 1 was plotted from equation (11).

$$E_g = 40.5 \tanh 0.22 I_f + 1.2 \quad (11)$$

Equation (11) was obtained by fitting (10) to the measured no-load saturation curve of Figure 1. The details are included in Appendix III as an example of the method.

Equation (11) is a direct application of (10) in which $KnA = 40.5$, $KnD = 1.2$, and $BN_f = 0.22$. The shunt-field current, I_f , measured at full load during the determination of the minimum speed, was 7.2 amperes. Thus, the value of I_f will not exceed 7.2 amperes over the load range. The calculated curve of Figure 1 over the load range is reasonably close to the measured value.

It has been demonstrated that the no-load saturation curve can be expressed in mathematical form. However, under conditions of load, equation (11) may or may not describe the relation between generated voltage and shunt-field current. The basic generator, as indicated in Figure 2, has compensating windings, series-field windings, and commutating poles.

VanValkenburg and Matthews⁶ have also developed an equation which expresses the total generator flux in terms of direct axis and quadrature axis excitation. The equation,

⁵ VanValkenburg, Ernest, and Matthews, Whitney, "Steady-State Analysis of Aircraft D-C Generators, Conclusion Section." (AIEE paper 48-250. AIEE Transaction, Vol. 67, 1948)

⁶ VanValkenburg, Ernest, and Matthews, Whitney, "Steady-State Analysis of Aircraft D-C Generators." Equation (6). (AIEE paper 48-250. AIEE Transactions, Volume 67, 1948)

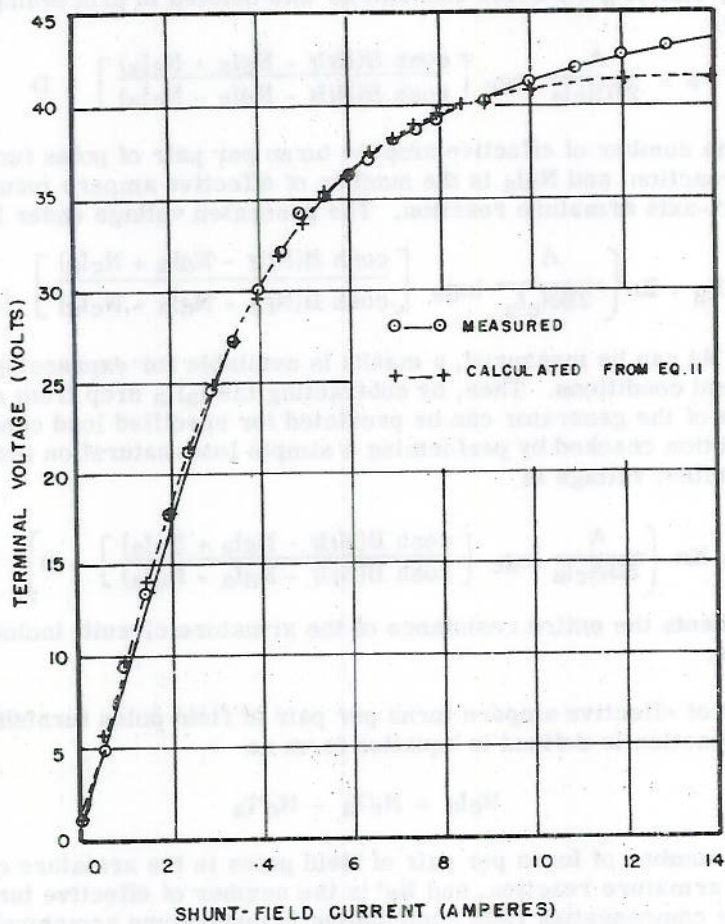


Figure 1 - Comparison of measured and derived no-load saturation curves for basic generator

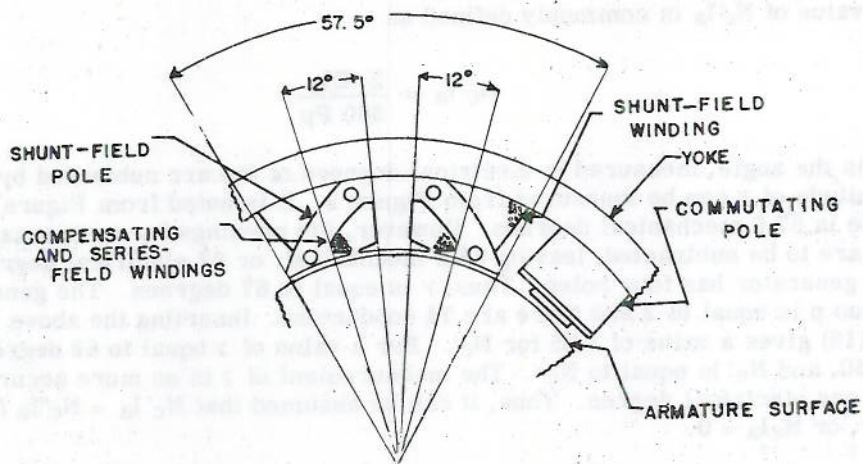


Figure 2 - Partial end view of basic generator

modified only by omitting the same constant as was deleted in proceeding from (8) to (9) in this report is,

$$\phi = \frac{A}{2BN_c I_a} \log_e \left[\frac{\cosh B(N_f I_f - N_d I_a + N_c I_a)}{\cosh B(N_f I_f - N_d I_a - N_c I_a)} \right] + D \quad (12)$$

where $N_c I_a$ is the number of effective ampere turns per pair of poles furnishing quadrature axis armature reaction, and $N_d I_a$ is the number of effective ampere turns per pair of poles furnishing direct-axis armature reaction. The generated voltage under load may now be expressed as

$$E_g = Kn \left\{ \frac{A}{2BN_c I_a} \log_e \left[\frac{\cosh B(N_f I_f - N_d I_a + N_c I_a)}{\cosh B(N_f I_f - N_d I_a - N_c I_a)} \right] + D \right\} \quad (13)$$

Thus, if N_c and N_d can be measured, a means is available for expressing the generated voltage under load conditions. Then, by subtracting the $I_a R_a$ drop from equation (13) the terminal voltage of the generator can be predicted for specified load conditions and the method of prediction checked by performing a simple load-saturation test. The resulting equation for terminal voltage is

$$E_t = Kn \left\{ \frac{A}{2BN_c I_a} \log_e \left[\frac{\cosh B(N_f I_f - N_d I_a + N_c I_a)}{\cosh B(N_f I_f - N_d I_a - N_c I_a)} \right] + D \right\} - I_a R_a \quad (14)$$

where R_a represents the entire resistance of the armature circuit, including the effective brush resistance.

The number of effective ampere turns per pair of field poles furnishing quadrature axis armature reaction is defined in equation form as

$$N_c I_a = N_c' I_a - N_c'' I_a \quad (15)$$

where N_c' is the number of turns per pair of field poles in the armature contributing to quadrature axis armature reaction, and N_c'' is the number of effective turns per pair of field poles in the compensating field contributing to quadrature armature reaction.

By observation the value of N_c'' was determined to be 3.50 turns for the basic generator.

The value of $N_c' I_a$ is commonly defined as

$$N_c' I_a = \frac{2\gamma Z I_a}{360 P p} \quad (16)$$

where γ is the angle, measured in electrical degrees of the arc subtended by a field pole. The magnitude of γ can be measured from Figure 2. It is noted from Figure 2 that the total angle is 57.5 mechanical degrees. However, the openings for compensating and series windings are to be subtracted, leaving 33.5 mechanical, or 67 electrical degrees since the basic generator has four poles. Thus, γ is equal to 67 degrees. The generator is wave wound; thus p is equal to 2 and there are 74 conductors. Inserting the above values in equation (16) gives a value of 3.45 for N_c' . For a value of γ equal to 68 degrees, N_c' becomes 3.50, and N_c' is equal to N_c'' . The measurement of γ is no more accurate than plus or minus one electrical degree. Thus, it can be assumed that $N_c' I_a = N_c'' I_a$ for the basic generator, or $N_c I_a = 0$.

For either the case of the armature current equal to zero, or for perfect compensation, equation (13) becomes indeterminate. The limits, however, as I_a or N_c approach zero are as follows:

$$\begin{aligned} \lim_{I_a \rightarrow 0} E_g &= \lim_{I_a \rightarrow 0} K_n \left[\frac{d(A \log_e X)}{dI_a} + D \right] \\ &= K_n (A \tanh BN_f I_f + D) = E_g \end{aligned} \quad (17)$$

where
$$X = \frac{\cosh B(N_f I_f - N_d I_a + N_c I_a)}{\cosh B(N_f I_f - N_d I_a - N_c I_a)}$$

Likewise,
$$\lim_{N_c \rightarrow 0} E_g = K_n [A \tanh B(N_f I_f - N_d I_a) + D] = E_g \quad (18)$$

Thus, these conditions are satisfied and equation (18) replaces equation (13) for the basic generator. Likewise, for the basic generator, equation (14) becomes:

$$E_t = K_n [A \tanh B(N_f I_f - N_d I_a) + D] - I_a R_a \quad (19)$$

The direct axis armature reaction $N_d I_a$ is also made up of two terms and is defined by

$$N_d I_a = N_d' I_a + N_d'' I_a \quad (20)$$

N_d' is the number of turns per pair of field poles in the armature contributing to direct axis armature reaction, and N_d'' is the effective series-field turns per pair of field poles contributing to direct axis armature reaction. $N_d' I_a$ is a function of the angular brush shift, and is expressed as

$$N_d' I_a = \frac{4\alpha Z I_a}{360 P_p} \quad (21)$$

where α is the brush shift angles expressed in electrical degrees. For the basic generator the brushes are shifted backward approximately one-half a mechanical degree. Thus, α is equal to -1 . The solution of equation (21) for the basic generator gives a value of $-0.103 I_a$ for $N_d' I_a$.

There are series demagnetizing windings on the basic generator composed of one-half turn on each pole which encloses only two-thirds of the pole. Therefore, $N_d'' I_a = (1/2 + 1/2)(2/3)(I_a/4) = 0.167 I_a$ ampere turns per pair of field poles. The value of one-fourth of the armature current is used because only one-fourth of the armature current flows in each winding.

For the basic generator, the solution of (20) becomes

$$N_d I_a = -0.103 I_a + 0.167 I_a = 0.064 I_a$$

There are 160 turns per pair of field poles in the shunt field, or N_f is equal to 160 turns. The total armature resistance R_a was measured to be 0.0231 ohms at operating temperature.

Referring to equation (11) it is noted that K_nA is equal to 40.5, K_nD is equal to 1.2, and BN_f is equal to 0.22. Therefore, for the basic generator (19) becomes

$$E_t = 40.5 \tanh 0.001375 (160 I_f - 0.064 I_a) + 1.2 - 0.0231 I_a. \quad (22)$$

Figure 3 is a plot of the measured full-load saturation curve and the curve calculated from (22). The correlation is good. Thus, it has been demonstrated that it is possible to express the full-load terminal voltage of the generator in mathematical form if the no-load saturation curve and measurable constants are known.

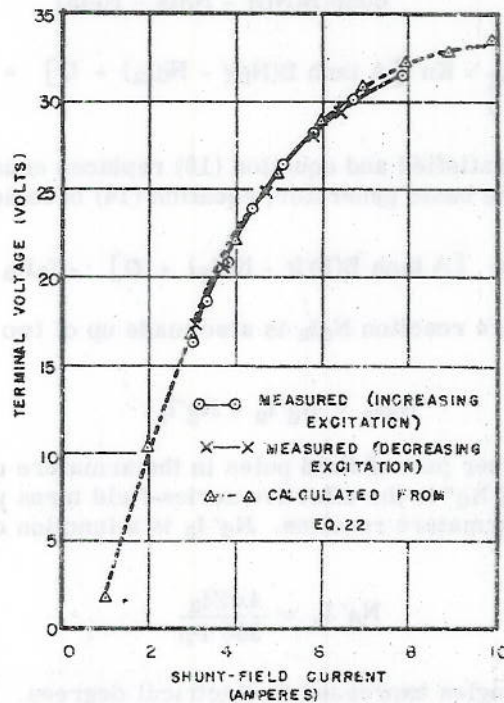


Figure 3 - Comparison of measured and derived full-load saturation curves for basic generator

Predicting Excitation

In the previous paragraph a knowledge of the no-load saturation curve was assumed. In the design of a generator the no-load saturation curve must be predicted. The design of another generator having essentially the same magnetic circuit as the basic generator will result in a no-load saturation curve of very similar shape. It is necessary, however, to be able to calculate the excitation required for some condition of load. From that one condition of load, and the determination of the constants discussed under the "Performance Equations," the no-load and full-load saturation curves can be plotted.

An analysis of the magnetic circuit of the basic generator was made to determine if the excitation required could be predicted from the physical dimensions of the generator and a knowledge of the physical properties of the materials. The analysis is included in:

Section IX, part A -1 of Appendix VI. The required excitation at full load and minimum speed from Appendix VI was 1146 ampere turns per pair of field poles. The required excitation at full load and minimum speed calculated from equation (22) was 1132 ampere turns per pair of field poles. The agreement is much better than was expected considering the assumptions made and indicates that the method of checking the division of excitation is within reason.

The method for calculating the equivalent air gap for the basic generator based on air-gap clearance, tooth dimensions, and slot dimensions is given in Appendix IV. The method of calculating slot and tooth dimensions when it is desired to retain both the air-gap clearance and the effective air-gap length is also included.

Temperature Rise of Cooling Air

As previously indicated, aircraft generators are cooled by forced convection. The rate at which heat is carried away by the cooling air is given by

$$W_{O1} = MkC_1 (T_2 - T_1) \quad (23)$$

where:

W_{O1} is the power carried away by the cooling air
 M is the weight of air flowing per unit of time
 C_1 is the specific heat of air
 T_2 is the exit air temperature
 T_1 is the entering-air temperature
 k is the constant depending on units

If W_{O1} is expressed in watts, M in pounds per minute, C_1 in British thermal units per pound per degree F, and T_2 and T_1 are expressed in degrees C, then k becomes 32.4. C_1 is 0.24 for air under all conditions studied. Thus, equation (23) becomes

$$W_{O1} = 7.78 M (T_2 - T_1) \text{ watts.} \quad (24)$$

For the basic generator, the mass rate of air flow, measured during the base-speed test in accordance with Appendix II, was 5.1 pounds per minute. T_2 and T_1 were measured during the same test and were 86 degrees C and 31 degrees C respectively. Inserting the above values in equation (24) results in a value of W_{O1} equal to 2180 watts for the basic generator. The total losses of the generator were determined by performing an efficiency test on the generator under the same conditions of load, speed, and mass rate of cooling air. The total losses measured 2850 watts. The total losses may also be expressed as the difference between the power developed, W_a , and the power output, W_o . Therefore, for the basic generator

$$W_{O1} = \frac{2180}{2850} (W_a - W_o) = 0.765 (W_a - W_o). \quad (25)$$

Cooling air flows through several parallel paths in the basic generator. The major path is bounded on the inside by the armature surface, and on the outside by the irregular surface made up of the yoke, the shunt-field poles, the commutating poles, the shunt-field winding, and the commutating winding.

Additional Data on Basic Generator

Considerable additional data were obtained from the basic generator. A tabulation of the data was made and is given in Appendix V.

DISCUSSION OF DESIGN

Parameters Retained

The design method is illustrated in Appendix VI. Certain parameters were measured on the basic generator and retained for each design. These parameters were: (1) current densities, (2) average air-gap flux density, (3) maximum rotor speed, (4) minimum rotor speed, and (5) temperature rise of cooling air.

Retaining the current densities insured that the insulation used would be adequate from a thermal standpoint provided the heat was carried away at the same rate. Retaining the flux densities permitted the use of the same type of magnetic materials. Retaining the same maximum rotor speed permitted the same type of structural design. Retaining the same minimum speed and flux densities furnished a constant core loss per unit of volume. Retaining the same temperature rise of cooling air insured essentially constant surface temperature over which the cooling air passed.

Current Density - The method used to retain current densities was to provide sufficient copper area so that the amperes per square inch of copper was the same as that measured on the basic generator.

Average Air-Gap Flux Density - The average air-gap flux density was retained by determining the required generated voltage for the minimum-speed condition and providing the proper combination of conductors, armature surface, and excitation to retain the value of E_{ave} . An analysis was made of the excitation required for the armature, armature teeth, air gap, shunt-field poles, and the yoke in order to provide the proper excitation windings. Appendix IV gives the method used in determining the air-gap length in terms of armature teeth and slot dimensions and the clearance between the armature and shunt-field pole shoe. Appendix IV also gives a method for adjusting the armature slot and tooth widths in order to retain the same values of clearance between the armature and the shunt-field pole shoe and at the same time retain the equivalent length of air gap.

Temperature Rise of Cooling Air - Maintaining the same temperature rise of cooling air required simplifying assumptions. The assumptions made, however, are considered practical for a theoretical design. One of the assumptions was that the percentage of the total losses carried away by the cooling air was the same for each generator. Therefore, (25) is applicable for each generator designed. Substituting (25) in (24),

$$0.765 (W_a - W_o) = 7.78 M (T_2 - T_1) \quad (26)$$

Since it is desired to hold $(T_2 - T_1)$ constant it is necessary only to calculate the losses for each generator and solve for the mass rate of air flow, M , which will hold $(T_2 - T_1)$ constant. By grouping all of the constant terms of (26) together and setting them equal to k

$$(W_a - W_o) = Mk_1 \quad (27)$$

The mass rate of air flow can also be expressed as

$$M = k_2 \rho A_V v_{ave} \quad (28)$$

in which k_2 is a constant depending on units, ρ is the air density, A_V is the cross sectional area for ventilation, and v_{ave} is the average velocity of the cooling air.

The average velocity of air flowing through an orifice is related to the pressure head across the orifice as follows:

$$v_{ave} = k_3 \left(\frac{h}{\rho} \right)^{1/2} \quad (29)$$

in which k_3 is a constant depending on units, the gravitational constant, and the discharge coefficient, and h is the pressure drop across the orifice. The assumptions were made that the air-flow paths through the armature can be treated as an orifice, and that each of the armatures designed had the same discharge coefficient. The small friction loss along the armature stack was assumed to be the same for each generator. It was further assumed that pressure drops resulting from other impedances to air flow in the generator were the same for each generator designed. Thus, the pressure head across each armature stack designed was the same as that across the basic generator stack. Therefore, for a fixed air density, the average velocity of the cooling air is constant, and (28) can be restated as

$$M = k_4 A_V \quad (30)$$

where $k_4 = k_2 k_3 v_{ave} (\rho)^{1/2}$. Also (27) becomes

$$(W_a - W_o) = k_5 A_V \quad (31)$$

where $k_5 = k_4 k_1$. Thus, it is noted that for the above assumptions to hold true, the area provided for forced ventilation for each generator designed must satisfy the equation,

$$\frac{A_V}{W_a - W_o} = \text{constant} \quad (32)$$

It was earlier stated that there were several parallel paths for cooling air through the armature of the basic generator, but that the major path for cooling air was bounded on the inside by the armature and on the outside by the yoke, the field poles, commutating poles, and windings. The assumption was made that the percentage of the total cooling air flowing through the major path was the same for all of the generators designed as that on the basic generator. Therefore, the major path for cooling air will be defined as A_V and is the only area which need be calculated in order to account for all of the cooling air.

Substituting the predetermined value of $A_V = 2.1$ sq. in. and $(W_a - W_o) = 2850$ watts for the basic generator into (32), the constant ratio is found to be 7.37×10^{-4} . Solving for A_V for any generator in terms of its value of $(W_a - W_o)$,

$$A_V = 7.37 \times 10^{-4} (W_a - W_o) \text{ sq. in.} \quad (33)$$

For each generator designed, the dimensions of the generator were adjusted until equation (33) was satisfied.

Determining Shunt-Field Conductor Size

The method used to select the specific conductor size for the shunt-field winding is given in Appendix VII. The method is quite general and permits an assumption for the value of excitation in order that the relations may be drawn. Changes in excitation requirements due to errors in assumptions may be later corrected by a fixed correction factor, which does not require redrawing the curves. The limitations on the power requirements of the voltage regulator are readily derived from the curves.

Performance Equations

The no-load saturation curve can be drawn as soon as the shunt-field excitation is established and the generator constant K is known. (See Appendix VI, Section IV). The full-load saturation curve requires that the excitation furnished by the commutating windings, compensating windings, and series-field winding be established in addition to the no-load saturation curve; also, that the armature circuit resistance R_a be established. Then, the full-load saturation curve can be expressed in the form of equation (14) or equation (19), whichever is applicable.

Generator Weight

The physical dimensions of the generator have been established in order to write the performance equations. Therefore, based on the physical dimensions and the previously determined weights of the component parts of the basic generator, the change in weight of the component parts can be calculated. The results can then be tabulated and combined with the weight of the basic generator to give the new generator weight.

DESIGN RESULTS

Change in Voltage with Power and Speed Constant

The basic generator was rated at 30 volts, 9 kilowatts, and had a minimum rated speed of 4510 rpm. The effect of changing the voltage with constant output power and constant minimum speed was first obtained. Generators were designed for 60 volts and 120 volts. The weight of the 60-volt generator was identical to that of the 30-volt generator, but the 120-volt generator was approximately two pounds heavier. For the 60-volt generator there was a reduction in commutator weight, but the poorer space factor in the armature caused increased armature volume. This, in turn, increased the over-all generator diameter. The increased volume caused a weight increase which compensated for the weight reduction of the commutator. As a result the over-all weight did not change. For the 120-volt generator, the commutator weight reduction was not achieved because of the increased number of commutator bars. Thus, the 120-volt generator was approximately two pounds heavier than the 30-volt or 60-volt generators. The 120-volt, 9-KW generator design details are included as Appendix VI, and a summary of the 120-volt, 9-KW, generator design results is given in Appendix VIII.

Change in Power with Voltage and Speed Constant

A 30-volt, 12-kilowatt generator was designed and the results are given in Appendix IX. Its weight for the base speed of 4510 rpm was 59.5 pounds. It should be kept in mind that all of the generators designed had the same speed range. This means the same maximum speed, which requires greater mechanical strength, and therefore, greater weight, than would be necessary if only the base speed were considered.

Two additional generators were designed, one at 120 volts, 20 KW, and 4510 rpm, and the other at 120 volts, 30 KW, and 4510 rpm. The results of the 20-KW design are given in Appendix X, and the results of the 30-KW design are given in Appendix XI. Their weights were 82 pounds and 106 pounds respectively.

EXPERIMENTAL VERIFICATION OF DESIGN RESULTS

Method

The most certain method of checking a design is to have the unit built to the design specifications, and then test to determine conformance with predicted performance. Such a procedure was not practical in this case. Therefore, a substitute method was devised. A number of 30-volt generators were available in the Laboratory (see Figure 4), but in order to conform with design data, only those having a maximum speed rating of 8000 rpm were selected. The minimum speed of each of the generators was obtained in accordance with the method described in Appendix II.



Figure 4 - Typical 30-volt, forced-air cooled, d-c aircraft generators; rated output of generators pictured ranges from 2.25 KW to 12 KW

From the minimum speed tests the watts output per minimum rpm W_0/n_1 was computed. The total weight of each generator was then obtained. The plot of generator weight, W , as a function of W_0/n_1 is shown in Figure 5.

The base or minimum speeds of the generators were obtained in order to furnish a common basis of comparison. Some of the generators furnished rated power output at speeds considerably below their nominally rated minimum; others would not furnish rated power at their nominally rated minimum speeds. Thus, a uniform method of obtaining the minimum speed was used which eliminated some variation in manufacturing and design. The resulting experimental points plotted in Figure 5 furnish data which is more nearly representative for comparison with theoretical design values. It is admitted that some variation exists in the flux densities, current densities, and cooling-air flow. The fact that several manufacturers' generators are represented will automatically result in some variation in the design parameters. However, attempts were made by the manufacturers to design for minimum weight in all cases, and the consistency of the data shown over a large weight range indicates that the method of comparison is valid.

Analysis of Experimental Weight Relations

The experimental data of Figure 5 is very close to linear. Equation (34) is written as an approximation of the relation in linear form.

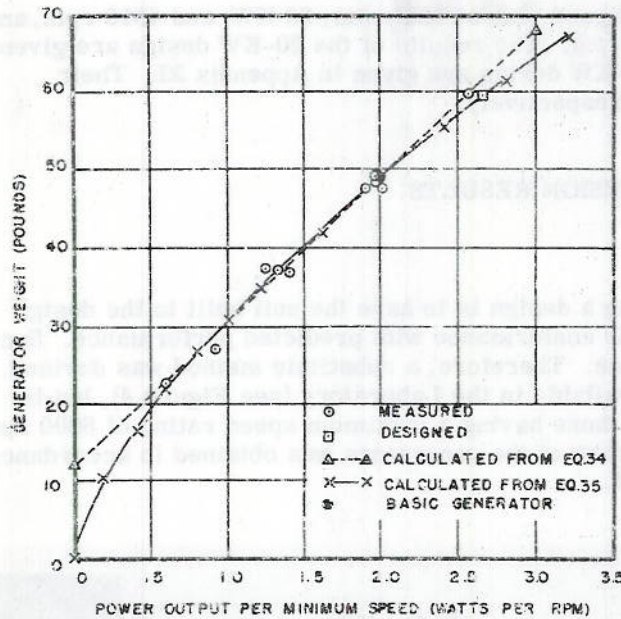


Figure 5 - Generator weight as a function of power output per minimum speed. Comparison of experimental results and derived results for 30-volt generators

pounds, and the value of W_0/n_1 is 2.66. Comparable values of W and W_0/n_1 are obtained from Appendix V for the basic generator. From the above two points and the origin, a curve was derived and is given as

$$W = 31 \left[\frac{W_0}{n_1} \right]^{2/3} \quad (35)$$

This equation represents the approximate weight curve for 30-volt, engine-mounted, aircraft generators whose maximum rated speed is 8000 rpm, and whose flux densities, current densities, and temperature rise of cooling air are the same as those of the basic generator. Equation (35) is plotted on Figure 5 and agrees with the experimental points as well as (34), but is more realistic at small and large values of output per rpm. It is generally conceded that 400 amperes is a practical upper limit of current which can be commutated successfully with no change in basic design. Therefore, 12 KW is an upper limit of power for which Equation (35) will be accurate.

It is recognized that (35) was derived from design values in which only the load current was varied. The terminal voltage was 30 volts and the minimum speed was 4510 rpm in each case. However, the experimental results of Figure 5 were obtained over a range of minimum speeds from 3800 to 4995 rpm. The agreement which exists between (35) and the experimental values plotted in Figure 5 is good. It can be concluded, therefore, that (35) will be valid for values of minimum speed varying between 3000 and 8000 rpm.

Checking 30-Volt Curve for Conformance with Basic Relations

Equation (35) is expressed in terms of power output, W_0 , and (7) in terms of the power developed, W_a . Efficiency tests were performed on the experimental generators at the

$$W = 18.7 \frac{W_0}{n_1} + 12 \quad (34)$$

where W is the total generator weight expressed in pounds. The linear relation is not valid, however, for low values of W_0/n_1 , for it is reasonable to expect that some power output should be available at less than twelve pounds. It is also reasonable to expect that as the value of W_0/n_1 is increased, the ratio of weight to W_0/n_1 should decrease, rather than remain a constant as (34) would require. Therefore, a curve is suggested in which W is proportional to some fractional power of W_0/n_1 .

DERIVATION INVOLVING DESIGN RESULTS

Weight Curve for 30-Volt Generators. Power Output and Minimum Speed Varied

From Appendix IX, the weight of the 12-KW, 30-volt generator is 59.5

TABLE 1
Data Measured on Experimental 30-Volt Generators

Generator Number	Base Speed (rpm)	Power Output (Watts)	Total Loss (Watts)	Friction and Windage Loss (Watts)	Core Loss (Watts)	Shunt-Field Loss (Watts)	$I_a^2 R_a$ Loss (Watts)	Percent Efficiency	$\frac{W_o}{n_1}$	Generator Weight (Pounds)
1	4600	9000	2720	413	265	216	1826	76.8	1.96	49.2
2	4580	6000	2220	176	147	210	1687	76.0	1.31	37.3
3	4280	6000	2480	439	137	204	1700	79.8	1.40	37.0
4	4600	9000	2380	236	265	201	1678	79.2	1.96	48.5
5*	4510	9000	2850	231	231	215	2173	76.0	2.00	49.2
6	4480	9000	2330	603	143	219	1365	79.3	2.01	47.8
7	4770	9000	2920	948	244	203	1525	75.5	1.89	47.8
8	4995	12000	3550	256	224	200	2870	77.2	2.40	60.4
9	4670	12000	3710	568	300	189	2652	76.3	2.57	59.8
10	3300	3000	1400	106	148	257	889	68.2	0.91	27.1
11	3760	2250	810	72	218	105	415	73.5	0.60	22.8
12	4825	6000	2000	217	186	210	1387	75.0	1.24	37.5

* Basic generator

previously determined minimum speeds. The results are given in Table 1. The losses are separated into components so as to better compare the units. The $I_a^2 R_a$ loss includes all losses other than core loss, friction and windage, and shunt-field losses. The output values are included in Table 1 to illustrate the range in power output.

Figure 6, a plot of output as a function of input (W_o/n_1 as a function of W_a/n_1) for the experimental generators, illustrates that efficiency is very nearly constant. An average equation for Figure 6 is

$$\frac{W_o}{n_1} = 0.76 \frac{W_a}{n_1} \tag{36}$$

The armature diameter, D_a , and the stack length, L_a , were obtained from a number of the experimental generators. The values of $D_a^2 L_a$ thus obtained are shown in Figure 7 plotted as a function of W_o/n_1 . The relation is essentially linear and an average equation for the relation is

$$D_a^2 L_a = 30 \frac{W_o}{n_1} \tag{37}$$

Substituting (36) in (37),

$$D_a^2 L_a = 22.8 \frac{W_a}{n_1} \tag{38}$$

Thus, for the experimental 30-volt generators, (38) is an example of (7) with the product of B_{ave} and q remaining constant. Substituting (37) in (35),

$$W = 3.22 \left[D_a^2 L_a \right]^{2/3} \tag{39}$$

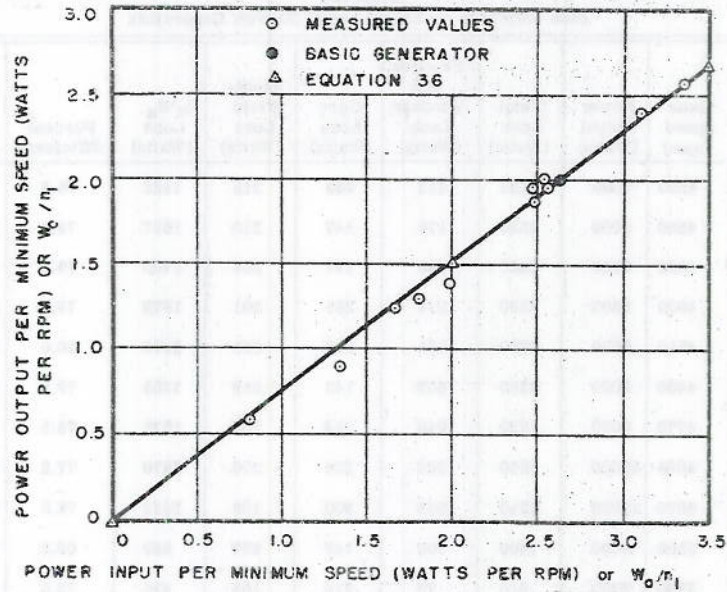


Figure 6 - Power output per minimum speed as a function of power developed per minimum speed (efficiency) for experimental generators

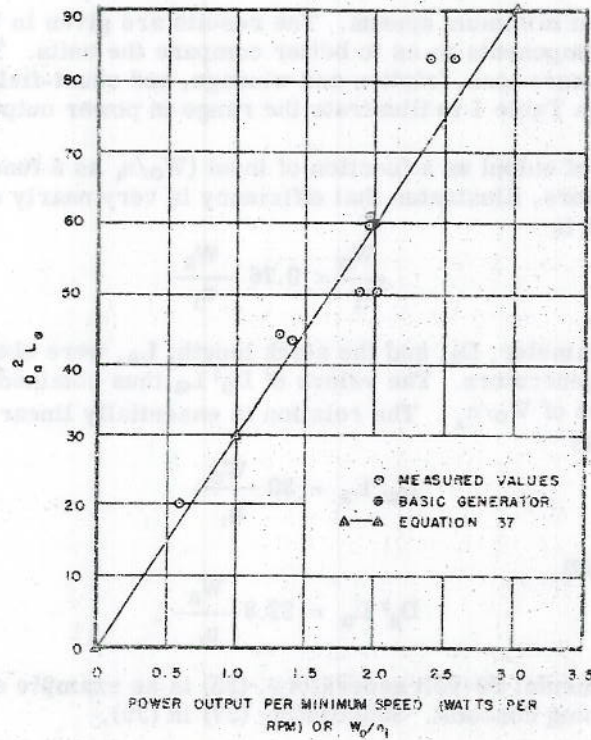


Figure 7 - Armature volume as a function of power output per minimum speed for experimental generators

It has been shown that the data for the experimental generators conforms with the basic relations of (7).

Experimental values of W as a function of $D_a^2 L_a$ are shown in Figure 8. Equation (39) is also plotted in Figure 8. Equation (39) agrees with the experimental points. Thus, additional strength is added to the previous conclusion that equation (35) represents the average relation between weight and power output per minimum speed for 30-volt d-c aircraft generators.

It has been demonstrated that it is possible to determine an empirical curve of the weight of aircraft 30-volt d-c generators as a function of power output per minimum rotor speed. The empirical curve was determined by the use of basic generator and the design of a second generator, holding certain parameters fixed. Experimental data on a series of 30-volt d-c aircraft generators check the empirical curve, and additional experimental information shows that the series of experimental generators and the designed generators conform with the basic performance relation expressed in (7). The empirical weight curve as a function of W_0/n_1 for 30-volt d-c aircraft generators having a maximum rated speed of 8000 rpm is given as (35) and is plotted in Figure 5.

Weight Curve for 120-Volt Generators - Power Output and Minimum Speed Varied

Appendix X gives the results of a 20-KW, 120-volt generator design holding the average air-gap flux density, the current densities, the cooling-air temperature rise, and the minimum rated speed the same as for the basic generator. The number of poles was increased to 6 in order to reduce weight. Very little weight difference will occur at the 9- or 12-KW level, but the 6-pole generator will be lighter than the 4-pole generator at the 20-KW level.

Appendix XI gives the results of a 30-KW, 120-volt generator design, holding the same values constant as for the 20-KW generator design.

Appendix VIII gives the results of a 9-KW, 120-volt generator design, and the design details are included in Appendix VI as an illustration of the design method.

The weights of the 9-KW, 20-KW, and 30-KW generators were 51, 82 and 106 pounds respectively. The values of W_0/n_1 were 2.00, 4.43 and 6.65 watts output per minimum rpm respectively. The above points and the origin were used to derive an equation in the same manner as for the 30-volt generators. The resulting equation is

$$W = 33.4 \left[\frac{W_0}{n_1} \right]^{0.51} \tag{40}$$

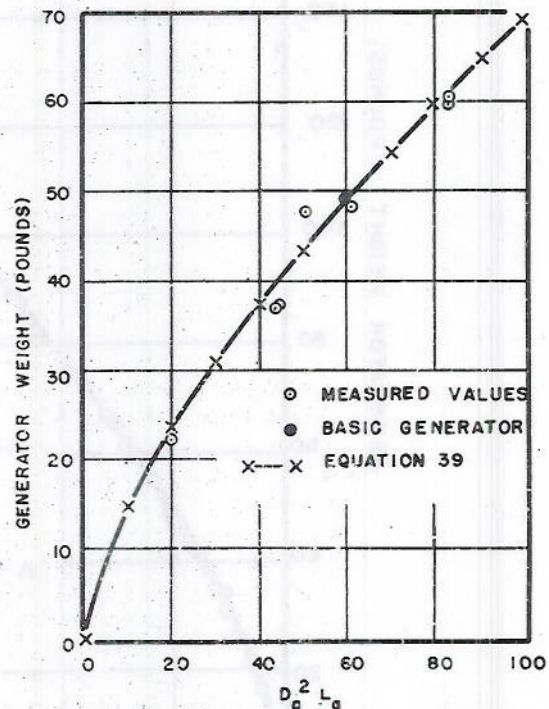


Figure 8 - Generator weight as a function of armature volume. Comparison of experimental values with derived equation

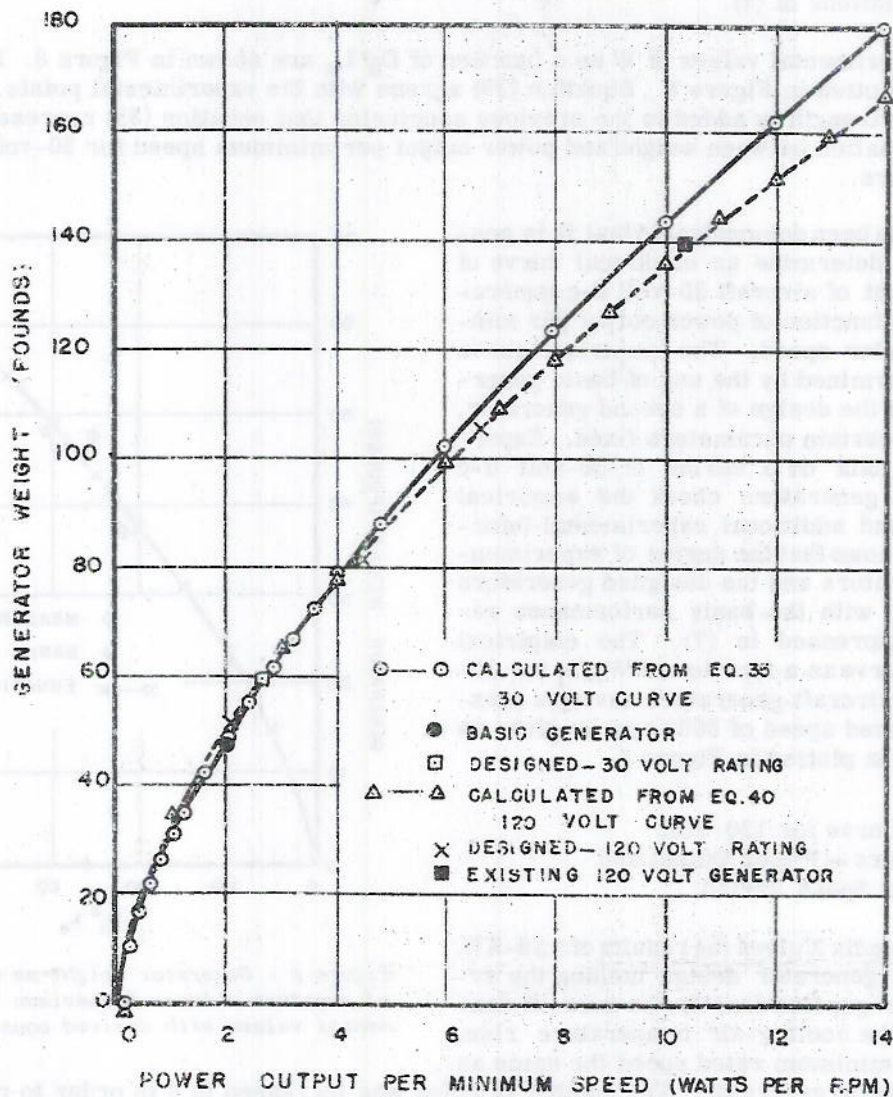


Figure 9 - Derived results of generator weight as a function of power output per minimum speed for 30-volt and 120-volt generators

Equation (40) is shown graphically in Figure 9 and represents the approximate relation between weight and watts output per minimum rpm for 120-volt, d-c aircraft generators which have the same average air-gap flux density, current densities, and cooling-air temperature rise as the basic generator. Equation (35) is also shown graphically in Figure 9 for comparison.

As in the 30-volt designs, the minimum speed was 4510 rpm for all of the 120-volt generators designed. The range of minimum speeds expected is the same as for the 30-volt generators. It was shown experimentally for the 30-volt generators that the effect of varying the speed was also included in (35). Therefore, speed appears in (40) as a vari-

Verification of 120-Volt Weight Curve

No generators were available to check (40) experimentally. However, the weight and minimum speed were obtained for one 30-KW generator, which was tested by the Air Force at Wright Field. The weight was 140 pounds and the minimum speed was 2900 rpm. Thus W_0/n_1 was 10.33. From Figure 9, the weight of a 30-KW, 120-volt generator whose watts output per minimum rpm is 10.33 should be between 138 and 139 pounds. Therefore, in this particular case, the variation in the predicted weight and the actual weight was less than 2 pounds, or a variation of less than 1.5 percent. The one test point may not be conclusive evidence. The same method of design, however, was used for both the 30-volt and 120-volt generators. The experimental verification of the 30-volt curve established the method. Therefore, less experimental evidence is required to prove the validity of the 120-volt curve.

CONCLUSIONS

It has been demonstrated that the effects of changes in terminal voltage, minimum rotor speed, and load current on the weight of d-c aircraft generators can be expressed in mathematical form. Equations (35) and (40) express these mathematical relations, which are also shown graphically in Figure 9.

For any fixed power output, the effect of minimum rotor speed can be determined by reading the values from the proper curve of Figure 9.

For low ratings of power output per minimum speed, the 30-volt generators will weigh less than the 120-volt units.

For higher ratings of power output per minimum speed, a weight saving will result from the use of 120-volt generators.

The effect of varying only the terminal voltage is available from Figure 9 by interpolating between the limits of 30 and 120 volts.

The relations of Figure 9* are approximate, and are limited to the type of design used in the basic generator, but it is believed that they are sufficiently representative to furnish practical values for use in aircraft electrical system weight studies. Such values have not heretofore been available.

* * *

* The following limitations should be noted in the use of the curves of Figure 9:

An upper limit of 400 amperes load current should not be exceeded.

An upper limit of 8000 rpm is the maximum rate speed; therefore, the minimum speed has an upper limit of 8000 rpm. A correction factor must be applied to the curves if it is desired to know the weight of generators whose maximum speed is greater than 8000 rpm.

No definite lower limit has been set on the minimum speed; in general, however, the interest is in minimum speeds in excess of 3000 rpm.

APPENDIX I

List of Symbols

A	- Arbitrary performance constant
A_a	- Equivalent area of armature for one flux path
A_b	- Area of one brush
A_f	- Average area of shunt-field-pole iron
A_g	- Equivalent area for air-gap flux under one pole
A_v	- Ventilation area
A_y	- Equivalent area of yoke per flux path
B	- Arbitrary performance constant
B_a	- Armature flux density
B_{ave}	- Average flux density over the armature surface
B_f	- Shunt-field-pole flux density
B_g	- Air-gap flux density
B_t	- Flux density at D_{at}
B_{te}	- Flux density at tooth crest
B_{tr}	- Flux density at tooth root
B_y	- Flux density in yoke
C	- Arbitrary performance constant
C_s	- Specific heat of air
C_{aa}	- Area of one armature conductor
C_{af}	- Area of one shunt-field conductor
C_{ai}	- Area of one commutating-winding conductor
C_{da}	- Depth of one armature conductor
C_{di}	- Depth of commutating-winding conductor
C_{wa}	- Width of one armature conductor
C_{wi}	- Width of commutating-winding conductor
D	- Arbitrary performance constant
D_a	- Diameter of armature at surface
D_{ar}	- Diameter of armature at the tooth root
D_{at}	- Armature diameter at average tooth width
D_c	- Commutator diameter at brush surface
D_{cr}	- Commutator-riser diameter
D_{yi}	- Inside diameter of yoke
D_{yo}	- Outside diameter of yoke
d_e	- $d_s + \delta - \delta_e$
d_s	- Slot depth
E_g	- Total generated voltage
E_t	- Generator terminal-voltage

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H_a	- Magnetic-field intensity in armature
H_f	- Magnetic-field intensity in shunt-field pole
H_t	- Magnetic-field intensity in armature teeth
H_{te}	- Magnetic-field intensity in armature teeth at crest
H_{tm}	- Magnetic-field intensity in armature teeth at average tooth width
H_{tr}	- Magnetic-field intensity in armature teeth at root
H_y	- Magnetic-field intensity in yoke
I_a	- Armature current
I_b	- Current in one brush
I_f	- Shunt-field current
K	- Generator constant
k	- Arbitrary constant
k_1	- Arbitrary constant
k_2	- Arbitrary constant
k_3	- Arbitrary constant
k_4	- Arbitrary constant
k_5	- Arbitrary constant
k_{ip}	- Peripheral space factor for commutating-winding conductors
k_{ir}	- Radial space factor for commutating-winding conductors
L_a	- Armature-stack length, gross
L_{aet}	- Average length of the end turn of one armature conductor
L_{ap}	- Length of armature conductor per path
L_{at}	- Length of armature conductor per turn
L_c	- Axial length of active commutator surface
L_{cet}	- Average length of the end turn of one compensating or series winding conductor
L_{cp}	- Length of compensating winding per path
L_{cr}	- Axial length of commutator riser
L_{ct}	- Length of compensating or series winding conductor per turn
L_f	- Axial length of shunt-field pole
L_i	- Axial length of commutating pole for calculating volume
L_{iet}	- Length per end turn of commutating winding
L_{ip}	- Length of commutating winding per path
L_{ma}	- Magnetic length of armature per flux path
L_{mf}	- Magnetic length of shunt-field poles per flux path
L_{mg}	- Magnetic length of air-gap per flux path
L_{my}	- Magnetic length of yoke per flux path
L_n	- Net armature stack length
L_{sp}	- Length of series winding per path
L_y	- Axial length of yoke
M	- Weight of air flowing per unit of time
N_{ap}	- Turns per path in armature
N_c	- Number of effective turns per pair of poles furnishing quadrature axis armature reaction
$N_{c'}$	- Number of turns per pair of poles in the armature contributing to quadrature axis armature reaction
$N_{c''}$	- Effective turns per pair of field poles in the compensating field contributing to quadrature axis armature reaction
N_{cp}	- Compensating winding turns per path

- N_d - Number of effective turns per pair of poles furnishing direct axis armature reaction
 N_d' - The number of turns per pair of poles in the armature contributing to direct axis armature reaction
 N_d'' - Effective series-field turns per pair of poles contributing to direct axis armature reaction
 N_f - Number of turns in shunt field per pair of poles
 N_{ip} - Turns per path of commutating winding
 N_{sp} - Series-field winding turns per path
 n - Speed of generator rotor
 n_1 - Base speed or minimum rated speed of generator rotor

 P - Number of shunt-field poles
 p - Number of current paths in generator armature

 q - Ampere conductors per inch of armature periphery

 R_a - Total resistance of armature circuit including brush drop and resistances of series field, commutating windings, and compensating windings
 R_{al} - R_a minus effective resistance due to brush drop
 R_{aa} - Resistance of armature only
 R_{aap} - Resistance of armature per path
 R_{ac} - Resistance of compensating- plus series field
 R_{acp} - Resistance of compensating- plus series field per path
 R_{ai} - Resistance of commutating winding
 R_{aip} - Resistance of commutating winding per path
 R_{cp} - Resistance in series with shunt field
 R_f - Resistance of shunt field

 S - Number of armature slots
 s - Slot width

 T_1 - Entering-air temperature
 T_2 - Exit-air temperature
 T_3 - Average temperature of windings during base-speed test
 t - Average tooth width
 t_i - Average insulation thickness between armature conductors and conductors to frame

 v_{ave} - Average velocity of cooling air

 W - Generator weight
 W_a - Power developed in armature rotor
 W_o - Power output of generator
 W_{oi} - Losses carried away by cooling air

 Z - Number of armature conductors
 Z_c - Number of commutator segments

GREEK SYMBOLS

α	- The brush shift angle expressed in electrical degrees
γ	- Angle in electrical degrees of the arc subtended by a shunt-field pole
Δs	- Change in slot width referred to slot width of basic generator
Δt	- Change in average tooth width referred to average tooth width of basic generator
$\Delta \lambda$	- $\Delta s + \Delta t$
δ	- Clearance between armature surface and shunt-field pole shoe
δ_e	- Effective air gap distance between armature surface and shunt-field pole shoe. Slightly greater than δ to include effect of armature slots
δ_i	- Clearance between armature surface and commutating pole shoe
λ	- $s + t$
μ	- Relative permeability of iron under definite conditions of flux density
ρ	- Density of air
ρ_a	- Armature mass density
ρ_c	- Commutator mass density
ρ_f	- Shunt-field pole mass density
ρ_{fw}	- Shunt-field winding mass density
ρ_i	- Commutating pole mass density
ρ_r	- Average mass density of brush rigging, brushes, and outboard end bell
ϕ	- Total armature surface flux
ϕ_a	- Armature flux per flux path
ϕ_f	- Flux in shunt-field pole
ϕ_g	- Air-gap flux per pole
ϕ_p	- Total armature surface flux divided by the number of poles
ϕ_y	- Flux per path in yoke

* * *

APPENDIX II

Minimum Rotor Speed

The minimum speed was defined in the report as the lowest speed at which the generator will furnish rated load current at rated terminal voltage under stabilized temperature conditions; with a definite resistance in series with the shunt field, and under definite conditions of flow of cooling air.

The resistance to be inserted in series with the shunt field has been set at 1.25 ohms for 30-volt generators, and 3.0 ohms for 120-volt generators. During operation, the terminal voltage of the generator is controlled by an automatic voltage regulator. The value of 1.25 ohms for the 30-volt generators or 3.0 ohms for the 120-volt generators is used to simulate the series combination of voltage-regulator minimum resistance and lead resistance between the generator field and the positive generator terminal.

The conditions of flow of cooling air are determined from several factors:

- (1) A chart relating the basic volume rate of air flow in cubic feet per minute to the kilowatt rating of the generator. The chart is given as Figure 10.
- (2) The impedance to air flow of the generator blast-tube through which the cooling air passes before reaching the generator.
- (3) The impedance to air flow of the generator itself.
- (4) The assumption that at minimum rotor speed the minimum pressure difference between the blast-tube inlet and the generator compartment will be 7 inches of water.

Figure 10 is the curve for determining basic cfm air flow at standard atmospheric conditions (29.92 inches of mercury and 15° centigrade) and is taken from AN-G-1a, the Air Force-Navy Aeronautical Specification for 30-volt Aircraft, Engine-Driven, D-C Generators. Specification AN-G-1a also imposes certain limitations on the blast cooling tube. During conditions of basic cfm air flow obtained from Figure 10 for the particular generator, the pressure drop from the blast-tube air inlet to the generator air entrance shall not exceed one inch of water; and under the conditions in which 130 percent of basic cfm is passing through the blast tube, the pressure drop from the blast-tube air inlet to the generator air entrance shall not exceed 1.69 inches of water. Thus, specification AN-G-1a does not say that all aircraft

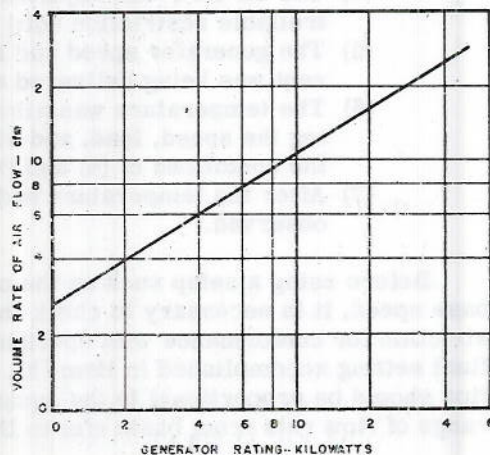


Figure 10 - Curve for determination of basic cfm for designed kilowatt output

generators at minimum flight speed will have the same value of pressure difference across the generator, such as 6 inches of water. The specification does suggest that for laboratory duplication of flight conditions a test setup similar to that illustrated in Figure 11 is required.

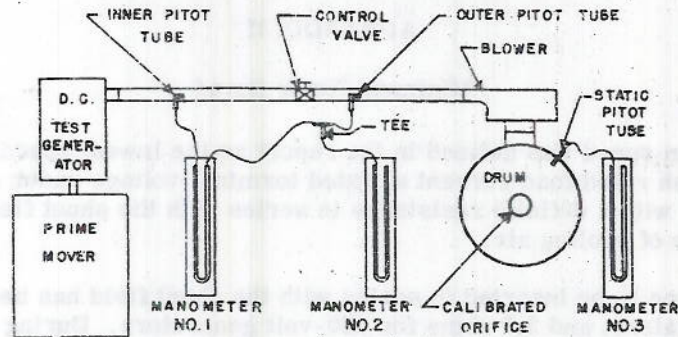


Figure 11 - Test setup for determining generator base speed

To determine the minimum speed of 30-volt aircraft d-c generators using the test setup of Figure 11 the procedure was as follows:

- (1) The generator speed was adjusted to a value below the expected minimum speed. (A value of 1.25 ohms fixed resistance had been previously inserted in the shunt field, and the generator was operating self-excited.)
- (2) The blower was started and the air flow adjusted until approximately basic cfm as determined from Figure 10 was observed on the air flow meter (calibrated orifice).
- (3) The controllable restriction (control valve) and the air flow were adjusted until water manometer No. 1 read 1 inch of water, and at the same time basic cfm of air was passing through the system. No further adjustment of the control valve was made.
- (4) The air flow was adjusted with no change in the setting of the controllable restriction until water manometer No. 2 read 7 inches.
- (5) The generator speed and load were adjusted until rated load current was being delivered at 30 volts terminal voltage.
- (6) The temperature was allowed to stabilize while constantly adjusting the speed, load, and air flow in order to continuously satisfy the conditions of (4) and (5) above.
- (7) After the temperature had stabilized, the minimum speed was observed.

Before using a setup such as the one indicated in Figure 11 for the measurement of base speed, it is necessary to check the combination of blast tube and controllable restriction for conformance with specification AN-G-1a. The requirement is that, after the final setting accomplished in item (3), the pressure drop across the controllable restriction should be proportional to the square of the volume rate of air flow for at least the range of flow rate from basic cfm to the value finally used.

APPENDIX III

Method of Fitting Equation (10) to Experimental Curve of Figure 1

The value of KnD in equation (10) represents the residual voltage. From Figure 1, $KnD = 1.2$ volts.

The value of KnA is obtained when the value of the hyperbolic tangent is equal to unity or:

$$\begin{aligned} KnA &= E_g - KnD \\ &= E_g - 1.2 \end{aligned} \quad (41)$$

It is recognized that (10) cannot be made to fit the experimental curve of Figure 1 much above the full-load excitation value. Thus, an initial approximation of a value for E_g equal to 2 or 3 volts more than E_g at full-load excitation will be a reasonable value.

A value of BN_f equal to approximately 0.2 is a good first approximation for 30-volt d-c aircraft generators.

The above values should be inserted in (10) and spot checked. The computation is not complicated; if the first approximation is not sufficiently accurate, slightly different values of BN_f and KnA are assumed until the curve is approximated. In some generators it may be necessary to use the more complicated equation given by VanValkenburg and Matthews⁷ which was suggested in (8).

* * *

⁷ VanValkenburg, Ernest, and Matthews, Whitney: "Steady-State Analysis of Aircraft D-C Generators." Equation (4) (AIEE paper 48-250. AIEE Transactions, Vol. 67, 1948.)

APPENDIX IV

Equivalent Air-Gap Equations

The following equation taken from Still⁹ was used to calculate the equivalent air gap for the basic generator in order to determine the excitation required to maintain the desired flux density in the air gap.

$$\delta_e = \frac{\lambda}{\frac{t}{\delta} + \frac{5s}{5\delta + s}} \quad (42)$$

where

- δ_e = the equivalent air gap
- δ = the clearance between the armature surface and shunt-field pole shoe
- s = the slot width
- t = the average tooth width
- $\lambda = s + t$

The teeth were tapered on the basic generator, and the average tooth width was obtained by measuring the tooth width at a point four-tenths of the distance from the root to the crest of the tooth.

It was desirable to retain the same clearance between the armature and pole face as well as the same equivalent air gap for the generators which were designed for the same power rating as the basic generator. Therefore, equation (42) was rewritten as

$$\delta_e (5\delta t + ts + 5\delta s) = \lambda (5\delta^2 + \delta s). \quad (43)$$

Consider now a small change in λ , s and t , but with δ_e and δ constant:

$$\delta_e \left\{ 5\delta (t + \Delta t) + (t + \Delta t)(s + \Delta s) + 5\delta (s + \Delta s) \right\} = \lambda \left\{ 5\delta^2 + \delta(s + \Delta s) \right\} + \Delta\lambda \left\{ 5\delta^2 + \delta(s + \Delta s) \right\} \quad (44)$$

Subtracting (43) from (44), and neglecting higher order differentials:

$$\delta_e (5\delta\Delta t + s\Delta t + t\Delta s + 5\delta\Delta s) = \lambda\delta\Delta s + 5\delta^2\Delta\lambda + \delta s\Delta\lambda. \quad (45)$$

Noting that $\Delta s + \Delta t = \Delta\lambda$, substituting in (45), dividing by $\Delta\lambda$, and solving for $\delta_e/\Delta\lambda$.

$$\frac{\delta_e}{\Delta\lambda} = \frac{5\delta\delta_e - 5\delta^2 - \delta s + s\delta_e}{-5\delta\delta_e + 5\delta^2 + \delta s - t\delta_e + \delta\lambda} \quad (46)$$

⁹ Alfred Still, "Elements of Electrical Design," Page 257 (New York and London, McGraw-Hill Book Co., Inc., 1932, 2nd Edition)

It is now possible to solve for the value of $\Delta s/\Delta t$ required to maintain the values of δ and δ_e . Values of s , t , δ and δ_e are obtained from Appendix V. Inserting the values thus obtained in (46):

Item
Number

$$\frac{\Delta s}{\Delta t} = 0.47 \quad (47)$$

35.

36. Equation (47) is useful in solving for new dimensions of slots and teeth by assuming values for either Δs or Δt . The values thus obtained will insure the same equivalent air-gap length for magnetic calculations.

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Item Number	Description	Symbol	Explanation or Numerical Value	Units
66.	Peripheral Dimension of Commutating Pole Shoe		0.47	inches
67.	Radial Dimension of Micarta Strip Between Commutating-Pole Shoe and Winding		0.047	inches
68.	Peripheral Dimension of Micarta Strip Between Commutating-Pole Shoe and Winding		0.95	inches
69.	Field-Pole Arc Length at D_{y1}		1.00	inches
70.	Nominal Wire Size of Shunt-Field Winding		No. 18	AWG
71.	Thickness of Mica Between Commutator Segments	-	0.017	inches
72.	Axial Length of Commutator Riser	D_{cr}	0.1563	inches
73.	Commutator Riser Diameter	D_{cr}	3.625	inches

* * *

APPENDIX VI

Design Details of 120-Volt, 9-KW Generator

I. DETERMINE D_a *

A. Given Data

1. $E_t = 120$ volts
2. $W_o = 9$ KW
3. $n_1 = 4510$ rpm
4. Wave winding. Thus $p = 2$
5. $\frac{\Delta s}{\Delta t} = 0.47$ (see Appendix IV)
6. $B_{ave} = 28,500$ lines per square inch
7. $C_{aa} = 0.0058$ square inch
8. $I_L = 75$ amps.

B. Assumptions

1. $P = 4$ poles
2. $\frac{Z}{S} = 6$ conductors per slot
3. $C_{da} = 0.242$ inch
4. $I_f = 2$ amps.
5. $t_1 = 0.010$ inch
6. $S = 41$ slots
7. $d_s = 0.610$ inch (d_s for basic generator = 0.6095 inch)

C. Calculate D_a

1. $C_{wa} = \frac{C_{aa}}{C_{da}} = \frac{0.0058}{0.242} = 0.024$ inch
2. $s = 3C_{wa} + 4t_1 = 3(0.024) + 4(0.010) = 0.112$ inch

* See Appendix I for definition of symbols

$$3. \Delta s = 0.115 - 0.112 = 0.003 \text{ inch}$$

$$4. \Delta t = \frac{\Delta s}{\Delta s / \Delta t} = \frac{0.003}{0.47} = 0.0064 \text{ inch}$$

$$5. t = 0.156 - 0.0064 = 0.1496 \text{ or } 0.150 \text{ inch}$$

6. Calculate D_{at}

$$a. s + t = 0.112 + 0.150 = 0.262 \text{ inch}$$

$$b. \text{Angle of slot plus tooth} = \frac{360}{41} = 8.78^\circ$$

c. Assume the ratio of the two chords are equal to the ratio of their sectors

$$(1) \text{Angle of slot} = \frac{0.112}{0.262} (8.78) = 3.75^\circ \text{ or } 0.0654 \text{ radians}$$

$$d. D_{at} = 2 \frac{s}{0.0654} = 2 \frac{0.112}{0.0654} = 3.43 \text{ inches (sin } \theta = \theta \text{ for } \theta = 3.75^\circ)$$

$$7. D_a = D_{at} + (0.6d_c) 2 = 3.43 + 0.732 = 4.162 \text{ or } 4.16 \text{ inches}$$

8. DETERMINE L_a

A. Determine $I_a R_a$ (approximate)

$$1. \text{Assume } R_{a1} = 16 (R_{a1} \text{ for basic generator)}^*$$

$$= 16(0.0166) = 0.266 \text{ ohm}$$

$$2. I_a R_a = I_a R_{a1} + 2 = 77(0.265) + 2 = 22.5 \text{ volts}$$

B. Calculate ϕ_p

$$1. \text{Assume } E_g = E_t + I_a R_a = 120 + 22.5 = 142.5 \text{ volts}$$

$$2. E_g = K \phi n$$

$$a. K = \frac{Z}{p(10^8)(60)}$$

$$b. E_g = \frac{Z \phi_p P n}{p(10^8)(60)}$$

$$3. \phi_p = \frac{E_g p(60)(10^8)}{Z n_1 P} = \frac{142.5(2)(60)(10^8)}{246(4510)(4)} = 385,000 \text{ lines}$$

C. Compute L_a (approximate)

$$1. L_a = \frac{\phi_p P}{\pi D_a B_{ave}} = \frac{385,000(4)}{\pi(4.16)(28,500)} = 4.12 \text{ inches}$$

* Assumption made in order to maintain the copper losses constant

D. Compute R_{aa} 1. Compute L_{aet}

a. Basic generator

$$(1) L_{ap} = 338 \text{ inches}$$

$$(2) N_{ap} = \frac{Z}{2p} = \frac{74}{4} = 18.5 \text{ turns}$$

$$(3) L_{at} = \frac{L_{ap}}{N_{ap}} = \frac{338}{18.5} = 18.3 \text{ inches}$$

$$(4) L_{aet} = \frac{L_{at} - 2L_a}{2} = \frac{18.3 - 2(3.91)}{2} = 5.24 \text{ inches}$$

b. 120-volt generator

$$(1) L_{aet} = (L_{aet} \text{ of basic generator}) \frac{D_a}{(D_a \text{ of basic generator})}$$

$$= 5.24 \frac{4.16}{3.919} = 5.56 \text{ inches}$$

2. Compute L_{at}

$$L_{at} = (L_{aet} + L_a)2 = (5.56 + 4.13)2 = 19.38 \text{ inches}$$

3. Compute L_{ap}

$$L_{ap} = L_{at} N_{ap}$$

$$(1) N_{ap} = \frac{Z}{2p} = \frac{246}{4} = 61.5 \text{ turns}$$

$$(2) L_{ap} = 19.38(61.5) = 1192 \text{ inches} = 99.3 \text{ feet}$$

4. Compute R_{aap} at 20°C

$$a. R_{aap}(20) = \frac{10.37(99.3)}{(C_{aa} \text{ in circular mills})} = \frac{10.37(99.3)}{(.0058)} \frac{\pi}{4} \times 10^{-6} = 0.14 \text{ ohm}$$

5. Compute R_{aap} at operating temperature, T_3

$$a. \text{ Assume } T_3 = 139^\circ\text{C} \text{ as in basic generator}$$

$$b. R_{aap} = 0.14 \left(\frac{234.5 + 139}{234.5 + 20} \right) = 0.14(1.467) = 0.21 \text{ ohm}$$

$$6. R_{aa} = \frac{R_{aap}}{2} = \frac{0.21}{2} = 0.105 \text{ ohm}$$

E. Compute R_{ac} 1. Compute L_{cet}

a. Basic generator

$$(1) (L_{cp} + L_{sp}) = 177 \text{ inches}$$

$$(2) N_{cp} = 7 \text{ turns}$$

$$(3) N_{sp} = 1/2 \text{ turn}$$

$$(4) L_{ct} = \frac{L_{cp} + L_{sp}}{N_{cp} + N_{sp}} = \frac{177}{7.5} = 23.6 \text{ inches}$$

$$(5) L_{cet} = \frac{L_{ct} - 2L_f}{2} = \frac{23.6 - 2(3.84)}{2} = 7.96 \text{ inches}$$

b. 120-volt generator

$$(1) L_{cet} = (L_{cet} \text{ of basic generator}) \frac{D_a}{(D_a \text{ of basic generator})}$$

$$= 7.96 \frac{4.16}{3.919} = 8.45 \text{ inches}$$

2. Compute L_{ct}

$$a. L_f = L_a - (L_a \text{ of basic generator} - L_f \text{ of basic generator})$$

$$= 4.13 - (3.91 - 3.84) = 4.06 \text{ inches}$$

$$b. L_{ct} = (L_{cet} + L_f)2 = (8.45 + 4.06)2 = 25.0 \text{ inches per turn}$$

3. Compute N_{cp}

a. From equation (13)

$$N_c' I_a = \frac{2\gamma Z I_a}{360 P_p} = \frac{2(67)(246)I_a}{360(4)(2)} = 11.45 I_a \text{ or } 11.5 I_a$$

b. For the basic generator

$$N_c'' I_a = \frac{(7 \times 2)}{4} I_a = 3.5 I_a$$

c. For the 120-volt generator

$$(1) N_c' I_a = \frac{23 \times 2}{4} I_a = 11.5 I_a$$

(2) For perfect compensation

$$N_c'' I_a = N_c' I_a$$

(3) Assume the value from (c) furnishes perfect compensation

$$d. N_{cp} = 11.5 \times 2 = 23 \text{ turns}$$

4. Compute L_{cp}

$$a. L_{cp} = L_{ct}(N_{cp}) = (25.0)23 = 575 \text{ inches or } 47.9 \text{ feet}$$

5. Compute L_{sp}

- a. Assume that $N_{sp} = \frac{1}{2}$ turn as in basic generator
- b. $L_{sp} = L_{ct} N_{sp} = 25(0.5) = 12.5$ inches or 1.04 feet

6. Compute R_{acp}

- a. Assume the same current density as in basic generator
 - b. Area per conductor = $\frac{\text{Amps}}{\text{Current Density}} = \frac{\frac{77}{4}}{10,360} = 0.00186$ sq.in.
= 2368 circular mils
 - c. R_{acp} at $20^{\circ}\text{C} = \frac{10.37(47.9 + 1.04)}{2368} = 0.214$ ohm
 - d. R_{acp} at $139^{\circ}\text{C} = 0.214(1.467) = 0.314$ ohm
7. $R_{ac} = \frac{R_{acp}}{4} = \frac{0.314}{4} = 0.0785$ ohm

F. Compute R_{ai}

- 1. Assume the same commutating-winding ampere-turns per pole as in basic generator = $\left[N_{ip} \frac{I_a}{4} \right]$ of basic generator = $12.5 \left(\frac{307.2}{4} \right) = 960$ ampere turns
 - a. $N_{ip} = \frac{960}{I_a/4} = \frac{960(4)}{77} = 49.9$ or 50 turns
- 2. Assume the same current density as in basic generator, or 9370 amperes per sq.in.
 - a. $C_{ai} = \frac{I_a/4}{9370} = \frac{77}{9370(4)} = 0.00205$ sq.in.
- 3. Assume $\delta_i = 0.0545$ inch as in basic generator
- 4. Assume a micarta strip 0.047 inch thick plus a pole shoe 0.06 inch thick, as in basic generator
- 5. Radial space for commutating-pole conductors and attached insulation for one commutating pole =

$$\frac{D_{yi} - [D_a + 2 \delta_i + 2(0.047 + 0.06)]}{2} = \frac{D_{yi} - [4.16 + 0.109 + 0.214]}{2} = \frac{D_{yi} - 4.48}{2}$$

- a. Assume $D_{yi} = 5.35$ inches*

- b. Radial space for commutating-pole conductors and attached insulation

for one commutating pole = $\frac{5.35 - 4.48}{2} = 0.435$ inch

* D_{yt} of basic generator = 5.328 inches

6. Assume $k_{ir} = 0.711$ as in basic generator

7. Radial space for conductor only = $0.435(0.711) = 0.309$ inch

8. Compute conductor dimensions

a. Assume 5 conductors per layer, 10 layers deep

$$b. C_{di} = \frac{0.309}{5} = 0.0618 \text{ inch}$$

$$c. C_{wi} = \frac{C_{aj}}{C_{di}} = \frac{0.00205}{0.0618} = 0.0332 \text{ inch}$$

9. Compute peripheral space of commutating pole and windings

a. Assume width of commutating pole plus 2 mica strips =

$$0.28 + 2(0.035) = 0.35 \text{ inch as in basic generator}$$

b. Assume $K_{ip} = 0.885$ as in basic generator

c. Peripheral space occupied by conductors = $C_{wi}(10)(2) = 0.0332(20) = 0.664$ in.

d. Peripheral space occupied by insulated conductors

$$= \frac{0.664}{k_{ip}} = \frac{0.664}{0.885} = 0.750 \text{ inch}$$

e. Peripheral space occupied by commutating pole and winding

$$= 0.750 + 0.35 = 1.10 \text{ inch}$$

10. Compute end view area of one commutating pole and winding

a. Area = commutating pole and winding area + mica strip area +
commutating-pole shoe area

(1) Assume mica strip = 1.10 inches wide

(2) Assume pole shoe = 0.47 inch wide as in basic generator

$$(3) \text{ Area} = 1.10(0.435) + 1.10(0.047) + 0.47(0.06) = 0.4785 + 0.0517 + 0.0282$$

$$= 0.5584 \text{ or } 0.558 \text{ sq. in.}$$

11. Compute L_{ip}

a. $L_{iet} = 0.85$ inch as in basic generator

b. Assume (L_2 - axial length of commutating-pole iron) is the same as in

$$\text{basic generator} = 3.91 - 3.15 = 0.76 \text{ inch}$$

c. Axial length of commutating-pole iron = $4.13 - 0.76 = 3.37$ inches

d. $L_{ip} = [3.37(2) + 0.85(2)] \frac{50}{12} = 36.8$ feet

12. Compute R_{aip}

a. R_{aip} at $20^{\circ}\text{C} = \frac{10.37(36.8)}{0.00205} \frac{\pi}{4} \times 10^{-6} = 0.146$ ohm

b. R_{aip} at $139^{\circ}\text{C} = 0.146(1.467) = 0.214$ ohm

13. $R_{ai} = \frac{R_{aip}}{4} = \frac{0.214}{4} = 0.0535$ ohm

G. Compute R_{a1}

1. $R_{a1} = R_{aa} + R_{ac} + R_{ai} = 0.105 + 0.0785 + 0.0535 = 0.237$ ohm

H. Compute $I_a R_a$

1. $I_a R_a = 77(0.237) + 2 = 20.25$ or 20.3 volts

I. Calculate ϕ_p at Base Speed and Full Load

1. Assume $E_g = E_t + I_a R_a = 120 + 20.3 = 140.3$ volts

2. $\phi_p = \frac{E_g P (60) (10^8)}{Z_n P} = \frac{140.3(2)(60)(10^8)}{246(4510)(4)} = 379,000$ lines

J. Check B_{ave}

1. $B_{ave} = \frac{\phi_p P}{\pi D_a L_a} = \frac{379,000(4)}{\pi(4.16)(4.13)} = 28,100$ lines/sq.in.

2. L_a must be decreased slightly

K. Correct Armature Stack Length

1. $L_a = \frac{\phi_p P}{\pi D_a B_{ave}} = \frac{379,000(4)}{\pi(4.16)(28,500)} = 4.07$ inches

III. CALCULATE SHUNT-FIELD POLE AREA

A. Calculate L_f

1. $L_f = 4.07 - 0.07 = 4.00$ inches

B. Calculate Peripheral Width for Flux

1. At pole-shoe face

a. Assume the same angular dimensions as in basic generator (see Figure 2)

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$$(1) \text{ Arc length} = \left(\frac{D_2}{2} + \delta \right) \times (\text{angle}) = \left(\frac{4.16}{2} + 0.0115 \right) \frac{33.5}{57.3} = 1.22 \text{ inches}$$

b. Peripheral distance for flux at pole-shoe face = 1.22 inches

2. At D_{y1}

a. Assume the same flux density at D_{y1} as in basic generator

(1) Calculate ϕ_f at yoke

(a) Assume 20% leakage flux

$$(b) \phi_f \text{ at yoke} = 1.2(379,000) = 454,800 \text{ lines}$$

$$(2) B_f \text{ at yoke} = \frac{\phi_f \text{ at yoke for basic generator}}{A_f \text{ at yoke for basic generator}} \\ = \frac{343,000(1.2)}{3.84} = 107,000 \text{ lines/sq.in.}$$

$$(3) A_f \text{ at yoke} = \frac{454,800}{107,000} = 4.25 \text{ square inches}$$

$$b. \text{ Peripheral width at } D_{y1} = \frac{4.25}{4.00} = 1.06 \text{ inches}$$

IV. DERIVE THE NO-LOAD SATURATION CURVE (Assuming $N_f I_f$ is the Same as in Basic Generator)

A. Basic Generator

$$1. E_t = 40.5 \tanh(0.001375 N_f I_f) + 1.2$$

$$2. K_n A = 40.5$$

$$a. K = \frac{74}{2(60)(10^3)} = 6.165 \times 10^{-7}$$

$$b. n_1 = 4510 \text{ rpm}$$

$$3. A = \frac{40.5}{K n_1} = \frac{40.5}{(6.165 \times 10^{-7})(4510)} = 1,456,000 \text{ lines}$$

$$4. K_n D = 1.2$$

$$5. D = 43,150 \text{ lines}$$

B. 120-Volt Generator

$$1. K_n = \frac{246(4510)}{2(60)(10^3)} = 3.24 \times 10^{-6}$$

$$2. A = \frac{A_p \text{ at air gap}}{A_p \text{ at air gap for basic generator}} \times A \text{ for basic generator}$$

$$= \frac{1.22(4.00)}{1.15(3.84)} (1,456,000) = 1,610,000 \text{ lines}$$

$$3. D = \frac{1.22(4.00)}{1.15(3.84)} (43,150) = 47,700 \text{ lines}$$

$$4. E_t = K_n A \tanh(0.001375 N_f I_f) + K_n D = 149 \tanh(0.001375 N_f I_f) + 4.4$$

V. RADIAL DEPTH OF ONE SHUNT-FIELD POLE

A. Compute Radial Depth

$$1. \text{ Radial depth} = \frac{D_{yi} - [(D_a + \delta (2))]}{2} = \frac{5.35 - [4.16 + (0.0115)(2)]}{2} = 0.583 \text{ in.}$$

VI. COMMUTATING POLE DIMENSIONS

A. Compute L_i

$$1. L_i = L_f = 4.00 \text{ inches}$$

B. Compute End View Area

$$1. \text{ End View Area} = 0.558(4) = 2.23 \text{ sq. in.}$$

C. Compute Total Volume

$$1. \text{ Total volume} = (2.23)(4.00) = 8.92 \text{ cu. inches for 4 commutating poles}$$

VII. COMPENSATING WINDING

A. Check Opening Space (see Figure 2)

1. 24 conductors per opening, each of 2368 circular mils

$$a. \text{ Copper area} = 24(2368) = 56,900 \text{ circular mils}$$

2. Basic generator had 32 conductors of 2260 circular mils

$$a. \text{ Copper area} = 32(2260) = 72,300 \text{ circular mils}$$

3. Space is adequate

B. Assume Perfect Compensation (from II, E, 3.)

VIII. SERIES WINDING

A. Compute Series Demagnetization per Pair of Poles

3. 1/2 turn per pole enclosing 2/3 of the pole and carrying $I_a/4$

$$a. N_d'' I_a = \left(\frac{1}{2} + \frac{1}{2} \right) \frac{2}{3} \frac{I_a}{4} = \frac{1}{6} I_a = 0.1667 I_a$$

2. Assume a negative brush shift to offset series demagnetization

$$a. N_d' I_a = \frac{4\sigma(Z)I_a}{360Pp} = \frac{-4(0.448)(246)I_a}{360(4)(2)} = -0.1667 I_a$$

b. $\alpha = -0.448$ electrical degree, or approximately $-1/4$ mechanical degree

IX. SHUNT-FIELD WINDING

A. Compute Division of Excitation

1. Basic generator

a. Air gap

$$(1) (N_f I_f)_g \text{ per pair of poles} = 0.313 B_g L_{mg}$$

$$(a) \phi_g = \phi_p = 343,000 \text{ lines}$$

$$(b) L_{mg} = 2 \delta_e = 2(0.0160) = 0.0320 \text{ inch}$$

$$(c) A_g = (L_a) \times (\text{equivalent air gap arc length}). \text{ From Figure 2,}$$

24 degrees are subtracted for calculation of compensation.

However, considerable fringing will take place. Subtract only

$$10 \text{ degrees. Arc length} = \left(\frac{57.5 - 10}{57.3} \right) \left(\frac{3.919}{2} \right) = 1.62 \text{ inches}$$

$$A_g = L_a (1.62) = 3.91(1.62) = 6.35 \text{ sq. in.}$$

$$(d) B_g = \frac{\phi_g}{A_g} = \frac{343,000}{6.35} = 54,000 \text{ lines per sq. in.}$$

$$(e) (N_f I_f)_g = 0.313 (54,000) (0.0320) = 542 \text{ ampere turns}$$

b. Shunt-field poles

$$(1) A_f = L_f \times (\text{equivalent arc length})$$

$$(a) \text{ Equivalent arc length} = \frac{\text{arc length at pole shoe} + \text{arc length at } D_y}{2}$$

$$\frac{1.15 + 1.0}{2} = 1.075$$

$$\underline{1.} \text{ Arc length at pole shoe} = \left(\frac{57.5 - 24}{57.3} \right) \left(\frac{3.919}{2} \right) = 1.15 \text{ in. (see Fig$$

$$(b) A_f = 3.84 (1.075) = 4.13 \text{ sq. in.}$$

(2) Assume 20% leakage flux

$$(a) \phi_f = 343,000(1.2) = 411,500 \text{ lines}$$

$$(3) L_{mf} = D_{yi} - (D_a + 2\delta) = 5.328 - (3.919 + 0.023) = 1.386 \text{ inches}$$

$$(4) B_f = \frac{\phi_f}{A_f} = \frac{411,500}{4.13} = 99,600 \text{ lines per sq. in.}$$

$$(5) \text{At } B_f = 99,600, H_f = 59 \text{ ampere turns per inch}^{\circ}$$

$$(6) (N_f I_f)_f = H_f L_{mf} = 59(1.386) = 81.8 \text{ ampere turns}$$

c. Armature

(1) Calculate L_{ma}

$$(a) D_{ar} = 2.70 \text{ inches}$$

$$(b) \text{Shaft diameter} = 0.875 \text{ inch}$$

$$(c) \text{Mean diameter} = \frac{D_{ar} + \text{shaft diameter}}{2} = 1.79 \text{ inches}$$

$$(d) L_{ma} = \frac{1.79}{2} (\text{arc length}) = \frac{1.79}{2} \cdot \frac{\pi}{2} = 1.41 \text{ inches}$$

(2) Calculate A_a

$$(a) \text{Diameter of lightening holes} = 0.375 \text{ inch}$$

$$(b) A_a = (L_a) (\text{effective iron measured along the radius})$$

$$= 3.91 \left[\frac{(2.70 - 0.875)}{2} - 0.375 \right]$$

$$= 3.91 (0.538) = 2.10 \text{ sq. inches}$$

$$(3) B_a = \frac{\phi_a}{A_a} = \frac{343,000}{2(2.10)} = 81,700 \text{ lines/sq. in.}$$

$$(4) \text{at } B_a = 81,700, H_a = 11 \text{ ampere turns per inch}^{\circ}$$

$$(5) (N_f I_f)_a = H_a L_{ma} = 11(1.41) = 15.5 \text{ ampere turns}$$

d. Armature teeth

(1) Tooth-density formula

^o Herbert C., *Rotors, Electromagnetic Devices*, Page 58, Curve 7a, (New York, John Wiley & Sons, Inc., London: Chapman & Hall, Limited, First Edition)

$$B_g = B_t \left[\frac{(d_e + \delta_e) + \frac{d_e}{\mu} \left(\frac{\lambda L_a}{t L_n} - 1 \right)}{d_e \left(\frac{\lambda L_a}{t L_n} \right) + \delta_e} \right] = B_t \left(0.559 \cdot \frac{0.441}{\mu} \right) \text{ gauss}$$

- (2) From the air-gap calculation, $B_g = 54,000$ lines per sq. in., or 8370 gauss
- (3) Assume values of B_t , read corresponding values of μ from Rotors, and solve the tooth-density formula¹⁰ graphically for $B_g = 8370$ gauss.
- (4) From graphic solution, at $B_g = 8370$ gauss, $B_t = 14,900$ gauss, or $B_t = 96,000$ lines per sq. in.

- (5) Correct for tooth taper¹¹

(a) Average $H_t = \frac{1}{6} H_{tr} + \frac{2}{3} H_{tm} + \frac{1}{6} H_{te}$

(b) At $B_t = 96,000$, $H_{tm} = 40$ ampere turns per inch⁹

- (c) H_{tr} and H_{te} are determined by first solving for the tooth density at the root and at the crest, and then reading the corresponding H values from the curve⁹

(d) Average tooth width = $t = 0.156$ inch

(e) Tooth width at root = 0.115 inch

(f) Tooth width at crest = 0.217 inch

- (g) Tooth data

1 At $B_g = 54,000$, $B_t = 96,000$, $H_{tm} = 40$

2 $B_{tr} = 96,000 \left(\frac{0.156}{0.115} \right) = 130,000$

a $H_{tr} = 900$

3 $B_{te} = 96,000 \left(\frac{0.156}{0.217} \right) = 69,000$

a $H_{te} = 6$

¹⁰ Alfred Still, "Elements of Electrical Design," Page 267, (New York and London, McGraw-Hill Book Company, Inc., 1932, 2nd Edition)

¹¹ Alfred Still, "Elements of Electrical Design," Page 102, (New York and London, McGraw-Hill Book Company, Inc., 1932, 2nd Edition)

$$(h) \text{ Average } H_t = \frac{1}{6}(900) + \frac{2}{3}(40) + \frac{1}{6}(6) = 178 \text{ ampere turns per inch}$$

$$(6) (N_f I_f)_{at} = 2 d_e (H_t) = 2(0.605)(178) = 217 \text{ ampere turns}$$

e. Yoke

$$(1) L_y = 6.25 \text{ inches}$$

$$(2) D_{y0} = 6.00 \text{ inches}$$

$$(3) D_{y1} = 5.328 \text{ inches}$$

$$(4) A_y = 6.25 \frac{(6.00 - 5.328)}{2} = 2.10 \text{ sq. in.}$$

$$(5) \text{ Calculate } \phi_y$$

(a) Assume 15% leakage

$$(b) \phi_y = \frac{\phi_g}{2} (1.15) = \frac{343,000}{2} (1.15) = 197,200 \text{ lines}$$

$$(6) B_y = \frac{\phi_y}{A_y} = \frac{197,200}{2.10} = 93,900 \text{ lines per sq. in.}$$

$$(7) \text{ Calculate } H_y$$

(a) Assume the yoke material fits Rotors' cast steel curve.¹²

$$(b) \text{ At } B_y = 93,900 \text{ lines per sq. in., } H_y = 65 \text{ ampere turns per inch}^{12}$$

$$(8) \text{ Calculate } L_{my}$$

$$(a) L_{my} = \frac{\pi}{4} \left(\frac{6.00 + 5.328}{2} \right) = 4.45 \text{ inches}$$

$$(9) (N_f I_f)_y = 4.45 (65) = 289 \text{ ampere turns}$$

f. Total excitation per pair of poles from above calculations

Air gap	542
Shunt-field poles	82
Armature	16
Armature teeth	217
Yoke	289

$$N_f I_f = 1146 \text{ amp. turns per pair of poles}$$

g. Total excitation from voltage equation (equation (22))

¹² Herbert C. Rotors, "Electromagnetic Devices," Page 48, Curve 5 (New York, John Wiley & Sons, Inc., London: Chapman & Hall, Limited, First Edition)

$$(1) E_t = 40.5 \tanh [0.001375(160 I_f + 0.064 I_a)] + 1.2 + 0.0231 I_a$$

$$(2) \text{At } I_f = 7.2 \text{ and } I_a = 307.2$$

$$(a) 160 I_f = 160(7.2) = 1152 \text{ ampere turns}$$

$$(b) 0.064 I_a = 0.064(307.2) = 19.65 \text{ or } 20 \text{ ampere turns}$$

$$(3) \text{Net excitation} = 1152 - 20 = 1132 \text{ ampere turns per pair of poles}$$

b. Calculate percent variation, considering equation value to be 100%

$$(1) \text{Percent variation} = \left[\frac{1146 - 1132}{1132} \right] (100) = \frac{1400}{1132} = 1.24\%$$

The agreement is much better than would be expected, considering the assumptions, but indicates that the method of checking the division of excitation is within reason.

2. 120 volt generator

a. Air gap

$$(1) (N_f I_f)_g = 0.313 B_g L_{m,g}$$

$$(a) \phi_g = \phi_p = 379,000 \text{ lines}$$

$$(b) A_g = (L_a) \times (\text{equivalent air gap arc length})$$

Assume the same angle as was used for basic generator arc

$$\text{length} = \text{radius} \times \text{angle} = \frac{4.16}{2} \left(\frac{47.5}{57.3} \right) = 1.72 \text{ inches}$$

$$A_g = 4.07(1.72) = 7.00 \text{ sq. in.}$$

$$(c) B_g = \frac{\phi_g}{A_g} = \frac{379,000}{7.00} = 54,100 \text{ lines per sq. in.}$$

$$(d) L_{m,g} = 2\delta_e = 2(0.0160) = 0.0320 \text{ inch}$$

$$(e) (N_f I_f)_g = 0.313(54,100)(0.0320) = 542 \text{ ampere turns}$$

b. Shunt-field poles

$$(1) A_f = L_f \times (\text{equivalent arc length})$$

$$(a) \text{Equivalent arc length} = \frac{\text{arc length at pole shoe} + \text{arc length at } L_f}{2}$$

$$= \frac{1.22 + 1.06}{2} = 1.14 \text{ inches}$$

$$\underline{1} \text{ Arc length at pole shoe} = \frac{33.5}{57.3} \frac{4.16}{2} = 1.22 \text{ inches}$$

$$(b) A_f = 4.00(1.14) = 4.56 \text{ sq. in.}$$

(2) Assume 20% leakage flux

$$(a) \phi_f = 379,000(1.2) = 455,000 \text{ lines}$$

$$(3) L_{mf} = 0.584(2) = 1.168 \text{ inches}$$

$$(4) B_f = \frac{\phi_f}{A_f} = \frac{455,000}{4.56} = 99,800 \text{ lines per sq. in.}$$

$$(5) \text{ At } B_f = 99,800, H_f = 60 \text{ ampere turns per inch}^9$$

$$(6) (N_f I_f)_f = H_f L_{mf} = 60(1.168) = 70.1 \text{ ampere turns}$$

c. Armature

(1) Calculate L_{ma}

$$(a) D_{ar} = 4.16 - 2(0.610) = 2.94 \text{ inches}$$

(b) Assume shaft diameter = 1.00 inch

$$(c) \text{ Mean diameter} = \frac{2.94 + 1.00}{2} = 1.97 \text{ inches}$$

$$(d) L_{ma} = \frac{1.97}{2} (\text{arc length}) = 0.985 \frac{\pi}{2} = 1.55 \text{ inches}$$

(2) Calculate A_a

(a) Assume same size lightening hole of 0.375 inch

$$(b) A_a = L_a \times (\text{effective iron measured along the radius}) \\ = 4.07 \left[\frac{2.94 - 1.00}{2} \right] - 0.375 = 2.42 \text{ sq. in.}$$

(3) Calculate B_a

$$(a) B_a = \frac{\phi_a}{A_a} = \frac{379,000}{2(2.42)} = 78,300 \text{ lines per sq. in.}$$

$$(4) \text{ At } B_a = 78,300 \text{ lines per sq. in.}, H_a = 9.5 \text{ ampere turns per inch}^9$$

$$(5) (N_f I_f)_a = H_a L_{ma} = 9.5(1.55) = 14.7 \text{ ampere turns}$$

d. Armature teeth

(1) Tooth density formula:¹⁰

$$B_g = B_t \left[\frac{(d_e + \delta_e) + \frac{d_e}{\mu} \left(\frac{\lambda L_a}{t L_n} - 1 \right)}{d_e \left(\frac{\lambda L_a}{t L_n} \right) + \delta_e} \right]$$

$$= B_t \left(0.556 + \frac{0.444}{\mu} \right) \text{ gauss}$$

(2) From the air-gap calculation, $B_g = 54,100$ lines per sq. in., or 8380 ga

(3) Assume values of B_t , read corresponding values of μ from Rotors,⁹ and solve the tooth-density formula graphically for $B_g = 96,000$ lines per sq. in.

(4) From graphic solution, at $B_g = 8380$ gauss,

$$B_t = 14,900 \text{ gauss, or } B_t = 96,000 \text{ lines per sq. in.}$$

(5) Correct for tooth taper¹¹

(a) Average $H_t = \frac{1}{6} H_{tr} + \frac{2}{3} H_{tm} + \frac{1}{6} H_{te}$

(b) At $B_t = 96,000$ lines per sq. in., $H_{tm} = 40$ ampere turns per inch⁹

(c) H_{tr} and H_{te} are determined by first solving for the tooth density at the root and at the crest, and then reading the corresponding H values from Rotors' curve.⁹

(d) Average tooth width = $t = 0.150$ inch

(e) Tooth width at root = 0.112 inch

(f) Tooth width at crest = 0.208 inch

(g) Tooth data

1. At $B_g = 54,100$, $B_t = 96,000$, $H_{tm} = 40$

2. $B_{tr} = 96,000 \left(\frac{0.150}{0.112} \right) = 129,000$

a. $H_{tr} = 850$

3. $B_{te} = 96,000 \left(\frac{0.150}{0.208} \right) = 69,500$

a. $H_{te} = 6$

(h) Average $H_t = \frac{1}{6} (850) + \frac{2}{3} (40) + \frac{1}{6} (6) = 170$ ampere turns per in.

(6) $(N_f I_f)_{at} = 2 d_e (170) = 2(0.606)(170) = 206$ ampere turns

a. Yoke

(1) Compute L_y

(a) Maintain the same difference $L_y - L_f$ as in basic generator

$$(L_y - L_f) \text{ in basic generator} = 6.25 - 3.84 = 2.41 \text{ inches}$$

(b) $L_y = 4.00 + 2.41 = 6.41$ inches

(2) $D_{yi} = 5.35$ inches

(3) Assume $B_y = 93,900$ lines per sq. inch in basic generator

(4) $\phi_y = \frac{279,000}{2} (1.15) = 218,000$ lines

(5) $A_y = \frac{\phi_y}{B_y} = \frac{218,000}{93,900} = 2.32$ sq. in.

(6) Wall thickness $= \frac{A_y}{L_y} = \frac{2.32}{6.41} = 0.362$ inch

(7) $D_{y0} = 5.35 + 2(0.362) = 6.07$ inches

(8) $L_{my} = \frac{\pi}{4} \frac{6.07 + 5.35}{2} = 4.48$ inches

(9) At $B_y = 93,900$ lines per sq. in., $H_y = 65$ ampere turns per inch¹²

(10) $(N_f I_f)_y = L_{my} H_y = 4.48(65) = 291$ ampere turns

f. Total excitation per pair of poles

Air Gap	542
Shunt-Field Poles	70
Armature	15
Armature Teeth	206
Yoke	291

$N_f I_f = 1124$ ampere turns per pair of poles

B. Discussion of Excitation

- The calculated excitation is approximately 2-1/2 percent less than the calculated excitation for the basic generator, which is better agreement than the degree of accuracy of calculations because of the assumptions made. Therefore, the same $N_f I_f$ (1152) will be used.

C. Determine N_f

- Figure 12 is a graph for determining wire size. The following assumptions were made in order to plot the figure:
 - $N_f I_f = 1152$ ampere turns per pair of poles
 - The shunt-field copper current density was held constant at 5340 amperes per sq. inch.
 - Space factors used were taken from Still¹³
 - $C_{af} = 0.00135$ sq. in. in basic generator
 - The total copper area was held constant
 - The value of $I_f^2 R_f$ was assumed constant

¹³ Alfred Still, "Elements of Electrical Design," Page 35. (New York and London, McGraw-Hill Book Company, Inc., 1932, 2nd Edition).

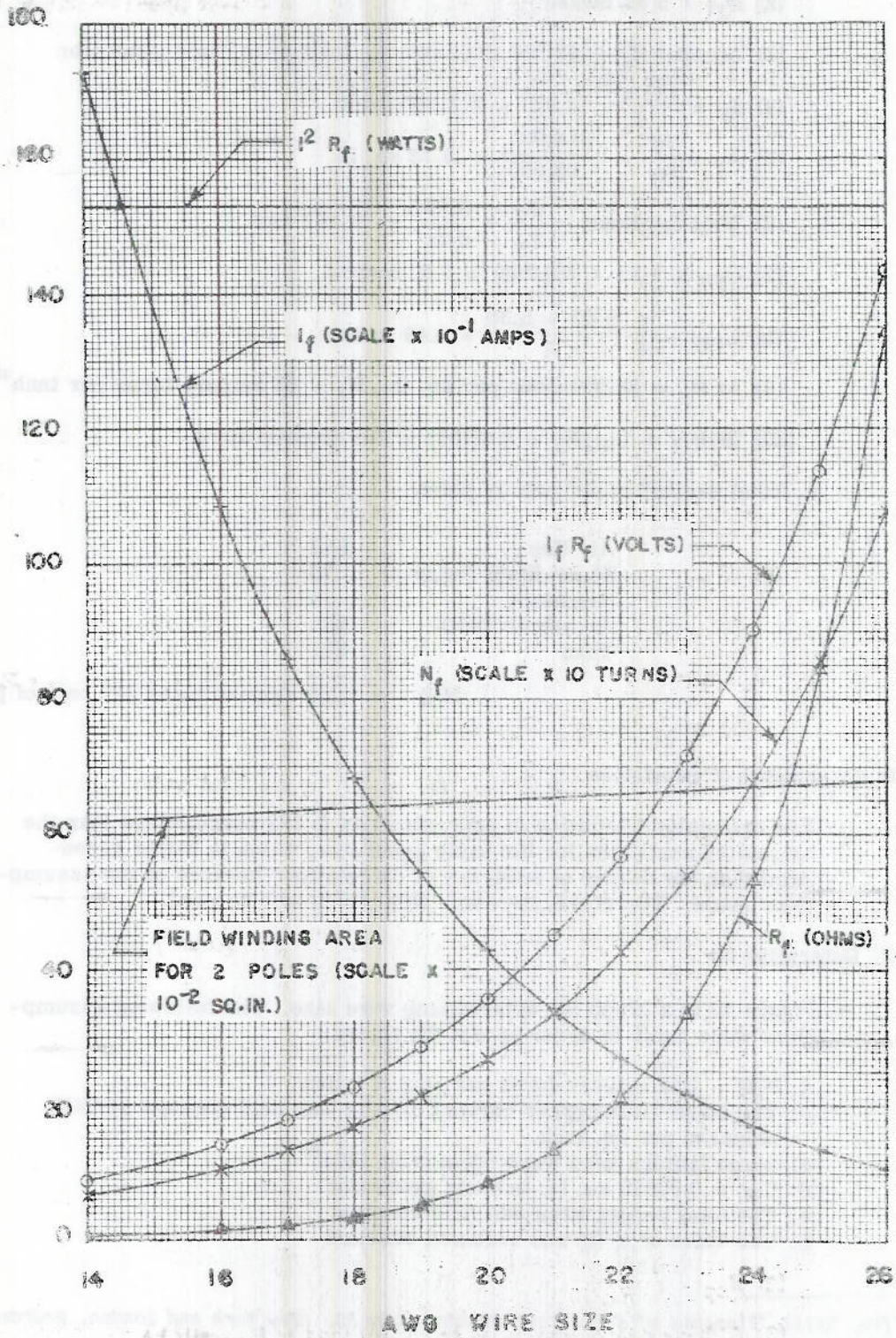


Figure 12 - Graph for determining shunt-field-conductor size

g. R_f was measured in the basic generator under base speed condition and was found to be 2.95 ohms.

h. $N_f = 160$ turns per pair of poles in basic generator

2. Determine the conductor size

a. Assume No. 25 wire

(1) From Figure 12:

(a) $I_f R_f = 113.7$ volts

(b) $R_f = 84.5$ ohms

(2) $I_f = 1.35$ amps. (calculated from (a) and (b) above)

(3) The resistance in series with the shunt field at no load will be assumed to be the voltage regulator carbon pile resistance = R_{cp}

(a) $R_{cp} = \frac{120 - 113.7}{1.35} = 4.68$ ohms

(4) At base speed conditions, the watts loss in R_{cp} will be

$$I_f^2 R_{cp} = (1.35)^2 (4.68) = 8.48 \text{ watts}$$

(5) The maximum watts loss in the carbon pile will be, when $R_{cp} = R_f$;

(a) At operating temperature,

$$\text{Maximum loss in carbon pile} = \frac{(60)^2}{84.5} = 42.6 \text{ watts}$$

(b) At $R_f = 0.75 \times (R_f \text{ at base speed temperature})$,

$$\text{Maximum loss} = \frac{(60)^2}{84.5(0.75)} = 56.8 \text{ watts}$$

1 0.75 R_f was arbitrarily selected to indicate a lower operation temperature.

b. Assume No. 24 wire

(1) From Figure 12:

(a) $I_f R_f = 90$ volts

(b) $R_f = 53.1$ ohms

(c) $I_f = 1.7$ ampere

(2) $R_{cp} = \frac{120 - 90}{1.70} = 17.7$ ohms

(3) At base speed conditions the watts loss in R_{cp} will be

$$I_f^2 R_f = (1.70)^2 (17.7) = 50.8 \text{ watts}$$

(4) The maximum watts loss in the carbon pile

(a) At operating temperature, maximum loss

$$\frac{(60)^2}{53.1} = 67.8 \text{ watts}$$

(b) At $R_f = 0.75$ (R_f at base speed), the maximum loss =

$$\frac{67.8}{0.75} = 90.5 \text{ watts}$$

1. $0.75 R_f$ arbitrarily used to denote lower operating temperature.

c. The use of No. 25 wire is marginal in that almost full rated voltage is absorbed by the shunt field at base speed.

d. The use of No. 24 wire is still within the value of 100 watts maximum loss in the voltage-regulator carbon pile

e. Use No. 24 wire in the shunt field.

3. $N_f = 678$ turns per pair of poles from Figure 12.

B. Determine the Cross-Sectional Area of the Shunt-Field Winding

1. From Figure 12, shunt-field area = 0.668 sq. in. per pair of poles.

E. COMPUTE THE AREA REMAINING FOR FORCED VENTILATION, A_v

A. Basic Generator

1. Compute annular area between yoke and armature

$$a. \text{ Area} = \frac{\pi}{8} (D_{y1}^2 - D_a^2)$$

$$= \frac{\pi}{4} [(5.328)^2 - (3.919)^2] = 10.26 \text{ sq. in.}$$

2. Compute annular area enclosing shunt-field poles

$$a. \text{ Area} = \frac{\pi}{4} [D_{y1}^2 - (D_a + 2r)^2]$$

$$= \frac{\pi}{4} [(5.328)^2 - (3.942)^2] = 10.11 \text{ sq. in.}$$

3. Angle used by shunt-field poles = 57.4° = 30° (See Figure 2)

4. Area of shunt-field poles (end view)

$$a. \text{ Area} = \frac{250}{360} (10.11) \quad (\text{area reserved for field winding})$$

$$= 6.46 - 1.92^*$$

$$= 4.54 \text{ sq. in.}$$

5. From Figure 12

a. Area of shunt-field winding = $0.637(2) = 1.27 \text{ sq. in.}$

6. Commutating pole area

a. Area = commutating pole and insulation area + Micarta-strip area
+ commutating-pole shoe area
= $0.95(0.543) + 0.95(0.047) + 0.47(0.06)$
= 2.35 sq. in.

7. $A_v = 10.26 - (\text{shunt-field pole area} + \text{shunt-field winding area} + \text{commutating-pole area})$
= $10.26 - (4.54 + 1.27 + 2.35)$
= $10.26 - 8.16$
= 2.10 sq. in.

8. It is true that some air passes through the lightening holes. However, it will be assumed that for the generators designed, the same percentage of A_v passes through the lightening holes as in the basic generator. Thus, A_v can be considered to be a true index of the ventilation area.

B. 120-Volt Generator

1. Annular area between yoke and armature

a. Area = $\frac{\pi}{4} [D_{yi}^2 - D_a^2] = \frac{\pi}{4} [(5.35)^2 - (4.16)^2] = 8.87 \text{ sq. in.}$

2. Annular area of field poles

a. Area = $\frac{\pi}{4} [D_{yi}^2 - (D_a + 2\delta)^2] = \frac{\pi}{4} [(5.35)^2 - (4.183)^2]$
= 8.71 sq. in.

3. Area of field poles

a. Area = $\frac{230}{360} (8.71) - (\text{area reserved for field winding})$
= $5.56 - 1.90^{**}$
= 3.66 sq. in.

4. From Figure 12

a. Area of winding = $0.668(2) = 1.34 \text{ sq. in.}$

5. Commutating pole area

a. From VI, B, area = 2.23 sq. in.

* Area of 1.92 sq. in. obtained by planimeter

** Area of 1.90 sq. in. obtained by planimeter

6. Compute A_V

$$\begin{aligned} \text{a. } A_V &= 3.87 \cdot (3.56 \cdot 34 + 2.23) \\ &= 3.87 \cdot 7.23 \\ &= 1.64 \text{ sq in} \end{aligned}$$

XI. CALCULATE LOSSES

A. Basic Generator

1. Shunt field = 215 watts
2. Friction and windage = 231 watts
3. Core loss = 231 watts
4. $I_a^2 R_a = 2176$ watts
5. Total loss = 2853 watts
6. Breakdown of $I_a^2 R_a$
 - a. $R_{aa} = 0.0073$ ohms, $I_a^2 R_{aa} = 692$ watts
 - b. $R_{ai} + R_{ac} = 0.00922$ ohms, $I_a^2 (R_{ac} + R_{ai}) = 870$ watts
 - c. Brush losses = $I_a(2) = 307(2) = 614$ watts
 - d. Total $I_a^2 R_a = 2176$ watts

B. 120-Volt Generator

1. $I_f = 1.7$ amperes, shunt-field losses = $1.7(120) = 204$ watts
2. Assume friction and windage same as basic generator = 231 watts
3. Compute core loss
 - a. Assume core loss is proportional to armature volume

$$(i) \text{ Core loss} = 231 \cdot \frac{\pi D_a^2 L_a}{(\pi D_a^2 L_a)_{\text{basic}}} = 231 \cdot \left[\frac{4.16^2 (4.07)}{5.316^2 (3.91)} \right] = 365 \text{ watts}$$

4. Compute $I_a^2 R_a$

- a. $I_a R_a = 20.3$ (from H. H.)
- b. $I_a = 76.7$
- c. $I_a^2 R_a = 76.7(20.3) = 1557$ watts
- d. Total loss = 2258 watts

XII. CHECK A_v

A. From Equation (34)

$$\begin{aligned} 1. A_v &= (\text{total loss in watts}) (7.37 \times 10^{-4}) \\ &= 2258 (7.37 \times 10^{-4}) \\ &= 1.66 \text{ sq. in.} \end{aligned}$$

B. Ventilation Area

1. The ventilation area of 1.64 sq. in. (from X, B,6) will be considered satisfactory

XIII. PERFORMANCE EQUATIONS

A. No-Load Saturation-Curve Equation

$$\begin{aligned} 1. E_t &= 149 \tanh(0.001375 N_f I_f) + 4.4 \text{ (from IV, B)} \\ \text{a. } N_f &= 678 \text{ turns (from IX, C)} \\ \text{b. } E_t &= 149 \tanh(0.932 I_f) + 4.4 \end{aligned}$$

B. Assumptions

1. Assume the brushes are shifted slightly backward so that the magnetization of armature reaction cancels the series-field demagnetization
 - a. The above assumption will not change the weight
2. Assume perfect compensation
 - a. Assumption made for simplicity and will not change the weight

C. Compute R_a

$$1. R_a = \frac{I_a R_a}{I_a} = \frac{20.3}{76.7} = 0.265 \text{ or } 0.27 \text{ ohm}$$

D. Full-Load Saturation-Curve Equation.

$$1. E_t = 149 \tanh(0.932 I_f) + 4.4 - 0.27 I_a$$

XIV. DESIGN COMMUTATOR

A. Determine Z_c

$$1. \text{ Assume } Z_c = 3S = 3(41) = 123 \text{ bars}$$

B. Determine D_c

$$1. \text{ Assume } D_{cr} = 4.16 \text{ inches}$$

$$3. D_c = D_{cr} = [4(C_{da}) + 0.03] = 3.16 = [4(0.242) + 0.03]; 3.16 \text{ inches}$$

Determine Volts per Bar

1. Basic generator

a. $Z_c = 37$; there are 4 brush sets and 2 bars under one brush

$$(1) \frac{37}{4} = 9.25$$

$$(2) 9.25 - 2 = 7.25$$

$$b. \text{ Volts per bar} = \frac{E_t}{7} = \frac{30}{7} = 4.3 \text{ volts}$$

2. 120-volt generator

a. $Z_c = 123$ and there are 4 brush sets; assume 2 bars under one brush

$$(1) \frac{123}{4} = 30.75$$

$$(2) 30.75 - 2 = 28.75$$

$$b. \text{ Volts per bar} = \frac{120}{28} = 4.28 \text{ volts}$$

D. Mica Thickness

1. Basic generator - 0.017 inch wide

2. 120-volt generator - 0.017 inch wide, since volts per bar is essentially the same as in basic generator

E. Determine Commutator Bar Width

1. Commutating surface

2. Compute angle which includes one bar plus one width of mica

$$(1) \text{ Angle} = \frac{2\pi}{123} = 0.0511 \text{ radians}$$

3. Compute surface width of one bar plus one width of mica

$$(1) \text{ Surface width} = \frac{D_c}{2} \times (\text{angle}) = \frac{3.16}{2} (0.0511) = 0.0807 \text{ inch}$$

$$\text{Surface for one bar} = 0.0807 - 0.017 = 0.0637 \text{ inch wide}$$

4. The value of 0.0637 is so thin that the commutator fabrication will be difficult. However, the value of 0.0637 inch will be used.

F. Determine Brush Thickness or Peripheral Distance of Brush Along Commutator Surface

1. Assume the brush covers 2 bars plus 1.5 widths of mica

$$\begin{aligned} \text{a. Brush thickness} &= 2(0.0637) + 1.5(0.017) \\ &= 0.1274 + 0.0256 \\ &= 0.153 \text{ inch} \end{aligned}$$

G. Determine Brush Width or Axial Distance of Brush Along the Commutator Surface

1. Assume 187 amperes per sq. in. as in basic generator

2. Assume 8 brushes total as in basic generator

$$3. I_b = \frac{I_a}{4} = \frac{76.7}{4} = 19.2 \text{ amperes}$$

$$4. A_b = \frac{I_b}{\text{amps/sq.in.}} = \frac{19.2}{187} = 0.103 \text{ sq. in.}$$

$$5. \text{Brush width} = \frac{A_b}{\text{Thickness}} = \frac{0.103}{0.153} = 0.673 \text{ inch}$$

H. Determine L_c

1. Basic generator commutator loss

a. Determine commutator loss

(1) Friction loss

(a) Assume 2 watts per sq. in. for each 1000 feet per minute of peripheral speed¹⁴

(b) Total brush contact area = 3.28 sq. in.

(c) Peripheral speed at 4510 rpm = 3100 ft. per minute

(d) Friction losses = $8(3.28)(3.1) = 81.3$ watts

(2) Contact loss

(a) $I_a(2) = 307(2) = 614$ watts

(3) Total loss = 695 watts

b. Commutator surface area

(1) Area = $\pi (2.625)(2.468) = 20.4$ sq. in.

c. Power-loss density = $\frac{695}{20.4} = 34.1$ watts per sq. in.

¹⁴ "AIEE Test Code for Direct-Current Machines," paragraph 23(c), (New York, The American Institute of Electrical Engineers, AIEE No. 501, July 1941)

2. 120-volt generator commutator loss

a. Friction loss

$$(1) \text{ Brush contact area} = 8 A_b = 8(0.1025) = 0.82 \text{ sq. in.}$$

$$(2) \text{ Peripheral speed at 4510 rpm} = \frac{\pi D_c}{12} (4510) = \frac{\pi(3.16)}{12} 4510 = 3730 \text{ ft./min.}$$

$$(3) \text{ Friction loss} = 8(0.82)(3.73) = 24.5 \text{ watts}$$

b. Contact loss

$$(1) I_a(2) = 76.7(2) = 153.4 \text{ watts}$$

$$c. \text{ Total commutator loss} = 24.5 + 153.4 = 178 \text{ watts}$$

3. Determine commutator surface area

a. Assuming the same loss per sq. in. as in basic generator,

$$(1) \text{ Commutator surface area} = \frac{178}{37.2} = 4.78 \text{ sq. in.}$$

(2) However, this area would not accommodate the brushes, and the brush width will establish the commutator area

b. L_c is dictated by brush width plus a reasonable clearance

$$a. L_c = 0.673(2) + 0.3 = 1.65 \text{ inches}$$

I. Assume $L_{cr} = 0.156$ inch

XV. CALCULATE WEIGHT CHANGE

A. Commutator

1. Basic generator

$$a. \text{ Active commutator volume} = \frac{\pi}{4} D_c^2 L_c = \frac{\pi}{4} (2.625)^2 (2.468) = 13.35 \text{ cu. in.}$$

$$b. \text{ Riser volume} = \frac{\pi}{4} D_{cr}^2 L_{cr} = \frac{\pi}{4} (3.625)^2 (0.1563) = 1.614 \text{ cu. in.}$$

$$c. \text{ Commutator weight} = 2.8 \text{ pounds}$$

$$d. \text{ Total commutator volume} = 13.35 + 1.61 = 14.96 \text{ cu. in.}$$

$$e. \rho_c = \frac{2.77}{14.96} = 0.185 \text{ pounds per cu. in.}$$

2. 120-volt generator

$$a. \text{ Active commutator volume} = \frac{\pi}{4} D_c^2 L_c = \frac{\pi}{4} (3.16)^2 (1.65) = 12.9 \text{ cu. in.}$$

$$b. \text{ Riser volume} = \frac{\pi}{4} D_{cr}^2 L_{cr} = \frac{\pi}{4} (4.16)^2 (0.156) = 2.12 \text{ cu. in.}$$

- c. Total volume = $12.9 + 2.12 = 15.0$ cu. in.
 - d. Commutator density will increase due to increased mechanical requirements. Assume $\rho_c = 0.20$ pounds per cu. in.
 - e. Commutator weight = $15.0(0.20) = 3.0$ pounds
3. Commutator weight change = $3.0 - 2.8 = 0.2$ pounds increase

B. Armature

1. Basic generator

- a. Volume = $\frac{\pi}{4} D_a^2 L_a = \frac{\pi}{4} (3.919)^2 (3.91) = 47.0$ cu. in.
- b. Weight = 11.7 pounds
- c. $\rho_a = \frac{11.7}{47} = 0.25$ pounds per cu. in.

2. 120-volt generator

- a. Volume = $\frac{\pi}{4} D_a^2 L_a = \frac{\pi}{4} (4.16)^2 (4.07) = 55.3$ cu. in.
- b. Change in volume = $55.3 - 47.0 = 8.3$ cu. in.

3. Weight change = $7.9(\rho_a) = 8.3(0.25) = 2.1$ pounds increase

C. Commutating Poles

1. Basic generator

- a. End-view area = 2.35 sq. in. for 4 commutating poles (from X, A)
- b. $L_i = 3.84$ inches
- c. Volume = $2.35(3.84) = 9.02$ cu. in.
- d. Weight = 1.81 pounds
- e. $\rho_i = \frac{1.81}{9.02} = 0.20$ pounds per cu. in.

2. 120-volt generator

- a. End-view area = 2.23 sq. in. for 4 commutating poles (from X, B)
- b. $L_i = 4.00$ inches
- c. Volume = $2.23(4.00) = 8.92$ cu. in.
- d. Change in volume = $9.02 - 8.92 = 0.10$ cu. in.

3. Weight change = $0.10 \rho_i = 0.10(0.20) = 0.02$ or 0.0 pound decrease

D. Shunt-Field Poles

1. Basic generator

a. Area = 4.54 sq. in. (from X, A)

b. Volume = 4.54 L_f = 4.54(3.84) = 17.4 cu. in.

c. Weight of 4 poles = 4.13 pounds.

d. $\rho_f = \frac{4.13}{17.4} = 0.24$ pounds per cu. in.

2. 120-volt generator

a. Area = 3.66 sq. in. (from X, B)

b. Volume = 3.66 L_f = 3.66(4.00) = 14.6 cu. in.

c. Volume = 17.4 - 14.6 = 2.8 cu. in.

d. Weight change = 2.8 ρ_f = 2.8(0.24) = 0.67 or 0.7 pounds decrease

E. Shunt-Field Winding

1. Basic generator

a. Area of winding per coil side = $\frac{0.637}{4} = 0.159$ sq. in. (from Figure 12)

b. Length per turn = 13.6 inches

c. Length per end turn = $\frac{13.6 - 2(3.84)}{2} = 2.96$ inches

d. Volume of windings = 0.159(13.6)(4) = 8.66 cu. in.

e. Weight of winding = 2.0 pounds

f. $\rho_{fW} = \frac{2.0}{8.66} = 0.231$ pounds per cu. in.

2. 120-volt generator

a. Area of winding per coil side = $\frac{0.668}{4} = 0.167$ sq. in. (from Figure 12)

b. Length per end turn = $2.96 \frac{4.16}{3.919} = 3.14$ inches

c. Length per turn = (3.14 + L_f)2 = (3.14 + 4.00)2 = 14.28 inches

d. Volume of winding = 0.167(14.28)(4) = 9.54 cu. in.

e. Change in volume = 9.54 - 8.66 = 0.88 cu. in.

f. Weight change = 0.88 ρ_{fW} = 0.88(0.231) = 0.2 pound increase

F. Compensating and Series-Winding End Turns

1. Basic generator

- a. $L_{cet} = 7.96$ inches (from II, E)
- b. Conductors per coil side = 30
- c. Conductor diameter = 0.0476 inch
 - (1) Space factor for 0.0476-inch diameter wire¹³ = 0.68
- d. Copper area per coil side = $\frac{\pi}{4} (0.0476)^2 (30) = 0.0534$ sq. in.
- e. Total area per coil side = $\frac{0.0534}{0.68} = 0.078$ sq. in.
- f. Volume of end turns = $0.078 (7.96) (8) = 4.97$ cu. in.
- g. Density same as shunt-field winding = 0.231 pounds per cu. in.

2. 120-volt generator

- a. $L_{cet} = 8.45$ inches (from II, E)
- b. Number of conductors per coil side = 24
- c. Conductor diameter = 0.0487
 - (1) Space factor¹³ = 0.68
- d. Copper area per coil side = $\frac{\pi}{4} (0.0487)^2 (24) = 0.0447$ sq. in.
- e. Total area per coil side = $\frac{0.0447}{0.68} = 0.0657$ sq. in.
- f. Volume of end turns = $0.0657 (8.45) (8) = 4.44$ cu. in.
- g. Change in volume = $4.97 - 4.44 = 0.53$ cu. in.

- 3. Weight change = $0.53 (0.231) = 0.12$, or 0.1 pound decrease

G. Yoke

1. Basic generator

- a. Volume = $\frac{\pi}{4} (D_{yc}^2 - D_{yi}^2) L_y = \frac{\pi}{4} (6.00^2 - 5.328^2) 6.25 = 37.3$ cu. in.
- b. Weight = 10.0 pounds
- c. $\rho_y = \frac{10.0}{37.32} = 0.268$ pounds per cubic inch

2. 120-volt generator

- a. Volume = $\frac{\pi}{4} (D_{yo}^2 - D_{yi}^2) L_y = \frac{\pi}{4} (6.07^2 - 5.35^2) 6.41 = 41.3$ cu. in.

b. Volume change = $41.3 - 37.3 = 4.0$ cu. in.

3. Weight change = $\rho_y (4.0) = 0.268(4.0) = 1.07$ or 1.1 pounds increase

H. Brush Rigging, Brushes, and Outboard End Bell

1. Basic generator

a. Axial length of brush rigging frame = 3.5 inches

b. Outside diameter of outboard end bell = 6.0 inches

c. Weight of brush rigging, end bell, and brushes = 2.5 pounds

d. Commutator length = 2.468 inches

e. Difference between axial length of end bell and length of commutator = $3.5 - 2.468 = 1.032$ or 1.03 inches

f. Volume of end bell = $\frac{\pi}{4}(6.0)^2 (3.5) = 99$ cu. in.

g. $\rho_r = \frac{3.5}{99} = 0.0357$ pounds per cu. in.

2. 120-volt generator

a. Commutator length = 1.64 inches

b. Axial length of end bell = $1.64 + 1.03 = 2.67$ inches

c. Diameter of end bell = 6.09 inches

d. Volume = $\frac{\pi}{4}(6.09)^2 (2.67) = 77.2$ cu. in.

e. Volume change = $99 - 77.2 = 21.8$ cu. in.

3. Weight change = $21.8(0.357) = 7.78$ or 7.8 pounds decrease

I. Tabulated Weight Change

	<u>Weight Increase</u>	<u>Weight Decrease</u>
Commutator	0.2	
Armature	2.1	
Commutating Poles		0.0
Shunt-Field Poles		0.7
Shunt-Field Winding	0.2	
Compensating and Series Winding End Turns		0.1
Yoke	1.1	
Brush Rigging and Outboard End Bell		<u>9.8</u>
	<u>3.6</u>	<u>1.6</u>

J. Net Weight Increase = $3.6 - 1.6 = 2.0$ pounds

K. Weight of 120-Volt Generator

1. Basic generator weighed 49.2 pounds
2. Net weight increase = 2.0 pounds
3. Weight of 120-volt generator = 51.2 or 51 pounds

APPENDIX VII

Determining Shunt-Field Conductor Size

Figure 12 (see Appendix VI, section IX, part C) is a graph for selecting the conductor size to use on a generator once the value of $N_f I_f$ is established. The method used to draw the curves of Figure 12 is given as follows:

- (1) The current density was assumed to be the same as for the basic generator. Thus, from standard tables giving dimensions of wire sizes, the current curve was plotted.
- (2) The value of $N_f I_f$ was assumed constant at 1152 ampere turns. Thus, from the ampere turns assumption and (1), the turns curve was plotted.
- (3) A standard space-factor table for enameled magnet wire was used. From the standard value of space factor and (2), the area per pair of poles was calculated and drawn.
- (4) The value of $I_f^2 R_f$ was measured for the basic generator and was assumed constant. From the value of $I_f^2 R_f$ and (1), R_f and $I_f R_f$ were calculated and plotted.

A set of curves was then available for determining the shunt-field conductor size and was used in Appendix VI, section IX, part C, as an illustration.

In the event the excitation requirements increase after drawing the curves, the new values of current and turns should be established. Then the new wire size can be selected. The new values of resistance, voltage, power, and area can be obtained by reading the number from Figure 12 for the wire size selected and multiplying by the ratio of the new to the old turns.

* * *

APPENDIX VIII

Tabulation of Data for 120-Volt, 9-KW Generator

Item number	Description	Symbol	Explanation or Numerical Value	Units
1.	Armature Diameter	D_a	4.16	inches
2.	Armature Stack Length	L_a	4.07	inches
3.	Number of Armature Slots	S	41	-
4.	Number of Armature Conductors	Z	246	-
5.	Armature Conductor Area	C_{aa}	0.0058	sq.in.
6.	Number of Current Paths Through Armature	p	2	-
7.	Armature Current at Base Speed	I_a	76.7	amps.
8.	Armature Conductor Width	C_{wa}	0.024	inches
9.	Armature Conductor Depth	C_{da}	0.242	inches
0.	Armature Slot Width	s	0.112	inches
1.	Armature Average Tooth Width	t	0.150	inches
2.	Average Flux per Pole at Base Speed	ϕ_p	379,000	lines
3.	Number of Shunt-Field Poles	P	4	-
4.	Axial Length of Shunt-Field Pole	L_f	4.00	inches
5.	Clearance between Armature and Shunt-Field-Pole Shoe	δ	0.0115	inches
6.	Inside Diameter of Yoke	D_{yi}	5.35	inches
7.	Outside Diameter of Yoke	D_{yo}	6.07	inches
8.	Axial Length of Yoke	L_y	6.41	inches
9.	Number of Commutating Poles	-	4	-
0.	Type of Compensating Winding	-	Concentrated	-
1.	Commutator Diameter	D_c	3.16	inches
2.	Axial Length of Commutator	L_c	1.65	inches
3.	Number of Commutator Bars	Z_c	123	-
4.	Volts per Bar (approximate)	-	4.3	volts
5.	Total Number of Brushes	-	8	-
6.	Brush Thickness	-	0.153	inches
7.	Brush Width	-	0.67	inches
8.	Brush Current Density	-	187	amps./sq.in.
9.	Current per Brush at Base Speed	I_b	19.2	amps.
0.	Shunt-Field Current at Base Speed	I_f	1.7	amps.
1.	Shunt-Field Resistance at Base-Speed Temperature	R_f	53.1	ohms
2.	Shunt-Field Turns per Pair of Poles	N_f	678	turns
3.	Armature Resistance at Base-Speed Temperature	R_{aa}	0.105	ohms
4.	Series-Field plus Compensating-Winding Resistances at Base-Speed Temperature	R_{ac}	0.0785	ohms
5.	Commutating-Winding Resistance at Base-Speed Temperature	R_{ai}	0.0535	ohms

Item Number	Description	Symbol	Explanation or Numerical Value	Units
36.	Total Armature Circuit Resistance at Base-Speed Temperature = $R_{aa} + R_{ac} + R_{ai}$	R_a	0.237	ohms
37.	Efficiency	-	79.9	percent
38.	Power Output per rpm at Base Speed	W_o/n_1	1.995	watts/rpm
39.	Total Generator Weight	W	51	pounds
40.	Base Speed	n_1	4510	rpm

APPENDIX IX

Tabulation of Data for 30-Volt, 12-KW Generator

Item Number	Description	Symbol	Explanation or Numerical Value	Units
1.	Armature Diameter	D_a	4.306	inches
2.	Armature Stack Length	L_a	4.07	inches
3.	Number of Armature Slots	S	31	-
4.	Number of Armature Conductors	Z	62	-
5.	Armature Conductor Area	C_{aa}	0.0305	sq.in.
6.	Number of Current Paths Through Armature	p	2	-
7.	Armature Current at Base Speed	I_a	407.2	amps.
8.	Armature Conductor Width	C_{wa}	0.126	inches
9.	Armature Conductor Depth	C_{da}	0.242	inches
10.	Armature Slot Width	s	0.146	inches
11.	Armature Average Tooth Width	t	0.217	inches
12.	Average Flux per Pole at Base Speed	ϕ_p	391,000	lines
13.	Number of Shunt-Field Poles	P	4	-
14.	Axial Length of Shunt-Field Pole	L_f	4.00	inches
15.	Clearance Distance between Armature and Shunt-Field-Pole Shoe	δ	0.0115	inches
16.	Inside Diameter of Yoke	D_{yi}	5.79	inches
17.	Outside Diameter of Yoke	D_{yo}	6.54	inches
18.	Axial Length of Yoke	L_y	6.41	inches
19.	Number of Commutating Poles	-	4	-
20.	Type of Compensating Winding	-	Concentrated	-
21.	Commutator Diameter	D_c	2.89	inches
22.	Axial Length of Commutator	L_c	3.00	inches
23.	Number of Commutator Bars	Z_c	31	-
24.	Volts per Bar (approximate)	-	4.5	volts
25.	Total Number of Brushes	-	8	-
26.	Brush Thickness	-	0.584	inches
27.	Brush Width	-	0.933	inches
28.	Brush Current Density	-	187	amps./sq.in.
29.	Current per Brush at Base Speed	I_b	101.8	amps.
30.	Shunt-Field Current at Base Speed	I_f	7.2	amps.
31.	Shunt-Field Resistance at Base-Speed Temperature	R_f	2.95	ohms
32.	Shunt-Field Turns per Pair of Poles	N_f	160	turns
33.	Armature Resistance at Base-Speed Temperature	R_{a2}	0.0050	ohms
34.	Series-Field plus Compensating-Winding Resistance at Base-Speed Temperature	R_{ac}	0.0041	ohms

Item Number	Description	Symbol	Explanation or Numerical Value	Units
35.	Commutating-Winding Resistance at Base-Speed Temperature	R_{ci}	0.0019	ohms
36.	Total Armature Circuit Resistance at Base-Speed Temperature = $R_{aa} + R_{ac} + R_{ai}$	R_{a1}	0.0110	ohms
37.	Efficiency	-	78.3	percent
38.	Power Output per rpm at Base Speed	W_0/n_1	2.66	watts/rpm
39.	Total Generator Weight	W	59.5	pounds
40.	Base Speed	n_1	4510	rpm

APPENDIX X

Tabulation of Data for 120-Volt, 20-KW Generator

Item Number	Description	Symbol	Explanation or Numerical		Units
			Value		
1.	Armature Diameter	D_a	5.62		inches
2.	Armature Stack Length	L_a	3.91		inches
3.	Number of Armature Slots	S	47		-
4.	Number of Armature Conductors	Z	188		-
5.	Armature Conductor Area	C_{aa}	0.0127		sq.in.
6.	Number of Current Paths Through Armature	p	2		-
7.	Armature Current at Base Speed	I_a	169.7		amps.
8.	Armature Conductor Width	C_{wa}	0.0525		inches
9.	Armature Conductor Depth	C_{da}	0.242		inches
10.	Armature Slot Width	s	0.135		inches
11.	Armature Average Tooth Width	t	0.193		inches
12.	Average Flux per Pole at Base Speed	ϕ_p	317,000		lines
13.	Number of Shunt-Field Poles	P	6		-
14.	Axial Length of Shunt-Field Pole	L_f	3.84		inches
15.	Clearance Distance between Armature and Shunt-Field-Pole Shoe	δ	0.023		inches
16.	Inside Diameter of Yoke	D_{yi}	7.37		inches
17.	Outside Diameter of Yoke	D_{yo}	8.08		inches
18.	Axial Length of Yoke	L_y	6.25		inches
19.	Number of Commutating Poles	-	6		-
20.	Type of Compensating Winding	-	Concentrated		-
21.	Commutator Diameter	D_c	4.20		inches
22.	Axial Length of Commutator	L_c	1.45		inches
23.	Number of Commutator Bars	Z_c	94		-
24.	Volts per Bar (approximate)	-	9.23		volts
25.	Total Number of Brushes	-	12		-
26.	Brush Thickness	-	0.263		inches
27.	Brush Width	-	0.577		inches
28.	Brush Current Density	-	187		amps./sq.in.
29.	Current per Brush at Base Speed	I_b	28.3		amps.
30.	Shunt-Field Current at Base Speed	I_f	3.0		amps.
31.	Shunt-Field Resistance at Base-Speed Temperature	R_f	36.4		ohms
32.	Shunt-Field Turns per Pair of Poles	N_f	538		turns
33.	Armature Resistance at Base-Speed Temperature	R_{aa}	0.0314		ohms
34.	Series-Field plus Compensating-Winding Resistance at Base-Speed Temperature	R_{ac}	0.027		ohms
35.	Commutating-Winding Resistance at Base-Speed Temperature	R_{ai}	0.0150		ohms

Item Number	Description	Symbol	Explanation or Numerical Value	Units
36.	Total Armature Circuit Resistance at Base-Speed Temperature = $R_{aa} + R_{ac} + R_{ai}$	R_{ai}	0.0734	ohms
37.	Efficiency	-	84.5	percent
38.	Power Output per rpm at Base Speed	W_o/n_1	4.43	watts/rpm
39.	Total Generator Weight	W	82	pounds
40.	Base Speed	n_1	4510	rpm

APPENDIX XI

Tabulation of Data for 120-Volt, 30-KW Generator

Item Number	Description	Symbol	Explanation or Numerical	
			Value	Units
1.	Armature Diameter	D_a	6.50	inches
2.	Armature Stack Length	L_a	4.07	inches
3.	Number of Armature Slots	S	75	-
4.	Number of Armature Conductors	Z	150	-
5.	Armature Conductor Area	C_{aa}	0.019	sq.in.
6.	Number of Current Paths Through Armature	p	2	-
7.	Armature Current at Base Speed	I_a	255.5	amps.
8.	Armature Conductor Width	C_{wa}	0.0872	inches
9.	Armature Conductor Depth	C_{da}	0.218	inches
10.	Armature Slot Width	s	0.107	inches
11.	Armature Average Tooth Width	t	0.136	inches
12.	Average Flux per Pole at Base Speed	ϕ_p	395,000	lines
13.	Number of Shunt-Field Poles	P	6	-
14.	Axial Length of Shunt-Field Pole	L_f	4.00	inches
15.	Clearance Distance between Armature and Shunt-Field-Pole Shoe	δ	0.040	inches
16.	Inside Diameter of Yoke	D_{yi}	8.52	inches
17.	Outside Diameter of Yoke	D_{yo}	9.28	inches
18.	Axial Length of Yoke	L_y	6.41	inches
19.	Number of Commutating Poles	-	6	-
20.	Type of Compensating Winding	-	Concentrated	-
21.	Commutator Diameter	D_c	4.35	inches
22.	Axial Length of Commutator	L_c	1.66	inches
23.	Number of Commutator Bars	Z_c	75	-
24.	Volts per Bar (approximate)	-	12	volts
25.	Total Number of Brushes	-	12	-
26.	Brush Thickness	-	0.341	inches
27.	Brush Width	-	0.669	inches
28.	Brush Current Density	-	187	amps/sq.in.
29.	Current per Brush at Base Speed	I_b	42.6	amps.
30.	Shunt-Field Current at Base Speed	I_f	5.5	amps.
31.	Shunt-Field Resistance at Base-Speed Temperature	R_f	17.2	ohms
32.	Shunt-Field Turns per Pair of Poles	N_f	452	turns
33.	Armature Resistance at Base-Speed Temperature	R_{aa}	0.0190	ohms
34.	Series-Field plus Compensating-Winding Resistance at Base-Speed Temperature	R_{ac}	0.0218	ohms

Item Number	Description	Symbol	Explanation or Numerical Value	Units
35.	Commutating-Winding Resistance at Base-Speed Temperature	R_{ai}	0.0069	ohms
36.	Total Armature Circuit Resistance at Base-Speed Temperature = $R_{aa} + R_{ac} + R_{ai}$	R_{ai}	0.0475	ohms
37.	Efficiency	-	85	percent
38.	Power Output per rpm at Base Speed	W_o/n_i	6.65	watts/rpm
39.	Total Generator Weight	W	106	pounds
40.	Base Speed	n_i	4510	rpm
