NRL REPORT 3679

FR-3679

A DEVICE FOR COMPUTING CORRELATION FUNCTIONS

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June 16, 1950

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ABSTRACT

An analogue device has been constructed for computing autocorrelation functions. On reproducing a function recorded on two channels of a magnetic tape, the spacing of two pick-ups can be varied to obtain the function, delayed in one relative to that in the other. An electrodynamometer wattmeter indicates the average product of the two functions for any value of delay, determining the autocorrelation function. Cross-correlation of two different functions can also be calculated. Autocorrelation functions and their transforms, the power spectra, are obtained for several types of functions. A comparison of this method of obtaining power spectra with that using an ordinary Fourier analysis indicates that the latter is preferable for the particular conditions outlined.

PROBLEM STATUS

This is an interim report on one phase of this problem; work is continuing.

AUTHORIZATION

NRL Problem R12-01D
NR 512-010
NO 284-609

A DEVICE FOR COMPUTING CORRELATION FUNCTIONS

INTRODUCTION

The autocorrelation function, $\phi(\tau)$, derived from a time function, f(t), is receiving considerable attention as an aid to the optimum design of circuits and as a means of obtaining power spectra. It is defined as

$$\phi(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)\, f(t+\tau)\, dt.$$

This indicates a process of multiplying the function continuously by its value at a later time, τ, and averaging. The even function, $\phi(\tau)$, is a maximum at $\tau = 0$ and is equal to the square of the rms value of f(t). If f(t) is nonperiodic, $\phi(\tau)$ approaches the square of the average of f(t) as τ increases; if f(t) is periodic, $\phi(\tau)$ has the same period. Any linearly additive component of f(t) produces its own linearly additive component of $\phi(\tau)$. A composite time function can be separated into linearly additive time functions and the autocorrelation function of each added linearly to give the composite autocorrelation function.

The power spectrum is the Fourier transform of the autocorrelation function, or

$$\Phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(\tau) \cos \omega\tau \, d\tau.$$

Again, linearly additive parts of $\phi(\tau)$ can be transformed separately and the separate transforms added to give the composite power spectrum.

METHOD OF OBTAINING $\phi(\tau)$

There have been built a few correlators[1,2] which sample a time function to obtain values of f(t) at a large number of discrete pairs of points separated by τ, multiply each

[1] Lee, Y.W., Wiesner, J.B., and Cheatham, T.C. "The Application of Correlation Functions in the Detection of Small Signals in Noise," MIT Research Lab. of Electronics, Technical Report 141

[2] MIT Research Lab. of Electronics, Quarterly Progress Report, July 15, 1948

pair together, sum the products, and average over time. The correlator to be described does not use discrete pairs, but continuously delays f(t) to obtain $f(t+\tau)$ and the product $f(t)\,f(t+\tau)$. The function f(t), recorded on magnetic tape, is reproduced by two pickup heads with variable spacing corresponding to τ. In order to obtain $\tau = 0$, which would mean coincidence of the heads on a single recording, two heads are separated a small distance along the tape; and each head records and reproduces on a separate half of the tape. The separation when recording is, then, the condition for $\tau = 0$. The multiplication is carried out by an electrodynamometer wattmeter with f(t) applied to one coil and $f(t+\tau)$ to the other. Averaging over the period of the wattmeter also occurs. In order to record and reproduce d-c, there is employed an amplitude-modulated carrier system which requires the upper frequency limit of f(t) to be less than one-half the carrier frequency.

While the device provides the delay, τ, and the averaged product, $\phi(\tau)$, it can also provide storage of f(t), which is particularly advantageous when f(t) occurs in a time too short for the desired $\phi(\tau)$ to be obtained. A continuous loop of tape can be formed to repeat the function until the complete autocorrelation function is found. Again, if f(t) is not completely stationary, so that $\phi(\tau)$ is a slowly changing function, a piece of f(t) can be repeated to provide a $\phi(\tau)$ representative of f(t) at a particular time.

If f(t) is stationary and continuously available, a loop of tape with two pairs of heads, one pair with fixed spacing for recording and one pair with variable spacing for reproducing, followed by an erase head, can provide the minimum of storage required. This more complicated arrangement offers no apparent advantage for the projected use of the correlator and has not been used. Cross-correlation between two functions can also be calculated by recording the functions on separate halves of the tape.

One disadvantage of magnetic recording is the variation in sensitivity from section to section of the tape, which introduces low-frequency amplitude noise. Frequency modulation, rather than amplitude modulation, would eliminate the noise but would require the maintaining of tape speeds relatively more constant and a wider frequency band. A factor favoring the use of amplitude modulation in the correlator is the apparently uncorrelated nature of much of the low-frequency noise, since the pick-up heads operate on different parts of the tape. Another disadvantage is that the bandwidth of f(t) has to be limited, inherently or by filter, to that available from the recording—0 to several thousand cps. This range of the spectrum, however, is quite adequate for use in many problems, and choice of tape speeds allows considerable flexibility.

EXAMPLES OF USE

Curve A of Figure 1 shows the correlation function of the form $\phi(\tau) = a^2/2 \cos \omega t$ measured experimentally from a recorded sinusoidal function of the form $f(t) = a \sin(\omega t + \alpha)$. $\phi(\tau)$ is independent of α, the phase of the time function, and is characteristically an even function with a maximum at $\tau = 0$.

Curve B of Figure 1 shows the correlation function measured experimentally from a recorded square wave of frequency 50 cps. $\phi(\tau)$ approximates the theoretical triangular waveform. The deviations are caused in part by the departure of the time function from a true square wave. The Fourier transform of $\phi(\tau)$ to $\Phi(\omega)$, by a mechanical wave analyzer, gives the following relative amplitudes of harmonics of the discrete spectrum compared with those read on an electronic wave analyzer:

Harmomic	Fourier Transform	Electronic Analyzer
1	1.00	1.00
2	0.02	0.12
3	0.28	0.33
4	0.02	0.11
5	0.17	0.20
6	0.00	0.10
7	0.14	0.13
8	0.00	0.10
9	0.08	0.11
10	0.14	0.05

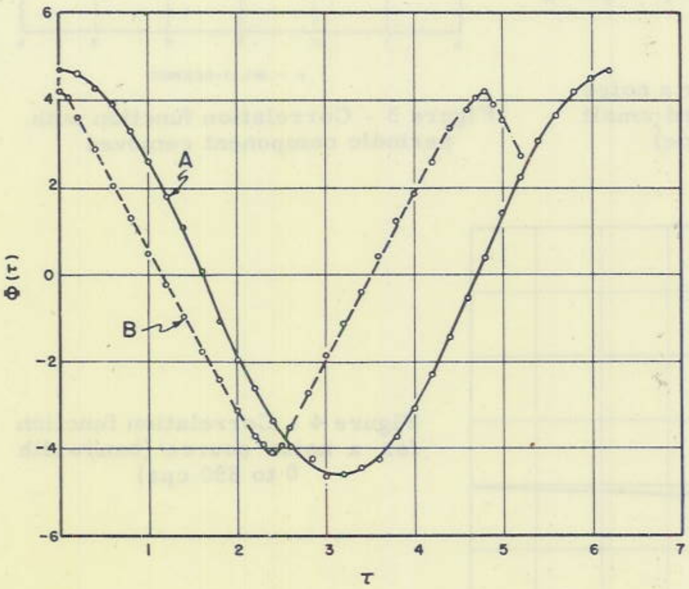


Figure 1 - Correlation functions of sine wave (A), and square wave (B)

The solid line of Figure 2 shows the correlation function for a noise source limited to the band 100 to 1000 cps. The presence of a small periodic component at 120 cps is indicated by the broken line. If this oscillatory part is removed, the remaining function, as shown in Figure 3, decreases rapidly from its initial maximum, approaching the theoretical form for white noise—a vertical line at $\tau = 0$. For comparison, the theoretical correlation function for white noise through an idealized 100 to 1000 cps filter is shown by the broken line of Figure 3.

Figure 4 is the correlation function obtained from a noise source limited by a filter to the band 0 to 850 cps. The power spectrum, obtained from $\phi(\tau)$ by a mechanical wave analyzer, is shown by the solid line in Figure 5. The maxima, harmonics of the power supply ripple at 120 cps, are superposed on the continuous noise spectrum. For comparison, the power spectrum as measured on an electronic wave analyzer from the reproduced waveform, is shown by the broken line. The shape of the continuous part of the spectrum is similar, but the inherently greater resolving power of the electronic wave analyzer defines the ripple frequencies better.

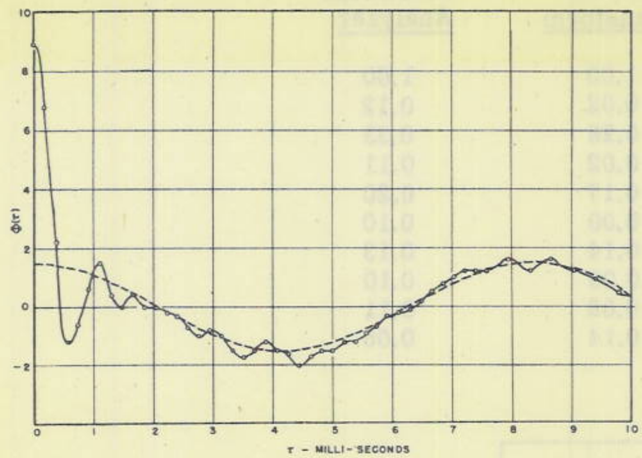


Figure 2 - Correlation function for a noise source (bandwidth 100 to 1000 cps) and small periodic component (broken line)

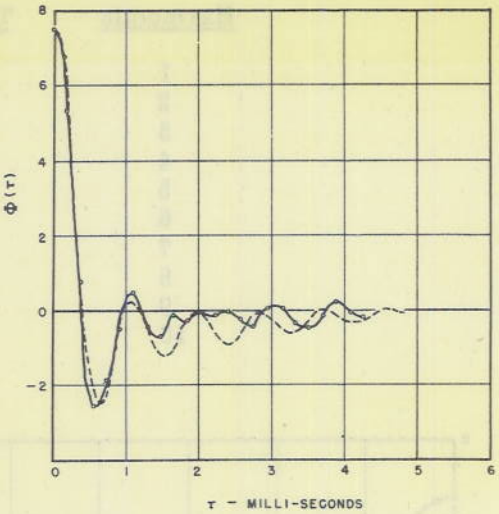


Figure 3 - Correlation function with periodic component removed

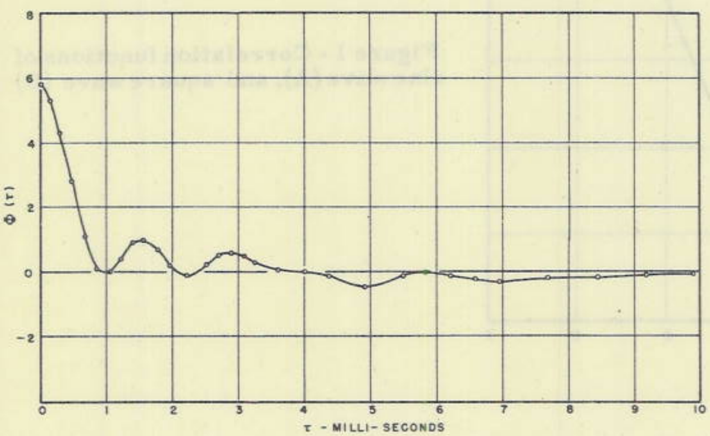
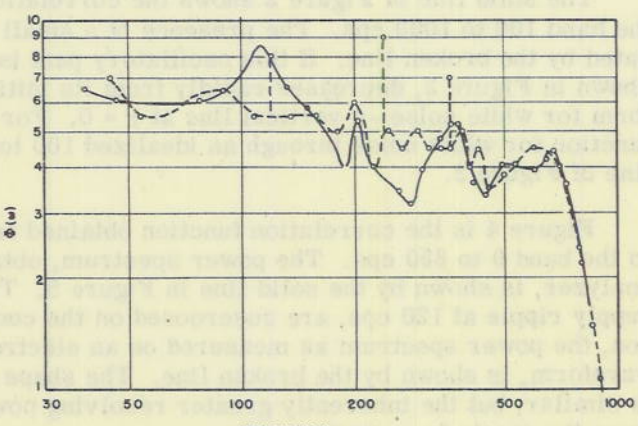


Figure 4 - Correlation function for a noise source (bandwidth 0 to 850 cps)

Figure 5 - Power spectrum obtained from correlation function of Figure 4, and power spectrum measured directly on an electronic analyzer (broken line)



DISCUSSION

Consideration is limited here to the specific problem of determining, by analogue methods of computation, the power spectrum of a time function over a limited range of frequency, from perhaps a few cps to several thousand cps. The discussion does not apply to digital methods of computation.

Each example shown was measured from a sample of tape on which the time function had been recorded and which was spliced to form a continuous loop. The tape speed was such that the wattmeter and the electronic wave analyzer (Figure 5), did not average over the whole loop. The small variations in indication over the loop length were averaged by eye, while the disturbance at the splice, generally small except for the periodic functions, was neglected. This method of reading gave the acceptably small scatter shown.

Some discussion of methods of obtaining power spectra, based on present limited experience, is desirable. One method, recently reported,[3] was studied and has since been used. This method consists of the repetition at a fixed frequency of a sample of the time function, $f_1(t)$, to form a new periodic function, $f_2(t)$. The resulting line spectrum of $f_2(t)$, easily found by an electronic wave analyzer, was shown to be closely related to the spectrum of $f_1(t)$, allowing for the change in frequency scales. For adequate resolving power, this method requires a large sample length which includes several periods, or a number of sample lengths to increase the number of frequencies at which measurements are made. The method has the advantages that no averaging of meter indications has to be done by eye and that the splice in the loop does not have to be neglected. A disadvantage is the somewhat limited resolving power of the method. All this discussion applies as well to a Fourier transform obtained from a sample of $f_1(t)$ by a mechanical wave analyzer or by computation.

Since the power spectrum from the autocorrelation function requires a Fourier transform, this procedure is subject to the limitations of resolving power just discussed, and is illustrated by the solid line of Figure 5.

If $f_1(t)$ is fairly stationary over a period long enough for a meter to indicate the desired value and also to recover from any effect of loop splice, and if $f_1(t)$ does not contain very low frequency components, the spectrum can be found more directly, without the auxiliary function, $f_2(t)$, with a wave analyzer. The broken line in Figure 5 was obtained in this manner. Here the resolving power is much greater than that obtained through the autocorrelation function, and the procedure is much simpler. Even when the spectrum is quite smooth or when a power spectrum has to be found with the auxiliary function, $f_2(t)$, there appears to be no advantage in going through the autocorrelation function, and a measurement of power spectrum using a mechanical or electrical wave analyzer is simpler.

The experimental work has been somewhat limited, and further experience may modify the very tentative conclusions. The correlation function has, moreover, other uses than that of an intermediate step in obtaining the power spectrum. For these uses the correlator described here offers the advantages of relative simplicity, storage of data, and rapidity of calculation.

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[3] Hastings, A. E., "Methods of Obtaining Amplitude-Frequency Spectra," NRL Report R-3466, May 16, 1949

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