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MONTEREY, CALIFORNIA

GAME THEORY AND THE NAVY BUDGET

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Game Theory and the Navy Budget

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DEFINITIONS OF MATH TERMS

Π_0	Bonus
ϕ	Production (Completed)
ϕ_f	Production (Forecasted)
λ_1	Constant with $\lambda_1 < 1$
λ_2	Constant with $\lambda_2 > 1$
β_i	Budget for Division i (e.g., \$100 million)
η_i	Performance Metric for Division i (e.g., 10 projects)
β_{uf}	Unit Budget Forecast (e.g., 0.5 project/\$ million) where $\beta_{uf} = \eta_i/\beta_i$
β_{NY}	Budget for the next Fiscal Year

ABSTRACT

This is Phase II follow-up from risk-based decision analysis in capital budgeting and stochastic portfolio optimization. The research purpose is to propose a novel, reusable and extensible advanced analytical process employing strategic game theory with Integrated Risk Management (IRM) to assist the DoD with its capital budgeting activities under uncertainty, with applications in Monte Carlo risk-simulation, forecasting and predictive analytics, as well as stochastic portfolio optimization of acquisitions and programs portfolios with multiple stakeholders (constraints are subject to budget, schedule, cost, risk, and other strategic constraints.) Game theory applications will include areas of repeated finite and infinite games with complete and incomplete information in strategic and extended forms. The research approach will include, when appropriate, theoretical mathematical formulations, recommended data collection/integration, and optimization approaches, as well as practical implementation recommendations to test the benefits and viability of the proposed approach to generate optimal budget allocations and strategies informed by the reactions of other nations or internal players (e.g., U.S. Congress, organizations within DoD). Whenever appropriate, strategic and extended forms will be applied. The assumption in game theory includes the rationality of the decision makers and updated information using Bayes' law. The idea is to analyze Bertrand, Cournot, and Nash equilibria with pure and mixed strategies with dominant and dominated conditions in repeated games to determine the recommended method. We use military SMEs to determine probabilistic payoff structures engaged in sequential and simultaneous moves to identify equilibrium states in the main game tree to subgame perfect Nash equilibrium as worked-out examples and illustrations of the approach. Deliverables include a written report, Excel files, and relevant scripts as they are generated in the research.

EXECUTIVE SUMMARY

Project Summary

Many agencies within the Navy are competing for a portion of the Navy budget to ensure their agency receives the maximum amount possible. With a budget of \$255.8 billion for FY24, the U.S. Navy must do everything possible to make the most informed decision possible. Consider a department with several divisions underneath that would receive funding from the department. The Mechanical Design Theory applied in this research allows us to create a game with the desired Nash equilibrium of an accurate budget prediction from each division. We create a game sequence that penalizes divisions for inaccurate budget predictions outside of a specified tolerance with an optional bonus for divisions that predicted their budgets accurately. To demonstrate a way to merge the ideas of the game sequence with those of traditional budget allocation, simulation is used to simulate actual decisions made during the game sequence. The output of these simulations provides a confidence interval for each division for the portion of the total budget to be allocated. Using additional metrics for military value and military downsides, stochastic optimization of the budget within the confidence interval from the game sequence simulations provides the final budget allocation output.

The recommended approach and methodology can be run standalone or in concert with Integrated Risk Management methodologies where Monte Carlo simulation, predictive forecasts, and stochastic portfolio optimization can be integrated to create a more robust and comprehensive approach. The research concludes that the inclusion of game theory may increase the robustness of what will be reported by the various departments as goals will be attainable. The methodology forces departments to report their best estimates and self-regulates the departments to improve their forecasts over time.

Background

Using game theoretic constructs and risk-based stochastic capital budgeting procedures within the Department of Defense, the current research employs several unique approaches to increase its performance in producing a believable and justifiable game theoretic stochastic portfolio allocation. The key point of relevance is that the research helps provide insights, direction, and actionable intelligence to senior decision-makers regarding the best program selection and

portfolio allocation of resources. The success criteria will be to determine a defensible portfolio allocation with uncertainty and employ game theory concepts from multiple points of view and approaches to triangulate a valid and reliable military value. The techniques can be used with a correlated portfolio of choice possibilities that can be stochastically optimized using Markowitz's efficient frontiers and within the framework of a game theory approach. This can be further enhanced with the previous research project's various portfolio and program-selection techniques to gauge how well they work with game theory, including Preference Ranking Organization Methods for Enrichment Evaluations, or PROMETHEE; Elimination and Choice Expressing the Reality, or ELECTRE; Multi-Criteria Analysis, or MCA; and Hierarchical Scoring-Ranking, or HSR, methods.

This research applies various methodologies such as Integrated Risk Management (risk-based Monte Carlo simulation, forecast and predictive modeling) and stochastic optimization (with multiple stakeholders, with budget, schedule, cost, risk, and other strategic constraints) with game theoretic concepts. Specifically, Nobel Prize-winning methods involving asymmetrical information in bargaining with the applications of Mechanism Design Theory and Revelation Principle will be applied to start from a Nash equilibrium and thereafter modified into a game designed to elicit the honest forecast of program managers for an efficient and effective budget allocation within the Department of Defense.

This research constitutes the Phase II follow-up from risk-based decision analysis in capital budgeting and stochastic portfolio optimization. The research purpose is to propose a novel, reusable and extensible advanced analytical process employing strategic game theory with Integrated Risk Management to assist the Department of Defense with its capital budgeting activities under uncertainty. Game theory applications will include areas of repeated finite and infinite games with complete and incomplete information in strategic and extended forms. The research approach includes theoretical mathematical formulations, recommended data collection and integration, and optimization approaches, as well as practical implementation recommendations to test the benefits and viability of the proposed approach to generate optimal budget allocations and strategies informed by the reactions of other players (different departments or organizations within the Department of Defense). The assumptions in game theory include the rationality of the decision-makers and updated information using Bayes' law. To determine the recommended method, the idea is to analyze Bertrand, Cournot, and Nash equilibria with pure and

mixed strategies with dominant and dominated conditions in repeated games. We use simulations and determine payoff structures engaged in sequential and simultaneous moves to identify equilibrium states in the main game tree to Subgame Perfect Nash equilibrium as worked-out examples and illustrations of the approach.

Findings and Conclusions

Based on the research, our main findings indicate that the recommended game theoretic approach needs to be a balanced methodology and practical to implement so decision-makers can easily understand it. As such, the proposed models and methods obey many desirable properties. The proposed approach is consistent with modern economic theory, and the model is consistent with modern Mechanism Design Theory and the Revelation Principle, which is state of the art in reducing Information Asymmetry problems. In practice, the model is simple, intuitive, and practical to implement. The proposed model and methods have only a few parameters with simple, practical interpretations. After negotiating the budget (first year, first round), program managers only need to estimate, honestly, one parameter (η). Models that reward truth are intuitive and look attractive.

In terms of implications of the research, we find that the proposed model and methodology have application flexibility, where the model is flexible in accommodating the preferences and specific conditions of headquarters. This model is flexible in dosing the degree of punishment, which can be symmetrical (with λ_1 and λ_2), and in allowing for tolerance to forecast error and to include an additional premium for honest reporting. Finally, there is a consideration of dynamic effects, where, over time, the methodology will reveal which are high-productivity program managers and which are low-productivity. In the case of low-productivity program managers, in addition to the alternative of replacing them, the rules of the game can be slightly changed after a certain year to provide an incentive to overcome their limitations. For high-productivity program managers, the opposite is necessary: always make it clear that the choice of the future parameter controls will be theirs and not that of headquarters, thus preventing the ratchet effect.

We recommend that game theory be considered when implementing budget allocation where there is fierce competition among various departments or divisions and sufficient uncertainty about the outcomes of the programs being implemented. Game theory helps reduce future uncertainty and makes the departments responsible for their programs' output forecasts. By holding departments accountable, the rolled-up portfolio has a better chance of hitting the anticipated goals.

I. INTRODUCTION

The Department of Defense (DoD) will benefit from the current research's application of several cutting-edge strategies in order to increase its success in developing a decision analytics paradigm that incorporates principles from strategic game theory and risk-based stochastic capital budgeting methods. The success criteria will be to establish a justifiable return on investment (ROI) from various perspectives and approaches, to triangulate to an ROI value that is valid and reliable, and to offer advice and information to DoD decision-makers regarding the best program selection and portfolio allocation of resources when those resources are in demand. The techniques will be based on game theory concepts, such as correlated choice possibilities that can be stochastically optimized (Markowitz efficient frontiers). The proposed methodology and applications, as well as a brief case study and illustrations of how the methodology can be used, will be covered in full in the report.

When applied in real-life conditions, depending on the data's accessibility and the decision makers' requirements, the following techniques may be included in the suggested approaches: Integrated Risk Management methodologies to run Monte Carlo simulations, advanced data analytical approaches (artificial intelligence and data science methods), strategic flexibility real options, and stochastic optimization are combined with tabletop game theory exercises with expected payoffs on repeated games (game theory is a decision analytics method with interacting strategic and rational decision makers). Economic data (total lifecycle cost, total ownership cost, acquisition cost, cost deferred, schedule, and risk); logistic data (e.g., inherent availability, effective availability, mission reliability, operational dependability, mean downtime, fielded capabilities, and Likert levels of innovative and creative technology); qualitative subject matter expert estimates (strategic value, value to society, command priorities, legal and regulatory impact scores, etc.); and market comparables to operationalize various elements of DoD benefit. This current research is a continuation of the previous research on Capturing Risk in Capital Budgeting, and as such, there will be some remnants and connections to the previous work (Mun, 2022).

Research Objectives, Methodology, and Questions

With the use of game theoretic constructs and risk-based stochastic capital budgeting procedures within the DoD, the current research employs a number of unique approaches to increase its performance in producing a believable and justifiable game theoretic stochastic portfolio allocation. In order to give direction and intelligence to DoD decision-makers regarding the best program selection and portfolio allocation of resources, the success criteria will be to determine a defensible portfolio allocation with game theory from multiple points of view and approaches to triangulate a valid and reliable military value. The techniques will be used with a correlated portfolio of choice possibilities that can be stochastically optimized (Markowitz efficient frontiers) and within the framework of a game theory approach. This can be further enhanced with the previous research project's various portfolio and program-selection techniques to gauge how well they work with game theory, including Preference Ranking Organization Methods for Enrichment Evaluations (PROMETHEE), Elimination and Choice Expressing the

Reality (ELECTRE), Multi-Criteria Analysis (MCA), and Hierarchical Scoring-Ranking (HSR) methods.

Contribution to Knowledge

This research applies Integrated Risk Management methods such as risk-based Monte Carlo simulation, forecast and predictive modeling, as well as the application of stochastic optimization with game theoretic concepts. Specifically, Nobel Prize-winning methods involving asymmetrical information in bargaining with the applications of Mechanism Design Theory and Revelation Principle will be applied to start from a Nash equilibrium into a game designed to elicit the honest forecast of program managers for the purposes of an efficient and effective budget allocation within the DoD.

II. LITERATURE REVIEW

In the simplest of terms, game theory concepts can form an analytical framework for the decision-making of multiple participants (“players”) in competitive situations in order to develop a strategy for achieving an optimal outcome. In capital budgeting, the evaluation and selection of projects or investments usually involve various players and their particular interests competing for a share of the money budgeted. The standard capital budgeting discounted and nondiscounted cash flow methods (e.g., payback period, net present value or NPV, profitability index, return on investment or ROI, and internal rate of return or IRR) generally do not take into account the preferences of those competing interests.

It is commonly acknowledged that game theory was introduced as a branch of applied mathematics in John von Neumann and Oskar Morgenstern’s 1944 book, *Theory of Games and Economic Behavior*. As the knowledge of the concept became more widespread, it, of course, became of interest for various applications, including political science, biology, economics, and finance. Capital budgeting falls within the purview of finance but was not quickly seen as an area where game theory could be usefully applied: “To say that game theory, in its more rigorous sense, still has no significant business applications does not, of course, mean that claims for its potential have been exaggerated. The day of orthodox game theory may well be on its way.” (Bennion, 1956). Thus, around this time, game theory was beginning to be considered a tool in the business world. Bennion’s paper provides examples of a two-player zero-sum game, in which the players are a businessman and the business cycle. This setup provides a foundation for strategies for capital budgeting in the DoD. However, as Bennion states, “[it] does not deal effectively with situations where there are more than two players or where the loser’s losses and the winner’s gains do cancel out” (Bennion, 1956).

As noted by Ng in 1968: if we accept the fact that computational problems can be solved in the near future by large-scale computing machine technology, we can foresee immense applicability of game theory. This implies that the idea of game theory as a tool for capital budgeting has been growing in relevance in the 12 years since the Bennion article. However, as more complex game models were being developed, the need for large-scale computing was a roadblock at the time since computers were still in their infancy. This is supported by Ng’s (1968) observation: “with the increasing capabilities of computers, such combinatorial problems could eventually be overcome.” An example the paper looked at was the Shell model or code name STRATCOM, which sought to look at a multi-person non-zero-sum game in continuous variable space. Without delving into the specifics of the model, it showed an improvement in the Bennion game by looking at more than two players and a non-zero-sum game, the two limitations stated by Bennion (1956). Ng (1968) states, “Nevertheless, it is generally agreed that the Shell model represents one of the first few concrete attempts at providing the sort of results that are meaningful to management contemplating game theory for complex, competitive decision problems.”

Thakor (1991) describes that game theory was not widely adopted in the economic world until the 1970s when economists began to realize that the limitations on the information possessed by individuals were important in understanding economic behavior, in part because such limitations induced people to alter their behavior. This statement further illustrates the evolution of the topic as relevant to economics and, thus, capital budgeting. The paper goes on to describe a variety of equilibrium concepts, including *Nash equilibrium*, *subgame perfection*, *sequential equilibrium*, the *Cho-Kreps intuitive criterion*, the *Banks-Sobel divinity* and *universal divinity refinements*. All of these concepts can be explored for further applications in capital budgeting for the DoD.

Looking further ahead in time, Boquist, Milbourn, and Thakor (1998) state that a company should estimate the likelihood of competition using game theory and probabilistic scenarios and determine a probability distribution for the competitive entry, along with its impact on project cash flows. The company should then integrate this information into its capital budgeting. Here we see a further extension of the multi-person game with the suggestion that probability distributions be assigned to the competition, implying there is a stochastic element to be introduced to account for uncertainty. This insight can prove to be valuable in the realm of capital budgeting in the DoD.

Huisman, Kort, Pawlina, and Thijssen (2003) state that at present, only a few contributions deal with the effects of strategic interactions on the option value of waiting associated with investments under uncertainty. The authors attribute this dearth to the application of game theory under continuous-time models not being well-developed, but they are optimistic, stating that additional future publications will appear due to the importance of the topic. The paper includes an examination of identical firms under a duopoly setting, which can have an application to capital budgeting in the DoD.

Often referred to as one of today's sophisticated capital budgeting tools (e.g., Kengatharan, 2016), the application of game theory to capital budgeting specifically is still relatively novel (Verbeeten, 2006; Siziba & Hall, 2021). Smit (2003) agrees, stating, "Only recently have strategic considerations to imperfect competition been treated in a formal and rigorous economics literature." Thus, we see a common theme throughout the history of the subject that game theory is viewed as a sophisticated tool for capital budgeting. However, a real breakthrough seems to always be right around the corner but is never quite realized. As stated by Li (2014), these techniques have been criticized as difficult to implement and marginal in their contribution to making better decisions.

Nevertheless, along with uncertainty, the existence of competition among multiple players sets up ideal conditions for the use of game theory (Nichols, 1994; Boquist et al., 1998; Thijssen et al., 2004; Araújo, 2015). While other theories, such as real options theory, might suggest waiting for more information to become available, game theory almost ignores this option entirely due to fear of competition pre-emption (Smit, 2003; Zhu & Weyant, 2003; Verbeeten, 2006). A general solution for investment strategies with the existence of competition can be found under a Nash-Cournot framework (Grenadier, 2002). Furthermore, Grenadier (2002) finds that the Nash equilibrium exercise strategies are identical to those obtained in an 'artificial' perfectly competitive equilibrium, with a slightly modified demand function. Therefore, we have shown the

necessity of game theory; equilibria can be found under a Nash-Cournot framework, and these equilibria are in the real-world setting when modifying the demand function versus the perfectly competitive model.

Game theory also reveals the importance of basing a budget on real reporting as opposed to inflated reporting by showing that the best payoff is fiscal management organization that approves a real budget [sic]” (Jiang & Gong, 2019). As noted by Anton (2019), it also validates opportunistic managerial behavior, noting that, under certain circumstances, managers misrepresent financial statements for their own advantage. Game theory is a very useful tool to understand the difficulties in implementing new accounting policies and procedures associated with low incentives/payoffs for management. Anton (2019) also states that game theory helps to design appropriate accounting standards that achieve the cooperation of all stakeholders. Because individual managers may misrepresent their financial statements, a more cooperative approach may benefit everyone involved without resorting to this misrepresentation. Such an approach can have ramifications in the DoD as well to cut down on artificial overspending, saving money that can be better spent elsewhere.

Gibbons (1992) provides an overall primer to game theory in economics, where his book was designed to introduce game theory to those who will later construct (or at least consume) game-theoretic models in applied fields within economics. The book provides a description of various games, including static games of complete information, dynamic games of complete information, static games of incomplete information, and dynamic games of incomplete information. This can provide a baseline for the game theory that can be referenced when looking at the games described in other sources.

Denault (2001) looks at the allocation of risk capital, stating that game theory provides an excellent framework on which to cast the allocation problem and an eloquent language to discuss it. The paper discusses the multiple coherent allocation principle approaches, such as the Shapley and Aumann-Shapley values, that can be useful for capital budgeting in the DoD.

Keppo (2002) developed a model that combines game theory and irreversible investment under uncertainty and postulates that the values follow something called geometric Brownian motion. The model then characterizes resulting Nash equilibrium under different assumptions on the information that the firms have about each other’s valuations for the project. This insight can be explored further for application in the DoD’s capital budgeting.

Thijssen et al. (2004) focus more on capital budgeting. The paper shows that it may be more beneficial to wait on some investments that may have a low positive return initially as they may yield more later. It also describes a way to measure error to indicate the “accuracy of capital budgeting rules” in a stochastic environment. The latter could be useful when investigating capital budgeting and the DoD.

Zhang, Peeta, and Friesz (2005) focus on a game theoretic model for infrastructure. However, it does introduce an “agent-based simulation solution structure to solve flow equilibrium” and,

more importantly, optimal budget allocation problems for the infrastructure model. An analogous model could prove to be useful when describing capital budgeting and the DoD.

Verbeeten (2006) investigates uncertainties and sophisticated budgeting practices in 189 Dutch organizations. The results of this study show that game theory is one of a handful of budgeting practices used by companies. The others are Monte Carlo simulation, certainty equivalents, and real options reasoning. Many of these methods are game theory adjacent and could also be considered when looking at capital budgeting and the DoD.

Ferreira, Kar, and Trigeorgis (2009) looked at competition in capital-intensive industries and stated that the option games will be of particular value to companies facing high-stakes decisions, such as those involving millions of dollars in capital investment, in a volatile environment in which their moves and those of their competitors clearly affect each other. This situation is applicable to capital budgeting in the DoD, where there is clear competition, whether it be internally for funding in the budget or externally, where contractors are competing for contracts.

In the transcript of a discussion of the government's budget negotiations, National Public Radio's White House correspondent, Ari Shapiro, reported on how game theory applies to budget negotiations in the White House (Shapiro, 2011). This shows a first-hand account of how game theory is currently being applied to capital budgeting in the national budget, in which a large portion is designated for the DoD.

Allen and Morris (2014) describe game theory models in finance, stating that game theory is an essential tool for describing the behaviors of investors, financial intermediaries, and corporate managers. The paper also concludes that game theory has provided a methodology that has brought insights into many previously unexplained phenomena by allowing asymmetric information and strategic interaction to be incorporated into the analysis. These models can be considered when looking into capital budgeting and the DoD.

Araújo (2015) describes capital budgeting in the economies of telecommunications. In addition to approaches connected with strategic game theory, the paper also explores Monte Carlo simulation with various statistical distributions to model a project's risk, as well as advanced capital budgeting algorithms with real options. A connection can then be made to extend beyond the world of telecommunications to the use of these techniques in the DoD's capital budgeting process.

Jiang and Gong (2019) analyze issues in game theory in the process of budgeting. The paper describes the process as a static and asymmetric game with incomplete information. It also describes that there are two management strategies, approve or disapprove, as well as two types of reports, inflated and real. It acknowledges that evaluating the authenticity of the budget is a serious problem of budget performance management. Finally, it concludes that the best payoff is a fiscal management organization that approves a real budget. In other words, it is not overall beneficial to overinflate the report to receive more funds, as this would be to the detriment of the organization as a whole. This lesson is certainly applicable to capital budgeting in the DoD.

While Anton (2019) mostly describes discount rate choices and agency cost minimization, his paper does have a section on game theory and capital budgeting. In this section, the author argued that game theory guides the process of choosing appropriate discount rates, accounting policies, accounting standards and managerial incentives and controls to achieve efficient capital investment decisions. It also acknowledges that financial statements can be misrepresented by managers for their own advantage. It goes on to say that game theory is a very useful tool to understand the difficulties in implementing new accounting policies and procedures associated with low incentives/payoffs for management. All of these takeaways can be applied to capital budgeting and the DoD.

Sikalo, Arnaut-Berilo, and Zaimovic (2022) describe a game theory-based model for efficient asset allocation. The paper finds fault with the best-known return-risk model, which is the Markowitz mean-variance model. The authors propose a minimax model for selecting the optimal portfolio based on the maximum loss as a measure of risk. They concluded that the new model was “particularly dominant over variance when the returns do not follow a normal distribution” (Sikalo et al., 2022). This insight can prove to be useful in capital budgeting and the DoD.

III. GAME THEORY MODEL IN BUDGET ALLOCATION

Incentive Mechanism to Reduce Information Asymmetry on Budget Allocation

Introduction to Asymmetric Information and Mechanism Design

In many negotiation situations, the strategic interaction between agents (“players”) is characterized by possessing private information where some agents have information that others do not. The most informed agents have natural incentives to take advantage of this informational disparity, for example, by saying that a larger budget to run a project is needed than it actually is or that a certain number of professionals would need to be hired when fewer would be sufficient. This knowledge imbalance is called information asymmetry.

Asymmetric Information is such an important problem in economics and game theory that its study has garnered Nobel Prizes in Economics. In 1996 it was awarded to James A. Mirrlees and William Vickrey “*for their fundamental contributions to the economic theory of incentives under asymmetric information*” and in 2001, to George A. Akerlof, A. Michael Spence, and Joseph E. Stiglitz “*for their analyses of markets with asymmetric information.*”

In 2007, the Nobel committee wrote that “an important goal of economic theory is to understand what institutions, or allocation mechanisms, are best suited to minimize the economic losses generated by private information” and awarded the prize in economics to Leonid Hurwicz, Eric S. Maskin, and Roger B. Myerson “*for having laid the foundations of **Mechanism Design Theory***.” Mechanism design has been widely used precisely as a tool to reduce asymmetric information problems. In this context, “mechanism” is defined as a game, or a set of rules, that specifies the strategies a player can choose from and the outcome for every choice. Mechanism design theory is a kind of reverse engineering of game theory. Instead of having a problem and determining the Nash Equilibrium (NE) as in the classical models of Cournot, Bertrand, Stackelberg, and so on, you ask the inverse question: what game and game rules would provide the NE we want? So, the idea is to design games with the desired outcome, such as efficient and fair budget allocation, which would be a good NE.

To that end, a mechanism (“game”) needs to be designed to meet the **Revelation Principle**, whereby the set of rules of the game makes it optimal for program managers to report the truth about how much they will actually spend and the number of projects they will execute.

Developed (1979) and named (1981) by Nobel Laureate R. Myerson, the Revelation Principal theorem states that one can constrain the identification for the optimal mechanism to those that are direct (ask for private information) and truth-telling incentive-compatible (i.e., it is optimal to reveal your truth, the hidden private information). It is proved that there is no loss of payoff by discarding mechanisms that do not meet the Revelation Principle. Thus, one can discard *all* mechanisms (rules) in which it is not optimal to tell the truth.

However, there are usually several (even infinite) mechanisms that meet the Revelation Principle. This makes it possible to design more flexible mechanisms, in which the client can dose the punishment or reward for telling the truth without the adjustments causing a failure to comply with the Revelation Principle. The proposed model will meet this Revelation Principle and will have the flexibility of applying different penalty *dosages*.

Mechanism Design and the Budget Problem

There is a vast literature on incentive mechanisms for program managers to reveal their private information. However after an extensive literature review, no articles were found that do not have monetary bonus incentives. For example, there are models where program managers earn higher bonuses when they reveal the truth about their unit's productive capacity. There are also fixed and variable salary mechanisms that make it best for program managers to report their hidden information and that still provide an incentive to exceed the expected result. Some mechanisms are linear (two-line segments), and others are more complex (nonlinear).

A simple example of a bonus (Π_0) for production (ϕ) in which it is optimal (highest bonus) to reveal the truth is given by the equation:

$$\Pi_0 = \kappa_1\phi - \kappa_2|\phi_f - \phi|$$

where κ_1 and κ_2 are constants, ϕ is the completed production, and ϕ_f is the forecasted production (Young 1985, Young & Lewis 1995). The second term is the penalty for deviating (both up and down) from the forecasted production. Only when production performed is equal to the projection is there no penalty.

Another example is presented in a microeconomics textbook by Pindyck and Rubinfeld(1995). Program managers (the players in the game, who are appointed and selected agents for the principal) forecast the year's production of a business unit, but they know more (private information) than the principal (e.g., key decision makers at headquarters who decide on how to allocate the budget such as the U.S. Navy's Office of the Chief of Naval Operations at the Pentagon) about the true production capacity with normal effort.

A simple mechanism is to ask the program manager for the production forecast and say that a bonus will be tied to an equation with two straight lines: one for production performed equal to or less than the forecast and another for values greater than the forecast. That is, the inflection point is the reported expected production. Suppose that, in this case, the truly expected production ("the truth") is 20,000 units. As seen in Figure 1, if the program manager reveals the truth (20,000), she or he will earn a bigger bonus than if she or he lies and says 10,000 or 30,000. There is thus an incentive, to tell the truth (20,000 with normal effort) and to try to outperform in order to get a higher bonus.

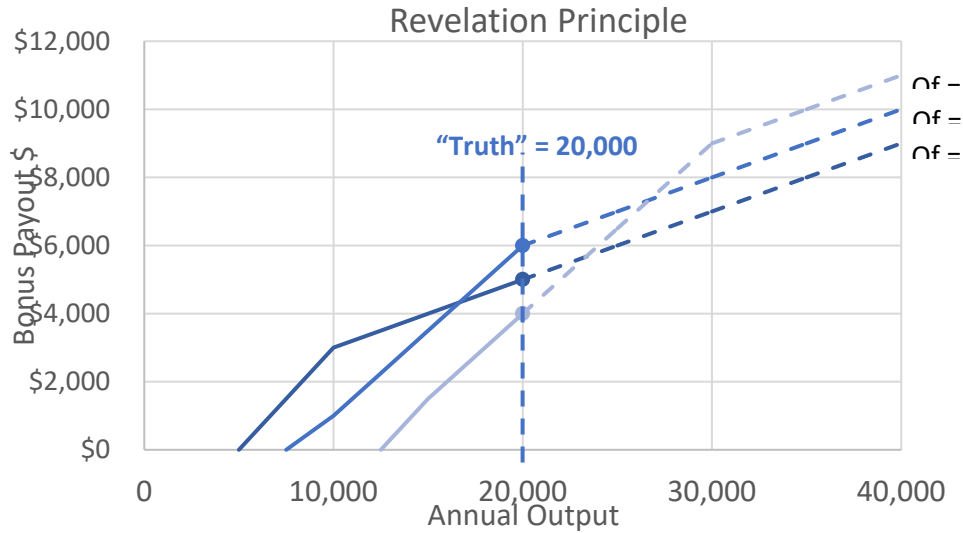


Figure 1: Linear Model Incentive: Highest Bonus Results from Telling the Truth (20,000 Production Units) (after Pindyck & Rubinfeld, 1995)

The DoD does not provide bonuses to government officials or staff as an incentive. Hence, the proposed model will use the next period's budget to incentivize. Although it is nonlinear, the proposed model will have an inflection point with the information revealed.

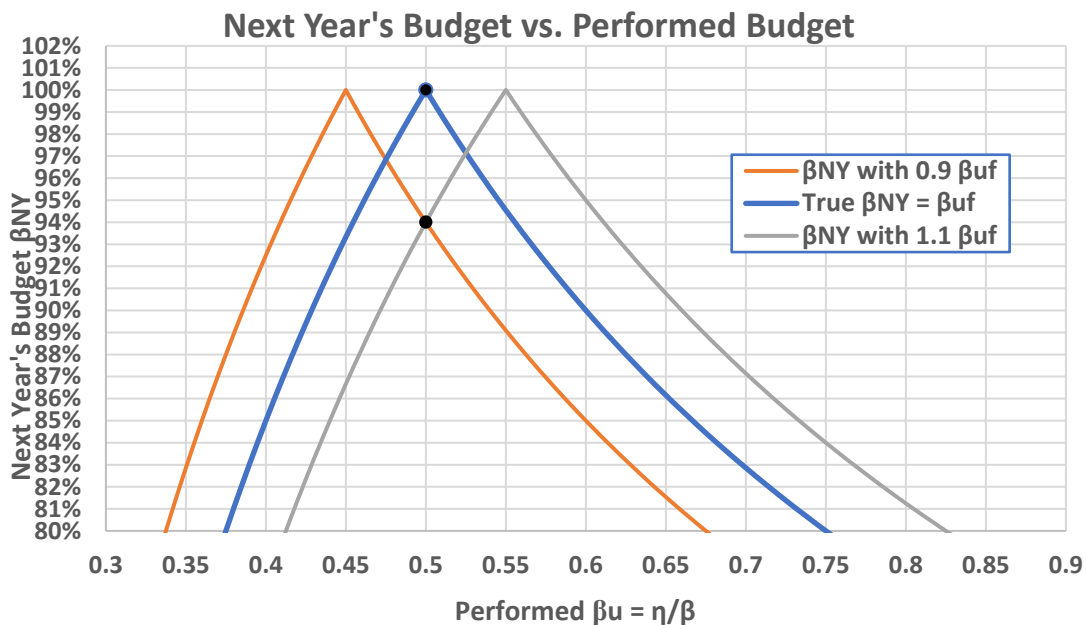


Figure 2: Proposed Truth-Telling Budget Model (truth: $\beta_{uf} = 0.5$)

In Figure 2, the truth is $\beta_u = \eta/\beta = 0.5$ project/\$ million. If 0.5 (blue line) were reported and 0.5 occurred, then the next year's budget will not be penalized (e.g., 100% of the previous year would be granted). But if, say, only 90% of the true value ($0.45 = 0.5 \times 0.9$, orange line) or if 110% of the true value ($0.55 = 0.5 \times 1.1$, gray line) were reported, then the division will be penalized if the expected outcome ("truth") occurs ($\eta/\beta = 0.5$): the next year's budget will only be 94% of the previous year.

Here the penalty was symmetrical (values 10% below or above the truth have the same penalty: a budget reduction to 94%). But, as described in the following sections, the proposed model is flexible enough to allow the punishment to be applied or "dosed" symmetrically or asymmetrically (e.g., more severe punishment for low predicted β_u). The model is also flexible in terms of being able to include an "additional truth premium," whereby the budget is increased (e.g., 105%) for telling the truth. Also, the model can include a tolerance. For example, with a 3% tolerance, if the forecast error is less than 3%, then the program manager keeps 100% of the budget for the next period.

The Proposed Model

1) Problem

The U.S. Navy budget for FY2022 is approximately \$212B, according to The President's Budget (2022). The budget is typically divided into:

- Infrastructure. \$3B is allocated to base realigning and closure, family housing, and military construction.
- Research & Development. \$22.6B is allocated to basic research, development of advanced technology, applied research, software pilots, management support, operations systems development, various system development and demonstration, and advanced component development.
- Military Personnel. \$56.5B is allocated to a permanent change of station, special and hazard pay, subsistence, health, reserve personnel, retired pay and thrift savings plan, and housing, as well as pay and allowances.
- Operation and Maintenance. \$71.2B is allocated to environmental protection, reserves, mobilization, training and education, service-wide support, combat weapons support, base operations, ship operations and prepositioning, aviation operations, and expeditionary.
- Procurement. \$52.8B is allocated to ammunition, Marine Corps, weapons, ships, aircraft, and other ancillary procurements.

Within each of these subcategories, for example, within ships, there are further subcategories such as operations and maintenance or upgrades. And within the subcategory of upgrades, for

example, there might be a Technology Insertion program whereby various classes of Navy ships will be allocated a certain budget to upgrade their hardware and software components (upgrades to MH60R Sikorsky Seahawk, NIFC-CA Naval Integrated Fire Control – Counter Air, Anti Surface Weapons, SPQ-9B/AN X-Band Pulse Doppler Radar for Littorals, CIWS-CEC Close-In Weapon System – Cooperative Engagement, RDDDL Radar Designated Capability Decoy Launch, SM-2 BLK II & SM-6 BLK I Interceptors and Anti Surface Missiles, just to name a few). Each of these programs will have one or more project or program managers responsible for their acquisition, development, deployment, testing, and follow-up.

These program managers ask for a budget for their respective projects, but they have private information about their division's ability to carry out projects with that budget. Thus, there is information asymmetry between these program managers and the key principals and decision makers at the Pentagon allocating the budget (e.g., Office of the Chief of Naval Operations), which we will collectively term "headquarters" for simplicity.

2) Purpose of the Model

The model is intended to be used to encourage program managers to tell the truth about their division's ability to execute projects within the requested budget. This is important because if each program manager puts forward their best efforts in forecasting their production outcomes, budget requirements, schedule and cost risks, and other pertinent program metrics, then the budget allocation process will be equitable and efficient. The idea is to reduce any suboptimal allocation where there is "game playing," where certain programs obtain more than they need while producing subpar results, chipping away at the opportunity costs where the excess budget could have been reallocated somewhere else that would provide better and optimal outcomes for the U.S. Navy, DoD, and the country's national security as a whole.

3) Punishment/Reward for Telling the Truth

Since the analysis is applied to divisions of the government and the government does not pay performance bonuses (assumption), the performance incentive mechanism (telling the truth) will be based on a budget for the next year (period). For example, program managers are warned that "The next year's budget allocation to your department will be LESS than what you asked for because you cheated." The reward can be structured as getting what they ask for next year. As a result, cheating on these strategic games would be greatly reduced.

Therefore, the model will be based on these assumptions for punishment and reward. If program managers' forecasts come true (the model allows for a tolerance), they will have 100% (or more than 100% because the model has this flexibility) of the previous year's budget (plus any legally binding adjustments to the cost of living, inflation, job-specific and geographic-specific requirements). Otherwise, they will have a smaller budget.

4) Assumption

The part of the budget allocated for mandatory expenses (salaries and benefits, health, subsistence, hazard pay, contracts already signed, etc.) is not the object of the model. In the model, it is assumed that the budget under discussion is for discretionary programs: new projects, staff training, optional upgrades, special procurement, and so on.

5) Program Managers' Input

It does not seem appropriate to ask the program managers to request the budget and verify that they have complied with it because they could spend money on nonsense items just to avoid spending less than requested.

So, it seems more reasonable in the first year to negotiate a budget with the divisions (see details of the “game” sequence below) by asking them *what they can do with* this budget.

Thus, some measure of performance is needed in the execution of the discretionary budget. Organizations usually have some measure of managerial performance. Here it is assumed that this measure is “units of projects done” or “units of work done,” but it can be any quantitative measure of performance that the organization is most comfortable using. For example, metrics can include operational and logistics such as Inherent Availability (IA), Effective Availability (EA), Mission Reliability (MR), Operational Dependability (OD), Mean Down Time (MDT), Mean Maintenance Time (MMT), Logistics Delay Time (LDT), Achieved Availability (AA), Operational Availability (OA), Mission Availability (MA), and others. Financial and economic metrics can, of course, be similarly used, such as net present value (NPV), internal rate of return (IRR), modified internal rate of return (MIRR), return on investment (ROI), profitability index (PI), payback period (PP), and discounted payback period (DPP).

In the model, then, it will be assumed that once the budget of division i (e.g., $\beta_i = \$100$ million) has been negotiated (in the first year), each program manager will report the number of project units that will be carried out with this budget (e.g., $\eta_i = 50$ projects), resulting in the unit budget forecast by the program manager (in the example, $\beta_{uf} = 50/100 = 0.5$ project/\$ million).

6) “Game” Sequence

6.1) Current Year

Each division program manager reports to headquarters the requested annual budget for projects and the number of projects that will be performed with this budget.

If the summation of budgets is higher than or, at most, equal to the overall budget (probable), headquarters negotiates (or imposes a proportional cut) with divisions to reduce the budgets until the sum of budgets equals the total available budget.

With the new budget, β_i (probably less than the requested budget), each division program manager will report the number, η_i , of projects expected to be implemented with that budget. So, the division i program manager forecasts $\beta_{uf} = \eta_i/\beta_i$.

Since it is η_i/β_i that must be met and not the assigned budget (β_i), if during the year the program manager feels it will not be possible to accomplish the number of forecasted projects, there will be an incentive to save money and so, even doing fewer projects than predicted (η_i), to match the predicted η_i/β_i with a lower β_i rather than using β_i for superfluous expenses. Excess unused budget will be returned to headquarters during the budget year.

6.2) The Second Year

For each division, decision makers will measure two units: the budget realized and the number of projects carried out with this budget spent. The assumption is that the budget review is annual. The model, however, can use different periods (e.g., triennial).

If the η_i/β_i ratio performed by division i is equal (or within tolerance) to that forecasted by the division, in the next period, this division may be allocated up to 100% of the previous budget (or more than 100% as the model allows an additional reward for telling the truth). Otherwise, the division will get a lower budget than the previous one. If the total budget increases (decreases) in the second year, the budget of division i will increase (decrease) proportionately (after adjusting for any requisite increases such as cost of living, inflation, contractual obligations, Congressional mandates, etc.).

Once they have the new budgets for the divisions (some benefited from correct/honest forecasting, others were penalized), top management again asks each division program manager what they can do (number of projects) with this budget. Again, a new η_i is reported, and $\beta_{uf} = \eta_i/\beta_i$ is calculated.

6.3) The Third and Subsequent Years

Repeat procedure 6.2.

Model Inputs: β and η

While budget (β) is, of course, one of the model's input variables, not all of the budget is used; a part of the budget is for *mandatory expenses* (salaries and benefits, contracts already signed, etc.) and is not the object of the model. Thus, in the model, it is assumed that the *budget under discussion is only for the discretionary programs*: new projects, staff training, discretionary budget, and so on.

As noted earlier, it does not seem appropriate to ask program managers to request the budget and then verify that they have complied with it because they could spend money on nonsense items just to avoid spending less than requested. For optimal results, it is necessary for program managers to also report what will be done with the budget, that is, what projects (or additional services) will be carried out with the requested budget.

In this model, we will use η for the “number of projects,” but it could represent “units of work,” net present value (NPV) of projects, real option values of those projects, social welfare value added by the projects, or even “man-hours” or similar metrics used in connection with projects. As mentioned previously, different organizations have different preferred performance metrics they can use. The model here will have a dimensionless result (a percentage), and it does not matter which performance measure is used for the realized budget. It is enough to have a *numerical value* of this realization that here we will call η_i (e.g., the number of projects predicted to be completed by division i).

The model uses two constants that calibrate the punishment for not reporting the truth, called λ_1 and λ_2 . We will see that to induce the truth about η and β . It is necessary that these constants obey $\lambda_1 < 1$ and $\lambda_2 > 1$. Before looking at the model equations, let’s look again at the proposed “game” sequence.

Game Sequence

a) The Current Year

First Round: Each division reports to headquarters the desirable annual budget, β_{i0} , for projects and the number of units of the project, η_{i0} , that will be performed with this budget. Note that η_i is the division program manager’s freely chosen input. Program managers *can choose* η_i where the η_i/β_i ratio is below (or above) the previous year’s ratio. Headquarters commit to not using the previous η_i/β_i as a lower limit for new predictions of η_i in this first round.

If the summation of budgets is higher than the overall budget, headquarters must cut budgets until the sum of budgets equals the total available budget. Depending on the performance promises (η_{i0}) in relation to the requested budget (β_{i0}), headquarters will make the cuts and determine the β_i budget of each division for the current year (period).

Before the second round, headquarters announced that a model would be used where everyone would enter new η_i for the budget (β_i) assigned to each program manager. Headquarters will show program managers the spreadsheet in which it will be optimal (no budget cuts in the following year) to make the most realistic prediction of η_i for this β_i and reveal at this point that $\lambda_1 < 1$ and $\lambda_2 > 1$ will be used. However, the exact values of λ_1 and λ_2 will only be revealed in the following period. That is, division program managers will know that making the best prediction (telling the truth) is the best option.

Second Round: With the new budget β_i (probably less than the requested budget), each division program manager will report the number η_i of projects expected to be implemented with that budget. So, the division i program manager forecasts $\beta_{if} = \eta_i/\beta_i$.

b) The Second Year

For each division, headquarters will measure two key composite metrics: the realized budget and the number of projects (or other performance measures) carried out with this budget spent. We

assume that the budget review is annual. However, the model can use different periods (e.g., triennial).

If the η_i/β_i ratio performed by division i is equal (or within tolerance) to that forecasted by the division, in the next period, this division will be given 100% of the previous budget (or more than 100% as the model allows an additional reward for telling the truth). Otherwise, the division will get a lower budget than the previous one. If the total budget increases (decreases) in the second year, the budget of division i could increase (decrease) proportionately.

Once they have the new budgets for the divisions (some benefited from correct/honest forecasting, others were penalized), top management again asks each division program manager what they can do (number of projects) with this budget. Again, new η_i values are reported, and $\beta_{uf} = \eta_i/\beta_i$ are calculated.

c) Third and Subsequent Years

Repeat procedure (b).

Corollary

In the first round of the first year, division program managers know that they will face cuts in requested budgets. There is *competition* among divisions for the (limited) budget. The program managers are aware of the competition and know that if they put a low η_{i0} (underachieving with β_{i0}) in the budget cut phase, they may end up with a bigger cut than in a division that reports a higher η_{i0}/β_{i0} . Thus, the division tendency would be to ask for a high budget (β_{i0}) but also to promise a high realization of projects (η_{i0}). While headquarters may have less information about projects than the division program managers, *they do have some information* and can *cut more budgets for low achievement*. Division program managers know that. And if their projects have some pre-assessment (e.g., NPV or social welfare value), this “threat” would be even more serious.

The model and spreadsheet will only be revealed to division program managers after the first round but before the second round. Headquarters reveals at this point (not before) that $\lambda_1 < 1$ and $\lambda_2 > 1$ will be used, but not the exact values of λ_1 and λ_2 . That is, division program managers will only know that making the best prediction (telling the truth) is the best option. But they don’t know if the biggest punishment is making η/β smaller or bigger than expected. This uncertainty tends to create an additional incentive to report what the program manager truly believes can be accomplished with β_i . Thus, the tendency is for program managers to make the best possible forecast for the available budget, that is, to report the “truth.”

Suppose in the first round of the competition among divisions for the budget, program managers tended to ask for a high β_i but also promised a high η_i so as not to have too many cuts. So, program managers previously promised an η_{i0}/β_{i0} number. In the second round, they may promise a different η_i/β_i ratio, but the tendency is that *if it is very different*, headquarters may be suspicious and adjust the exact values of λ_1 and λ_2 accordingly. For example, if a program manager reports a much smaller η_i/β_i than η_{i0}/β_{i0} , top management can use a low value of λ_1 to punish the

more likely achievement greater than the low projected value of η_i . Uncertainty about the size of the punishment ($\beta\%$ reduction next year) tends to prevent program managers from reporting η_i too low.

What if division program managers report a very high η_i ? For example, having reported a high η_{i0}/β_{i0} (to have a higher β_i budget in the competition among divisions), a program manager may not then want to put in a much lower η_i/β_i ratio (headquarters would notice) and thus ends up reporting a η_i/β_i greater than is reasonable. But the program manager can still fulfill the projection! Since it is η_i/β_i that must be met and not the assigned budget (β_i), if during the year the program manager feels it will not be possible to accomplish the number of forecasted projects, there will be an incentive to save money and, even doing fewer projects than predicted (η_i) to match the predicted η_i/β_i with a lower β_i rather than using β_i for superfluous expenses. In this sense, it is less risky to promise high η_i because if you cannot deliver the promised η_i , you have the option of performing less β_i to maintain η/β and not have cuts in the next year.

What if program managers forecast a very low η_i ? For example, having reported a low η_{i0}/β_{i0} value (perhaps due to an evaluation error), the program manager may not want to give a very different ratio than before and ends up indicating a low η_i . In this case, in the competition among the divisions, this program manager has probably already been allocated with a low β_i . If this program manager puts in a low η_i/β_i , the option would be to spend money on superfluous items to try to fulfill η_i/β_i , but this could be identified by headquarters, and the program manager (probably already with a low β_i) could lose his or her position for bad performance. An alternative to headquarters would be, from the second or third year onwards, to change the incentive scheme so that this kind of program manager is encouraged to exceed the η_i/β_i that she or he projects (may even make $\lambda_1 > 1$).

Basic Model Equations

After establishing the budget, β_i , for division i and the program manager reporting the number of expected projects η_i with this budget, headquarters calculates the forecast using $\beta_{fi} = \eta_{fi}/\beta_{fi}$.

In the next period or year, headquarters will measure the budget and the number of projects carried out for each division, that is, $\beta_{ui} = \eta_i/\beta_i$. The budget for the next period of the i division will depend on whether β_{ui} is greater than or less than β_{fi} . If it is equal, division i will have 100% of the budget for the next year (or period) β_{NY} . Otherwise, it will have a budget cut ($< 100\%$) according to the equations:

$$\beta_{NY} = \frac{\beta_{fi}}{\beta_{ui}} + \lambda_1 \frac{(\beta_{ui} - \beta_{fi})}{\beta_{ui}} \quad \text{If } \beta_{ui} > \beta_{fi}$$

$$\beta_{NY} = \frac{\beta_{fi}}{\beta_{ui}} - \lambda_2 \frac{(\beta_{fi} - \beta_{ui})}{\beta_{ui}} \quad \text{If } \beta_{ui} \leq \beta_{fi}$$

with a truth-telling incentive of $\lambda_1 < 1$ and $\lambda_2 > 1$ (λ_1 can even be negative, indicating a higher punishment).

Note that if $\beta_{uf} = \beta_{ui} \Rightarrow \beta_{NY} = 1$ (or 100%), otherwise $\beta_{NY} < 1$.

The parameters λ_1 and λ_2 are constants set by headquarters. These constants make it possible to change the “dosage” level of punishment as long as $\lambda_1 < 1$ and $\lambda_2 > 1$, symmetrically or asymmetrically.

The *lower* the constant $\lambda_1 < 1$, the greater the punishment for predicting *fewer* projects than what was done. It is even possible to have a negative λ_1 to increase the punishment.

The *greater* the constant $\lambda_2 > 1$, the greater the punishment for having predicted *more* projects than were done.

For symmetrical punishment, choose λ_1 and λ_2 so that the average is 1, that is, $(\lambda_1 + \lambda_2)/2 = 1$.

While Figure 2 shows a case with symmetrical punishment (with $\lambda_1 = 0.4$ and $\lambda_2 = 1.6$), Figure 3 presents an *asymmetrical* punishment case ($\lambda_1 = 0.5$ and $\lambda_2 = 2$).

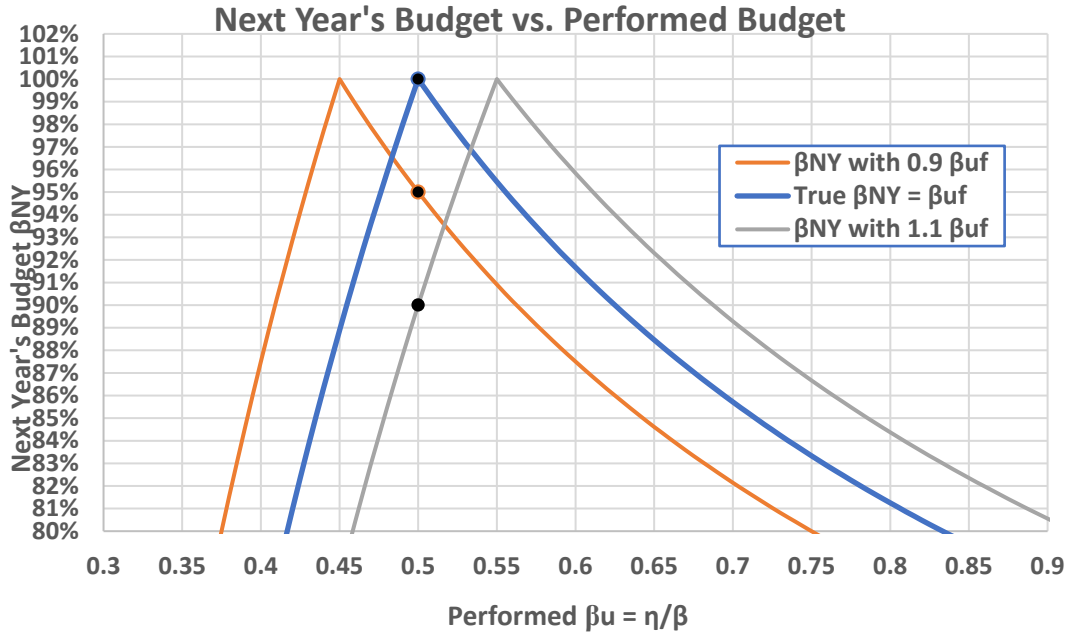


Figure 3: Incentive Model with Asymmetric Punishment ($\lambda_1 = 0.5$ and $\lambda_2 = 2$)

Here (assuming the truth is 0.5), whoever reports less (e.g., 0.45) and does more (the expected = 0.5) will get a lower punishment (95% budget) than whoever reports more (e.g., 0.55) and achieves less (the expected = 0.5), as they would get only get 90% (see highlighted points on the graph at 0.5).

Headquarters will establish the λ_1 and λ_2 constants, that is, the dosage of punishment for η forecasts above or below what actually happened. Therefore, while maintaining the incentive to

induce the truth, the model has the flexibility to be able to dose the punishment symmetrically or asymmetrically.

To encourage the truth, it is necessary to obey the inequalities $\lambda_1 < 1$ and $\lambda_2 > 1$. If $\lambda_1 = 1$; there will be an incentive to report a low η and obtain 100% of the budget if the division manages to fulfill at least the low number of projects reported (any $\beta u_i \geq \beta u_{fi}$ will have 100%). See Figure 4.

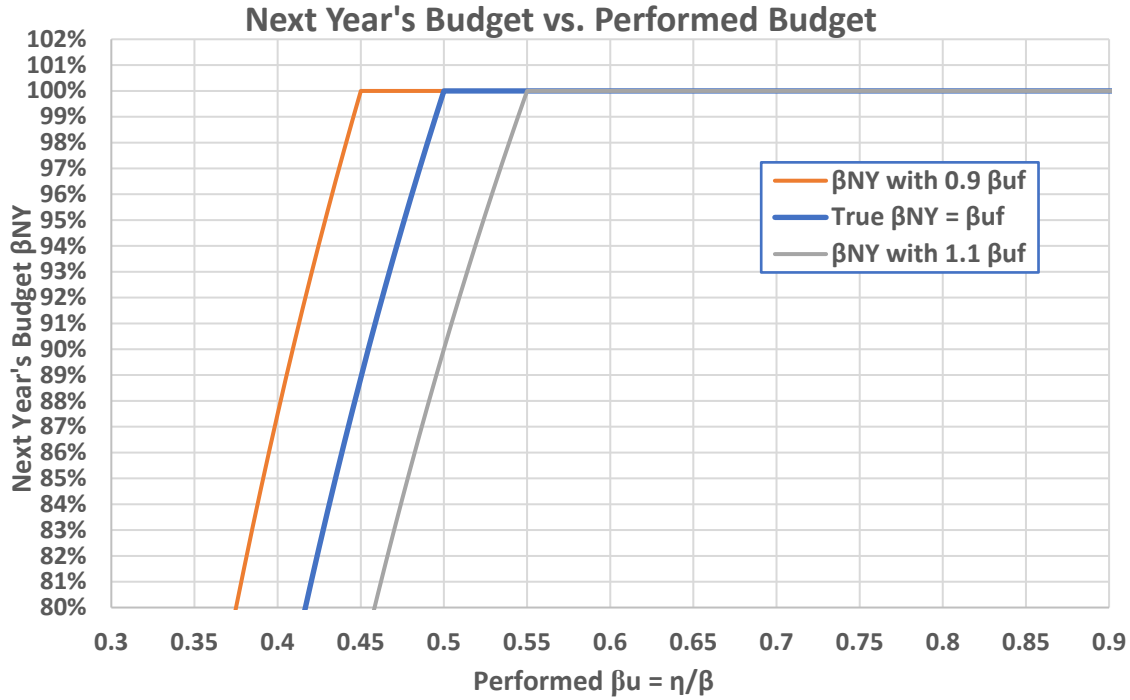


Figure 4: Model with $\lambda_1 = 1$: Incentive to Lie (Forecast Low η)

Even worse, if $\lambda_1 > 1$, divisions would get more than 100% if they report an unrealistically low value for η and do equal to or more than the forecast. To summarize, if $\lambda_1 \geq 1$, program managers will underpromise, overdeliver, and receive 100% of the requested budget going forward, perpetuating a suboptimal condition.

If $\lambda_2 = 1$, there will be an incentive to report a high η and get 100% if the predicted or *fewer* projects are done. With $\lambda_2 = 1$, any $\beta u_i \leq \beta u_{fi}$ will get 100%. See Figure 5.

Even worse, if $\lambda_2 < 1$, divisions would get more than 100% if they report an unrealistically high value for η and do equal to or less than the forecast. To summarize, if $\lambda_2 \leq 1$, program managers always overpromise and will maintain 100% budget requested even if production is less than promised, also perpetuating a suboptimal condition.

Therefore, it is recommended to headquarters that the truth-telling incentive setting be $\lambda_1 < 1$ and $\lambda_2 > 1$.

An interesting and related question is how much information should top management pass on to division program managers? The recommendation here is to *provide a spreadsheet for program managers* to see that by telling the truth, they are more likely to have at least the same amount as the current budget next year. Therefore, it is recommended that top management tell divisional program managers that $\lambda_1 < 1$ and $\lambda_2 > 1$.

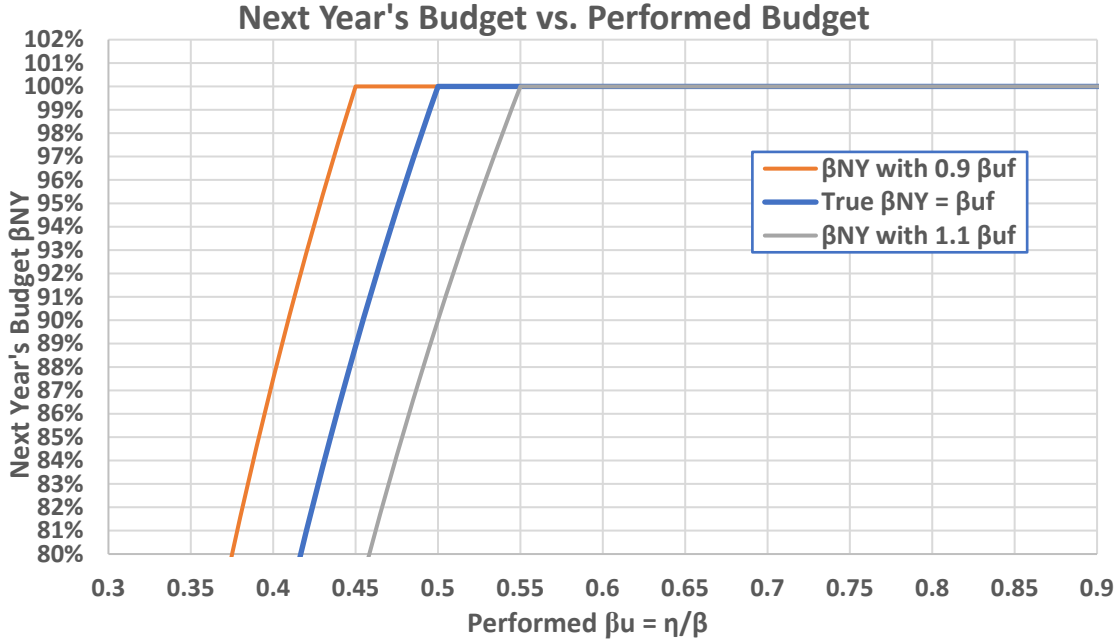


Figure 5: Model with $\lambda_2 = 1$: Incentive to Lie (Forecast Higher η)

But what are the *exact values* of λ_1 and λ_2 that will be used? At first, it seems best to keep this information from the division program managers and just say that $\lambda_1 < 1$ and $\lambda_2 > 1$. That makes it optimal for each program manager to tell the truth about η because the program manager's uncertainty about what the biggest punishment (either exaggerating or underestimating the number of projects) can be an additional inducement for the program manager to truthfully report what she or he really thinks is possible to do with normal effort when running the budget. On average, program managers are risk averse, and greater uncertainty about λ_1 and λ_2 would be an additional reason to report what they really think will happen ("the truth").

Tolerance and Additional Truth Premium

In order to inject more flexibility into the model, two other parameters are introduced: tolerance and an additional premium for telling the "truth." Headquarters will also establish these parameters.

Although division program managers have more information than headquarters, there is always some uncertainty about how many projects can actually be done on budget. So even if the program manager wants to tell the truth about η , this uncertainty can make η bigger or smaller. Thus, it seems fair to consider a tolerance in the forecast. For example, a tolerance of 5% allows errors of

up to 5% (over- or underrealized) to not be punished with a lower budget next year. What tolerance level to set depends on the assessment by headquarters of division program managers' difficulty in correctly predicting budget achievement. Headquarters may also want to give an additional premium to those who make the correct predictions. The model also allows for the placement of an additional premium (e.g., 3%) for reporting the truth within the tolerance. Figure 6 illustrates a case of 5% tolerance and 3% additional premium.

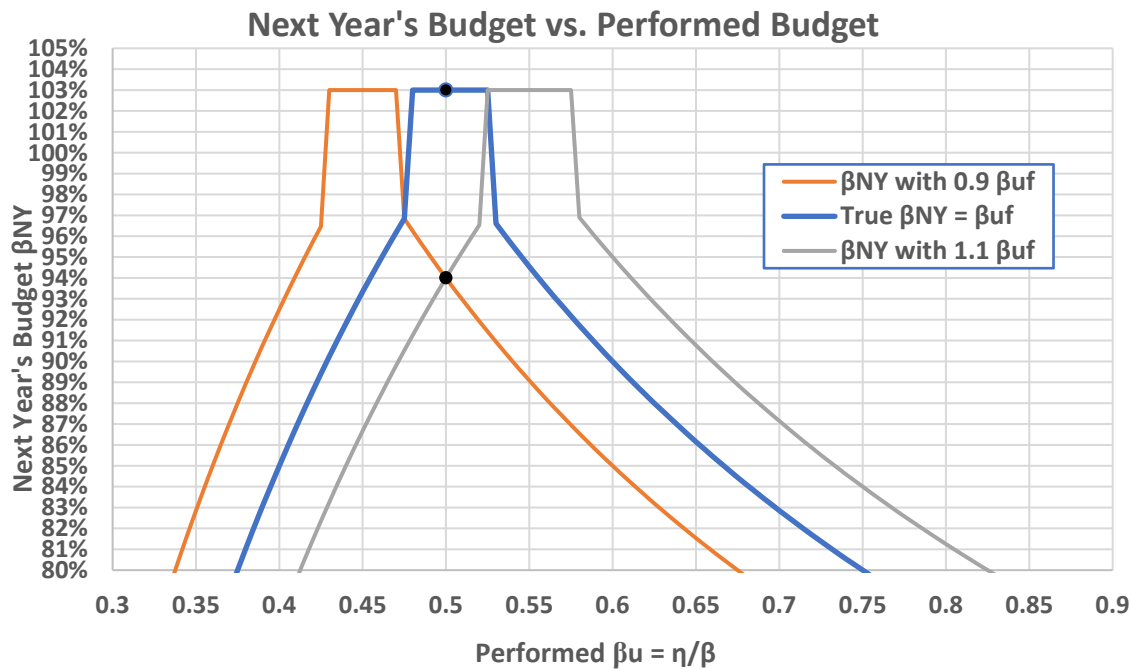


Figure 6: Symmetric Punishment Model with Tolerance (5%) and Extra Premium (3%)

Note that those who reported 0.5 within the tolerance (between 0.475 and 0.525) received no penalty (100%) and the additional premium (getting 103% for the next year). Outside the tolerance (curves 10% lower or 10% higher), there is a penalty: if 0.5 occurs, they only get 95% of the budget.

This model can be implemented in a simple Excel spreadsheet. It has a tutorial tab, which recalls the game rules and equations and gives some insights, and a tab for headquarters input data and for the division program manager to enter the number of projects and view the result (in a cell and on a graph). This allows the division program manager to realize that he or she must report the best forecast (“the truth”) in order to not be penalized the following year with a lower budget.

Dynamic Effects

The model encourages program managers to tell the truth about their ability to carry out projects. In game theory, it is said that it is optimal for the player to reveal his or her “type” if the mechanism meets the Revelation Principle. In this application, the “type” can be either a high-working, hard-working program manager or a low-working, lazy program manager.

Over the years, headquarters may want to use the revealed η_i/β_i , for example, and not allow the following year to use a lower η_i/β_i than the previous year, thereby changing the suggested game rule (η_i would always be the program manager's choice in the proposed rule; it is not headquarters' choice). Changing the rule can be tempting, but it can lead to problems. Recall that the constants λ_1 and λ_2 can always be adjusted (headquarters can even use different constants for different divisions).

For example, a highly productive division program manager might naturally report a high η_i/β_i . But if the program manager thinks that this η_i/β_i will be used in subsequent years as a minimum value, then she or he would become more conservative and would set a lower η_i/β_i value from the start, fearing that this level would become the minimum level of performance. This dynamic effect is known as the *Ratchet Principle* (Weitzman 1980). Program managers must weigh an uncut budget for good performance in one year against being assigned more ambitious targets in the future.

To avoid this ratchet effect, headquarters must make it clear before the first round of the first year that future η_i values will *always* be the choice of division program managers, given the assigned budget β_i . In other words, η_i will *not* be used as a minimum performance level in subsequent years.

Another dynamic problem is discovering that the “truth” of some program managers is low productivity. Of course, senior leadership can replace this type of program manager. But headquarters can also try to provide incentives for this program manager to try to outdo him- or herself over the years. Suppose division program manager j is stuck at a low η_j/β_j . In year 3 or 4, then, headquarters can change λ_1 to a value greater than 1 (instead of less than 1). For example, using $\lambda_1 = 1.1$ (instead of a value $\lambda_1 < 1$) and $\lambda_2 = 1.3$, there is an incentive to do more projects than forecasted (higher η_i earns more than 100% for the next year's budget; see Figure 7). But in the case of $\lambda_1 > 1$, headquarters cannot allow these low-productivity program managers to report lower η_i/β_i than the previous ones. The graph makes the reason clear (see the orange curve). For headquarters to put $\lambda_1 > 1$ for these program managers, it is necessary that the η_i/β_i value is not lowered by these low-productivity program managers.

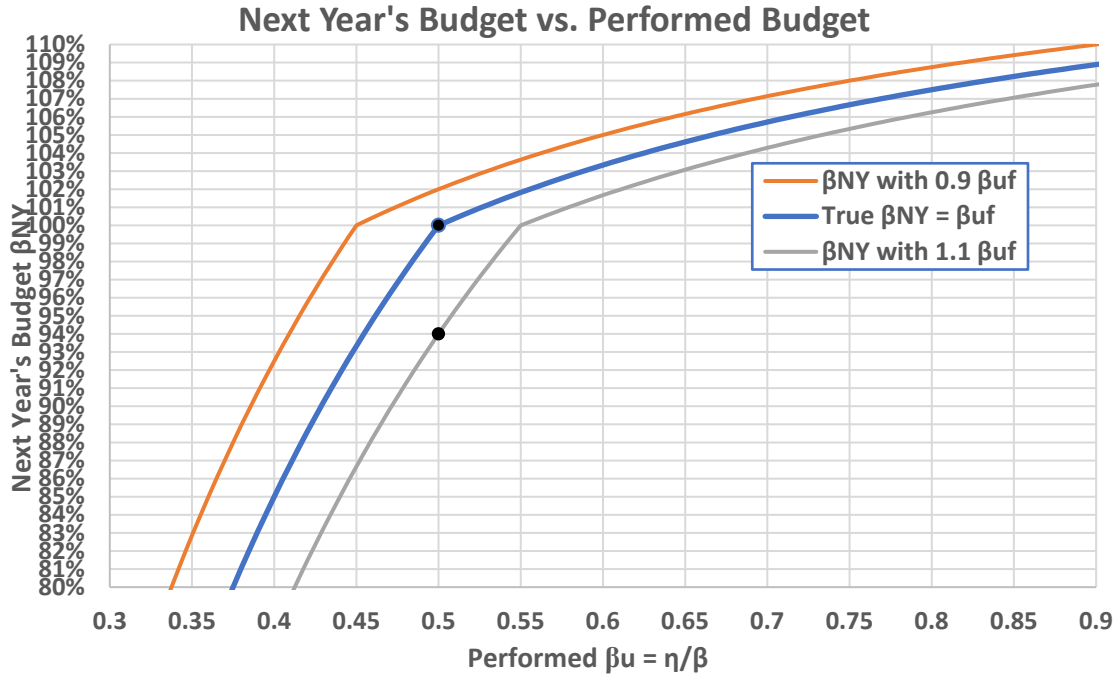


Figure 7: Special Case of Low Productivity Program Managers with $\lambda_1 > 1$

Stochastic Portfolio Optimization with Monte Carlo Simulation on Game Theory Model

This section illustrates how Game Theory and Monte Carlo simulation with stochastic portfolio optimization can be applied to budget allocation. This is done in order to illustrate how the Integrated Risk Management process can be integrated into Game Theory but also to perform a test run of the proposed Game Theory model in this research. Because of the lack of live data, and in lieu of a tabletop war gaming experiment, simulations can be run instead to recreate possible outcomes in the model. Monte Carlo simulation was first applied in Year 1. Each division can either be simulated independently of the other divisions or together in a correlated portfolio. However, for simplicity, the distributions for penalty parameters (x_j and y_j), tolerance (t_j) and bonus (u_j) remain the same. Ideally, having historical data is preferred to having the most accurate simulation where we can first perform a distributional fitting routine to determine the best probability distributions and parameters to use. However, in the absence of historical data, sample stylized data can be used as an alternative. The simulation described in this section uses the latter.

The example simulation specifications are as follows. For $B_{i,j Plan}$ and $B_{i,j Act}$, each is given one of three distributions, all with a mean of \$100 Million. The first is the triangular distribution with a minimum of \$90 Million, maximum of \$110 Million, and mode of \$100 Million. The second distribution is the normal distribution with a mean of \$100 Million and standard deviation of \$10 Million. The third and final distribution is the uniform distribution with a minimum of \$90 Million and maximum of \$110 Million. The value of $B_{i,j Plan}$ and $B_{i,j Act}$ is assigned one of these three distributions for each division.

For $n_{i,j \text{ Plan}}$ and $n_{i,j \text{ Act}}$, each is given one of two distributions with a mean of 50 projects. Since the number of projects must be a whole number, each is discrete distributions. The first distribution is the discrete uniform distribution with a minimum of 45 projects and a maximum of 55 projects. The other distribution used is the Poisson distribution with a mean of 50 projects. The value of $n_{i,j \text{ Plan}}$ and $n_{i,j \text{ Act}}$ is assigned one of these two distributions for each division.

The final four parameters are typically set by the department. For the simulation a distribution is assigned to cover the range of values that the department could assign to them. Regarding the penalty parameters x_j and y_j , the normal distribution is used for both. For x_j , the mean is 0.9 and standard deviation is 0.01, while for y_j , the mean is 1.1 and standard deviation is 0.01. Additionally, an upper bound of 1.0 is imposed on x_j and a lower bound of 1.0 is imposed on y_j to ensure they stay penalties and do not turn into bonuses as previously discussed. Based on the distributions currently assigned, it is highly unlikely that the bonds will come into play. However, it is important to highlight this point if the distributions are changed, potentially increasing the likelihood the bounds come into play for the tolerance t_j and bonus u_j , the exponential distribution with a mean of 0.02 is used for both. Figure 8 illustrates the example simulated portfolio while Figure 9 shows the optimization results, with Figure 10 illustrating the forecast simulated probability distributions of the allocations for each division. The methodology, results, and conclusions that can be obtained from this trial run is the same as the previous two sections of this research report.

BUDGET ALLOCATION STOCHASTIC OPTIMIZATION MODEL												
Description	WA Military Value	WA Downsides	Allocation Weights	Allocation (\$000)	Requested	95% Confidence		Game Theory Min	Game Theory Max	Game Theory Min	Game Theory Max	Game Allocated
						Game Theory Min	Game Theory Max					
Division 1	11.21%	12.39%	11.60%	\$11,599	\$11,616	\$11,212.62	\$11,615.68	96.53%	100.00%	99.86%		
Division 2	11.33%	16.30%	6.58%	\$6,577	\$7,038	\$6,510.07	\$7,178.67	92.50%	102.00%	93.45%		
Division 3	11.78%	15.35%	7.84%	\$7,842	\$8,000	\$7,040.00	\$8,400.00	88.00%	105.00%	98.02%		
Division 4	10.67%	12.33%	11.01%	\$11,007	\$11,390	\$10,994.51	\$11,455.80	96.53%	100.58%	96.64%		
Division 5	13.22%	13.09%	12.09%	\$12,089	\$12,622	\$11,359.81	\$12,874.45	90.00%	102.00%	95.78%		
Division 6	14.09%	14.35%	11.09%	\$11,087	\$11,378	\$10,410.56	\$11,718.99	91.50%	103.00%	97.45%		
Division 7	15.48%	14.14%	12.77%	\$12,766	\$12,551	\$11,107.89	\$12,802.31	88.50%	102.00%	101.71%		
Division 8	15.12%	16.40%	9.44%	\$9,437	\$9,031	\$8,037.33	\$9,482.24	89.00%	105.00%	104.50%		
Division 9	14.40%	16.63%	7.90%	\$7,900	\$8,624	\$7,762.01	\$8,969.43	90.00%	104.00%	91.60%		
Division 10	9.75%	12.40%	9.70%	\$9,696	\$10,502	\$9,661.83	\$10,712.03	92.00%	102.00%	92.33%		
Portfolio Total	12.79%	4.51%	100.00%	\$100,000	\$102,751	\$94,097	\$105,210					
Value to Risk Ratio	2.8341		Target Allocation	\$100,000								

Figure 8: Example Portfolio

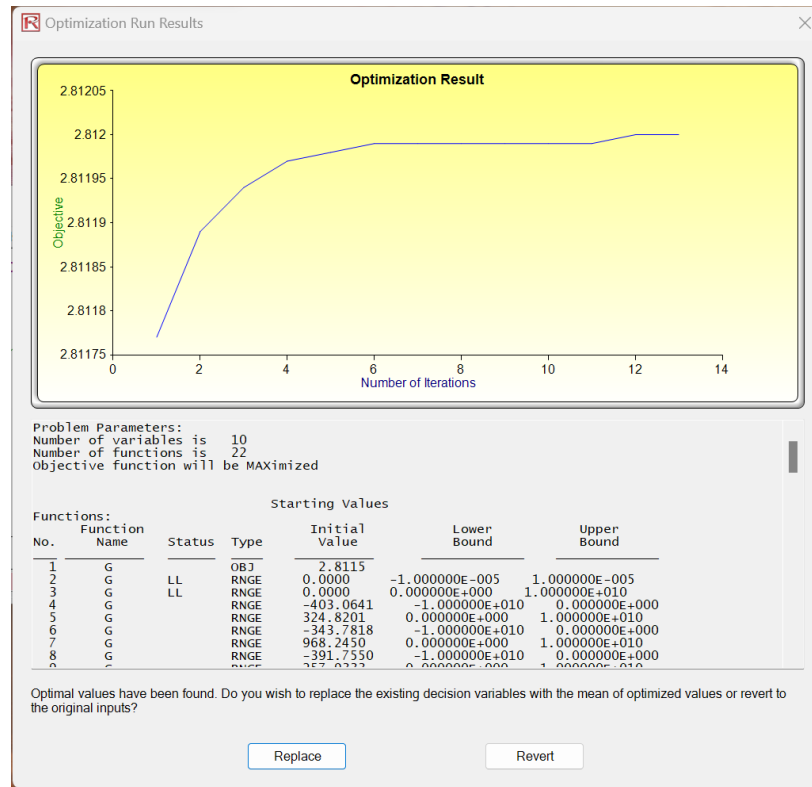


Figure 9: Portfolio Efficient Frontier

Conclusions

The presented model obeys many desirable properties:

Theory: Models must be consistent with modern economic theory. This model is consistent with modern Mechanism Design Theory and the Revelation Principle, which is state of the art in reducing Information Asymmetry problems.

Practice: The model has to be simple and intuitive to be practical. This model has a few parameters, all of them with simple, practical interpretations. After negotiating the budget (first year, first round), program managers only need to estimate, honestly, one parameter (η). Models that reward truth are intuitive and look attractive.

Application Flexibility: The model has to be flexible to accommodate the preferences and specific business conditions of headquarters. This model is flexible in dosing the degree of punishment, which can be symmetrical or not (with λ_1 and λ_2). The model is also flexible in allowing for tolerance to forecast error and to include an additional premium for honest reporting.

Dynamic Effects: Over the years, it will become clearer which are high-productivity program managers and which are low-productivity. It is the dynamic effect of the model.

In the case of low-productivity program managers, in addition to the alternative of replacing them, the rules of the game can be slightly changed after a certain year to provide an incentive to overcome their limitations (with $\lambda_1 > 1$, but not being able to reduce the previous year's η_i/β_i).

For high-productivity program managers, the opposite is necessary: always make it clear that the choice of the future η_i will be theirs and not that of headquarters, thus preventing the ratchet effect.

Finally, the example in this research used monetary variables and the number of projects as outcomes to be measured. There are a slew of additional or alternate metrics that can be used.



Figure 10: Simulation Results for Each Department's Budget Allocation

Besides monetary values, various operational, logistical, and other military values of interest can be developed and utilized in the proposed model, as explained in this report. Below are a few examples of alternative value metrics that are taken directly from *Risk-Based ROI, Capital Budgeting, and Portfolio Optimization in the Department of Defense* by Dr. Jonathan Mun (2020, p. 84-85). These metrics can be employed in future studies with real data gathered. They can be utilized independently or combined into a composite weighted metric that can be used as $n_{i,j}$.

- **Inherent Availability (IA).** Measures operational percentage in an ideal support environment per design specifications. Mean Time Between Failure (MTBF) and Mean Time to Repair (MTTR) are used in this calculation.
- **Effective Availability (EA).** Probability a ship's system is available at any instant during the maximum operational period, accounting for all critical failures, repairable and nonrepairable at sea, and preventive maintenance. Additional metrics here are Mean Down Time (MDT), Maintenance Time (MT) and Mean Time to Failure (MTTF).
- **Mission Reliability (MR).** **Operational Ready Rate (ORR)** at the start of a mission compared to its **Inherent Reliability (IR).**
- **Operational Dependability (OD).** Probability a system can be used to perform a specified mission when desired.
- Mean Down Time (MDT), Mean Maintenance Time (MMT), Logistics Delay Time (LDT), and their combinations.
- Achieved Availability (AA), Operational Availability (OA), and Mission Availability (MA)
- **Cost Deterrence and Avoidance.** Soft or shadow revenue (cost savings) over the economic and operational life of the program or system. Milestones A, B, C.
- Traditional Financial Metrics. **Net Present Value (NPV)**, **Internal Rate of Return (IRR)**, **Return on Investment (ROI)**, and other metrics, as long as there are financial and monetary values.
- **Budget Constraint.** FY Budget limitations and probabilities of budgetary overruns.
- **Total Ownership Cost (TOC)** and **Total Lifecycle Cost (TLC).** Accounting for the cost of developing, producing, deploying, maintaining, operating, and disposing of a system over its entire lifespan. Uses **Work Breakout Structures (WBS)**, **Cost Estimating Categories (CEC)**, and **Cost Element Structures (CES).**
- Knowledge Value Added (KVA). Monetizing Learning Time, Number of Times Executed, Automation, Training Time, and Knowledge Content.
 - **Strategic and Capability.** Multiple value metrics can be determined by Subject Matter Experts (SME) including **Expected Military Value**, **Strategic Value**, **Future Weapon Strategy**, and others
- **Capability Measures (CM).** Difficult to quantify and needs SME judgment:
 - Innovation Index, Conversion Capability, Ability to Meet Future Threats
 - Force Structure (size/units), Modernization (technical sophistication), Combat Readiness, Sustainability
 - Future Readiness (ability to meet evolving threats, ability to integrate future weapons systems)
- **Domain Capabilities (DC)**
 - Portfolios are divided into different domains, and each domain is optimized separately and then combined at the enterprise level and reoptimized; example domains include Coastal Defense, Anti-Air Surface Warfare, Anti-Surface Warfare, Anti-Submarine Warfare, Naval Strike, Multi-Mission Air Control, Sea Control, Deep Strike, Missile Defense, and so on.

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IV. REFERENCES

- Afsordegan, A., Sánchez, M., Agell, N., Zahedi, S., & Cremades, L. (2016). Decision making under uncertainty using a qualitative TOPSIS method for selecting sustainable energy alternatives. *International Journal of Environmental Science and Technology*, 13(6), 1419–1432.
- Allen, F., & Morris, S. (2014). Game theory models in finance. In K. Chatterjee and W. Samuelson (eds.), *Game theory and business applications* (pp. 17-41). Springer, Boston, MA. https://d1wqtxts1xzle7.cloudfront.net/50121826/game_theory_in_finance-with-cover-page-v2.pdf?Expires=1666116626&Signature=ITAQKm~PZBjHoqZGj077vNRSTKMbWAIaPBrw4uf4QiUHyYhi9VibGonCAjGLjIefXezs6rwlUreWl16lK71ZLn8aSFBIYA9jVbwsLNGCe3htm~5zzxjlW6vxxvU6VbK-0bcNo~mw9Uedxby-V~IwOkwcSc9unNQVhF1ucAlPacTuSJk6~58KJvJbYTHxoRFv60-K82t2UuC0pT~J6pAaigEKQ9y08Wl3Xwy7F24Glvj5O62khL8HyDZhCgqexLnFTHuyW-2Ig~LckHduZMzLTRgIwJVvODOw6qPzG1MrgIUyVP630TtLGci6pS0McFoflAVoTKvAGeqfUHi6YdEPEg__&Key-Pair-Id=APKAJLOHF5GGSLRBV4ZA
- Al-Sharrah, G. (2010). Ranking using the Copeland Score: A comparison with the Hasse Diagram. *Journal of Chemical Information and Modeling*, 50, 785–791.
- Angelou, G., & Economides, A. (2008). A real options approach for prioritizing ICT business alternatives: A case study from broadband technology business field. *Journal of the Operational Research Society*, 59(10), 1340–1351.
- Anton, A. V. (2019). *Choice of discount rate and agency cost minimization in capital budgeting: analytical review and modeling approaches* (Doctoral dissertation, Victoria University). https://vuir.vu.edu.au/40447/1/ANTON%20Arun-thesis_nosignature.pdf
- Araújo, M. (2015). Economics of telecommunications: capital budgeting under competitive markets using strategic game theory and real options approach. <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.306.9046&rep=rep1&type=pdf>
- Asbeck, E., & Haimes, Y. Y. (1984). The partitioned multiobjective risk method. *Large Scale Systems*, 6(1), 13–38.
- Association of the United States Army. (2016, September 15). Army review to rank 780 programs by priority. Retrieved from <https://www.ausa.org/news/army-review-rank-780-programs-priority>
- Barrett, G. F., & Donald, S. G. (2003). Consistent tests for stochastic dominance. *Econometrica*, 71(1), 71–104.
- Beaujon, G. J., Marin, S. P., & McDonald, G. C. (2001). Balancing and optimizing a portfolio of R&D projects. *Naval Research Logistics*, 48(1), 18–40.

- Behzadian, M., Kazemzadeh, R. B., Albadvi, A., & Aghdasi, M. (2010). PROMETHEE: A comprehensive literature review on methodologies and applications. *European Journal of Operational Research*, 200(1), 198–215.
- Bennett, S. (2017, July). Optimization aids in the quest for government efficiency. Retrieved from <https://federalnewsradio.com/commentary-analysis/2017/07/optimization-aids-in-the-quest-for-government-efficiency/>
- Bennion, E. G. (1956). Capital-Budgeting and Game-Theory, *Harvard Business Review*, 34(6), 115–123.
- Bodie, Z., Kane, A., & Marcus, A. J. (2009). *Investments* (8th ed.). New York, NY: McGraw-Hill.
- Boquist, J. A., Milbourn, T. T., & Thakor, A. V. (1998). How do you win the capital allocation game? *MIT Sloan Management Review*, 39(2), 59.
<http://apps.olin.wustl.edu/faculty/milbourn/MilbournSloanMgtRev.pdf>
- Botkin, B. (2007, June). *Applying financial portfolio analysis to government program portfolios* (Master's thesis, Naval Postgraduate School). Retrieved from <https://calhoun.nps.edu/handle/10945/3481>
- Boucher, C. M., Danielsson, J., Kouontchou, P. S., & Maillet, B. B. (2014). Risk models-at-risk. *Journal of Banking and Finance*, 44, 72–92.
- Bouyssou, D., Marchant, T., Pirlot, M., Tsoukias, A., & Vincke, P. (2006). *Evaluation and decision models with multiple criteria* (Operational Research and Management Sciences). New York, NY: Springer's International Series.
- BP. (2015). *Sustainability report 2015*. Retrieved February 15, 2017, from <https://www.bp.com/content/dam/bp/pdf/sustainability/group-reports/bp-sustainability-report-2015.pdf>
- Brans, J. P., & Vincke, P. H. (1985). A preference ranking organization method: The PROMETHEE method for multiple criteria decision making. *Management Science*, 31(6), 647–656.
- Brans, J.-P., & Mareschal, B. (2005). Multicriteria decision aid. The PROMETHEE-GAIA solution in multiple criteria decision analysis: State of the art surveys. *International Series in Operations Research & Management Science*, 78, 163–186.
- Brito, A. J., de Almeida, A. T., & Mota, C. M. M. (2010). A multicriteria model for risk sorting of natural gas pipelines based on ELECTRE TRI integrating utility theory. *European Journal of Operational Research*, 200(3), 812–821.
- Brown, B., & Anthony, S. (2011, June). How P&G tripled its innovation success rate. *Harvard Business Review*, 89(6), 64–72.
- Bruggemann, R., & Carlsen, L. (2011). An improved estimation of averaged ranks of partial orders. *Match Communications in Mathematical and in Computer Chemistry*, 65, 383–414.

- Bruggemann, R., & Patil, G. P. (2011). *Ranking and prioritization for multi-indicator systems: Introduction to partial order applications*. New York, NY: Springer.
- Bruggemann, R., Bücherl, C., Pudenz, S., & Steinberg, C. (1999). Application of the concept of partial order on comparative evaluation of environmental chemicals. *Acta hydrochimica et hydrobiologica*, 27, 170–178.
- Bruggemann, R., Schwaiger, J., & Negele, R. D. (1995). Applying Hasse diagram technique for the evaluation of toxicological fish tests. *Chemosphere*, 30, 1767–1780.
- Bruggemann, R., Sorensen, P., Lerche, D. & Carlsen, L. (2004). Estimation of averaged ranks by a local partial order model. *Journal of Chemical Information and Computer Sciences*, 44, 618–625.
- Burk, R. C., & Parnell, G. S. (2011). Portfolio decision analysis: Lessons from military applications. In A. Salo, J. Keisler, & A. Morton (Eds.), *Portfolio decision analysis* (pp. 333–357). New York, NY: Springer.
- Cavallaro, F. (2005) An integrated multi-criteria system to assess sustainable energy options: An application of the Promethee method. *European Journal of Operational Research*, 182(2), 844–855.
- Chong, Y. Y. (2004). *Investment risk management*. Oxford, UK: John Wiley & Sons.
- Clinger–Cohen Act of 1996, 40 U.S.C. § 1401 (1996).
- Cliville, V., Berrah, L., & Mauris, G. (2007). Quantitative expression and aggregation of performance measurements based on the MACBETH multi-criteria method. *International Journal of Production Economics*, 105(1), 171–189.
- Corrente, S., Figueira, J. R., & Greco, S. (2014). The SMAA-PROMETHEE method. *European Journal of Operational Research*, 239, 514–522.
- Costa, C., De Corte, J. M., & Vansnick, J. C. (2012). MACBETH. *International Journal of Information Technology & Decision Making*, 11(2), 359–387.
- Davendralingam, N., & DeLaurentis, D. (2015, May). A robust portfolio optimization approach to system of system architectures. *Systems Engineering*, 18(3), 269–283.
- De Loof, K., De Baets, B., & De Meyer, H. (2011). Approximation of average ranks in posets. *Match Communications in Mathematical and in Computer Chemistry*, 66, 219–229.
- Denault (2001, Jan.). Coherent allocation of risk capital. *Journal of Risk*, 4, 1–34.
<https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.72.4317&rep=rep1&type=pdf>
- Department of Defense (DoD). (2005, October 10). *Information technology portfolio management* (DOD Directive 8115.01). Retrieved from
<http://www.dtic.mil/whs/directives/corres/pdf/811501p.pdf>
- Department of Defense (DoD). (2006). (DOD Directive 8115.bb).

- Department of Defense (DoD). (2017, June 9). *DOD space enterprise governance and principal DOD space advisor (PDSA)* (DOD Directive 5100.96). Retrieved from https://fas.org/irp/doddir/dod/d5100_96.pdf
- Desai, S., Bidanda, B., & Lovell, M. R. (2012). Material and process selection in product design using decision-making technique (AHP). *European Journal of Industrial Engineering*, 6(3), 322–346.
- Dickinson, M. W., Thornton, A. C., & Graves, S. (2001, November). Technology portfolio management: Optimizing interdependent projects over multiple time periods. *IEEE Transactions on Engineering Management*, 48(4), 518–527.
- Dorini G., Kapelan, Z., & Azapagic, A. (2011). Managing uncertainty in multiple-criteria decision making related to sustainability assessment. *Clean Technologies and Environmental Policy*, 13, 133–139.
- Dunlop, J. (2004). Modern portfolio theory meets wind farms. *The Journal of Private Equity*, 7(2), 83.
- ExxonMobil. (2015). *2015 summary annual report*. Retrieved from http://cdn.exxonmobil.com/~media/global/files/summary-annual-report/2015_summary_annual_report.pdf
- Fabozzi, F. J. (2010). *An introduction to financial markets, business finance, and portfolio management*. Hoboken, NJ: John Wiley & Sons.
- Ferreira, N., Kar, J., & Trigeorgis, L. (2009). Option Games: The Key to Competing in Capital-Intensive Industries. *Harvard Business Review*, 87(3), 101-107. <https://hbr.org/2009/03/option-games-the-key-to-competing-in-capital-intensive-industries>
- Flynn, B., & Field, J. (2006, January 1). Transformation of analytical tools: Using portfolio analysis techniques in defense applications. *Armed Forces Comptroller*. Retrieved from <https://www.thefreelibrary.com/Transformation+of+analytical+tools%3A+using+portfolio+analysis+...-a0145158636>
- General Accounting Office (GAO). (1997, February). *Assessing risk and returns: A guide for evaluating federal agencies' IT investment decision-making* (GAO/AIMD-10.1.13). Retrieved from <http://www.gao.gov/special.pubs/ai10113.pdf>
- George A. Akerlof, A. Michael Spence, and Joseph E. Stiglitz (2001). “For their analyses of markets with asymmetric information.”
- Georgopoulou, E., Lalas, D., & Papagiannakis, L. (1997). A multicriteria decision aid approach for energy planning problems: The case of renewable energy option. *European Journal of Operational Research*, 103(1), 38–54.
- Ghafghazi, S., Sowlati, T., Sokhansanj, S., & Melin, S. (2010). A multicriteria approach to evaluate district heating system options. *Applied Energy*, 87(4), 1134–1140.
- Gibbons, R. S. (1992). *Game theory for applied economists*. Princeton University Press. https://dlwqtxts1xzle7.cloudfront.net/57034411/Robert_Gibbons-

Game_Theory_for_Applied_Economists-Princeton_University_Press_1992-with-cover-page-v2.pdf?Expires=1664119234&Signature=SIPVSorJwkzb94IFFav81AodHAoATWHj6VxXXy7kePnvV3-YoOArbScVVFq5pO5V8eRK5MXegxzgFB5kuLkER-2GmAZ6FWgXYnr9EI7Y0WaHFcU7zL~G7IRNWJjlW2nylKqtkMrTOXWVufjryMYMnoDQdf3Wa5fPksjRBFg0Ubck2BNv6A-JFy2KLDa4CCgbYE47WXgu2Pg8ZN3JWu~bQHYZ9fR~-14pgiN1LQj-AkAYrxa2dwYGTRPRHUrWkj6uWvgDB8sAIpwPOoctvBQ997bDKRN2z4fbol32DZ6ag7GVH0H9-QBko7maaP6PKUrJOXBvEi0lok5eFErK7epNg__&Key-Pair-Id=APKAJLOHF5GGSLRBV4ZA

- Girotra, K., Terwiesch, C., & Ulrich, K. T. (2007). Valuing R&D projects in a portfolio: Evidence from the pharmaceutical industry. *Management Science*, 53(9), 1452–1466.
- Goumas, M., & Lygerou, V. (2000). An extension of the PROMETHEE method for decision making in fuzzy environment: Ranking of alternative energy exploitation projects. *European Journal of Operational Research*, 123(3), 606–613.
- Government Accountability Office (GAO). (2007). *Best practices: An integrated portfolio management approach to weapon system investments could improve DOD's acquisition outcomes* (GAO-07-388). Washington, DC: Author.
- Greiner, M. A., McNutt, R. T., Shunk, D. L., & Fowler, J. W. (2001). Selecting military weapon systems development portfolios: Challenges in value measurement. In *Proceedings of the Portland International Conference on Management of Engineering and Technology, 2001 (PICMET '01)* (pp. 403–410). doi:10.1109/PICMET.2001.952153
- Grenadier, S.R. (2002). “Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms,” *Review of Financial Studies* 15, 691-721
- Gurgur, C., & Morley, C. T. (2008, July–August). Lockheed Martin Space Systems Company optimizes infrastructure project portfolio selection. *Interfaces*, 38(4), 251–262.
- Haimes, Y. Y. (2009). *Risk modeling, assessment, and management* (3rd ed.). Hoboken, NJ: John Wiley & Sons.
- Haq, R., Gandhi, A., & Bahl, S. (2012, Fall). Advanced physical portfolio optimization: Improving margins in a tight market. *CROSSINGS: The Sapient Journal of Trading & Risk Management*, 42–47. Retrieved from http://www.sapient.com/content/dam/sapient/sapientglobalmarkets/pdf/thought-leadership/crossingsfall2012_advanced_physical_portfolio_optimization.pdf
- Hopkins, A. (2011). Risk-management and rule-compliance: Decision-making in hazardous industries. *Safety Science*, 49, 110–120.
- Huisman, K. J. M., Kort, P. M., Pawlina, G., & Thijssen, J. (2003). Strategic Investment Under Uncertainty: Merging Real Options with Game Theory. *SSRN Electronic Journal*. <https://doi.org/10.2139/ssrn.545543>

- Hyde, K. M., & Maier, H. R. (2006). Distance-based and stochastic uncertainty analysis for multi-criteria decision analysis in Excel using Visual Basic for Applications. *Environmental Modelling & Software*, 21, 1695–1710.
- Hyde, K. M., Maier, H. R., & Colby, C. B. (2004). Reliability-based approach to multicriteria decision analysis for water resources. *Journal of Water Resource Planning Management*, 130(6), 429–438.
- James A. Mirrlees and William Vickrey (1996). “For their fundamental contributions to the economic theory of incentives under asymmetric information.”
- Janiga, M., & Modigliani, P. (2014, November–December). Think portfolios, not programs. *Defense AT&L Magazine*, 12–16. Retrieved from www.dtic.mil/get-tr-doc/pdf?AD=ADA612303
- Jiang, Z., & Gong, X. (2019). Research on Issues of Budget Performance Management on the Process of Budgeting by Game Theory. *Journal of Financial Risk Management*, 8(04), 193. https://www.scirp.org/pdf/jfrm_2019111915105540.pdf
- Jocic, L., & Gee, J. (2013, May). Developing planning and decision support. *Crosslink*. Retrieved from <http://www.aerospace.org/crosslinkmag/spring2013/development-planning-and-decision-support/>
- Johannessen, S. (2015). *Portfolio optimization of wind power projects* (Master’s thesis). Retrieved from <https://brage.bibsys.no/xmlui/handle/11250/2352877>
- Jorion, P. (2007). *Value at risk: The new benchmark for managing financial risk* (3rd ed.). New York, NY: McGraw-Hill.
- Kaya, T., & Kahraman, C. (2011). Multicriteria decision making in energy planning using a modified fuzzy TOPSIS methodology. *Expert Systems with Applications*, 38(6), 6577–6585.
- Kengatharan, L. (2016). Capital budgeting theory and practice: a review and agenda for future research. *Research Journal of Finance and Accounting*, 7(1). <https://core.ac.uk/download/pdf/234631219.pdf>
- Lazar, A., Bryant, J. A., Baynes, M. R., & Dissinger, R. L. (2011, September 8). Management discussion section. In *Barclays Capital Back to School Consumer Conference* [Transcript]. Retrieved from <http://investor.kelloggs.com/~media/Files/K/Kellogg-IR/reports-and-presentations/2011/Transcript-K-BTSconf-09-08-11.pdf>
- Leonid Hurwicz , Eric S. Maskin , and Roger B. Myerson (2007). “For having laid the foundations of mechanism design theory.”
- Levy, H. (2006). *Stochastic dominance: Investment decision making under uncertainty*. Berlin, Germany: Springer Science+Business Media.
- Li, Y. (2014). Value of more sophistication: capital investment decision-making with competitive dynamics in the mining industry. [MS thesis, MIT] <https://www.semanticscholar.org/paper/Value-of-more-sophistication-%3A-capital-investment-Li/42dd25b0b1b3154baac5bc63c0deee90179ab143>

- Mansini, R., Ogryczak, W., & Speranza, M. G. (2007). Conditional value at risk and related linear programming models for portfolio optimization. *Annals of Operations Research*, 152(1), 227–256.
- Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance*, 77–91.
- Matos, M. A. (2007). Decision under risk as a multicriteria problem. *European Journal of Operational Research*, 181(3), 1516–1529.
- Mun, J. (2015). *Modeling risk: Applying Monte Carlo risk simulation, strategic real options, stochastic forecasting, portfolio optimization, data analytics, business intelligence, and decision modeling* (3rd ed.). CA: Thomson-Shore and ROV Press.
- Mun, J. (2016). *Real options analysis: Tools and techniques for valuing strategic investments and decisions with integrated risk management and advanced quantitative decision analytics* (3rd ed.). CA: Thomson-Shore and ROV Press.
- Mun, J. (2022). Capuring Risk in Capital Budgeting. *Naval Research Program*. Retrieved from: <https://apps.dtic.mil/sti/trecms/pdf/AD1184536.pdf>
- Mun, J., Ford, D., & Housel, T. (2012, October 10). *Naval ship maintenance: An analysis of the Dutch shipbuilding industry using the Knowledge Value Added, Systems Dynamics, and Integrated Risk Management methodologies* (NPS-AM-12-204). Retrieved from <https://calhoun.nps.edu/bitstream/handle/10945/33851/NPS-AM-12-204.pdf?sequence=1>
- Mun, J., Hernandez, E., & Rocco, C. (2016). A combined lexicographic average rank approach for evaluating uncertain multi-indicator matrices with risk metrics. In M. Fattore & R. Brüggemann (Eds.), *Partial order concepts in applied sciences*. Retrieved from <http://www.springer.com/la/book/9783319454191>. eBook ISBN (978-3319454214)
- Mun, J., Hernandez, E., & Rocco, C. (2017). Active management in state-owned energy companies: Using real options and multicriteria analysis to make companies sustainable. *Applied Energy*, 195, 487–502. <https://doi.org/10.1016/j.apenergy.2017.03.068>
- Murto, P., & Keppo, J. (2002). A game model of irreversible investment under uncertainty. *International Game Theory Review*, 4(02), 127–140. <http://lib.tkk.fi/Diss/2003/isbn9512264455/article3.pdf>
- Myerson, R. (1979) “Incentive Compatibility and the Bargaining Problem.” *Econometrica*, 47(1), p. 61–73, January 1979.
- Myerson, R. (1981). “Optimal Auction Design,” *Mathematics of Operations Research*, 6(1), 58–73.
- Ng, K. S. (1968). *The relevance of game theory in its application to decision making in competitive business situations* (Doctoral dissertation, University of British Columbia). <https://open.library.ubc.ca/soa/cIRcle/collections/ubctheses/831/items/1.0102410>
- Nichols, N. A. (1994). Scientific management at Merck: An interview with CFO Judy Lewent. *Harvard Business Review*, 72(1), 88–99. <https://hbr.org/1994/01/scientific-management-at-merck-an-interview-with-cfo-judy-lewent>

- Pindyck, R. S., and Rubinfeld, D. L. (1995). *Microeconomics*, Third Edition. Prentice-Hall.
- Pohekar, S., & Ramachandran, M. (2004). Application of multi-criteria decision making to sustainable energy planning—A review. *Renewable and Sustainable Energy Reviews*, 8(4), 365–381.
- Pulido, F. J., Mandow, L., & de la Cruz, J. L. P. (2014). Multiobjective shortest path problems with lexicographic goal-based preferences. *European Journal of Operational Research*, 239(1), 89–101.
- Restrepo, G., Brüggemann, R., Weckert, M., Gerstmann, S., & Frank, H. (2008). Ranking patterns, an application to refrigerants. *Match Communications in Mathematical and in Computer Chemistry*, 59, 555–584.
- Rocco, C. M., & Tarantola, S. (2014). Evaluating ranking robustness in multi-indicator uncertain matrices: An application based on simulation and global sensitivity analysis. In R. Brüggemann, L. Carlsen, & J. Wittmann (Eds.), *Multi-indicator systems and modeling in partial order* (pp. 275–292). : Springer.
- Rockafellar, R. T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of Risk*, 2, 21–41.
- Roy, B. (1968). Classement et choix en presence de points devue multiples (la methode ELECTRE). *Revue d'Informatique et de recherché opérationelle*, 6(8), 57–75.
- Roy, B. (1996). *Multicriteria methodology for decision aiding*. Berlin, Germany: Springer Science+Business Media.
- Saaty, T. (2013). The modern science of multicriteria decision making and its practical applications: The AHP/ANP approach. *Operations Research*, 61(5), 1101–1118.
- Saban, D., & Sethuraman, J. (2014). A note on object allocation under lexicographic preferences. *Journal of Mathematical Economics*, 50, 283–289.
- Sakthivel, G., Ilangkumaran, M., Nagarajan, G., & Shanmugam, P. (2013). Selection of best biodiesel blend for IC engines: An integrated approach with FAHP-TOPSIS and FAHP-VIKOR. *International Journal of Oil Gas and Coal Technology*, 6(5), 581–612.
- Scarlat, E., Chirita, N., & Bradea, I. A. (2012). Indicators and metrics used in enterprise risk management (ERM). *Economic Computation and Economic Cybernetics Studies and Research*, 46(4), 5–18.
- Setter, O., & Tishler, A. (2007). Theory and application to the U.S. military. *Defense and Peace Economics*, 18(2), 133–155.
- Shapiro, A. 2011. Budget Negotiations: A Study of Game Theory
<https://www.npr.org/2011/04/06/135186332/budget-negotiations-a-study-of-game-theory>
- Sidiropoulos, L., Sidiropoulou, A., & Lalagas, S. (2014). Defense portfolio analysis. *Journal of Computations & Modelling*, 4(1), 327–347.

- Sikaló, M., Arnaut-Berilo, A., & Zaimovic, A. (2022). Efficient Asset Allocation: Application of Game Theory-Based Model for Superior Performance. *International Journal of Financial Studies*, 10(1), 20. <https://www.mdpi.com/2227-7072/10/1/20>
- Silbergliitt, R., Sherry, L., Wong, C., Tseng, M., Ettegui, E., Watts, A., & Stothard, G. (2004). *Portfolio analysis and management for naval research and development*. Retrieved from https://www.rand.org/content/dam/rand/pubs/monographs/2004/RAND_MG271.pdf
- Siziba, S., & Hall, J. H. (2021). The evolution of the application of capital budgeting techniques in enterprises. https://repository.up.ac.za/bitstream/handle/2263/74586/Siziba_Evolution_2021.pdf?sequence=1&isAllowed=y
- Smit, H.T.J. (2003). Winter. Infrastructure investment as a real options game: the case of European airport expansion. *Finance. Manage.* 32 (4), 27–57.
- Szolgayová, J., Fuss, S., Khabarov, N., & Obersteiner, M. (2011). A dynamic CVaR-portfolio approach using real options: An application to energy investments. *European Transactions on Electrical Power*, 21(6), 1825–1841.
- Tarantino, A. (2008). *Governance, risk, and compliance handbook: Technology, finance, environmental, and international guidance and best practices*. Hoboken, NJ: John Wiley & Sons.
- Tavana, M., Behzadian, M., Pirdashti, M., & Pirdashti, H. (2013). A PROMETHEE-GDSS for oil and gas pipeline planning in the Caspian Sea basin. *Energy Economics*, 36, 716–728.
- Thakor, A. V. (1991). Game theory in finance. *Financial Management*, 71-94. <https://www.jstor.org/stable/3666098>
- The President's Budget FY2022. Accessed: <https://www.navy.mil/Resources/Blogs/Detail/Article/2638885/department-of-the-navy-fy-2022-presidents-budget/>
- Thijssen, J. J. J., Huisman, K. J. M., & Kort, P. M. (2004). The effect of information streams on capital budgeting decisions. *European Journal of Operational Research*, 157(3), 759-774. <https://pure.uvt.nl/ws/portalfiles/portal/608157/EJOR.pdf>
- Tian, G. (2013). *Lecture Notes on Microeconomic Theory*. Texas A&M University. <http://econweb.tamu.edu/tian/micro1.pdf>
- Tolga, A. C. (2011). Fuzzy multi-criteria method for revaluation of ERP system choices using real options. In *Proceedings of the World Congress on Engineering 2011* (Vol. 2). London, England: International Association of Engineers.
- Tolga, A. C. (2012) A real options approach for software development projects using fuzzy Electre. *Journal of Multiple-Valued Logic and Soft Computing*, 18(5–6), 541–560.
- Tolga, A. C., & Kahraman, C. (2008). Fuzzy multiattribute evaluation of R&D projects using a real options valuation model. *International Journal of Intelligent Systems*, 23(11), 1153–1176.

- Vascik, P., Ross, A., & Rhodes, D. (2015). *A method for exploring program and portfolio affordability tradeoffs under uncertainty using Epoch-Era Analysis: A case application to carrier strike group design*. Paper presented at 12th Annual Acquisition Research Symposium, Naval Postgraduate School, Monterey, CA.
- Verbeeten, F. H. (2006). Do organizations adopt sophisticated capital budgeting practices to deal with uncertainty in investment decision?: A research note. *Management accounting research*, 17(1), 106-120. https://www.researchgate.net/profile/Frank-Verbeeten/publication/222427649_Do_organizations_adopt_sophisticated_capital_budgeting_practices_to_deal_with_uncertainty_in_the_investment_decision_A_research_note/inks/5a6ed5f0458515d4075893b2/Do-organizations-adopt-sophisticated-capital-budgeting-practices-to-deal-with-uncertainty-in-the-investment-decision-A-research-note.pdf
- Von Neumann, J., & Morgenstern, O. (1944). *Theory of games and economic behavior*. Princeton, NJ: Princeton University Press.
- Weber, C. A., Balut, S. J., Cloos, J. J., Frazier, T. P., Hiller, J. R., Hunter, D. E., . . . Tran, D. (2003, October). *The acquisition portfolio schedule costing/optimization model: A tool for analyzing the RDT&E and production schedules of DoD ACAT I systems* (IDA Document D-2835). Alexandria, VA: Institute for Defense Analyses. Retrieved from www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA421123
- Weitzman, M.L. (1980). "The 'Ratchet Principle' and Performance Incentives." *Bell Journal of Economics*, vol. 11, pp. 116-123.
- Wismeth, J. (2012, March 9). *Improving Army information technology asset visibility*. Carlisle, PA: Army War College, Strategic Studies Institute. Retrieved from www.dtic.mil/get-tr-doc/pdf?AD=ADA562133
- Yaman, F., Walsh, T. J., Littman, M. L., & Desjardins, M. (2011). Democratic approximation of lexicographic preference models. *Artificial Intelligence*, 175(78), 1290–1307.
- Yang, S. C., Lin, T. L., Chang, T. J., & Chang, K. J. (2011, March). A semi-variance portfolio selection model for military investment assets. *Expert Systems with Applications*, 38(3), 2292–2301.
- Yang, S., Yang, M., Wang, S., & Huang, K. (2016, September). Adaptive immune genetic algorithm for weapon system portfolio optimization in the military big data environment. *Cluster Computing*, 19(3), 1359–1372. doi:10.1007/s10586-016-0596-3
- Young (1985). "Participative Budgeting: The Effects of Risk Aversion and Asymmetric Information on Budgetary Slack." *Journal of Accounting Research*, vol. 23(2), 1985, pp. 829-842.
- Young and B. Lewis (1995). "Experimental Incentive-Contracting Research in Management Accounting." *Judgment and Decision-Making Research in Accounting and Auditing*, edited by R. H. Ashton, and A. A. Ashton, 55-75. Cambridge, U.K.: Cambridge University Press.

- Yu, O.-Y., Guikema, S. D., Briaud, J.-L., & Burnett, D. (2012). Sensitivity analysis for multi-attribute system selection problems in onshore environmentally friendly drilling (EFD). *Systems Engineering*, 15(2), 153–171.
- Zacks Equity Research. (2015, January 27). DuPont's (DD) Q4 earnings in line, revenues miss. Retrieved from <http://www.talkmarkets.com/content/us-markets/duponts-dd-q4-earnings-in-line-revenues-miss?post=57461>
- Zandi, F., & Tavana, M. (2010). A hybrid fuzzy real options analysis and group ordinal approach for knowledge management strategy assessment. *Knowledge Management Research & Practice*, 8(3), 216–228.
- Zanoli, R., Gambelli, D., Solfanelli, F., & Padel, S. (2014). Assessing the risk of non-compliance in UK organic agriculture. *British Food Journal*, 116(8), 1369–1382.
- Zhang, P., Peeta, S., & Friesz, T. (2005). Dynamic game theoretic model of multi-layer infrastructure networks. *Networks and Spatial Economics*, 5(2), 147-178.
https://d1wqtxts1xzle7.cloudfront.net/46543536/Dynamic_game_theoretic_model_of_multi-la20160616-8279-1pp45yd-with-cover-page-v2.pdf?Expires=1664032430&Signature=Ajgm-Ad0h6bHeo8~vTBSv4jDP1sngqv5dhj~1MdX93JQPM4Obf3EY9WRVYaQDNeOAwe3hqXQRQazEjVQuvhtjMnxy8nI~X8ySywSryGGhEEjQ7LwomZchk7FwESQ5N7Lceee5zWOEzcs3DAiz6D0XReGZooTwQ3HjHbIRAD~afOQvkAcUPpiQfgpvGuqZi6QlSID7V~t2zxnfQMrj1rc6vrXfgK78KJkr97bPINAmXqPkYoNsaMViYMs5wuIOJap5t9y-NXfLgxb9TR7snCWdXbtXUfgH4uawJCp8FOJe2XqkH6EDSC1U459i4OgVNYduGONWrKYD-DoPv6Vx-W~Q__&Key-Pair-Id=APKAJLOHF5GGSLRBV4ZA
- Zhu, K., & Weyant, J.P. (2003). Strategic decisions of new technology adoption under asymmetric information: a game-theoretic model. *Decision Sci.* 34 (4), 643–675.