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Toward Modeling of Coulomb Explosions of an Elastic Plate

by Stephan Bilyk and Michael Grinfeld

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14. ABSTRACT
Under the action of electromagnetic irradiation or high inertial forces accompanying some ballistic phenomena, the solids are able to accumulate net electric charges. These charges can, in turn, generate sufficiently high macroscopic forces, leading to damage and even explosions of macroscopic and nanoscale aggregates. Corresponding phenomena were coined as the Coulomb explosions. We suggest a quantitative model of the stresses, generated due to the Coulomb forces, and explicitly calculate stresses in the isotropic elastic plate in the framework of linear elasticity.

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1. Introduction

Under the action of electromagnetic irradiation or high inertial forces accompanying some ballistic phenomena, the solids are able to accumulate net electric charges. These charges can, in turn, generate sufficiently high macroscopic forces, leading to damage and even explosions of macroscopic and nanoscale aggregates. Corresponding phenomena were coined as the “Coulomb explosions.”

There are, at least, several groups of phenomena that are associated with the Coulomb explosions. One group concerns the behavior of solids under the action of high-energy laser irradiation of solids. The second group concerns macro-phenomena and even celestial phenomena with the sky bodies moving with supersonic speeds.

Brief sketches of the problems dealing with the modeling and experimental observations of the so-called “Coulomb explosion” can be found in the references of the recently published Technical Note “Toward Modeling of Coulomb Explosions” (Bilyk and Grinfeld 2022). Also, we can recommend the popular review article Marakhtanov and Marakhtanov (2002).

According to the current views, Coulomb explosions are the result of interaction between positively charged ionic lattices and the negatively charged subsystem of electrons. The modern modeling of such phenomena relies on quantum mechanics. Fortunately, it allows also (macroscopic) interpretation in terms of classical physics (Marakhtanov and Marakhtanov 2002). Per Marakhtanov and Marakhtanov (2002), after losing sufficiently large amounts of electrons, the positively charged ions of the lattices experience very strong Coulomb repulsion. The distributed electronic gas, however, produces the screen for the ionic repulsion, thus supporting the macroscopic integrity of the combined system. However, if considerable amounts of electrons are eliminated from the system, the positively charged lattice subsystem experiences huge electrostatic forces that are able to destroy the integrity of the solids and causes an explosion. Also, there are conjectures that the Coulomb explosions happen when the falling metallic meteors hit rock-solid surfaces. The rocky part of the Earth stops the lattice-carrying part of the meteors, whereas elements of the electronic gas continue to move by inertia. As a result, the body of the meteor accumulates non-compensated positive charges, which are able to generate the Coulomb explosion. Thus, in addition to the original kinetics impulse, the meteor acts on the rocks by the additional impulse due to the Coulomb explosion.

Another example is delivered by the action of supersonic projectiles hitting the solid armor. In this example, the pulses due to the kinetic energy of the projectile comprise only a fraction of the total influence of the projectile—the remaining part is explained by the Coulomb explosion caused by the electrons leaving (due to inertia) the originally neutral projectile.

In our publication (Bilyk and Grinfeld 2022), we suggested one of the simplest continuum models that allows combining concepts of stresses and fracture with the concept of electrostatic forces. For that purpose, we introduced a model of an electrostatically charged bubble. That model recalls the classical liquid drop model of atomic nuclei and their fission. The suggested model, based on simple ideas of classical physics, was deliberately oversimplified just to get the most robust features of the Coulomb explosion, which includes the competition between electrostatic forces and the mechanical forces supporting the integrity of the substances. The mechanical forces are modelled as the surfaces tension.

In this report, we take the next step in our modeling efforts and model the solid as the elastic thick plate and use the linear elasticity model. When describing this model theoretically, we use the concept of the Aleph tensor, which is defined in a series of our publications (Grinfeld and Grinfeld 2020). The Aleph tensor repeats both the Maxwell tensor of electromagnetic field and the energy–momentum tensor, which are wildly used in the classical field theory.

2. The Continuum Model

The body is modelled as an isotropic elastic solid of a parallelepipedal shape in its undeformed configuration. We assume that under the action of electromagnetic irradiation or the inertia, part of the electrons leave their original positions in the lattice; the body, afterwards, carries a distributed electric charge with the density $q_+(z)$ per unit volume. For the sake of simplicity, we assume that the charge is distributed uniformly over the body of the plate. Also, for the sake of simplicity, we neglect the changes in the charge density due to the deformability of the plate.

The Aleph-tensor $\aleph^{ij}$ for the nonpolarizable elastic system reads (compare with Grinfeld and Grinfeld 2020)

$$\aleph^{ij} \equiv P^{ij} - \frac{1}{8\pi} E_k E^k Z^{ij} + \frac{1}{4\pi} E^i E^j \tag{1}$$

where P^{ij} is the Cauchy symmetric stress tensor and E_k is the electrostatic field.

In terms of the Aleph tensor, the exact bulk equation reads

$$\nabla_j \left(P^{ij} - \frac{1}{8\pi} E_k E^k Z^{ij} + \frac{1}{4\pi} E^i E^j \right) = 0 \tag{2}$$

The bulk equation (Eq. 2) can be rewritten as the following one:

$$\nabla_j P^{ij} = -\frac{1}{4\pi} E^i \nabla_j E^j \tag{3}$$

The exact boundary condition at the boundary of nonpolarizable substances reads

$$\left[P^{ij} - \frac{1}{8\pi} E_k E^k Z^{ij} + \frac{1}{4\pi} E^i E^j \right]_-^+ N_j = 0 \tag{4}$$

where N_j is the unit normal to the boundary.

Since the electric field is assumed continuous across the boundary of nonpolarizable substances, the condition (Eq. 4) reduces to the following one:

$$[P^{ij}]_-^+ N_j = 0 \tag{5}$$

We notice from the simple reduction from Eq. 4 to Eq. 5 that the bodies are either polarizable or charges can accumulate indefinitely at the interfaces (like, for example, in elementary theory of electrostatics).

The system (Eqs. 1–5) should be amended with the bulk equations of electrostatics for the electric potential φ and the electric field E_i:

$$E_i = -\nabla_i \varphi \tag{6}$$

$$\nabla_k E^k = -4\pi q_+ \tag{7}$$

Also, the bulk equations (Eqs. 6 and 7) should be amended with the boundary conditions

$$\left[\varphi \right]_-^+ = 0 \tag{8}$$

$$\left[\nabla_i \varphi \right]_-^+ N^i = 0 \tag{9}$$

3. Analysis of a Uniformly Charged Elastic Plate

Let us apply the master system (Eqs. 1–7) to analysis of a thick parallelepiped, occupying the domain $-H \leq z \leq H$ with the uniformly distributed charge; let $q_+^\circ = const$ be the charge density per unit spatial volume. In the 1D case, we get $\vec{E}(z) = E(z)\vec{N}$. Equation 6 implies the following 1D equation:

$$\frac{dE(z)}{dz} = -4\pi q_+^\circ \tag{10}$$

We consider solutions, which are symmetric with respect to the plane $z=0$; we get the solution:

$$E(z)=-4\pi q_+^\circ z \ \ for\ \left|z\right|\leq H \tag{11}$$

The continuity conditions imply:

$$E(z)=-4\pi q_+^\circ H \ \ for\ \left|z\right|\geq H \tag{12}$$

For the electrostatic potential, we get

$$\varphi(z)=2\pi q_+ z^2, \ \ for\ \left|z\right|\leq H\ , \tag{13}$$

The linear 1D Hooke's law reads

$$P=G\varepsilon=G\frac{dU}{dz} \tag{14}$$

where G is the effective elasticity model.

The 1D force equation reads

$$\frac{dP}{dz}=-4\pi q_+^2 z \tag{15}$$

The symmetric with respect to the plane $z=0$ solution of Eq. 15 reads

$$P(z)=-2\pi q_+^2 z^2+C \tag{16}$$

where C is a constant to be determined.

Combining Eqs. 14 and 16, we get

$$G\frac{dU}{dz}=-2\pi q_+^2 z^2+C \tag{17}$$

Equation 18, we find the following anti-symmetric solution of $U(z)$:

$$U(z)=-\frac{2\pi q_+^2}{3G}z^3+\frac{C}{G}z \tag{18}$$

The constant C can be found from the boundary conditions.

3.1 Plate with Fixed Endpoints

If the end points are fixed, we get $U(H)=0$, and Eq. 18 implies

$$-\frac{2\pi}{3}\frac{q_+^2}{G}H^3+\frac{C}{G}H=0 \tag{19}$$

and then,

$$C=\frac{2\pi}{3}q_+^2H^2 \tag{20}$$

Using Eq. 20, we can rewrite Eq. 18 as follows

$$U(z)=\frac{2\pi q_+^2H^3}{3G}\left(\frac{z}{H}-\frac{z^3}{H^3}\right) \tag{21}$$

as implied by the chain:

$$\begin{aligned}U(z)=-\frac{2\pi q_+^2}{3G}z^3+\frac{C}{G}z=&\\-\frac{2\pi q_+^2}{3G}z^3+\frac{2\pi}{3}\frac{q_+^2}{G}H^2z=\frac{2\pi q_+^2H^3}{3G}\left(\frac{z}{H}-\frac{z^3}{H^3}\right)\end{aligned} \tag{22}$$

In view of Eq. 21, we arrive at the following formulas of the deformation $\varepsilon(z)\equiv dU(z)/dz$:

$$\varepsilon(z)=\frac{2\pi}{3}\frac{q_+^2H^2}{G}\left(1-\frac{3z^2}{H^2}\right) \tag{23}$$

and the stress distribution $F(z)\equiv G\varepsilon(z)$

$$F(z)=\frac{2\pi}{3}q_+^2H^2\left(1-\frac{3z^2}{H^2}\right) \tag{24}$$

Equation 24 shows that the stress vanishes at

$$z_{neu}=\frac{\sqrt{3}}{3}H \tag{25}$$

3.2 Plate with Stress-Free Endpoints

In this case, the bulk equation read

$$\frac{dP}{dz}=-4\pi q_+^2z \tag{26}$$

Equation 26 implies

$$P(z) = -2\pi q_+^2 z^2 + D \tag{27}$$

where D is a certain constraint.

Since $P(z)$ vanishes at $z = \pm H$, we get

$$D = 2\pi q_+^2 H^2 \tag{28}$$

and, eventually, Eq. 27 reads

$$P(z) = 2\pi q_+^2 (H^2 - z^2) \tag{29}$$

Using Eq. 29, we get

$$\frac{dU}{dz} = \frac{2\pi q_+^2}{G}(H^2 - z^2) \tag{30}$$

Integrating Eq. 30, we get

$$U(z) = \frac{2\pi q_+^2 H^3}{G}\left(\frac{z}{H} - \frac{1}{3}\frac{z^3}{H^3}\right) \tag{31}$$

An analysis of a nonuniformly charged elastic plate is in the Appendix.

4. Discussion and Conclusion

In conclusion, we formulated a continuum model of plate with distributed electric charges. We analyzed a similar problem in Bilyk and Grinfeld (2022). The current model qualitatively differs from the model of Grinfeld and Grinfeld (2020). In the model of Bilyk and Grinfeld (2022), the charges distributed over the surface of the spherical ball, and the forces supporting the integrity of the ball are not the bulk forces but rather the surface-like forces. For the current model, the electric charged are distributed over the bulk of the ball, and the forces supporting integrity of the ball are rather the bulk forces than the surface-like forces. In principle, the suggested model can be applied to the elastic bodies of different shapes, but the exact analytical solutions are possible only for the bodies of simple geometries. In particular, they can be obtained for the plate-like bodies. These rather simple analytical solutions appear to be particularly useful for the purposes of validation and verification. Such explicit solutions have been for the cases of fixed boundaries and stress-free boundaries.

5. References

Bilyk S, Grinfeld M. Toward modeling of Coulomb explosions. DEVCOM Army Research Laboratory (US); 2022 June. Report No.: ARL-TN-1125.

Grinfeld M, Grinfeld P. Magneto-solid mechanics and the aleph tensor. Appl Math Phys. 2020;8(1):8–13.

Marakhtanov M, Marakhtanov A. Coulomb explosion of metal. Science and Life. 2002;4. http://www.nkj.ru/archive/articles/4072/.

Appendix. Analysis of a Nonuniformly Charged Elastic Plate

If q_+ is a function of z, we get

$$\frac{\partial^2\varphi}{\partial z^2} = 4\pi q_+(z) \tag{A-1}$$

In the symmetric case, Eq. A-1 implies

$$\frac{\partial\varphi(z)}{\partial z} - \frac{\partial\varphi(0)}{\partial z} = 4\pi\int_0^z d\xi q_+(\xi) \tag{A-2}$$

or

$$E(z) = -4\pi\int_0^z d\xi q_+(\xi) \tag{A-3}$$

and then Eq. A-3 implies

$$\frac{dP(z)}{dz} = 4\pi q_+(z)\int_0^z d\xi q_+(\xi) \tag{A-4}$$

Integrating Eq. A-4, we get

$$P(z) - P(0) = 4\pi\int_0^z d\eta q_+(\eta)\int_0^\eta d\xi q_+(\xi) \tag{A-5}$$

If the plate's edges are free of stress, that is, if $P(H) = 0$, Eq. A-5 implies

$$P(0) = -4\pi\int_0^H d\eta q_+(\eta)\int_0^\eta d\xi q_+(\xi) \tag{A-6}$$

Using Eq. A-6, we can rewrite Eq. A-5 as

$$P(z) = 4\pi\int_0^z d\eta q_+(\eta)\int_0^\eta d\xi q_+(\xi) + 4\pi\int_0^H d\eta q_+(\eta)\int_0^\eta d\xi q_+(\xi) \tag{A-7}$$

List of Symbols, Abbreviations, and Acronyms

1D	one-dimensional
ARL	Army Research Laboratory
DEVCOM	US Army Combat Capabilities Development Command

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