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INCREASE IN SENSITIVITY BY BASE CLIPPING

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INCREASE IN SENSITIVITY BY BASE CLIPPING

S. F. George and H. P. Birmingham

June, 18, 1951

DECLASSIFIED: By authority of - Group 4
Declassified After 12 years
Cite Authority Date 7-1963
E.R. Lauria 5340-A
Entered by NRL Code

Approved by:

Dr. R. M. Page, Superintendent, Radio Division III

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ABSTRACT

Theoretical and experimental research has been done on the use of base clipping of a signal confused by random noise to enhance the signal-to-noise ratio and hence to improve the detection probability. This investigation comprised a mathematical analysis of just how base clipping can be used to increase the sensitivity and an experimental verification based on both photographic and electronic methods. The theory and experiment agree quite closely and both indicate that an improvement of from 10 to 12 db in ultimate sensitivity can be achieved by base clipping.

PROBLEM STATUS

This work describes an exploratory research project which is completed by this report.

AUTHORIZATION

NRL Problem R99-01R
RDB NR 599-010

Manuscript submitted for publication: May 10, 1951

INCREASE IN SENSITIVITY BY BASE CLIPPING

THE THEORY OF BASE CLIPPING

To realize an improvement in sensitivity[1] by the use of base clipping[2] or biasing, an indicating device must be employed which can take advantage of the instantaneous values of the signal and noise rather than a long-time average. That the mean signal-to-noise ratio is unaffected by the bias voltage can easily be demonstrated as follows. Let the instantaneous voltage be of the form

$$V = A \cos \omega_s t + V_N - V_B , \qquad (1)$$

where A is the peak signal, $\omega_s = 2\pi f_s$ for a signal frequency of f_s, V_N is random noise, and V_B is the dc bias level. The mean square value of V is readily found to be

$$\overline{V^2} = A^2/2 + \overline{V_N^2} + V_B^2 . \qquad (2)$$

From Equation (2) the mean power signal-to-noise ratio is $A^2/2\,\overline{V_N^2}$ and hence is independent of V_B. The factor V_B^2, a constant dc bias on the measuring equipment, can be zeroed out and does not change the relationship between the original signal and noise.

One of the simplest methods of indication which takes advantage of instantaneous voltages uses the positive cross-over points of the combined signal and noise to form pulses which are then placed on a cathode-ray tube (crt) with a suitable time sweep. For the sake of mathematical expediency, let us consider the following method of presentation on a crt.[3] First, let each positive crossing of the adjustable bias voltage trigger a short pulse. The sequence of pulses will then be used to intensity-modulate the grid of the crt. Second, let the vertical sweep be a sawtooth synchronized in frequency with the input signal. Third, let the horizontal sweep be very slow relative to the vertical sweep. The result of such a method of indication will be a horizontal line on the crt, representing a signal.

The electronic method, described later, demonstrates experimentally the advantage of base clipping, and differs somewhat in the method of using the clipped voltage. Instead of generating constant amplitude pulses at the positive bias crossings, the instantaneous amplitudes of the signal plus noise are applied directly to the vertical deflection plates

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[1] The word sensitivity is used throughout this report to mean signal-to-noise discrimination.

[2] The original concept of improving the signal-to-noise ratio by base clipping was entered by R. M. Page in NRL Logbook No. 7768, p. 65, September 30, 1949.

[3] This method of presentation was first suggested by Mr. George F. Asbury of the Sound Division of the Naval Research Laboratory.

and the horizontal sweep is synchronized to the input signal frequency. It is believed that the two methods of presentation should differ only slightly in their ability to make use of the sensitivity improvement.

The pulses used to intensity-modulate the grid of the crt are generated at those times when the instantaneous input voltage exceeds the adjustable bias level V_B. If we let f_c represent the input-signal-carrier frequency, and V_N represent the random noise voltage in the bandwidth $\Delta f = f_b - f_a$ centered about f_c, f_b, and f_a being the upper and lower bandwidth extremities, then the instantaneous input voltage will be

$$V = A \cos 2\pi f_c t + V_N \quad . \tag{3}$$

What the problem essentially reduces to is that of determining just when and how frequently the voltage, V, exceeds the bias level, V_B. To understand this, it will be helpful to discuss briefly the two limiting cases: (1) pure signal, no noise, and (2) no signal, pure random noise.

In the case of pure signal and no noise, the crossings of the bias, V_B, by the instantaneous signal voltage, V, will be everywhere equidistant; i.e., they will be at the same point on the vertical sweep. Hence, the sequence of spots on the crt will form a horizontal line as swept by the slow horizontal sweep. Looking at this from the probability viewpoint, we might say that it is a degenerate case wherein the probability distribution of spots on the vertical sweep is a single line with zero dispersion.

In determining the crossings for the second case, that of pure random noise only, we must examine the situation more closely. If the noise were truly random, infinite bandpass, then the distance between crossings would also be random. This would mean that the spots would be completely scattered along the vertical sweep and show no tendency to center about a mean value. Hence, of course, no signal indication could be observed. This is the condition considered in this investigation—Δf must be sufficiently wide for this assumption to hold. It should be borne in mind, however, that for a very narrow band, high-Q filter, the noise tends to behave like the center frequency and a false target indication might result.

After examining the two limiting cases, we see that in the general case the probability distribution of crossings of V_B will vary from zero dispersion to a relatively high dispersion depending upon both the bias level and the input signal-to-noise ratio. The first step in the analysis will thus be to determine these probability distributions. Fortunately, the theory required has already been treated by S. O. Rice.[4] He has developed a formula for the probability that the instantaneous value of V will increase through V_B during the interval $(t, t + dt)$. If we let this probability be denoted by $P(V_B, t)\,dt$ then

$$P(V_B, t) = \pi N_0 \varphi(\gamma - r_i \cos \omega t) \left[-\frac{r_i'}{2} \sin \omega t + \varphi(r_i' \sin \omega t) + r_i' \sin \omega t \int_0^{r_i' \sin \omega t} \varphi(x)\, dx \right] \tag{4}$$

where N_0 is the average number of zero crossings per second of V_N alone, γ is the bias V_B as a multiple of the rms noise, r_i is the input signal-to-noise ratio defined as peak signal over rms noise, $\omega = 2\pi f_c$, and $r_i' = 2 r_i f_c / N_0$. The function $\varphi(x)$ represents the

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[4]Rice, S. O., "Statistical Properties of a Sine Wave Plus Random Noise," Bell Sys. Tech. J. 27: 109-157, January 1948

normal law function as usually defined. By the use of Equation (4) the probability density of crossing of V_B can be computed for various values of γ and r_i. Since the densities also depend upon N_0, this must be specified. It has been shown[5] that the expected number of crossings per second, N_0, is a function only of the power spectrum, $W(f)$, of the random noise V_N. For the well-behaved spectrum $W(f)$ found in practice, the number of zeros is given by

$$N_0 = 2\left[\frac{\int_0^\infty f^2 W(f)\,df}{\int_0^\infty W(f)\,df}\right]^{1/2} \tag{5}$$

For wide band applications, it will not be incorrect to assume an ideal rectangular band-pass filter of width $\Delta f = f_b - f_a$ centered about f_c. Equation (5) then reduces to

$$N_0 = 2\left[\frac{1}{3}\cdot\frac{f_b^3 - f_a^3}{f_b - f_a}\right]^{1/2} \tag{6}$$

since $W(f) = 1$ for $f_b \leq f \leq f_a$, and $W(f) = 0$ otherwise. It can readily be established that for bandwidths not exceeding 50 percent of the center frequency, N_0 reduces to essentially $2f_c$ and hence the r_i' of Equation (4) becomes simply r_i. This assumption of an ideal filter has been used in all of the computed curves. Mention will be made later of the effect of a Gaussian-type filter on the number of zeros and on the probability densities.

Now, by the use of Equations (4) and (6), we are able to plot the probability density curves as functions of the parameters γ and r_i. The distribution of the crossings at various points on the sawtooth reference or on $\sin \omega t$ may be obtained by plotting $P(V_B, t)$ against t or ωt. However, it is perhaps of more practical significance to plot $P(V_B, t)$ against vertical displacement on the crt, centered about the mean value line. Such plots are given in Figures 1 through 4 showing $P(V_B, t)$ for $\gamma = 0, 1, 2,$ and 4, and r_i ranging from 0.1 through 2.0. The probability densities have been normalized so that the areas under the curves are all equal. One can see immediately that the curves become steeper and narrower as the input sensitivity increases.

Also, it is quite noticeable from comparing Figures 1 through 4 that, for a given input signal-to-noise ratio, the curves become steeper and narrower as the bias value is increased. This indicates, as will be more clearly and precisely demonstrated later, that we can exchange input signal-to-noise ratio for bias level. It appears that we are getting something for nothing here since changing the bias is not a function of either time or bandwidth. We seem to be violating the fundamental principle of information theory. This dilemma is easily resolved if we inquire into the expected number of crossings of V_B per second as a function of γ and r_i. A formula for this average number has also been derived[6] and is

$$N(V_B) = N_0 \int_0^\pi \varphi(\gamma - r_i \cos \omega t)\left[\varphi(r_i' \sin \omega t) + r_i' \sin \omega t \int_0^{r_i' \sin \omega t} \varphi(x)\,dx\right] d\omega t\,, \tag{7}$$

where $N(V_B)$ gives the average number of crossings of V_B per second. A plot of $N(V_B)$ for various values of γ and r_i is given in Figure 5. These points are based on a center

[5] Rice, S. O., "Mathematical Analysis of Random Noise," Bell Sys. Tech. J. 24: 51-57, January 1945

[6] Rice, S. O., 1948, loc. cit.

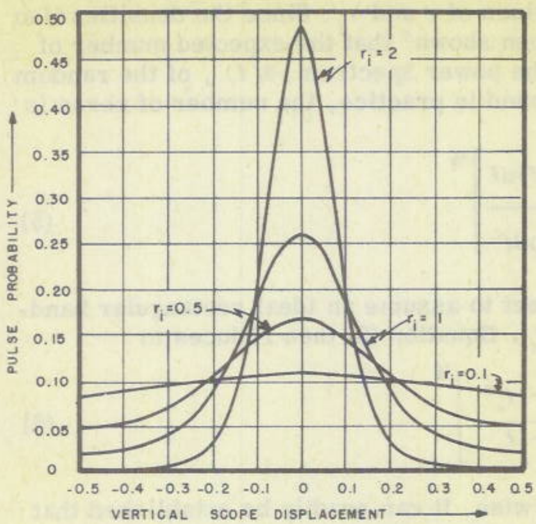

Figure 1 - Pulse probability distribution vs. vertical displacement on oscilloscope —bias = 0

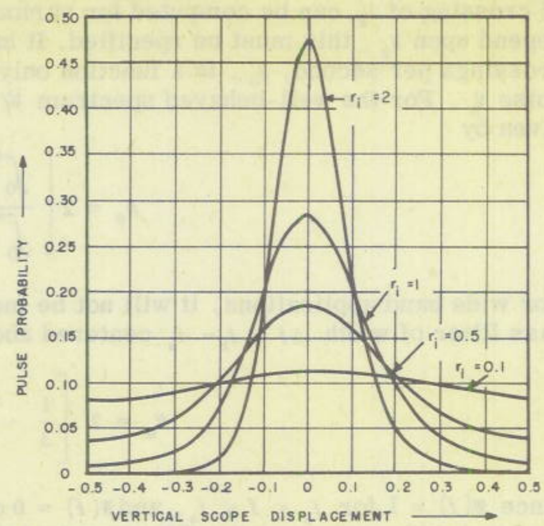

Figure 2 - Pulse probability distribution vs. vertical displacement on oscilloscope —bias = RMS noise

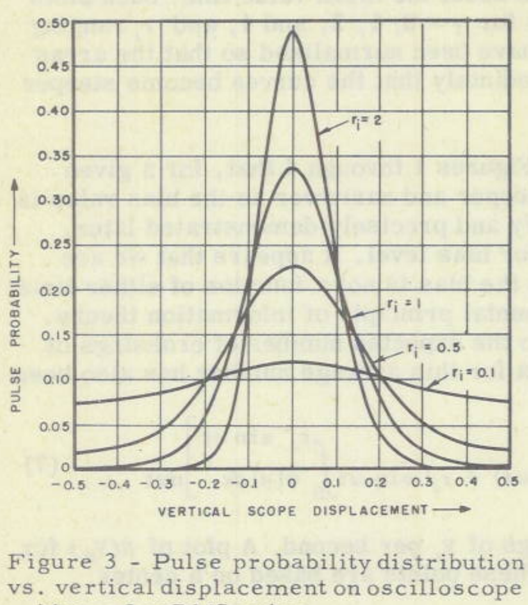

Figure 3 - Pulse probability distribution vs. vertical displacement on oscilloscope —bias = 2 x RMS noise

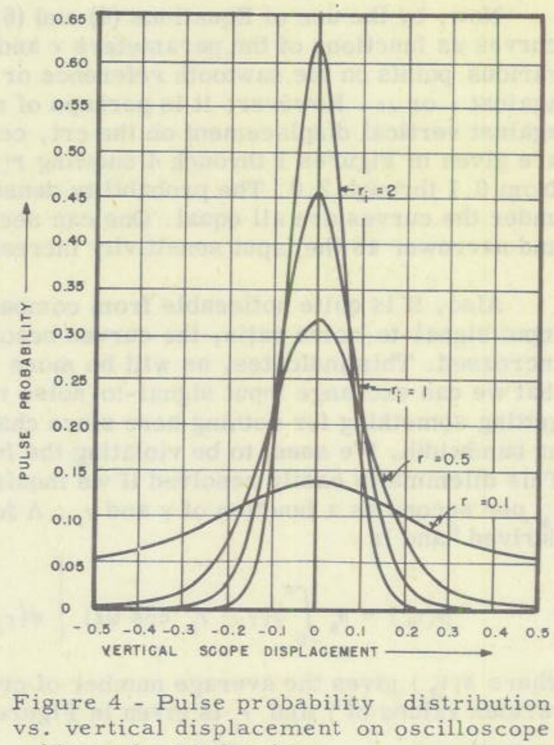

Figure 4 - Pulse probability distribution vs. vertical displacement on oscilloscope —bias = 4 x RMS noise

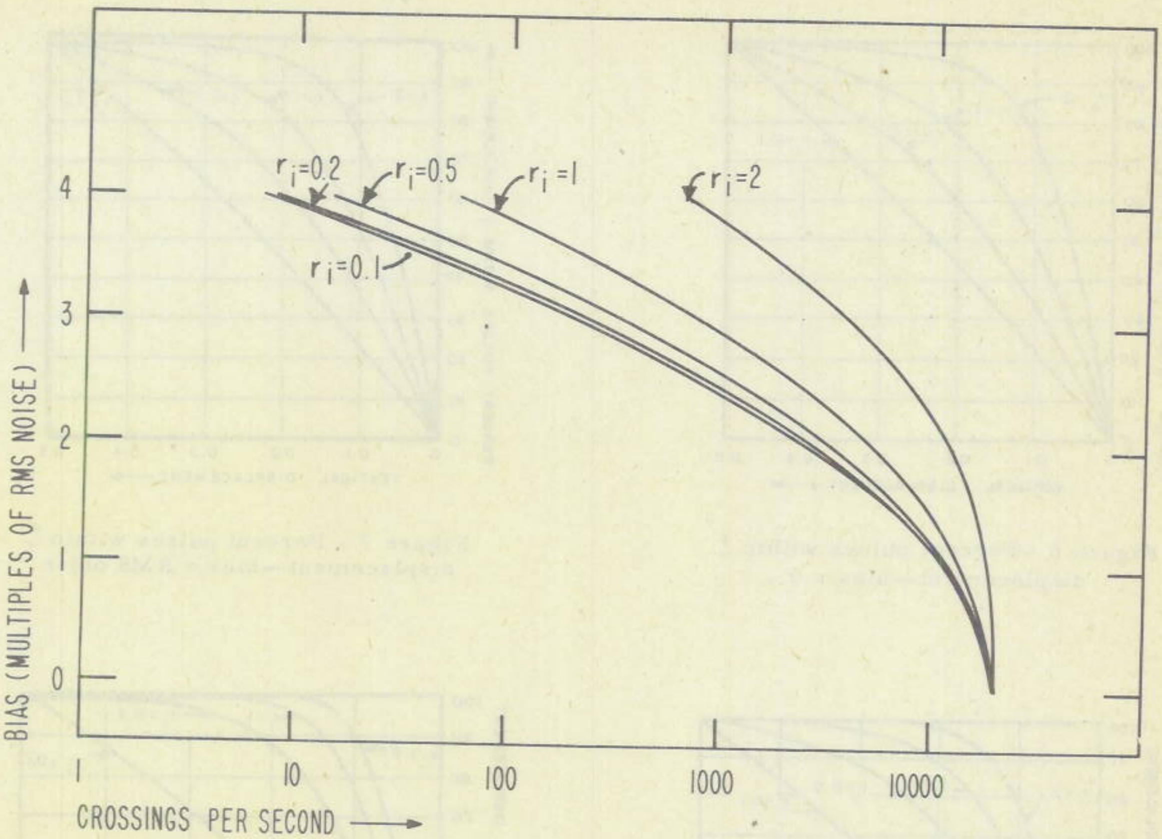

Figure 5 - Crossings of bias value vs. bias based on center frequency of 20 kc

frequency of 20 kc. It is at once obvious that we are actually exchanging time for signal-to-noise ratio as we increase the bias level. Hence the dilemma is resolved.

IMPROVEMENT IN SENSITIVITY

Figures 1 through 4 indicate that there is an improvement in sensitivity due to the increased bias level V_B. However, it is rather difficult to make any very definite quantitative statements as to the amount of improvement from the curves. Some measure of dispersion would be more suitable. The first step in determining dispersion is to integrate the probability curves to obtain ogive or cumulative probability curves (Figures 6 through 9) which show the cumulative percentages of pulses lying within certain distances of the mean. From these curves we can obtain any suitable measure of deviation, but cannot use the standard deviation since the probability curves are not normal. However, we can use the semi-interquartile range or quartile deviation which will be the interval of 50 percent probability; i.e., the interval within which half of the pulses lie. Figure 10 is a plot of the quartile deviation vs. r_i for various values of γ. Another interesting measure of dispersion is the interval within which 90 percent of all pulses lie. This is plotted in Figure 11.

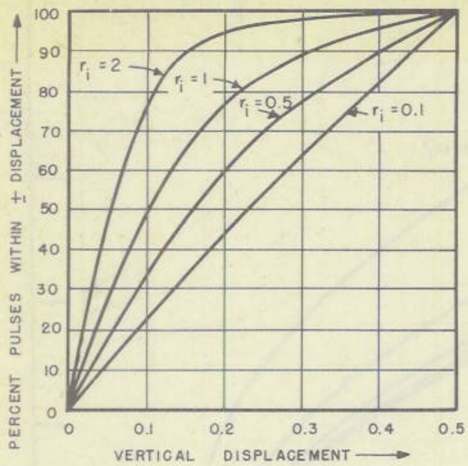

Figure 6 - Percent pulses within ± displacement—bias = 0

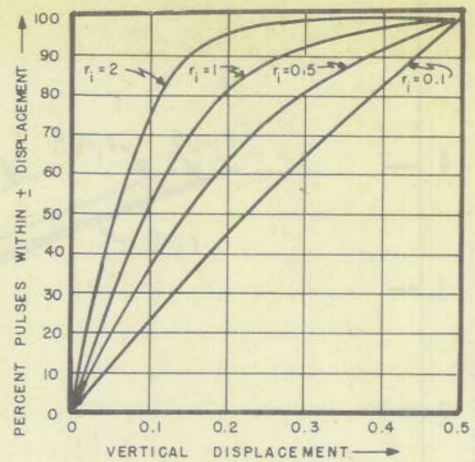

Figure 7 - Percent pulses within ± displacement—bias = RMS noise

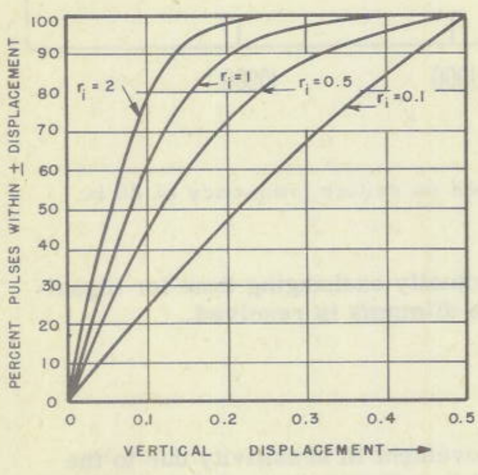

Figure 8 - Percent pulses within ± displacement—bias = 2 x RMS noise

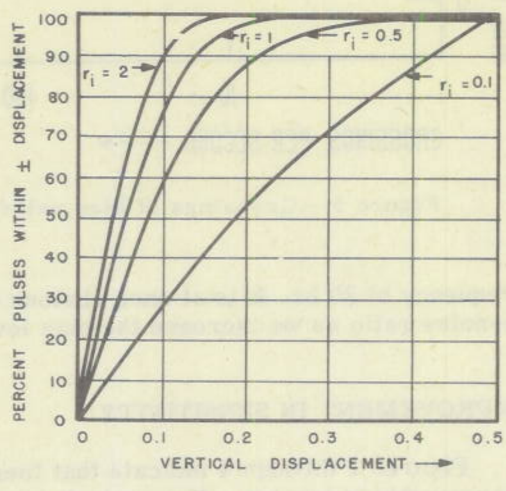

Figure 9 - Percent pulses within ± displacement—bias = 4 x RMS noise

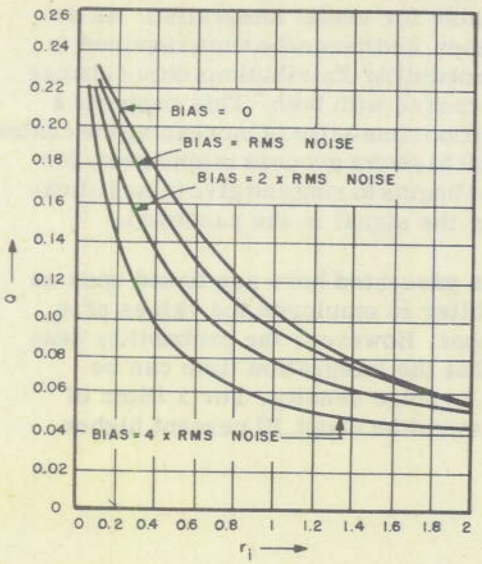

Figure 10 - Quartile deviation vs. input signal-to-noise ratio

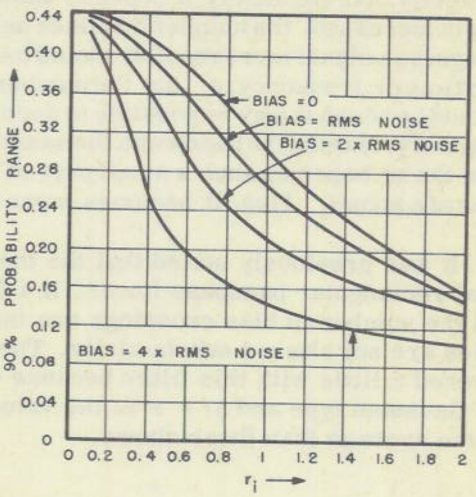

Figure 11 - Ninety percent probability range vs. input signal-to-noise ratio

Although a quantitative value for the improvement in sensitivity can be obtained (Figures 10 and 11), just how much dispersion can be tolerated, before positive recognition of the signal becomes possible, is not known. However, it doesn't really make too much difference on the improvement in sensitivity. Whatever value of dispersion can be tolerated, that same value can be tolerated whether $\gamma = 0$ or $\gamma = 4$ or any other value. It can be seen from Figures 10 and 11 that to maintain any given dispersion, r_i can be reduced as γ is increased. To be specific, for example, from Figure 10, for $Q = 0.160$ at $\gamma = 0$, the r_i required is $r_i = 0.5$ whereas for $\gamma = 4$ the r_i reduces to $r_i = 0.156$, representing an increase in sensitivity of about 10 db. Similarly, if one accepts the 90 percent probability criterion of Figure 11, for $\gamma = 0$, and $P(90\%) = 0.30$, the $r_i = 1$; and for $\gamma = 4$ and the same dispersion, $r_i = 0.325$ or an increase in sensitivity of nearly 10 db.

It can be seen from the figures that as r_i approaches zero the advantage of base clipping becomes less significant. The value of increasing the bias is most important in the range of $0.3 \le r_i \le 1$. Also, it should be noted that $\gamma = 4$ is not the upper limit to the bias level, for even greater improvement can be expected for $\gamma > 4$. However, it is recalled that as γ is increased, it takes more and more time to obtain enough pulses to sufficiently define the probability distributions. In other words, the visual integration time must be increased as γ is increased.

LIMITATIONS OF METHOD

It might be well to emphasize the limitations of this method of improving sensitivity by base clipping. The input signal-to-noise ratio required for signal recognition does decrease as the bias level is increased, representing a definite increase in sensitivity. However, the improvement depends upon a type of instantaneous indication which can take full advantage of the change in probability density. An averaging device will not yield any improvement from base clipping.

The amount of improvement in sensitivity depends upon the bias level directly, but indirectly and ultimately it depends upon the time available for visual integration. As the bias is increased, the number of pulses per second decreases, and hence the time required to recognize a signal increases. It should be noted that the probability distributions do not change with time or frequency, f_c, but the number of pulses do increase with both. This suggests a method by which it may be possible to decrease the recognition time—that of increasing the center frequency, leaving the bandwidth the same. The upper limit to such a process is apparent, for soon the Δf becomes such a small part of f_c that the noise begins to ring and give false indications of a signal. Also, it becomes more difficult to keep the signal in the passband.

It was previously stated that the theoretical results presented here are based upon an ideal rectangular bandpass for Δf. If a Gaussian-type filter is employed the values of N_0 and the number of bias crossings are increased somewhat. However, the probability densities are not altered substantially. The net result is that the integration time can be lowered a little with this filter because of the increase in pulse density. For a filter of the Gaussian type and $\Delta f = 8$ *kc* the values of Figure 5 would be about 10 percent higher on the average than those shown.

EXPERIMENTAL VERIFICATION

This phase of the work took place in two basic stages.

1. Photographic demonstration

2. Electronic demonstration
   a. On cathode-ray tubes
   b. On sonar plotter

The Psychology Branch was asked what methods might be available for increasing detectability of weak targets on a sonar plotting device. On electrically sensitive paper, this device plotted target range against time. The presence of a target is indicated as the darkening of the trace. A sample plot from this device shows a barely visible simulated target, varying in range, and having a signal amplitude such as to provide a 1-1 ratio with the simulated noise.

It was apparent that the contrast between the target indication and the background noise was insufficient to be detected by the eye. Therefore, base clipping as a means of increasing the contrast appeared to be a feasible method of increasing target detectability.

Photographic Demonstration

Before investigating the electronic methods of providing improvement in contrast, the principle was demonstrated photographically. Figure 12 is a reproduction of the original plot, and Figure 13 shows the improvement gained after several stages of the photographic process. The signal is indicated by the dark trace starting near the lower right of the plot and moving toward the upper left, reversing its direction of movement near the top. Both figures were photographed at once for purposes of this report so that any change due to the processing of the report has affected both equally.

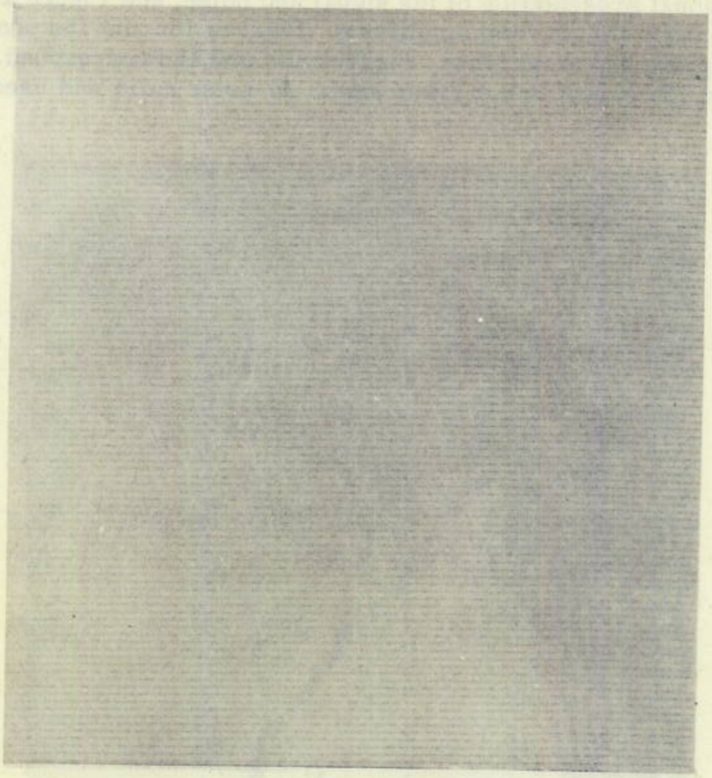

Figure 12 - Original plot of signal embedded in noise

The photographic process was one of copying and recopying through the use of "line," high-contrast film. Figure 13 is the end result after going through the copying process six times. At the third copying, the plot was defocused somewhat to take advantage of the fact that a signal causes a thickening of the line as well as an increase in blackness. The dark area to the left of Figure 13 is caused by a slight difference in the sensitivity of the paper used in making the original record.

Electronic Demonstration

Electronic equipment (Figure 14) was designed and constructed to demonstrate the increase in signal detectability gained through base clipping where the signal and its accompanying noise is presented on a cathode-ray tube, plotted against time. A wideband (up to 2 megacycles) noise generator was used, and a signal of 1,000 cycles was mixed with this noise. Care was taken in the base-clipping circuit and its associated amplifier to maintain a bandwidth of at least 2 megacycles. The clipping tube is biased as a function of the signal and noise impressed upon its grid, so that only the positive peak voltages impressed upon its grid are reflected in its output. The horizontal sweep circuit of the oscilloscope was synchronized with the signal generator.

Results of the demonstration are shown in Figures 15a, b, c,and d. The horizontal sweep on the oscilloscope is adjusted to show 2 cycles of the generated signal. Figure 15a shows the conventional presentation of the signal and noise, and Figure 15b shows the presentation after base clipping and amplifying. Figures 15c and 15d show the presentation after base clipping and amplifying. Figures 15c and 15d correspond, respectively, to Figures 15a and 15b except that a lower signal-to-noise ratio was used.

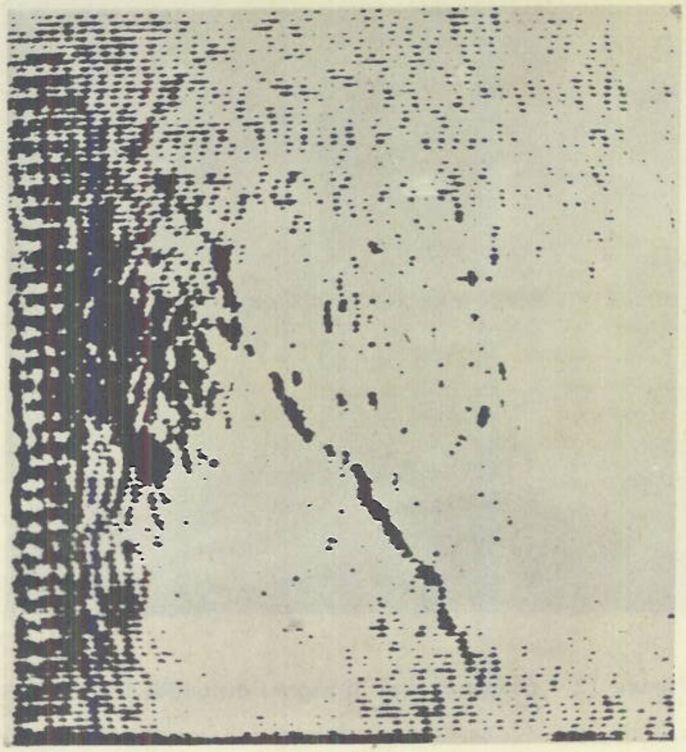

Figure 13 - Plot improved after several stages of photographic process

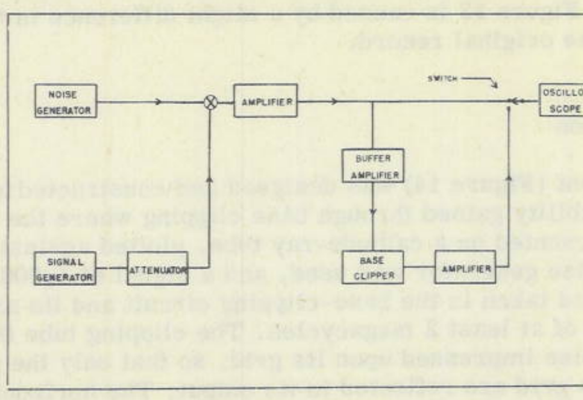

Figure 14 - Arrangement of electronic equipment

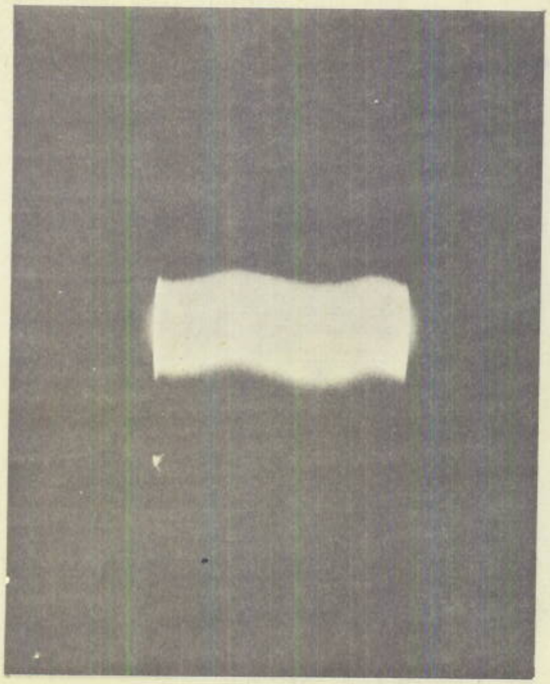

Figure 15a - Conventional presentation of signal and noise

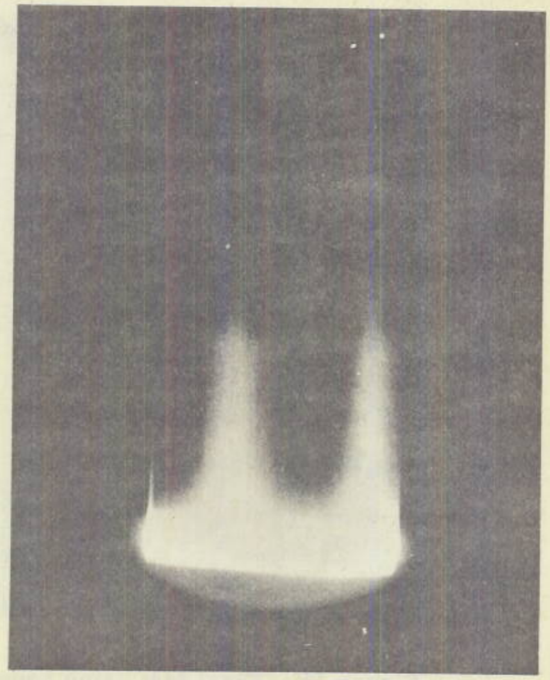

Figure 15b - Presentation after base clipping

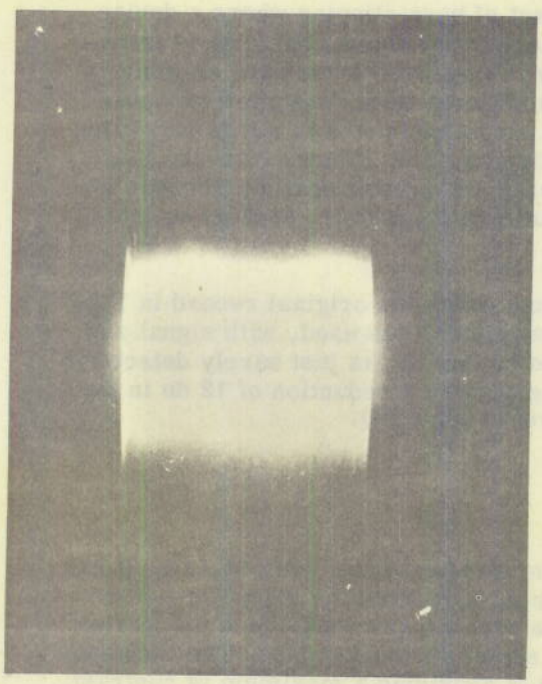

Figure 15c - Conventional presentation of signal and noise—corresponds to 15a except that a lower signal-to-noise ratio was used

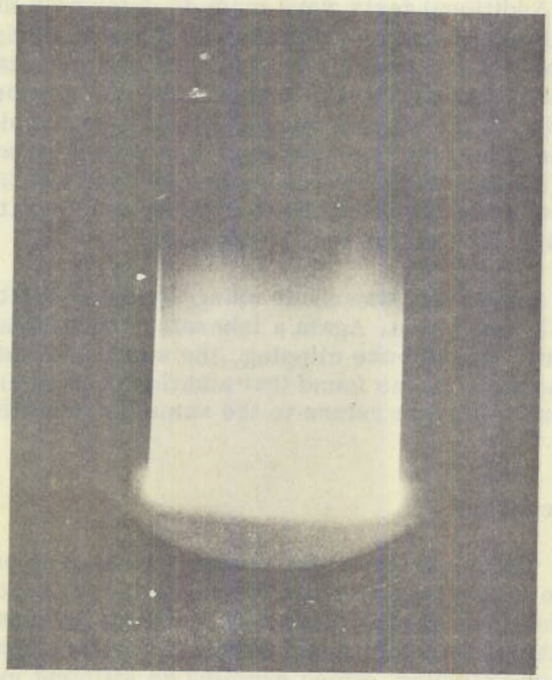

Figure 15d - Presentation after base clipping—corresponds to 15b except that a lower signal-to-noise ratio was used

Figure 15 - Results of electronic demonstration

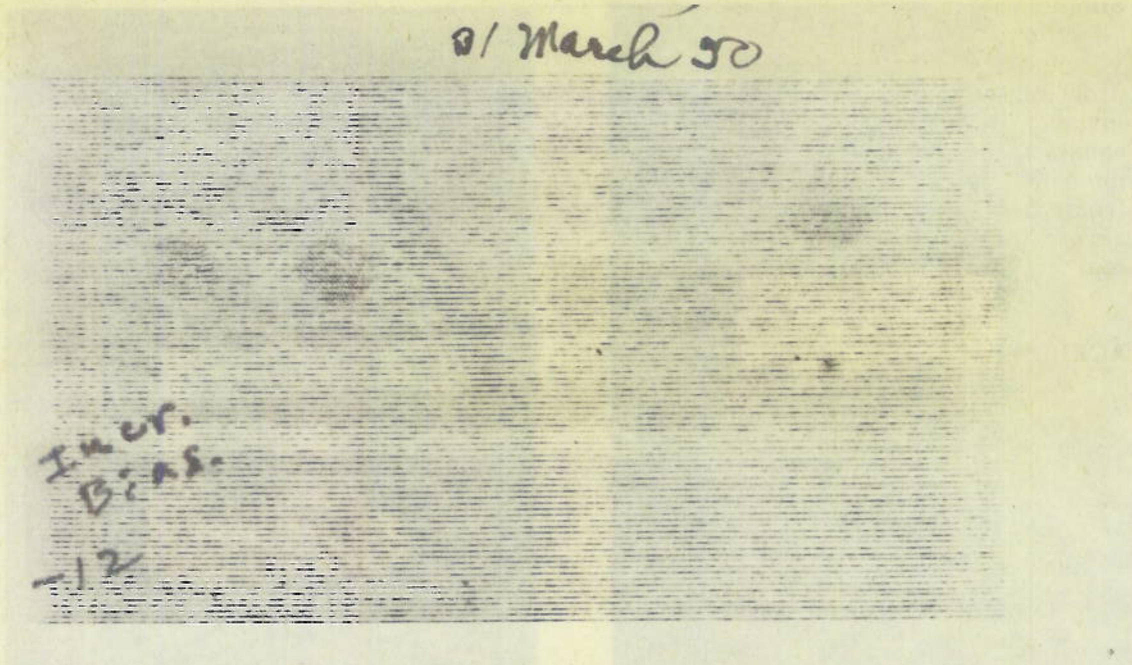

Figure 16 - Final test using sonar plotter — with base clipping

Additional tests were made to determine the effect of base clipping where a device which sharply narrows the bandwidth of the original combined signal and noise is introduced into the system. It was found that detectability is improved if the base clipping occurs after narrowing of the bandwidth, providing additional time is allowed for integration of the clipped signal. When base clipping is performed prior to sharp narrowing of the bandwidth, no advantage was shown by base clipping. This follows from the fact that the bandwidth narrowing device used was an energy-integrating device, whereas to derive advantage from base clipping, an amplitude-integrating device, such as an integrating peak voltmeter, is required.

A final test was made using the sonar plotter from which the original record in Figure 12 was taken. Again a laboratory equipment arrangement was used, with signal and noise. Without base clipping, the signal was adjusted so that it was just barely detectable in the plot. It was found that addition of base clipping enabled a reduction of 12 db in the signal voltage to return to the same detectability level (Figure 16).

Discussion

The sonar-type plot presents a common problem in detection of weak signals. It has been shown by mathematical analysis how base clipping increases the signal-to-noise ratio, thus making the target more easily seen. The psychologist looks upon the matter of signal detectability in terms of contrast between signal and background. The technique of base clipping, which supplies an efficient method of providing a maximum of contrast, takes a small range of amplitude difference in a critical area and spreads this range over the white to black range of indication. Thus, the difference between signal and background is increased in contrast, making the signal more easily seen.

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SUMMARY OF RESULTS

The theoretical analysis presented, shows that the signal-to-noise ratio can be improved by base clipping, as a result of the change in the probability distributions of the instantaneous amplitudes of signal plus noise. As the clipping level is increased, for a constant input signal-to-noise ratio, the dispersion of amplitudes decreases, and hence the output sensitivity or the detection probability increases. Both photographic and electronic methods employed to take advantage of this phenomenon proved successful and agreed with the theory that the maximum improvement to be expected in a practical system was of the order of 10 to 12 db.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to C. E. Corum and J. S. Potts of the Mathematics Group for their assistance in the preparation of the theoretical part of this report. A large part of the experimental research on this project was conducted by J. J. MacGregor and G. P. Walker, without whose untiring efforts the work would not have been successful. R. M. Page, Superintendent of Radio Division III, originally conceived the idea of base clipping as a means of increasing sensitivity. He personally supervised the first experimental verification and demonstration of the principle in September 1949.

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