# *Review and Assessment of Failure Models Applicable to Ballistic Impact Simulations of Ductile Materials*

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**14. ABSTRACT**

A literature review was conducted to evaluate failure models for ductile materials. The focus was on failure models that could be used in numerical simulations of ballistic impact. The assessment examined the number of constitutive parameters required by the model, the types of characterization tests required, and the underlying physics of the model. The Johnson-Cook damage model because of its general acceptance and wide use, was used as a baseline for comparisons. A recommendation is made concerning possible improvement in replicating failure modes observed in ballistic experiments.

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# 1.0 Introduction

A literature review has been conducted to evaluate failure models for ductile materials. The focus was on failure models that could be used in numerical simulations of ballistic impact. The review is by no means exhaustive with regard to the extensive failure model field, but was limited to models that showed promise in ballistic applications, and in particular, might be able to reproduce ballistic limits and failure patterns. Simple criteria like maximum principal stress or strain were omitted since they provide accurate failure patterns only for very specific examples. Similarly, very "sophisticated" failure models that have many "fundamental" parameters that are difficult to ascertain (and which tend to be data-fit parameters) were not included. Rather, the intent was to examine models that have material parameters that can be obtained/measured in fundamental laboratory characterization experiments.

The Johnson-Cook (J-C) failure model [1] was selected as the reference model for this study owing to its widespread use and the fact that constants have been determined for many materials. Additionally, the laboratory tests for obtaining the material model constants are relatively straightforward. Consequently all the other models are evaluated relative to the J-C model in terms of ease of numerical implementation, ability and ease of obtaining material model constants, etc.

Models not widely used in the community but that, from the authors' perspective, could be considered as appropriate candidates for implementation in hydrocodes are also discussed. The primary objective of this report is to summarize succinctly the models and relevant fundamental features, the model parameters and the experiments required to determine the parameters, and maturity of the model for ballistic applications. Then, we conclude with a recommendation on a path forward that might lead to more accurate modeling of failure processes in ballistic applications.

## 2.0 Summary of Models

Essential features or characteristics of the failure models examined are summarized in the tables below. Table 1 provides an overall description of the failure models reviewed, and theoretical features that are implicitly or explicitly included into the model framework. Brief descriptions of the columns of the table are provided below.

Column 1. *Model*: The first column lists the name of author or authors of the model together with the bibliographic reference for the model.

Column 2. *Year*: This column lists the year the model was published.

Column 3. *Model Equations*: A succinct summary of the governing equations is provided. It is not the objective to describe each model in detail; readers are referred to the respective publications. However, it was felt that a succinct summary of the equations highlights the model constants that need to be determined. Examination of the governing equations also shows the commonality and/or relationships to other models.

Column 4. *Soft*: Some failure models like J-C use a damage parameter $D$ as a "switch" to suddenly fail material (a computational element) when $D = 1.0$, but without changing material properties while damage is evolving. Failure of a computational element means that the element no longer can support shear or tension, but it can still support compression through the equation of state. Other failure models couple damage with the plasticity model, leading to softening (i.e., decrease in flow stress) as damage evolves. A "Y" in this column means that the flow surface of the model softens as damage increases. Although damage softening might be physically realistic, it adds constants to the calibration process, adding complexity to both the experimental and numerical procedure. Further, while damage softening may be applicable/important in creep (for example), some researchers believe it is minor importance in very rapid loading events such as ballistic impact.

Column 5. *Triaxiality*: In isotropic homogeneous materials, both the plasticity and failure models should only depend on the three invariants of the stress tensor[1] ($I_1$, $I_2$, and $I_3$). In metal plasticity a further simplification is to assume that flow only depends on $J_2$, an invariant of the deviatoric stress tensor. It is commonly accepted in failure models, since Bridgman's [13] pioneering studies, that the larger the triaxiality:

$$\sigma^* = \frac{I_1/3}{\sqrt{3J_2}} = \frac{-P}{\sigma_{eff}} \qquad (1)$$

the smaller the strain to failure of a material; or in other words, the larger the pressure applied, the larger the strain required to fail the

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[1] The Appendix provides a brief synopsis of the invariants of the stress and stress deviator tensors.

material. Models where failure does not depend on triaxiality have a "No" in this column.

Column 6. *$J_3$*: $J_3$ is another invariant of the deviatoric stress tensor. Recently, some papers analyze carefully how $J_3$ (or, equivalently, the Lode angle) affects the strain to failure. These models are identified with a "Y" in this column.

Column 7. *Comments*: These are comments by the authors of this report that highlight significant features or distinguishing characteristics of the model.

Table 2 focuses on the model constants that must be determined. A brief explanation of the information in each of the columns of Table 2 is provided below. The models are listed in chronological order, i.e., the order is the same as for Table 1.

Column 1. *Model:* Model name, from Table 1.

Column 2. *Constants*: The constants needed by the failure model.

Column 3. *Characterization Tests*: These are the tests needed to calibrate the constants of the model. A model with many constants does not necessarily mean that many tests are required. For example the Gurson-Tveergard-Needleman (GTN) model has 8 constants and only one type of test is required for calibration. When evaluating the model the preference is for a small number of characterization tests, i.e., a relatively inexpensive procedure for determining the constitutive parameters. The symbols, designating various types of tests, are defined in Table 3.

Column 4. *Hydrocodes*: Four codes were considered in this column: AUTODYN (now ANSYS), CTH, EPIC and LS-DYNA. If the model is implemented in the code, it is listed in this row. Additionally, for LS-DYNA, the corresponding material number is listed.

### Table 1. Essential Features of Failure Models Reviewed

| Model | Year | Model Equations | Soft.[2] | Tri-axiality | $J_3$ | Comments |
|---|---|---|---|---|---|---|
| **Cockcroft-Latham [2]** | 1968 | $W = \int_0^{\varepsilon_{eq}} \langle \sigma_I \rangle \, d\varepsilon_{eq} \le W_{crit}$ | N | N | N | Essentially a plastic work (energy) criterion. |
| **Tuler-Butcher [3]** | 1968 | $D = \int (\sigma - \sigma_o)^n dt \rangle D_{crit}$ | N | N | N | Energy per unit area criterion; developed for spall. |
| **Gurson [4]** | 1975 | $\phi = \frac{\sigma_{eq}^2}{Y^2} + 2f \cosh\left(\frac{\sigma_{kk}}{2Y}\right) - 1 - f^2$<br>$f$ = void volume fraction | Y | Y | N | Evolution of ductile voids that reduce the strength of the yield surface; void growth occurs in tension. Coalescence of voids not realistic. |
| **Wilkins [5]** | 1980 | $D_p = \int w_1 w_2 d\varepsilon_{eq}^p \ge D_c \ , \ w_1 = \left(\frac{1}{1+aP}\right)^{\alpha} ; w_2 = \left[2 - \max\left(\frac{s_2}{s_3}, \frac{s_2}{s_1}\right)\right]^{\beta}$ | N | Y | Y | Damage as a function of hydrostatic pressure and shear stress state. |
| **Johnson-Cook [1]** | 1983 | $\varepsilon_f = \left(D_1 + D_2 \, e^{D_3 \sigma^*}\right)\left(1 + D_4 \ln \dot{\varepsilon}\right)\left(1 + D_5 T^*\right)$ | N | Y | N | Critical strain for failure as a function of hydrostatic pressure (triaxiality), strain rate, and temperature. |
| **Gurson-Tvergaard-Needleman (GTN) [6]** | 1984 | $\phi = \frac{\sigma_{eq}^2}{Y^2} + 2f^* q_1 \cosh\left(\frac{\sigma_{kk}}{2Y}\right) - \left\{1 + \left(q_1 f^*\right)^2\right\}$<br>$f^* = f$ if $f \le f_c$; $f^* = f_c + K(f - f_c)$ if $f > f_c$, $K \equiv (f^*_U - f_c)(f_F - f_c)$<br>$f^*_U$ ultimate void vol. fraction, $f_c$ critical, $f_F$ failure | Y | Y | N | Improved Gurson void growth model. |
| **Borvik, et al. [7]** | 2001 | $\varepsilon_p = \left[D_1 + D_2 \, e^{D_3 \sigma^*}\right]\left[1 + \dot{\varepsilon}_p\right]^{D_4}\left[1 + D_5 T^*\right]$ | Y | Y | N | Similar to the J-C model, but with a different strain rate dependence. |

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[2] This is softening of the yield surface. None of the papers reviewed mentions softening of the stiffness matrix; however, some probably soften the elastic modulus as the material is damaged.

| Model | Year | Model Equations | Soft.[2] | Tri-axiality | $J_3$ | Comments |
|---|---|---|---|---|---|---|
| **Bao-Wierzbicki [8]** | 2004 | Piecewise curve-fit for failure strain as a function of triaxiality | N | Y | Y | Empirical curve fit for the strain to failure vs. triaxiality curve. |
| **Xue-Wierzbicki [9]** | 2005 | $\bar{\varepsilon}_f = c_1 e^{-c_2\eta} - \left(c_1 e^{-c_2\eta} - c_3 e^{-c_4\eta}\right)\left(1-\xi^{1/n}\right)^n$<br>$\eta$ is triaxiality (i.e., $\sigma^*$), $\xi = \frac{27}{2}\frac{J_3}{\bar{\sigma}^3}$ | N | Y | Y | Failure depends both on triaxiality and Lode angle; also, replaced piecewise curve fit of Ref. [8] with analytic function. |
| **Bai-Wierzbicki [10]** | 2008 | $\hat{\varepsilon}_f\left(\eta,\bar{\theta}\right) = \left[\frac{1}{2}\left(D_1 e^{-D_2\eta} + D_5 e^{-D_6\eta}\right) - D_3 e^{-D_4\eta}\right]\bar{\theta}^2 +$<br>$+\frac{1}{2}\left(D_1 e^{-D_2\eta} - D_5 e^{-D_6\eta}\right)\bar{\theta} + D_3 e^{-D_4\eta}$<br>$\bar{\theta} = 1 - \frac{6\theta}{\pi}$ where $\theta$ is the Lode angle | N | Y | Y | Both plasticity and failure depend on the Lode angle. |
| **Xue-GTN [11]** | 2008 | $\phi = \frac{\sigma_{eq}^2}{Y^2} + 2D\cosh\left(\frac{q_2\sigma_{kk}}{2Y}\right) - \left(1+D^2\right) = 0$<br>$D = K_D(q_1\dot{f} + \dot{D}_{shear})$ | Y | Y | Y | Modification of GTN model to improve shear failure prediction. |
| **Nahshon-Hutchinson [12]** | 2008 | GTN with $\dot{f} = \left(1-f\right)D_{kk}^p + k_\omega f\ \ \omega\left(\bar{\bar{\sigma}}\right)\frac{s_{ij}D_{ij}^p}{\sigma_{eq}}$ | Y | Y | Y | Modification of GTN model to improve shear failure prediction. |

Table 2. Model Constants, Characterization Tests Required, and Availability

| Model | Constants | Characterization Tests[3] | Availability in Hydrocodes |
|---|---|---|---|
| **Cockcroft-Latham** | $W_{cr}$ | ST | LS-DYNA-107 |
| **Tuler-Butcher** | $\sigma_o, n, D_{cr}$ | Spall | LS-DYNA (Erosion) |
| **Gurson** | $f_o, f_c, f_f$ | ST | LS-DYNA-120, CTH[4], EPIC[3] |
| **Wilkins** | $a, \alpha, b, D_c, r_c$ | ST, EN, FP, FPN, TO, TTO | LS-DYNA-81 |
| **Johnson-Cook** | $D_1, D_2, D_3, D_4, D_5$ | ST, EN, BN, TO | LS-DYNA-15, CTH, EPIC, AUTODYN |
| **Gurson-Tvergaard-Needleman (GTN)** | $q_1$=1.5, $q_2$=1, $f_o$=0, $f_c$=0.15, $f_f$=0.25, $f_N$, $\varepsilon_N$, $S_N$ | ST | LS-DYNA-120 |
| **Borvik et al.** | $D_2, D_3, D_c, \varepsilon_{threshold}$ | ST, EN, BN, | LS-DYNA-104 & 107 |
| **Bao-Wierzbicki** | $A, B, C, D_1, D_2, D_3, F, G$ | CO, ST, EN, BN, TO | No |
| **Xue-Wierzbicki** | $C_1, C_2, C_3, C_4, m$ | ST, EN, BN, TO | No[5] |
| **Bai-Wierzbicki** | $D_1, D_2, D_3, D_4, D_5, D_6$ | ST, EN, BN, TO, CO, FN | No |
| **Xue-GTN** | $q_1, q_2, q_3, q_4, f_o, f_c, f_f, f_N, \varepsilon_N, S_N$ | ST, TO | No |
| **Nahshon-Hutchinson** | $q_1, q_2, q_3, q_4, f_o, f_c, f_f, f_N, \varepsilon_N, S_N, k_\omega$ | ST, TO | No |

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[3] See Table 3 for definition of the various types of characterization tests.

[4] The TEPLA model has a Gurson flow surface; the TEPLA model is implemented into CTH and EPIC.

[5] According to LSTC implementation into LS- DYNA is ongoing.

**Table 3. Specimen Geometries**

| | |
|---|---|
| ST | Smooth Tensile |
| EN | E-Notch |
| BN | B-Notch |
| TO | Torsion |
| CO | Compression |
| FN | Flat-Notched |
| FP | Flat Plate Tension |
| FPN | Flat Plate Notched Tension |
| TTO | Tension-Torsion |

## 3.0 Evaluation of Models

Table 4 compares each model with the reference model (J-C) and includes the authors' evaluation of the model. Thus, Table 4 will be used as a "decision" table that can highlight which failure models have potential for further exploration. We emphasize, however, that without actual implementation and thorough validation of each model, it is difficult to evaluate fully a specific model's ability to predict ballistic response accurately. Therefore, some of the evaluations presented below are based on the authors' experience. We acknowledge that until a model is fully implemented, exercised, and validated, there will exist some uncertainty in our evaluation, whether it be negative or favorable. The Recommendations Section will summarize the findings in this section.

An explanation of the information provided in the various columns of Table 4 is given below.

Column 1. *Model*: Model names, listed in the same order as Tables 1 and 2.

Column 2. *Validated:* All the models in the table have been validated to some extent. The most common validation consisted of reproducing, typically, tensile and/or shear tests that have been conducted. Since model constants are often adjusted so that the simulation reproduces the test, this is really an examination of self-consistency, and not a true validation. Not all the models have been used in ballistic impact applications. This column summarizes whether ballistic simulations, to the authors' knowledge, have been compared with ballistic test results. An "N" in this column means that no comparison with ballistic data was found in the literature. "Limited" implies that only a few comparisons have been conducted (maybe one or two papers). "Significant'" is stated for models that have been used regularly by one or more researchers in the ballistics community. "Extensive" is reserved for the J-C model which has been used in hundreds of ballistic applications by many different researchers and groups worldwide.

Column 3. *Maturity Level:* This column is a qualitative assessment how often the model has been used within the ballistics community and estimates the "maturity level" of the model with respect to ballistic applications. A model that has never been used will have in this column a "None" qualification. Models with minimal experience in ballistics will have a "Low" qualification. This column relates closely to the previous column.

Column 4. *Additional Tests.* As mentioned above, the J-C model is the reference model in this report. Assuming that J-C model constants are already available for the material (as well as the data from tests performed to obtain the constants, see Tables 2 and 3), this column provides the number of *additional* characterization tests needed for the new model. For example, a Gurson model[6], in principle, only needs a tensile test to calibrate the constants. Since the tensile test is also needed in the J-C model, no additional tests should be required to build a Gurson model for the same material. Hence, the column lists "0" as the number of additional tests.

Column 5. *Additional Computations.* This column assesses the *additional* computational runs required, given a J-C model of the material, to obtain constitutive constants for a different model. For example, the J-C model requires numerical simulations to estimate triaxiality on smooth-tensile and notched specimen tensile tests. Only the Wilkins [5] and Bai-Wierzbicki [11] models require additional computations to calibrate constants.

Column 6. *Additional Work.* One of the objectives of this report is to answer the question: How much additional experimental and computational work is required to switch from the J-C model to a different model? This column summarizes the additional work estimated to migrate from a J-C model to a different failure model. The estimate is based on an assessment of additional experimental and numerical work required; thus, this column summarizes the two previous columns. But in addition, this column reflects an estimate of the effort to implement the model in a computer code; that is, write the material subroutine. The work involved in writing the code increases with the complexity of the model.

Column 7. *Potential.* Failures modes in ballistic problems are very diverse. For example, targets can fail in pure shear (plugging), tensile extension, petalling, spall, compressive failure, etc. Similarly, when testing with an MTS machine in compression or tension, depending on the specific material, a variety of failure patterns can arise. An accurate model should be able to capture the rich patterns of failure modes seen in quasistatic and ballistic tests. This column provides our best estimate on how well the model might be able to replicate failure patterns. This estimate is based on the available papers and also on the fundamental framework of the model. For example, a failure model that depends on $J_3$ and triaxiality will have a higher likelihood at capturing/replicating more failure modes than a model that depends only on triaxiality. Nevertheless, it is not possible to know if the correct failure patterns are actually captured until the model is implemented into a computer code and tested against experimental data.

Column 8. *Recommendation for Further Exploration.* The finding in this column is subjective, but it provides our summary recommendation

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[6] It might be assumed that micromechanical examination of void growth and coalescence would be required for the Gurson model and other void growth models. However, there is no indication in the papers that this was done. Rather, void growth and coalescence appear to be calculated from assumptions combined with theoretical considerations.

concerning the model. A "Low" consideration is given for a model that might be difficult to implement into a computer program, and/or requires many characterizations experiments, and/or requires many material constants, and/or cannot reproduce desired failure modes. A "High" consideration would reflect opposite characteristics. The assessment attempts to balance these various characteristics. Essentially, this column provides our assessment of whether it might be worthwhile to explore further a new failure model for ballistic applications.

Table 4. Model Evaluation Relative to Johnson-Cook Failure Model

| Model | Validated w/ Ballistic Experiments | Maturity Level in Ballistics | Additional Tests | Additional Comps. | Additional Work to Migrate from J-C | Potential in Reproducing Failure Modes | Recommendation for Further Exploration |
|---|---|---|---|---|---|---|---|
| **Cockcroft-Latham** | Limited[7] | Low | 0 | None | Low | Low | Low |
| **Tuler-Butcher** | N | Low | 0 | None | Low | Low | Low |
| **Gurson** | N | None | 0 | None | High | Low | Low |
| **Wilkins** | Significant | Medium | 3 | Many | High | High | Medium |
| **Johnson-Cook** | Extensive | High | – | – | – | High | – |
| **GTN** | N | None | 0 | None | High | Medium | Low |
| **Borvik, et al.** | Significant | Medium | 0 | None | Medium | High | Medium |
| **Bao-Wierzbicki** | Limited | Low | 1 | None | Medium | High | Medium |
| **Xue-Wierzbicki** | N | None | 0 | Few[8] | Medium | High | High |
| **Bai-Wierzbicki** | N | None | 2 | Many | High | Very high | Medium |
| **Xue-GTN** | N | None | 0 | None | High | Very high | Medium |
| **Nahshon-Hutchinson** | N | None | 0 | None | High | Very High | Medium |

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[7] See Borvik, et al. [16-17.

[8] Only optimization of parameters

# 4.0 Recommendations

As mentioned previously, it is difficult to assess the applicability and merit of the models to ballistic impact without actually testing them and comparing them with ballistics data. Nevertheless, from the physics incorporated into the model, it is possible to assess if the model is capable of reproducing certain types of failure patterns. For example, if the plasticity and failure surfaces are von Mises, i.e., they depend only on the second invariant of the stress deviator tensor, $J_2$, the model will not be able to reproduce slanted failure surfaces in simple compression or tensile tests. The reason is that a von Mises surface, which is a circle in the principal stress plane, has no preferential direction for plastic deformation or failure. However, a characteristic failure angle can be predicted if the plasticity or failure model depends on the Lode angle (i.e., the third invariant $J_3$).

The physics of the model, or in other words, its capability to capture failure modes, was one of the primary decision parameters for the recommendation provided in the last column of Table 4. With respect to two (or more models) that seem to incorporate similar physics, preference was given to the simpler model.

Finally, when providing our recommendation, the "maturity" level of the model as well as any data comparing the model with actual ballistics results were also taken into account. A model is expected to work better if many groups in the ballistics community have used it successfully to reproduce different ballistic tests.

There is only one model, Xue-Wierzbicki [9], among the models studied in this report that receives a "High" recommendation to be implemented and tried in hydrocodes. Actually the recommendation is to implement a slight variation of the model that will be explained below. Even though the model has not been validated with ballistics tests and hence has no "maturity," it is thought that the potential of this model in the ballistics community is high for the following reasons:

1. The model is simple since it only introduces a Lode angle dependence on the failure model.
2. The model does not soften the yield surface with damage (that is, the plasticity model and the failure model are not explicitly coupled)[9]; this makes the model simpler with respect to numerical implementation and limits the number of parameters to be determined experimentally.
3. The model can satisfactorily reproduce the experimental data of strain to failure versus triaxiality, as well as differences in strain to failure as a function of stress state. Specifically, the model is able to capture the fact that torsion specimens often fail at smaller strains than smooth tensile specimens. The J-C model cannot replicate this observation.
4. Given the J-C plasticity and failure constants for a material, it is possible to construct readily a Xue-Wierzbicki model from the same set of tests and constants.

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[9] Damage development can depend upon the stress state—for example, the strain to failure in the Johnson-Cook damage model depends on the triaxiality—but explicit coupling of the failure and plasticity models is where damage development softens the yield surface, which then can change the rate of damage evolution.

5. The Xue-Wierzbicki model was successfully implemented by Lidén [14] to model the fracture of a long rod impacting oblique steel plates.[10]

Although the recommendation is to implement the Xue-Wierzbicki (X-W) model, preliminary computations show that incorporating the Lode angle only on the failure part of the numerical model is not sufficient to capture all the failure patterns seen in the experiments. Consequently, the following modifications of the X-W model are also recommended:

1. Use the J-C strength model as the plasticity model, but modify it to incorporate the Lode angle.
2. X-W calculates material failure constants from the experimental data. We recommend use of the J-C failure model and constants, but modify the failure envelope by the Lode angle function used in the X-W model [9].
3. It is known than the strain hardening response can be different in tension and torsion, see for example Wilkins [5]. Preliminary computations show that taking into account the differences in the hardening responses leads to appropriate failure patterns.

A model based on the recommendations above was implemented as a user subroutine in LS-DYNA. Preliminary results are very promising because the model reproduces failure patterns like cup and cone failure in tensile specimens, and slanted failure in compression specimens. The model also satisfactorily reproduces final length on Taylor anvil tests and failure patterns observed in ballistic impact on aluminum 6061 plates [18].

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[10] Although this is a ballistic test, to the authors' knowledge, it is the only example of the application of the Xue-Wierzbicki model to ballistics. Further, the application was solely to the projectile, and not the target. For this reason the Xue-Wierzbicki model maturity was graded as "None".

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# Appendix: Invariants

The following provides a synopsis of the stress invariants and the stress deviator invariants. Tensor notation is used. The reader is referred to Khan and Huang [15] or similar textbook on plasticity for a more detailed description.

**Stress Invariants:**

$$I_1 = \sigma_{kk} = \sigma_1 + \sigma_2 + \sigma_3 = -3P \qquad \text{(A-1)}$$

$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \qquad \text{(A-2)}$$

$$I_3 = \sigma_1\sigma_2\sigma_3 \qquad \text{(A-3)}$$

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the principal stress, and $P$ is the pressure (negative of the mean stress).

**Stress Deviator Invariants:**

It is common in plasticity theory to decompose the stress tensor into two parts,

$$\sigma_{ij} = -P\delta_{ij} + s_{ij} \qquad \text{(A-4)}$$

where $s_{ij}$ is the deviator stress tensor and $P\delta_{ij}$ is the spherical or hydrostatic stress tensor. Then, the stress deviator invariants are (with $s_1$, $s_2$, and $s_3$ being the principal values of the stress deviator tensor):

$$J_1 = s_{ii} = s_1 + s_2 + s_3 = 0 \qquad \text{(A-5)}$$

$$\begin{aligned} J_2 &= \frac{1}{2}s_{ij}s_{ij} = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right] \\ &= \frac{1}{2}\left(s_1^2 + s_2^2 + s_3^2\right) = \frac{1}{3}\left(I_1^2 - 3I_2\right) \end{aligned} \qquad \text{(A-6)}$$

$$\sigma_{eff} = \sqrt{3J_2} \qquad \text{(A-7)}$$

where $\sigma_{eff}$ is the effective (von Mises) stress.

$$\begin{aligned} J_3 &= \det(s_{ij}) = \left(\frac{1}{6}\right)e_{ijk}e_{mnl}s_{im}s_{jn}s_{kl} = s_1s_2s_3 \\ &= \frac{1}{27}\left(2I_1^3 - 9I_1I_2 + 27I_3\right) \end{aligned} \qquad \text{(A-8)}$$