

UNCLASSIFIED

***Characterization of Soda-Lime Glass
by Confined Compression Testing
with Numerical Validation***

Sidney Chocron
Charles E. Anderson, Jr.
Arthur E. Nicholls
Kathryn A. Dannemann
Rory P. Bigger

Southwest Research Institute®
P.O. Drawer 28510
San Antonio, TX 78228

Contract: W56HZV-06-C-0194

SwRI® Report 18.12544/021

Prepared for:

US Army RDECOM-TARDEC
RDTA-RS
Warren, MI 43897-5000

June 2010

UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED

REPORT DOCUMENTATION PAGE			<i>Form Approved</i> OMB No. 0704-0188		
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Service, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington, DC 20503.					
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.					
1. REPORT DATE (DD-MM-YYYY) 30-06-2010		2. REPORT TYPE Technical		3. DATES COVERED (From - To) June 2009 – June 2010	
4. TITLE AND SUBTITLE Characterization of Soda-Lime Glass by Confined Compression Testing with Numerical Validation			5a. CONTRACT NUMBER W56HZV-06-C-0194		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) Sidney Chocron, Charles E. Anderson, Jr., Arthur E. Nicholls, Kathryn A. Dannemann, and Rory P. Bigger			5d. PROJECT NUMBER 18.12544		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Southwest Research Institute, P.O. Drawer 28510, San Antonio, TX 78228			8. PERFORMING ORGANIZATION REPORT NUMBER 18.12544/021		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) US Army Tank-Automotive Research, Development, and Engineering Center, Warren, MI 48397-5000			10. SPONSOR/MONITOR'S ACRONYM(S) RDECOM-TARDEC/RDTA-RS		
			11. SPONSORING/MONITORING AGENCY REPORT NUMBER		
12. DISTRIBUTION AVAILABILITY STATEMENT Approved for Public Release; Unlimited Distribution					
13. SUPPLEMENTARY NOTES The views, opinion, and/or findings contained in this report are those of the authors and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documents.					
14. ABSTRACT This report describes two different techniques used to characterize intact and damaged soda-lime glass at pressures up to ~2 GPa: triaxial compression and confined sleeve. The results of the characterization experiments—for intact and damaged glass as a function of confinement pressure—are described; the results are interpreted in terms of two pressure-dependent constitutive models, Drucker-Prager and Mohr-Coulomb. An observation is that the slopes of the two models appear to be independent of the degree of damage (intact, predamaged and severely damaged specimens). It is also observed that there is a maximum strength for the damaged glass, i.e., there is a cap on the strength. The Drucker-Prager model is then used to compute the position-time and projectile residual length versus time for impact of tungsten-alloy, long-rod projectiles into glass targets, using the constitutive constants inferred from the characterization experiments, although the zero-pressure value has to be considerably lower to represent highly damaged (comminuted) glass.					
15. SUBJECT TERMS soda lime glass, Mohr-Coulomb, Drucker-Prager, constitutive modeling, characterization, numerical simulations, ballistic experiments, damage, triaxial compression, confined sleeve					
16. SECURITY CLASSIFICATION OF: Unclassified, Unlimited Distribution		17. LIMITATION OF ABSTRACT None	18. NUMBER OF PAGES 58	19a. NAME OF RESPONSIBLE PERSON Frederick C. Rickert	
a. REPORT Unlimited	b. ABSTRACT Unlimited	c. THIS PAGE Unlimited	19b. TELEPHONE NUMBER (Include area code) 586-282-3914		

UNCLASSIFIED

Table of Contents

	Page
1.0 Introduction.....	1
2.0 Experimental Technique.....	3
2.1 Materials	3
2.2 Overview	3
2.3 “Hydraulic Bomb” Technique	4
2.4 “Confined Sleeve” Technique.....	5
3.0 Results and Interpretation	7
3.1 Interpretation of the Hydraulic Bomb Experiments.....	7
3.1.1 Intact Specimens	7
3.1.2 Predamaged Specimens	7
3.2 Interpretation of the Confined Sleeve Experiments.....	9
3.2.1 Intact Specimens	9
3.2.2 Predamaged Specimens	11
4.0 Constitutive Model Parameters.....	15
4.1 Constitutive Models.....	15
4.1.1 Drucker-Prager Model	15
4.1.2 Mohr-Coulomb Model	15
4.2 Hydraulic Bomb Tests Results.....	16
4.2.1 Drucker-Prager Analysis.....	16
4.2.2 Mohr-Coulomb Analysis	17
4.3 Confined Sleeve Tests Results.....	18
4.3.1 Drucker-Prager Analysis.....	18
4.3.2 Mohr-Coulomb Analysis	19
4.4 Summary of Constitutive Model Results.....	20
5.0 Comparison of Starphire and Borofloat Response.....	21
5.1 Drucker-Prager Representation.....	21
5.2 Mohr-Coulomb Representation	23

Table of Contents (Cont'd)

	Page
6.0 Numerical Simulations of Ballistic Penetration.....	25
6.1 Introduction.....	25
6.2 The Experiments	25
6.3 Numerical Simulations: Previous Modeling Efforts.....	28
6.4 Numerical Simulations: New Simulations.....	29
6.4.1 Constitutive Model for Projectile.....	29
6.4.2 Glass Constitutive Modeling.....	30
6.4.3 Simulation Results	31
7.0 Characterization Data and Flyer-Plate Impact Data	35
7.1 Introduction.....	35
7.2 Plate-Impact Experiments.....	35
7.3 Comparison of Characterization Experiments with Flyer-Plate Impact Experiments	37
7.3.1 Predamaged Compression Tests on Soda-Lime Glass.....	38
7.3.2 Mohr-Coulomb Representation	39
8.0 Summary	41
9.0 Acknowledgement	43
10.0 References	45

List of Figures

	Page
Figure 1. Hydraulic bomb experimental set-up.....	4
Figure 2. Specimen placement before being inserted into the pressure vessel	5
Figure 3. Confined sleeve test placed in the split-Hopkinson bar.....	6
Figure 4. Two intact specimens (SP-13 and SP-15) and two predamaged specimens (SP-5 and SP-10) tested in the bomb at 100 and 250 MPa nominal confinement pressures.....	7
Figure 5. Predamaged specimen tested in the bomb at 100 MPa confinement pressure. The brass sleeve has been lifted to show the damaged glass in the right figure. Shear failure is evident in both figures	8
Figure 6. Close-up photographs of specimen SP-8 (hydraulic bomb test).....	8
Figure 7. Stress vs. axial strain for an initially intact specimen in a confined sleeve test under monotonic load (test SP-33)	9
Figure 8. Stress vs. sleeve hoop strain for an initially intact specimen in a confined sleeve test under monotonic load (test SP-33).....	10
Figure 9. Volumetric decrease vs. axial stress (test SP-33)	10
Figure 10. Stress vs. axial strain for a predamaged specimen in a confined sleeve test under monotonic load (test SP-21)	11
Figure 11. Stress vs. sleeve hoop strain for a predamaged specimen in a confined sleeve test under monotonic load (test SP-21).....	11
Figure 12. Cut-away view of showing shear planes in a confined sleeve experiment on predamaged glass, Exp. SP-25.....	12
Figure 13. Equivalent stress vs. pressure for a predamaged specimen in a confined sleeve test under cyclic loading (test SP-25)	13
Figure 14. Interpretation of the sleeve tests with predamaged specimens	13
Figure 15. Equivalent stress at failure for different intact specimens tested in the hydraulic bomb	16

List of Figures (Cont'd)

	Page
Figure 16. MC failure points for intact specimens tested in the bomb. σ_1 and σ_3 are the maximum and minimum principal stresses respectively	18
Figure 17. DP constitutive model inferred for predamaged confined-sleeve specimens	19
Figure 18. MC constitutive models inferred for predamaged confined-sleeve specimens	19
Figure 19. Comparison of confined intact strength of two glass materials	21
Figure 20. Comparison of confined predamaged strength of two glass materials	22
Figure 21. Comparison of Mohr-Coulomb representation for intact glass	24
Figure 22. Comparison of Mohr-Coulomb representation for predamaged glass.....	24
Figure 23. Schematic of experimental arrangement.....	25
Figure 24. Position-time data for experiments into glass. The dashed lines represent regression fits through the data	26
Figure 25. Schematic of modified target arrangement (600 kV X-ray is the same as in Fig. 23).....	27
Figure 26. Time differentiation of the position-time regression fits in Fig. 24 give the tail and nose (penetration) velocities vs. time.....	27
Figure 27. Normalized residual rod length vs. time; data with centered dot represents measurements of the residual rod after recovery from the impact tank.....	27
Figure 28. Nose and tail positions versus time for impact velocity of 1.25 km/s: various Y_0 's, $\beta = 1.2$, $Y_{cap} = 1.6$ GPa	32
Figure 29. Nose and tail positions versus time for impact velocity of 1.70 km/s: various Y_0 's, $\beta = 1.2$, $Y_{cap} = 1.6$ GPa	32
Figure 30. Normalized rod length versus time for various Y_0 's, $\beta = 1.2$, $Y_{cap} = 1.6$ GPa	33
Figure 31. Equivalent stress vs. hydrostatic pressure for flyer-plate impact	35
Figure 32. Comparison of characterization data with flyer-plate impact data	38

List of Figures (Cont'd)

	Page
Figure 33. Mohr-Coulomb representation of a comparison of characterization experiments and flyer-plate impact experiments.....	39

UNCLASSIFIED

List of Tables

	Page
Table 1. Mechanical Properties for Starphire	3
Table 2. DP Parameters for Intact and Predamaged Soda-Lime Glass ($Y = Y_0 + \beta P$)	20
Table 3. MC Parameters for Intact and Predamaged Soda-Lime Glass ($\tau = c + \mu \tilde{\sigma}_n$)	20
Table 4. Mechanical Properties for Two Glasses	21
Table 5. Johnson-Cook Strength Parameters for Tungsten Alloy	30
Table 6. Johnson-Cook Damage Parameters for Tungsten Alloy.....	30

UNCLASSIFIED

UNCLASSIFIED

1.0 Introduction

Southwest Research Institute is involved in a multi-year program to investigate the ballistic response and to characterize two transparent materials that have potential application in transparent armor. Much of the work has focused on borosilicate glass; the main activities are summarized in the following paragraphs.

Impact (reverse ballistic) experiments were conducted for a gold long rod into intact lead and borosilicate glass specimens at impact velocities from 0.4 km/s to 2.8 km/s [1]. Very high-speed digital imaging and flash radiography were used to measure the nose position and rod length as a function of time, and high-speed photography was used to measure the position of the failure front (FF) as a function of time. It was found that the failure front, which propagates at a speed much faster than the penetrating rod, quickly outdistances the projectile-target interface. Thus, except for the first few moments after impact, the rod presumably penetrates failed glass.

A series of reverse ballistic experiments using short gold rods revealed further properties of the FF [2]. It was shown that the FF ceases to propagate a short time after the rod is fully eroded. The termination of the FF can reasonably be predicted in most cases with a simple model that assumes a rarefaction wave, originating at the time of complete rod erosion, propagates from the bottom of the penetration channel to the FF at a speed equal to the bulk wave speed of undamaged glass.

Using multiple short rods (again using reverse ballistics), it was shown that propagation of the FF is reinitiated if a second co-axial rod, spaced a distance from the first, impacted the glass at the bottom of the penetration channel [3]. In some experiments both rods were gold; in other experiments, one rod was copper and the other gold. Much of the observed phenomenology could be modeled assuming that the rod, either first or second, "communicates" with the FF at a speed corresponding to the bulk sound speed of the undamaged glass.

The work of Refs. [2-3] strongly suggests that the propagation of the failure front is not governed by the wave equation. A more detailed discussion is provided in Ref. [4].

Laboratory characterization of borosilicate glass was conducted at high pressures by performing compression tests under confinement. Specifically, predamaged and intact specimens were tested by two methods: triaxial compression and confined sleeve [5-7]. The experiments were analyzed and interpreted using two pressure-dependent constitutive models—Drucker-Prager and Mohr-Coulomb—and constants for the two models were derived from the results of the laboratory experiments [6-7]. Numerical simulations of the ballistic tests [1] validated the applicability of the derived constitutive models for representing the penetration response of borosilicate glass [8], under the assumption that the rod penetrates failed glass.

Experiments were performed and interpreted to elucidate the physics of material failure when glass is impacted by a rod-like projectile [9]. Partially penetrated glass targets were sectioned and examined to observe the fracture damage. Finely comminuted material in advance of arrested penetrators suggested that frictional properties of fragments influence penetration. A test was devised and applied to measure shear resistance of a bed of fragments at high pressure. The results formed the basis for a physics-based model of fracture and flow of glass ahead of a penetrator and, hence, the basis for computational simulations of projectile/armor encounters that can be used for armor design.

Dwell and interface defeat on borosilicate glass [10-11] has also been investigated. It was shown in experiments using long, gold rods, that dwell could be sustained on bare glass at impact velocities up to approximately 450 m/s. The presence of a copper buffer on the impact surface increased the velocity required for penetration by a factor of approximately 2, with a significant period of dwell at impact velocities as high as 890 m/s.

Glass impact experiments were designed at three different scales—0.22-cal, 0.375-cal, and 0.50-cal—named after the diameter of the bullets [12]. The experiments were conducted to obtain residual velocity as a function of impact (striking) velocity, including sufficient partial penetrations to calculate a V_{50} . Eroded lengths of the bullets were also measured. The objective of the experiments was to investigate whether a time dependency exists in glass damage/failure for ballistic experiments, and if so, try to quantify this dependency. A variety of scale effects were observed in the glass impact experiments, suggesting that there does exist a time dependency to failure that is important within the timeframe of ballistic events.

It was desired to characterize also a second type of transparent material, soda-lime glass. This report focuses on characterization of soda-lime glass, similar to what was performed for borosilicate glass. Triaxial compression and confined sleeve experiments were conducted on intact and predamaged material. Results are interpreted in terms of two pressure-dependent constitutive models, Drucker-Prager and Mohr-Coulomb. The response of soda-lime glass is compared and contrasted to that of borosilicate glass.

The constitutive constants inferred from the characterization experiments are then used in numerical simulations of two experiments. In these two experiments, tungsten-alloy, long-rod projectiles impacted soda-lime glass targets at impact velocities of 1.25 km/s and 1.70 km/s. Position-time and projectile residual length versus time were recorded using flash radiography. The simulations are shown to reproduce the experiments very well.

Lastly, the characterization data are compared to flyer-plate impact data. This comparison raises some questions—not resolved here—but the comparison also indicates that there is little to no strain-rate sensitivity of the strength of failed glass.

2.0 Experimental Technique

2.1 Materials

The characterization tests discussed below were performed on both intact and predamaged specimens of soda-lime glass. The brand name is Starphire[®], which is manufactured by PPG using a float process. The X-ray fluorescence analysis performed on the test samples [13] indicates an approximate composition (by weight) of: 73.2 SiO₂, 14.7 Na₂O, 1.44 Al₂O₃, 0.20 SrO, 0.08 MgO, 0.03 ZrO₂, and 0.01 or less of other trace compounds. The elastic mechanical properties of Starphire were determined by ultrasound measurements [13] and are shown in Table 1. In Table 1, the symbols denote, respectively, density ρ , elastic modulus E , the shear modulus G , Poisson's ratio ν , the longitudinal sound speed c_L , and the shear wave speed c_s .

Table 1. Mechanical Properties for Starphire

ρ (g/cm ³)	E (GPa)	G (GPa)	ν (-)	c_L (km/s)	c_s (km/s)
2.50	72.1	29.5	0.222	5.75	3.44

2.2 Overview

Triaxial compression (or confined compression) is a fairly well established technique to characterize geologic materials; for example, see Desai and Siriwardane [14]. This technique allows a straightforward determination of Drucker-Prager or Mohr-Coulomb plasticity models. This is the first time, to the authors' knowledge, that the methods have been used to characterize soda-lime glass at high pressures.

The sleeve technique, i.e., confining the specimen with a thick steel sleeve, has also been used in the past to determine pressure dependence of the yield strength. For example, Chen and Ravichandran [15-16] characterized ceramics at high strain rates and high pressures by confining them in metallic sleeves. Also, Ma and Ravi-Chandar [17], and Lu and Ravichandran [18], characterized aluminum and a metallic glass, respectively, at slow strain rates. More recently Chen and Luo [19] characterized intact and damaged ceramics under low confinement pressures at high strain rates. A confinement sleeve was also used by Forquin, *et al.* [20], in combination with numerical simulations, to characterize concrete at high pressures. Other work, all with ceramics, are described in Refs. [21-23]. In general, the above references confine the specimens at low to moderate pressures (100-300 MPa). In the present work, confinement pressures achieved—on the order of 1 GPa—are significantly higher.

Triaxial compression and confining sleeve techniques are complementary since one explores lower pressures than the other. The fact that they overlap at confining pressures of 300 to 400 MPa increases the confidence in the interpretation of the confining sleeve technique. Also, the high pressures achieved in the tests allow comparison of results with flyer-plate impact tests. Flyer-plate impact experiments interrogate material response at very high strain rates, and thus, the comparison can provide insight to strain-rate effects.

2.3 “Hydraulic Bomb” Technique

The triaxial compression test is a “classic” test used to characterize pressure-dependent materials like sands or concrete, for example, see Ref. [14]. A specimen is placed inside a thick-wall steel pressure vessel (the pressure “bomb”). The pressure bomb is placed in an MTS machine. A steel piston runs from the loading platen of the MTS machine to the specimen through an alumina-loading anvil. A hydraulic fluid, controlled by a pump, is used to load the specimen at different constant fluid pressures. An axial load is applied from the MTS machine. For simplicity, this test will be referred as “the bomb technique,” or “hydraulic bomb,” in the text.

Both intact and predamaged specimens were tested in compression inside a pressure vessel. The intact glass specimen is a cylinder with radius 3.175 mm and length 12 mm. The predamaged specimen was obtained by exposing the intact specimen to two cycles of 500° C in a furnace and suddenly placing the specimen into ice water. The predamaged specimens have non-contiguous cracks, but have strength (structural integrity) and can be handled readily without disintegrating into pieces. Dimensions (measured with a caliper) remain unchanged from the intact specimen [24].

The bomb set-up is illustrated in Fig. 1. The intact or predamaged specimens are introduced in a pressure vessel where they are loaded by a piston through two alumina anvils. The specimen is placed inside a pliable plastic (shrink tubing) sleeve to protect the specimen from the hydraulic fluid. A very thin (2-mil-thick) brass element is wrapped around the plastic sleeve to help preclude easy tearing of the plastic sleeve by glass fragments. The load is measured by a load cell placed inside the pressure vessel and wired directly to provide the equivalent stress acting on the specimen. For this simple load configuration with cylindrical symmetry the equivalent stress is:

$$\sigma_{eq} = |\sigma_z - \sigma_r| = \tilde{\sigma}_z - \tilde{\sigma}_r \quad (1)$$

where σ_z is the axial load applied through the piston and σ_r is the fluid pressure ($\sigma < 0$ in compression, $\tilde{\sigma} > 0$ in compression).

Figure 2 shows the specimen, the anvils and the load cell before being placed inside the pressure vessel. The maximum pressure reachable in the vessel is 500 MPa due to limitations of the feed-through seals. Tests were performed at fluid pressures of 25, 50, 100, 250, 400, and 500 MPa. During a preload phase the specimen axial stress and the fluid pressure are kept equal

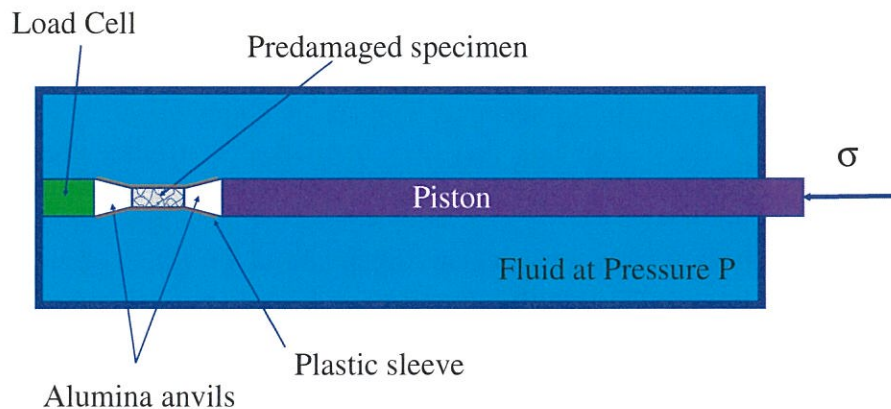


Figure 1. Hydraulic bomb experimental set-up.

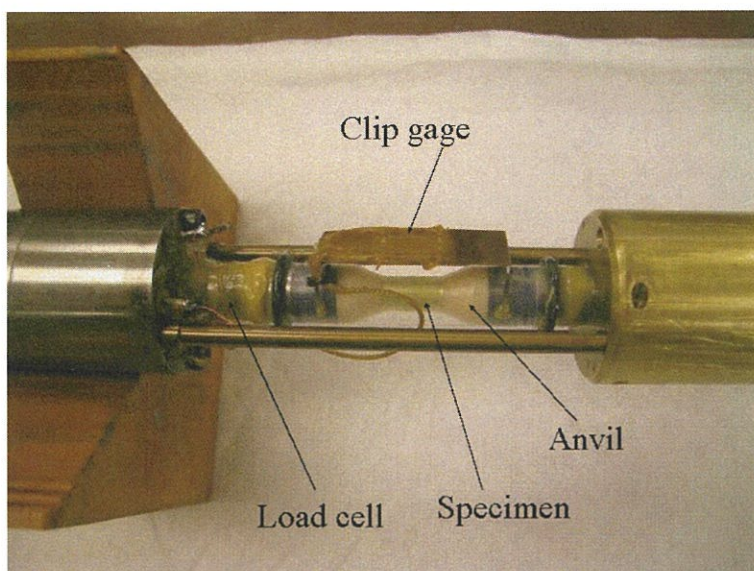


Figure 2. Specimen placement before being inserted into the pressure vessel.

(hydrostatic stress in the specimen) to avoid premature failure. When the desired confinement pressure is attained the test starts by increasing the axial load while leaving the fluid pressure constant. The nominal strain rate in this test is 0.001 s^{-1} . The axial strain of the specimen is measured during the test with a calibrated clip gage.

2.4 “Confined Sleeve” Technique

A second testing technique consists of placing the specimens inside a confinement sleeve. The experimental technique is described in Ref. [25] but is briefly presented here for completeness. The specimen is inserted into a Vascomax steel sleeve that is honed to fit the specimen. The sleeve outer diameter is 12.70 mm. An axial compressive stress is applied to the specimen with an MTS servohydraulic machine by means of two tungsten carbide (or SiC-N) platens.

The variables recorded during the test are the axial stress in the specimen measured by a load cell in the MTS machine; axial strain in the specimen, measured by a clip gage placed on the top and bottom platens; and axial and hoop strain in the sleeve, measured by a vertical and annular strain gages, respectively, on the sleeve. The hoop strain is used to infer the “internal pressure” of the sleeve, i.e., the radial stress that the specimen exerts on the sleeve, which is also the confinement pressure of the specimen. It is important to point out that the confinement pressure increases during the test. Acoustic emission was independently recorded to assist in determination of “events” that occur within the specimen during the test. The confined sleeve test is limited by the yield strength of the steel sleeve (the analysis for interpreting the data assumes that the sleeve remains elastic). Extensive discussion of interpretation and the results of this technique can be found in [24, 26-27].

The sleeve technique can also be applied to the split-Hopkinson pressure bar. In this case, the sleeved specimen is placed between the incident and output bar of the split-Hopkinson bar. The steel confining sleeve, with the hoop strain gage in place, is shown in Fig. 3. Alumina and silicon carbide anvils are used to transfer the load from the bars to the specimen.

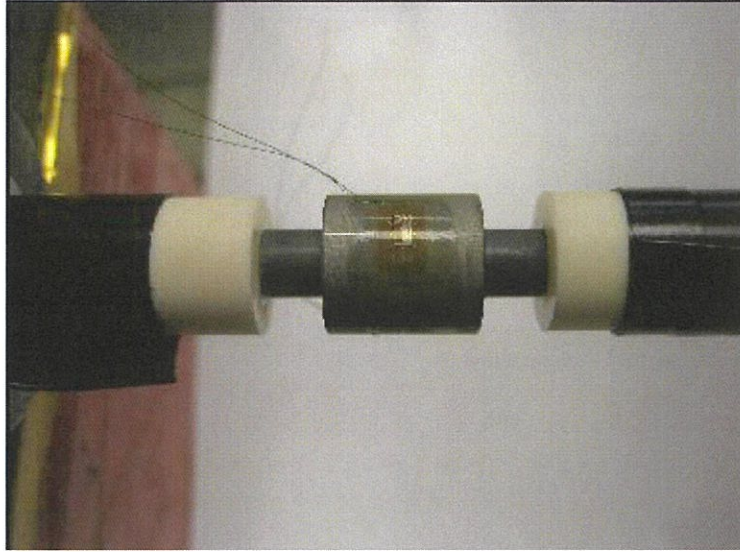


Figure 3. Confined sleeve test placed in the split-Hopkinson bar.

3.0 Results and Interpretation

3.1 Interpretation of the Hydraulic Bomb Experiments

The bomb tests are difficult but their interpretation is, in principle, unambiguous. The axial stress is directly derived from the load measured with the load cell ($\tilde{\sigma}_z = \sigma_{eq} + \tilde{\sigma}_r$) and the hydrostatic pressure on the specimen is inferred from $P = (\tilde{\sigma}_z + 2\tilde{\sigma}_r)/3$.

The primary objective of this work is to evaluate how the failure stress of intact and damaged glass varies with pressure. As it will be discussed in Section 4, two well known constitutive models—the Drucker-Prager (DP) and Mohr-Coulomb (MC) models—can be used to interpret the data. For the DP model, the failure stress of the different specimens is plotted in a σ_{eq} versus P graph. For the MC model, following Desai's [14] recommendation, the failure points are plotted on a $(\sigma_1 - \sigma_3)/2$ versus $(\sigma_1 + \sigma_3)/2$ graph, where σ_1 and σ_3 are the maximum and minimum principal stress, respectively.

3.1.1 Intact Specimens

Twenty-one tests were performed on soda-lime glass in the pressure bomb. Ten of the tests were on intact specimens; the other eleven were performed on predamaged glass. The results of four typical tests are shown in Fig. 4. Tests SP-15 and SP-13 were performed on intact specimens at confinement pressures of 100 and 250 MPa, respectively. The response is nominally linear until failure, denoted by the vertical arrows. Failure occurs suddenly (vertical arrows), and after failure, the load carrying stress is zero since the specimens “exploded” in compression.

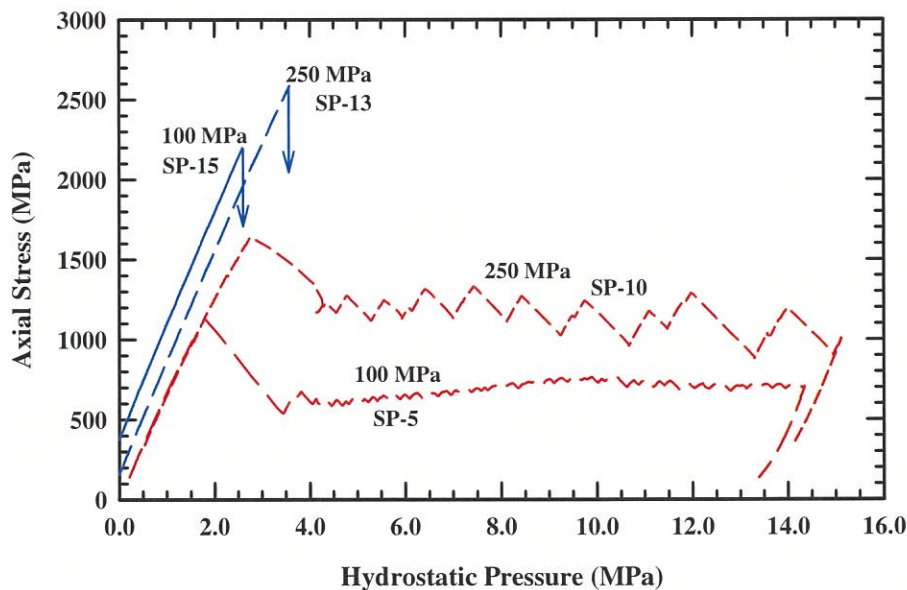


Figure 4. Two intact specimens (SP-13 and SP-15) and two predamaged specimens (SP-5 and SP-10) tested in the bomb at 100 and 250 MPa nominal confinement pressures.

3.1.2 Predamaged Specimens

Failure of predamaged specimens is very different, as shown in Fig. 4 for tests SP-5 and SP-10. Upon reaching some maximum axial stress, the load drops, but the specimen still supports a

significant amount of load for large strains. The initial drop in load carrying capability (at 2-3% axial strain) results from the formation of a shear plane, as shown in Fig. 5. The stress-strain curve after this initial failure has a sawtooth shape (particularly at the higher confinement pressure), probably because the failure surfaces of the shear plane slide over each other, occasionally “catching” and then releasing, creating the sawtooth pattern.

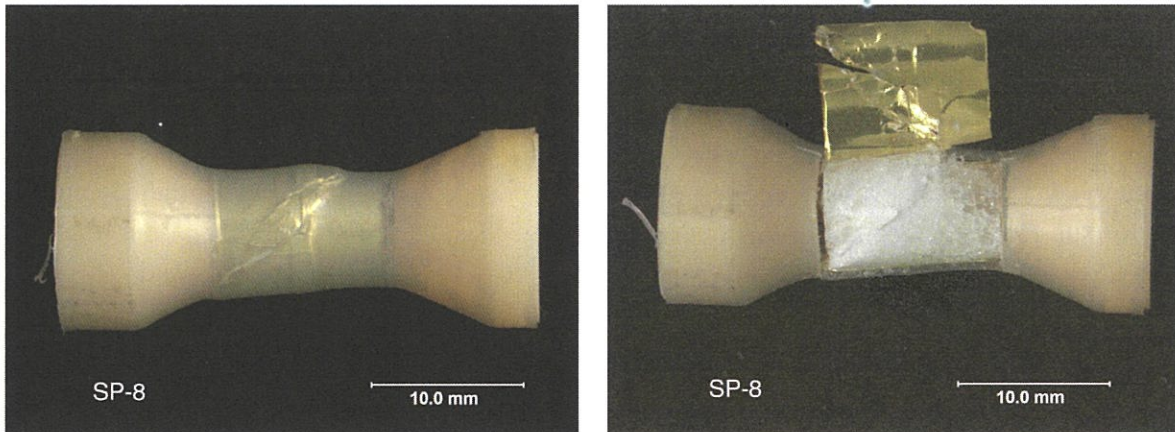
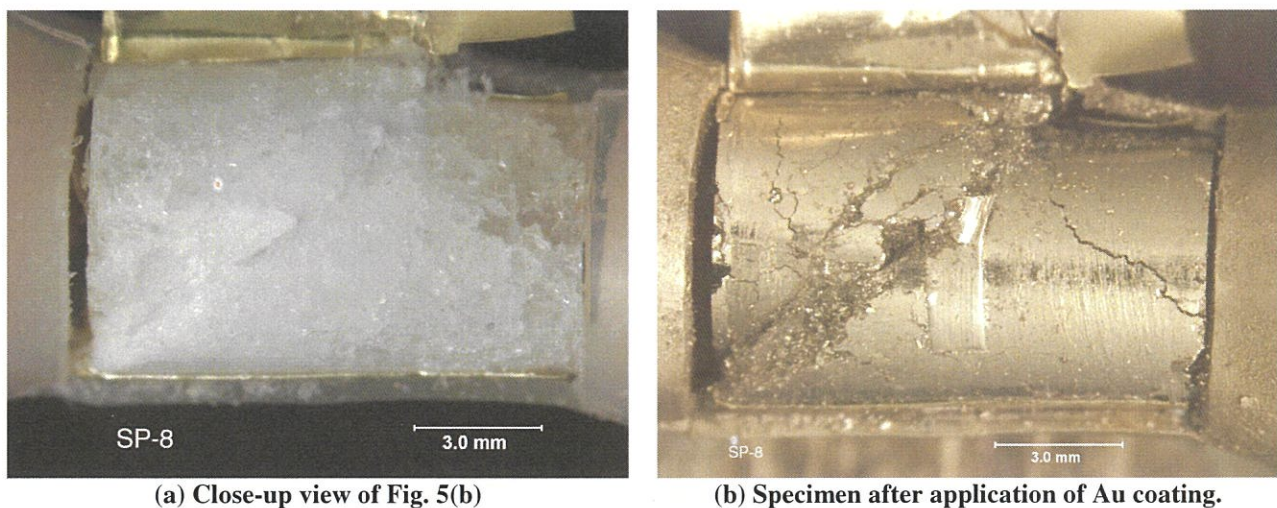


Figure 5. Predamaged specimen tested in the bomb at 100 MPa confinement pressure. The brass sleeve has been lifted to show the damaged glass in the right figure. Shear failure is evident in both figures.

Shear angles were very similar—typically 50° to 60° —for the entire 25 to 500 MPa range of confinement pressures. A close-up view of the damaged glass in Fig. 5 is shown in Fig. 6(a). The specimen is shown again in Fig. 6(b), but after being coated with gold prior to SEM evaluation. The gold highlights the shear plane as well as other cracks. Also, it is observed that much of the damage seen in Fig. 6(a) is interior to the surface (light is reflected off many fracture faces interior to the specimen), since the surface of the gold-coated specimen appears considerably more intact than the uncoated specimen.¹



(a) Close-up view of Fig. 5(b)

(b) Specimen after application of Au coating.

Figure 6. Close-up photographs of specimen SP-8 (hydraulic bomb test).

¹ A piece of tape is visible in Fig. 6(b), but is not readily apparent in the photo of Fig. 6(a). The tape was used to hold the thin brass sleeve in place as it was wrapped around the glass specimen.

3.2 Interpretation of the Confined Sleeve Experiments

A total of 15 specimens were tested in the sleeve. Four tests were on intact specimens; the other eleven were performed on predamaged glass. The equivalent stress is again $\sigma_{eq} = \tilde{\sigma}_z - \tilde{\sigma}_r$ but now the confinement pressure $\tilde{\sigma}_r$ varies during the test. If the sleeve deforms elastically and uniformly, $\tilde{\sigma}_r$ can be calculated from the elastic solution (see for example Ref. [28]):

$$\tilde{\sigma}_r = \frac{E_{sl}}{2} \frac{b^2 - a^2}{a^2} \varepsilon_\theta \quad (2)$$

where E_{sl} is the elastic modulus of the sleeve, a and b are the internal and external radii, respectively, and ε_θ is the hoop strain of the sleeve.

3.2.1 Intact Specimens

An example of a test with monotonic load of the specimen inside the steel sleeve is shown in Fig. 7. In this case there is no absolute maximum, but jumps or “failure avalanches” [29] occur. The specimen initially responds linearly elastic, as indicated by the dashed line (the slope of this line is not Young’s modulus because the specimen is under confinement, i.e., it is not uniaxial stress conditions). The maximum load attained in these tests was controlled by the operator, not the specimen strength. The jumps probably indicate the formation of a shear plane that is suddenly propagated. The planes slide against each other, but after a few microns of motion, propagation is stopped by the sleeve. Slip planes are apparent in the post-test analysis of the specimens; see Ref. [24] for an example. Note that after a stress jump, i.e., formation of a shear plane, loading occurs nominally at the same elastic slope as the initial loading. Although unloading of the specimen is not linearly elastic, it is observed that much of the unloading has a slope similar to the elastic loading response.

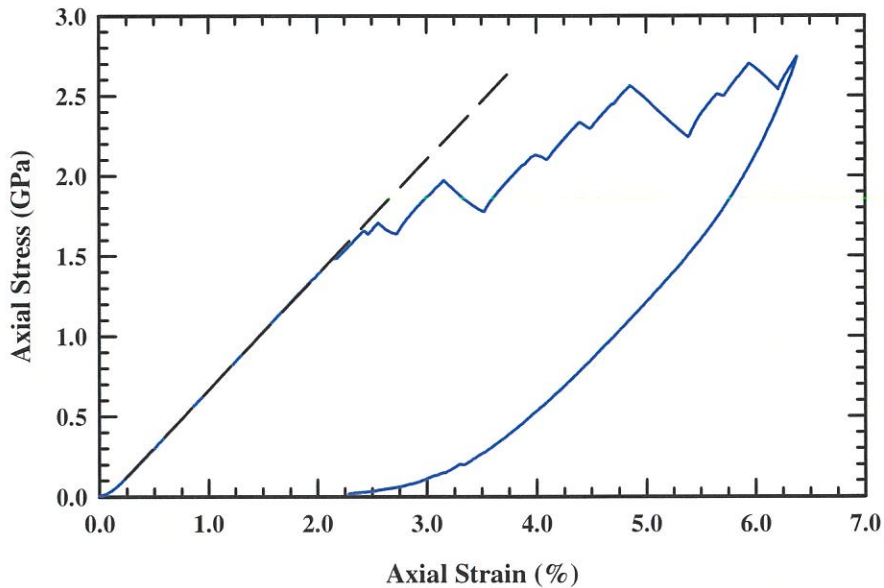


Figure 7. Stress vs. axial strain for an initially intact specimen in a confined sleeve test under monotonic load (test SP-33).

The stress jumps observed in the axial stress versus axial strain response are usually amplified on an axial stress versus hoop strain plot, Fig. 8. The hoop strain in the sleeve is very

sensitive to specimen motion, so when failure occurs in the specimen and two planes slide, the relative change in the hoop strain is large.

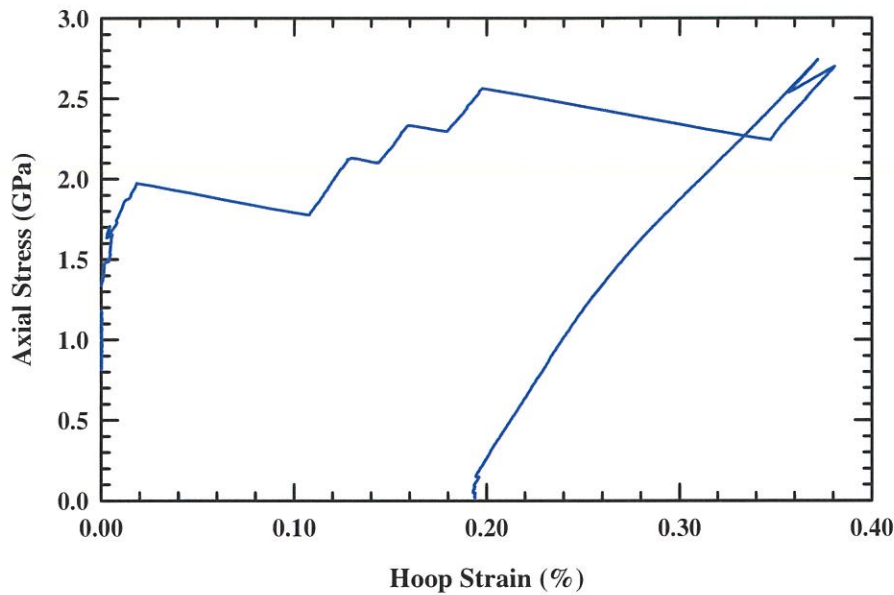


Figure 8. Stress vs. sleeve hoop strain for an initially intact specimen in a confined sleeve test under monotonic load (test SP-33).

Although the axial stress returns to zero upon unloading, there is some permanent axial and hoop strain. The change in volume can be computed from the axial and hoops strains. This is shown in Fig. 9. It is noted that after unloading, the volume has *decreased* by approximately 1%. We do not observe an increase in volume as is often proposed for bulking (an increase in volume at constant pressure, or an increase in pressure for constant volume).

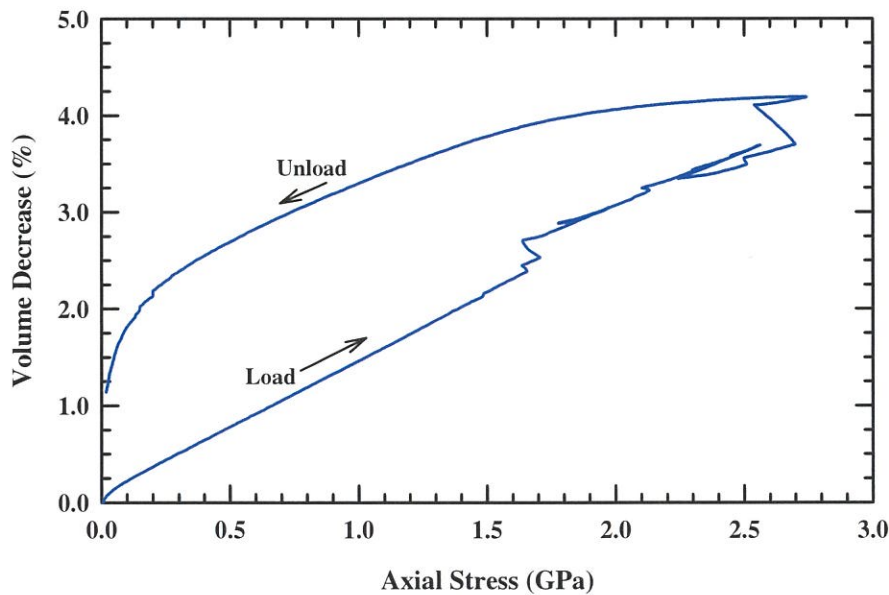


Figure 9. Volumetric decrease vs. axial stress (test SP-33).

3.2.2 Predamaged Specimens

Confined sleeve experiments were conducted on 11 predamaged specimens that were subjected to a maximum of 7 load cycles. Figures 10 and 11 show the axial stress as a function of axial strain and hoop strain, respectively, for a monotonic load. The dashed lines again provide an estimate for the onset of a nonlinear response. Sixteen load jumps were identified

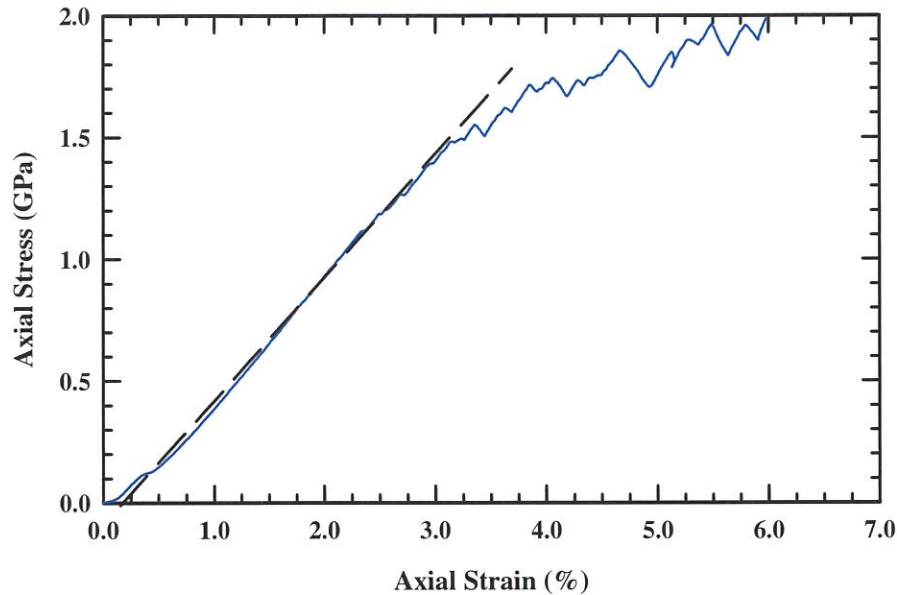


Figure 10. Stress vs. axial strain for a predamaged specimen in a confined sleeve test under monotonic load (test SP-21).

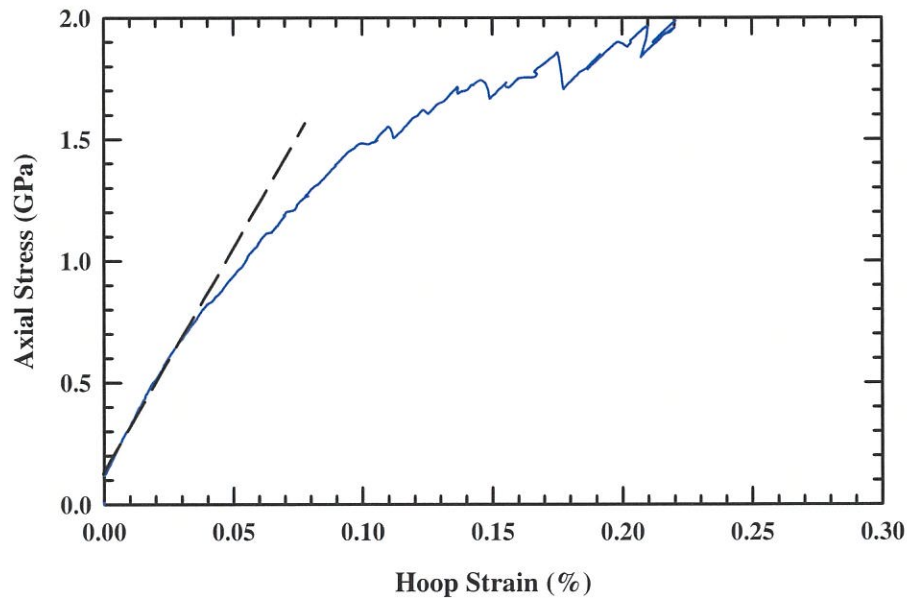


Figure 11. Stress vs. sleeve hoop strain for a predamaged specimen in a confined sleeve test under monotonic load (test SP-21).

in the test on specimen SP-21. A jump is a sudden discontinuity in the pressure applied to the specimen, probably due to the creation and slippage of an internal shear plane. The jumps occur while the applied axial load is increasing. It is thought that the jumps provide fundamental information regarding how the specimen fails at different confinement pressures. Figure 12 is a photograph that shows some of the shear planes from the test on specimen SP-25. A section of the sleeve was removed after the test to allow examination of the glass. Predamaged specimens tested in the bomb systematically showed a shear plane at an angle between 50 and 60 degrees, and seemed to be independent of the confinement pressure.

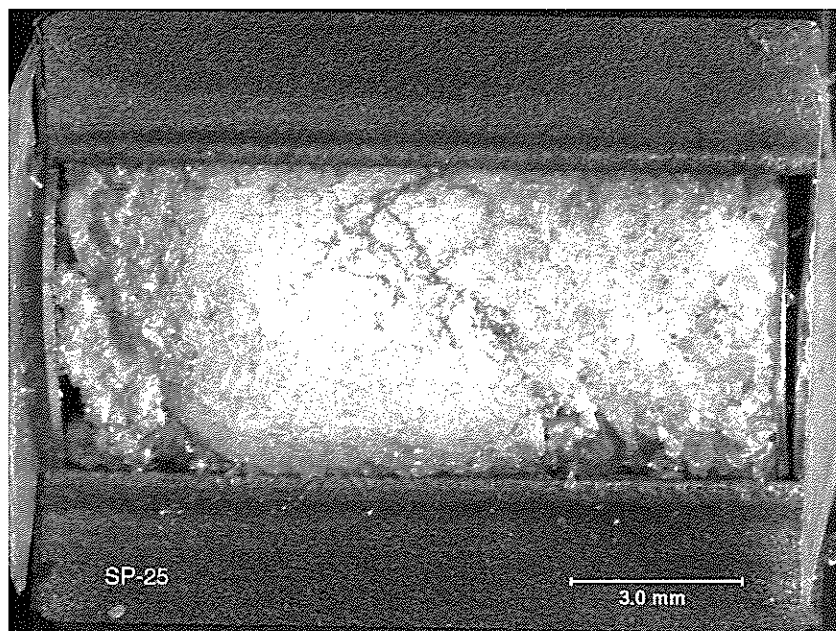


Figure 12. Cut-away view of showing shear planes in a confined sleeve experiment on predamaged glass, Exp. SP-25.

The interpretation of the jumps and how these jumps are “translated” to a constitutive model are described next. The results for a multi-load test (5 load cycles) are shown in Fig. 13, which plots the data in terms of equivalent stress versus hydrostatic pressure; there are 7 jumps shown in Fig. 13 for test SP-25. The equivalent stress is computed from Eqn. (1) once the radial stress is calculated from the hoop strain gage, Eqn. (2). The right plot of Fig. 14 is a summary of the stress-pressure jumps recorded in the confined-sleeve experiments conducted on predamaged specimens. In the example, three of the 7 jumps in test SP-25 (left figure) are used to populate the right figure. It is noted that the hydrostatic pressure is a combination of the confining pressure and the pressure generated by the axial load. Data from seven of the eleven tests are plotted (there were gage issues in four of the tests, which invalidated these data).

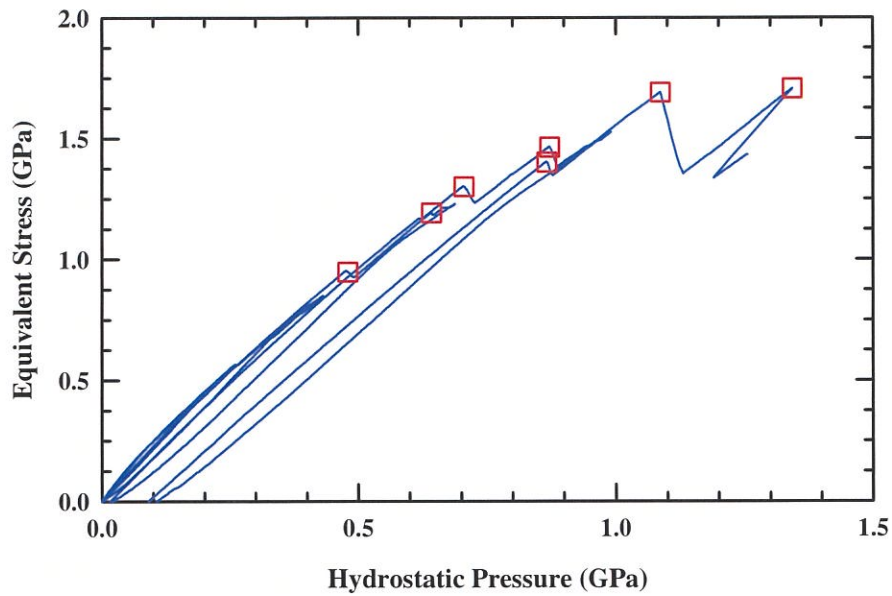


Figure 13. Equivalent stress vs. pressure for a predamaged specimen in a confined sleeve test under cyclic loading (test SP-25).

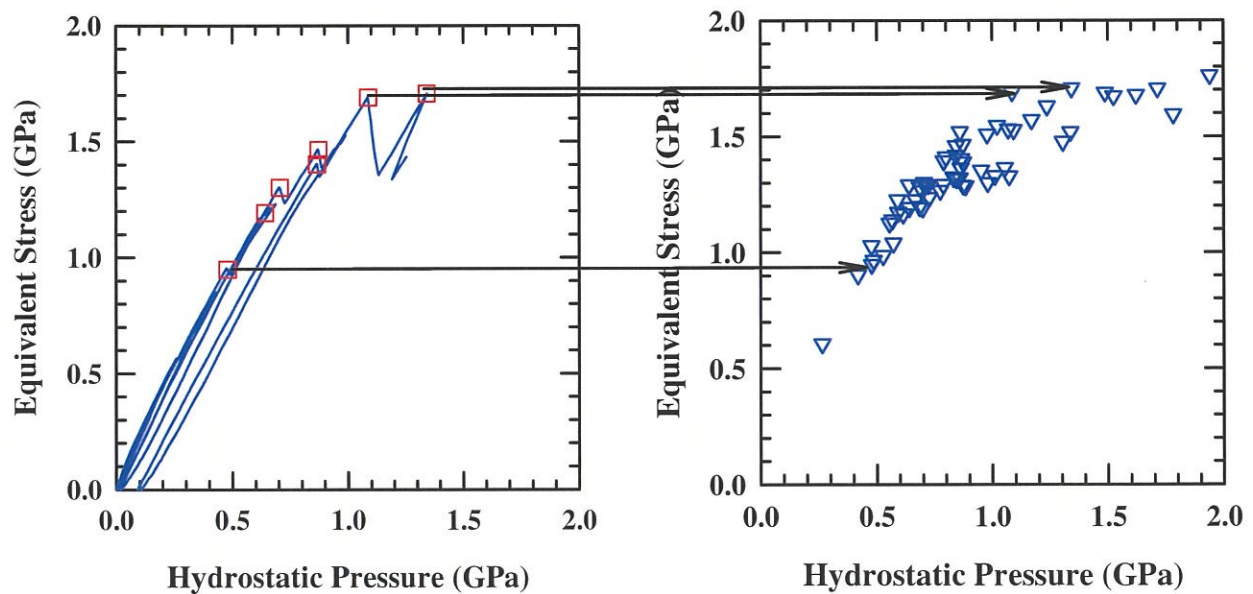


Figure 14. Interpretation of the sleeve tests with predamaged specimens.

UNCLASSIFIED

4.0 Constitutive Model Parameters

4.1 Constitutive Models

It is known that some glasses or ceramics under confinement can flow plastically. For example, Peter [30] mentions that not all the flow phenomena in glasses can be explained as densification and argues that some glasses show properties compatible with classic plasticity. Other arguments for plastic flow can be found in Refs. [31-32]. Here, however, whenever the Drucker-Prager or Mohr-Coulomb models are mentioned, it is understood that they represent a surface that, when used in numerical simulations, are used to limit material strength. For intact material, this might be thought of as a failure surface; for predamaged material, it might be a flow surface. We use the term “failure” to signify a change in load-carrying capability of the material. We do not provide a prescription for how the strength might change as a function of damage, as done by others, e.g., Holmquist and Johnson [33-34]. Rather, the data will be analyzed to determine constants that describe the limits in load-carrying capability of intact glass and for damaged glass.

4.1.1 Drucker-Prager Model

The Drucker-Prager (DP) model [35] has the form:

$$Y = \begin{cases} Y_0 + \beta P & P < (Y_{cap} - Y_0)/\beta \\ Y_{cap} & P \geq (Y_{cap} - Y_0)/\beta \end{cases} \quad (3)$$

where Y_0 is the zero-pressure strength, β is the slope of Y versus P , P is the hydrostatic pressure (negative of the mean stress), and Y_{cap} is the limiting flow stress.

4.1.2 Mohr-Coulomb Model

Predamaged specimens tested in the bomb systematically showed a shear plane at an angle between 50 and 60 degrees. The angle seems to be independent of the applied confinement pressure. The DP model is based on the first invariant of the stress tensor, I_1 , and the second invariant of the stress deviator tensor, J_2 . The flow surface in the π -plane is a circle, and thus, the DP model can never have a characteristic failure angle. Incorporating the third invariant J_3 into the description of failure results in flow surface on the π -plane being a polygon, which then has a characteristic failure angle. The Mohr-Coulomb (MC) model incorporates J_3 , and has a characteristic failure angle independent of the confinement pressure (see Ref. [36]); thus, it was felt that the MC model could be an appropriate candidate for describing the response of glass.

The MC model gives the maximum shear stress, τ , that the glass can support on any plane:

$$\tau = c + \mu \tilde{\sigma}_n \quad (4)$$

where c is the cohesion, $\mu \equiv \tan(\phi)$ is the friction coefficient (ϕ is the friction angle), and $\tilde{\sigma}_n$ the normal stress (positive in compression). If there is a cap, then the MC model has a form similar to Eqn. (3):

$$\tau = \begin{cases} c + \mu \tilde{\sigma}_n & \tilde{\sigma}_n < (\tau_{cap} - c)/\mu \\ \tau_{cap} & \tilde{\sigma}_n \geq (\tau_{cap} - c)/\mu \end{cases} \quad (5)$$

4.2 Hydraulic Bomb Tests Results

4.2.1 Drucker-Prager Analysis

Failure data obtained from tests like the ones shown in Fig. 4 were plotted in an equivalent stress versus hydrostatic pressure graph, as shown in Fig. 15. The solid squares indicate intact unconfined tests, i.e., uniaxial stress tests, for which the load path is a straight line with a slope of three. For unconfined specimens the scatter in equivalent stress is very large, approximately from 1.3 GPa to 2.2 GPa. This scatter is inherent to brittle materials like glass. All the intact confined tests are denoted by the solid triangles. In general, the scatter seems to decrease with increasing confinement. Two of the datum points, at hydrostatic pressures of approximately 670 and 900 MPa, have strengths below that of one of the unconfined intact points. These two datum points were generated with the lowest confinement pressures of 25 MPa. However, as the confinement pressure is increased, the strength of the glass is nominally linear with hydrostatic pressure, as denoted by the long dashed line in Fig. 15 (the two lowest datum points were not included in the linear regression analysis). The data in Fig. 15 suggests that an appropriate DP model for the intact specimens is:

$$\text{Confined Intact Strength: } Y = 1.20 + 1.26P \quad (\text{GPa}) \quad (6)$$

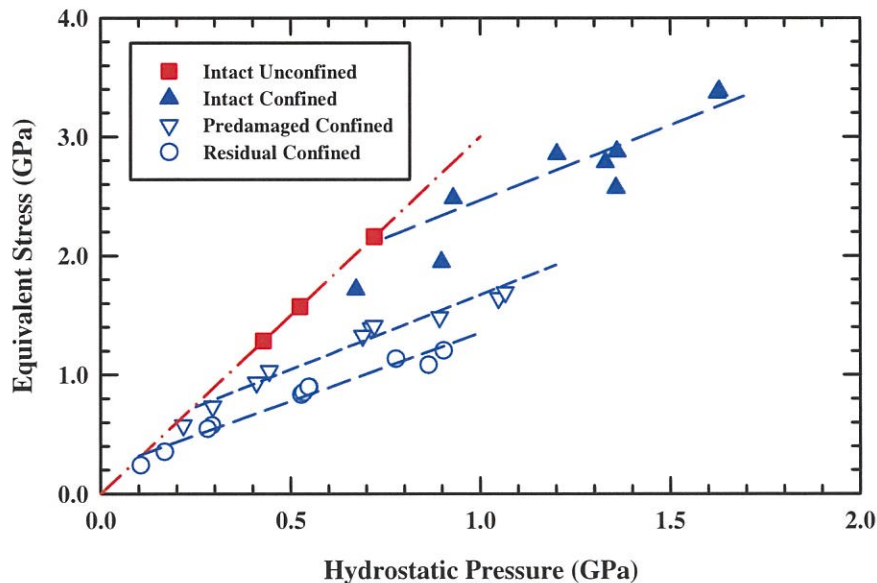


Figure 15. Equivalent stress at failure for different intact specimens tested in the hydraulic bomb.

Predamaged specimens have a very different behavior, since after failing the specimen is still able to carry load², as shown in Fig. 4. The peak loads of the 11 tests performed are shown in Fig. 15 as the inverted open triangles. A linear regression fit performed through these data provides the following DP model for predamaged specimens:

$$\text{Confined Predamaged Strength: } Y = 0.416 + 1.25P \quad (\text{GPa}) \quad (7)$$

The regression fit is shown by the short dashed line.

² Technically, the specimens that were initially intact and then fail could also carry a load, but failure is so catastrophic that the plastic sleeve is ripped apart and the comminuted specimen is dispersed in the hydraulic fluid.

The residual strength constants for the hydraulic bomb tests were obtained from the same tests that were used to estimate the predamaged constants, but using the sawtooth portions of the response. Analysis of the sawtooth portion of the curves of Fig. 4 is not straightforward since the specimen at this stage probably has a large amount of damage. It was decided to select an average of the sawtooth response; these points are plotted as σ_{eq} versus P in Fig. 10 as the open circles. A linear regression was performed on these data giving:

$$\text{Confined Residual Strength: } Y = 0.20 + 1.15P \quad (\text{GPa}) \quad (8)$$

As was noted for borosilicate glass [6-7], the slopes for the intact, predamaged and residual strength are very similar (~ 1.2), but the zero-pressure intercept depends upon the degree of damage.

4.2.2 Mohr-Coulomb Analysis

When performing numerical simulations of the bomb test it was realized that a DP model can never have a characteristic failure angle since the DP model is based only on I_1 and J_2 , the first and second invariants of the stress deviator tensor. A characteristic angle can arise only if the flow surface on the π -plane is not a circle but a polygon, i.e., the third invariant J_3 enters in the flow surface equation. The Mohr-Coulomb model includes J_3 , and a characteristic failure angle independent of the confinement pressure arises naturally from the model (see Ref. [36]).

The same intact data shown on Fig. 15 are now analyzed and plotted from a MC perspective in Fig. 16. The MC model gives the maximum shear stress τ a solid can support on any plane, Eqn. (4). The unconfined intact data has a slope of 1.0. A linear least squares regression³ in the $(\sigma_1 - \sigma_3)/2$ versus $(\sigma_1 + \sigma_3)/2$ representation was performed on the confined intact data (ignoring the same two datum points as was done for the DP analysis), giving an intercept of $a = 0.415$ GPa and the slope $b = 0.546$. The relation between these curve-fit constants and MC parameters, Eqn. (4), is given by:

$$\phi = \arcsin(b) \quad \mu = \tan(\phi) \quad c = a/\cos(\phi) \quad (9)$$

so for the intact confined specimens an MC model can be written as:

$$\text{Confined Intact Strength: } \tau = 0.495 + 0.652 \tilde{\sigma}_n \quad (\text{GPa}) \quad (10)$$

Similarly, regression analysis of the confined predamaged (the inverted open triangles) gives a value of 0.162 GPa for the intercept and 0.522 for the slope. Regression analysis of the confined residual strength data (the open circles) gives a value of 0.0842 GPa for the intercept and 0.484 for the slope. Using Eqn. (9) to convert these curve fit parameters to the form of Eqn. (4) gives:

$$\text{Confined Predamaged Strength: } \tau = 0.190 + 0.612 \tilde{\sigma}_n \quad (\text{GPa}) \quad (11)$$

$$\text{Confined Residual Strength: } \tau = 0.096 + 0.553 \tilde{\sigma}_n \quad (\text{GPa}) \quad (12)$$

As with the DP analysis, it is observed that the friction coefficients for the intact, predamaged, and residual glass are very similar (~ 0.6), while the cohesion decreases with increasing levels of damage.

³ Regression fits are done in the form of $y = a + bx$.

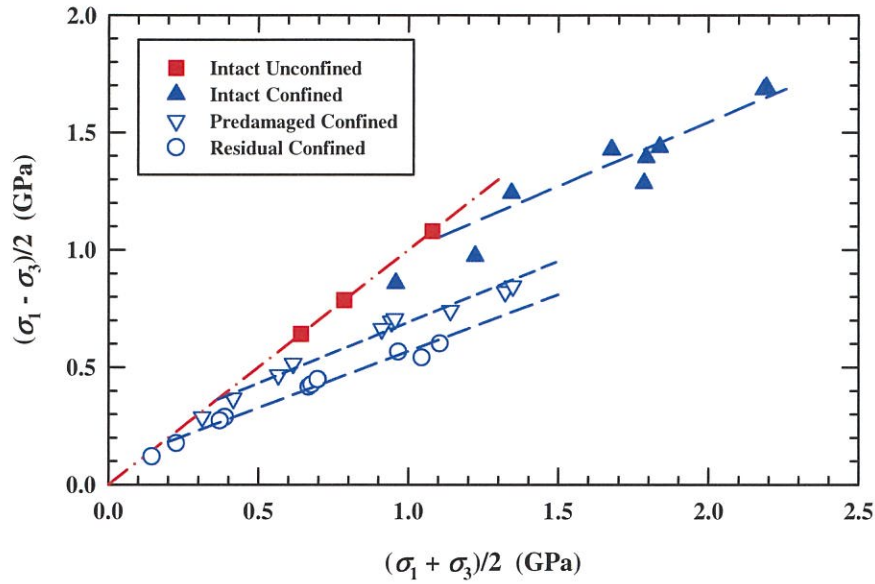


Figure 16. MC failure points for intact specimens tested in the bomb. σ_1 and σ_3 are the maximum and minimum principal stresses respectively.

4.3 Confined Sleeve Tests Results

4.3.1 Drucker-Prager Analysis

As described in Section 4.2, the jumps recorded in the confined sleeve tests were transferred to an equivalent stress versus pressure plot for 7 experiments. Results for predamaged specimens confined in the sleeve are shown in Fig. 17.⁴ The results of one test, SP-21, appear to have an anomaly. The data for pressures lower than approximately 0.9 GPa fall within the scatter of the other specimens. However, for whatever reason, the strength reaches a plateau (a cap) of approximately 1.3 GPa (the solid inverted triangles), which lies below the strength inferred from the other test data. The SP-21 data represented by the solid inverted triangles were ignored in the subsequent analysis.

It is observed that the strength (equivalent stress) reaches a plateau at a pressure of approximately 1.0 GPa, after which the strength is independent of the pressure, i.e., a cap. There is scatter in the data, but this cap has a value of 1.61 ± 0.08 GPa, where the 0.08 GPa represents one standard deviation from the average.

Linear regression was used to obtain an $\sigma_{eq} - P$ relationship for pressures below 1.0 GPa. The resultant regression fit is:

$$\text{Confined Predamaged Strength: } Y = \begin{cases} 0.411 + 1.19P & P \leq 1.01 \text{ GPa} \\ 1.61 \text{ GPa} & P > 1.01 \text{ GPa} \end{cases} \quad (13)$$

⁴ A few intact specimens were tested in the sleeve, but as found previously for borosilicate glass [6], any small gap between the intact specimen and sleeve (eccentricity, misalignment, etc.) can make a large difference in the confinement pressure and hence strength, and the glass can fail before it contacts the sleeve. Thus, no further analysis was conducted on intact sleeve specimens.

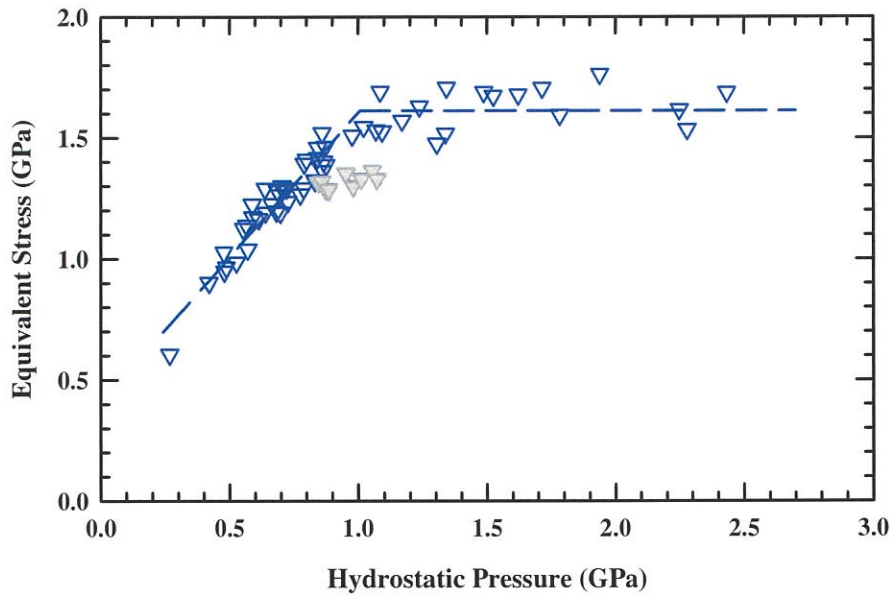


Figure 17. DP constitutive model inferred for predamaged confined-sleeve specimens.

4.3.2 Mohr-Coulomb Analysis

Figure 18 shows the same data presented in the previous section but analyzed from the MC perspective. A linear least squares regression in the $(\sigma_1 - \sigma_3)/2$ versus $(\sigma_1 + \sigma_3)/2$ representation, ignoring the same solid inverted triangles as was done for the DP analysis, gives 0.143 GPa for the intercept and 0.535 for the slope. The shear stress achieves a maximum value (a cap) in

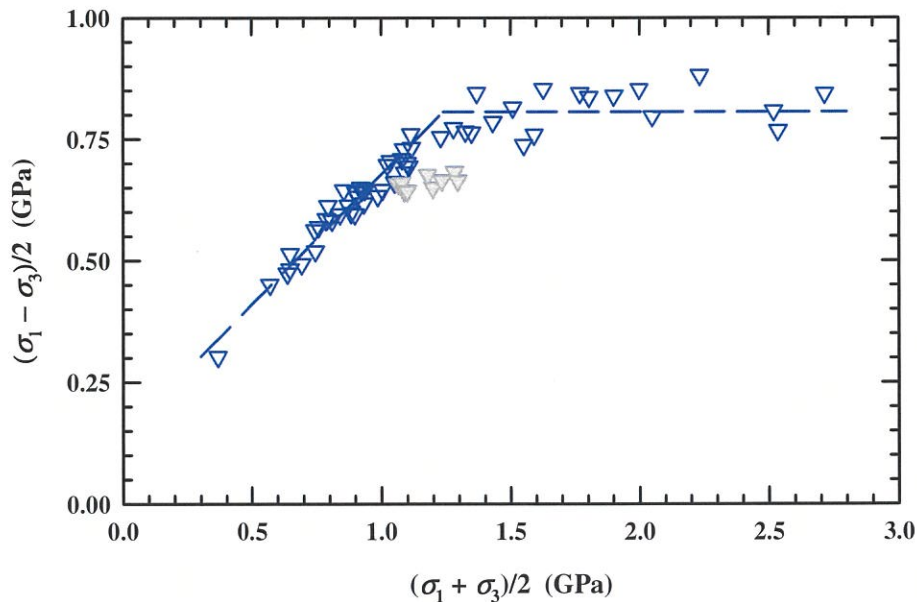


Figure 18. MC constitutive models inferred for predamaged confined-sleeve specimens.

Fig. 18 of 0.81 GPa at a normal stress of approximately 1.0 GPa—or $\frac{1}{2}(\sigma_1 + \sigma_3) = 1.25$ GPa. Using Eqn. (9) to convert these curve fit parameters to the form of Eqn. (4) gives for the confined predamaged strength:

$$\text{Confined Predamaged Strength: } \tau = \begin{cases} 0.169 + 0.633\tilde{\sigma}_n & \tilde{\sigma}_n \leq 1.01 \text{ GPa} \\ 0.81 \text{ GPa} & \tilde{\sigma}_n > 1.01 \text{ GPa} \end{cases} \quad (14)$$

4.4 Summary of Constitutive Model Results

The constitutive parameters for the Drucker-Prager model and the Mohr-Coulomb model are summarized for intact, predamaged, and residual strength soda-lime glass in Tables 2 and 3, respectively. In the tables, P_{max} and $\tilde{\sigma}_{max}$ represent the pressure and normal stress, respectively, for the onset of the cap. A few observations can be made. First, the parameters were determined by least-squares regression fits to the experimental data. Although three significant figures are given for the various parameters, the parameters should only be considered to be accurate to—at best—two significant figures.

It is quite satisfying that the two experimental methods (the bomb tests and the confined sleeve tests) yield comparable values for the constitutive parameters. This provides confidence in the results of the confined sleeve experimental procedures and interpretation. Additionally and as already pointed out, it is rather remarkable that β for the Drucker-Prager model and μ for the Mohr-Coulomb model seem to be relatively independent of the damage, whereas Y_0 and c decrease with increasing damage.

Table 2. DP Parameters for Intact and Predamaged Soda-Lime Glass ($Y = Y_0 + \beta P$)

Specimen	Bomb Tests		Sleeve Tests			
	Y_0 (GPa)	β	Y_0 (GPa)	β	P_{max} (GPa)	Y_{cap} (GPa)
Intact	1.20	1.26	—	—	—	—
Predamaged	0.416	1.25	0.411	1.19	1.01	1.61
Residual	0.20	1.15	—	—	—	—

Table 3. MC Parameters for Intact and Predamaged Soda-Lime Glass ($\tau = c + \mu \tilde{\sigma}_n$)

Specimen	Bomb Tests		Sleeve Tests			
	c (GPa)	μ	c (GPa)	μ	$\tilde{\sigma}_{max}$ (GPa)	τ_{cap} (GPa)
Intact	0.495	0.652	—	—	—	—
Predamaged	0.190	0.612	0.169	0.633	1.01	0.81
Residual	0.096	0.553	—	—	—	—

5.0 Comparison of Starphire and Borofloat Response

The preceding section describes the pressure-dependent response results for the soda-lime glass, Starphire[®]. Previously, the pressure-dependent response for a borosilicate glass, Borofloat[®]33, was determined [6-7]. This section provides a direct comparison of the two materials. Some of the information already presented for Starphire will be repeated here to facilitate direct comparison of the two glasses.

The composition of the two glasses is as follows:

Starphire[®]: 73.2 SiO₂, 14.7 Na₂O, 1.44 Al₂O₃, 0.20 SrO, 0.08 MgO, 0.03 ZrO₂

Borofloat[®]33: 80.5 SiO₂, 12.7 B₂O₃, 2.5 Al₂O₃, 3.5 Na₂O, 0.64 K₂O

The mechanical properties are summarized in Table 4.

Table 4. Mechanical Properties for Two Glasses

	ρ (g/cm ³)	E (GPa)	G (GPa)	ν (-)	c_L (km/s)	c_s (km/s)
Starphire [®]	2.50	72.1	29.5	0.222	5.75	3.44
Borofloat [®] 33	2.22	62.3	26.0	0.20	5.61	3.41

5.1 Drucker-Prager Representation

The intact strengths (equivalent stress) as a function of hydrostatic pressure are shown in Fig. 19. The dashed-dot line, which goes through the unconfined tests, has a slope of 3. There is considerable scatter in the compressive strength of the unconfined glasses, but on average, the

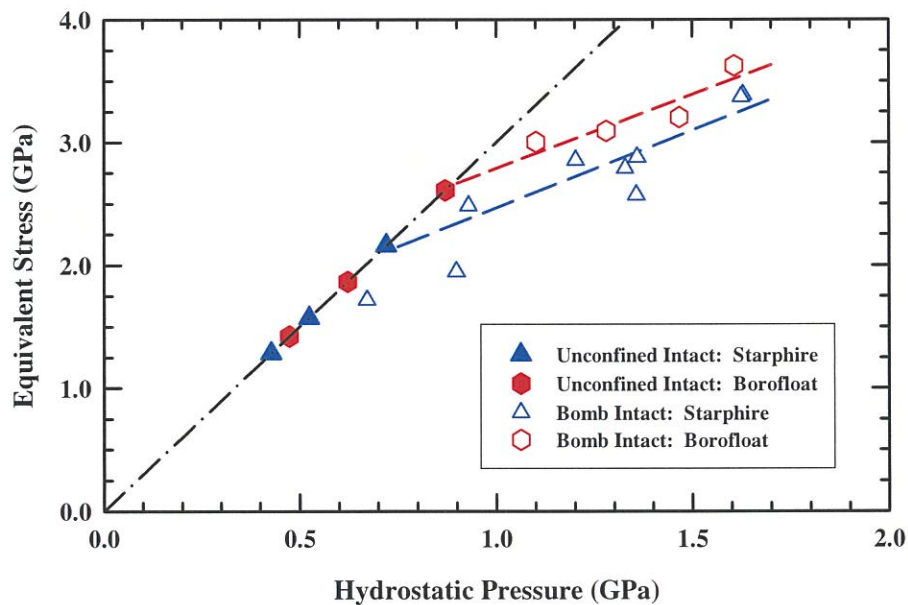


Figure 19. Comparison of confined intact strength of two glass materials.

unconfined strength of Borofloat is larger than that of Starphire. Also, it is seen that the confined strength, as a function of pressure, is larger for Borofloat than for Starphire. The linear

regression fits to the intact specimens tested in the hydraulic bomb are given in Eqns. (15) and (16). The pressure dependency is similar for the two glasses, i.e., the slopes are approximately the same, but because Borofloat is stronger, it has a larger “zero-pressure” intercept.

$$\text{Intact Starphire:} \quad Y = 1.20 + 1.26P \quad (\text{GPa}) \quad (15)$$

$$\text{Intact Borofloat:} \quad Y = 1.59 + 1.20P \quad (\text{GPa}) \quad (16)$$

It is interesting to note that the regression fits closely go through the highest measured unconfined intact strength for each of the two glasses.

The strengths of the damaged glass are shown in Fig. 20. These are the data for the predamaged specimens; the data from the hydraulic bomb and the confined sleeve are plotted. Again, it is noted that damaged Borofloat glass is stronger than damaged Starphire, both in the pressure-dependent region and the cap. Regression fits for the bomb data and the confined sleeve data are given in Eqns. (17) and (18), respectively, for Starphire, and Eqns. (19) and (20) for Borofloat.

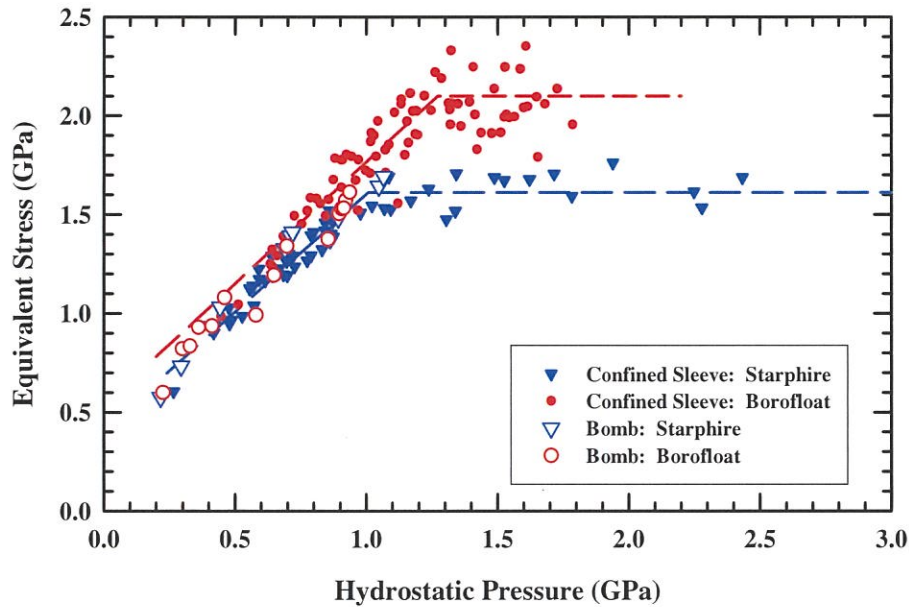


Figure 20. Comparison of confined predamaged strength of two glass materials.

Starphire Confined Predamaged Strength:

$$\text{Hydraulic Bomb:} \quad Y = 0.416 + 1.25P \quad (\text{GPa}) \quad (17)$$

$$\text{Confined Sleeve:} \quad Y = \begin{cases} 0.411 + 1.19P & P \leq 1.01 \text{ GPa} \\ 1.61 \text{ GPa} & P > 1.01 \text{ GPa} \end{cases} \quad (18)$$

Borofloat Confined Predamaged Strength:

$$\text{Hydraulic Bomb:} \quad Y = 0.423 + 1.22P \quad (\text{GPa}) \quad (19)$$

$$\text{Confined Sleeve:} \quad Y = \begin{cases} 0.535 + 1.23P & P \leq 1.3 \text{ GPa} \\ 2.10 \text{ GPa} & P > 1.3 \text{ GPa} \end{cases} \quad (20)$$

There are several additional observations that can be made. Although there is less scatter in the Starphire data than for the Borofloat data (most obvious in the cap), this is probably largely due to a more refined analysis of the Starphire data; that is, elimination of data points that were “questionable” after further analysis of the data.⁵ In general, the data from the confined sleeve tests are in good agreement (within data scatter) of the hydraulic bomb tests in the region of overlap. Confinement pressures in the hydraulic bomb are limited to approximately 500 MPa (at higher pressures, the seals leak). However, higher confinement pressures can be achieved in the confined sleeve test (to the yield limit of the Vascomax steel).⁶ Because the cap is lower for the Starphire material, larger hydrostatic pressures can be achieved in the confined sleeve test.

The regression fits have uncertainties associated with the coefficients because of data scatter. Although the regression fits are given to three significant figures, the numbers are, at best, accurate to two significant figures. It is observed that the slopes of the intact and predamaged curves are very similar, with the primary difference being that the zero-pressure intercept is lower for the material that has more damage.⁷ Similar behavior has been observed in intact and damaged marble [37]. It is thus concluded that the pressure-dependent slope (β) is independent of damage, but that the zero-pressure intercept (Y_0) decreases as damage increases. As was seen in numerical simulations of ballistic experiments for Borofloat, and will be seen in the next Section in order to match data for ballistic penetration of soda-lime glass, Y_0 had to be significantly decreased for highly damaged (comminuted) glass.

Another observation, which is perhaps only coincidence, is that the pressure-dependent slopes of the two glasses are very similar. Perhaps this is because the chemical composition of the two materials is similar (~75% SiO₂, with various amounts of Na₂O and Al₂O₃, along with other chemical constituents), and the process for making the glasses is similar. On the other hand, the chemical constituents and/or processing techniques do make a difference in the strength of the cap of the failed material.

5.2 Mohr-Coulomb Representation

The same data comparisons, but in the Mohr-Coulomb representation, are shown in Figs. 21 and 22, along with a summary of the equations that provide regression fits to the data:

$$\text{Intact Starphire:} \quad \tau = 0.495 + 0.652 \tilde{\sigma}_n \quad (\text{GPa}) \quad (21)$$

$$\text{Intact Borofloat:} \quad \tau = 0.755 + 0.587 \tilde{\sigma}_n \quad (\text{GPa}) \quad (22)$$

Starphire Confined Predamaged Shear Strength

$$\text{Hydraulic Bomb:} \quad \tau = 0.190 + 0.612 \tilde{\sigma}_n \quad (\text{GPa}) \quad (23)$$

$$\text{Confined Sleeve:} \quad \tau = \begin{cases} 0.169 + 0.633 \tilde{\sigma}_n & \tilde{\sigma}_n \leq 1.01 \text{ GPa} \\ 0.81 \text{ GPa} & \tilde{\sigma}_n > 1.01 \text{ GPa} \end{cases} \quad (24)$$

⁵ Many of the analysis procedures, particularly for the confined sleeve test, were developed while testing Borofloat. These analysis procedures were considerably more mature when they were applied to the Starphire material.

⁶ As a reminder, the hydrostatic pressure is larger than the confinement pressure since the hydrostatic pressure is a combination of the confinement pressure and the axial load.

⁷ There are comparable curves for the residual strength, as in the lower curve (the open circles) in Fig. 15; although not shown, these results support the general conclusions and observations.

Borofloat Confined Predamaged Shear Strength

$$\text{Hydraulic Bomb:} \quad \tau = 0.201 + 0.594 \tilde{\sigma}_n \quad (\text{GPa}) \quad (25)$$

$$\text{Confined Sleeve:} \quad \tau = \begin{cases} 0.246 + 0.609 \tilde{\sigma}_n & \tilde{\sigma}_n \leq 1.3 \text{ GPa} \\ 1.05 \text{ GPa} & \tilde{\sigma}_n > 1.3 \text{ GPa} \end{cases} \quad (26)$$

The same comments made in the previous subsection concerning the response of Borofloat also apply to the Mohr-Coulomb representation.

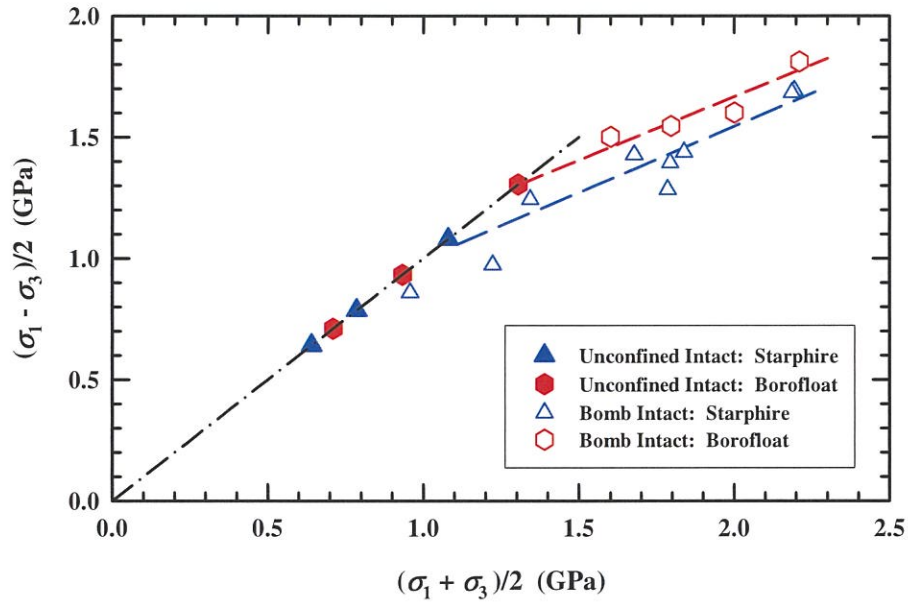


Figure 21. Comparison of Mohr-Coulomb representation for intact glass.

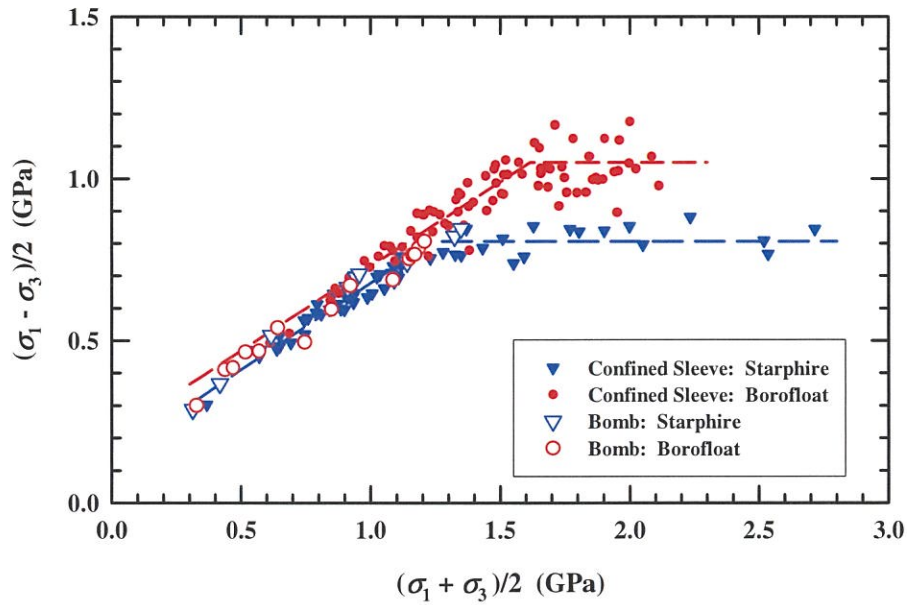


Figure 22. Comparison of Mohr-Coulomb representation for predamaged glass.

6.0 Numerical Simulations of Ballistic Penetration

6.1 Introduction

The ultimate objective of this project is to understand the behavior of glass and to be able to simulate penetration phenomena. Numerical simulations require constitutive models and material constants for the materials typically in the form of an equation of state and a plasticity model. In the preceding sections, constitutive constants were determined for two pressure-dependent constitutive models using laboratory characterization experiments. We now examine how well these constants, when used in numerical simulations, replicate ballistic experiments.

6.2 The Experiments

The experiments were conducted by Hohler and Stilp at the Fraunhofer Institut für Kurzzeiddynamik, Ernst-Mach-Institut (EMI) [38-39]. The blunt-nose, tungsten-alloy (density 17.6 g/cm^3) projectiles had a length-to-diameter ratio (L/D) of 12.5, and a diameter of 5.8 mm. The targets were fabricated from sheets of soda-lime (float) glass, density 2.5 g/cm^3 , with a compressive strength of 0.90 GPa. Ten sheets of 1.9-cm-thick glass and one sheet of 1.0-cm-thick glass were bonded together using double-sided adhesive (0.014-cm thick), for a total thickness of 20.0 cm. The glass was backed by a block of mild steel, as shown in Fig. 23. The lateral dimensions for most of the glass targets were 15 cm by 15 cm. Two of the experiments were performed with 30-cm x 30-cm wide glass plates to investigate whether there was any influence of the lateral dimensions. The experimental results were independent of this doubling of the lateral dimensions.

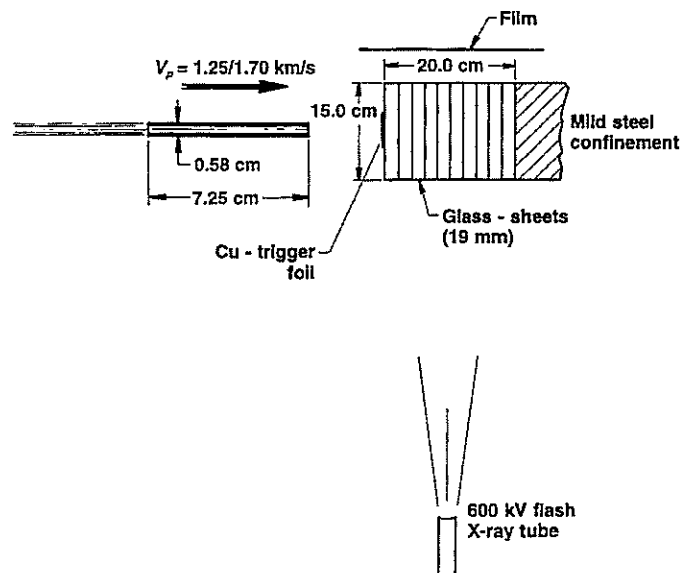


Figure 23. Schematic of experimental arrangement.

Flash radiography was used to record the positions of the front and tail of the projectile, as well as the length of the projectile, as a function of time after impact. The nominal impact velocities for two sets of experiments were 1.25 km/s and 1.70 km/s.

The position-time of the nose and tail of the rods are shown in Fig. 24 for the two sets of experiments. Each pair of points (nose and tail positions) represents one experiment. The

experimental spread in the impact velocities was ± 20 m/s. The experimental data were adjusted by a simple proportionality to bring all data to a common impact velocity of 1.25 or 1.70 km/s. In neither set of experiments did the projectile penetrate to the mild steel block. Near the end of penetration, the eroded rods began tumbling (two of the last three data pairs at 1.70 km/s, and the last three data pairs at 1.25 km/s). Penetration depths for a “non-tumbling” rod were estimated for these points.

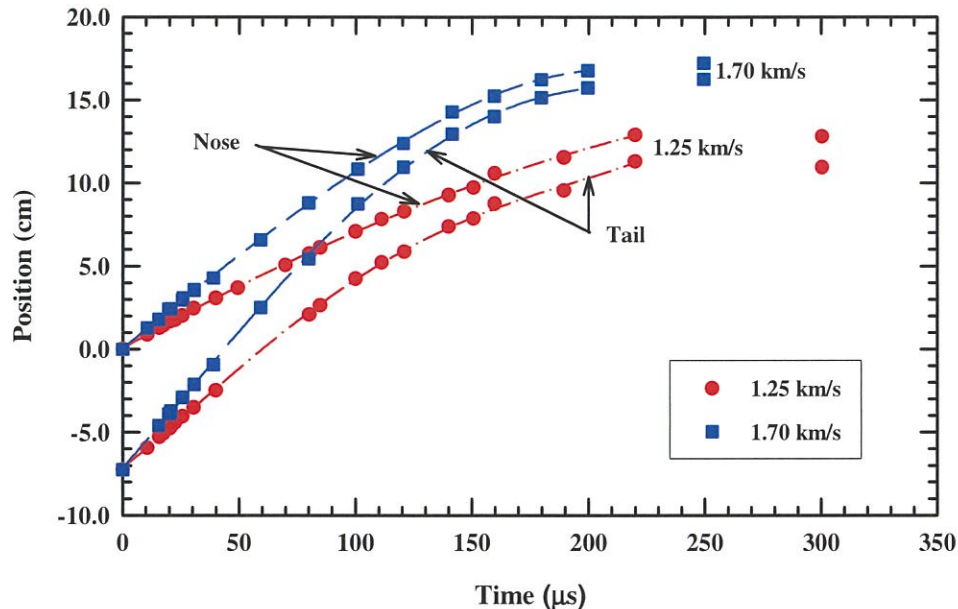


Figure 24. Position-time data for experiments into glass. The dashed lines represent regression fits through the data.

A modified target was constructed for some of the tests to provide an estimate of the projectile velocity at different depths of penetration. This experimental configuration is shown in Fig. 25. The glass portion of the target had different thicknesses, and the projectile was allowed to exit the target through a 3.0-cm wide cavity drilled into the mild steel backup plate. A 4.5-cm wide, 1.5-cm thick, honeycomb sheet was placed at the rear of the glass plates to provide confinement but minimize penetration/perforation resistance. Flash radiography was then used to obtain the residual velocity and projectile length behind the target.

Least-squares polynomial regression fits to the position-time data are shown as the dashed lines in Fig. 24. Differentiation of these regression fits with respect to time gives the tail and nose (penetration) velocities vs. time for the two sets of data. These are plotted in Fig. 26. Also plotted in the figures are the measured rod velocities using the experimental arrangement of Fig. 24. The residual rods nominally have the same velocity as the tail since most of the projectile is moving with the tail velocity [40].

The normalized lengths of the rod vs. time are shown in Fig. 27. The closed symbols denote data using the experimental arrangement shown in Fig. 23, while the open symbols denote data using the arrangement shown in Fig. 25 (after the rods have left the target). Additionally, some of the residual rods were recovered in the impact tank; these are plotted as open symbols with a center dot beyond 300 μ s. The transition from eroding to rigid-body penetration occurs around 130 μ s - 140 μ s for both impact velocities. Thus, penetration into glass can be divided into two distinct phases: eroding penetration and rigid-body penetration.

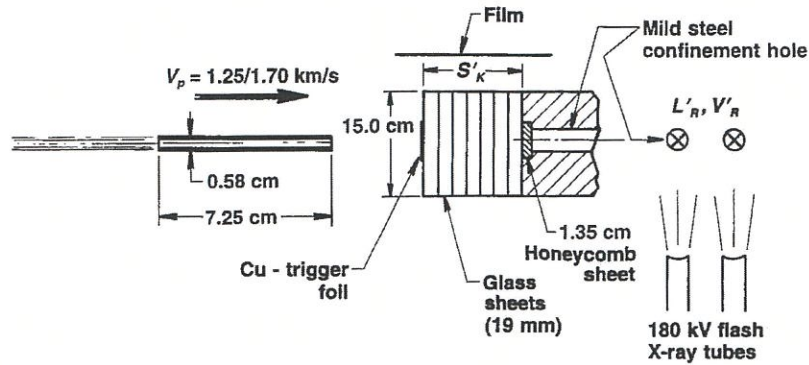


Figure 25. Schematic of modified target arrangement (600 kV X-ray is the same as in Fig. 23).

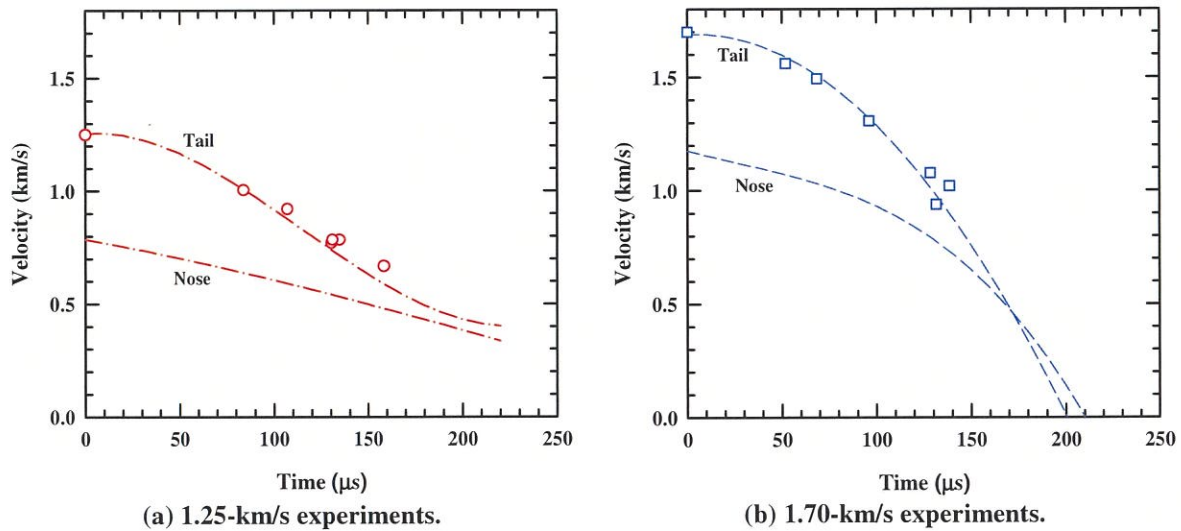


Figure 26. Time differentiation of the position-time regression fits in Fig. 24 give the tail and nose (penetration) velocities vs. time.

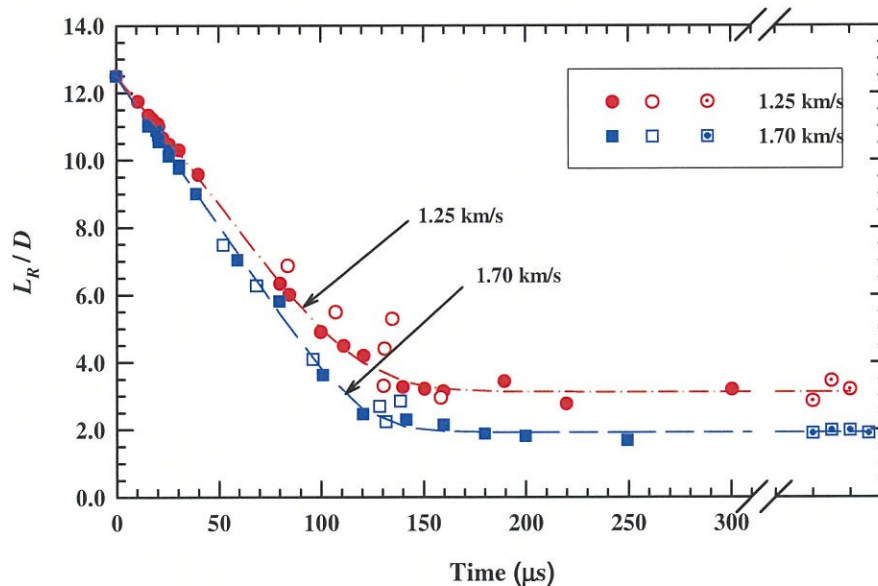


Figure 27. Normalized residual rod length vs. time; data with centered dot represents measurements of the residual rod after recovery from the impact tank.

6.3 Numerical Simulations: Previous Modeling Efforts

Anderson, *et al.* [38], initially conducted numerical simulations of the glass experiments in 1993, using the nonlinear Eulerian wavecode CTH [41]. The Johnson-Holmquist model [42] was used to model the constitutive behavior of the glass; parametric studies were conducted to assess the influence of constitutive parameters. With the parameters that were being used, the glass went from intact to damage very rapidly; and thus, penetration resistance was dominated by the failed material curve. Five computational cells were used to resolve the radius of the projectile. It was found that the treatment of mixed cells—how to homogenize the flow stress when there is more than one material within a computational cell—could have as much, if not more, influence on total penetration than the selection of the constitutive constants. Further, the simulations could not capture the transition to rigid-body penetration, which was observed in the experiments. The simulations considerably overpredicted projectile erosion.

Holmquist, *et al.* [43], obtained some glass from EMI that was used in the ballistic experiments and conducted a number of characterization experiments to determine constitutive constants for the Johnson-Holmquist (JH-2) model [44], including the hydrostat for the equation of state. Characterization experiments included compression and biaxial tests, and flyer-plate impact tests. The constants were then used in CTH to simulate the two ballistic experiments. Penetration was overpredicted at both impact velocities, and rod erosion was excessive [43].

The number of zones used to resolve the projectile radius was increased to 10 to minimize the influence of mixed cells in subsequent work [39]. It was shown that an elastic-perfectly plastic constitutive model, with a flow stress of 1.0 GPa, seemed to capture the early penetration-time response reasonably well; however, the simulations overpredicted projectile erosion, and underpredicted total penetration. A Drucker-Prager constitutive model was used to describe the response of failed glass:

$$Y = \min(\beta P, Y_{cap}) \quad (27)$$

One set of constants, $\beta = 2.0$ and $Y_{cap} = 1.5$ GPa, provided approximate agreement between the simulations, a newly developed analytical model [45], and experimental data for both impact velocities. Importantly, it was shown that the data could be reproduced assuming that the projectile penetrated failed glass, i.e., the details of going from intact to failed glass were ignored.

Interest in glass has increased during the last few years because of the observation of the phenomenon referred to as a failure wave, e.g., Ref. [46-49], and because of the importance of transparent “armor” for vehicles to provide adequate ballistic protection against a variety of threats. In addition, computational power has increased tremendously over the last 12 or so years. Therefore, in 2005-2006 [50-51], it was decided to re-examine the work conducted earlier [38-39]. The Drucker-Prager model was used for these simulations. In Ref. [50], parametric studies on β and Y_{cap} were conducted, and it was determined that the response at 1.70 km/s is controlled by the cap, and the response at 1.25 km/s is controlled by the slope for the first 150 μ s of penetration (during the erosion phase of penetration). The slope and cap have a secondary role in controlling penetration resistance at 1.70 and 1.25 km/s, respectively.

The objective of the work in Ref. [51] was to conduct parametric studies to optimize the Drucker-Prager constitutive parameters. Initially, twenty-five zones were used to resolve the radius of the projectile. With this resolution, the treatment of mixed cells should have less than a 4% effect on the penetration results; thus, avoiding the problem observed in Ref. [38].

Subsequently, a convergence study showed that 7.5 zones across the radius gave the same results as 25 zones, with a substantial speed up in execution time. Concurrent with the numerical study, characterization experiments for borosilicate glass (but not yet soda-lime glass) had been initiated [52-53] (and subsequently fully documented in [6-7]). These characterization experiments provided insights to possible values for Eqn. (27). One of the major findings was that fractured glass has some intrinsic strength at zero axial load [53]. This implies that failed glass has tensile carrying capabilities under confinement. Thus, the failed glass can develop tensile hoop stresses in the divergent stress field in front of the projectile during penetration. The ability of comminuted material to support tensile stresses, at first, seems counterintuitive; but, the comminuted glass particles are interlocked, and cannot necessarily move (apart) with an applied tensile stress. Additionally, as the pressure decreases, so does Y in Eqn. (27); and thus, the resistance to penetration also decreases. Without a positive, non-zero value for Y at zero pressure, Y_0 , penetration will continue forever, albeit at lower and lower penetration velocities. In the EMI ballistic experiments, the remnant projectile never penetrated to the steel backup plate. Thus, an alternative form of the Drucker-Prager model was adopted:

$$Y = Y_0 + \min(\beta P, Y_{cap}) \quad (28)$$

where Y_0 provides a minimum resistance to penetration.⁸ Preliminary analysis of the borosilicate data also suggested that β might be approximately 1.6.⁹ In addition to analyzing the influence of projectile strength, parametric studies were conducted on Y_0 and Y_{cap} . Nominally optimized values were determined for the three Drucker-Prager constitutive model parameters in Eqn. (28): $\beta = 1.6$, $Y_{cap} = 1.5$ GPa, and $Y_0 = 0.015$ GPa. Overall, the comparisons were quite good for the first ~ 130 μ s, the time duration of eroding penetration. Rod length was slightly underpredicted at 1.25 km/s, and slightly overpredicted at 1.70 km/s. After 130-150 μ s, the rod penetrated the failed glass in a rigid-body mode.

6.4 Numerical Simulations: New Simulations

6.4.1 Constitutive Model for Projectile

The constitutive response of the tungsten (W)-alloy projectile was modeled using the Johnson-Cook strength and Johnson-Cook damage models [54-55]. The strength model [54] is expressed as:

$$\sigma = [A + B\varepsilon_p^n][1 + C \ln \dot{\varepsilon}^*][1 - T^{*m}] \quad (29)$$

where ε_p is the equivalent plastic strain, $\dot{\varepsilon}^*$ is the dimensionless total strain rate (where the strain rate is normalized by a reference strain rate of 1.0 s^{-1}), and T^* is the homologous temperature, defined as:

$$T^* = \frac{T - T_{room}}{T_{melt} - T_{room}} \quad (30)$$

The Johnson-Cook damage model [55] is based on accumulated damage. When the damage reaches 1.0, the computational cell is failed. After failure, the material no longer can support shear or tension, but it can still support hydrostatic compression. Damage (D) is defined as:

⁸ All previous numerical simulations assumed $Y_0 = 0$; however, a non-zero Y_0 was assumed in the analytical model of Ref. [45].

⁹ After further analysis, the slope for damaged Borofloat glass was determined to be approximately 1.2 [6-7].

$$D = \sum \left(\frac{\Delta \varepsilon_p}{\varepsilon_p^f} \right) \quad (31)$$

where $\Delta \varepsilon_p = \dot{\varepsilon}_p \Delta t$ is the increment of equivalent plastic strain that occurs during an integration cycle. Here, $\dot{\varepsilon}_p$ is the equivalent plastic strain rate and Δt is the integration time increment. ε_p^f is the equivalent plastic strain to failure, which is determined from:

$$\varepsilon_p^f = [D_1 + D_2 \exp(D_3 \sigma^*)][1 + D_4 \ln \dot{\varepsilon}_p][1 + D_5 T^*] \quad (32)$$

where the dimensionless pressure-stress ratio is defined as $\sigma^* = \sigma_m / \sigma_{eq}$, where σ_m is the average of the three principal stresses (tension positive) and σ_{eq} is the von Mises equivalent stress.

The specific tungsten alloy used in the EMI ballistic experiments has not been characterized, but another tungsten alloy (92.5%W-4.85%Ni-1.15%Fe-1.5%Co) that we believe is similar to the one used in the experiments has been characterized [56]. This specific W-alloy has high density ($\rho = 17.68 \text{ g/cm}^3$). Constitutive constants for the W-alloy are given in Tables 5 and 6. Poisson's ratio for this alloy is 0.28.

Table 5. Johnson-Cook Strength Parameters for Tungsten Alloy

A (GPa)	B (GPa)	n	C	m	T_{melt} (K)
1.197	0.580	0.05	0.025	1.90	1730

Table 6. Johnson-Cook Damage Parameters for Tungsten Alloy

D_1	D_2	D_3	D_4	D_5
0	1.158	-6.80	0	0

The Mie-Grüneisen equation of state was used to model the thermodynamic response of the W-alloy. The linear shock-particle velocity relationship has the bulk sound speed as 4.066 km/s, with a slope of 1.368. The Mie-Grüneisen coefficient was 1.74.

6.4.2 Glass Constitutive Modeling

The Mie-Grüneisen equation of state was used to describe the thermodynamic response of the glass and the tungsten-alloy rod. The bulk modulus for soda-lime glass, as determined by Holmquist, *et al.* [43] is 45.4 GPa; it was assumed that the glass had no nonlinear compressibility effects, i.e., $P = \kappa(\rho / \rho_o - 1)$, where κ is the bulk modulus, and the subscript “o” refers to the initial density. This is clearly an oversimplification, but it is believed that the penetration velocity would only be marginally affected by the inclusion of nonlinear terms. The Grüneisen coefficient (Γ) was set to 1.0.

It is assumed that the projectile penetrates failed glass. This is not exactly true at the very beginning of penetration, and details of the transition of intact to failed glass might be important at early times. However, for the range of impact velocities studied here, the failure front propagates at least twice as fast as the rod is penetrating, e.g., Refs. [1-4]; thus, it would appear that the assumption that the rod penetrates failed material is reasonable. We thus avoid needing a description of how the glass fails with respect to simulating these experiments.

The baseline constitutive model for the damaged glass—using the form of Eqn. (27)—is based on the characterization experiments described in Sections 3 and 4:

$$Y = 0.038 + \min(1.2P, 1.6) \quad (\text{GPa}) \quad (33)$$

The Y_0 that is initially assumed is considerably smaller than determined from the characterization experiments. However, it is observed that Y_0 decreases with increasing damage for soda-lime glass, as it did for borosilicate glass [6-7]. It was shown in Ref. [6,8] that Y_0 needed to be made smaller (representing highly comminuted glass) for ballistic penetration problems. Therefore, as an initial estimate, it was assumed that Y_0 for soda-lime glass has the same value as determined for borosilicate glass [6,8].

In addition to the inelastic response of the glass, as represented by either the DP or MC models, the elastic behavior of the damaged material must be modeled. It has been shown that if the material is confined, a severely cracked specimen has elastic constants nearly the same as an intact specimen [27]; thus, the elastic response of failed material is taken as same as that of the intact material.

6.4.3 Simulation Results

The nonlinear Eulerian wave propagation and material response computer program CTH [41] was used to conduct the numerical simulations. Although the DP model was resident within CTH, the MC model was implemented as described in Ref. [8]. Ten zones were used to resolve the radius of the projectile (cylindrical symmetry). Square zoning was used throughout the computational mesh.

The penetration-time results, using the DP constitutive constants of Eqn. (33) are compared to the experimental data in Figs. 28 and 29 for the 1.25-km/s and the 1.70-km/s experiments, respectively. There are several observations. It is noted that the tail position as a function of time is captured quite well until approximately the time that the projectile goes to rigid-body penetration. That implies that the strength of the W-alloy is being modeled nominally correctly since deceleration of the tail is governed by the dynamic flow stress of the projectile material [57]. It is also observed, with a finite value for Y_0 , that the projectile comes to rest in the glass (the glass target is 20-cm thick), as observed in the experiments. A parametric study was conducted to examine the influence of Y_0 , with values approximately 10% and 20% lower than the 0.038 GPa initially assumed. The value of Y_0 has a relatively significant effect on late-time penetration, as is evident from the parametric study on Y_0 .

The normalized rod lengths (L_R/D) versus time at the two impact velocities are shown in Fig. 30. Y_0 has only a small effect on L_R/D at 1.70 km/s, but there is a much larger dependence for the 1.25-km/s case. Probably the largest discrepancy between simulations and experiments is the region where the rod is transitioning from eroding to rigid-body penetration, during the time interval between about 130 μs and 140 μs . But this is also where the experimental data have the largest variation, particularly for the experiments conducted using the arrangement shown in Fig. 25.

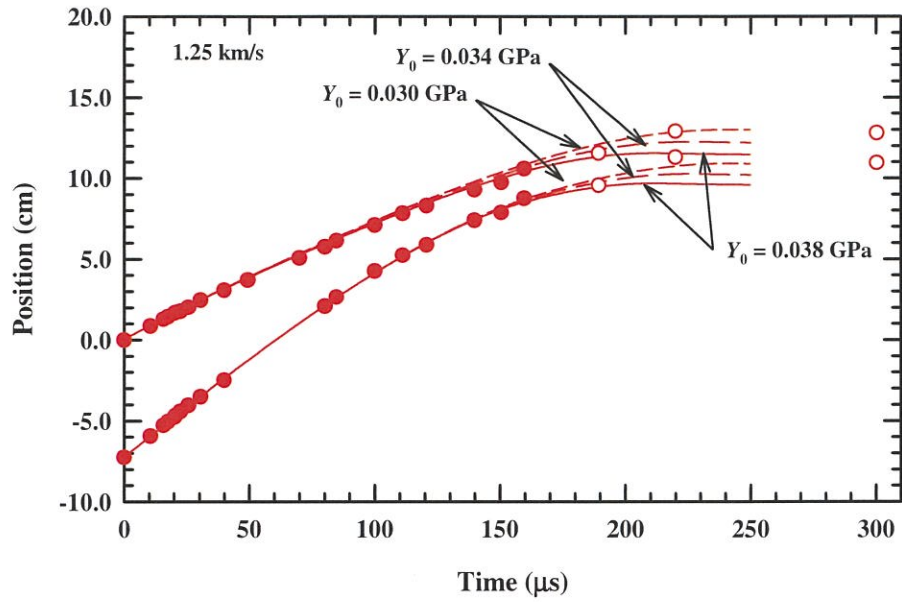


Figure 28. Nose and tail positions versus time for impact velocity of 1.25 km/s: various Y_0 's, $\beta = 1.2$, $Y_{cap} = 1.6$ GPa.

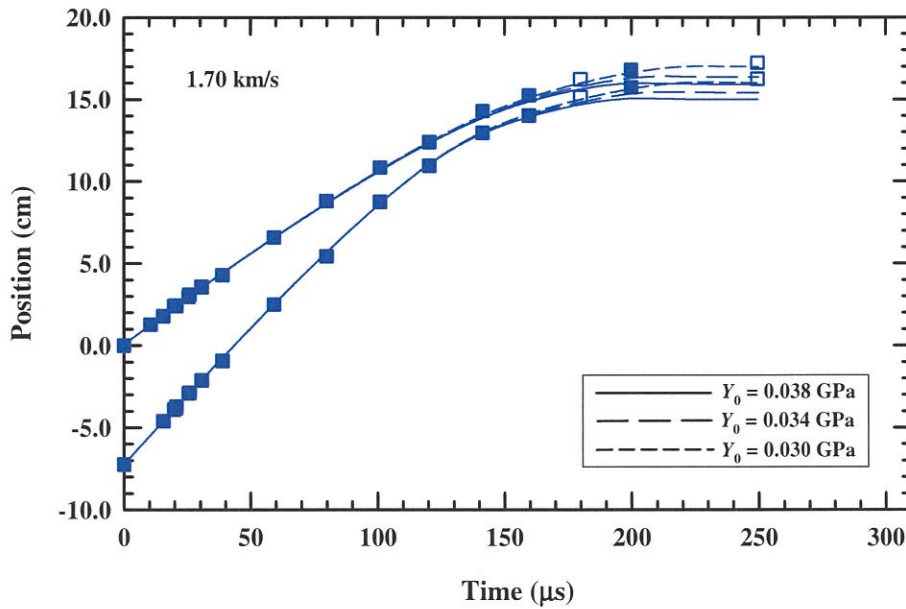


Figure 29. Nose and tail positions versus time for impact velocity of 1.70 km/s: various Y_0 's, $\beta = 1.2$, $Y_{cap} = 1.6$ GPa.

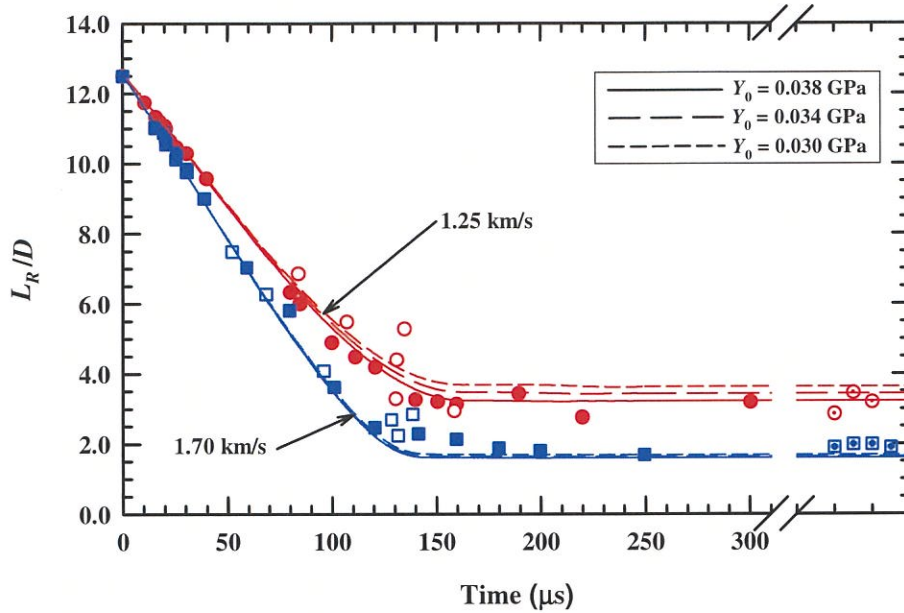


Figure 30. Normalized rod length versus time for various Y_0 's, $\beta = 1.2$, $Y_{cap} = 1.6$ GPa.

It is clear, from Fig. 30, that $Y_0 = 0.038$ GPa provides a better estimate of final rod length than do the other two values for the 1.25-km/s experiments. It would seem that a value of 0.030 GPa provides for a more accurate final depth of penetration, Figs. 28 and 29. Previously mentioned, however, is that the residuals rods were clearly tumbling in the X-ray images near the end of penetration, and that estimates were made for nose and tail positions. Although the lengths could be measured accurately in the X-ray images, rod rotation is so severe (tumbling) that the penetration position could not be estimated with great reliability. These data points are denoted with the open symbols (in contrast to the solid symbols) in Figs. 28 and 29 (the open symbols in Fig. 30 denoted experiments that used the test configuration shown in Fig. 25). Given the uncertainties in position, and the good agreement in L_R/D , it is concluded that the appropriate value for the zero-pressure intercept is $Y_0 = 0.038$ GPa.

Thus, for comminuted glass, the Drucker-Prager relationship is:

$$\text{Comminuted soda-lime glass} \quad Y = \begin{cases} 0.038 + 1.2P & P \leq 1.27 \text{ GPa} \\ 1.61 \text{ GPa} & P > 1.27 \text{ GPa} \end{cases} \quad (34)$$

It was shown in Ref. [8] that the Mohr-Coulomb constitutive model provides the same response as the DP model for penetration into glass; thus, calculations using the MC representation are not repeated here. For completeness, however, the Mohr-Coulomb representation is given in Eqn. (35).

$$\text{Comminuted soda-lime glass} \quad \tau = \begin{cases} 0.012 + 0.6\tilde{\sigma}_n & \tilde{\sigma}_n \leq 1.66 \text{ GPa} \\ 0.81 \text{ GPa} & \tilde{\sigma}_n > 1.66 \text{ GPa} \end{cases} \quad (35)$$

Since Y_0 did not change from what was reported for Borofloat, it was assumed that the cohesion, c , for the MC model would also not change from that reported in Refs. [6,8], i.e., $c = 0.012$ GPa for Starphire.

UNCLASSIFIED

UNCLASSIFIED

7.0 Characterization Data and Flyer-Plate Impact Data

7.1 Introduction

In the previous work with borosilicate glass [6-7], the response of borosilicate glass was compared to flyer-plate impact data. In this section we compare characterization data for soda-lime glass to flyer-plate data. Much of the discussion was presented in Ref. [6], but will be repeated here except with a focus on soda-lime glass. Brar, *et al.* [47], estimated ≈ 2 GPa as the strength of damaged soda-lime glass at pressures of 4 to 6 GPa. Bourne, *et al.* [48-49], showed tests with strength values of ≈ 1.8 GPa for damaged borosilicate and soda-lime for pressures from 4 to 8 GPa. Recently Alexander, *et al.* [58], presented Hugoniot Elastic Limit (HEL) and equation of state data for borosilicate and soda-lime glass. The flyer-plate impact data greatly extends the confining pressures that can be achieved with the hydraulic bomb and confined sleeve techniques. Therefore, we will summarize the results of Bourne, *et al.*, and Alexander, *et al.*, and then provide comparisons between these data and our data.

7.2 Plate-Impact Experiments

The flyer-plate data from Bourne, *et al.* [48-49], and the HEL reported Alexander, *et al.* [58], are shown in Fig. 31. In this figure, the black squares represent soda-lime glass, the white squares represent borosilicate (Pyrex) glass, and the solid hexagons are the HEL data (to be discussed further). Some explanation of the Bourne, *et al.*, data is required.¹⁰ Bourne, *et al.*, used lateral and longitudinal gauges to measure the stresses in the glass specimens. As they increased the impact velocity, the lateral stress shows a second increase, which is interpreted as the arrival of a failure front [48-49], which lowers the strength. The arrival of the second increase in the lateral stress corresponds to the arrival of a longitudinal recompression wave observed in a VISAR signal in the experiments by Rasorenov, *et al.* [46].

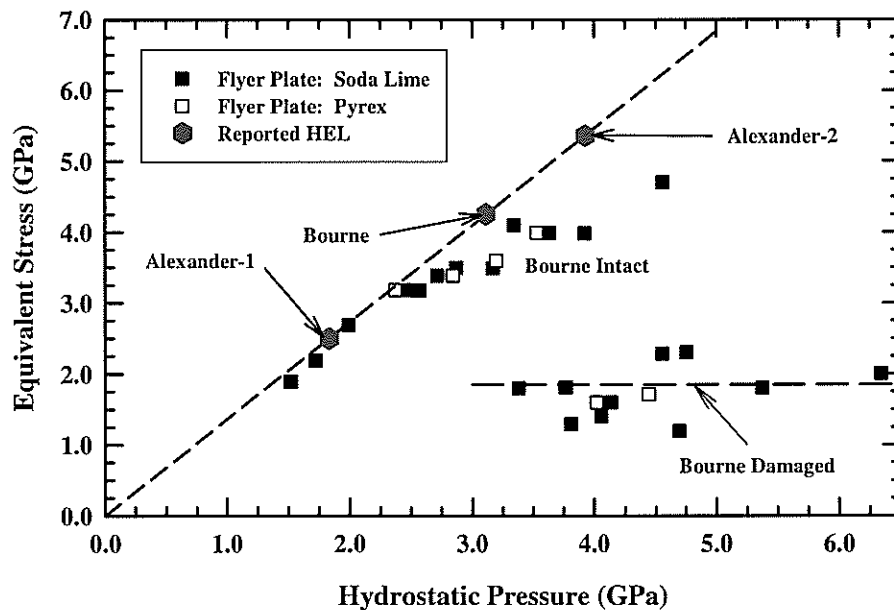


Figure 31. Equivalent stress vs. hydrostatic pressure for flyer-plate impact.

¹⁰ Bourne, *et al.* [48-49], show uncertainty ("error") bars of approximately 0.5 GPa centered on their data values. For clarity, we have omitted these error bars.

The equivalent stress, for uniaxial strain conditions, is $\sigma_{eq} = \sigma_x - \sigma_y$, where σ_y is the lateral stress. The pressure (P), for these uniaxial strain experiments, is given by¹¹:

$$P = \frac{1}{3}(\sigma_x + 2\sigma_y) \quad (36)$$

and the lateral stress is related to the longitudinal stress through Poisson's ratio, ν :

$$\sigma_y = \frac{\nu}{1-\nu} \sigma_x \quad (37)$$

Using Eqn. (37), the equivalent stress is related to the longitudinal stress by:

$$\sigma_{eq} = \sigma_x - \sigma_y = \frac{1-2\nu}{1-\nu} \sigma_x \quad (38)$$

Bourne, *et al.* [49], use the lowest value of σ_x (the longitudinal stress) where there is a jump in the lateral stress gage (the arrival of the failure wave) to obtain an estimate of the HEL.¹² The long dashed horizontal line through the damaged material in Fig. 31 is $\sigma_{eq} = 1.85$ GPa, an average of the strength of the damaged glass. Letting the symbol σ_{fail} represent the dashed line (i.e., 1.85 GPa), then the lateral stress is given by (since $\sigma_{eq} = \sigma_x - \sigma_y = \sigma_{fail}$):

$$\text{Failed Material: } \sigma_y = \sigma_x - \sigma_{fail} \quad (39)$$

Then, inserting Eqn. (39) into Eqn. (36) and solving for the longitudinal stress gives:

$$\text{Failed Material: } \sigma_x = P + \frac{2}{3} \sigma_{fail} \quad (40)$$

Thus, the estimate for the HEL of soda-lime glass is ~ 4.5 GPa ($P = \sim 3.25$ GPa). This value for the HEL is slightly higher than the value of 3.5 GPa reported by Alexander, *et al.* [58], denoted with the callout "Alexander-1". However, this value of ~ 4.5 GPa for the HEL is less than the 6 GPa reported in Table 1 of Refs. [48-49]. Holmquist, *et al.* [43], also report a value of 6 GPa for the HEL of soda-lime glass.

The slope of the elastic response in a σ_{eq} versus P graph is derived by manipulations of Eqns. (36-38):

$$\sigma_{eq} = \sigma_x - \sigma_y = 3 \left(\frac{1-2\nu}{1+\nu} \right) P \quad (41)$$

The corresponding pressure is calculated from:

$$P = \frac{1}{3} \left(\frac{1+\nu}{1-\nu} \right) \sigma_{HEL} \quad (42)$$

Poisson's ratio for Starphire is 0.222, which by Eqn. (42), gives a σ_{eq} - P slope of 1.37, which is plotted as the short dashed line in Fig. 31. Bourne, *et al.*'s soda-lime glass data fall on this line up to σ_{eq} of approximately 3 GPa, at which point the data begin to deviate from the line.

¹¹ For convenience, for this discussion of flyer-plate impact, we assume that the stress is positive in compression.

¹² Presumably, the glass remains elastic below the HEL, but fails upon reaching the HEL.

Several general remarks are appropriate. An elastic analysis has been assumed, i.e., the bulk modulus and shear modulus—and equivalently, Poisson's ratio—are assumed constant. The bulk modulus and shear modulus for Borofloat appear to soften as the pressure increases, as reported by Holmquist [59], based on discussions with Alexander [60]. However, Alexander, *et al.* [58], report that the response of soda-lime glass is linear up to the point denoted by "Alexander-2", i.e., soda-lime glass responds "pseudo-elastically" up to this point. The Bourne, *et al.*, intact data are inconsistent with this observation. This could, perhaps, mean that the glass Bourne, *et al.*, were testing had some predamage, caused by the placement of the lateral stress gages.

Alexander, *et al.* [58,61], also report that soda-lime glass has a complete loss of strength at the HEL (the Alexander-1 datum point). This statement is at odds with our experimental data, as will be seen subsequently. The difficulty is that although the net axial stress is measured in flyer-plate impact experiments, the net stress is the sum of the hydrostatic pressure and strength. If the hydrostat has not been determined independently, then assumptions are required to estimate the hydrostat. Typically, the hydrostat is represented by a quadratic or cubic expression for the pressure in terms of the density. For many materials, the P - ρ response is concave upwards. Holmquist, *et al.* [43], represent the hydrostat for soda-lime glass with a much more complicated curve. What does seem to be clear is that a distinct HEL for soda-lime glass is not readily discerned from flyer-plate impact data.

7.3 Comparison of Characterization Experiments with Flyer-Plate Impact Experiments

The data in Fig. 31 are replotted in Fig. 32, but with the unconfined compression tests (solid triangles), confined compression tests from the hydraulic bomb (open triangles), and the predamaged confined sleeve (small inverted solid triangles) characterization tests. An important distinction between these data and the flyer-plate data is that the characterization tests are used to define a failure surface; whereas some of the responses of the flyer-plate experiments (particularly at the lower pressures) appear to be elastic.

The dash-dot line, with a slope of 3, designates the elastic response of an unconfined uniaxial *stress* specimen (σ_x = axial stress; $\sigma_y = \sigma_z = 0$; and $P = [\sigma_x + \sigma_y + \sigma_z]/3 = \sigma_x/3$). The three solid triangle data points fall on this curve. These points are plotted where the specimens failed. As mentioned previously, unconfined tests on brittle materials have large scatter in their compressive strengths.

The hydraulic bomb tests on intact material (open triangles) provide very reliable failure data as a function of hydrostatic pressure, although again there is scatter at the lowest confining pressures. The long dashed line through the data is given by Eqn. (6); also see Table 2. A linear extrapolation of the long dashed curve intercepts the short dashed line considerably above the "Alexander-2" datum point, but there is no requirement for the response at higher pressures to remain linear with increasing pressure. However, it is unclear whether the long dashed line representing the intact strength for soda-lime glass should go through the "Bourne" datum point or the "Alexander-2" datum point. But, it is also evident, from the open triangles, that soda-lime glass, under confinement, can be stronger than that implied by the "Alexander-1" datum point.

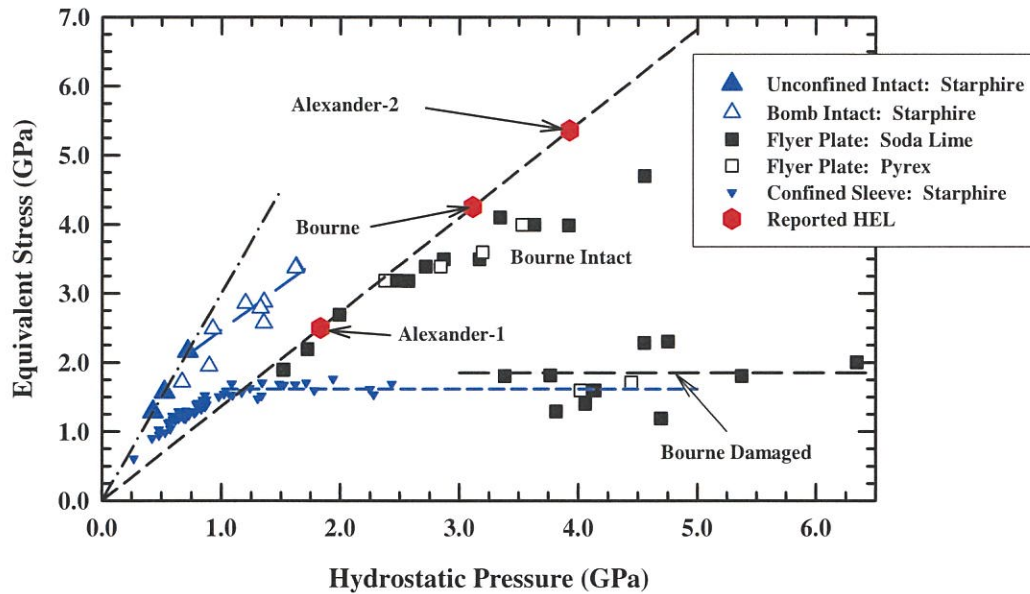


Figure 32. Comparison of characterization data with flyer-plate impact data.

7.3.1 Predamaged Compression Tests on Soda-Lime Glass

The characterization data from the confined sleeve experiments (small inverted solid triangles) are plotted with the other data in Fig. 32 (for clarity, predamaged bomb data are not plotted). The confined sleeve data are triaxial tests; thus, the data fall between the uniaxial stress loading path (the dash-dot line) and the uniaxial strain line (the short dashed line).

The hydraulic bomb data and the confined sleeve data overlap in pressure, and where they overlap, the equivalent stresses as a function of pressure are in agreement (see Fig. 20). Much higher confining pressures, and hence hydrostatic pressures, can be achieved with the confined sleeve experimental technique. As seen in Fig. 20, the confining pressures achieved were sufficiently high to observe a maximum stress that can be carried by the damage material. This is denoted by the short horizontal dashed line in Fig. 32. This cap is in very good agreement with the cap observed in the Bourne, *et al.*, data. The dashed line through the Bourne “damaged” data is an average of the strength of all the damaged data (whether soda-lime glass or Pyrex). The two lines differ by only ~15% (1.85 GPa versus 1.6 GPa). The difference between these two values lies within the scatter of the experimental data, particularly since Bourne, *et al.*, estimated that their uncertainty is ± 0.5 GPa.

There are several significant conclusions resulting from a comparison of the data in Fig. 32: 1) there exists a maximum load carrying capability, i.e., a cap, for the damaged glass; 2) the failure surface for the damaged glass is independent of strain rate; and 3) there is not any significant decrease in the value of the cap as a function of damage. This last conclusion results from the observation that a cap generated from the quasi-static laboratory experiments does not decrease as a result of further load-reload cycling (and hence, comminution) of the glass during testing; and from the flyer-plate experiments, which presumably, result in a very high degree of damage.

7.3.2 Mohr-Coulomb Representation

The data in Figs. 31-32 are plotted as σ_{eq} versus P , which is convenient for representing the experimental results in terms of a Drucker-Prager model. These data can be readily converted to a Mohr-Coulomb representation by the following expressions:

$$\frac{(\sigma_1 + \sigma_3)}{2} = P + \frac{\sigma_{eq}}{6} \quad (43)$$

$$\frac{(\sigma_1 - \sigma_3)}{2} = \frac{\sigma_{eq}}{2} \quad (44)$$

The results are shown in Fig. 33. As before, the solid squares represent the flyer-plate results from Bourne, *et al.* [48-49] for soda-lime glass and the open squares represent the Bourne data for Pyrex. The hexagons represent various possible values for the HEL.

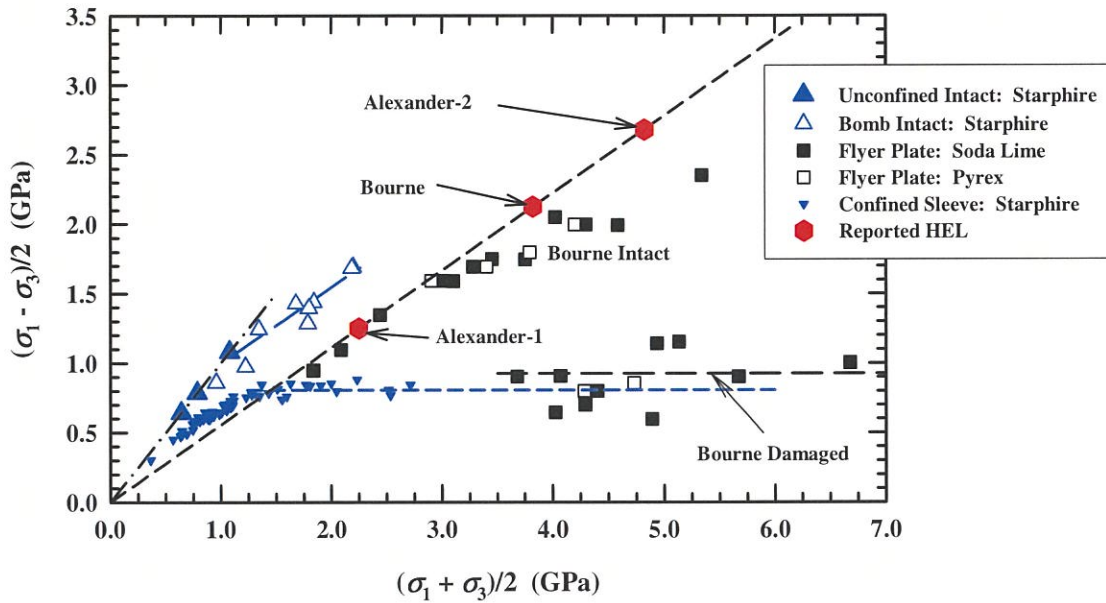


Figure 33. Mohr-Coulomb representation of a comparison of characterization experiments and flyer-plate impact experiments.

UNCLASSIFIED

UNCLASSIFIED

8.0 Summary

Laboratory characterization experiments were conducted to gather experimental data that could be used for developing a numerical constitutive model for soda-lime glass that would be applicable to ballistic experiments. The strength of glass is pressure dependent, so the characterization experiments were devised to explore the strength of the glass as a function of confining pressure. Interest was in both intact and damaged glass since simulations require a description of both intact and failed material. Three types of characterization experiments were conducted:

- Unconfined compression on intact specimens (uniaxial stress experiments);
- Hydraulic compression using the hydraulic bomb apparatus (triaxial compression, giving equivalent stress versus hydrostatic pressure) on intact and predamaged specimens;
- Confined sleeve compression experiments (triaxial compression, where the confining pressure increases with axial load) on intact and predamaged specimens.

The focus of the characterization experiments was largely on the latter two test procedures, with the objective of developing failure maps, as a function of pressure, for intact and predamaged material. The confined sleeve test, in principle, can explore higher pressures than possible with the hydraulic bomb test. However, for intact material, small gaps and/or misalignment of the specimen with the sleeve typically resulted in glass failure before full potential confinement could be exerted by the steel sleeve; thus, data from these tests are not reliable. However, the confined sleeve test worked very well for predamaged specimens, and high confinement pressures were achieved. The results for the hydraulic bomb and the confined sleeve overlapped at pressures below 1.0 GPa, providing increased confidence in the results.

The experimental data were interpreted in terms of two pressure-dependent constitutive models, the Drucker-Prager (DP) model and the Mohr-Coulomb (MC) model. Constitutive parameters were determined from the characterization experiments. The slope (β) for the DP model and the friction angle (ϕ) for soda-lime glass, determined from the characterization experiments, were found to be nominally independent of the degree of damage to the glass (intact, predamaged, residual damage). However, the zero-pressure strength (Y_0) and the cohesion (c) for the DP and MC models, respectively, do depend upon the degree of damage, with these parameters decreasing as damage increased. Additionally, the confining pressures in the confined sleeve experiments were sufficiently high to achieve a saturation of the load-carrying ability of the damaged glass, i.e., a cap.

The data from the characterization experiments were compared with flyer-plate impact experiments conducted on borosilicate and soda-lime glass. There appear to be some inconsistencies between the results of the characterization experiments and flyer-plate impact experiments for intact material. However, the equivalent stress derived for failed glass from the flyer-plate impact experiments is in very good agreement with the cap determined for damaged glass from the characterization experiments. This agreement over many orders of magnitude in strain rate indicates that a cap exists for damaged (failed) glass, and that this cap is strain rate independent.

The results of the characterization experiments were used to simulate numerically tungsten-alloy, long-rod projectiles impacting a soda-lime glass target at two impact velocities, 1.25 km/s and 1.70 km/s. The impact experiments had been completed previously [38-39]. It was assumed that the projectile penetrated failed material; thus, transition of intact glass to failed glass was ignored. The rationale for this approach was that the failure front propagates much more rapidly than the projectile penetrates; thus, the projectile penetrates failed material, e.g., Refs. [1-4].

Since the zero-pressure strength (Y_0) and the cohesion (c) for the DP and MC models depend upon the degree of damage, it was decided for the simulations to use the zero-pressure strength deduced for borosilicate glass [8] as an initial estimate. Then values of Y_0 10% and 20% less were also used in simulations to examine the sensitivity of the results to this parameter. It was shown that rod position and residual rod length versus time are very sensitive to the value of Y_0 . It was concluded—after examination of the position-time data and residual length-time data, combined with the uncertainty in position of the nose because of rod tumbling at late times—that $Y_0 = 0.038$ GPa was an appropriate value for highly comminuted soda-lime glass. Thus, the DP and MC constitutive constants for *failed* (highly comminuted) soda-lime glass are:

$$\text{Drucker-Prager: } Y = \begin{cases} 0.038 + 1.2P & P \leq 1.27 \text{ GPa} \\ 1.61 \text{ GPa} & P > 1.27 \text{ GPa} \end{cases} \quad (44)$$

$$\text{Mohr-Coulomb: } \tau = \begin{cases} 0.012 + 0.6\tilde{\sigma}_n & \tilde{\sigma}_n \leq 1.66 \text{ GPa} \\ 0.81 \text{ GPa} & \tilde{\sigma}_n > 1.66 \text{ GPa} \end{cases} \quad (45)$$

It was shown in Ref. [6] that at low impact velocities, particularly at very early times, the assumption of penetrating solely failed glass is not valid. That is, *the details of the transition of intact to damage glass are important at the lower impact velocities, and that a more comprehensive glass model (intact, damage initiation, damage propagation) is required in order to model projectile penetration over the full range of impact velocities.* Development of such a model is the subject of on-going research.

9.0 Acknowledgement

The authors would like to thank Dr. Doug Templeton from TARDEC for funding this work, and the administrative support provide by Mr. Rick Rickert of TARDEC. Also thanks to the ARL for providing ultrasonic modulus measurements of intact samples. Additionally, the authors thank Mr. Dennis Orphal (International Research Associates) for his careful reading and his many helpful comments and suggestions.

UNCLASSIFIED

UNCLASSIFIED

10.0 References

1. Th. Behner, C. E. Anderson, D. L. Orphal, V. Hohler, M. Moll, and D. W. Templeton. Penetration and failure of lead and borosilicate glass against rod impact. *Int. J. Impact Engng.*, **35**(6): 447-456 (2008).
2. C. E. Anderson, Jr., D. L. Orphal, T. Behner, and D. W. Templeton. Failure and penetration response of borosilicate glass during short-rod impact. *Int. J. Impact Engng.*, **36**(6), 789-798 (2009).
3. D. L. Orphal, C. E. Anderson, Jr., T. Behner, and D. W. Templeton. Failure and penetration response of borosilicate glass during multiple short-rod impact. *Int. J. Impact Engng.*, **36**(10-11): 1173-1181 (2009).
4. D. L. Orphal. Some recent results on the propagation of the failure front associated with rod penetration of borosilicate glass. *Shock Compression of Condensed Matter—2009*, (M. L. Elert, *et al.*, Eds.), AIP Conf. Series 1195, pp. 1425-1430, AIP, Melville, NY (2009).
5. S. Chocron, A. E. Nicholls, K. D. Dannemann, Susan Martinez, and C. E. Anderson, Jr. A database for the material response of a confined borosilicate glass. SwRI Report 18.12544/007, prepared for RDECOM-TARDEC, AMSRD-TAR-R, Warren, MI, October (2008).
6. S. Chocron, C. E. Anderson, Jr., K. A. Dannemann, and A. E. Nicholls. Characterization of borosilicate glass by confined compression testing and numerical validation. SwRI Report 18.12544/010, prepared for RDECOM-TARDEC, AMSRD-TAR-R, Warren, MI, November (2009).
7. S. Chocron, C. E. Anderson, Jr., A. E. Nicholls, and K. A. Dannemann. Characterization of confined intact and damaged borosilicate glass. *J. Am Ceram. Soc.*, in press (2010).
8. S. Chocron and C. E. Anderson, Jr. Numerical simulations of the penetration of glass using two pressure-dependent constitutive models. In *Predictive Modeling of Dynamic Processes: A Tribute to Klaus Thoma*, (S. Hiermaier, Ed.), pp. 167-187, Springer Science+Business Media LLC, New York (2009).
9. D. A. Shockey, D. R. Curran, J. W. Simons, and D. Bergmannshoff. Penetration physics of armor glass. SwRI Report 18.12544/019, prepared for RDECOM-TARDEC, AMSRD-TAR-R, Warren, MI, November (2009).
10. C. E. Anderson, Jr., T. Behner, Dennis L. Orphal, T. J. Holmquist, V. Hohler, and M. Wickert. Interface defeat of long rods impacting borosilicate glass: experimental results. SwRI Report 18.12544/009, prepared for RDECOM-TARDEC, AMSRD-TAR-R, Warren, MI, February (2009).
11. C. E. Anderson, Jr., T. J. Holmquist, D. L. Orphal, and T. Behner. Dwell and interface defeat on glass targets. *Int. J. Appl. Ceramic Tech.*, in press (2010).
12. C. E. Anderson, Jr., C. E. Weiss, and S. Chocron. Glass impact experiments into borosilicate glass at three scale sizes. SwRI Report 18.12544/018, prepared for RDECOM-TARDEC, AMSRD-TAR-R, Warren, MI, November (2009).
13. P. Patel and M. Matoya. US Army Research Laboratory, Aberdeen, MD, personal communication (2005).
14. C. S. Desai and H. J. Siriwardane. **Constitutive Laws for Engineering Materials with Emphasis on Geologic Materials**. Prentice-Hall, Englewood Cliffs, NJ (1984).
15. W. Chen and G. Ravichandran. Static and dynamic compressive behavior of aluminum nitride under moderate confinement. *J. Am. Ceram. Soc.*, **79**: 579-584 (1996).
16. W. Chen and G. Ravichandran. Dynamic compressive failure of a glass ceramic under lateral confinement. *J. Mech. Phys. Solids*, **45**: 1303-1328 (1997).

17. Z. Ma and K. Ravi-Chandar. Confined compression: A stable homogeneous deformation for constitutive characterization. *Exp. Mech.*, **40**(38): 38-45 (2000).
18. J. Lu and G. Ravichandran. Pressure-dependent behavior of $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$ bulk metallic glasses. *J. Mater. Res.*, **18**: 2039-2049 (2003).
19. W. Chen and H. Luo. Dynamic compressive responses of intact and damaged ceramics from a single split Hopkinson pressure bar experiment. *Exp. Mech.*, **44**(3): 295-299 (2004).
20. P. Forquin, A. Árias, and R. Zaera. An experimental method of measuring the confined compression strength of high-performance concretes to analyse their ballistic behaviour. *J. Physique IV*, **134**: 629-634 (2006).
21. W. Chen and G. Ravichandran. Failure mode transition in ceramics under dynamic loading. *Int. J. Fract.*, **101**: 141-159 (2000).
22. W. Chen and H. Luo. Dynamic compressive responses of intact and damaged ceramics from a single split Hopkinson pressure bar experiment. *Exp. Mech.*, **44**: 295-299 (2004).
23. H. Luo, W. Chen and A. M. Rajendran. Dynamic compressive response of damaged and interlocked SiC-N ceramics. *J. Amer. Ceram. Soc.*, **89**: 266 - 273 (2005).
24. K. A. Dannemann, S. Chocron, A. E. Nicholls, and C. E. Anderson, Jr. Compressive damage development in confined borosilicate glass. *Mat. Sci. and Engng. A - Structures*, **478**: 340-350 (2008).
25. K. A. Dannemann, S. Chocron, A. E. Nicholls, J. D. Walker, and C. E. Anderson, Jr. Compression testing and response of SiC-N ceramics: Intact, damaged and powder. *Ceramic Engineering and Science Proc., Advances in Ceramic Armor, 29th Int. Conf. on Advanced Ceramics and Composites* (J. J. Swab, Ed.), **26**: 109-116; American Ceramic Society (2005).
26. S. Chocron, J. D. Walker, A. E. Nicholls, K. A. Dannemann, and C. E. Anderson, Jr.. Analytical model of the confined compression test used to characterize brittle materials. *J. of Appl. Mech.*, **75**: 021006-1-7 (2008).
27. S. Chocron, K. A. Dannemann, J. D. Walker, A. E. Nicholls, and C. E. Anderson, Jr. Constitutive model for damaged borosilicate glass under confinement. *J. of Amer. Cer. Soc.*, **90**(8): 2549-2555 (2007).
28. S. Timoshenko and J. N. Goodier. **Theory of Elasticity**. Mc. Graw-Hill, New York, first edition (1951).
29. D. R. Curran, D. A. Shockey, and J. W. Simons. Mesomechanical constitutive relations for glass and ceramic armor. *32nd Int. Conf. on Advanced Ceramics and Composites*, Daytona Beach, January (2008).
30. K.W. Peter. Densification and flow phenomena of glass in indentation experiments. *J. Non-Crystalline Solids*, **5**: 103-115 (1970).
31. F.M. Ernsberger. Mechanical properties of glasses. *J. Non-Crystalline Solids*, **25**: 293-321 (1977).
32. J. Lankford, C. E. Anderson, Jr., A. J. Nagy, J. D. Walker, A. E. Nicholls, and R. A. Page. Inelastic response of confined aluminum oxide under dynamic loading conditions. *J. Mat. Sci.*, **33**(6): 1619-1625 (1998).
33. T. J. Holmquist and G. R. Johnson. Characterization and evaluation of silicon carbide for high velocity impact. *J. Appl. Phys.*, **97**: 093502-1-12 (2005).
34. T. J. Holmquist and G. R. Johnson. The failed strength of ceramics subjected to high-velocity impact. *J. Appl. Phys.*, **104**(013533):013533-1-11 (2008).

35. D. C. Drucker and W. Prager. Soil Mechanics and plastic analysis of limit design. *Q. Appl. Math.*, **10**(2): 157-175 (1952).
36. R.M. Nedderman. **Statics and Kinematics of Granular Materials**. Cambridge University Press, Cambridge, UK (1992).
37. J. C. Jaeger and N. G. W. Cook. **Fundamentals of Rock Mechanics**. Chapman and Hall, Ltd. and Science Paperbacks, London (1969).
38. C. E. Anderson, Jr., V. Hohler, J. D. Walker, and A. J. Stilp. Penetration of long rods into steel and glass targets: experiments and computations. *Proc. 14th Int. Symp. on Ballistics*, Québec City, Canada, 24-29 September (1993).
39. C. E. Anderson, Jr., V. Hohler, J. D. Walker, and A. J. Stilp. Modeling long-rod penetration into glass targets. *14th U. S. Army Symp. on Solid Mech.*, (K. R. Iyer and S-C Chou, Eds.), pp. 129-136, Battelle Press, Columbus, OH (1996).
40. C. E. Anderson, Jr. and J. D. Walker. An examination of long-rod penetration. *Int. J. Impact Engng.*, **11**(4), 481-501 (1991).
41. J. M. McGlaun, S. L. Thompson, and M. G. Elrick. CTH: A three-dimensional shock wave physics code. *Int. J. Impact Engng.*, **10**(1-4): 351-360 (1990).
42. G. R. Johnson and T. J. Holmquist. A computational constitutive model for brittle materials subjected to large strains, high strain rates, and high pressures. *Shock Waves and High-Strain Rate Phenomena in Materials* (M. A. Myers, L. E. Murr, and K. P. Staudhammer, Eds.), pp. 1075-1081, Marcel Dekker, NY (1992).
43. T. J. Holmquist, G. R. Johnson, D. E. Grady, C. M. Lopatin, and E. S. Hertel Jr. High strain rate properties and constitutive modeling of glass. *Proc. 15th Int. Symp. Ballistics*, Jerusalem, Israel, pp. 237--244 (1995).
44. G. R. Johnson and T. J. Holmquist. An improved computational constitutive model for brittle materials. *High Pressure Science and Technology-1993* (S. C. Schmidt, J. W. Shaner, G. A. Samara, and M. Ross, Eds.) pp. 981-984, AIP Conf. Proc. 309, AIP Press, Woodbury, NY (1994).
45. J. D. Walker and C. E. Anderson, Jr. An analytic penetration model for a Drucker-Prager yield surface with cutoff. *Shock Compression of Condensed Matter-1997* (S. C. Schmidt, D. P. Dandekar, and J. W. Forbes, Eds.) pp. 897-900, AIP Conf. Proc. 429, AIP Press, Woodbury, NY (1998).
46. S. V. Rasorenov, G. I. Kanel, and V. E. Fortov. The fracture of glass under high pressure impulsive loading. *High Press. Res.*, **6**: 225 - 232 (1991).
47. N. S. Brar, S. J. Bless, and Z. Rosenberg. Impact-induced failure waves in glass bars and plates. *J. Appl. Phys.*, **59**(26): 3396-3398 (1991).
48. N. Bourne, J. Millett, Z. Rosenberg, and N. Murray. On the shock induced failure of brittle solids. *J. Mech. Phys. Solid*, **46**(10): 1887-1908 (1998).
49. N. K. Bourne, J. C. F. Millet, and J. E. Field. On the strength of shocked glasses. *Proc. R. Soc. Lond. A*, **455**: 1275-1282 (1999).
50. C. E. Anderson, Jr., I. S. Chocron, and J. D. Walker. Analysis of time-resolved penetration into glass targets. *Ceramic Engng. & Sci. Proc.*, **26**(7), *Proc. 29th Int. Conf. Advanced Ceramics & Composites*, 27-34, American Ceramics Society (2005).
51. C. E. Anderson, Jr., I. S. Chocron, and C. E. Weiss. Analysis of time-resolved penetration of long rods into glass targets—II. *Advances in Ceramic Armor II, A Collection of Papers Presented at the*

- 30th Int. Conf. & Exp. on Advanced Ceramics and Composites* (L P Franks, Ed.), 27(7), 107-118, The American Ceramics Society, John Wiley & Sons, Inc., NJ (2006).
52. K. A. Dannemann, A. E. Nicholls, C. E. Anderson, Jr., S. Chocron, and J. D. Walker. Response and characterization of confined borosilicate glass: intact and damaged. *30th Int. Conf. & Exp. on Advanced Ceramics and Composites*, The American Ceramics Society, Cocoa Beach, FL, 22-27 January (2006).
 53. S. Chocron, J. D. Walker, A. E. Nicholls, C. E. Anderson, Jr., and K. A. Dannemann. Constitutive model for damaged borosilicate glass. *30th Int. Conf. & Exp. on Advanced Ceramics and Composites*, The American Ceramics Society, Cocoa Beach, FL, 22-27 January (2006).
 54. G. R. Johnson and W. H. Cook. A constitutive model for data for metals subjected to large strains, high strain rates and high temperatures. *Proc. 7th Int. Symp. Ballistics*, 541-548, The Hague, The Netherlands, April (1983).
 55. G. R. Johnson and W. H. Cook. Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures. *Eng. Fract. Mech.*, 21(1): 31-48 (1986).
 56. I. Rohr, H. Nahme, K. Thoma, and C. E. Anderson, Jr. Material characterization and constitutive modeling of a tungsten-sintered alloy for a wide range of strain rates. *Int. J. Impact Engng.*, 35(8): 661-673 (2008).
 57. J. D. Walker and C. E. Anderson, Jr. A time-dependent model for long-rod penetration. *Int. J. Impact Engng.*, 16(1): 19-48 (1995).
 58. C. S. Alexander, L. C. Chhabildas, W. D. Reinhart, and D. W. Templeton. Changes to the shock response of fused quartz due to glass modification. *Int. J. Impact Engng.*, 35(12): 1376-1385 (2008).
 59. T. J. Holmquist, private communication (2009).
 60. C. S. Alexander, private communication with T. J. Holmquist (2009).
 61. C. S. Alexander, L. C. Chhabildas, and D. W. Templeton. The Hugoniot elastic limit of soda-lime glass. *Shock Compression of Condensed Matter-2007* (M. Elert, et al., Eds.), pp. 733-738, AIP Conf. Proc. 955, AIP Press, Melville, NY (2007).