



**US Army Corps  
of Engineers®**  
Engineer Research and  
Development Center

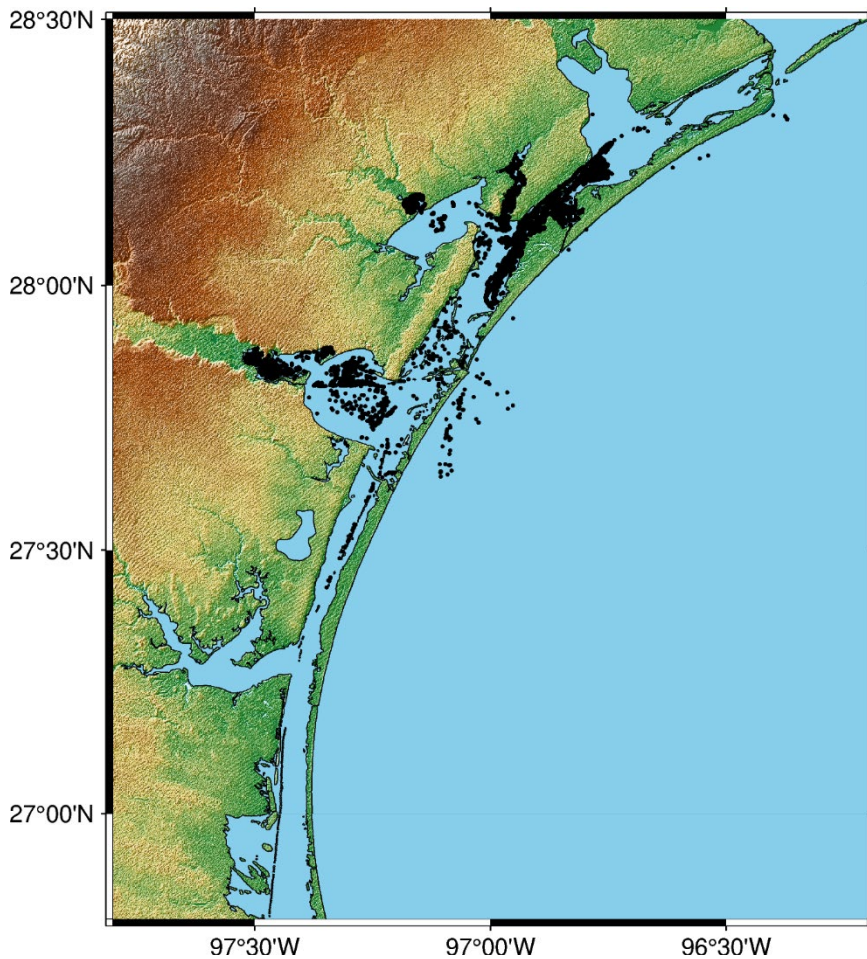


*Engineered Resilient Systems*

## **ERDC-PT: A Multidimensional Particle Tracking Model**

Corey J. Trahan, Jing-Ru C. Cheng, and Amanda Hines

January 2024



**The US Army Engineer Research and Development Center (ERDC)** solves the nation's toughest engineering and environmental challenges. ERDC develops innovative solutions in civil and military engineering, geospatial sciences, water resources, and environmental sciences for the Army, the Department of Defense, civilian agencies, and our nation's public good. Find out more at [www.erdclibrary.on.worldcat.org/discovery](http://www.erdclibrary.on.worldcat.org/discovery).

To search for other technical reports published by ERDC, visit the ERDC online library at <http://www.erdclibrary.on.worldcat.org/discovery>.

# **ERDC-PT: A Multidimensional Particle Tracking Model**

Corey Trahan, Jing-Ru C. Cheng, and Amanda Hines

*US Army Engineer Research and Development Center  
Information Technology Laboratory  
3909 Halls Ferry Road  
Vicksburg, MS 39180-6199*

Distribution Statement A. Approved for public release; distribution is unlimited.

Prepared for Headquarters, US Army Corps of Engineers  
Washington, DC 20314-1000

Under Funding Account Code D2H545, AMSCO Code 008284

## Abstract

This report describes the technical engine details of the particle- and species-tracking software ERDC-PT. The development of ERDC-PT leveraged a legacy ERDC tracking model, “PT123,” developed by a civil works basic research project titled “Efficient Resolution of Complex Transport Phenomena Using Eulerian-Lagrangian Techniques” and in part by the System-Wide Water Resources Program. Given hydrodynamic velocities, ERDC-PT can track thousands of massless particles on 2D and 3D unstructured or converted structured meshes through distributed processing. At the time of this report, ERDC-PT supports triangular elements in 2D and tetrahedral elements in 3D. First-, second-, and fourth-order Runge-Kutta time integration methods are included in ERDC-PT to solve the ordinary differential equations describing the motion of particles. An element-by-element tracking algorithm is used for efficient particle tracking over the mesh. ERDC-PT tracks particles along the closed and free surface boundaries by velocity projection and stops tracking when a particle encounters the open boundary. In addition to passive particles, ERDC-PT can transport behavioral species, such as oyster larvae. This report is the first report of the series describing the technical details of the tracking engine. It details the governing equation and numerical approaching associated with ERDC-PT Version 1.0 contents.

**DISCLAIMER:** The contents of this report are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such commercial products. All product names and trademarks cited are the property of their respective owners. The findings of this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

**DESTROY THIS REPORT WHEN NO LONGER NEEDED. DO NOT RETURN IT TO THE ORIGINATOR.**

# Contents

<b>Abstract</b> .....	<b>ii</b>
<b>Contents</b> .....	<b>iii</b>
<b>Figures</b> .....	<b>iv</b>
<b>Preface</b> .....	<b>v</b>
<b>1 Introduction</b> .....	<b>1</b>
1.1 Background.....	1
1.2 Objectives.....	1
1.3 Approach .....	2
<b>2 Computational Strategy and Features</b> .....	<b>4</b>
<b>3 Governing Equations and Numerical Solutions</b> .....	<b>6</b>
3.1 Governing Equation .....	6
3.2 Time Integration.....	6
3.3 Interpolation of Velocity.....	6
3.4 Element-by-Element (EBE) Tracking .....	7
3.5 Tracking along a Closed Boundary .....	9
3.5.1 Velocity Projection onto a 2D Boundary Edge .....	10
3.5.2 Velocity Projection onto a 3D Boundary Face .....	11
<b>4 Summary</b> .....	<b>14</b>
<b>References</b> .....	<b>15</b>
<b>Abbreviations</b> .....	<b>17</b>
<b>Report Documentation Page</b>	

## Figures

1. Element-by-element particle tracking (PT) diagram. .... 7
2. Plot to demonstrate how PT timestep size is reduced when the end location is outside of the active element. .... 8
3. Projection of velocity onto a 2D boundary segment..... 10
4. Projection of velocity onto a 3D boundary face..... 11

## Preface

This study was conducted for the US Army Corps of Engineers under Funding Account Code D2H545, AMSCO Code 008284.

This work was performed by the Computational Analysis Branch of the Computational Science and Engineering Division, US Army Engineer Research and Development Center (ERDC), Information Technology Laboratory (ITL). At the time of publication, Mr. David C. Stuart was branch chief; Dr. Jeffrey L. Hensley was division chief; and the technical director was Dr. Robert M. Wallace. The deputy director of ERDC-ITL was Dr. Jackie S. Pettway, and the director was Dr. David A. Horner.

Investigators for the original PT123 model were from the Hydrologic Systems Branch, ERDC Coastal and Hydraulics Laboratory, and the Department of Defense Supercomputing Resource Center, ERDC-ITL.

The commander of ERDC was COL Christian Patterson, and the director was Dr. David W. Pittman.

This page intentionally left blank.

# 1 Introduction

## 1.1 Background

The particle tracking (PT) technique has a wide range of applications in environmental sciences and engineering. This technique typically uses the output from hydrodynamic or advection-diffusion models to predict particle movements in a Lagrangian manner. Given the velocity field, PT methods can track the migration of chemicals, biological species, sediments, and so on in surface water and groundwater systems. For example, PT methods can be used to understand, visualize, and analyze flow fields (Pokrajac and Lazic 2002). They can be also used to study sediment transport (MacDonald et al. 2006), oil spill (Liu et al. 2011), and natural or man-induced retardation mechanisms that may be used for the remediation or prevention of environmental pollution. PT methods can be used to understand and predict fish behavior (Goodwin et al. 2006) for ecosystem restoration and preservation. They can also be applied in the Eulerian-Lagrangian (EL) approximation to solve transport equations numerically, which is a crucial modeling practice to help deal with environmental issues concerning water quality. The quality of particle tracking dictates much of the accuracy of the whole EL approximation (Russell and Celia 2002) as well as efficiency on serial and parallel platforms (Cheng and Plassman 2004). Efficient numerical methods for transport are necessary for large-scale modeling in achieving the US Army Corps of Engineers' (USACE's) mission. Therefore, having accurate and efficient PT can help USACE engineers and scientists to carry out reimbursable and research and development projects. These methods have been implemented in a computer model called ERDC-PT.

## 1.2 Objectives

The purposes of this effort are twofold. One is to develop accurate, efficient, and distributable PT software for solving multidimensional transport problems using EL methods; the other is to develop a library-type computer program that can be incorporated agnostically into any existing flow models (e.g., ERDC's adaptive hydraulics model [AdH] or advanced circulation model). This work modernized and extended the legacy PT123 particle tracking software to be both distributable on high-performance computing systems and have biological species-tracking

capabilities. During modernization, the software was stripped down to its basic engine and the core components rewritten from FORTRAN to C. Only those fundamental engine components necessary for trajectory propagation were included in the new software. After updating the engine to C, the following enhancements (not included in PT123) were made to the new ERDC-PT software:

- Message Passing Interface distribution of trajectories for large-scale applications (note here that each processor retains the whole hydrodynamic grid but only some fraction of trajectories)
- Free surface tracking in 3D applications
- Oyster larvae behavioral tracking
- Sensor point tracking for biological species such as fish who have a nonlocal tracking component
- An extensive, internal geometric and search engine 2D/3D verification test suite
- Extensible Data Model and Format (XDMF) binary output options for both the hydrodynamic grid and particle trajectory results for easy import into Paraview

### 1.3 Approach

All particle trajectory methods require driving field velocities on background spatial grids. Of course, any inaccuracies in these velocity values will introduce an error into the tracking simulation.

Spatial/temporal interpolation of the velocities by the tracking algorithm from the background grid to the particle location may also introduce interpolation errors into the particle tracker. This is true even if the nodal velocities are exact (Pokrajac and Lazic 2002). Analytical PT solutions are limited to cases with simple geometry and velocity fields. Semianalytical PT is used in the Pollock's method, where linear interpolation of velocity enables the analytical calculation of path lines and travel times over an element (Pollock 1988).

By assuming that the given velocity field is accurate and velocity interpolation error is negligible, the ERDC-PT implementation reduces to techniques for solving ordinary differential equations (ODEs). These ODEs describe the motion of particles along their path lines. A summary of the ERDC-PT computational strategy for solving these ODEs along with model input/output follows next. It is worth noting that the standard numerical

methods for time discretization in particle trackers are the first-order Euler, the second-order Euler, predictor-corrector methods, and higher-order Runge-Kutta (RK) methods. For convenience, RK1 and RK2 are used to represent the first- and second-order methods (Press et al. 1992). Studies have shown that higher-order RK methods (e.g., fourth-order RK or RK4) are superior to the lower-order RK methods regarding accuracy, and adaptive spatial or temporal steps may improve the efficiency of PT algorithms in velocity fields containing wide spectrums of velocity magnitude and element size (Cash 1989; Press et al. 1992; Bensabat et al. 1998; Oliveira and Baptista 1998; Cheng and Plassman 2004).

## 2 Computational Strategy and Features

The ERDC-PT software utilizes an element-by-element (EBE) tracking algorithm and linear/bilinear space/time interpolation to estimate continuous fields such as velocity, salinity, and others during the tracking process. Version 1.0 of ERDC-PT includes the following computational features:

- PT in multiple dimensions: 2D (triangles) or 3D (tetrahedrons)
- Flexible time and length units: any time and length units are valid if consistent
- RK1, RK2 and RK4 schemes
- Large-scale particle transport: There is no constraint on the number of tracked particles since the particles are distributed via magnetic particle imaging. The user specifies the number and locations of the tracked particles in the input data.
- Flexible start time and time duration for tracking: The time parameters can be assigned any values in the range that the given velocity field covers.
- Boundary and free surface projection-based tracking
- Oyster larvae behavioral options
- Sensor point tracking for nonlocal species interactions

ERDC-PT does not require any specification of time and length units in the input file. Any combination of time and length units can be utilized in the PT computation as long as consistency is maintained throughout the input data. The output assumes the same time and length units implied in the input data.

The input data that ERDC-PT requires for PT computation include

- element indices and nodal coordinates of the hydrodynamic grid (i.e., geometry of the computational domain);
- flow, salinity, and water quality fields from hydrodynamic models;
- open/closed boundary information; and
- specifics for the PT computation (e.g., particle data, computation parameters, etc.).

ERDC-PT uses several input files to accommodate this input data. As mentioned, the software is agnostic to the hydrodynamic model used to

calculate the flow fields; however, a “reader” for inputting hydrodynamic output must be included in the particle tracker. Currently, ERDC-PT includes a reader for the AdH software suite.

ERDC-PT outputs the trajectory of each tracked particle from the start time through the time duration (i.e., time versus location for each particle at a desired frequency). For example, if a particle is tracked for a time duration of 30,000 s and the user wants to trace the locations of the tracked particle every 100 sec, then the trajectory will include 301 points, where 301 is equal to  $(30,000/100) + 1$ . ERDC-PT stores the PT output in both ASCII and binary XDMF format for inspection and post-processing, respectively. If XDMF output is chosen, one output file is created that stores the grid data and another for the PT data. Both are easily imported into Paraview, for example.

### 3 Governing Equations and Numerical Solutions

#### 3.1 Governing Equation

In the ERDC-PT software, the following ODE in vector form is discretized in time solved for, effectively defining a particle path:

$$\frac{d\mathbf{x}}{dt} = \mathbf{V}(\mathbf{x}, t). \quad (1)$$

Here, the following are defined:

- $\mathbf{x}$  = location of a tracked particle [ $L$ ]
- $t$  = time [ $t$ ]
- $\mathbf{V}$  = tracking velocity [ $L/t$ ]

Given the initial location of a particle, that is,  $\mathbf{x}(t_0)$ , one can compute the particle path through time integration over the specified velocities, as shown in Equation (2):

$$\mathbf{x}(t) = \mathbf{x}(t_0) + \int_{t_0}^{t'} \mathbf{v}(\mathbf{x}, t) dt, \quad (2)$$

where

- $t_0$  = start time for PT [ $t$ ], and
- $t'$  = a dummy variable used for time integration.

#### 3.2 Time Integration

ERDC-PT includes options for first-, second-, or fourth-order RK for computation, where the user provides a specified timestep size parameter. The computed particle trajectory is composed of many tracking segments, and each segment is associated with a successful RK computation. The timestep may be cut as the particle approaches an external boundary.

#### 3.3 Interpolation of Velocity

The given velocity, salinity, and water quality fields can vary in both time and space. Since the analytical fields are not available in complex real-

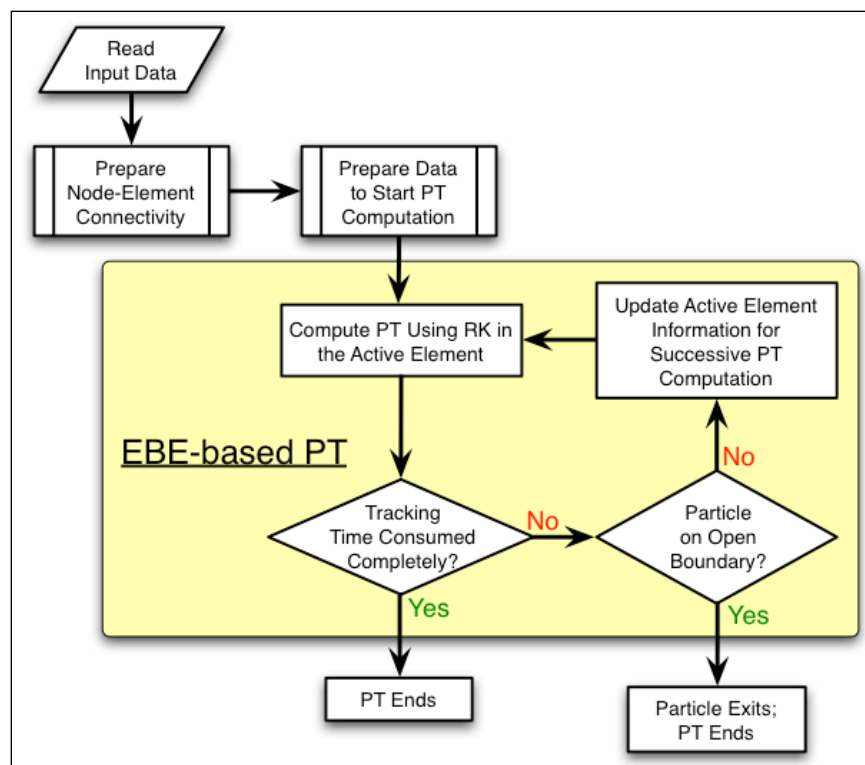
world systems, interpolation becomes essential to estimate velocity at various times and locations in the PT computation. ERDC-PT conducts a linear temporal interpolation and the following spatial interpolation schemes, depending on the shape of the active element, for its background field computation:

- Linear 2D triangular and 3D tetrahedral elements
- Bilinear for 2D quadrilateral elements

### 3.4 Element-by-Element (EBE) Tracking

ERDC-PT conducts all PT on an EBE basis (Cheng et al. 1996), where each tracking segment computed uses the designated RK scheme within an element. Figure 1 presents this EBE tracking concept.

Figure 1. Element-by-element particle tracking (PT) diagram.

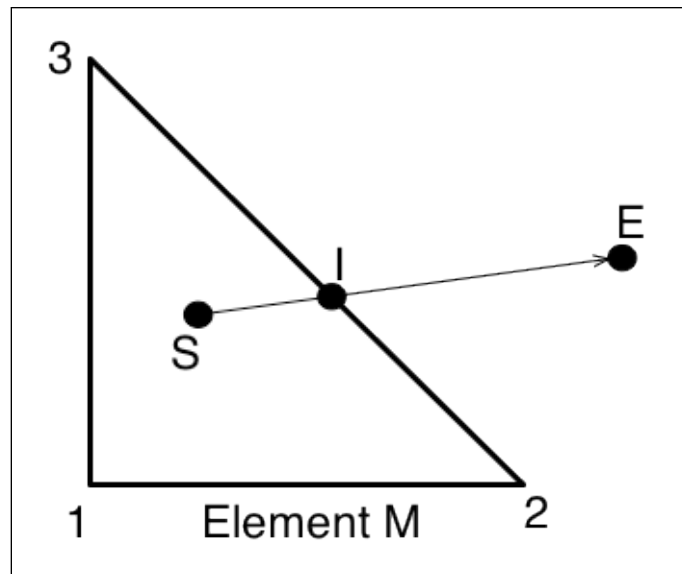


As shown in Figure 1, ERDC-PT reads domain geometry, velocity, and necessary information for particle tracking. It uses the information of domain geometry to prepare node-element connectivity, where the elements connecting at each global node are identified and stored. To track a particle, ERDC-PT first locates the element where the particle has entered. This element is called the active element. Then it conducts PT

computation within this active element using the designated RK scheme (if the particle is on an interface between elements, all elements owning this particle are potential active elements and will be examined one by one until a successful PT computation is performed). ERDC-PT uses the user-specified initial timestep size for the first PT computation within the active element.

If the timestep size used is too large such that the particle will go outside the active element, the timestep size will be reduced so that the particle would reach the boundary of the active element. This reduction of timestep size is enforced in the EBE tracking even if a non-adaptive RK scheme is employed. Figure 2 demonstrates how this reduction of tracking timestep size is achieved in ERDC-PT.

Figure 2. Plot to demonstrate how PT timestep size is reduced when the end location is outside of the active element.



In Figure 2, points S and E represent the start and the end locations of a PT computation associated with the active element M, and point I is the intercept of segment SE and the element boundary. If the PT timestep size from point S to point E is  $\Delta t_{old}$ , then the new PT timestep size to prevent the particle from going outside the element is estimated using the following equation:

$$\Delta t_{new} = \Delta t_{old} \cdot \frac{l_{SI}}{l_{SE}}, \quad (3)$$

where

$$\begin{aligned}\Delta t_{new} &= \text{new PT timestep,} \\ l_{SI} &= \text{distance between points S and I, and} \\ l_{SE} &= \text{distance between points S and E.}\end{aligned}$$

Both the end location and the available tracking time are examined after each successful PT segment is computed. If the end location is still within the active element and the available tracking time is not zero, successive PT computations are conducted until either the tracking time is completely consumed or the particle reaches the boundary of the active element.

The cumulative tracking of a particle is considered complete when either the available tracking time becomes zero or the particle exits from an open boundary before tracking time is consumed completely. An open boundary is a boundary through which particles are permitted to enter or leave the domain of interest. When the particle reaches the boundary of the active element that is not an open boundary and the available tracking time is not zero, tracking will continue. In this case, all active element candidates are tested one by one until a successful tracking is conducted as mentioned before. This forms the EBE-based tracking as highlighted with the shading in Figure 1.

### 3.5 Tracking along a Closed Boundary

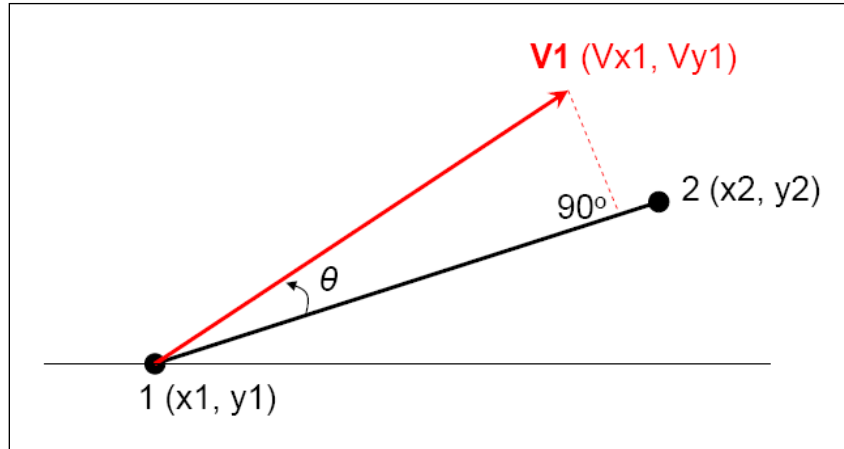
An open boundary is a boundary through which a particle can enter or exit the domain of interest. A boundary is a closed boundary if it is not an open boundary. Conceptually, the flow velocity associated with a closed boundary is parallel or tangential to the boundary (i.e., zero normal velocity at the closed boundary). However, both mesh resolution and numerical error can contribute to non-tangent flow velocity at the closed boundary. This is common in the simulations of real-world problems. As a result, the computed PT results can become misleading if the PT computation does not proceed when the tracked particle reaches a closed boundary.

ERDC-PT uses the projected velocity on the closed boundary to continue the PT computation until the tracked particle reaches an open boundary or the tracking time is completely consumed. The computation of projected velocity on the closed boundary is now described.

### 3.5.1 Velocity Projection onto a 2D Boundary Edge

As shown in Figure 3, the computed velocity at node 1 is  $\mathbf{V}_1$ , which is nonparallel to the closed boundary edge between nodes 1 and 2.

Figure 3. Projection of velocity onto a 2D boundary segment.



The length of edge 1-2 ( $l_{12}$ ) is

$$l_{12} = \sqrt{(\Delta x)^2 + (\Delta y)^2}, \quad (4)$$

where

$$\begin{aligned} \Delta x &= x_2 - x_1, \\ \Delta y &= y_2 - y_1, \\ (x_1, y_1) &= \text{coordinates of node 1,} \\ (x_2, y_2) &= \text{coordinates of node 2.} \end{aligned}$$

The unit vector parallel to edge 1-2,  $\mathbf{u}$ , is

$$\mathbf{u} = \left[ \frac{\Delta x}{l_{12}}, \frac{\Delta y}{l_{12}} \right]. \quad (5)$$

The projected magnitude of  $\mathbf{V}_1$  onto edge 1-2 can be computed as

$$|v_{1p}| = \mathbf{v}_1 \cdot \mathbf{u}, \quad (6)$$

where

$$\mathbf{V}_{1p} = \text{projected velocity of } \mathbf{V}_1 \text{ onto edge 1-2.}$$

Then the projected velocity of  $\mathbf{V}_1$  onto edge 1-2 (i.e.,  $\mathbf{V}_{1p}$ ) is computed as

$$\mathbf{V1p} = |\mathbf{V1p}| \mathbf{u} = (\mathbf{V1} \cdot \mathbf{u}) \mathbf{u} = \left( V_{x1} \cdot \frac{\Delta x}{l_{12}} + V_{y1} \cdot \frac{\Delta y}{l_{12}} \right) \left( \frac{\Delta x}{l_{12}}, \frac{\Delta y}{l_{12}} \right), \quad (7)$$

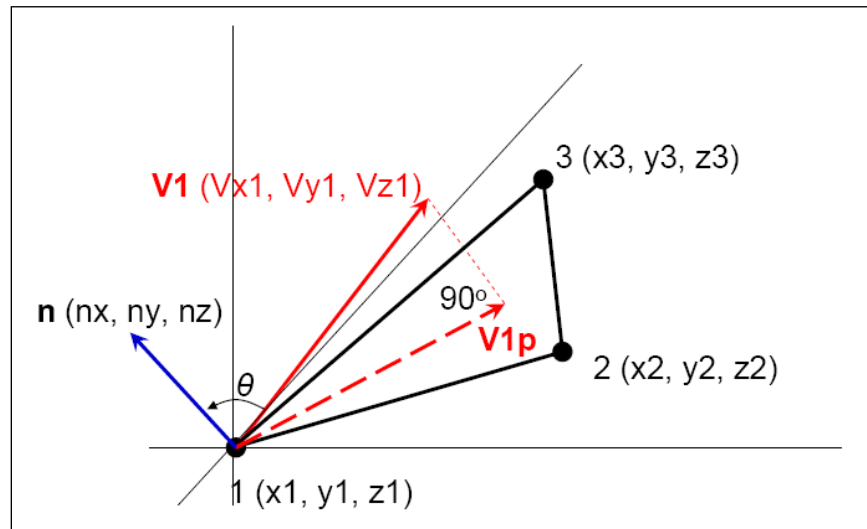
where

$V_{x1}, V_{y1}$  =  $x$ - and  $y$ -components of  $\mathbf{V1}$ .

### 3.5.2 Velocity Projection onto a 3D Boundary Face

Figure 4 shows the geometric relationship of the velocity at node 1 (i.e.,  $\mathbf{V1}$ ) and a 3D triangular boundary face 1-2-3. The equation describing the plan containing face 1-2-3 can be represented by  $ax + by + cz + d = 0$ , where the normal vector of the plane is  $(a, b, c)$ .

Figure 4. Projection of velocity onto a 3D boundary face.



The normal vector of the plane,  $\mathbf{n}$ , can be computed using Equation (8) as

$$\mathbf{n}=(a,b,c) = \Delta \mathbf{L}_{12} \times \Delta \mathbf{L}_{13} = (\Delta x_{12}, \Delta y_{12}, \Delta z_{12}) \times (\Delta x_{13}, \Delta y_{13}, \Delta z_{13}), \quad (8)$$

where

$$a = \begin{vmatrix} \Delta y_{12} & \Delta z_{12} \\ \Delta y_{13} & \Delta z_{13} \end{vmatrix},$$

$$b = \begin{vmatrix} \Delta z_{12} & \Delta x_{12} \\ \Delta z_{13} & \Delta x_{13} \end{vmatrix},$$

$$c = \begin{vmatrix} \Delta x_{12} & \Delta y_{12} \\ \Delta x_{13} & \Delta y_{13} \end{vmatrix},$$

$$\Delta x_{12} = x_2 - x_1,$$

$$\Delta y_{12} = y_2 - y_1,$$

$$\Delta z_{12} = z_2 - z_1,$$

$$\Delta x_{13} = x_3 - x_1,$$

$$\Delta y_{13} = y_3 - y_1,$$

$$\Delta z_{13} = z_3 - z_1,$$

$(x_1, y_1, z_1)$  = coordinates of node 1,

$(x_2, y_2, z_2)$  = coordinates of node 2, and

$(x_3, y_3, z_3)$  = coordinates of node 3.

The unit normal vector is calculated as

$$\mathbf{u} = \left( \frac{a}{|\mathbf{n}|}, \frac{b}{|\mathbf{n}|}, \frac{c}{|\mathbf{n}|} \right), \quad (9)$$

where

$$\begin{aligned} \mathbf{u} &= \text{the unit normal velocity of the plan containing face 1-2-3, and} \\ |\mathbf{n}| &= \sqrt{a^2 + b^2 + c^2}. \end{aligned}$$

The projected velocity of  $\mathbf{V}_1$  parallel to the unit normal velocity  $\mathbf{u}$  is

$$\mathbf{V}_{1n} = |\mathbf{V}_{1n}| \mathbf{u} = (\mathbf{V}_1 \cdot \mathbf{u}) \mathbf{u} = \left( V_{x1} \cdot \frac{a}{|\mathbf{n}|} + V_{y1} \cdot \frac{b}{|\mathbf{n}|} + V_{z1} \cdot \frac{c}{|\mathbf{n}|} \right) \left( \frac{a}{|\mathbf{n}|}, \frac{b}{|\mathbf{n}|}, \frac{c}{|\mathbf{n}|} \right), \quad (10)$$

where

$$V_{x1}, V_{y1}, V_{z1} = x, y \text{ and } z \text{ components of } \mathbf{V}_1.$$

The projected velocity of  $\mathbf{V}_1$  onto Face 1-2-3 is

$$\begin{aligned} \mathbf{V}_{1p} &= \mathbf{V}_1 - \mathbf{V}_{1n} = \mathbf{V}_1 - |\mathbf{V}_{1n}| \mathbf{u} \\ &= (V_{x1}, V_{y1}, V_{z1}) - \left( V_{x1} \cdot \frac{a}{|\mathbf{n}|} + V_{y1} \cdot \frac{b}{|\mathbf{n}|} + V_{z1} \cdot \frac{c}{|\mathbf{n}|} \right) \left( \frac{a}{|\mathbf{n}|}, \frac{b}{|\mathbf{n}|}, \frac{c}{|\mathbf{n}|} \right). \end{aligned} \quad (11)$$

It is important to note that each boundary element should have only one closed boundary edge/face when the unstructured mesh is constructed. Otherwise, the velocity at nodes associated with two closed boundary edges/faces will not be projected properly.

ERDC-PT closed boundary tracking can be applied to the grid surface to track movement of particles along a transient free surface.

## 4 Summary

This report describes the initial effort of modernizing and enhancing the PT123 particle tracking model to form a new, more powerful tracking tool, ERDC-PT. PT123 was designed to perform accurate and efficient particle tracking of massless particles for (1) solving multidimensional transport problem using the Eulerian-Lagrangian Localized Adjoint Methods as proposed in the Civil Works Basic Research project titled “Efficient Resolution of Complex Transport Phenomena Using Eulerian-Lagrangian Techniques” and (2) enhancing ERDC’s modeling capability through linkage to or incorporation into existing flow, transport, and individual-based particle tracking models. ERDC-PT modernized the legacy software to the more commonly used C language. It also now includes distributed computing, free surface tracking, XDMF output, and many more advanced particle trajectory options, such as specific behavior capabilities.

Much like its predecessor, ERDC-PT can track particles on 2D and 3D unstructured meshes. The elements currently used to construct ERDC-PT meshes are triangular elements in 2D and tetrahedral elements in 3D. Various RK schemes are available in ERDC-PT to solve the ordinary differential equations describing the motion of massless particles. ERDC-PT also implements an EBE-based tracking method to minimize the element searching effort when tracking can go beyond one element. ERDC-PT also conducts velocity projection to perform smooth tracking along closed boundaries.

Future advancements may include more species behavior options, GUI development, development of auxiliary tools to convert data from finite difference or finite volume models to the ERDC-PT format, and including more element shapes.

## References

- Adaptive Hydraulics (AdH). 2010. Adaptive Hydraulics Modeling. <https://adh.usace.army.mil/>.
- Bensabat J., Q. Zhou, and J. Bear. 1998. "An Adaptive Path Line-Based Particle Tracking Algorithm for the Eulerian–Lagrangian Method." *Advances in Water Resources* 23: 383–97.
- Cash, J. R. 1989. "A Block 6(4) Runge–Kutta Formula for Nonstiff Initial Value Problems." *ACM Trans Math Software* 15 (1): 15–28.
- Cheng, H.-P., M. W. Farthing, K. D. Winters, S. E. Howington, and J.-R. Cheng. Hines, A. M. 2010. *PT123 A Multi-Dimensional Particle Tracking Computer Program: Version 1.0*. ERDC TR-11-10. Vicksburg, MS: US Army Engineer Research and Development Center.
- Cheng, J.-R., H.-P. Cheng, and G.-T. Yeh. 1996. "A Particle Tracking Technique for the Lagrangian-Eulerian Finite Element Method in Multi-Dimensions." *International Journal for Numerical Methods in Engineering* 39: 1115–36.
- Cheng, J.-R. C., and P. E. Plassman. 2004. "Parallel Particle Tracking Framework for Applications in Scientific Computing." *The Journal of Supercomputing* 28: 149–64.
- Farthing, M., and C. Kees. 2009. *Evaluating Finite Element Methods for the Level Set Equation*. ERDC/CHL-TR-09-11. Vicksburg, MS: US Army Engineer Research and Development Center.
- Goodwin, R. A., J. M. Nestler, J. J. Anderson, L. J. Weber, and D. P. Loucks. 2006. "Forecasting 3-D Fish Movement Behavior Using a Eulerian-Lagrangian-Agent Method (ELAM)." *Ecological Modelling* 192: 197–223.
- Jury, W. A., W. R. Gardner, and W. H. Gardner. 1991. *Soil Physics, 5th ed.* Hoboken, NJ: John Wiley & Sons.
- Liu, Y., R. H. Weiberg, C. Hu, and L. Zheng. 2011. "Tracking the Deepwater Horizon Oil Spill: A Modeling Perspective." *EOS, Transactions, American Geophysical Union* 92 (6): 45–46.
- MacDonald, N. J., M. H. Davies, A. K. Zundel, J. D. Howlett, Z. Demirbilek, J. Z. Gailani, T. C. Lackey, and J. Smith. 2006. *PTM: Particle Tracking Model, Report 1: Model Theory, Implementation, and Example Applications*. ERDC/CHL TR-06-20. Vicksburg, MS: US Army Engineer Research and Development Center.
- Oliveira, A., and A. M. Baptista. 1998. "On the Role of Tracking on Eulerian-Lagrangian Solutions of the Transport Equations." *Advances in Water Resources* 21: 539–54.

- Pokrajac, D., and R. Lazic. 2002. "An Efficient Algorithm for High Accuracy Particle Tracking in Finite Elements." *Advances in Water Resources* 25: 353–369.
- Pollock, D. W. 1988. "Semianalytical Computation of Path Lines for Finite Difference Models." *Ground Water* 26 (6): 743–50.
- Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. 1992. *Numerical Recipes in C, 2nd ed.* Cambridge, UK: Cambridge University Press.
- Russell, T. F., and M. A. Celia. 2002. "An Overview of Research on Eulerian-Lagrangian Localized Adjoint Methods (ELLAM)." *Advances in Water Resources* 25 (8–12): 1215–31.
- Tate, J. N., T. C. Lackey, and T. O. McAlpin. 2010. *Seabrook Fish Larval Transport Study*. ERDC/CHL TR-10-12. Vicksburg, MS: US Army Engineer Research and Development Center.

## Abbreviations

AdH	Adaptive hydraulics
EBE	Element-by-element
EL	Eulerian-Lagrangian
ERDC	Engineer Research and Development Center
ODE	Ordinary differential equations
PT	Particle tracking
RK	Runge-Kutta
USACE	US Army Corps of Engineers
XDMF	Extensible Data Model and Format

# REPORT DOCUMENTATION PAGE

<b>1. REPORT DATE</b> January 2024		<b>2. REPORT TYPE</b> Technical Report		<b>3. DATES COVERED</b>	
				<b>START DATE</b> FY22	<b>END DATE</b> FY23
<b>4. TITLE AND SUBTITLE</b> ERDC-PT: A Multidimensional Particle Tracking Model					
<b>5a. CONTRACT NUMBER</b>		<b>5b. GRANT NUMBER</b>		<b>5c. PROGRAM ELEMENT</b>	
<b>5d. PROJECT NUMBER</b>		<b>5e. TASK NUMBER</b>		<b>5f. WORK UNIT NUMBER</b>	
<b>6. AUTHOR(S)</b> Corey J. Trahan, Jing-Ru C. Cheng, and Amanda Hines					
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> US Army Engineer Research and Development Center Information Technology Laboratory 3909 Halls Ferry Road Vicksburg, MS 39180-6199				<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b> ERDC/ITL TR-24-2	
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> Headquarters, US Army Corps of Engineers Washington, DC 20314-1000			<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b> HQUSACE		<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b>
<b>12. DISTRIBUTION/AVAILABILITY STATEMENT</b> Distribution Statement A. Approved for public release; distribution is unlimited.					
<b>13. SUPPLEMENTARY NOTES</b> Funding Account Code D2H545, AMSCO Code 008284					
<b>14. ABSTRACT</b> This report describes the technical engine details of the particle- and species-tracking software ERDC-PT. The development of ERDC-PT leveraged a legacy ERDC tracking model, "PT123," developed by a civil works basic research project titled "Efficient Resolution of Complex Transport Phenomena Using Eulerian-Lagrangian Techniques" and in part by the System-Wide Water Resources Program. Given hydrodynamic velocities, ERDC-PT can track thousands of massless particles on 2D and 3D unstructured or converted structured meshes through distributed processing. At the time of this report, ERDC-PT supports triangular elements in 2D and tetrahedral elements in 3D. First-, second-, and fourth-order Runge-Kutta time integration methods are included in ERDC-PT to solve the ordinary differential equations describing the motion of particles. An element-by-element tracking algorithm is used for efficient particle tracking over the mesh. ERDC-PT tracks particles along the closed and free surface boundaries by velocity projection and stops tracking when a particle encounters the open boundary. In addition to passive particles, ERDC-PT can transport behavioral species, such as oyster larvae. This report is the first report of the series describing the technical details of the tracking engine. It details the governing equation and numerical approaching associated with ERDC-PT Version 1.0 contents.					
<b>15. SUBJECT TERMS</b> Computer simulation; Fluid dynamics--Mathematical models; Hydrodynamics--Mathematical models; Sediment transport; Water quality					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>		<b>18. NUMBER OF PAGES</b>
<b>a. REPORT</b> Unclassified	<b>b. ABSTRACT</b> Unclassified	<b>c. THIS PAGE</b> Unclassified	SAR		26
<b>19a. NAME OF RESPONSIBLE PERSON</b>			<b>19b. TELEPHONE NUMBER (include area code)</b>		