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INSTRUMENTATION AND MEASUREMENT OF AIRCRAFT CARRIER DECK MOTION

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INSTRUMENTATION AND MEASUREMENT OF AIRCRAFT CARRIER DECK MOTION

INTRODUCTION

The Automatic Carrier Controlled Approach System (ACCA)^{1,2} requires the control of aircraft along a control line tied to an unstable platform, namely the flight deck of an aircraft carrier. The vertical motion of the fantail of a carrier is of particular interest because of the response requirements it imposes on the vertical control loop of ACCA. Any logical extension of ACCA to an Automatic Carrier Controlled Landing System will make it imperative to study these motions and in some way compensate for them. The information obtained on this motion will be used to develop an equipment capable of sensing and possibly predicting future deck positions. These predicted positions will in turn be employed as an integral part of ACCA to compensate the altitude coordinate of the aircraft under control.

It is not expected that aircraft vertical frequency response characteristics will be sufficiently broad that instantaneous ship motion can be followed even if this were desirable. However, whatever information can be obtained about the instantaneous coordinate of the fantail position with respect to the sea as an inertial reference will either be predicted or smoothed for aircraft position regulation. To determine the feasibility of prediction requires a complete knowledge of the characteristics of the data. This report contains a description of techniques and results of measurement of fantail motion.

BACKGROUND

Investigation³ indicates that little experimental work has been directed toward determining the complete motions of a ship under various sea conditions and particularly those of an aircraft carrier in landing attitude. At the advent of these studies some data on the deck motion of the SS OCEAN VULCAN, a 10,000-ton cargo steamer, was available to the authors; since the Laboratory started work on this problem, the David Taylor Model Basin has begun an experimental study of the various hull motions of ships at sea.

¹ Reynolds, J. B., Jr., and Saffitz, I. M., "Analysis of the Automatic Carrier Controlled Approach System," NRL Report 3687 (Confidential), 25 May 1950

² Reynolds, J. B., Jr., and Huntley, K. L., "Instrumentation and Preliminary Tests of the Automatic Carrier Controlled Approach System," NRL Report 3806 (Confidential), 18 May 1951

³ Bohne, Q. R., "Literature Search on the Subject of Ship Motion," Minneapolis-Honeywell Regulator Co., Aeronautical Division, Research Report AD 5060 A-RI (Unclassified), 1 November 1951

The six components of motion of a ship at sea are surge, sway, and heave in translation; plus pitch, roll, and yaw in rotation.⁴ Of these six only two are of principle interest, namely, heave and pitch. Roll has been measured to determine its influence on pitch and heave measurement. The evaluation of pitch and total deck displacement allows for the determination of heave alone assuming a linear superposition of heave and linear vertical motions due to pitch. The relative magnitudes of linear pitch motion and heave is of vital concern for two reasons. If it can be shown, for example, that heave motion is of an order of magnitude less than linear pitch motion at the fantail then only one of these two variables need be sensed in the final system. Secondly, the relative magnitudes of the two variables determine the nature of the prediction equipment. A point of vital interest in data interpretation will be to determine under what conditions, if any, deck motion approximates the properties of a stationary random time process.

OBJECTIVES

The deck motion study program has three main objectives:

- (1) Instrumentation and measurement of all deck motions pertinent to the control problem of ACCA.
- (2) Data reduction and analysis.
- (3) Incorporation of features in the design of the ACCA vertical control loop, which will eliminate or minimize the uncertainties resulting from ship motion.

This report concerns itself entirely with instrumentation and measurement of deck motion which is the first major phase of the program.

TECHNIQUE

Roll, pitch, and displacement were sensed and recorded as shown in Figure 1. The aircraft carriers on which various measurements were made were all equipped with stable elements as part of their fire control systems. These stable elements provided excellent information sources for pitch and roll motions with angular accuracies in the order of 4 minutes. This accuracy yields a pitch displacement resolution approximately equal to the six-inch displacement resolution designed into the displacement indicator used in conjunction with the accelerometer. The roll and pitch information were simultaneously recorded using frequency multiplexing on a single magnetic tape. This permanent tape record made data available in electrical form for laboratory analysis.

The accelerometer (Figure 2), mounted near the fantail, provided electrical signals proportional to the instantaneous vertical deck accelerations. This signal is fed into the displacement indicator (Figures 3 and 4) where it is double integrated in the strict mathematical sense to yield y , the vertical deck displacement. This signal in turn is frequency multiplexed and recorded along with roll and pitch.

The separation filters and demodulators (Figure 1) comprise the decoding portions of the instrumentation. Their principle utility is in the laboratory though on shipboard they provide a convenient means of checking the quality and acceptability of data being recorded.

⁴ Manning, G. C., and Schumacher, T. L., "Principles of Warship Construction and Damage Control," pp. 116-117, Annapolis, Maryland: United States Naval Institute, 1935

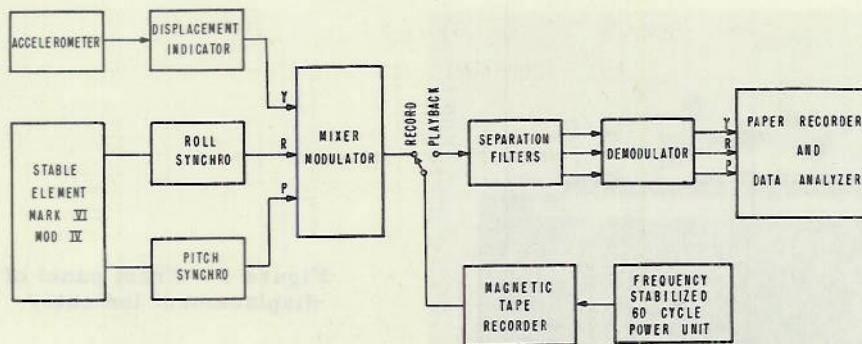


Figure 1 - Block diagram of instrumentation for ship motion studies

INSTRUMENTATION

Accelerometer

The accelerometer (Figure 2) used for these measurements was a model DA-1 manufactured by Schaevitz which operates on a differential transformer principle. This accelerometer is capable of responding without "droop" to frequencies above 10 cps and maintains linearity for accelerations up to $\pm 7g$. Since it responds only to lineal accelerations along its axis it is well suited to deck motion measurement. As a result of the axially constrained response of the accelerometer plus the constraint of the accelerometer to the ship deck, it is apparent that the presence of roll and pitch motion will introduce some spurious acceleration indications. An analysis made of this error figure (Appendix A) indicates that its rms value is no greater than the experimentally evaluated over-all displacement resolution. In view of these results, no steps were taken to eliminate this source of error.

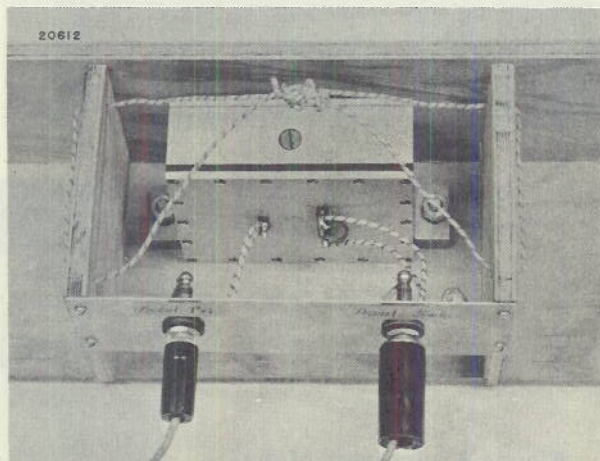


Figure 2 - Schaevitz model DA-1 accelerometer

Displacement Indicator

A phase shift oscillator V-1A and V-1B (Figure 5), tuned to 1050 cps, feeds a power amplifier V-2 where it is transformer coupled through the cable pictured in Figure 3 to the primary of the accelerometer. The potentiometer P-1 provides drive control and meter M-1, a drive monitor. As a consequence of the orders of magnitude of accuracy involved it was necessary to shock mount the oscillator against vibration due to microphonics to prevent impulsive and long time drifts in the double-integrator output. The accelerometer amplitude modulates the 1050-cycle carrier in direct proportion to those accelerations impressed on it. P-2 is an accelerometer output gain control, fed from the accelerometer secondary by a cable pictured in Figure 3, and is adjusted for an output calibration in the order of 1/20 volt per foot. The accelerometer output is amplified and full-wave detected in V-3, V-4A, and V-5. V-4B provides a low impedance output for metering and calibration purposes.

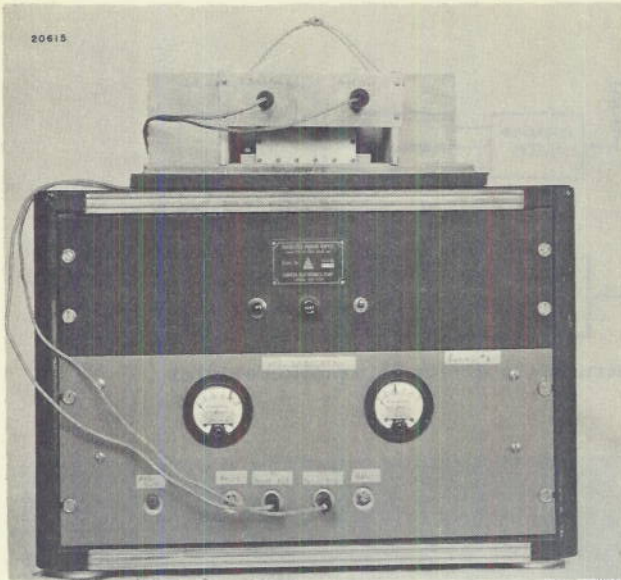


Figure 3 - Front panel of displacement indicator

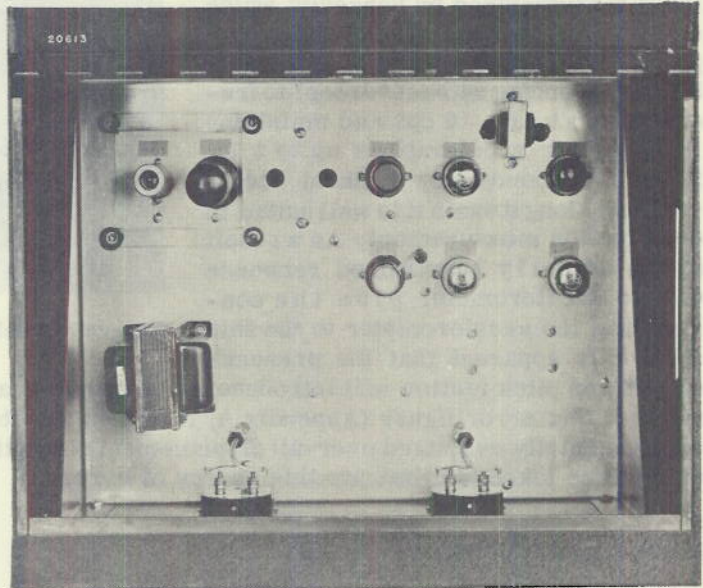


Figure 4 - Interior of displacement indicator

The detected acceleration is double integrated by passage through a succession of cascaded RC networks of 10-second time constant. These networks are actively coupled by cathode followers V-6A, V-6B, and V-8A and voltage amplifier 6F6 to the output Minibox. The db-log frequency plot (Figure 6) reveals the frequency sensitive character of the circuitry. For frequencies an octave above the corner, corresponding to periods of 30 seconds or less, the slope of the characteristic is -12 db per octave consistent with the mathematical double integration. The RC networks effect a double integration of the acceleration signal to yield displacement for ship heave, pitch, and roll periods which are 30 seconds or less. For frequencies an octave below the corner, corresponding to

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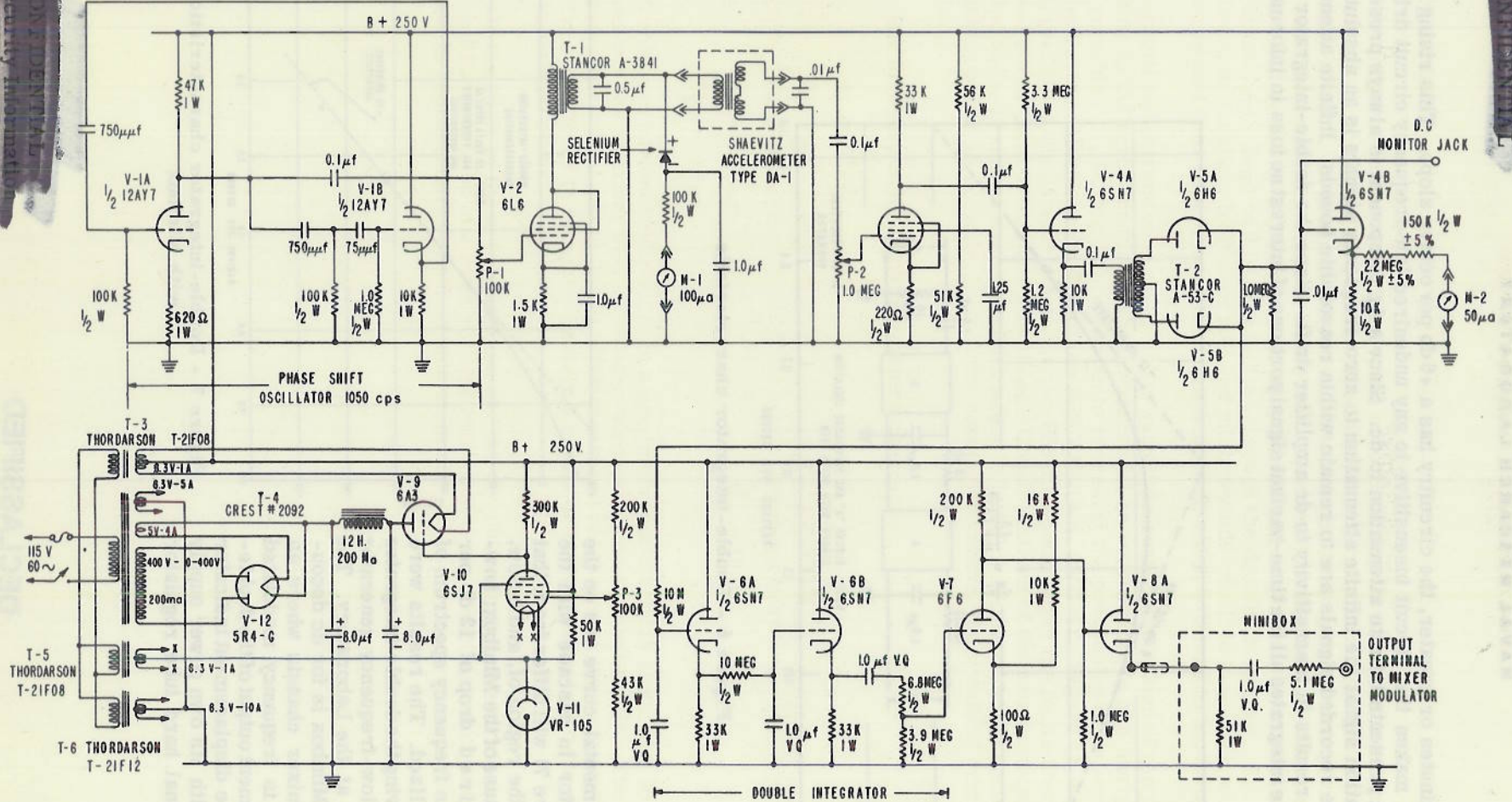


Figure 5 - Schematic of accelerometer and displacement indicator circuits

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periods of 2 minutes or greater, the circuitry has a +6 db per octave slope. This rising characteristic makes the circuit insensitive to any undesired low-frequency circuit drift and in addition presents infinite attenuation to dc. Since a 1g component is always present in the acceleration signal the infinite attenuation to zero-frequency signals is an absolute necessity if the recorded signals are to remain within reasonable bounds. Infinite attenuation to dc also results in insensitivity to dc amplifier drift. Since the double-integrator circuitry double integrates all the time-variant signal portions of interest no loss in information results.

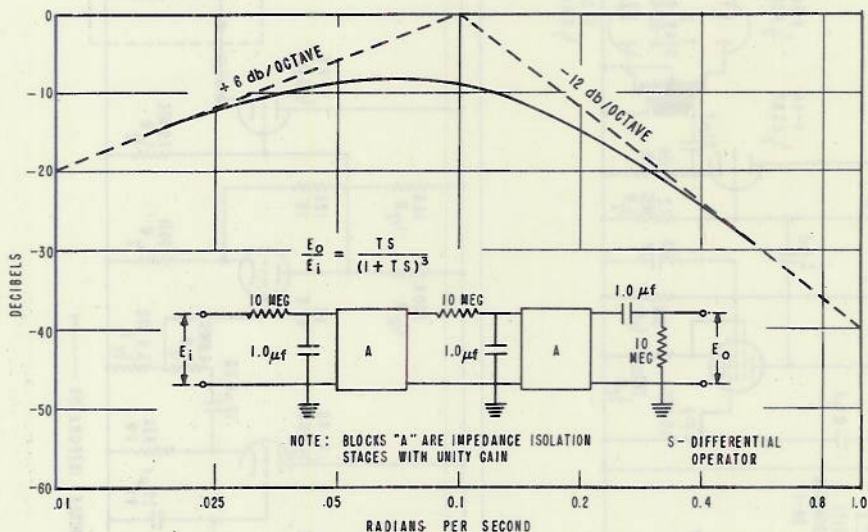


Figure 6 - Double-integrator characteristic

An experimental curve run on the double-integrator in cascade with the Minibox (Figure 7) will differ from that of Figure 6 in the region of, and below, the corner because of the Minibox; however, the desired drop of 12 db per octave in the frequency spectrum of interest is realized. The results were obtained by driving the double-integrator with an ultra low-frequency generator designed here at the Laboratory. The purpose of the Minibox is for dc decoupling to the mixer chassis where an audio carrier is frequency modulated by the displacement output of the double-integrator. The displacement indicator is equipped with its own power supply with conventional hard tube regulator circuitry.

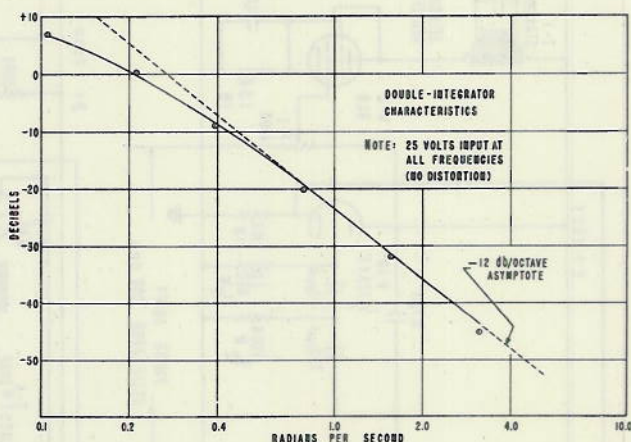


Figure 7 - Double-integrator characteristic with Minibox

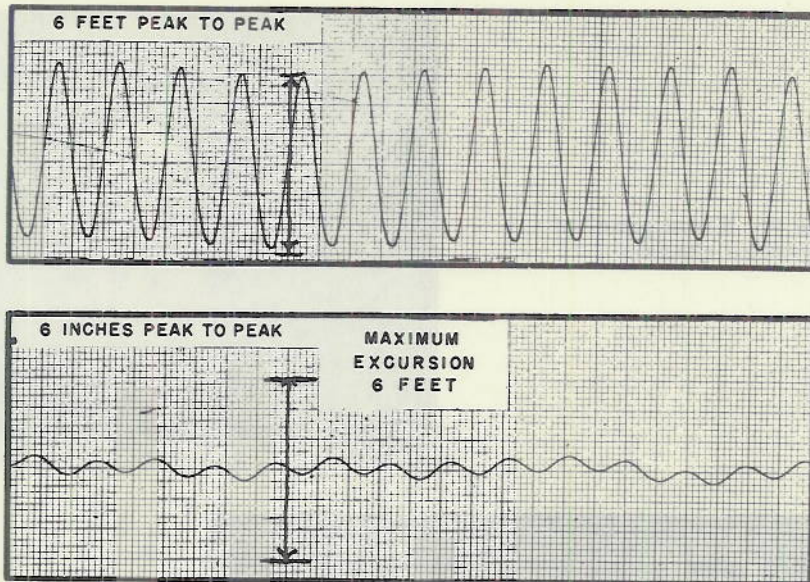


Figure 8 - Records made during an experimental check of the accelerometer (ser. No. 2) and displacement indicator on January 2, 1952

In Figure 8 are records made during an experimental check out of the accelerometer and displacement indicator. The primary purpose of these checks was to determine the minimum resolution of the system in feet. The experimental setup consisted of driving the accelerometer alternately through a 6-foot and 6-inch excursion at a ten-second period with the aid of a boom and low speed drive mechanism. It is apparent from the 6-inch peak-to-peak record that the "base line drift" or minimum resolution is in the order of 6 inches or less. The output calibration for this particular displacement indicator was set at approximately .05 volt per foot. A 1/10-percent regulation of accelerometer drive signal is necessary to keep base line drift within 6 inches.

Roll and Pitch Instrumentation

The front and interior of the synchro chassis employed in the sensing and recording of instantaneous pitch and roll are shown in Figures 9 and 10. The rotary switch provides a means of monitoring by the front panel meter any one of the three variables. The toggle switch following the batteries from which the synchro driven potentiometers are energized is used for the prevention of unnecessary battery drain. Conversion from rotary stable element information to electrical signals is achieved by mechanically coupling Navy 5-F synchros to microtorque potentiometers. The output of these potentiometers is a dc signal directly proportional to the roll and pitch motions. The rather elaborate networks for the conversion (Figure 11) were necessitated by low battery drain requirements plus the need for averaging outputs to zero prior to modulation. Potentiometer P-1 is for amplitude control while P-2, P-3, and P-4 serve as centering controls. The 5-F synchros electrically connected with the stable element are not included in Figure 11. Figures 10 and 11 also show a yaw sensing channel. Initially yaw measurements were made but after careful consideration of problem needs these measurements were discontinued. The stable element available for instrumentation was the Mark VI-Mod IV.

Figure 9 - Front panel of synchro chassis

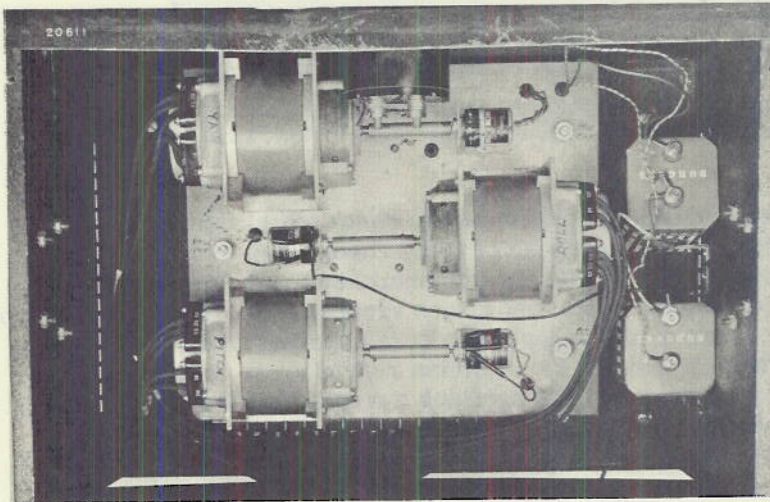
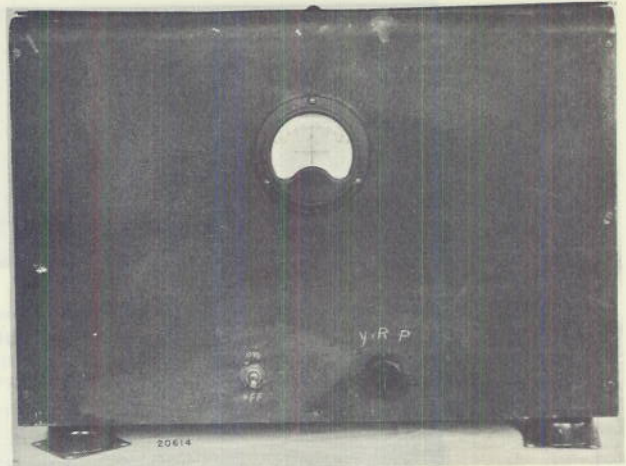
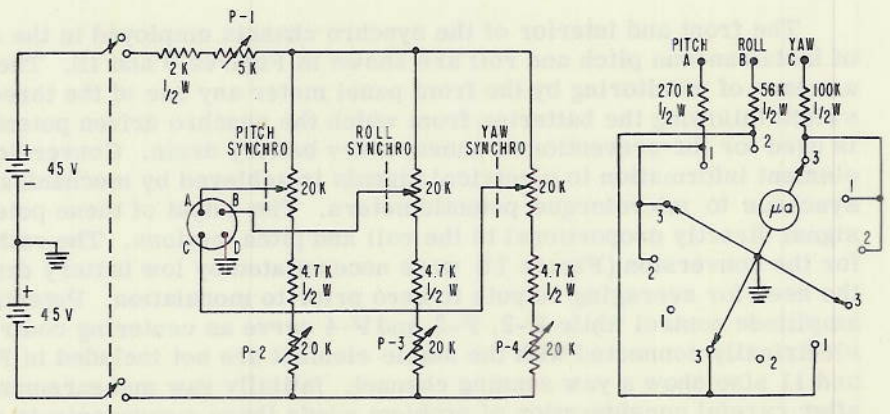


Figure 10 - Interior of synchro chassis

Figure 11 - Schematic of synchro circuit



Mixer-Modulator and Recording

Extensive present day telemetering applications have made telemetering-recording equipment commercially available and hence no development of a modulating recording system was necessary. One of the choices necessary in synthesizing such a system is whether to use FM or AM prior to tape recording. The advantage of AM over FM is its freedom from recorder "wow" disturbances. Also the frequency spread required of the separation filters will in general be less for AM than FM. However, AM suffers from so called tape noise plus noise and distortions inherent to the recording process itself. Low noise for AM requires low-frequency carriers. In general it can be stated that, where using recorders with low wow figures, FM is more desirable from a noise standpoint. This fact plus commercial availability of FM telemetering equipment dictated the choice of mixing-modulating techniques. Due to varying power line load on board ship, it proved expedient to use a Sola ac regulator ahead of the various instrumentation equipments.

The Mixer-Modulator block is made up of commercially available FM telemetering equipment developed by the Applied Physics Laboratory of The Johns Hopkins University. In brief, it consists of multichannel mixer-modulator circuits with a common output suited to feeding an Ampex Tape Recorder. The separation filters unscramble the signal played back from the magnetic recorder after which it is demodulated by a set of three FM discriminators. The three carrier frequencies used were 5400, 7350, and 12,300 cps modulated with a 15-percent frequency deviation. Figure 12 shows a record of the same data simultaneously recorded at the output of the roll channel of the synchro chassis and the output of the demodulators. This was accomplished by placing a jumper across the record-playback switch. Its purpose was to check the fidelity of the modulation-demodulation technique exclusive of the tape recorder. The fidelity can be seen to be excellent.

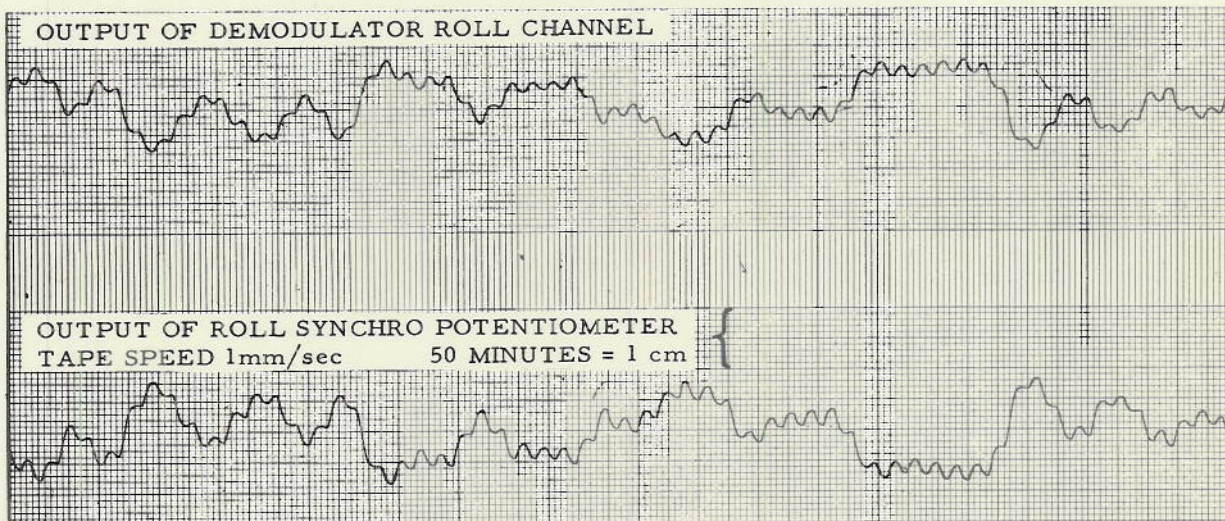


Figure 12 - Telemetering response characteristics. Data taken November 9, 1951 aboard USS TARAWA (CV40)

Since the data are being frequency-modulated, speed variation in the recorder is highly undesirable. This may be minimized by the use of an AC power source frequency controlled by a 60-cycle tuning fork and power amplifier. Thereafter any variations in tape speed are solely a consequence of mechanical eccentricities in the recorder drive mechanisms commonly called wow. Since the wow figure for the Ampex is a fraction of a percent at a 4-cps rate, the noise resulting from these effects is either negligible or

may readily be abstracted from the data depending on signal levels. The Sanborn Cardiographic Recorder was used for making a visual record of the various pieces of data. On shipboard it was primarily used for monitoring and recording directly from the data sources themselves ahead of the mixer-modulator.

RESULTS

Samples of data which were Sanborn recorded on the USS TARAWA and USS KULA GULF are shown in Figures 13 and 14. The longest run was 60 minutes. In statistical analysis on an analogue computer run it is desirable to have as long runs as possible. The pitch and heave periods cluster around 10 sec. Much knowledge and technique were acquired as a result of experience gained on successive trips. Since the data is intermediate to the goals sought, the results themselves are not spectacular.

CONCLUSIONS

The program of Deck Motion Instrumentation has resulted in the development of a practical device for direct measurement of displacement. Continuation of this program should involve data taking on different classes of carriers during landing operations. The influence of various sea conditions on the character of the data must be investigated. If prediction of deck motion is to afford a solution such prediction must be accurate under all sea conditions. Therefore, in order to obtain sufficient data to analyze and attempt to solve the Automatic Carrier Landing problem, the entire program of deck motion measurement should be prosecuted on a continuing basis.

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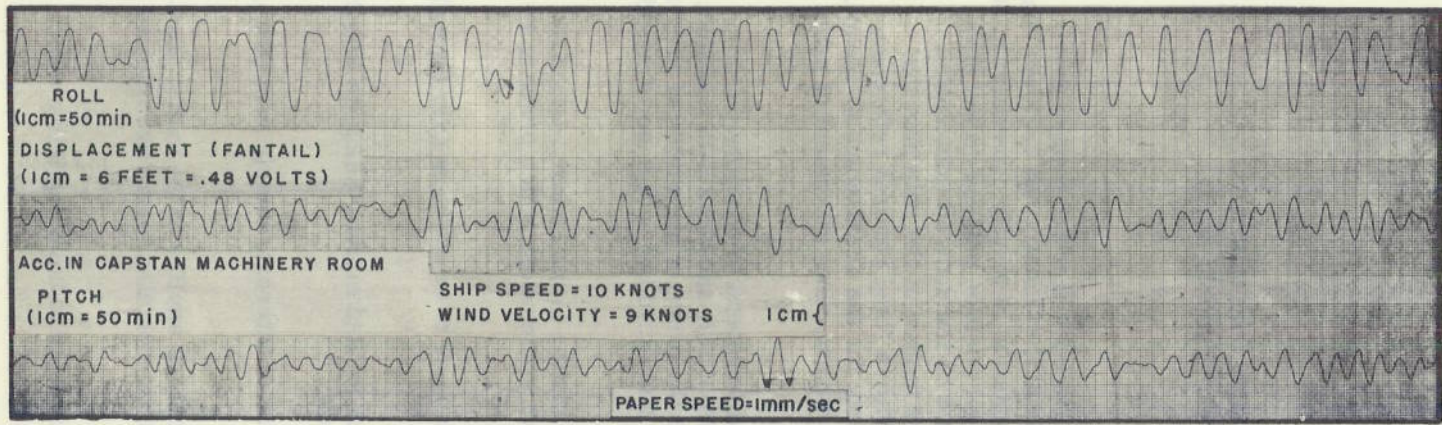


Figure 13 - Roll, pitch, and displacement recording made in plotting room of USS TARAWA (CV40) on October 5, 1951

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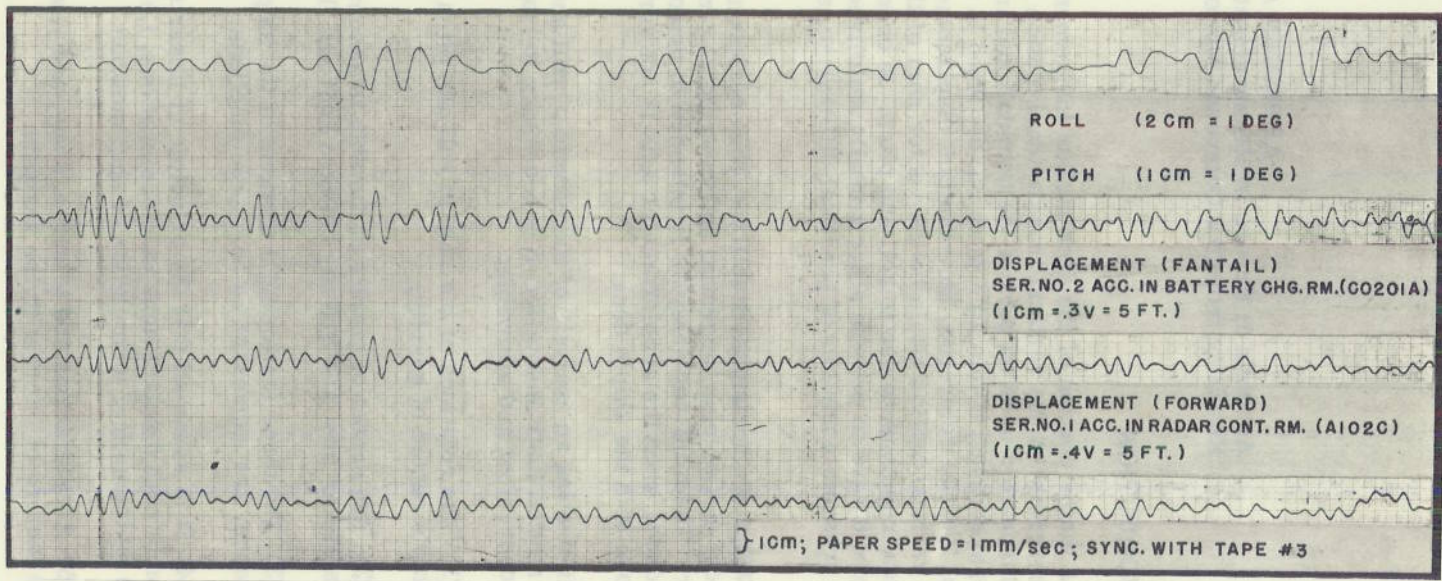


Figure 14 - Roll, pitch, and displacement recording on USS KULA GULF (CVE108)

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APPENDIX A
Analysis of Errors in Measurement of Vertical Ship Motion
Resulting from the Compound Motions of the Accelerometer

INTRODUCTION

A major portion of the deck motion instrumentation has been directed toward the measurement of the vertical translational motion of the fantail of an aircraft carrier. The accelerometer used as part of the translational instrumentation is axially constrained. It is apparent that, if this axis were rigidly fixed in a vertical position relative to the horizontal inertial reference, the resultant displacement would be the desired information. This vertical motion would be the vector sum of the heave motion of the ship plus the vertical components due to roll and pitch. In order to obtain a fixed vertical reference, one would have to provide a horizontally stabilized platform for the accelerometer mount. This presents a number of practical difficulties, such as weight, size, placement of gear, etc.

Without a horizontal stabilizer, the accelerometer will respond to components of vertical motion plus tangential and centripetal components of roll and pitch with resultant error. The following analysis is intended to demonstrate the order of magnitude of errors introduced when the accelerometer axis is not vertically stabilized but rather constrained to move with the ship's deck.

In making an analysis of the effects of roll and pitch motion in the chosen method of deck motion instrumentation, it is necessary to make some assumptions which prove to be both convenient and allowable:

- (1) The rotations of the ship in roll and pitch are assumed to be about an axis through the ship's center of gravity.
- (2) The center of gravity of the ship is assumed to be located midway along its length, on the center line, and halfway between keel and flight deck.
- (3) The three fundamental motions of the ship that are of interest, namely roll, pitch, and heave, are expressed as harmonic functions.
- (4) It is assumed that the principle of linear superposition applies in the vector summation of both displacement and accelerations, i.e., no cross-coupling terms are allowed in the mathematical formulation of ship motions.
- (5) Accelerations due to surge, sway, and yaw are assumed to have negligible effect on the factors of interest in the analysis.
- (6) All angular excursions are restricted to 10° or less.

There exists in the literature on the subject of naval architecture¹ ample precedent for assumption (1). Carrier specifications indicate (2) to be valid within a few percent.² Assumption (3), although an idealization insofar as true ship motion is concerned, does not result in any departure from reality insofar as this analysis is concerned. This follows from assumption (4) and the fact that relative amplitudes and periods are the chief factors affecting error. Assumption (4) may be made as result of the small angular excursions in pitch and roll.³ Assumption (5) is considered valid for two reasons. During carrier landings there is little perturbation in the form of surge accelerations or sway motions. Secondly, the instrumentation is relatively insensitive to horizontal translational motion. Assumption (6) is merely a restriction upon the admissible data, beyond which limits the data will not be considered valid. Experimentally, the great majority of data do not exceed this limit.

The general tack of this analysis is to determine the net accelerations to which the accelerometer is subject and to double integrate these accelerations yielding an indicated displacement. This indicated vertical displacement is then subtracted from the true vertical displacement to give the instantaneous error. The following list defines all parameters and variables used in this analysis:

- H - Heave of the center of gravity of the ship.
- ϕ - Roll angle measured about the center of roll.
- ψ - Pitch angle measured about the center of pitch.
- Y_T - True vertical displacement of the accelerometer.
- Y_i - Indicated vertical displacement of the accelerometer.
- γ - Net angle made by the axis of the accelerometer with the vertical.
- $T_H, T_R,$ and T_P - Heave, roll, and pitch periods respectively.
- P - Pitch moment arm.
- R - Roll moment arm.
- β_0 - Angle between pitch moment arm and longitudinal axis of the carrier.
- g - Acceleration of gravity.
- H_0 - Maximum heave displacement.
- ϕ_0 - Maximum roll angle.
- ψ_0 - Maximum pitch angle.
- $\omega_H, \omega_R,$ and ω_P - Angular rate of heave, roll, and pitch respectively.

¹ Rossell, H. E., and Chapman, L. B., "Principles of Naval Architecture," Vol. II, pp. 10-11, New York: The Society of Naval Architects and Marine Engineers, 1939

² "Aircraft Carrier CVB-42," Booklet of General Plans CVB42, S0103-328322 (Restricted), New York Naval Shipyard, New York, 1 November 1945

³ Weinblum, G., and St. Dennis, M., "On the Motions of Ships at Sea," Advance Paper No. 2 for the Annual Meeting of The Society of Naval Architects and Marine Engineers, 9-10 November 1950

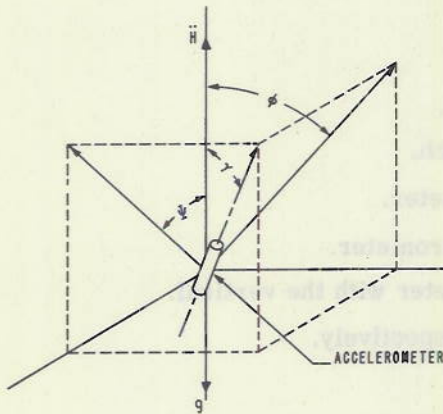
The first phase of the analysis is to establish the total accelerations acting on the accelerometer. The accelerometer is so designed as to respond only to accelerations along its axis. Therefore, the problem reduces to one of superimposing all axial components of acceleration. Since \ddot{H} and g are always directed vertically (Figure A1), it is apparent that the accelerometer's instantaneous response to these two components will be given by $(\ddot{H} - g) \cos \gamma$. The remaining components of acceleration along the axis of the accelerometer result from roll and pitch (Figures A2 and A3) and may be summed as

$$P \cos \beta_0 \ddot{\psi} - P \sin \beta_0 \dot{\psi}^2 - R \dot{\phi}^2.$$

$R \dot{\phi}$ does not appear since it is normal to the accelerometer axis. Then, for the total acceleration,

$$\ddot{y}_1 = (\ddot{H} - g) \cos \gamma - R \dot{\phi}^2 + P \cos \beta_0 \ddot{\psi} - P \sin \beta_0 \dot{\psi}^2 \quad (A1)$$

where \ddot{y}_1 is the instantaneous indicated acceleration.



$$\gamma = (\phi^2 + \psi^2)^{1/2} \quad (\phi \text{ AND } \psi \text{ SMALL})$$

Figure A1 - Instantaneous accelerometer inclination with the vertical

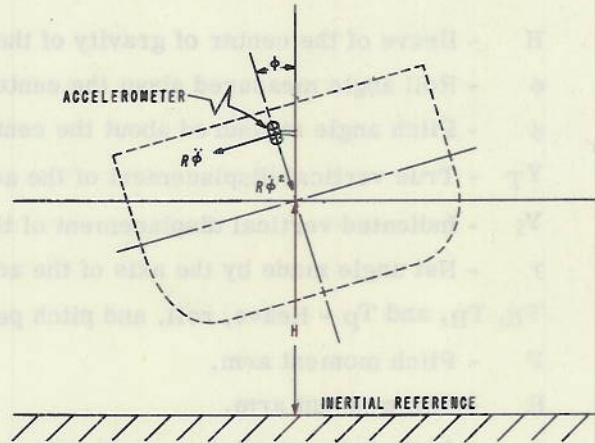


Figure A2 - Instantaneous acceleration component due to roll

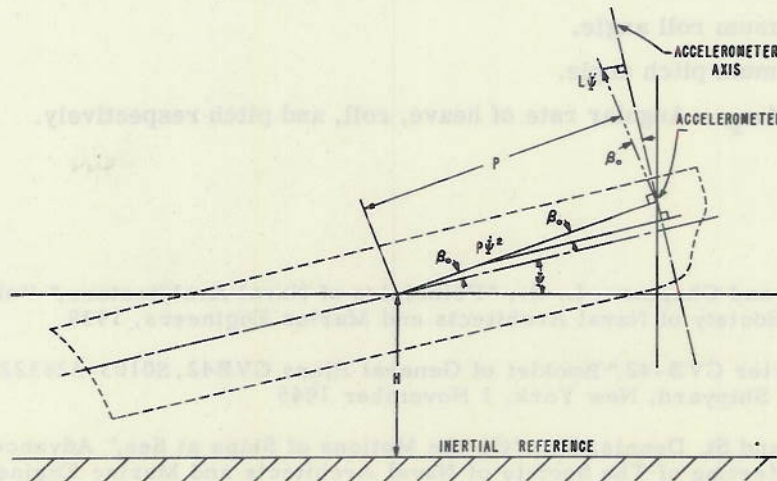


Figure A3 - Instantaneous acceleration component due to pitch

The true vertical displacement of the accelerometer is, assuming linear superposition of motions,

$$y_T = H + R \cos \phi + P \left[\sin (\psi + \beta_0) - \sin \beta_0 \right]$$

and after expanding,

$$y_T = H + R \cos \phi + P \left[\sin \psi \cos \beta_0 + \sin \beta_0 (\cos \psi - 1) \right] \quad (A2)$$

The remainder of the analysis involves developing the error function in terms of y_1 and y_T in the form

$$e(t) = y_T - \iint \ddot{y}_1 (dt)^2. \quad (A3)$$

In performing these operations the following approximations are made:

(a) In expanding and integrating the acceleration, all terms in constant accelerations are eliminated.

(b) Since ψ and ϕ are sufficiently small, $\sin \phi = \phi$ and $\cos \phi = 1 - \phi^2/2$, and similarly for ψ .

$$(c) \gamma = (\phi^2 + \psi^2)^{\frac{1}{2}}.$$

The approximation in (a) is based on the transmission characteristics of the electronic double-integrator incorporated in the instrumentation. This characteristic presents infinite attenuation to dc components and double integrates only those frequencies of interest. Approximation (c) follows from the fact that

$$\cos^2 \gamma = \frac{1}{\tan^2 \phi + \tan^2 \psi + 1} \approx \frac{1}{1 + \psi^2 + \phi^2}.$$

Using a binomial expansion,

$$\cos \gamma = 1 - \frac{1}{2} (\psi^2 + \phi^2),$$

and for a small γ

$$\cos \gamma = 1 - \frac{1}{2} \gamma^2.$$

The following substitutions are made in Equation (A1):

$$\ddot{H} = -\omega_H^2 H_0 \sin \omega_H t$$

$$\dot{\phi}^2 = \omega_R^2 \phi_0^2 \cos^2 \omega_R t$$

$$\ddot{\psi} = -\omega_P^2 \psi_0 \sin \omega_P t$$

$$\dot{\psi}^2 = \omega_P^2 \psi_0^2 \cos^2 \omega_P t$$

The result, after making the appropriate substitutions and double integrating, is:

$$\begin{aligned}
 y_i = & H_0 \sin \omega_H t + \frac{\psi_0^2 \omega_H^2 H_0}{4} \left[\frac{\sin (\omega_H + 2\omega_P)t}{2(\omega_H + 2\omega_P)^2} + \frac{\sin (\omega_H - 2\omega_P)t}{2(\omega_H - 2\omega_P)^2} - \frac{\sin \omega_H t}{\omega_H^2} \right] \\
 & + \frac{\omega_H^2 H_0 \phi_0^2}{4} \left[\frac{\sin (\omega_H + 2\omega_R)t}{2(\omega_H + 2\omega_R)^2} + \frac{\sin (\omega_H - 2\omega_R)t}{2(\omega_H - 2\omega_R)^2} - \frac{\sin \omega_H t}{\omega_H^2} \right] \\
 & + \frac{g}{16} \left[\frac{\psi_0^2 \cos 2\omega_P t}{\omega_P^2} + \frac{\phi_0^2 \cos 2\omega_R t}{\omega_R^2} \right] + \frac{R \phi_0^2 \cos 2\omega_R t}{8} \\
 & + P \cos \beta_0 \psi_0 \sin \omega_P t + P \frac{\psi_0^2}{4} \sin \beta_0 \cos 2\omega_P t. \quad (A4)
 \end{aligned}$$

Referring to Equation (A2), the true displacement

$$\begin{aligned}
 y_T = & H_0 \sin \omega_H t + \frac{R \phi_0^2}{4} \cos 2\omega_R t + P \psi_0 \cos \beta_0 \sin \omega_P t \\
 & + \frac{\psi_0^2}{4} P \sin \beta_0 \cos 2\omega_P t. \quad (A5)
 \end{aligned}$$

Substituting the appropriate quantities in Equation (A3), the error function

$$\begin{aligned}
 e(t) = & \frac{H_0}{4} (\psi_0^2 + \phi_0^2) \sin \omega_H t - \frac{g T_P^2}{640} \psi_0^2 \cos 2\omega_P t \\
 & + \left(\frac{R}{8} - \frac{g T_P^2}{640} \right) \phi_0^2 \cos 2\omega_R t \\
 & - \frac{\psi_0^2 H_0}{8} \left[\left(\frac{T_P}{T_P + 2T_H} \right)^2 \sin (\omega_H + 2\omega_P)t + \left(\frac{T_P}{T_P - 2T_H} \right)^2 \sin (\omega_H - 2\omega_P)t \right] \\
 & - \frac{\phi_0^2 H_0}{8} \left[\left(\frac{T_R}{T_R + 2T_H} \right)^2 \sin (\omega_H + 2\omega_R)t + \left(\frac{T_R}{T_R - 2T_H} \right)^2 \sin (\omega_H - 2\omega_R)t \right]
 \end{aligned}$$

CONCLUSIONS

As might be inferred intuitively, the error function increases with increased values of roll, pitch, and heave amplitude. The interplay of roll, pitch, and heave periods is not directly obvious. In general, for heave periods equal to or greater than pitch period, greater heave periods result in less error. Presently available data indicate that roll period will be 1/3 to 2/3 longer than pitch period and in general, terms containing the argument $(\omega_H - 2\omega_R)$ will be transmitted through the double-integrator. From inspection of the error function, increased roll and pitch periods result in larger error. However, heave period does depend on roll and pitch period, which will tend to moderate this effect.

It will be instructive at this point to substitute reasonable values of the parameters making up the error function. These values will be representative of a heavy sea, and the quantities β_0 , P, and R are chosen to represent the positioning of an accelerometer in the Capstan Machinery Room of the USS TARAWA, CVB-41. Figure 13, illustrating roll, pitch, and displacement data for the USS TARAWA shows the parameters chosen indeed correspond to extreme conditions. Considerations of the relationships of heave period to the period of wave encounter seem to justify a choice of heave period in the neighborhood of 15 seconds. Also from conclusions above, the resulting error will be larger.

The following is a list of parameters chosen:

$\psi_0 = 5^\circ$	$\beta_0 = 2^\circ$
$\phi_0 = 10^\circ$	P = 500 ft
$T_R = 20$ sec	R = 20 ft
$T_H = 15$ sec	$H_0 = 10$ ft
$T_P = 10$ sec	

This choice of parameters results in a maximum instantaneous error of 0.90 ft. This represents the summing of the absolute values of all error coefficients. The root-mean-square error calculates to 0.63 ft. The measured resolution of the deck displacement indicator was found to be approximately 6 inches or better, which is not far different from the calculated error.

On the basis of these calculations, and the resolution of the measuring device, it is felt that the provision of a fixed vertical reference is not necessary, and that data taken to date is representative of actual ship motion.

* * *

As might be inferred intuitively, the error function increases with increased values of roll, pitch, and heave amplitude. The intensity of roll, pitch, and heave periods is not directly involved. In general, for heave periods equal to or greater than pitch period, greater heave periods result in less error. Presently available data indicate that roll period will be 1/3 longer than pitch period and in general, terms containing the argument $(\omega - \omega_0)$ will be transmitted through the double-integrator. From inspection of the error function, increased roll and pitch periods result in larger error. However, heave period does depend on roll and pitch period, which will tend to moderate this effect.

It will be instructive at this point to substitute reasonable values of the parameters making up the error function. These values will be representative of a heavy sea, and the quantities A , R , and H are chosen to represent the positioning of an accelerometer in the Captain Machinery Room of the USS YARAWA, CVE-41. Figure 13, illustrating labeled correspond to extreme conditions. Consideration of the relationships of heave period to the period of wave encounter seem to justify a choice of heave period in the neighborhood of 15 seconds. Also from conclusion above, the resulting error will be larger.

The following is a list of parameters chosen:

$V_0 = 7$	$V_0 = 7$
$\phi = 10^\circ$	$\phi = 10^\circ$
$T_H = 20 \text{ sec}$	$T_H = 20 \text{ sec}$
$T_R = 15 \text{ sec}$	$T_R = 15 \text{ sec}$
$T_P = 10 \text{ sec}$	$T_P = 10 \text{ sec}$
$A = 2$	
$R = 500 \text{ ft}$	
$H = 20 \text{ ft}$	
$H_0 = 10 \text{ ft}$	

This choice of parameters results in a maximum instantaneous error of 0.85 ft. This represents the summing of the absolute values of all error coefficients. The root-mean-square error calculated to 0.53 ft. The measured resolution of the deck displacement indicator was found to be approximately 5 inches or better, which is not far different from the calculated error.

On the basis of these calculations, and the resolution of the measuring device, it is felt that the provision of a fixed vertical reference is not necessary, and that data taken to date is representative of actual ship motion.