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A PROPOSED METHOD FOR THE PRODUCTION OF SHORT R-F PULSES

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ABSTRACT

This report describes and analyzes a proposed method for the production of short, high-power r-f pulses.

Using a relatively low-power c-w or comparatively long-pulse r-f generator, r-f energy is built up and stored in a resonant cavity in the form of a standing wave. After the stored energy has reached a high level it is discharged into the load in the form of a short, high-power r-f pulse. If the c-w charging source is used, it should be possible to obtain phase-coherent pulses.

Analysis and tests show that a significant increase in peak pulse power over that of the charging source can be expected with the proposed storage system. The principal problem to be solved is the construction of a low-loss, high-speed switch which will confine the r-f energy to the cavity during the charging period and yet permit the rapid and complete discharge of this energy to the load at the desired time.

PROBLEM STATUS

This is an initial report; work is continuing.

AUTHORIZATION

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A PROPOSED METHOD FOR THE PRODUCTION OF SHORT R-F PULSES

INTRODUCTION

Background

The production of very short, high-power r-f pulses by the envelope modulation of an r-f generator becomes increasingly difficult as the length of the pulse is made shorter and shorter. The problem is twofold; a short pulse modulator has to be devised, and then an r-f generator which has inherently the wide bandwidth required to permit modulation by a very short pulse has to be constructed. At the present stage of the art the production of a 0.01-microsecond pulse at X-band by pulse modulation of certain types of magnetrons represents about the limit to this sort of device.

A new approach to the problem of short r-f pulse production has been suggested,* which, if successful, would permit the production of high-power r-f pulses of an order of magnitude shorter than those now obtainable. Simply stated, this scheme consists of the following. R-f energy from a c-w or a comparatively long-pulse signal generator of low or medium power is coupled into a short length of resonant waveguide. Through resonance there will be a build-up and storage of r-f energy in the guide of considerable magnitude providing losses in the guide are held to a small value. When this energy level reaches a near maximum value a switching phenomenon takes place, probably in the form of a spark-gap discharge, which simultaneously changes the length of the guide to one of anti-resonance and couples a load to the guide. The guide discharges into the load putting, ideally, an r-f pulse of relatively high power and approximately twice as long as the transit time of r-f energy through the resonant guide section. If a c-w r-f generator is used as the charging source, it should be possible to obtain phase-coherent output pulses.

An exploratory investigation has been made of the many aspects and characteristics of some possible forms of a device to produce very short high-power r-f pulses through r-f storage. The object of this investigation has been to identify, as far as possible, the problems involved, the limitations and limiting factors present, and to obtain an estimate of the over-all feasibility of the scheme. This investigation has involved a number of theoretical calculations which are supported by some experimental data where such data were obtainable.

Basic Schematic

The idea, as originally set forth was given in terms of a transmission line analogy which is illustrated in Figure 1.

* The basic ideas behind the proposed method for the production of short r-f pulses were suggested by Dr. R. M. Page, Superintendent, Radio Division III.

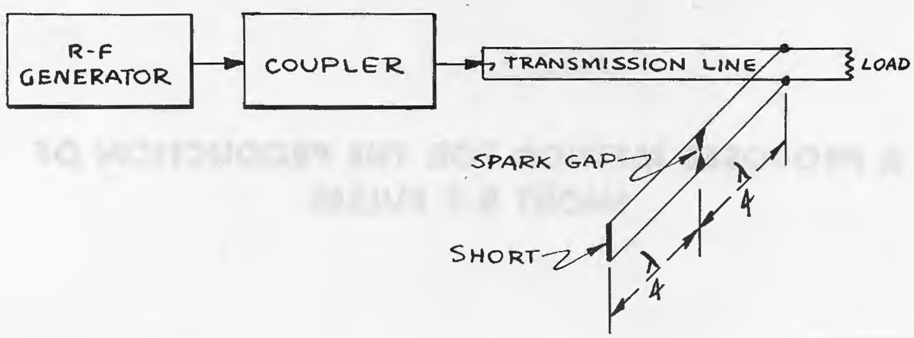

Figure 1 - Basic schematic

Operation is as follows: A transmission line is terminated in a matched load. To prevent r-f energy from reaching the load a half-wave shorted stub is placed across the line near the load end which places a virtual short circuit across the line in front of the load. The stubbing point is so chosen that the effective length of line between the stub and the r-f source is an integral number of half wavelengths long and is, therefore, resonant. The function of the coupler is to match the r-f source to the losses in the line (this is equivalent to making the line of resonant length). A standing wave will be set up along the line and its amplitude will grow until a point is reached where the power supplied is equal to the power dissipated along the line. If the line loss factor is held to a very small value, the amplitude of the standing wave will be very great and a large amount of stored energy will be present on the line.

A spark gap or similar device is placed in the shorting stub at a distance of a quarter wavelength from the line. When the standing wave voltage builds up during the charging period to a sufficiently high value, the gap will arc across, placing a short at a quarter wavelength from the line and cause a virtual open circuit to appear at the stubbing point. The stored energy is now free to proceed into the load. Ideally the time required for a complete discharge will be equal to twice the length of the resonant section divided by the velocity of r-f propagation.

After the line is discharged the arcing at the gap stops and the whole process begins over again.

If a c-w generator is used as the charging source and the arc discharge can be made to take place on the same part of the charging cycle, there will be phase-coherence from pulse to pulse.

Variations of this basic scheme are possible. From the standpoint of loss, waveguide would probably be superior to transmission line. The resonant line or cavity might well be used to control the frequency or be made a part of the r-f generator.

Assumptions

Because of great number of possible forms that such a device might take, complete analysis would be difficult, if not impossible. Consequently, the scope of the subsequent analysis is limited by making certain restrictive assumptions which appear reasonable. These are:

1. Energy storage will take place in waveguide. Waveguide is superior to transmission line or coaxial cable, having significantly lower loss in the band of frequencies where such a device is likely to operate. Open-wire transmission line has the disadvantage of not confining the stored energy to a definite space, and may even radiate appreciably.

2. A dielectric medium other than air can not be used to advantage in the storage cavity. Although a dielectric might reduce cavity dimensions and possibly wall losses, it would be unsatisfactory since the loss factor of any known dielectric is so high that it would more than cancel any gain made in this respect.

3. The storage cavity must be a relatively simple structure physically. It can not have any abrupt discontinuities, or tuning and matching devices. Such structures set up reflections and standing-waves and may give rise to unwanted modes of energy storage. Reflections, by their very nature, would prevent quick and complete discharge of the stored energy and give rise to a prolonged transient following the r-f pulse.

4. Energy is stored principally in one mode. Since ultimately the stored energy must be delivered to the load in one mode, it appears unlikely that such a transfer could be quick and complete if the storage were in several modes. It is probable also that the stored energy should be in a "lowest order" mode corresponding to the TE_{10} mode in a rectangular guide or TE_{11} or TM_{01} in circular guide. These are the most easily controlled ones.

Problems to be Solved

In order to make this device operate successfully a number of problems must be solved. The major problems appear to be:

1. The construction of a high Q cavity to permit a large resonant build-up of stored energy and power multiplication.

2. The prevention of leakage of energy to the load during the build-up period, i.e., load isolation.

3. The rapid and complete discharge of the stored energy into the load without undue transients following and without significant energy loss.

These problems are, of course, interrelated but it is convenient to trust them separately.

POWER MULTIPLICATION

It is a well-known fact that energy may be stored in a resonant cavity. For a given amount of power supplied to such a cavity, the level to which the amount of stored energy rises is dependent on the Q of the cavity. As a general proposition, other things being equal, the greater the physical size of the cavity, the greater the Q.

However, the total amount of the stored energy is not the important factor for the device under consideration. The important factor is the ratio of the amount of stored energy to the time required for a complete discharge of this energy into a load. This ratio represents power. The ratio of power, thus defined, to the power supplied to the cavity during the charging period may be taken as the <u>Power Multiplication Factor</u> of the cavity. The power multiplication factor is, therefore, the ratio of the peak pulse power to the constant power supplied to the cavity during the charging period.

Although a high Q is desirable for high-power multiplication it does not follow that increasing the Q of the cavity will result in a higher power multiplication factor. Consider, for example, a cavity consisting of a section of waveguide. The cavity Q and the total energy storage can be increased by increasing the length of the guide. Now if one end of the cavity is opened and the stored energy is discharged into a load, the time required for complete discharge of the cavity will be approximately proportional to the length of the cavity. Therefore, unless the amount of stored energy increases faster with cavity length than the length does itself, the power into the load will decrease with increase in length.

If one considers the energy to be put into the cavity by a generator having limited power output capability, the amount of stored energy will increase until the power losses in the walls of the cavity, and elsewhere, just equal the power supplied by the generator. For any given mode of energy storage, increasing cavity length must entail a lowering of the stored energy density although the total amount of stored energy may increase. Cavity dimensions and density of energy storage are the principal factors which determine power loss. Thus, so far as power multiplication is concerned, for a given cross-sectional geometry, greater power multiplication will be obtained for short cavities than for long ones.

An expression giving the power multiplication factor for a cavity of uniform cross section and having a length, L, may be derived. Consider Figure 2.

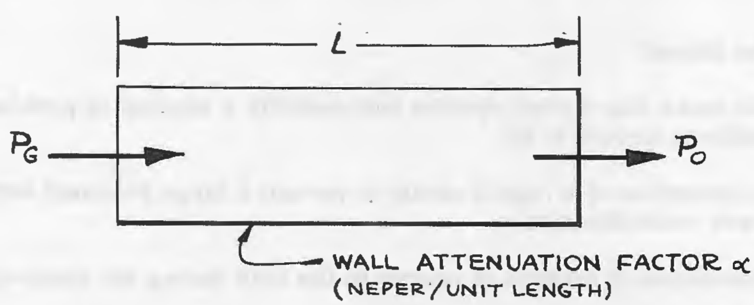

Figure 2 - Cavity power relations

P_G is the power supplied to the cavity by a generator during charging and P_o is the power delivered by the cavity on discharge into a matched load. As was stated, the density of stored energy in the cavity will increase during charging until cavity losses just equal the power supplied, i.e., P_G.

Electrically this energy will be in the form of a standing wave or standing waves. When the output of the cavity is opened the cavity discharges into the load. This discharge will take a certain length of time. The minimum possible time for complete discharge will be the time it takes a wave to travel from one end of the cavity and back.

Now if the cavity were used for straight through transmission purposes without internal reflections and standing waves, a wave would suffer some attenuation due to wall losses in traversing the length of the cavity.

In fact the attenuation constant can be defined by the expression,

$$\alpha \text{ (nepers per unit length)} = \frac{\text{Ave. power lost per unit length}}{2 \text{ x power transmitted}} . \quad (1)$$

[redacted]

Since there is no net transmission during the cavity charging period but a very high ratio standing wave is established, the average power lost will be approximately twice that which would be lost if there were transmission. Moreover, if α is small, which is tacitly assumed, the amplitude of the standing wave will be substantially constant over the entire length of the cavity and the average power lost for unit length will be very closely constant over the cavity length. The average power lost per unit length will, therefore, be equal approximately to the total energy lost divided by the cavity length L. Designating the total power lost by P_G, which is the power supplied by the generator, calling the power into the load P_O, and making the correction for the increase in loss due to standing waves, one can write Equation (1) as

$$\frac{P_O}{P_G} = \frac{1}{4\,\alpha\,L}. \qquad (2)$$

Equation (2) represents the maximum possible power multiplication. Losses due to the cavity ends as well as certain other losses yet to be discussed are not included. However, symbolically these too may be included if the quantity αL is replaced by a new total attenuation constant, α_T, i.e.,

$$\frac{P_O}{P_G} = \frac{1}{4\,\alpha_T} \qquad (3)$$

or

$$\frac{P_O}{P_G} = \frac{8.686}{4\,\alpha'_T} \qquad (4)$$

where α'_T is the total attenuation constant expressed in db.

As a theoretical check of the validity of Equation (2) an alternate derivation based upon the fields existing in the cavity was made for a resonant section of rectangular guide in the TE_{10} mode. This derivation, which is set down in the Appendix, gives the same result.

A number of experimental checks on the magnitude of the resonant build-up in cavities of various dimensions and configurations were made. The experimental setup for these measurements is shown in Figure 3.

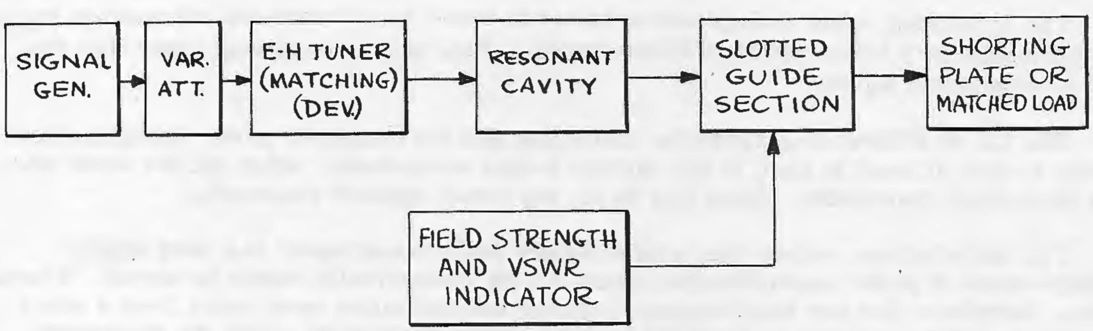

Figure 3 - Test setup for measuring potential power multiplication factor

Measurements were made as follows. With a matched load terminating the slotted section, the E-H tuner was adjusted for maximum power transmitted to the load, as indicated by the vswr indicator. The matched load was then replaced by a shorting plate and the E-H tuner was readjusted for maximum standing-wave-ratio. With the probe of the vswr indicator set at a voltage maximum, the amount of attenuation which had to be inserted between the signal generator and tuner to bring the voltage in the slotted section to the same value as that for the matched condition was noted. This attenuation, minus 6 db due to voltage doubling in the standing wave pattern, was taken as the potential power multiplication factor of the cavity. It was recognized, however, that the figure so obtained would be probably lower than the theoretical one, since no correction is made for the power extracted by the probe in the slotted section or for that lost in the E-H tuner due to very unfavorable matching requirements.

The data for a typical measurement made in the manner described are the following:

TABLE 1
Data of a Typical Power Multiplication Factor Measurement

L (Including slotted guide section)	2.025 meters
Cross Section	0.400″ x 0.900″ (Std. X-Band Guide)
Material	Copper
Signal wavelength	3.2 cm
Power Multiplication Factor	
Measured	9.0 db
Calculated	10.3 db

The calculated power multiplication factor is based on a theoretical attenuation Figure of approximately 3.5 db per 100 ft of X-band guide. This will be somewhat lower than the actual attenuation figure.

The 1.3 db difference between the calculated and the measured power multiplication factor is due, at least in part, to the various losses enumerated, which did not enter into the theoretical calculation. Since this is so, the result appears reasonable.

The use of silver, rather than copper cavity walls would result in a very slight improvement in power multiplication (about 0.2 db theoretically, which is trivial). It would seem, therefore, that any improvement in power multiplication must come from a more favorable cavity geometry or from the use of a signal wavelength where the dissipative attenuation is less. Practically, this means that the cavity must be short in length and large in cross section, and that the frequency be not too high.

In order to obtain a 100 to 1 multiplication in power (20 db) the factor α_T in Equation (3) must be 0.0025 or $\alpha'_T = 0.0217$ db. To permit an easier conception of what power multiplications are possible for cavities of various dimensions, Table 2 has been computed.

[redacted]

In Table 2, rectangular, silver waveguides of more or less standard dimensions are considered at various wavelengths. Propagation and storage are assumed to be in the TE_{10} mode. Losses in the device which matches the generator to the cavity are not considered.

TABLE 2
Calculated Power Multiplication Factor
for Silver-Walled Rectangular Cavities

Cavity Length	λ (cm)	Cavity Cross Section a x b	Power Multi. (db)
1.5 Meters (0.01 μ sec pulse approx.)	1	0.280″ x 0.140″	4.0
		0.900″ x 0.400″	11.3
	3	0.900″ x 0.400″	11.4
		2.840″ x 1.340″	19.0
	10	2.840″ x 1.340″	18.9
		6.500″ x 3.250″	25.3
	30	9″ x 4″	26.4
0.15 Meters (0.001 μ sec pulse approx.)	1	0.280″ x 0.140″	14.0
		0.900″ x 0.400″	21.3
	3	0.900″ x 0.400″	21.4
		2.840″ x 1.340″	29.0
	10	2.840″ x 1.340″	28.9
		6.500″ x 3.750″	35.3

Table 2 represents only a few of the possible cavity configurations. Moreover, for some of the cavity configurations, other than the TE_{10}, mode of propagation is possible. However, the power multiplication figure listed is representative. It will be shown later that the ultimate power multiplication possible in a practical case will depend, very probably, on factors other than the inherent cavity loss so that the power multiplication figure as listed is undoubtedly very optimistic.

Certain probable limitations to the system may be deduced from Table 2. It seems likely that wavelengths shorter than 1 cm can not be used successfully due to rather high losses and consequently low power multiplication.

For wavelengths longer than 30 cm, the cavity dimensions become rather large and are a practical disadvantage.

At the shorter wavelengths cavities having a relatively large cross section are not considered feasible and are not included in the table. So many different modes of energy are possible that it would be difficult to control the nature of the storage and, in particular, the nature of the discharge.

LOAD ISOLATION

During the cavity charging period, the load should be isolated from the storage cavity as well as possible. Power escaping into the load during this period reduces the maximum possible power multiplication of the cavity since this leakage power must be subtracted from that supplied by the generator during the energy storage interval. Even if the power multiplication factor is not reduced below a satisfactory level, it seems very likely that an uncontrolled leakage of power to the load would be undesirable, and possibly not permissible, in a practical application.

A half-wave stub has been suggested as a possible device for load isolation. It has the obvious advantage of mechanical and electrical simplicity. For these reasons, an analysis will be made of a waveguide form of the half-way stub used as a load isolation device. From this analysis certain deductions can be made which have general application.

Schematically the isolation stub may be represented as in Figure 4.

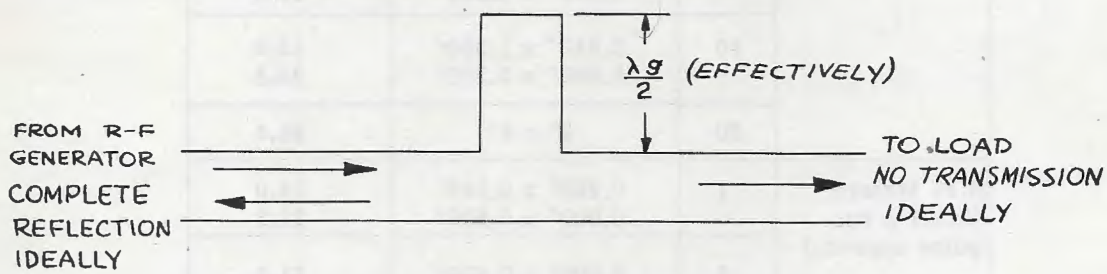

Figure 4 - Stub schematic

The electrical equivalent of the junction is a four terminal network which is given approximately in Figure 5.

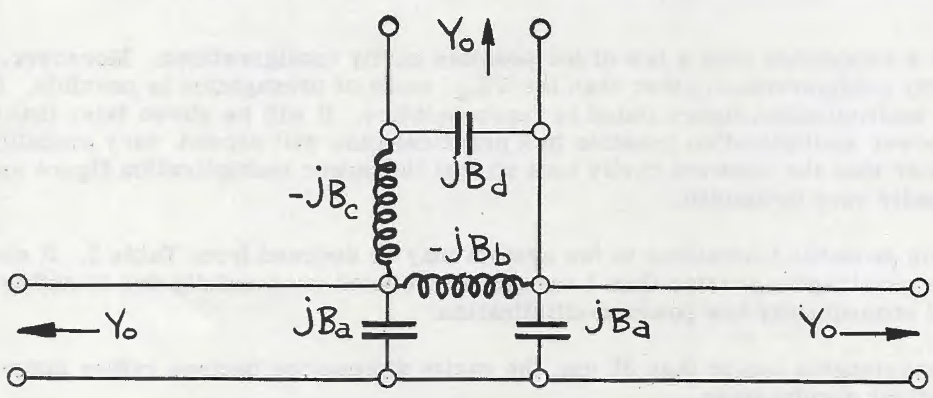

Figure 5 - Equivalent circuit of a tee junction

It is assumed here that each arm of the tee has the same dimensions and consequently the same characteristic admittance, Y_o. In a practical case this may not be so but the validity of the results is not greatly affected. The susceptances, B_a, B_b, B_c, and B_d depend upon the guide dimensions.

[redacted]

Through transmission can be prevented by putting a susceptance across the terminals of the series arm which when combined with $-jB_c$ and jB_d will result in jB_b. This is shown in Figure 6.

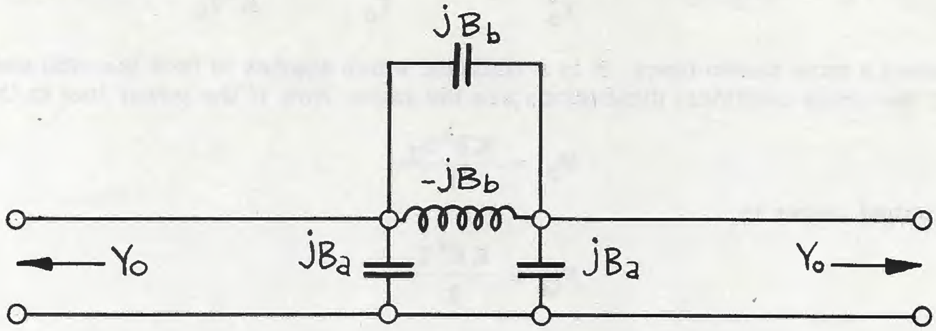

Figure 6 - Equivalent circuit of a shorted half-wave stub

The series arm is now parallel resonant and will prevent through transmission. The susceptance $+jB_b$ is obtained, of course, by making the series arm a short-circuit section of waveguide approximately a half wavelength long. On the basis of Figure 6, it would appear that it is theoretically possible to obtain zero transmission at some one frequency. This is not quite the case. The equivalent circuit does not take into account the stub losses. A more realistic equivalent circuit would be in Figure 7.

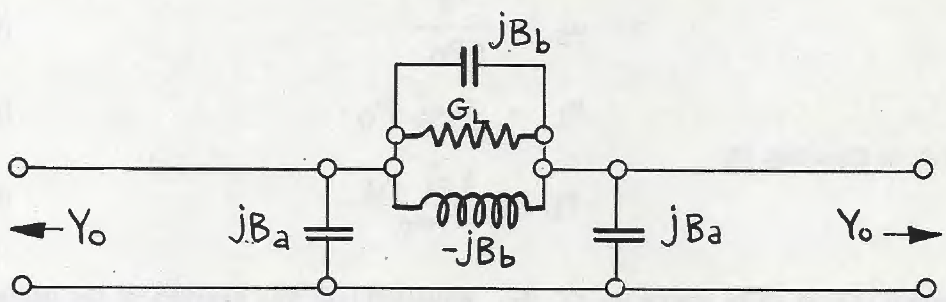

Figure 7 - Equivalent circuit of a shorted half-wave stub including stub losses

Here, G_L represents the equivalent loss conductance. As a consequence of this loss there will be some leakage of power at all frequencies.

The two shunt susceptances, $+jB_a$, represent discontinuity capacitances. These should, ideally, be held to a small value in order to prevent large reflections during transmission.

If one assumes that the susceptance, jB_a, is small compared to the admittance Y_o and that the loss conductance G_L is also comparatively small (as it must be in a practical system), the field intensity transmitted compared to the incident field intensity will be

$$\frac{E_L}{E} = \frac{G_L}{Y_o + G_L} = \frac{G_L}{Y_o} \text{ (approx.)}. \tag{5}$$

The power lost P_L will be

$$P_L = KE_L^2 Y_o = \frac{KE^2 Y_o G_L^2}{Y_o^2} = \frac{KE^2 G_L^2}{Y_o} = \frac{KE^4 G_L^2}{E^2 Y_o}. \qquad (6)$$

Y_o assumed a pure conductance. K is a constant which applies to both the stub and the cavity if the cross-sectional dimensions are the same. Now if the power lost in the stub is

$$P_S = \frac{KE^2 G_L}{2} \qquad (7)$$

and the output power is

$$P_O = \frac{KE^2 Y_o}{2} \qquad (8)$$

then

$$P_L = \frac{2 P_S^2}{P_O}. \qquad (9)$$

Equation (9) shows that the leakage power goes up as the square of the stub losses for a given constant output power.

If the attenuation constant of the stub is defined as

$$\alpha_S = \frac{P_S}{4 P_O} \qquad (10)$$

then

$$P_L = 32 \alpha_S^2 P_O. \qquad (11)$$

Going back to Equation (3)

$$P_L = \frac{8 \alpha_S^2 P_G}{\alpha_T}. \qquad (12)$$

Equation (12) is valid where $\alpha_S^2 \ll \alpha_T$. Equation (12) was derived on the basis of a half-wave stub. It is apparent from its form and manner of derivation that similar results would be obtained for other load-isolating devices. The important fact to be gained is that the isolating device losses must be kept at a very low value not only for the consideration of power multiplication but also in order to keep the leakage to the load small during the charging period.

Some tests were made of the effectiveness of a half-wave stub as a load isolating device. For these tests the setup of Figure 3 was used with the modification that the shorting plate was replaced by a half-wave stub followedby a crystal detector. This is shown in Figure 8.

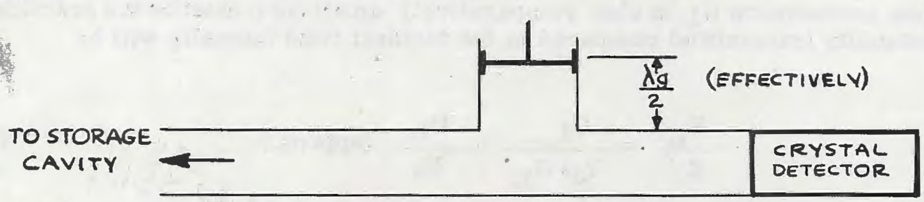

Figure 8 - Test setup for measuring leakage of power past a half-wave stub

The stub was adjusted for minimum crystal current while at the same time the storage cavity was adjusted to resonance. Under these conditions it was found that the power multiplication was reduced approximately 1 db. This reduction indicates an effective loss of 10 percent of the generator power, part of which escaped to the crystal and part of which was consumed as stub losses.

If one assumes a power leakage of 10 percent and a power multiplication of 20 db, one obtains a theoretical ratio of 30 db between power to the load during the charging period and the power during discharge. For the simple stub arrangement used, and with more careful adjustment, it seems probable that an even greater ratio might be obtained. However, it must be emphasized that this ratio depends on low stub losses. Some of the practical difficulties arising in this respect when a cavity discharge device is added to stub will be discussed in the next section.

CAVITY DISCHARGE SWITCH

After the energy has been built up to a high level in the storage cavity, it must be rapidly and completely discharged into the load. A practical mechanism for accomplishing this result may not be easily obtained in view of the extremely short time involved. To be effective, the switching time must be a small fraction of the output pulse duration, which itself may be extremely small. Not only must be switching time be small, but also the character of the switching must be such as to yield the desired result of complete cavity discharge into the load. The switching problem presents many great practical difficulties.

Before taking up the question of practical switching devices, an ideal switch will be considered. An ideal switch will be defined as one which has zero loss and zero switching time. This does not mean, however, that the ideal switch will necessarily produce the desired switching action of quick and complete discharge of the stored energy in the cavity into the load. On the contrary, as the subsequent analysis will show, a transient is inevitably tied in with the switching process.

It has been suggested that a possible switching mechanism might consist of a spark gap placed at the midpoint of the half-wavelength stub. By adjusting the spark gap to fire at the end of the cavity charging cycle, a short will appear at the spark gap and cause the effective length of the stub to be a quarter wavelength. A short-circuited quarter-wave stub has an infinite input impedance and would presumably switch the stored energy into the load. An analysis will be made of such a switching mechanism on a transmission line basis. Consider Figure 9.

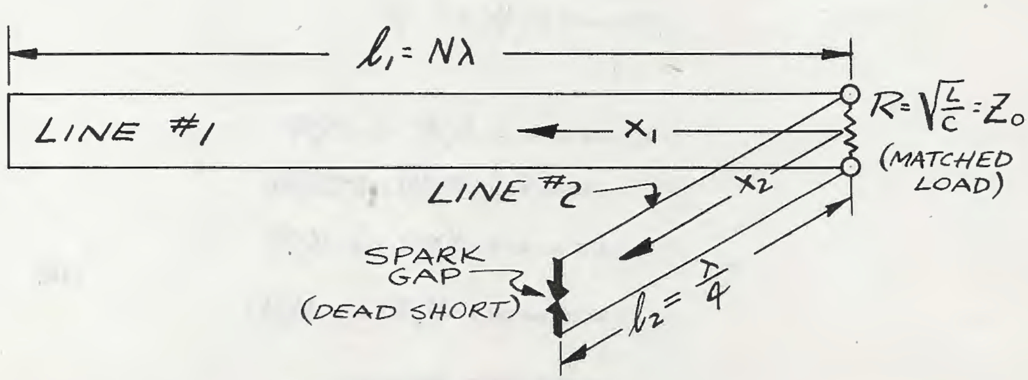

Figure 9 - Switching analysis based on transmission line analogy

The stub was adjusted for minimum crystal current while at the same time the storage cavity was adjusted to resonance. Under these conditions it was found that the power multiplication was reduced approximately 1 db. This reduction indicates an effective loss of 10 percent of the generator power, part of which escaped to the crystal and part of which was consumed as stub losses.

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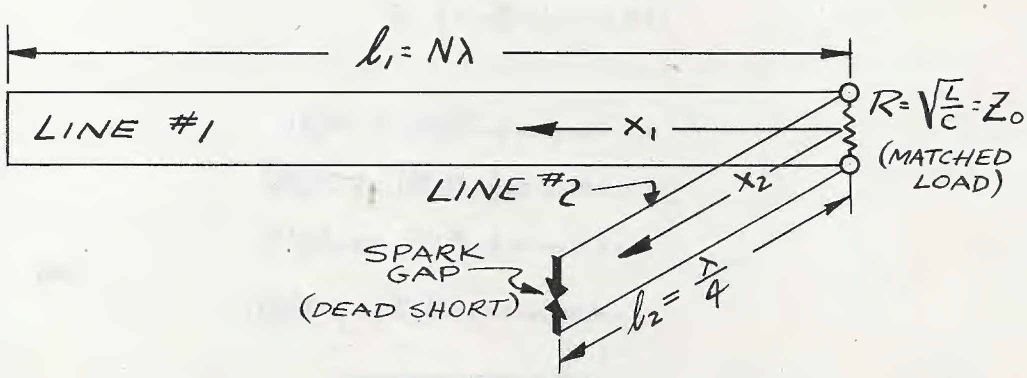

Figure 9 - Switching analysis based on transmission line analogy

The resulting output voltage is a rather complicated series of terms. A reflection on the physical nature of the circuit will explain why this should be so. First of all, it must be realized that the infinite input impedance of a short-circuited quarter-wave stub is a steady-state condition only. It represents a resonance effect. Certainly a wave incident on the input terminals of the stub has no way of knowing whether the output is shorted or not. One may, therefore, expect transient phenomena as a result of the switching action.

Figures 10 and 11 are partial plots of the load voltages. Figure 10 is the plot of the beginning of the r-f pulse and Figure 11 is a plot of the end of the pulse.

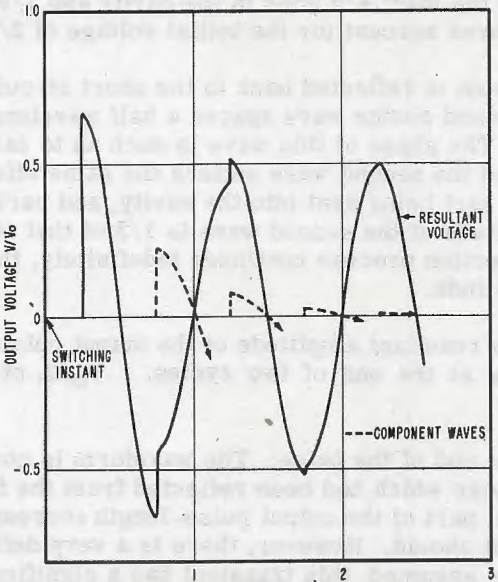

Figure 10 - Load voltage after switching (pulse beginning)

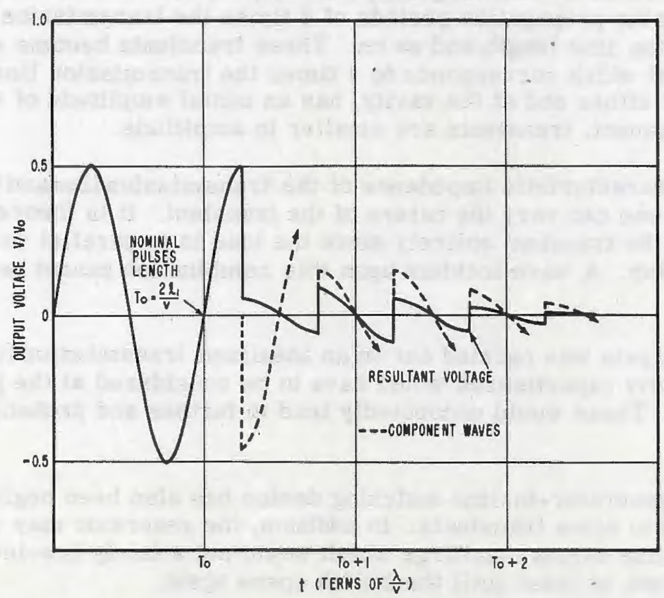

Figure 11 - Load voltage after switching (pulse end)

A qualitative explanation of Figure 10 is fairly simple. First of all it should be noted that the effect of the gap arcing over is not felt at the load until a time later, corresponding to a quarter wave of propagation on the line. This, obviously, is due to the fact that the short is physically a quarter wave from the load. It should also be noted that the output starts as a cosine wave because the short occurs at a voltage maximum point and thus creates a cosine wave which is reflected from the short. The amplitude of the cosine wave is V_0. However, when the wave reaches the point at which the load is located it sees not only the load but also the main cavity in parallel with the load, which is not a matched condition. Under the conditions assumed the voltage reflection coefficient is (1-2)/1+2) or -1/3, which means that 4/9 of the power is dissipated in the load; 4/9 goes in the cavity and 1/9 is reflected back toward the short circuit. These figures account for the initial voltage of 2/3 V_0 at the load.

The 1/9 of the power which is reflected back to the short circuit is in turn reflected back toward the load, giving a second cosine wave spaced a half wavelength (two quarter waves) from the first cosine wave. The phase of this wave is such as to cancel out part of the first wave (Figure 10). At the load the second wave suffers the same effect as the first, part of it being dissipated in the load, part being sent into the cavity, and part being reflected back to the short circuit. The amplitude of the second wave is 1/3 of that of the first; i.e., the voltage is 2/9 V_0. This reflection process continues indefinitely, the waves becoming smaller and smaller in amplitude.

Figure 10 shows that the resultant amplitude of the output pulse approaches $V_0/2$ rapidly, differing very little from this at the end of two cycles. $V_0/2$, of course, represents the steady-state condition.

Figure 11 represents the end of the pulse. The waveform is composed of waves reflected from the short circuit and those which had been reflected from the far end of the cavity. It can be seen that the principal part of the output pulse-length corresponds to twice the length of the transmission line, as it should. However, there is a very definite transient following this pulse. For the constants assumed, this transient has a significant amplitude for a time corresponding to several cycles.

In addition to the transient following the pulse there will be a succession of transients at times corresponding the propagation periods of 4 times the transmission line length, 6 times the transmission line length, and so on. These transients become smaller and smaller. The transient which corresponds to 4 times the transmission line length is due to two reflections, one at either end of the cavity, has an initial amplitude of 4/27 V_0. The higher order, or subsequent, transients are smaller in amplitude.

By changing the characteristic impedance of the transmission line and the stub relative to the load impedance one can vary the nature of the transient. It is theoretically impossible, however, to eliminate the transient entirely since the load is in parallel with both the transmission line and the stub. A wave incident upon this combination cannot be completely absorbed by the load.

The foregoing analysis was carried out on an idealized transmission line analogy. In an actual case, discontinuity capacitances would have to be considered at the junction of the cavity, stub, and load. These would undoubtedly lead to further and probably higher-amplitude transients.

The effect of the generator-to-line matching device has also been neglected. This device would also lead to some transients. In addition, the generator may continue to attempt to charge the line during discharge which would put a fairly low-level c-w or long pulse signal into the load, at least until the switch opens again.

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Switching has been assumed to take place at a voltage maximum point and at the time of the absolute voltage maximum. Analysis shows that switching at other points and/or at other times, at least under most circumstances, leads to longer transients, and in some cases to higher-amplitude transients. The nature of the switching assumed appears to be the most favorable.

Because the transient, is at least several cycles in length, it would appear impractical to attempt to produce a pulse which is shorter than several cycles even though rapid enough switching could be obtained.

In the preceding analysis an ideal switch was assumed. It is, of course, unlikely that any device approaching an ideal switch in characteristics can be found in practice. Undoubtedly some delay in complete switching will be encountered, and undoubtedly some loss will be present. These factors can be counted upon to eliminate some of the sharp corners in the output-pulse waveform and reduce the amplitude and length of the transients. However, an accurate estimate of the output waveform depends on a good knowledge of the exact switching characteristics.

The search for a practical switch has been unsuccessful. Measurements have been made on two possible devices, and both have been found unsatisfactory. These were the gas tube and the spark gap.

Gas tubes were considered first. These are used successfully in radar antenna-duplexing systems and some exhibit rather short ionizing times. One serious limitation to the use of such a device in the application discussed here is the relatively high dissipative loss which it adds to the half-wave stub. The through transmission on these devices exhibit an attenuation on the order of 1.0 db. Some of the better ones may have an attenuation perhaps as low as 0.5 db, but a search revealed none appreciably better. A loss of 0.5 db in the stub is prohibitively high both with respect to power multiplication and with respect to load isolation.

A type 1b60 TR, chosen as a typical TR, was placed in the stub and tuned to the frequency of operation. The load isolation became approximately 15 db poorer and the power multiplication figure dropped almost to unity. Quite obviously these figures represent a thoroughly unacceptable situation.

Even if the loss of the TR had been very low it is not certain that it would have been acceptable. The Q of the TR would have to be decidedly low in order to permit a rapid firing time. This is equivalent to an extremely wide bandwidth. It is doubtful that a TR of conventional design would be satisfactory even in this respect since most TR's consist of a tuned iris or several tuned irises. As such they would present a significant discontinuity to an incident wave front.

To estimate the efficiency of a spark gap as a switching device a rod approximately 3/32-inch in diameter was used as a short across a section X-band of waveguide. It did not prove to be, however, a very effective short. Measurements indicated that approximately one sixth of the power escaped around this short and proceeded down the guide. Theoretical calculations give a result in substantial agreement with this figure. A short to be effect has to be area extensive, i.e., across the entire guide.

A small gap could probably not be used very successfully. A gap of 1/64-inch using half-round points of 3/64-inch radius was measured and found to cause a 2 db loss at X-band. With a 1/32-inch gap, the loss was 1 db. Sharp points would undoubtedly yield even higher losses.

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The loss using a closely spaced gap is due to the high concentration of field in the neighborhood of the points. Such a high concentration of field is necessary for rapid arc-over so it would appear that a dilemma exists. The loss in the TR tube can also be attributed to the narrow gap present.

CONCLUSIONS

The exploratory study made on the feasibility of producing very short pulses through r-f storage cannot be regarded as giving a definite answer to practicability of the method. Admittedly the study has been restricted to fairly narrow limits, and in some cases the analysis has been rather superficial. However, on the basis of the study made, certain things appear to be true.

1. It appears possible to obtain a fairly high-power multiplication through r-f storage in the cavity, at X-band and below.

2. Fairly complete load isolation seems possible providing losses in the switching mechanism can be kept low.

3. The major problem to be solved is the switching problem. All devices considered have much too high losses. It seems probable also that an attainment of a sufficiently high switching speed would be difficult. No adequate test of switching speed can be made until a low-loss switch is found.

4. Switching devices, at least of the form considered, give rise to switching transients several cycles in length. Consequently, the minimum attainable pulse length must consist of at least several cycles of energy.

It is, of course, possible that a device for producing very short coherent r-f pulses operating on the stated principle but having some quite different physical form than now envisioned may exist which would reduce, if not entirely eliminate, the switching problem and the associated switching transients. The direct generation of phase-coherent, high-power r-f pulses from a relatively much lower power c-w oscillator is of sufficient value to justify and stimulate thinking in this direction for long as well as for short pulses.

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APPENDIX A
Alternate Derivation of the Power Multiplication Factor

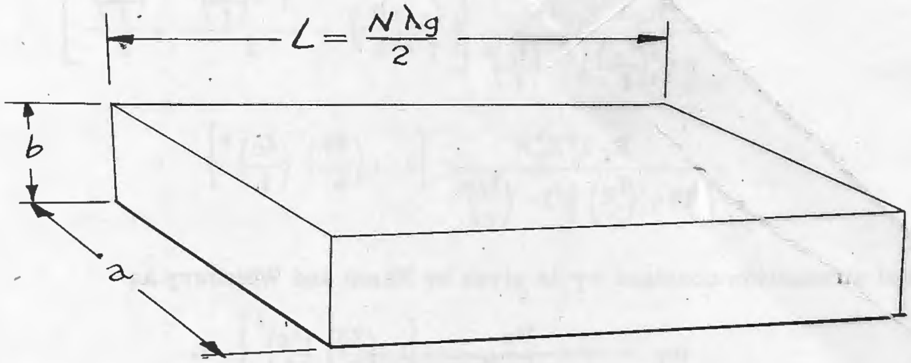

Figure 12

Let the cavity of Figure 12 be resonant in the TE_{10N} mode, i.e.,

$$L = \frac{N\lambda_g}{2}.$$

The power loss, P_G, neglecting the ends is given by Ramo and Whinnery as

$$P_G = \frac{R_S \lambda_1^2 E_0^2}{8\,\eta_1^2}\left[\frac{b\lambda_g}{2a^2} + \frac{a}{\lambda_g} + \frac{\lambda_g}{4a}\right] \qquad (17)$$

where R_S is the skin resistance

$\eta_1 = \sqrt{\dfrac{\mu_1}{\epsilon_1}}$ is the wave impedance

λ_1 is the free-space wavelength

λ_g is the guide wavelength

E_0 is the electric field maximum.

Now

$$\lambda_g = \frac{\lambda_1}{\sqrt{1-\left(\frac{\lambda_1}{2a}\right)^2}} = \frac{\lambda_1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{2a\left(\frac{f_c}{f}\right)}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad (18)$$

where f is the frequency
f_c is the cutoff frequency of the cavity as a waveguide.

$$P_G = \frac{R_S \lambda_1^2 E_O^2 N}{8 \eta_1^2}\left[\frac{\left(\frac{f_c}{f}\right)\left(\frac{b}{a}\right)}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} + \frac{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{2\left(\frac{f_c}{f}\right)} + \frac{\left(\frac{f_c}{f}\right)}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\right]$$

$$= \frac{R_S \lambda_1^2 E_O^2 N}{8 \eta_1^2 \left(\frac{f_c}{f}\right)\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\left(\frac{f_c}{f}\right)\left(\frac{b}{a}\right) + \frac{1-\left(\frac{f_c}{f}\right)^2}{2} + \frac{\left(\frac{f_c}{f}\right)^2}{2}\right]$$

$$= \frac{R_S \lambda_1^2 E_O^2 N}{16 \eta_1^2 \left(\frac{f_c}{f}\right)\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[1 + \left(\frac{2b}{a}\right)\left(\frac{f_c}{f}\right)^2\right] . \qquad (19)$$

The total attenuation constant α_T is given by Ramo and Whinnery as

$$\alpha_T = \frac{R_S}{b\,\eta_1 \sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[1 + \left(\frac{2b}{a}\right)\left(\frac{f_c}{f}\right)^2\right] L \qquad (20)$$

putting $N\lambda_g/2$ for L and combining (4) with (3)

$$P_G = \frac{E_O^2 \lambda_1^2 b\, \alpha_T}{16\eta_1 \left(\frac{f_c}{f}\right)\left(\frac{\lambda_g}{2}\right)} = \frac{E_O^2 ab\, \alpha_T}{4\eta_1}\sqrt{1-\left(\frac{\lambda_1}{2a}\right)^2} , \qquad (21)$$

Also by Ramo and Whinnery the power, P_O, through the waveguide is

$$P_O = \frac{E_1^2 a b}{4 Z_{TE}} = \frac{E_1^2 a b}{4 \eta_1}\sqrt{1-\left(\frac{\lambda_1}{2a}\right)^2} \qquad (22)$$

where

$$Z_{TE} = \frac{\eta_1}{\sqrt{1-\left(\frac{\lambda_1}{2a}\right)^2}} .$$

Since E_O is the result of a standing-wave, $E_O = 2E_1$,

$$P_O = \frac{E_O^2 ab}{16\eta_1}\sqrt{1-\left(\frac{\lambda_1}{2a}\right)^2} . \qquad (23)$$

Dividing (23) by (21)

$$\frac{P_O}{P_G} = \frac{1}{4\alpha_T} \quad \text{Q.E.D.} \qquad (24)$$

* * *

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APPENDIX B
Load Voltage After Switching

The double Laplace transforms of the voltage and current along a dissipationless transmission line are given by

$$\bar{\bar{V}}(S_1,S_2) = \frac{S_1[L\bar{I}(S_1,0) + \bar{V}(0,S_2)] - LS_2[C\bar{V}(S_1,0) + \bar{I}(0,S_2)]}{S_1^2 - LCS_2^2} \qquad (25)$$

$$\bar{\bar{I}}(S_1,S_2) = \frac{S_2[C\bar{V}(S_1,0) + \bar{I}(0,S_2)] - CS_2[L\bar{I}(S_1,0) + \bar{V}(0,S_2)]}{S_1^2 - LCS_2^2} \qquad (26)$$

where

$$\bar{I}(S_1,0) = \mathcal{L}_x\, I(x,t) \quad \text{at } t = 0$$

$$\bar{V}(S_1,0) = \mathcal{L}_x\, V(x,t) \quad \text{at } t = 0$$

$$\bar{I}(0,S_2) = \mathcal{L}_t\, I(x,t) \quad \text{at } x = 0$$

$$\bar{V}(0,S_2) = \mathcal{L}_t\, V(x,t) \quad \text{at } x = 0$$

and $\mathcal{L}_x$ and $\mathcal{L}_t$ are the Laplace transforms with respect to the variables x and t respectively.

Referring to Figure 9 and noting that on line (1)

$$\bar{\bar{I}}_1(S_1, 0) = 0$$

$$V(S_1, 0) = \mathcal{L}_x\, V_0 \sin\left[\frac{2\pi}{\lambda} x_1\right] = V_0 \frac{\left(\frac{2\pi}{\lambda}\right)}{S_1^2 + \left(\frac{2\pi}{\lambda}\right)^2}$$

(25) and (26) become for line (1)

$$\bar{\bar{V}}(S_1,S_2) = \frac{S_1\bar{V}(0,S_2) - LS_2\left[\frac{CV_0\left(\frac{2\pi}{\lambda}\right)}{S_1^2 + \left(\frac{2\pi}{\lambda}\right)^2} + \bar{I}_1(0,S_2)\right]}{S_1^2 - LCS_2^2} \qquad (27)$$

$$\bar{\bar{I}}(S_1,S_2) = \frac{S_1\left[\frac{CV_0\left(\frac{2\pi}{\lambda}\right)}{S_1^2 + \left(\frac{2\pi}{\lambda}\right)^2} + \bar{I}_1(0,S_2)\right] - CS_2\bar{V}_1(0,S_2)}{S_1^2 - LCS_2^2}. \qquad (28)$$

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Taking the inverse transforms of (3) with respect to S_1

$$\begin{aligned}\overline{V}_1(x_1,S_2) &= \overline{V}_1(0,S_2)\cosh\sqrt{LC}\,S_2x_1 \\ &+ \frac{V_0\sqrt{LC}}{LCS_2^2+\left(\frac{2\pi}{\lambda}\right)^2}\left[\sqrt{LC}\,S_2\sin\frac{2\pi x_1}{\lambda} - \frac{2\pi}{\lambda}\sinh\sqrt{LC}\,S_2x_1\right] \\ &- \overline{I}_1(0,S_2)\sqrt{\frac{L}{C}}\sinh\sqrt{LC}\,S_2x_1 \end{aligned} \qquad (29)$$

noting that $\overline{V}_1(\ell_1,S_2) = 0$ and $\cos\frac{2\pi\ell_1}{\lambda} = 1$ one obtains from (29)

$$\overline{I}_1(0,S_2) = \overline{V}_1(0,S_2)\sqrt{\frac{L}{C}}\,\frac{\cosh\sqrt{LC}\,S_2\ell_1}{\sinh\sqrt{LC}\,S_2\ell_1} - V_0C\,\frac{\left(\frac{2\pi}{\lambda}\right)}{LCS_2^2+\left(\frac{2\pi}{\lambda}\right)^2}. \qquad (30)$$

Similarly for the second line,

noting that $\cos\frac{2\pi\ell_2}{\lambda} = 0$ and

$\sin\frac{2\pi\ell_2}{\lambda} = 1$

one obtains as a result of $\overline{V}_2(\ell_2,S_2) = 0$

$$\overline{I}_2(0,S_2) = \overline{V}_2(0,S_2)\sqrt{\frac{L}{C}}\,\frac{\cosh\sqrt{LC}\,S_2\ell_2}{\sinh\sqrt{LC}\,S_2\ell_2} - \frac{V_0C}{LCS_2^2+\left(\frac{2\pi}{\lambda}\right)^2}\left[\frac{\sqrt{LC}}{\sinh\sqrt{LC}\,S_2\ell_2} - \frac{2\pi}{\lambda}\right]. \qquad (31)$$

Now the sum of the currents at the load must be zero, hence

$$\overline{I}_1(0,S_2) + \overline{I}_2(0,S_2) + \frac{\overline{V}_1(0,S_2)}{R} = 0. \qquad (32)$$

Since $R = \sqrt{\frac{L}{C}}$ by (30), (31), and (32)

$$\begin{aligned}&\overline{V}_1(0,S_2)\sqrt{\frac{C}{L}}\left[\frac{\cosh\sqrt{LC}\,S_2\ell_1}{\sinh\sqrt{LC}\,S_2\ell_1} + \frac{\cosh\sqrt{LC}\,S_2\ell_2}{\sinh\sqrt{LC}\,S_2\ell_2} + 1\right] \\ &- \frac{V_0C}{LCS_2^2+\left(\frac{2\pi}{\lambda}\right)^2}\left[\frac{2\pi}{\lambda} + \frac{\sqrt{LC}}{\sinh\sqrt{LC}\,S_2\ell_2} - \frac{2\pi}{\lambda}\right] = 0\end{aligned} \qquad (33)$$

and

$$\bar{V}_1(0, S_2) = \frac{V_0 S_2 \sinh\sqrt{LC}\, S_2 \ell_1}{\left[S_2^2 + \frac{1}{LC}\left(\frac{2\pi}{\lambda}\right)^2\right]\left[\sinh\sqrt{LC} S_2(\ell_1 + \ell_2) + \sinh\sqrt{LC} S_2\, \ell_1\, \sinh\sqrt{LC} S_2 \ell_2\right]} \cdot \qquad (34)$$

The inversion of (34) to obtain $V_1\,(0,t)$, the load voltage can best be accomplished by converting the hyperbolic functions into a series of exponentials making use of the fact that

$$\frac{1}{2\pi j}\int_{Br} \frac{S_2 e^{aS_2} dS_2}{S^2 + \gamma^2} = \cos a\gamma \cdot \mathcal{U}(a)$$

where $\mathcal{U}(a) = 0,\ a < 0$
$= 1,\ a > 0$.

Now,

$$\frac{\sinh\sqrt{LC}\, S_2 \ell_1}{\sinh\sqrt{LC}\, S_2(\ell_1 + \ell_2) + \sinh\sqrt{LC}\, S_2 \ell_1 \cdot \sinh\sqrt{LC}\, S_2 \ell_2}$$

$$= \frac{2 e^{-\sqrt{LC}\, S_2 \ell_2}(1 - e^{-2\sqrt{LC}\, S_2 \ell_1})}{3 - e^{-2\sqrt{LC}\, S_2 \ell_2} - e^{-2\sqrt{LC}\, S_2 \ell_1} - e^{-2\sqrt{LC}\, S_2(\ell_1 + \ell_2)}}$$

$$= \frac{2}{3} e^{-\sqrt{LC}\, S_2 \ell_2} + \frac{2}{9} e^{-3\sqrt{LC}\, S_2 \ell_2} + \frac{2}{27} e^{-5\sqrt{LC}\, S_2 \ell_2} + \ldots$$

$$- \frac{4}{9} e^{-\sqrt{LC}\, S_2(2\ell_1 + \ell_2)} + \frac{4}{27} e^{-\sqrt{LC}\, S_2(2\ell_1 + 3\ell_2)} + \ldots$$

$$+ \text{etc.} \qquad (35)$$

The typical inverse transform which must be found is out of the form

$$\frac{1}{2\pi j}\int_{Br} \frac{K V_0 e^{S[t - \sqrt{LC}(a\ell_1 + b\ell_2)]} dS_2}{S_2^2 + \left(\frac{1}{LC}\right)\left(\frac{2\pi}{\lambda}\right)^2}$$

$$= K V_0 \cos 2\pi f\,[t - \sqrt{LC}\,(a\ell_1 + b\ell_2)] \cdot \mathcal{U}\,[t - \sqrt{LC}(a\ell_1 + b\ell_2)] \qquad (36)$$

where $f = \frac{1}{\lambda\sqrt{LC}}$, and K, a, and b are constants.

The inversion of the terms of (35) will yield the result in the section devoted to the cavity discharge switch.

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