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APPROXIMATIONS FOR SHORT-RANGE SHALLOW-WATER SOUND TRANSMISSION

R. J. Urick, Consultant

Sound Division

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CONTENTS

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ABSTRACT

A knowledge of the sound field of a low-frequency shallow-water sound source is useful in determining the sweep widths of an acoustic minesweeper and in adjusting the sensitivity of acoustic mines. Unfortunately, exact theory is at present of little practical utility, not only because of the cumbersome nature of the theoretical result, but more important, because of uncertainty about the appropriate acoustic properties of the bottom in the area where the prediction is desired. Thus, in practice there is little recourse but to estimates based on guesses and approximate considerations.

One method for estimating the shallow-water short-range sound field of a low-frequency source is given in this report. The variation of sound level with distance from a point source in the far field is determined by the relation of the operating frequency to the cutoff frequency for any particular area. For frequencies far above the cutoff frequency, the pressure variation is as the inverse square-root of the distance; for frequencies far below cutoff, as the inverse square. Close to the source, on the other hand, the variation is in all cases as the inverse first power. Curves are given to determine the estimated cutoff frequency for a given bottom type and water depth, and simple formulas permit the easy computation of a distance, R_0, which effectively separates the near and the far sound field. In an actual computation of transmission loss, the loss out to range R_0 is found, using the inverse first power variation, and added to the loss between R_0 and the field point.

Comparisons are given between the levels so computed and four examples of measured field data found in the literature. Although the method is incapable of giving an accurate prediction of the sound level of a source in shallow water, it does give a rough estimate that may be useful for many purposes.

PROBLEM STATUS

This is an interim report; work on this problem is continuing.

AUTHORIZATION

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APPROXIMATIONS FOR SHORT-RANGE SHALLOW-WATER SOUND TRANSMISSION

INTRODUCTION

The principal application for a knowledge of how sound is transmitted during the first thousand feet or so from a shallow-water source is in mine warfare. In pro-mine warfare, it is one of the determining factors for sensitivity settings in mine laying. In anti-mine warfare, short range transmission governs the range of effectiveness of sweeping by acoustic means.

There are two theoretical approaches to the shallow-water transmission problem that are formally exact and correct. One is the ray approach, using the method of images. In this method, the infinite series of images formed by the source in the surface and bottom, and the successive images of the images, are summed, with due care for their phase shifts and source levels to give the sound field at a point in the liquid layer. A formal expression for this sum can be written easily, and the method in addition has a good deal of physical and intuitive appeal. The trouble lies in evaluating the infinite sum to obtain a numerical answer. This stumbling block to obtaining a definite result has made the image solution of the wave equation practically useless, except for extremely short ranges or high reflection loss at the boundaries, in which cases the number of images that need be summed is very small.

The other theoretical approach to the shallow-water problem, the normal mode theory, is in sharp contrast to the ray, or image, method in the complicated nature of the solution and its real lack of intuitive appeal -- or at least the difficulty of grasping the real significance of the apparently artificial normal modes that result. Nevertheless, the theory has been successfully worked out and has been roughly verified by field measurements. The "classic" paper on the subject is that by Ide, Post, and Fry,[1] dating from 1943. More recent treatments are those by Pekeris,[2] a group at the British Admiralty Mining Establishment,[3] and a group at the U. S. Navy Mine Countermeasures Station, Panama City, Fla.[4]

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[1] Ide, J. M., Post, R. F., Fry, W. J., "The Propagation of Underwater Sound at Low Frequencies as a Function of the Acoustic Properties of the Bottom," NRL Report S-2113, 1943

[2] Pekeris, C. L., and others, "Propagation of Sound in the Ocean," USGS Memoir 27, 1948

[3] Merbst, H., Flint, F., and others, "The Normal Mode Theory of Propagation of Sound in a Shallow Sea" -- "Theoretical Application and Experimental Results for Low Frequencies," Great Britain A. M. E. Informal Report January 1950

[4] Elliott, M. A., Melchor, J. L., Young, J. M., "Underwater Sound Transmission at Low Frequencies...," USNMCS Report 26, Apr. 1950

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The normal-mode theory gives the sound field for a shallow water source as an infinite sum of normal modes. The propagation constant for each mode is related in a definite manner to the characteristics of the bottom. If the bottom is a homogeneous fluid, or can be considered as such, the theory considered in the papers mentioned above gives the real and imaginary parts of the propagation constant in terms of the density and sound velocity of the bottom. If the latter are known, or can be assumed, this theory is excellent, provided that the cumbersome arithmetic involved is tolerated.

The essential point, however, is that at present we do not know, and have no way of knowing, enough about the bottom characteristics in a particular location to make it worthwhile to use the normal mode theory, or any exact theory. Even if we did, it is likely that the normal-mode theory, in its present state of development, would be useless, since sea and river bottoms are seldom simple structures.

That is to say, normal-mode theory for a bottom having a certain impedance, or perhaps reflection coefficient (rather than a simple density and velocity), has not been developed in a practical form. To apply the theory in its present state requires us to specify a simple density and velocity for the bottom. These cannot be measured in any way, since low frequency sound penetrates a long way in a bottom material. We are thus in the position of using an elaborate theory with inexact, and indeed, perhaps meaningless, parameters.

Until useful theory has been developed for a bottom of arbitrary impedance or reflection coefficient, and ways of measuring these factors developed for field use, it seems desirable to consider rough approximate methods of finding the sound field from a shallow water source. Very simple considerations suffice, and it will be seen that they give results that are in fair agreement with field data.

APPROXIMATE CONSIDERATIONS

Let us give principal attention to the manner of variation of pressure with distance from the source. Quite close to the source, spherical spreading holds, and the pressure varies as the inverse first power of the range. On a plot of water depth and frequency (Fig. 1a) the pressure is inversely proportional to the first power of the range ($p \propto r^{-1}$) for all depths and frequencies at close enough distances from the source. Figure 1b is the same sort of plot at a greater distance. It is divided into three parts or regions. In Region A, for shallow water depths and low frequencies, energy is not trapped by the bottom, but escapes into the earth as though the bottom were not present. The sound field is given by the interference pattern of the source and its image in the surface. As will be seen later, this dipole source adds another r^{-1} to the free-field pressure-distance relationship so that $p \propto r^{-2}$. In Region C, at great water depths and high frequencies, all the energy emitted by the source is trapped within the water layer, and cylindrical spreading occurs.* In the image theory, this corresponds to an infinite number of equally bright images lying on a line above and below the source. When cylindrical or one-dimensional spreading takes place, $p \propto r^{-1/2}$. Region B is an intermediate region centering about the locus of the "cutoff frequency" of the first mode in the normal mode theory. In this transitional region, we may by interpolation expect that $p \propto r^{-1}$. Thus, the exponent of the distance in the pressure-distance relationships varies from -1/2 to -2, depending on the frequency and water depth. Close to the source the exponent is -1.

*...if the range is not excessive. At really long ranges, appreciable energy leaks out of any channel.

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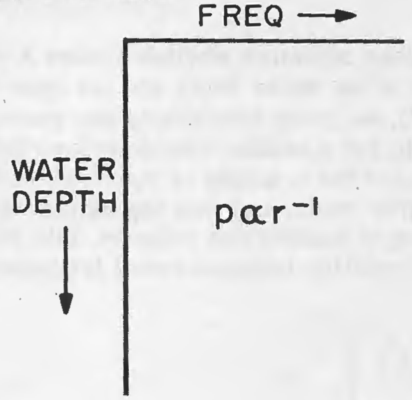

Figure 1a - Near enough to the source, the pressure is inversely proportional to the first power of the range, regardless of frequency and water depth.

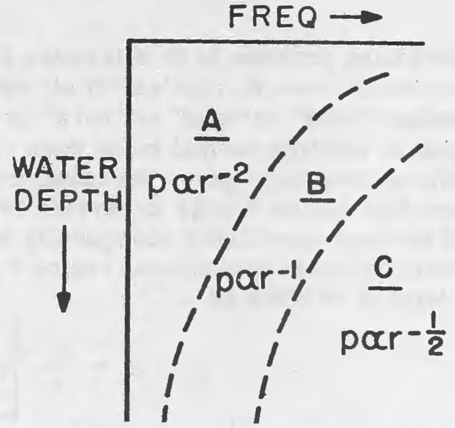

Figure 1b - At longer ranges, the pressure varies with range in a manner determined by the frequency and water depth. In region A, the variation is as the inverse square; in region B, as the inverse first power; in region C, as the inverse square root.

Figure 2 shows level plotted against range. At unit small distance from the source the pressure is P_0, and falls off as r^{-1} with increasing distance. At long ranges, the spreading laws considered above will apply. In between will be a transitional region, indicated by the dashed lines, in which we may expect to find peaks and troughs due to interference between modes (in the mode theory) or images (in the image theory). The dashed lines will represent a smoothed average of these variations. The problem then becomes that of finding the range R_0 at which the r^{-1} spreading close to the source changes to $r^{-1/2}$ for complete trapping (Case A), or to r^{-2} for no-trapping (Case C).

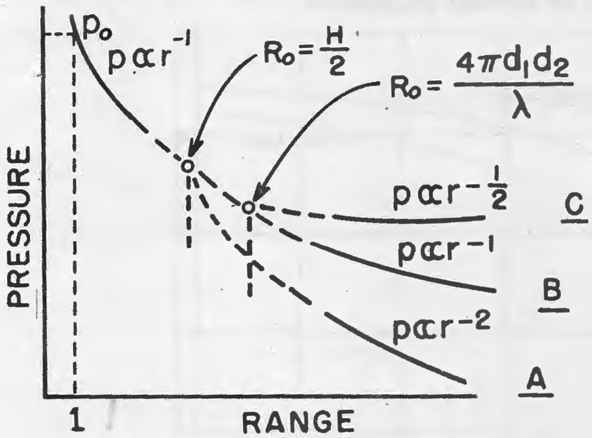

Figure 2 - Pressure vs. range for the regions A, B, and C of Figure 1. The transition ranges R_0 can be found approximately by simple methods.

As shown in the Appendix, this range R_0 is

$$4\pi \frac{d_1 d_2}{\lambda} \quad \text{for Case A,}$$

$$H/2 \quad \text{for Case C,}$$

where H is the water depth; d_1 and d_2 are the depths of source and receiver; and λ is the wavelength. The pressure level p_R at range R is then related to the level P_0 at unit small distance as follows:

$$p_R = \left(\frac{p_0}{R_0}\right)\left(\frac{R}{R_0}\right)^{-2} = \frac{p_0 R_0}{R^2} \quad \text{for Case A,}$$

$$p_R = \left(\frac{p_0}{R_0}\right)\left(\frac{R}{R_0}\right)^{-1/2} = \frac{p_0}{R^{1/2} R_0^{1/2}} \quad \text{for Case C.}$$

The intermediate case B of partial trapping can be approximated by

$$p_R = p_0/R.$$

The remaining problem is to determine for a given situation whether Cases A or C, or the intermediate case B, applies. If all we know is the water depth and the type of bottom (whether "sand" or "mud" or "hard" or "soft"), we must essentially use guess work and an appeal to existing normal mode theory, which, for a bottom characterized by a (real) density and velocity, gives the cutoff frequencies for trapping of the first mode. If we assume that all the energy is carried by the first mode, and that the bottom is a single fluid medium specifiable acoustically by a single density and velocity, this cutoff frequency will lie in the transitional region B. The relation between cutoff frequency f_c and water depth H is given by

$$f_c \cdot H = \frac{c_1}{4}\left[1 - \left(\frac{c_2}{c_1}\right)^2\right]^{-1/2},$$

where c_1 and c_2 are the sound velocities in water and the bottom, respectively. It is noteworthy that the cutoff frequency does not depend on the density, but only on velocity. The following table gives estimated values of the ratio c_2/c_1 for various types of bottom:

Soft Mud	1.03 (or less)
Mud	1.07
Sand	1.2
Firm sand, rock	1.4 (or more)

Figure 3 shows the above relationships plotted for these four values of c_2/c_1. The basis for these values lies in the fragmentary published information on sediment sound velocities and the homogeneous-mixture theory for velocity as a function of density. No credence is given to the astonishingly low values of c_2/c_1 assumed in several phases in the Ide, Post, and Fry report, based on their measurements of normal impedance.

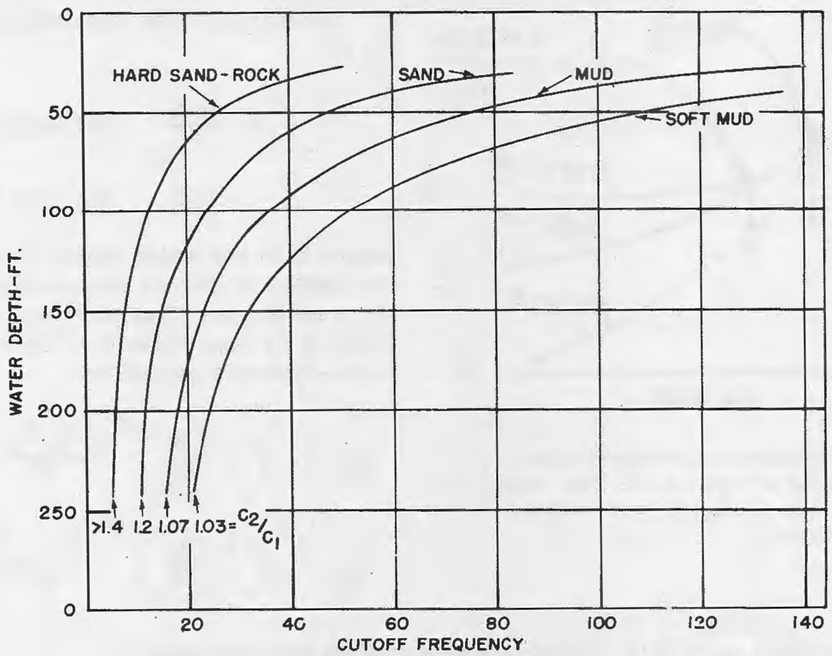

Figure 3 - Cutoff frequencies for several bottom types, based upon estimated values of the velocity ratio c_2/c_1

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Figure 3 can be used to determine whether no trapping (Case A), complete trapping (Case C), or partial trapping (Case B) is expectable for a given water depth and bottom type by means of the ratio of the actual operating frequency to the cutoff frequency. If this ratio is much greater than unity, say more than 2, C will apply; if it is much less than unity, say less than 1/2, A will apply.

As an example, suppose we have a 120-cycle sound source in 100 feet of water over a sandy bottom. The cutoff frequency from Figure 3 is 24 cycles. The ratio $f/f_c = 120/24 = 5.0$, and complete trapping may be assumed to exist (Case A). The pressure at range R will be roughly $p_o/(RR_o)^{1/2}$ where $R_o = H/2$. If, however, the water shoals to 25 feet, the cutoff frequency rises to 90 cycles. Now only partial trapping exists (Case B) and a closer approximation to the field at range R will be p_o/R.

COMPARISON WITH FIELD OBSERVATIONS

In all of the foregoing, the large number of approximations and poorly justified assumptions will be evident. If, however, we have at hand only qualitative statements of the nature of the bottom (e.g., "soft mud"), and will remain content with approximate values for the sound field, it is believed that the foregoing method is adequate. The real test of its value lies in the comparison with field data. Some examples of such comparisons follow.

Plate 9 of Ide, Post, and Fry gives measured transmission curves over a "hard" and a "soft" bottom for two frequencies (70 and 100 cps). These were obtained by running a ship-mounted sound source (depth: 12′) toward and away from a bottom-mounted hydrophone (1-1/2 feet above the bottom) in 55 feet of water. The "hard" bottom was at the mouth of the Potomac River, and was described as a hard sandy mud; the "soft" bottom was soft mud at the Potomac River Bridge. Referring to Figure 3 for a depth of 55 ft, and calling the "hard" bottom "sand" in Figure 3, and the "soft" bottom "soft mud," we obtain the following for the ratio f/f_c:

	f_c	f = 100	f = 70
"Hard" (Sand)	43	$f/f_c = 2.3$	1.6
"Soft" (Soft Mud)	96	1.0	0.7

These ratios indicate that these are all partial-trapping situations. The 100 cps "hard" case with ratio 2.3 may perhaps be considered nearly complete trapping (Case C); the others will be transitional (Case B) between A and C. Figure 4 shows the transmission data of the Ide report traced along with the results of computations of present theory. It will be seen that fair agreement exists. The measured curves all lie between the trapped and not-trapped theoretical curves; the 100 cps "hard" data is close to the complete trapping curve.

Another example is provided by the recent British data of Reference 3. This data, reproduced in Figures 5 and 6 of this report, is in the form of loss contour curves for about 80 feet of water and a frequency of 10 cps, at two locations which we can designate Area 1 and Area 2, having different bottom types.* Reference to Figure 3 shows that for 80 ft of water, 10 cps is far below the cutoff frequency for any type of bottom.

* Area 1 is stated to have a bottom of "firm coral sands above hard sandstone." Area 2 had "several inches of mud over shingle and light sands; this was expected to represent a much less hard bottom" than Area 1.

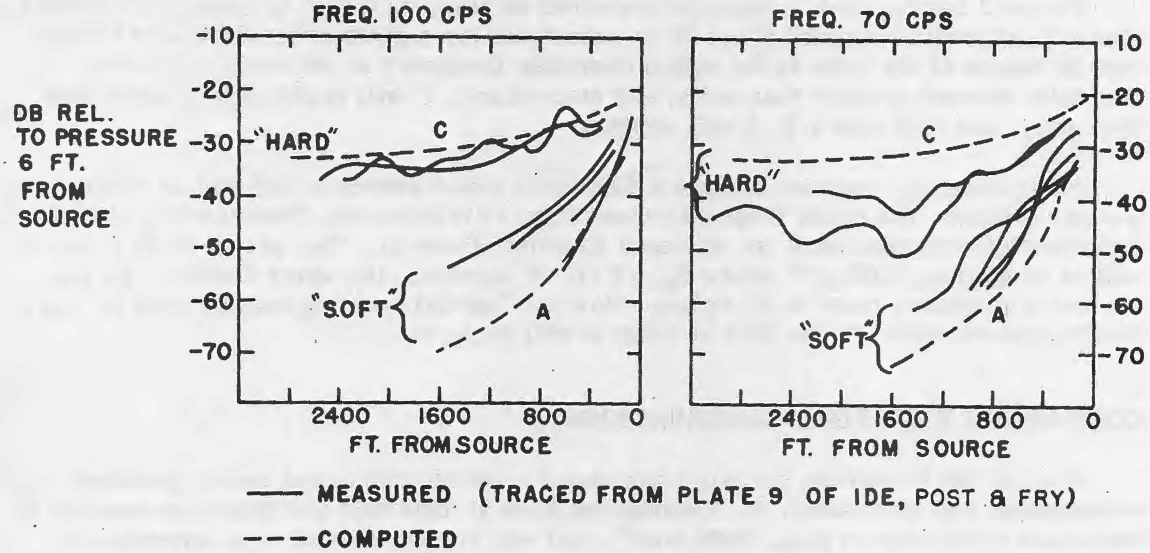
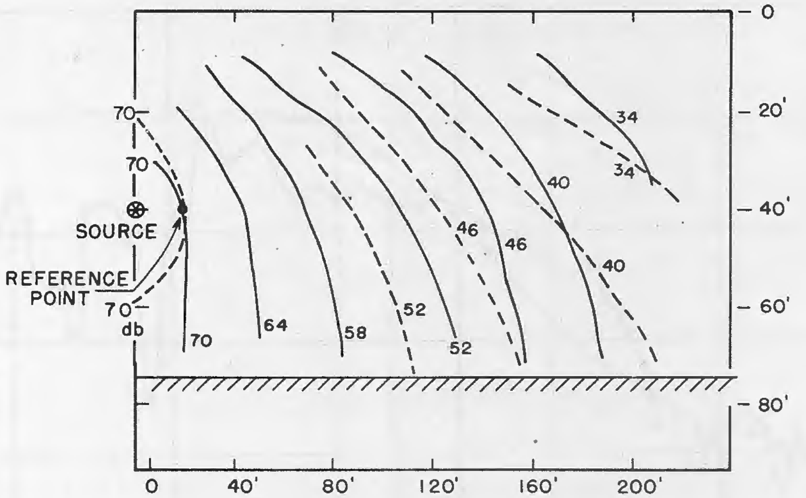


Figure 4 - Comparison of the field data of Plate 9 of Reference 1 with approximate computations

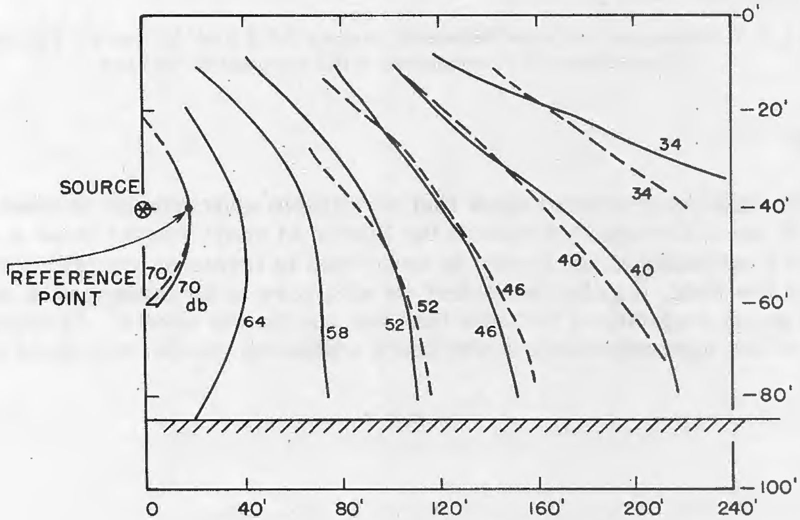
Thus, no trapping is possible, and the sound field will be given by the interference of the source and its surface image. Figures 5 and 6 show loss contours computed from Case A, relative to the loss at a distance of 20 feet as given by the data. The agreement is seen to be excellent between the field contours and the superposed computed contours. This British report, incidentally, contains a fine theoretical treatment of normal-mode theory. However, the field data used in its support do not require the normal-mode theory at all except as the simple limit of an elaborate theory.

One final comparison with observed data is obtained from Reference 4. One of the figures in this report shows for a wide range of frequency the measured attenuation between two hydrophones, one at a distance of 3 yards and the other at 95 yards from the source. The water depth was 41 feet, the source depth was 20 feet, and the distance hydrophone was located 1-1/2 feet above the bottom, which is stated to be "sandy mud." If we use the curve labeled SAND in Figure 3, we find for a water depth of 41 feet a cutoff frequency of 56 cycles. For frequencies of less than half this, Case A applies, no trapping exists and the range r_0 is given by $4\pi d_1 d_2/\lambda$. For frequencies greater than twice 56 cycles, complete trapping occurs, and r_0 is simply H/2. At the cutoff frequency, spherical spreading may be taken to apply. Figure 7 shows the computed losses together with the observed data traced from Figure 1 of Reference 4. It will be noticed that good agreement between theory and measurement exists below about 80 cycles. At higher frequencies the approximation yields losses that are too small. No explanation is at hand for the fact that the peaks of the intermodal interference pattern observed at high frequencies fall below (i.e., have a greater loss than) the measured data at about 170 cycles.



—— MEASURED (REF. 3). AREA 1, FREQ: 10 cps
– – – COMPUTED

Figure 5 - Measured and computed level contours, using field data of Reference 3



—— MEASURED (REF. 3). AREA 2, FREQ: 10 cps
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Figure 6 - Measured and computed level contours, using field data of Reference 3

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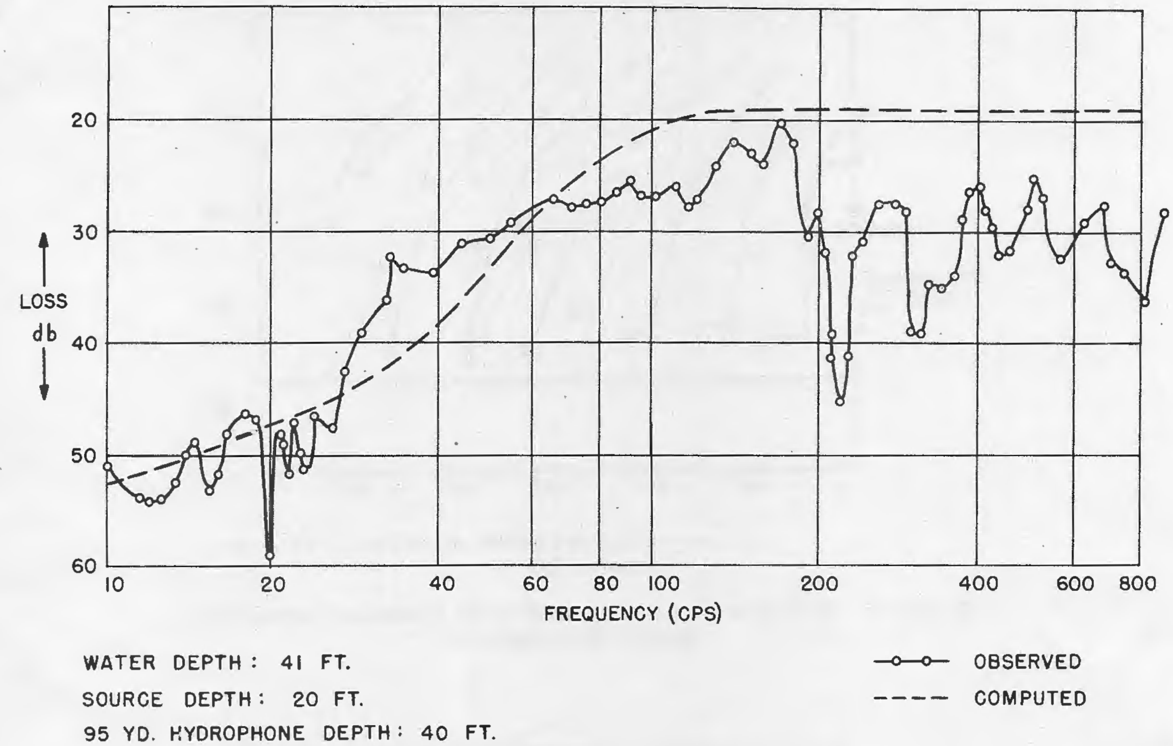

Figure 7 - Transmission loss between ranges of 3 and 95 yards vs. frequency (Reference 4) compared with computed values

CONCLUSION

The above examples seem to show that the simple approximate method outlined here will provide at least a rough estimate of the losses at short ranges from a shallow water source. Until a workable exact theory is developed in terms of acoustic bottom parameters measurable in the field, it is believed that we will have to be content with such approximations for that great majority of bottoms that are not simple media. An estimate of the real validity of the approximations must await additional checks with field data.

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APPENDIX

Derivation of R_o

Case A: No Trapping

In this case the field is given by the interference of source and its image

$$p = p_o \left[\frac{e^{ikr_1}}{r_1} - \frac{e^{ikr_2}}{r_2} \right]$$

where $k = 2\pi/\lambda$, r_1 and r_2 are the distances of the field point to the source and its image, and the minus sign expresses the phase reversal of pressure on reflection from the free surface. By expansion we can show that at a remote distance from the source

$$p_{far} = \frac{p_o}{r} \cdot 2 \sin \frac{k(r_1 - r_2)}{2}$$

if $r = r_1 \approx r_2$. If the depth of the source is d_1 and the depth of the field point d_2, we can further find that

$$r_1 - r_2 = \frac{2d_1 d_2}{r}$$

Let us assume that for a rough approximation we can replace the sine by its argument if the angle is less than 45°. If we do this, we find that

$$p_{far} = \frac{p_o}{r} \left[2k \frac{2d_1 d_2}{r} \right] = p_o \cdot \frac{4\pi}{\lambda} \cdot \frac{d_1 d_2}{r^2}$$

The 45° limit means that r must be greater than $8d_1 d_2/\lambda$ for this last approximation to apply. Since the pressure at a unit small distance from the source is p_o, the pressure at range r near the source will be

$$p_{near} = \frac{p_o}{r}$$

At range R_o, P_{near} and P_{far} will be equal. Hence,

$$p_{far} = p_{near} = p_o \cdot \frac{4\pi}{\lambda} \frac{d_1 d_2}{R_o^2} = \frac{p_o}{R_o}$$

so that

$$R_o = 4\pi \frac{d_1 d_2}{\lambda}$$

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Case C: Complete Trapping

Let the power output of the nondirectional source be P. Assume that at a long range the power flux is distributed equally throughout the thickness H of water. This implies that the intensity and pressure are independent of depth. Although this is not true near the surface, a pressure-release boundary, it may not be too bad an approximation if many modes are trapped. With this assumption, the intensity at r is

$$I_{far} = \frac{P}{2\pi r H}$$

The intensity at a short range r from the source is

$$I_{near} = \frac{P}{4\pi r^2}$$

Equating I_{far} and I_{near} at range R_o, we have

$$I_{far} = I_{near} = \frac{P}{2\pi R_o H} = \frac{P}{4\pi R_o^2}$$

Hence

$$R_o = \frac{H}{2}$$

* * *

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