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AN ACOUSTIC MODELING SYSTEM FOR PHYSICS-GUIDED MACHINE LEARNING

SAN DIEGO STATE UNIVERSITY

JANUARY 2024

FINAL TECHNICAL REPORT

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AN ACOUSTIC MODELING SYSTEM FOR PHYSICS-GUIDED MACHINE LEARNING

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Summary

This study aims to establish an acoustic modeling cyber infrastructure (CI) that can use physics of sound to simulate the sound wave propagation in real-life environments and provide tools to generate realistic datasets for training the deep learning algorithms. Unmanned air systems (UAS) are being increasingly manufactured and used in different applications. Their high technical capabilities allow adversaries to exploit these technologies and conduct different types of attacks against critical infrastructures. Acoustic sensors can realize a resilient low-cost and power sensing system that enables detection, localization, and classification of these UAS through capturing and processing their propeller noise in a variety of environments. However, the state-of-the-art sound source detection and classification techniques do not provide accurate prediction of sound wave propagation in complex environments. Deep learning-based methods have shown promises, but the costs of collecting large training datasets to cover a range of possible scenarios are prohibitive. A reliable solution to successfully detect a UAS flying in outdoor needs reproducing and testing a range of realistic scenarios. These include different atmospheric conditions (wind, temperature, turbulence, absorption), noise barriers, types of UAS and maneuvers, number of sound sources, ground effects, and irregular terrains. What if we had a highly capable CI that could realistically simulate the sound propagation under different conditions in outdoor environments and generate reliable data? Establishing such CI is a goal of this research. If successful, the CI can help researchers to generate realistic datasets that can be used for training and testing deep learning algorithms.

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I. Introduction

1.1 Problem Statement

This research aims to address the lack of a simulation platform which can help the researchers to produce reliable datasets for training of deep learning algorithms to develop intelligent acoustic sensing and detection systems. Currently, the researchers use experimental datasets which are expensive and challenging to collect. Existence of a simulation platform to complement those experimental results can lead to more robust and reliable sound wave propagation models.

1.2 Background and Novelty

The presence of unauthorized UAS near critical infrastructures might lead to different types of attacks (cyber, illegal surveillance, and kinetic) or cause mid-air collisions between these UAS and other aerial vehicles (e.g., aircrafts in the airports). A resilient low-cost sensing system that enables detection, localization, and classification of these UAS is required now more than ever. Acoustic sensors can provide such capabilities through capturing and processing the UAS propeller noise in dark and cluttered environments and in different meteorological conditions (fog, cloud, mist). However, the state-of-the-art sound source detection techniques do not provide accurate prediction of sound wave propagation in complex environments.

1.2.1 Literature Survey

The existing methods are mostly limited to the conventional signal processing-based and/or geometric acoustic approaches, both of which can only allow understanding of simple scenes [1, 2, 3, 4, 5, 6, 7, 8, 9]. Simple scenes include controlled lab environments that involve limited number of sound sources, high signal-to-noise ratio, and direct sounds (as opposed to NLOS or indirect sound). Also, most of these methods are only able to detect the direction of arrival (DOA) of the sound sources rather than the 3D positions (i.e., DOA and distance). Although deep learning-based techniques are highly capable of modeling complex nonlinear phenomena, they have received little attention in the acoustic community due to difficulties involved in collecting training datasets that represent real-life scenes. Highly accurate models (used for localization and classification) require significant amount of training data to

cover a wide range of sound propagation scenarios that is extremely challenging and mostly impossible to recreate in lab environments.

On the other hand, the currently existing software packages[10, 11, 12] for acoustics simulations are mainly intended for architectural acoustics purposes and can simulate a single room acoustics which is mostly useful for room acoustics design tasks. These tools do not allow studying the effect of realistic receivers such as the microphone arrays typically used in real-life environments. They do not as well allow simulating low-frequency sound effects such as diffraction or simulating the sound propagation in complex inhomogeneous outdoor environments which involve effects such as refraction.

1.2.2 Novelty

This study aims to address these challenges through building a CI that can simulate the sound wave propagation in real-life environments and provide tools to generate realistic datasets for training the deep learning algorithms. The CI will provide the capability of modeling the following: wave propagation in outdoor spaces considering meteorological conditions and sound effects such as reflections, diffraction, and refraction. Integration of many of these capabilities — such as propagation in inhomogeneous outdoor spaces — requires developing an in-depth understanding of the spatial and temporal propagation of sound waves and establishing methods to model these nonlinear phenomena. If successful, the proposed CI will result in bi-directional advances in acoustics science and robotic perception.

1.3 Project Overview

This project leverages the state-of-the-art methods for modeling sound in atmosphere and develops a universal framework that can use the available experimental data and generate new realistic data through simulations of the sound propagation phenomena in the atmosphere. Figure 1 presents a brief overview of the project. The goal is to develop a modeling system that can take as an input the experimental measurement of a UAS sound under certain conditions and return the predicted sound signal of the same UAS at a different atmospheric and ground conditions and/or at a different location with respect to the receiver (microphone).

1.4 Current Progress and Report Outline

The two tasks required for developing the CI described in Section 1.3 are:

Task 1: Modeling the sound propagation phenomena;

Task 2: Comparing the model results with experiments and addressing the discrepancies.

There are multiple sound propagation phenomena that effect the acoustic signal at the microphones such as illustrated in Figure 2. The dominate factor for sound propagation is the refraction of the acoustic wavefront. Changes in the speed of sound spatially in the atmosphere will deform the acoustic wavefront so that normal rays from the source appear to deform and bend. This bending creates upward and downward refracting rays which

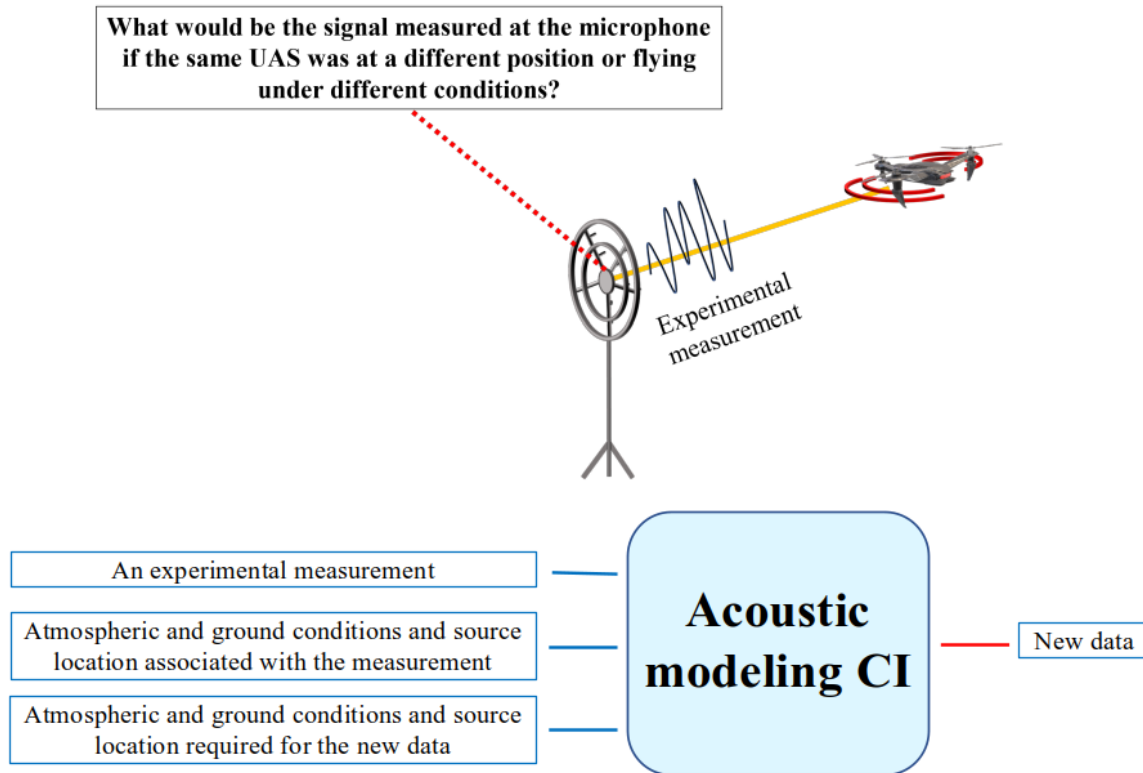


Figure 1: Project Overview

will either increase or decrease respectively the sound level at the receiver. Additionally, the upwards refraction creates regions called "shadow zones" where no rays reach which, in theory, means that the signal does not reach the receiver in some geometric relationships. In reality, acoustic signals do exist in the shadow zone due to turbulence in the atmosphere but at a significantly diminished strength.

There are multiple methods that handle atmospheric refraction, in both the time and frequency domain. For computational speed and simplicity, we only consider the frequency domain methods. There are numerous frequency based methods discussed in [13] that would handle refraction such as ray tracing, the Fast Field Program (FFP), the Crank-Nicholson Parabolic Equation (CNPE), and the Green's Function Parabolic Equation (GFPE). Ray tracing is geometrically straight forward, treating the acoustic signal as rays emanating from an acoustic source. The acoustic signal strength is determined from interpolation of nearby rays. These rays may lead to complicated and intersecting fields in downward environments that make the interpolation difficult. Alternatively, other methods attempt to solve the acoustic equations in the frequency domain across the spatial domain. The CNPE and GFPE are based on a parabolic equation approximation of the wave propagation but required solutions marching outwards from the source radially. The advantage of the radial marching is that these models are able to capture 2D variations of the ground but the parabolic approximation limits the angle of elevation for accuracy i.e. these models do not perform well for source and detector pairs that are near vertical to each other. Lastly, FFP does not deal in the spatial horizontal domain but rather a transformed horizontal domain.

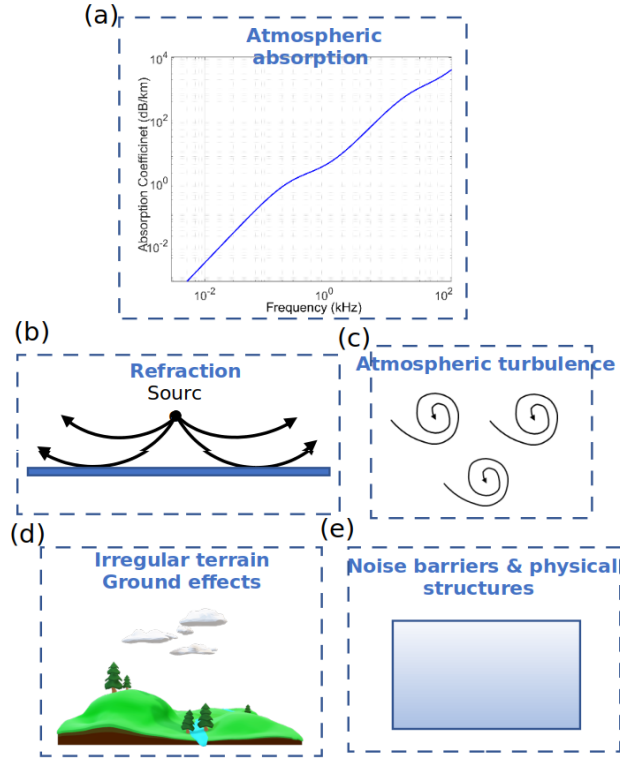


Figure 2: Different sound effects in outdoor environments

FFP determines the solution vertically in this transformed domain and performs the inverse transform to give the pressure level in the 3D spacial domain. The disadvantage is that unless the atmosphere properties and ground effects are also transformable, FFP can only deal with horizontally homogeneous media.

In this study, we focused on implementing GFPE and FFP. FFP was perceived as the simplest method and implemented to give us insight on implementing the GFPE. Moreover, FFP does not have any limitations in terms of the elevation angle. It can complement the GFPE in homogeneous media and cover elevation angles over 60 degrees that the original version of GFPE is not capable of accurately covering. However, GFPE has been found to be more frequent in modern literature than FFP and alterations to GFPE have allowed it to handle modeling complexities like irregular terrain. Thus, one can combine both FFP and GFPE to handle long range sound propagation of acoustic signals.

The following sections of this report discuss the steps taken for Task 1 of this project. A general description of the physics and mathematical modeling of the source-detector acoustic problem are discussed in Section 2.1. This section will describe some notation, general assumptions of the atmosphere, handling of any ground terrain, and the relevant equations for linear acoustics with reflections. Next, Section 2.2 outlines the approach used to solve the acoustic problems with or without refractions. The methods have been incorporated into a code base being developed for Task 1 which is compared to other results in the literature within Section 2.3. Given the validation of our solver implementations, there are some theoretical explorations in Section 3.1 into acoustic models and parameters on the

broadband frequency sound propagation. Additionally, it is also explored to see if one can estimate missing parameter values from experimental datasets using optimization. Some of the lessons from the initial exploration into some experimental datasets are discussed in Section 3.4 for Task 2.

II. Methods, Assumptions, and Procedures

2.1 Problem Description

2.1.1 Geometry

The most natural coordinate system for the acoustic source-detector problem is Cartesian coordinate system with a point being described as the vector $\mathbf{x} = [x, y, z]^T$ where x, y are the orthogonal horizontal components and z is the vertical component above the ground plane. For ease of reference in this work, we refer to scalar quantities in a normal font (x) but vectors are bold ($\mathbf{x}$) with $(\cdot)^T$ being the vector transpose. The unit vectors along a primary direction of the coordinate system are given as $\mathbf{e}_{(\cdot)}$. For convenience, we assume throughout this work that the source is a stationary object located at $\mathbf{x}_s = [0, 0, z_s]^T$ while the detector is more arbitrary at $\mathbf{x}_d = [x_d, y_d, z_d]^T$. Here we use the subscript s to indicate any quantities related to the acoustic source while d is for quantities related to the detector or receiver. The convenience of the source being located along the unit vector $\mathbf{e}_z$ gives rise to the cylindrical coordinate system being a useful coordinate system. This coordinate system is defined with $\mathbf{r} = [r, \theta, z]^T$, a tuple of the range (or radial component) r , polar angle θ , and again height z . The two coordinate systems are related by the transform $x = r \cos \theta$ and $y = r \sin \theta$ and share some terminology used later. The range of the detector from the source is given as either x or r although the cross-range only refers to y . Both systems will be used in later sections and subsections.

Given the coordinate system, we can now define the acoustic modeling problem. As all of the methods discussed in Section 2.2 are numerical methods, the acoustic field is solved numerically on a grid as in Figure 3. This requires defining bounds of the problem dimensions as well as the appropriate grid size of the coordinate systems used in the methods. The specifics of the grid segmentation and boundary conditions depend on the method being used. Normally, the methods only consider the maximum range and height, and potentially the maximum polar angle, but some specific problem geometry may have to be introduced. For example, the methods for solving Parabolic Equations (PE) often require the introduction of an artificial absorption layer starting at z_{abs} . This means that certain methods need to consider not only the maximum height of acoustic interest for the problem, but also the additional height required for accuracy of the methods. Here are a couple of considerations when determining the appropriate problem dimensions:

1. Region of Interest for placement of the microphones;
2. maximum height achieved by acoustic rays in downward refracting atmospheres;
3. and, finally, any limitations on the region of accuracy for methods used.

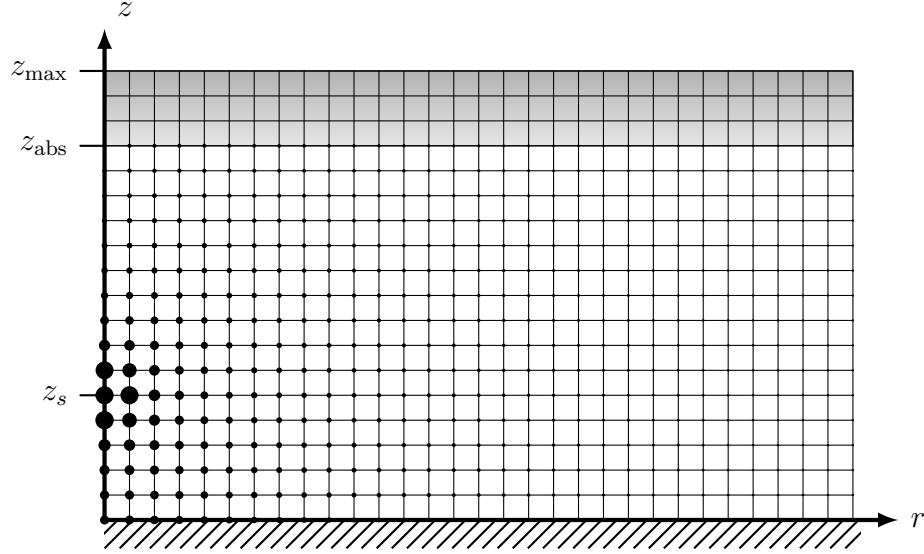


Figure 3: Atmosphere Grid for Numerical Propagation of the Acoustic Source

Figure 4 illustrates these considerations for the PE methods, where the region of interest is where the microphones may be placed for detecting the Unmanned Aerial System (UAS). The region of interest must lie entirely within numerical grid, which is cylindrical for PE methods but may be rectangular for other methods. Importantly, the region of interest must lie entirely within a known region of accuracy for the methods. As discussed later in Section 2.2.3, the accuracy of some methods rely on the method order for initialization. High order initialization allows for a higher maximum elevation angle of validity $\gamma_{\max}$. Depending on the distance of separation between the UAS and the region of interest, the choice in order of starting field that can be used is limited. For vertically offset source and detectors, the elevation angle increases as the range between them decreases. For very short ranges but high vertical displacement, only the higher order methods can be used. Furthermore, the domain must be large enough to accurately capture all significant contributing acoustic phenomenon such as downwards refracting rays. To ensure this, the maximum height of the atmosphere and if applicable the start of the absorption layer must exceed the maximum height achieved by any relevant downwards refracting rays. All of these scenario characteristics are required to be described before a sufficient numerical domain can be defined.

2.1.1.1 Circular Conformal Mapping

Often in real environments, the ground is not level but instead has a profile associated with it. One option for the two dimensional pressure solving methods is to transform the profile into a flat shape through conformal mapping as discussed in [13]. The idea is to represent the ground profile as a series of either circular arcs with a radius of curvature R_c or straight lines like shown in Figure 5.

We will refer to the real domain as (x, y) and the conformal domain as (r, z) . The relevant

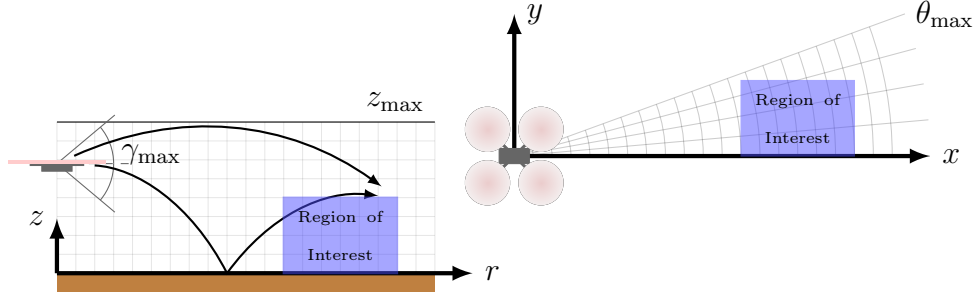


Figure 4: Determining Numerical Domain Boundaries

transforms between the real domain and the conformal domain is given by

$$r = R_c \left(\phi_0 - \arctan \left(\frac{y}{x} \right) \right) \quad (1)$$

$$z = R_c \ln \left(\frac{\sqrt{x^2 + y^2}}{|R_c|} \right) \quad (2)$$

$$x = |R_c| \exp \left(\frac{z}{R_c} \right) \sin \left(-\frac{r}{R_c} + \phi_0 \right) \quad (3)$$

$$y = |R_c| \exp \left(\frac{z}{R_c} \right) \cos \left(-\frac{r}{R_c} + \phi_0 \right) \quad (4)$$

where ϕ_0 is an initial angle on the circle. For convention, it is assumed that a convex ground has $R_c > 0$ while a concave ground has $R_c < 0$. The speed of sound in the conformal domain is also different than in the original domain to account for the spatial stretching of the conformal transformation. The speed of sound is now $cJ^{-1/2}$ where J is the Jacobian of the transformation given by Eq. (5). This new speed of sound artificially adds in upwards or downwards refraction into the conformal domain that may not have been in the original domain.

$$J = \left| \frac{\partial x}{\partial r} \frac{\partial y}{\partial z} - \frac{\partial y}{\partial r} \frac{\partial x}{\partial z} \right| = \exp \left(\frac{2z}{R_c} \right) \quad (5)$$

While the above works for any given circular arc, we need to be able to do it for hills which can be thought of as successive circular arcs, each with their own solution domain. This leads to the conformal mapping being limited to when the problem domain is meant to be solved for some finite maximum height for accuracy of the solution. From the transforms given, the height will be extended for the convex ground and reduced for the concave ground. This means that there is a minimum radius of curvature that is acceptable for a required maximum atmospheric height. As a result, the method is limited to relatively slow changing ground profiles.

Another technical consideration is the starting field for the GFPE solver on each computational domains. By its means of marching, the GFPE solver will solve each domain consecutively and care should be taken to ensure that the final range step solved for in a domain, is at the interface between the domain and its successor domain. It should be noted

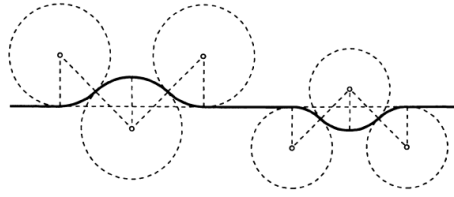


Figure M.2. Approximation of a terrain profile by a chain of circle segments.

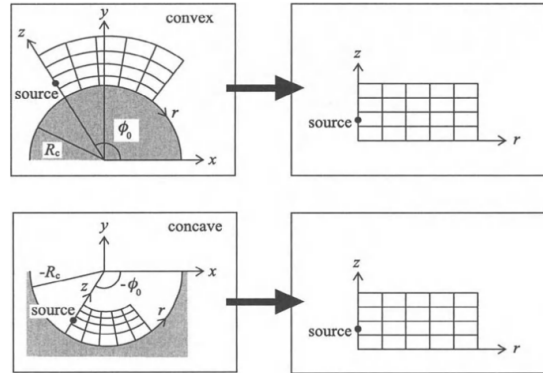


Figure 5: Circular Conformal Mapping Figures M2 and M3 in [13]

that the computational grids between successive domains are never perfectly aligned, so the successor domain starting solution should be interpolated from the previous domain's solution. Additionally, there may be some points for the new domain that may lie outside of the previous problem domain. In this work, we have found that simply assigning the value of those points to be zero gives an accurate solution as these points are often in the absorption layers.

We will present an example of how the domains are established and solved for by considering a hill that is 600 meters wide and has a maximum height of 10 meters. The acoustic source in this example is a monopole emitting an acoustic signal at 500 Hz. The establishment of the problem domains are shown in Figure 6. Notice how at the top of each domain, the domains are ever so slightly misaligned. In particular, the domains following concave ground segments always have portions of their starting solution that must be assigned to be zero. However, assigning these values seems to be acceptable as the solved sound pressure level in Figure 7 does not show any adverse artifacts of this round away from the maximum atmospheric height.

2.1.1.2 Rotating Rectangles

An alternative to attempting to find conformal mappings, is to simply approximate the ground profile as straight lines at angles of inclination and declination. This means that the computational domain consists of smaller, overlapping rectangular domain. This technique has been used previously in [14, 15] and found to give reasonable results with using GFPE solvers. Unlike the circular conformal mapping techniques, there is no worry about maximum

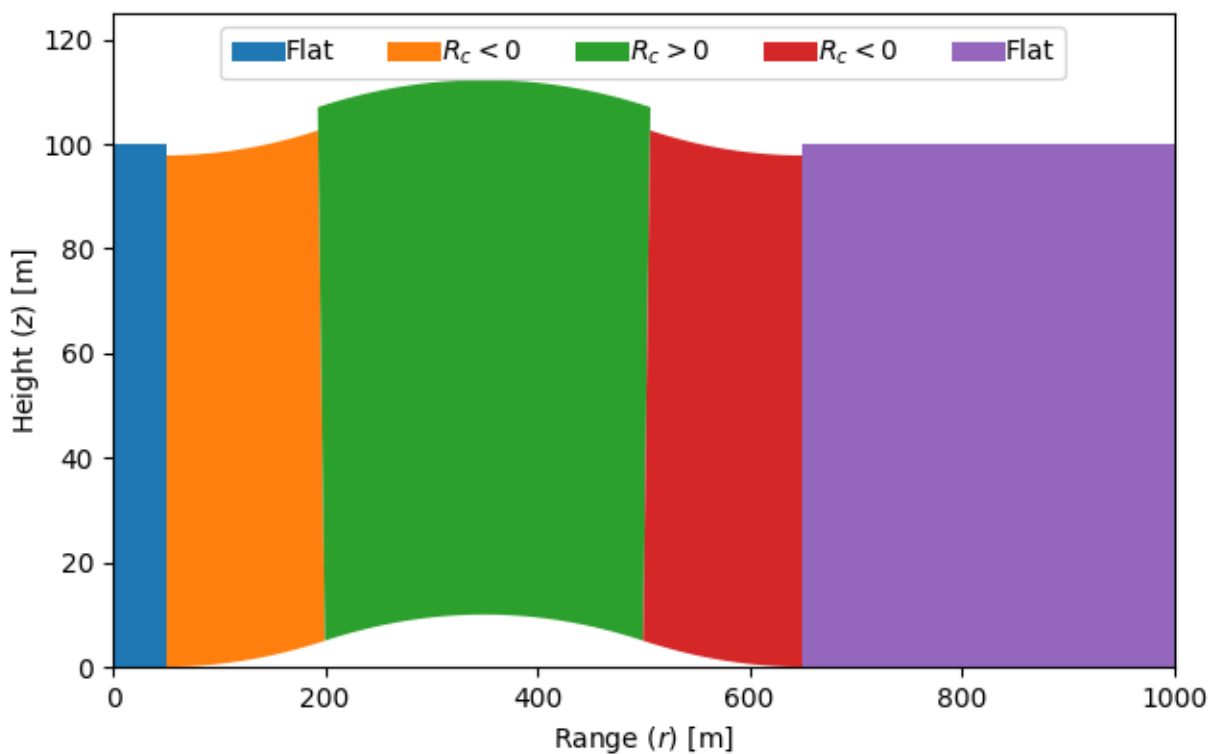


Figure 6: Example Round Hill Solution Domains

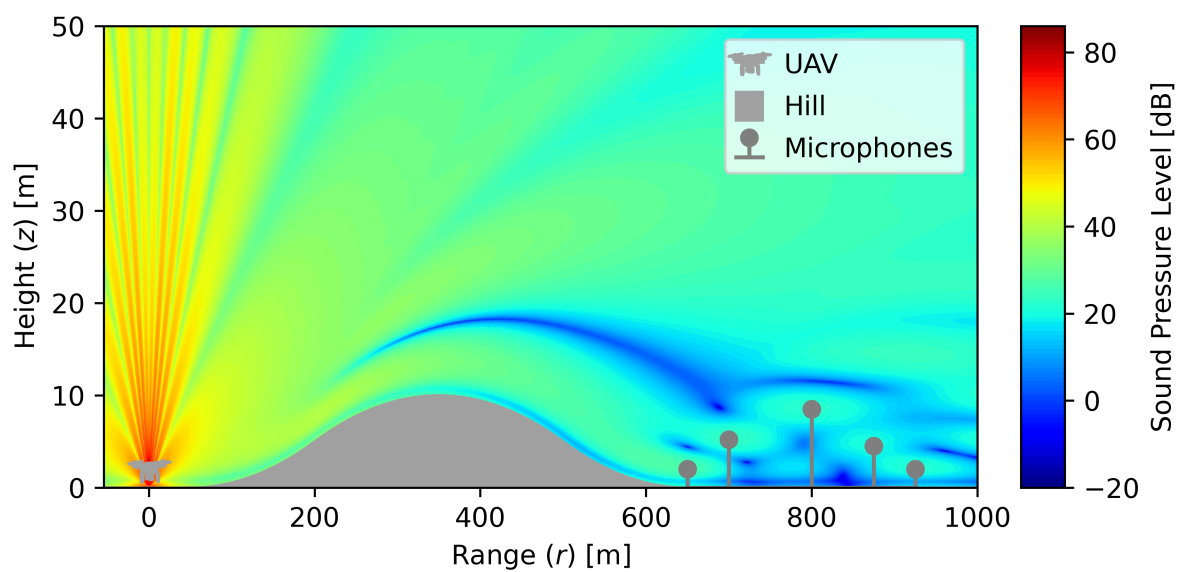


Figure 7: Example Soundfield in Downwards Refracting Atmosphere over a Round Hill

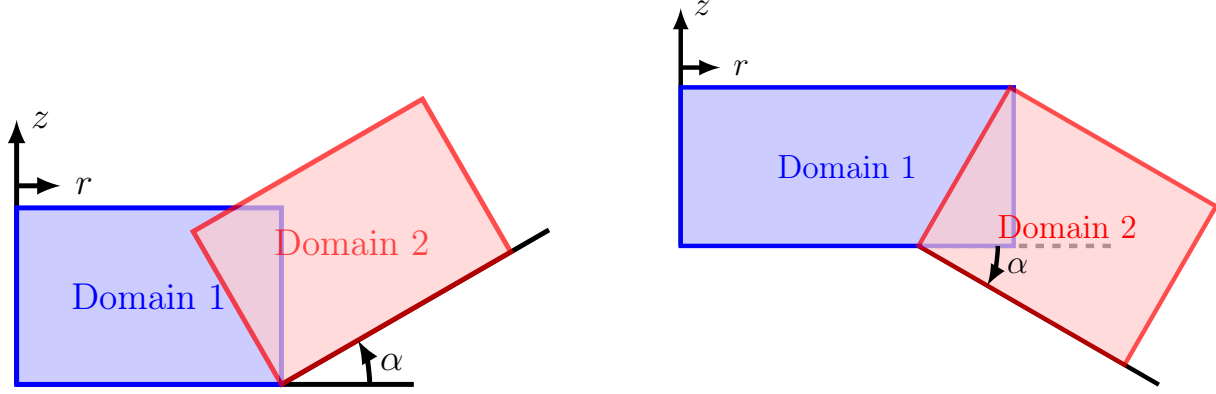


Figure 8: Rotating Rectangular Domains

height required for an accurate solution as the height of each rectangle and the maximum range can be arbitrarily extended. These extensions are necessary to ensure that successor domain starting edges are fully within an already solved domain and can be interpolated. Hence, at the expense of some extra computations, the parabolic equation solvers like GPFE never have to assign zero for starting solutions outside of the previous domain. It has been noted in [14] that this technique is more accurate for smaller angles of inclination of declination. Figures 9 and 10 show the computational domains as well as a solution for the same example fill problem discussed in Section 2.1.1.1. .

2.1.2 Atmosphere Model

The acoustic propagation through a medium depends on the composition and property of the medium. Physical properties are the pressure p , density ρ , and temperature T of the liquid while the composition determines the specific gas constant R and ratio of specific heats γ . We will assume that the atmosphere is an ideal gas (6) and acoustic waves propagate at the adiabatic speed of sound c given in (7). Another simplifying assumption is that the physics of the fluid flow within the atmosphere is dictated by the conservation of mass and momentum given in (8) and (9) respectively. The conservation of momentum uses the total derivative operator $D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$. Since the governing equations (6)-(9) can be used for any fluid and some equations to describe waves in other media, we will use the subscript 'a' to describe atmospheric properties.

$$p = \rho RT \quad (6)$$

$$c^2 \equiv \frac{\partial p}{\partial \rho} = \gamma RT \quad (7)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (8)$$

$$\frac{D(\rho \mathbf{v})}{Dt} = -\nabla p - \rho g \mathbf{e}_z \quad (9)$$

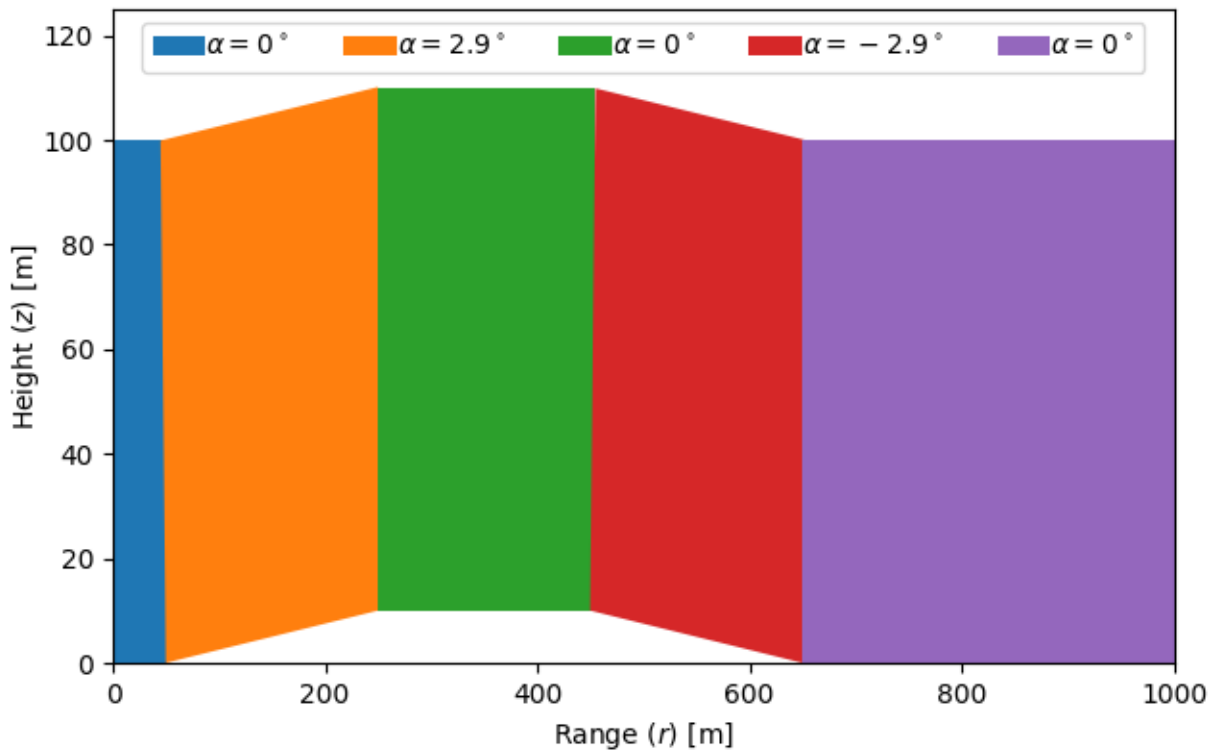


Figure 9: Example Sharp Hill Solution Domains

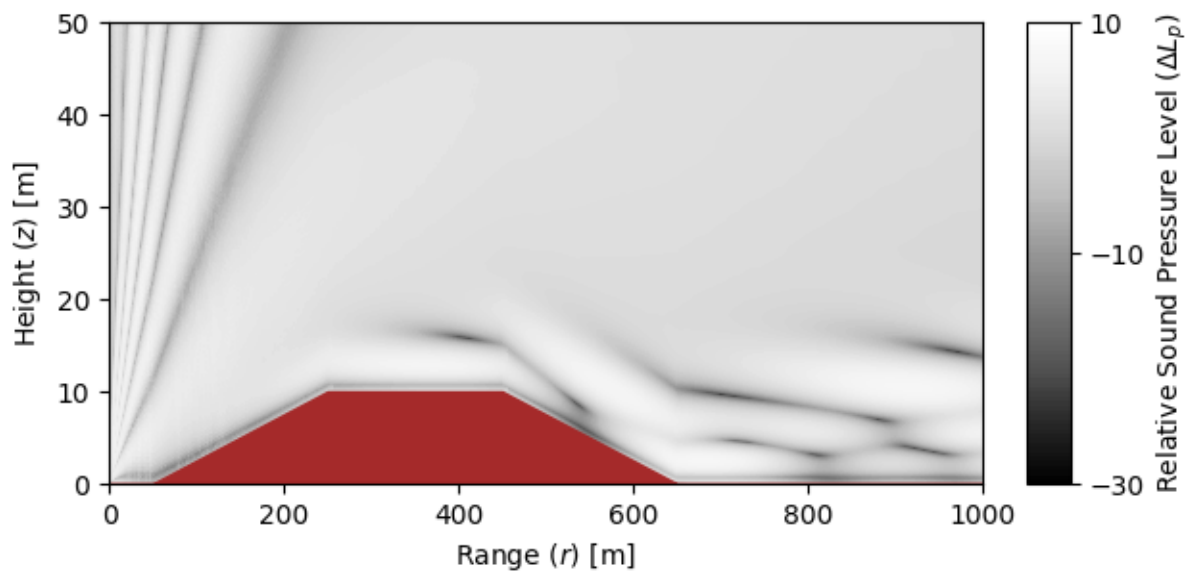


Figure 10: Example Soundfield in Downwards Refracting Atmosphere over a Sharp Hill

While the above governing equations can be used to describe many fluids, we will use it to define an underlying average atmosphere for the acoustic problem. This atmosphere will have a nominal velocity field $\bar{\mathbf{v}}$ where $\bar{(\cdot)}$ indicates the nominal, unperturbed value. The mean velocity field for an atmosphere near the surface can be assumed to be in the shear layer, and thus the nominal vertical velocity is zero. Hence the atmosphere can be modeled as a layered atmosphere where the fluid properties are only height dependent such as notionally represented in Figure 11. Generally the nominal distributions required to be specified for the acoustic modeling are the density, speed of sound, and the horizontal velocity profiles. These distributions are used when discussing the methods outlined in Section 2.2 and solving for the acoustic signal in the atmosphere. One further assumption to note is that while the atmospheric properties are continuous, the layered atmosphere of Figure 11 is a step-wise approximation with the i th layer having properties sub-scripted as such to avoid an excessive amount of parenthesis. Properties at the source and detector levels will be accompanied the s and d subscripts accordingly. For the simplicity of construction, there will always be a layer interface at the source and detector heights. Indicating layer s or layer d simply indicates the layer being the layer immediately below the source or detector, respectively. Additionally, there are times when we consider an specific region of the domain rather than just a single detector. In this case, we do not use d but rather the layer index to refer to properties.

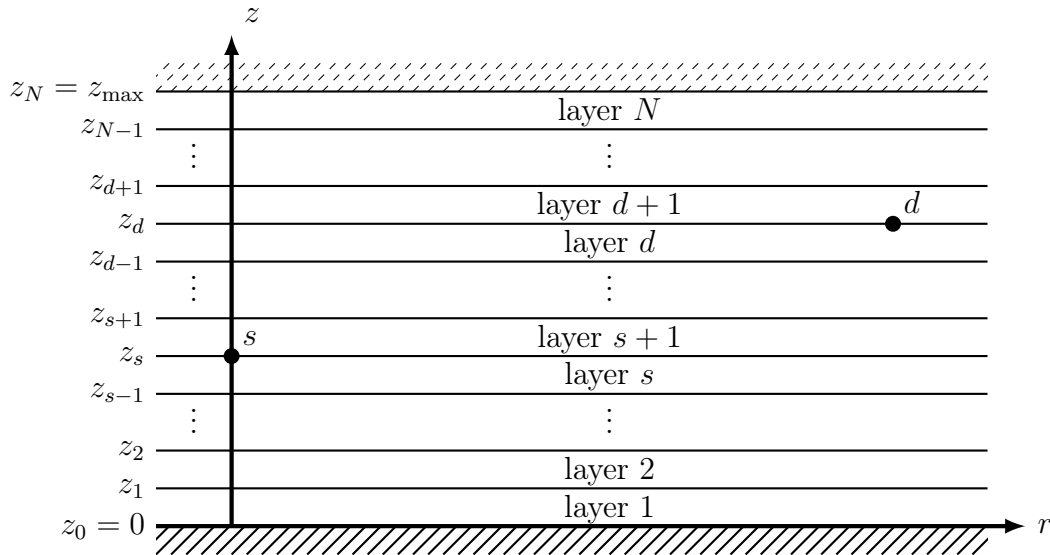


Figure 11: Stratification for a Layered Atmosphere

2.1.3 Atmospheric Profile Modeling

While Section 2.1.2 gave the general equations for the fluid media, one still needs to define the mean wind speed and temperature profiles. There are some profiles which may exist but are not stable, mainly in convection modes. An important parameter is the Obukhov length L , where a stable atmosphere has $L^{-1} > 0$ while an unstable atmosphere has $L^{-1} < 0$. Intuitively, a stable atmosphere has temperature increasing with height, such as in nighttime,

while an unstable atmosphere has temperature decreasing with height, such as in daytime. A neutral atmosphere has $L^{-1} \approx 0$ with temperature slightly decreasing with height, with a drop of one degree Celcius per 100 meters. One can determine L from [13] by

$$L = -\frac{\bar{\theta} u_*^2}{\kappa g \theta_*} \quad (10)$$

$$\theta_* = \frac{-(w'\theta')_s}{u_*} \quad (11)$$

$$u_*^2 = [(u'w')_s^2 + [(v'w')_s^2]^{1/2} \quad (12)$$

where $\theta = T([50 \text{ kPa}]/p)^{(\gamma-1)/\gamma}$ is the potential temperature, κ is Kármán's constant, and u_* is the friction velocity. These equations do require some knowledge about the turbulence strength of the atmosphere and we note $(\cdot)'$ for turbulent quantities different from the nominal and $(\bar{\cdot})$ for average values. The subscript s indicates the eddy flux values evaluated near the ground surface.

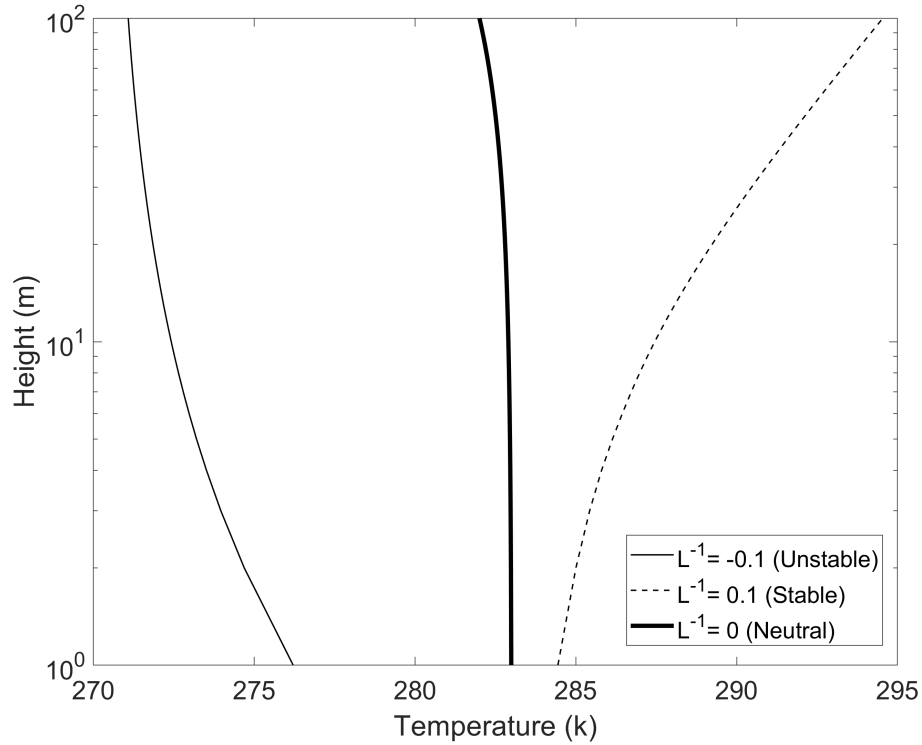


Figure 12: Temperature gradients for variously stable atmospheres.

With the notion of stability, we can estimate various profile using Businger-Dyer relations for stable and unstable atmosphere. One such profile is the temperature profile given in Eq. (13)

$$\bar{\theta}(z) = \theta_0 + \frac{\theta_*}{\kappa} [\ln \frac{z}{z_0} - \Psi_t] \quad (13)$$

where Ψ_t is correction term. The correction term is

$$\Psi_t = 2 \ln \frac{1 + (1 - 16z/L)^{(1/2)}}{2}$$

for unstable atmosphere and

$$\Psi_t = \begin{cases} -5z/L & \text{for } z/L \leq 0.5 \\ -7 \ln(z/L) - \frac{4.25}{z/L} + \frac{0.5}{(z/L)^2} - 0.852 & \text{for } z/L > 0.5 \end{cases}$$

for stable atmospheres. The required parameters θ_* , θ_0 , L , and sometimes z_0 are usually unknown but by having temperature at four different heights, we might be able to obtain them using the temperature profile equation. Once those parameters are estimated, we can obtain the rest of the temperature profile. For $L^{-1} > 0$, this can be more complex because Ψ_t is a piecewise continuous function of z/L . In continuation of profiles, the wind profile using Businger-Dyer relations for unstable and stable atmosphere is

$$\bar{u}(z) = \frac{u_*}{\kappa} \left[\ln \frac{z}{z_0} - \Psi_w \right] \quad (14)$$

with a different correction factor Ψ_w . While $\Psi_w = \Psi_t$ for $L > 0$ in a stable atmosphere, it has a different form in an unstable atmosphere

$$\Psi_w = 2 \ln \frac{1+x}{2} + \ln \frac{1+x^2}{2} - 2 \arctan(x) + \pi/2, \quad x = (1 - 16z/L)^{(1/4)}$$

Again, there are estimation issues as the friction velocity u_* is usually unknown. However, by having the wind velocity at one height, we can obtain it using the wind profile equation above and then estimate the rest of the wind profile.

2.1.4 Acoustics

The acoustic signal is pressure variations from the nominal atmospheric pressure. Neglecting random effects in the atmosphere, the natural perturbation within the medium follows the wave equation in (15).

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad (15)$$

Sources that emit a constant acoustic signal generate a source field that can be represented as

$$\begin{aligned} p &= \text{Re} [p_c \exp(-j\omega t)] \\ \mathbf{v} &= \text{Re} [\mathbf{v}_c \exp(-j\omega t)] \end{aligned}$$

Here, the pressure and velocity have an oscillatory time component with a complex coefficient that is spatially dependent. Since the wave equation must hold true for all time, then the wave equation is true in the complex sense too. We can consider the pressure and velocity as complex variables and drop the subscript c for the remainder of this report.

2.1.4.1 Linear Acoustic Equations

All of the methods presented within this work use linear acoustics to solve the Helmholtz equation. The 3D version in Cartesian coordinates is shown in (16) while the 3D version in cylindrical coordinates is shown in (17). If the source in (17) is located at $[0, 0, z_s]^T$ and the domain is axisymmetric, then we can change the source term on the RHS to $-2\delta(r)\delta(z-z_s)/r$.

$$\nabla^2 p + k_{\text{eff}}^2 p = -4\pi\delta(\mathbf{x}) \quad (16)$$

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} + k_{\text{eff}}^2 p = -\frac{4\pi}{r} \delta(r-r_s) \delta(\theta-\theta_s) \delta(z-z_s) \quad (17)$$

The important characteristic of the Helmholtz equation is the wavenumber k . In a still, lossless atmosphere, the wavenumber is simply $k = \omega/c$ which may exhibit variations throughout the domain. However, with real atmospheres, there are several effects to be incorporated into an effective wavenumber k_{eff} . The incorporations depend on the transforms used to solve the Helmholtz equation; a more indepth discussion on the appropriate effective wavenumber usage is discussed in Section 2.2.1.

2.1.4.2 Wave Impedance

An important notion in modeling of the acoustics is the impedance of the media. The three important parameters defined in (18) are the dimensional impedance ζ , the impedance of air ζ_a , and the normalized impedance Z . The behavior of the waves at the boundaries between different media such as between the atmosphere and the ground are discussed further in Section 2.1.4.3.

$$\zeta \equiv \frac{p}{v}, \quad \zeta_a = \rho_a c_a, \quad Z = \frac{\zeta}{\zeta_a} \quad (18)$$

Generally acoustic models specify the normalized impedance as well as the complex wavenumber within the media as a function of the wavenumber in the atmosphere. For the upper boundary condition of the layered atmosphere, we have the free atmosphere and liquid half-space boundary conditions although only the liquid half-space is defined with an impedance model. The liquid half-space boundary condition assumes an infinitely thick, still, homogeneous media which is assumed to be a continuation of the layered atmosphere with the properties given at z_N . This implies the relation

$$\begin{aligned} Z_t &= 1 \\ k_t &= k(z_N) \end{aligned} \quad (19)$$

For the bottom boundary condition, there are numerous ground models available in the literature. One that will be used in this work is the Delany-Bazley-Chessel model reproduced in 20 as this is the only ground model used in Section 2.3.1. This model depends on a single boundary parameter, the specific flow resistivity σ , as well as the acoustic source frequency f_s . The specific flow resistance parameter is often in the range of 100 to 300 kPa·s·m⁻² for matching experimental data of sound propagation over grasslands.

$$\begin{aligned}
Z_b &= 1 + 0.0511 \left(\frac{\sigma}{f_s} \right)^{0.75} + j0.0768 \left(\frac{\sigma}{f_s} \right)^{0.73} \\
k_b &= \frac{\omega}{c_0} \left[1 + 0.0858 \left(\frac{\sigma}{f_s} \right)^{0.70} + j0.175 \left(\frac{\sigma}{f_s} \right)^{0.59} \right]
\end{aligned} \tag{20}$$

Another common acoustic ground model is the Attenborough 4 parameter model shown in (21) and is used in Section 2.3.2. This model has a couple of atmospheric properties as well as more boundary parameters. The atmospheric properties for the ground model are the density ρ_0 , the ratio of specific heats γ , and the Prandtl number N_{pr} . The ratio of specific heats and Prandtl number are approximately 1.4 and 0.7 for air and constant through the stratified atmosphere. The density can change throughout the atmosphere so the model only considers the density immediately above the ground. The ground parameters for this model are the porosity Ω , pore shape factor ratio s_f , and the tortuosity q . The tortuosity is not an independent quantity but related to the porosity by $q^2 = \Omega^{-g}$ where g is the grain shape factor. Reference [13] lists typical values of 0.5, 0.3, and 0.75 for the grain shape factor, porosity, and pore shape factor ratio, respectively.

$$\begin{aligned}
Z_b &= \left(\frac{4q^2}{3\Omega} + j \frac{s_f^2 \sigma}{\rho_0 \omega_s} \right) \frac{\omega_s / c_0}{k_b} \\
k_b &= \frac{\omega_s}{c_0} \sqrt{\gamma \Omega} \sqrt{\left(\frac{4}{3} - \frac{\gamma - 1}{\gamma} N_{pr} \right) + j \frac{s_f^2 \sigma}{\rho_0 \omega_s}}
\end{aligned} \tag{21}$$

Lastly, two more additional ground impedance models [16] investigated in this work are the Hamet model

$$\omega_\mu = \frac{\sigma \Omega}{q \rho}, \quad F_0 = 1 + j \frac{\omega_\mu q}{N_{pr} \omega}, \quad F_\mu = 1 + j \frac{\omega_\mu}{\omega} \quad Z = \frac{1}{\Omega} \sqrt{\frac{q}{\gamma} F_\mu} \left(1 + \frac{\gamma - 1}{\gamma} F_0^{-1/2} \right) \tag{22}$$

and the Zwikker and Kosten (ZK) model

$$q_c = \frac{c_s}{\Omega} + j \frac{\sigma}{\rho \omega}, \quad Z = \sqrt{\frac{q_c}{\Omega}} \tag{23}$$

which uses a material structural constant $c_s \approx 5$. A comparison of these models can be seen in Fig. 13. Notice that the imaginary component of the impedance of all these models tends towards zero the frequency increases while the real component converges to a non-zero value.

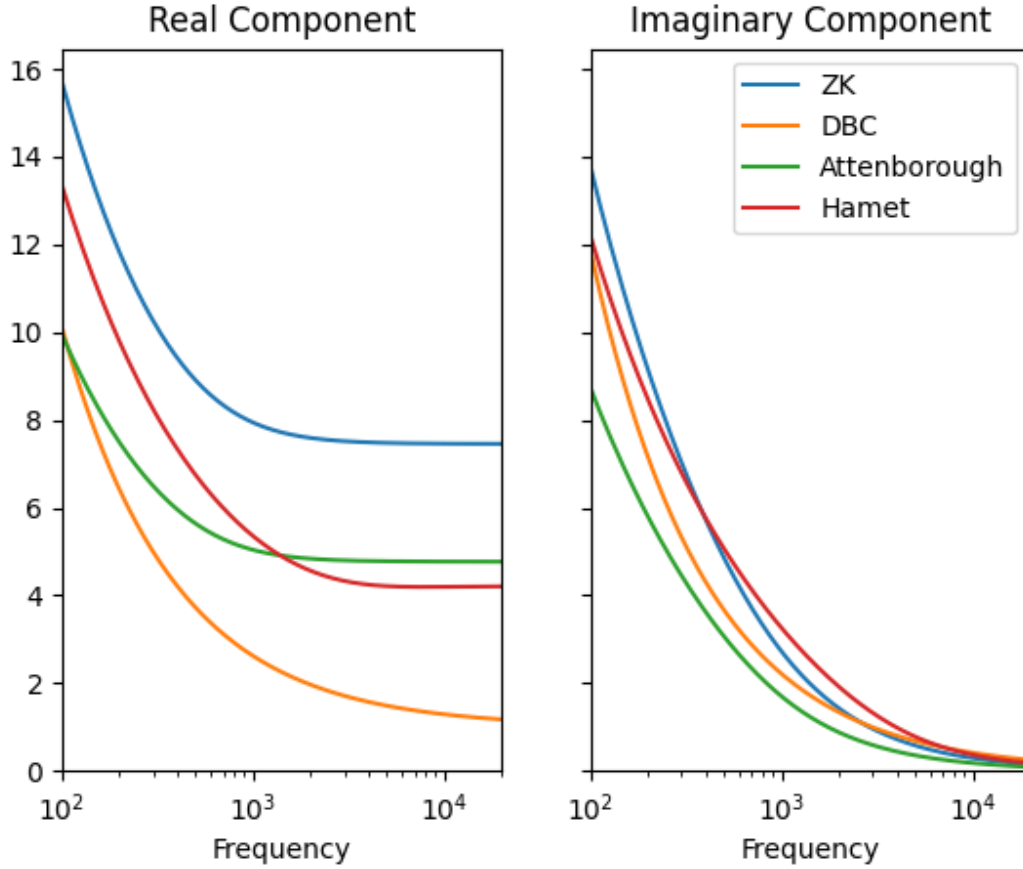


Figure 13: Normalized Impedance over Frequency for $\sigma = 100 \text{ kPa}\cdot\text{s}\cdot\text{m}^{-2}$

2.1.4.3 Reflections

While the previous section has handled the modeling of the impedance of the boundaries from a material standpoint, this section will show the modeling from a physical standpoint. The general setup of the wave propagation is shown in Figure 14 where an incoming wave is reflected off a ground surface. While we show the ground boundary condition, one can easily imagine the setup vertically flipped for reflections on the top of the atmosphere. Throughout this section, we will only discuss the bottom or ground boundary layer quantities such as the wavenumber k_b by the subscript b . All equations are valid for the top layer boundary by using the subscript t in place of b in the subsequent equations and Figure 14 reflected across the horizontal axis.

The 2D plane wave can be decomposed into its radial and vertical components with the wave arriving with an angle θ_i before being reflected at an angle θ_r and transmitted through the air-ground interface at an angle θ_b . Using the manipulations from [13], one can show that boundary conditions

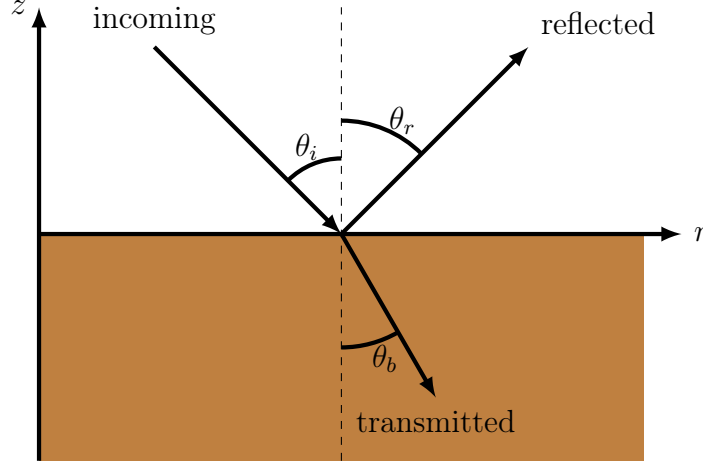


Figure 14: Acoustic Reflections at a Ground Interface

$$Z_{b,n} = \frac{Z_b}{\sqrt{1 - (k_r/k_b)^2}} \quad (24)$$

$$R_b = \frac{Z_{b,n}k_z - k}{Z_{b,n}k_z + k} \quad (25)$$

where the pressure wave in the layer just above the ground in the atmosphere is determined to be

$$\begin{aligned} p(r, z) &= A \exp(jk_r r + jk_z z) + B \exp(jk_r r - jk_z z) \\ &= A [\exp(jk_r r + jk_z z) + R_b \exp(jk_r r - jk_z z)] \end{aligned}$$

2.2 Solver Methods

2.2.1 Pressure Transforms

Rather than solving the pressure directly in Cartesian coordinates, the nature of the problem is often easier to handle in a transformed space. The two common transforms to handle this are the Fourier Transform as well as the Hankel transform. In both, we refer to the original variables as lower case (p) while the transformed variables are upper case (P). Since both transforms used in Section 2.2.2 are only applied to the horizontal plane, we have the following relationship between the pressure and particle velocity as follows in 26.

$$W = \frac{j}{\omega \rho} \frac{dP}{dz} \quad (26)$$

2.2.1.1 Spatial Fourier Transforms

One option to handle the domain is to use a Fourier transform which is a common transform for acoustic waves. For reference we show the transform and inverse transform in (27) and (28) respectively.

$$P(k_x, k_y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x, y, z) \exp(-jk_x x - jk_y y) dx dy \quad (27)$$

$$p(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(k_x, k_y, z) \exp(jk_x x + jk_y y) dk_x dk_y \quad (28)$$

The corresponding Helmholtz equation under some mild simplifications given in [13] can be shown as

$$k_{\text{eff}}^2 \frac{\partial}{\partial z} \left(k_{\text{eff}}^{-2} \frac{\partial P}{\partial z} \right) + k_z^2 P = -4\pi\delta(z - z_s) \quad (29)$$

where

$$\begin{aligned} k_{\text{eff}} &= k - k_x \frac{\bar{u}_a}{c} - k_y \frac{\bar{v}_a}{c} \\ k_z^2 &= k_{\text{eff}}^2 - k_x^2 - k_y^2 \\ k_{\text{eff}}^2 \frac{\partial}{\partial z} \left(k_{\text{eff}}^{-2} \frac{\partial P}{\partial z} \right) &= \frac{\partial^2 P}{\partial z^2} \quad \text{within layers} \end{aligned}$$

Note that in this model, there is only nominal horizontal wind effects with no attenuation terms present. This leaves something to be desired as waves naturally attenuate traveling through air but other works use a different transform with an integrated attenuation effect as discussed in the next section.

2.2.1.2 Hankel Transforms

The Hankel transform is used for the 2D case or when the problem is axisymmetric. While the Hankel transform can be of various orders, we specifically used the zeroth order for both the transform (30) and inverse transform (31). Occasionally this transform may be referred to as the Fourier-Bessel transform in literature.

$$P(k_r, z) = \int_0^{\infty} p(r, z) J_0(k_r r) r dr \quad (30)$$

$$p(r, z) = \int_0^{\infty} P(k_r, z) J_0(k_r r) k_r dk_r \quad (31)$$

The transforms use the zeroth order Bessel function of the first kind which satisfies the differential equation (32) and is sometimes defined as (33). It is both the differential equation and the alternative definition that allows for one to show the equivalence between the Fourier transform in 2.2.1.1.

$$x^2 \frac{d^2 J_0}{dx^2} + x \frac{dJ_0}{dx} + x^2 J_0(x) = 0 \quad (32)$$

$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} \exp(-jx \cos(\theta - \phi)) d\theta \quad (33)$$

First we show the equivalence of the Fourier transform to the Hankel transform in an axisymmetric domain. If the domain is axisymmetric then $p(r, z)$ with $(x, y) = (r \cos(\theta), r \sin(\theta))$ and similarly $P(k_r, z)$ with $(k_x, k_y) = (k_r \cos(\phi), k_r \sin(\phi))$. If we apply these substitutions to the Fourier transform (27) we see that

$$\begin{aligned} P(k_r, z) &= \int_0^\infty \int_0^{2\pi} p(r, z) \exp(jk_r r [\cos(\theta) \cos(\phi) + \sin(\theta) \sin(\phi)]) r d\theta dr \\ &= \int_0^\infty p(r, z) r \int_0^{2\pi} \exp(jk_r r \cos(\theta - \phi)) d\theta dr \\ &= 2\pi \int_0^\infty p(r, z) J_0(k_r r) r dr \end{aligned}$$

The difference between what was just shown above and the given Hankel transform in (30) is a factor of 2π . This factor is a result of the source term in (16) being -4π rather than -2 in the axisymmetric, cylindrical coordinate system (17). As another check, we can take the immediate result above with inverse Fourier transform (28) and similar logical steps to arrive at (31). If we examine the cylindrical Helmholtz with an axisymmetric domain, we can simplify the equation to

$$\begin{aligned} \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \underbrace{\frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2}}_0 + \frac{\partial^2 p}{\partial z^2} + k_{\text{eff}}^2 p &= \underbrace{-\frac{4\pi}{r} \delta(r - r_s) \delta(\theta - \theta_s) \delta(z - z_s)}_{-\frac{2}{r} \delta(r) \delta(z - z_s)} \\ \int_0^\infty P k_r^3 \left[\overbrace{\left[\frac{d^2 J_0}{d(k_r r)^2} + \frac{1}{k_r r} \frac{dJ_0}{d(k_r r)} \right]}^{-J_0(k_r r)} + \left[\frac{\partial^2 P}{\partial z^2} + k_{\text{eff}}^2 P \right] J_0(k_r r) k_r dk_r \right. &= -\frac{2}{r} \delta(r) \delta(z - z_s) \\ \left. \int_0^\infty \left[\frac{\partial^2 P}{\partial z^2} + (k_{\text{eff}}^2 - k_r^2) P \right] J_0(k_r r) k_r dk_r \right. &= -\frac{2}{r} \delta(r) \delta(z - z_s) \\ \frac{\partial^2 P}{\partial z^2} + (k_{\text{eff}}^2 - k_r^2) P &= -\int_0^\infty \frac{2}{r} \delta(r) \delta(z - z_s) J_0(k_r r) r dr \\ \frac{\partial^2 P}{\partial z^2} + (k_{\text{eff}}^2 - k_r^2) P &= -2\delta(z - z_s) \end{aligned}$$

Here we can see that the Hankel transform is a special set of assumptions for the Fourier transform. However in literature, there is a different formulation of the vertical number to account for the attenuation effects so we say that the Hankel transformed Helmholtz equation under symmetry is (34):

$$\frac{\partial^2 P}{\partial z^2} + (k_{\text{eff}}^2 - j2\mu k_{\text{eff}} - k_r^2) P = -2\delta(z - z_s) \quad (34)$$

where

$$k_{\text{eff}} = \frac{1}{c + \bar{u}_a}$$

$$\mu > 0$$

In the above, inclusion of the term with the positive, real attenuation coefficient μ adds in an imaginary component for the vertical wavenumber. Literature using the Fourier transform does not have a direct term incorporation of attenuation although there is a scaling method in [13] which accomplishes the equivalent. Additionally the effective wavenumber has changed to include a component of the forward wind component under assumptions that the wind is taken along the radial distance component only, with minimal cross-range wind.

There is also an asymptotic form to this transform, which depends on the Hankel functions of first and second kind representing the incoming and outgoing waves.

$$J_0(k_r r) = \frac{1}{2} \left[\overbrace{H_0^{(1)}(k_r r)}^{\text{incoming}} + \overbrace{H_0^{(2)}(k_r r)}^{\text{outgoing}} \right]$$

Neglecting the incoming wave and utilizing a first order asymptotic approximation, we can approximate the inverse transform of the Hankel transform in the farfield.

$$H_0^{(2)}(k_r r) \sim \sqrt{\frac{2}{\pi k_r r}} \exp \left(-jk_r r + j\frac{\pi}{4} \right), \quad k_r r \rightarrow \infty$$

$$p(r, z) \sim \frac{1+j}{2\sqrt{\pi r}} \int_0^\infty P(k_r, z) \exp(-jk_r r) \sqrt{k_r} dk_r, \quad k_r r \rightarrow \infty \quad (35)$$

2.2.2 Fast Field Program

Algorithm 1: 2D FFP

- Data:** $z_i, \rho_i, c_i, Z_b, Z_t, f_s, z_s, z_d$
- 1 Determine $k_{r,\text{max}}$ for numerical integration
 - 2 Discretize $k_{r,i}, r_i$ using rules
 - 3 Calculate transformed pressure

$$P_i(k_r) = f(k_r, z_s, z_d, f_s; z_i, \rho_i, c_i, Z_{b,t})$$

- 4 Perform inverse transform

$$p(r; z_d) = F(k_{r,i}, r_i, P_i)$$

The Fast Field Program (FFP) algorithm used for this project is notionally defined in Algorithm 1 for the 2D case. The 3D case is the same algorithm except for k_x and k_y are used instead of k_r where appropriate. The main idea behind this method is to transform the problem to a simpler one using the transforms discussed in 2.2.1. This simpler problem then

represent the pressure and the particle vertical velocity as purely upward and downward moving waves as shown in (36).

$$\begin{aligned} P_i(z_i) &= A_i \exp(jk_{z,i}z_i) + B_i \exp(-jk_{z,i}z_i) \\ W_i(z_i) &= -\frac{k_{z,i}}{\omega\rho_i}A_i \exp(jk_{z,i}z_i) + \frac{k_{z,i}}{\omega\rho_i}B_i \exp(-jk_{z,i}z_i) \end{aligned} \quad (36)$$

Since the atmosphere properties are constant within the layers, the waves pressure and vertical velocity are oscillatory functions of the height above the lower layer interface for a constant vertical wavenumber. For example, pressure is shown to be

$$\begin{aligned} P_i(z_i + \Delta z_i) &= \cos(k_{z,i}\Delta z_i) P_i(z_i) + j \sin(k_{z,i}\Delta z_i) \{A_i \exp(jk_{z,i}z_i) - B_i \exp(-jk_{z,i}z_i)\} \\ &= \cos(k_{z,i}\Delta z_i) P_i(z_i) - \frac{j\omega\rho_i}{k_{z,i}} \sin(k_{z,i}\Delta z_i) W_i(z_i) \end{aligned}$$

A similar result occurs if the same is done for the particle vertical velocity. Composing the pressure and vertical velocity into a vector, we show that we can use a layer transfer matrix d_i to determine wave attributes at the layer interfaces. The transfer matrix for moving upwards through the atmosphere and its inverse for moving downwards is shown in (37).

$$\begin{aligned} \begin{bmatrix} P_i \\ W_i \end{bmatrix} (z_i + \Delta z_i) &= \overbrace{\begin{bmatrix} \cos(k_{z,i}\Delta z_i) & -\frac{j\omega\rho_i}{k_{z,i}} \sin(k_{z,i}\Delta z_i) \\ -\frac{j k_{z,i}}{\omega\rho_i} \sin(k_{z,i}\Delta z_i) & \cos(k_{z,i}\Delta z_i) \end{bmatrix}}^{d_i} \begin{bmatrix} P_i \\ W_i \end{bmatrix} (z_i) \\ \begin{bmatrix} P_i \\ W_i \end{bmatrix} (z_i) &= \underbrace{\begin{bmatrix} \cos(k_{z,i}\Delta z_i) & \frac{j k_{z,i}}{\omega\rho_i} \sin(k_{z,i}\Delta z_i) \\ \frac{j\omega\rho_i}{k_{z,i}} \sin(k_{z,i}\Delta z_i) & \cos(k_{z,i}\Delta z_i) \end{bmatrix}}_{d_i^{-1}} \begin{bmatrix} P_i \\ W_i \end{bmatrix} (z_i + \Delta z_i) \end{aligned} \quad (37)$$

Some other literature has used a different transfer matrix, such as [13] who uses the assumption that the density variation in layers can be neglected. Instead, the vertical derivative of the transformed pressure is used which gives an alternative transform matrix shown in (38).

$$\begin{bmatrix} P_i \\ dP_i/dz \end{bmatrix} (z_i + \Delta z_i) = \overbrace{\begin{bmatrix} \cos(k_{z,i}\Delta z_i) & k_{z,i}^{-1} \sin(k_{z,i}\Delta z_i) \\ -k_{z,i} \sin(k_{z,i}\Delta z_i) & \cos(k_{z,i}\Delta z_i) \end{bmatrix}}^{d_i} \begin{bmatrix} P_i \\ dP_i/dz \end{bmatrix} (z_i) \quad (38)$$

Given the layer transfer matrices of the atmospheres, one needs to select appropriate boundary conditions for top and bottom of the atmosphere. The bottom of the atmosphere occurs at the interface with the ground, so the required boundary conditions is obtained from the normal impedance equation (24) for a selected ground impedance model. The upper atmosphere boundary condition is a little more nuanced since the impedance depends

on the choice between a liquid halfspace given by Eq. (19) or a free surface where $Z_t = 0$. The choice of a liquid halfspace gives the vertical impedance of

$$Z_{t,n}(k_r) = \begin{cases} \left| \frac{1}{\sqrt{1-(k_r/k_t)^2}} \right| & k_r < k_t \\ j \left| \frac{1}{\sqrt{1-(k_r/k_t)^2}} \right| & k_r > k_t \end{cases} \quad (39)$$

whereas the free surface has $Z_{t,n} = 0$. Either choice has a corresponding reflection coefficient given by Eq. (25) where t is substituted for b in the equation.

2.2.2.1 Determining Transform

The two transforms were discussed in Section 2.2.1. In the presence of wind in the cross-range direction, the literature indicates that there are azimuthal variations in the detector pressure magnitude. These effects require the Fourier transform which is a numerically larger task as it will require two axes of integration rather than one. Alternatively, if one only cares about the downrange direction with minimal cross-range wind or if there is minimal wind, then the Hankel transform is a more efficient solution. In addition, only the Hankel transform has consideration for attenuation within the atmosphere in the literature. It is unclear at this time as to whether the attenuation can easily be incorporated into the Fourier transform for additional accuracy.

2.2.2.2 Variable Discretization

With the transform selected, the wavenumber(s) of integration need to be discretized. The simplest idea is by assignment which is used in some work such as [13] and [17]. The suggestion in [17] is to have the maximum integration wavenumber to be roughly 3 times the critical wavenumber. An alternative approach is to find a reasonable bounds based on the attenuation of the sound through the layered atmosphere. As introduced in [18], the Algorithm 2 uses a specified tolerance to find the cutoff or maximum wavenumber $k_{r,\max}$ although [18] shows that there are some issues with the cutoff being too close to the singularity to accurately evaluate the inverse transform.

2.2.2.3 Solving Transformed Pressure

There are two methods investigated for solving the transformed Helmholtz equation, the Direct Matrix Method and the Thompson-Haskell Matrix Method. The Direct Matrix Method is from [18] although we have derived the equations in Appendix A as they are not derived in [18]. The advantage of the Direct Matrix Method is that the pressure has already been analytically solved.

However, in the formulation presented in [18], z_0 is defined as the top of the atmosphere and the formulation is limited to the case where the detector is above the source. In the present work, we expand the method to include the case of the detector located below the source. For that purpose, we show that the solutions for layers both above and below the source is

Algorithm 2: Bisection Method

Data: A_{tol} , $k_{z,i}$, Δz_i , $\mu_{\max}$, $c_{\max}$, ω_s

- 1 Have an assigned tolerance to the attenuation, typically $A_{tol} = 10^{-3}$, where the total attenuation throughout the atmosphere is

$$A(k) = \prod_{i=1}^N \exp(-\Delta z_i \operatorname{Im} k_{z,i})$$

- 2 Bound the maximum wave number using k_a and k_b

$$\begin{aligned} \bar{k}_z &= -\ln(A_{tol}) / \left(\sum_{i=1}^N \Delta z_i \right) \\ k_a &= \left\{ \bar{k}_z^2 + (\omega_s / c_{\max})^2 [1 - (\mu_{\max} / \bar{k}_z)^2] - \mu_{\max}^2 \right\}^{1/2} \\ k_b &= [\bar{k}_z^2 + (\omega_s / c_{\min})^2]^{1/2} \end{aligned}$$

- 3 Perform a bisection search to find $k_{r,\max} \in [k_a, k_b]$.
-

$$\begin{aligned} P_i(z_i \leq z_s) &= \frac{-2j}{\omega \rho_s} \cdot \frac{[\zeta(z_0)(D_d)_{11} + (D_d)_{12}][(D)_{12} - \zeta(z_N)(D)_{22}]}{\{\zeta(z_0)(D)_{11} + (D)_{12}\} - \zeta(z_N)\{\zeta(z_0)(D)_{21} + (D)_{22}\}} \\ P_i(z_i > z_s) &= \frac{2j}{\omega \rho_s} \cdot [\zeta(z_N)(D_d)_{11} + (D_d)_{12}] \\ &\quad \times \frac{[\{\zeta(z_0)(D)_{21} + (D)_{22}\}(D_s)_{12} - \{\zeta(z_0)(D)_{11} + (D)_{12}\}(D_s)_{22}]}{\{\zeta(z_0)(D)_{11} + (D)_{12}\} - \zeta(z_N)\{\zeta(z_0)(D)_{21} + (D)_{22}\}} \end{aligned} \quad (40)$$

where

$(\cdot)_{ij}$ indicates element of matrix

$$D = d_N d_{N-1} \cdots d_1$$

$$D_s = d_N d_{N-1} \cdots d_s$$

$$D_d = \begin{cases} d_d d_{d-1} \cdots d_1 & \text{if } z_i \leq z_s \\ d_{d+1}^{-1} d_{d+2}^{-1} \cdots d_N^{-1} & \text{otherwise} \end{cases}$$

From here we can see the advantage of this method that only a single pass through the layered atmosphere is required to calculate the transfer matrices. However, we have sometimes found that this method has numerical issues. The full atmosphere transfer matrix D often encounters overflow numerical errors when $k_r \approx k_{\text{eff}}$ i.e. around the singularities. An alternative is the Thompson-Haskell-Matrix Method as used in [13]. The method is shown in Algorithm 3 and involves marching from the boundaries in towards the source.

The advantage of this two step method is a higher degree of numerical stability. One of the disadvantages of the Direct Matrix method is the matrix D in the solution. The individual transfer matrices have a singularity as the vertical wavenumber approaches zero,

Algorithm 3: Thompson-Haskell-Matrix Method

Data: $\omega, k_{z,i}, \rho_i, \Delta z_i, Z_{n,b}, Z_{n,t}$

- 1 Use boundary conditions to initialize the boundaries. Note the $\pm$ is for matching radiation requirement and $A_{N+1} = B_0 = 1$ is arbitrary.

$$A_{N+1}, B_0 \geq 0, A_0 = B_{N+1} = 0, \quad (\text{Radiation requirement})$$

$$P_{1/N} = R(k_{z,1/N}) + 1, P'_{1/N} = \pm j k_{z,1/N} [R(k_{z,1/N}) - 1]$$

- 2 Propagate the boundary conditions using the transfer matrix until the source layer using (37). For numerical stability, potentially rescale intermediate values.
- 3 Solve the mismatch from just above (+) and below (−) source layer to satisfy the appropriate Hemholtz equation, either (16) or (17).

$$P_s = \frac{-S_\delta}{(W_s/P_s)^+ - (W_s/P_s)^-}, \quad S_\delta \text{ is source term being } 4\pi \text{ in 3D and } 2 \text{ in 2D.}$$

- 4 Rescale the pressure layers above and below the source by $P_s/P_s^\pm$
-

i.e. k_r approaches k_{eff} , which often leads to overflows during numerical calculations. The effect of this singularity can be minimized in the Thompson-Haskell-Matrix Method since the boundary values are arbitrary assigned. During the march towards the source layer, if the vector norm of the wave pressure and velocity as you would see in (37) exceeds a value, all layers from the current interface to the boundary can be rescaled by a common factor so that the current interface has a unit vector norm. This is a valid approach as a scalar term does not affect the validity of (37) and (38).

2.2.2.4 Numeric Considerations

We have already discussed numerical stability within calculating the transformed pressure throughout the atmosphere with the usage of the Thompson-Haskell-Matrix Method. However, in addition to checking the vector norm of the wave, we can also introduce a scaling factor as suggested in [13] when k_z is complex. This scaling factor is $\exp(jk_{z,0}\Delta z_i)$ for upwards marching and $\exp(-jk_{z,0}\Delta z_i)$ given a reference atmosphere vertical wavenumber $k_{z,0}$. This approach is similar to the suggestion in Section 2.2.2.3 although the previous layers are not additionally rescaled.

The previous suggestions do not solve all of the numerical issues due to existence of singularities in the transfer matrices. These singularities exist whenever k_z is zero, leading to divisions by zero in (37) and (38). For stability, it is advisable to move the points within the grid or line of integration away from singularities. There are various ways to do this such as introducing an artificial attenuation and scaling the inverse transformed pressure appropriately [18] or moving points of integration slightly away from the singularity [13].

Algorithm 4: 2D GFPE

Data: $z_i, r_j, \rho_{ij}, c_{ij}, Z_{b,j}, f_s, z_s$

- 1 Determine atmosphere and range property discretization
- 2 Initialize the starting field $\psi(0, z)$
- 3 **for** $r = 0, \Delta r, 2\Delta r, \dots, r_{\max} - \Delta r$ **do**
- 4 Update range dependent parameters.
- 5 Solve Fourier Transformed Field $\Psi(r, k_r)$.
- 6 Extrapolate field $\psi(r + \Delta r, z)$ using inverse Fourier Transform.
- 7 **end**

2.2.3 Green's Function Parabolic Equation

While the previous method of FFP in Section 2.2.2 focused on the vertical waves, parabolic equation methods are centered around marching the wave front in the range direction. Under assumptions that the acoustic waves far from the source have a slow variation in range direction, the pressure can be rewritten in terms of a complex field variable ψ . The field variable is the pressure corrected for the cylindrical spreading and a constant reference radial wavenumber for downrange propagation. This is shown by (41)

$$\psi(r, z) = pr^{1/2} \exp(-jk_r r) \quad (41)$$

where k_r is the radial reference wavenumber. This work considers the wavenumber just above the ground to be the default reference wavenumber unless otherwise stated. Under this change of variables and an assumption that $\frac{\partial^2 \psi}{\partial r^2} \approx 0$, the governing Cylindrical Helmholtz equation (17) becomes the Narrow Angle Parabolic Equation 42 as derived in [19, 13]. The curvature of the field $\frac{\partial^2 \psi}{\partial r^2} \approx 0$ is consistent for acoustic waves in the far field resembling planar waves as well as reducing the partial differential equation from a second order to first order equation in range.

$$\frac{\partial \psi}{\partial r} = \frac{j}{2k_r} \left(\frac{\partial^2}{\partial z^2} + (k_{\text{eff}}^2(z) - k_r^2) \right) \psi \quad (42)$$

2.2.3.1 Determining Grid

The Green's Function Parabolic Equation algorithm, as shown in Algorithm 4, is used to propagate the field variable downrange of the acoustic source while satisfying the governing PE. Before the marching of the field variable, the numerical grid for GFPE relies on some necessary limitations for grid step sizes. Unlike the previously discussed FFP algorithm in Section 2.2.2, GFPE bases its discretization on an average wavelength λ of the acoustic source. The general criteria from [13, 19] is that the vertical grid step should satisfy $\Delta z \leq \lambda/10$ and that the maximum range step that can be supported is $\Delta r_{\max} \approx 5\lambda$. While the vertical grid step criteria agrees with the Crank-Nicholson Parabolic Equation (CNPE) vertical grid step, GFPE supports a much larger range step than CNPE. With the numerical domain predetermined from operation considerations discussed in Section 2.1.1 and the grid steps determined in range and height, we can establish the numerical grid in the Fourier Domain.

With an atmosphere that has a maximum height of $z_{\max} = M\Delta z$, we use $N = 2M$ points with $z_i = i\Delta z$ for $i = 0, \dots, N - 1$. Heights with an index $i > M$ represent the negative heights for the Fourier Transform [19] rather than intuitive positive height. There are two reasons for doing this, to ensure periodicity and allow for numerical speedups by means of a Fast Fourier Transform. The negative heights will be identically zero for across all range values as to not contribute any acoustic transmissions between ground and the top of the atmosphere through the negative heights. The zeros do ensure that the source field is periodic along the height axis, allowing for the validity of the Fourier Transform. It should also be noted here that as the ground reflects the traveling wave, we not only have to consider Fourier Transforms associated with $+k_n$ but also $-k_n$ for the reflection. Without using the negative heights, this would have required two transforms. However, this can be reduced to one transform as the vertical wavenumber for Fourier Transforms is $k_n = n\Delta k$ with $n = 0, 1, \dots, N/2, -N/2+1, \dots, -1$. This ordering is convenient as it allows for finding the Fourier Transform of the negative wavenumbers simply by reordering the results of the positive wavenumber Fourier Transform. Finally, by enforcing the relationship $\Delta z \Delta k = 2\pi/N$, discrete first order approximations of continuous Fourier Transform integrals can be replaced with the Fast Fourier Transforms for computational speed.

Another concern with the Fourier Transforms is the reflection at the maximum height of the atmosphere downwards. Physically this does not happen as the atmosphere is assumed to be infinitely high for acoustic problems but numerically it must be truncated. To overcome this, a portion of the top of the atmosphere has artificial attenuation added to it to remove the vast majority of acoustical energy that may be reflected. An imaginary term of α is added to the effective wavenumbers for all grid points within the desired absorption layer, with heights ranging from z_{abs} to $z_{\max}$. Throughout this work, the absorption layer model used is (43) where A is a frequency dependent coefficient interpolated from Table 1. This technique of adding in α and the numerical model was found in [19] and appears common in practice as a slightly different interpolation table appears in [13]. The depth of the absorption layer is recommended to be 50λ from [13], although larger absorption layers may remove more artificial oscillations at the expense of reducing the region of validity for GFPE.

$$\alpha(z) = A \left(\frac{z - z_{\text{abs}}}{z_{\max} - z_{\text{abs}}} \right)^2 \quad (43)$$

Table 1: Artificial Attenuation Coefficient from [19]

f [Hz]	≤ 16	31.5	63	125	250	500	$\geq 1,000$
A [m^{-1}]	0.1	0.2	0.3	0.4	0.5	0.5	1.0

2.2.3.2 Approximate Starting Field

With the computational grid determined, we can propagate the acoustic field across the numerical domain given some initial or starting source field. While the complex pressure at $r = 0$ is non-zero due to the source, a naive substitution into (41) will not work as then the field will be identically zero along the height axis. This will not work as the starting field

since the GFPE propagation step will only propagate zero across the range values. Instead, we need an approximation of the field to allow for GFPE to propagate meaningful values. First we consider only the vertical acoustic field in (44).

$$\psi(0, z \leq z_{\max}) = \psi_0(z - z_s) + \frac{\overbrace{Z_b - 1}^{R(k_{\text{eff},0})}}{Z_b + 1} \psi_0(z + z_s) \quad (44)$$

The source field consists of the values generated by a monopole source and its reflection but we need a starting field function ψ_0 . If we assume that this starting field has a plane wave representation

$$\psi_0 = \int_{-\infty}^{\infty} S(k_z) e^{jk_z z + jr \sqrt{k_r^2 - k_z^2}} dk_z$$

which depends on an unknown function S . Using results from stationary phase and matching the starting field to the analytical monopole source in the free atmosphere, the critical wavenumber for evaluating the integral in the plane wave is $k_z^* = zk_r / \sqrt{r^2 + z^2}$ and the function S is

$$S(k_z) = \sqrt{\frac{j}{2\pi k_r}} \left(1 - \frac{k_z^2}{k_r^2}\right)^{-1/4}.$$

There is a singularity that arises in S when the vertical wavenumber approaches the reference range wavenumber which means that the integral of the inverse Fourier Transform cannot be analytically evaluated. This corresponds to the case of the elevation angle of the outgoing wave approaching verticality. To approximate S without the singularity, a number of approximations for the required stationary phase results is shown in [13] in the form of (45).

$$S(k_z) = \sqrt{\frac{j}{2\pi k_r}} \left(a_0 + a_2 \frac{k_z^2}{k_r^2} + a_4 \frac{k_z^4}{k_r^4} + a_6 \frac{k_z^6}{k_r^6} + a_8 \frac{k_z^8}{k_r^8} \right) \exp \left(-b_2 \frac{k_z^2}{k_r^2} \right) \quad (45)$$

A closeness of fit of the approximation to the singularity can be seen in Figure 15, where the elevation angle is related to the wavenumbers as $\sin \gamma = k_z / k_r$. As the approximation order increases, more terms are kept which leads to the approximations having better results for higher elevation angles. It should be noted that higher order approximations are not always better. There are some distinct oscillations around zero for the 8th order approximation in the relative error. Reference [13] indicates that this would cause some numerical errors due to surface waves and instead recommends the 4th order approximation for most practical cases. After an approximation for S has been determined, we can perform the inverse Fourier Transform to arrive at (46). A full derivation of going from the wavenumber domain to the spatial domain is shown in B but the optimal numerical values are listed in Table 2 from [13].

$$\psi_0(z) = \sqrt{k_r} (A_0 + A_2 k_r^2 z_z^2 + A_4 k_r^4 z_z^4 + A_6 k_r^6 z_z^6 + A_8 k_r^8 z_z^8) \exp \left(-\frac{k_r^2 z_z^2}{B} \right) \quad (46)$$

Table 2: Optimal Coefficients from [13]

Order	A_0	A_2	A_4	A_6	A_8	B	$\gamma_{\max}$
0	1	0	0	0	0	2	10°
2	1.3717	-0.3701	0	0	0	3	30°
4	1.9705	-1.1685	0.0887	0	0	3	40°
8	9.6982	-20.3785	6.0191	-0.4846	0.0105	3	60°

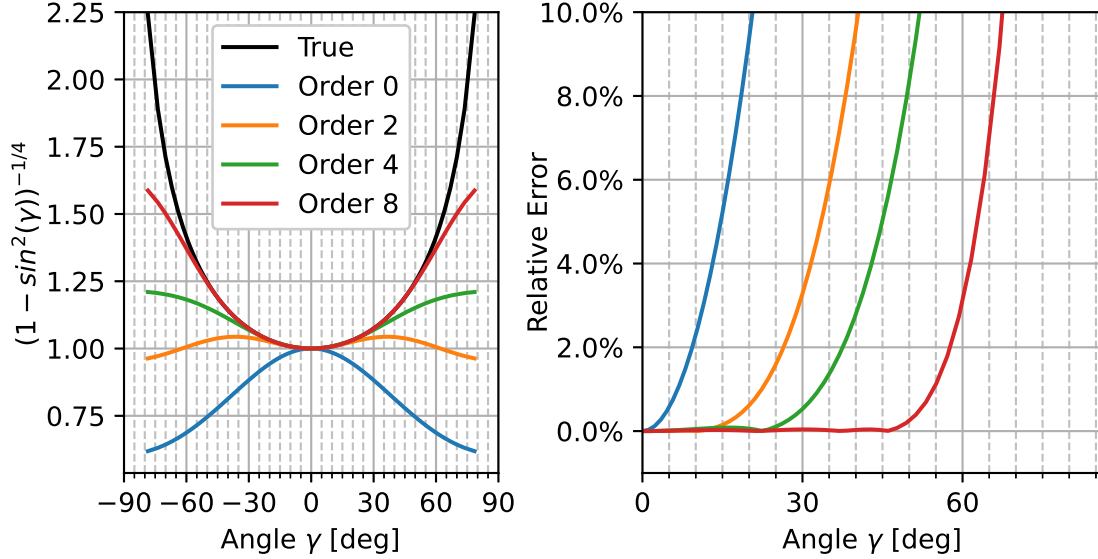


Figure 15: Approximations to the Singularity.

2.2.3.3 Field Propagation

After the starting field has been initialized, the field is updated in the range direction in accordance with Algorithm 4. At each new range step that we consider, the atmospheric and boundary models are updated. Typically, this will include the speed of sound along the vertical direction as well as the vertical wavenumber using the relation $k_z^2 = k_{\text{eff}}^2 - k_r^2$. After these quantities have been updated, we use the midpoint Fourier Transform (47) which has been indicated to be more accurate in [19].

$$\Psi(r, k_n) = \left[\sum_{i=0}^{N-1} e^{-jk_n z_i} \psi(r, z_i + \Delta z/2) \right] \times e^{(-jk_n \Delta z/2)} \Delta z \quad (47)$$

After the Fourier Transform has been obtained, we can use the marching equation in two dimensional form (48) from [19].

$$\psi(r + \Delta r, z_i) = e^{j \frac{\Delta r k_z^2}{2k_r}} \left\{ \frac{\Delta k_n}{2\pi} \left[\sum_{n=0}^{N-1} \{ \Psi(r, k_n) + R(k_n) \Psi(r, -k_n) \} e^{j \Delta r (\sqrt{k_r^2 - k_n^2} - k_r)} e^{jk_n z_i} \right] \right\}$$

$$+2j\beta\Psi(r, \beta)e^{j\Delta r(\sqrt{k_r^2-\beta^2}-k_r)}e^{jz_i\beta}\} \quad (48)$$

Gleaning from (48), there are three components contributing to the field propagation mechanics: the transmitted wave $\Psi(r, k_n)$; the reflected wave $R(k_n)\Psi(r, -k_n)$; and the surface wave $\Psi(r, \beta)$ where $\beta = k_b/Z$. While the transmitted wave and reflected wave components are contributing throughout GFPE, there are circumstances where the surface wave does not contribute. In environments with acoustically rigid ground surfaces, i.e. Z being infinite, β is zero and removes the surface wave component. Most applications for GFPE will not have acoustically rigid ground surfaces and so surface waves will contribute to the acoustic source propagation. There is some spreading of acoustic waves from these three components from the positive heights to the negative heights that must be corrected. After the field has been marched to the next range step using (48), we assign $\psi(r + \Delta r, z > z_{\max}) = 0$ to eliminate any ground coupling. Additionally, this allows for ψ to maintain its periodicity before continuing the for loop in Algorithm 4.

2.2.3.4 Exact Starting Field

An extension of the GFPE is to improve the starting field used. While the standing wave approximations of the starting field in the previous section gave good agreement up till sixty degrees of inclination from the horizontal plane, there is a way to improve the starting field until near vertical ($\approx 86.6^\circ$). If we consider starting not at $r = 0$ but at $r = \Delta r$, then [20] gives an exact starting field of

$$\psi(\Delta r, z) = e^{j(k_{\text{eff}}-k_r)} \left(\sqrt{\frac{j}{2\pi k_r}} \int_{-\infty}^{\infty} [e^{-jk_z z_s} + R(k_z)e^{jk_z z_s}] \frac{e^{j\sqrt{k_r^2-k_z^2}\Delta r}}{(1 - k_z^2/k_r^2)^{1/4}} e^{jk_z z} dk_z - \sqrt{\frac{2\pi}{jk_r}} 2\beta e^{-j\beta(z+z_s)} \frac{e^{j\sqrt{k_r^2-\beta^2}\Delta r}}{(1 - \beta^2/k_r^2)^{1/4}} \right) \quad (49)$$

This exact starting field does have some numerical considerations when evaluating the indefinite integral. To avoid a singularity, the reference wave number k_r must contain a small positive imaginary component similar to the case of an attenuating atmosphere. Additionally, the indefinite integral must be evaluated using a window to be numerically calculated. The suggested window is $|k| > 2k_r$ with smaller windows corresponding to lower accuracy at higher angles of elevation [20]. A validation of our implementation matching the original work presented in [20] is shown in Fig. D.16.

2.2.3.5 3D GFPE Algorithm

While the 2D GFPE works well for modeling transmission of sound in the source-detector plane, there is sometimes a need to incorporate other dimensions for more complex acoustic phenomenon. Turbulence is an example of such a phenomenon, with the acoustic scattering not being symmetric in the polar angle. To capture such phenomenon, we must first extend the 2D GFPE algorithm by including an angular dependence ϕ in the cylindrical coordinate

system. This new coordinate dimension will be discretized as $\phi = m\Delta\phi$ and $k_\phi = m\Delta k_\phi$ where $m = 0, \dots, M-1$ and generally $M\Delta\phi = 2\pi$. Again, similar to the vertical wavenumber, we require the condition $\Delta\phi\Delta k_\phi = 2\pi/M$ in order to use the Fast Fourier Transform. After establishing the new dimension of the grid, we need to alter (48) to account for the dependence on ϕ . This requires adding in another Fourier and inverse Fourier transform and change the arguments in the square roots to account for a new effective range wavenumber. This inclusion results in

$$\begin{aligned} \psi(r + \Delta r, \phi, z_i) = e^{j\frac{\Delta r k_z^2}{2k_r}} & \left\{ \frac{\Delta k_\phi}{2\pi} \sum_{m=0}^{M-1} e^{jk_{\phi,m}\phi} \left\{ \frac{\Delta k_n}{2\pi} \left[\sum_{n=0}^{N-1} \{ e^{(jk_{z,n}z_i)} \Psi(r, k_{\phi,m}, k_{z,n}) + \right. \right. \right. \\ & R(k_{z,n}) \Psi(r, k_{\phi,m}, -k_{z,n}) \} e^{\sqrt{k_r^2 - k_{z,n}^2 - k_{\phi,m}^2/r^2 - k_r}} \left. \right] \\ & \left. \left. + 2j\beta \Psi(r, k_{\phi,m}, \beta) e^{\sqrt{k_r^2 - \beta^2 - k_{\phi,m}^2/r^2 - k_r}} e^{jz_i\beta} \right\} \right\} \end{aligned} \quad (50)$$

where the $\Delta k_\phi^2/r^2$ accounts for the spherical spreading. It is important to note is that the 3D version of GFPE is significantly slower than the 2D GFPE since not only does it have to account for azimuthal angles, but that the division by r in terms means calculation of more terms at every range step. Lastly, one should use $r = \Delta r/2$ for the first range step instead of $r = 0$ in the extrapolation [13]. A validation of our implementation is shown in a Fig. D.17.

2.3 Solver Method Validations

2.3.1 FFP Theoretical Comparisons

The main theoretical comparisons for FFP of this work are based on [18] which was one of the earliest direct method cases. In this work, there are three types of atmospheres as well as some examples of the transformed pressure spectrums. Matching the presented examples should give some confidence as the validity of the methods used in predicting the acoustic sound levels. In all subsequent test cases, an atmosphere with uniform density is shown and the ground model used is the Delany-Bazley-Chessel model with varying specific flow resistivity constants. Table 3 outlines the validation tests from this source below. While there is some uncertainty as to the exact atmosphere used in the neutral atmosphere cases, the best estimation of the maximum height can be obtained from reversing the Bisection Method similar to [18] using the cutoff wavenumbers $k_{r,\max}$ and the attenuation tolerance A_{tol} . Throughout this section, we will refer to the cases in [18] as they are referred to in the original work to maintain consistency. The exclusion of case (a) is simply due to the fact that (a) is a theoretical comparison with another work. The comparison of this work using the code written by the authors and resulting plots are shown Appendix C. Any work that is from the authors will have "(our work)" in the plot legend. The two methods used throughout the comparisons are the Direct Method and Thomson-Haskell-Matrix Method referred to in plot legends by "DM" and "THM" respectively.

Table 3: Theoretical Validation Tests from [18]

Case	$z_{\max}$	N Layers	Refraction
(b)-(d)	1.8(?) meters	2(?)	Neutral
(e)-(f)	100 meters	10, each 10 meters	Downwards
(g)	8 meters	16, each 0.5 meters	Upwards

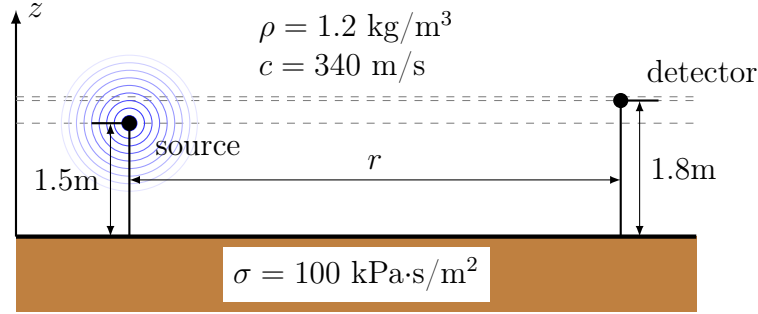


Figure 16: Test geometry for the neutral atmosphere cases.

2.3.1.1 Neutral Atmosphere

The first set of cases is the neutral atmosphere cases. These test cases involve a low, medium, and high frequency acoustic source while all still using the same geometry and atmospheric properties shown in Figure 16. The tests vary in frequency of the acoustic source so that cases (b) through (d) are 40, 160, and 640 Hz respectively. In addition to its own work, [18] also presents some theoretical solutions from other works. Lines with the "Attenborough" handle refer to [21] and while "Donato" refers to [22]. Both of these theoretical works are used to justify the accuracy of the transmission loss in [18].

As shown within Appendix C in Figures C.1 through C.6, the current implementation of the FFP used in this work has some discrepancies with the existing literature. Of the two, the DM method best represents the results presented from [18]. Ideally, the transformed pressure should be completely matched the results from [18] but there are still some discrepancies. As it was one of the first FFP references, the notation used in [18] differs dramatically from current notation. Better agreement with previous literature results came from manually tuning certain parameters, such as N and $k_{\max}$, than using directly using values reported in [18]. This shows as the high sensitivity of FFP to the method parameters such as the maximum integration wavenumber. In future, we would suggest to use a pole multiplicity instead of bisection method. The pole multiplicity seems to be more applicable to 3D.

2.3.1.2 Downwards Refracting Atmosphere

There are two downward refracting cases studies in [18], referred to as cases (e) and (f). The geometry of these two cases are shown in Figure 17 with both cases being the same acoustic frequency. These two cases achieve a downwards refracting atmosphere by having a positive speed of sound vertical gradient although case (f) has a larger gradient than case

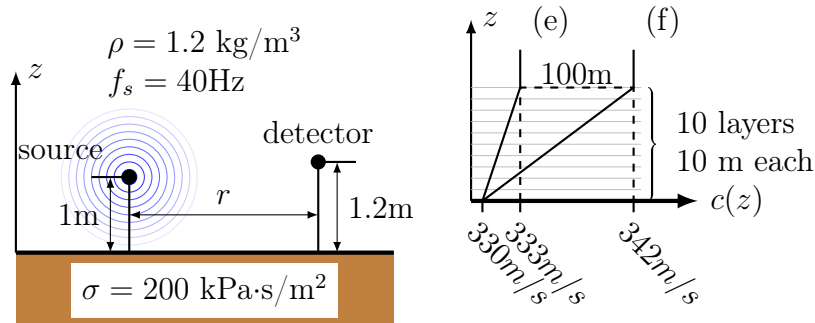


Figure 17: Test geometry for the downward refraction cases.

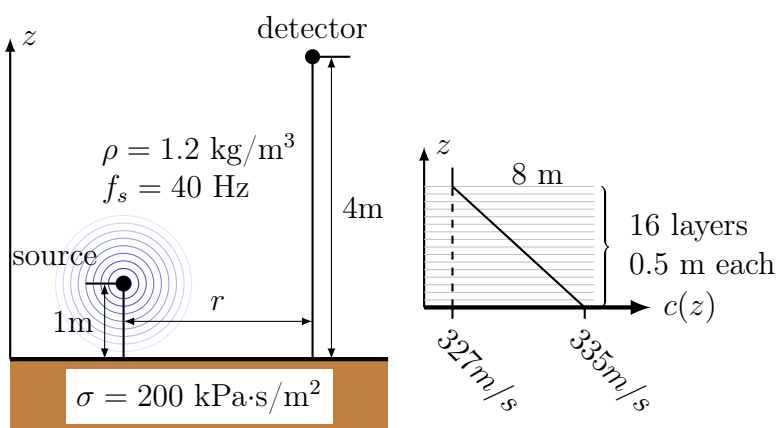


Figure 18: Test geometry for the upward refraction cases.

(e). Comparison results are plotted in Figures C.7 through C.10 of Appendix C. We found that our results after incorporation of the "clipping" technique also matches the previous work of [18].

2.3.1.3 Upwards Refracting Atmosphere

Case (g) is only one upward refracting tests presented in [18] and the test geometry is shown in Figure 18. As noted in the [18], there is a limiting ray case where a detector above a certain distance is in the acoustic "shadow" zone. Both methods used in this work seem to give slightly different results from the presented previous work of [18] as displayed in Figure C.12. However, predictions of FFP in upward refraction is a known difficult problem due to the "shadow" zone. Mismatch of the pressure ratio could potentially be a result of spectrum parameter mismatch or the non-zero tail of the spectrum. We know from previous sections that the pressure is highly sensitive to the transformed pressure spectrum. One potential method to improve the robustness of FFP is to use spectrum tail corrections as in [17].

2.3.2 GFPE Theoretical Comparisons

Two previous works, [13] and [19], have been used for the theoretical validation of the GFPE method developed in this work. These references contain results for both a combination of neutral, downwards refracting, and upwards refracting atmospheres as well as both rigid and absorbing ground mediums. While neither reference was the seminal work for GFPE, the combination contains enough results to ensure the validity of the current two dimensional GFPE implementation. Figures in this section will have "Salomons" in the plot legend to indicate results [19] and "CAA" for results from [13]. The central focus of these works are different, as [19] worked on development of GFPE whereas [13] is more of a survey on computational acoustic frequency methods. For this reason, the frequencies tested within [19] range from 10, 100, and 1,000 Hz while [13] was more consistent. Reference [13] used 500 Hz for all results shown here with the exception of Figure 22 which used 300 Hz.

All of the results presented within this section will use the Attenborough 4 parameter ground model for absorbing grounds with nominal values as discussed in Section 2.1.4.2. There is a distinction of specific flow resistance between the two works. The comparisons with [13] use a specific flow resistivity of $200 \text{ kPa}\cdot\text{s}\cdot\text{m}^{-2}$ while comparisons with [19] are done with $300 \text{ kPa}\cdot\text{s}\cdot\text{m}^{-2}$. For validation cases with refracting atmospheres, the majority of cases considered will use a logarithmic profile (51). This profile is parameterized by a given speed of sound immediately above the ground c_0 , a velocity gradient parameter b , and a roughness length z_0 . Both works use a baseline speed of sound of 340 meters with downwards refraction models as $b = 1$ meter per second and $z_0 = 0.1$ meters with one exception that will be noted later.

$$c(z) = c_0 + b \log \left(\frac{z}{z_0} + 1 \right) \quad (51)$$

The numerical grid for the GFPE algorithm dictates the accuracy of the results. Since [19] was developing GFPE, this work is able to match the numerical grid used to achieve the results in the previous work. The numerical grid used a vertical grid spacing of $\lambda/10$ and a range step of 3λ . The maximum height of the atmosphere was 4096 layers for 10 and 100 Hz but 8192 for 1,000 Hz. Other system heights were included in the previous work and shown in this work to discuss the various effects due to inadequate system heights. The number of absorption layers were kept constant throughout these validation cases at 500 absorption layers which is 50λ . In contrast, [13] did not specify any grid spacing or number of absorption layers. This work used a vertical grid spacing of $\lambda/10$ again but a larger range step of 5λ . In contrast with the other previous work, the absorption layer was 500λ thick for all results but the maximum system height was approximately $z_{\text{max}} = 200$ meters. We found that this larger absorption layer was necessary for smooth numerical results, which may be a consequence of the starting field methods used. Reference [19] used a 0th order starting field, also known as a Gaussian Beam starting field while [13] used an 8th order for the starting field. The 8th order method is known to have some numerical oscillations.

Throughout this section, we will give results in the relative sound pressure level. This is a measurement of the change in sound pressure level from an idealized monopole source in an infinite, lossless space without any refraction. The relative sound pressure level is calculated from (52)

$$\begin{aligned}\Delta L_p &\triangleq 10 \log_{10} \left(\frac{|p(r, z)|^2}{|p_{\text{free}}(r, z)|^2} \right) \\ &= 20 \log_{10} |p(r, z)| + 10 \log_{10} (r^2 + (z - z_s)^2) + \alpha \sqrt{r^2 + (z - z_s)^2}\end{aligned}\tag{52}$$

where α is an absorption coefficient in decibels per meters. It should be noted that this absorption coefficient is the one that is intrinsic to the atmosphere and not the artificial component used in Section 2.2.3. As none of the validations within this section use an absorptive atmosphere, $\alpha \equiv 0$ for this section and the third term vanishes from the relative sound pressure level.

2.3.2.1 Neutral and Downwards Refracting Atmospheres

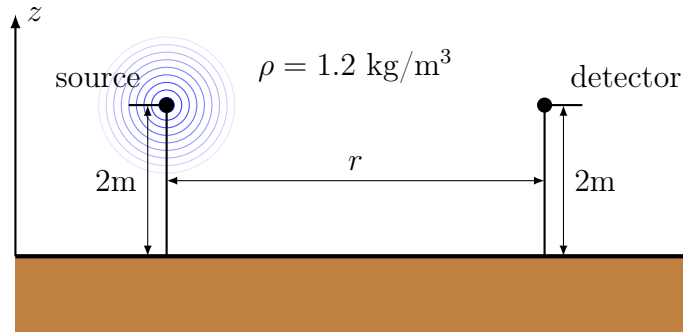


Figure 19: Test Geometry of GFPE Neutral and Downwards Refraction Atmosphere

This section will go over the GFPE validation cases for neutral and downwards refraction atmosphere. The validation cases involve an acoustic source at two meters above the ground are shown in Figure 12. For cases from [19], the detector is on the level with the source but otherwise the results are displayed in the vertical plane. The rigid ground and neutral atmosphere cases are shown in Figures D.1 - D.4. The GFPE code developed in this work shows good agreement with the previous work with the relative sound pressure level rapidly converging to the analytical solution of 6 decibels. An insufficient maximum height for the atmosphere does show a divergence from the known solution, with higher frequencies showing divergence at smaller ranges. Changing the ground from an acoustic rigid surface to an absorbing medium for a neutral atmosphere does lead to a reduction in the relative sound pressure level. This is shown in Figures D.5 - D.7 with higher frequencies displaying a greater reduction in relative sound pressure level. Again, the higher frequencies show a divergence from other solutions as the maximum height of the atmosphere is reduced. This gives some indication that a potential test for the choice of the system height can be achieved by testing some frequencies with atmospheres that are twice as higher.

In contrast to the neutral atmosphere, the downwards refracting atmosphere show more local maximums in the relative sound pressure level as seen in Figures D.8 - D.11. For a rigid ground, the relative sound pressure level continuously increases. Since the ground is not absorbing acoustic energy as the acoustic rays are reflecting off the ground, the energy is not

lost. In contrast, Figures D.12 - D.14 show the absorbing ground case where acoustic energy is dissipated by the ground. While the 10 Hz case does continue to increase in relative sound pressure level, the higher frequencies converge over the range to oscillate around 0 relative decibels. There are some effects of too small of a maximum atmospheric height. Figure D.14 shows that a small atmosphere height has a reduction in pressure level. This is due to the highest arcing rays which should contribute to the pressure level being artificially attenuated by the absorption layer. This phenomenon leads to another potential suggestion for determining the maximum atmospheric height, which is to use acoustic ray tracing. While some analytical solutions exist for the maximum height achieved by acoustic rays for logarithmic profiles [19, 13], this is not available for potentially complicated speed of sound profiles.

In addition to singular frequencies, we also show a broadband frequency result from [19] in Figure 20. This grid is different from the other results presented alongside. Rather than a fixed maximum height, the atmosphere changed from 1,024 to 2,048 to 4,096 layers for frequencies in the range of 10 to 100 Hz, 100 to 1,000 Hz, and over 1,000 Hz. The range step is also dynamic, being $6\lambda(1 + f_s/500)$ for all frequencies considered. Again, the results from the code developed from this work shows good agreement. While there is noticeable relative pressure levels below 1 kHz, the higher frequencies show a much narrow band around the free atmosphere solution.

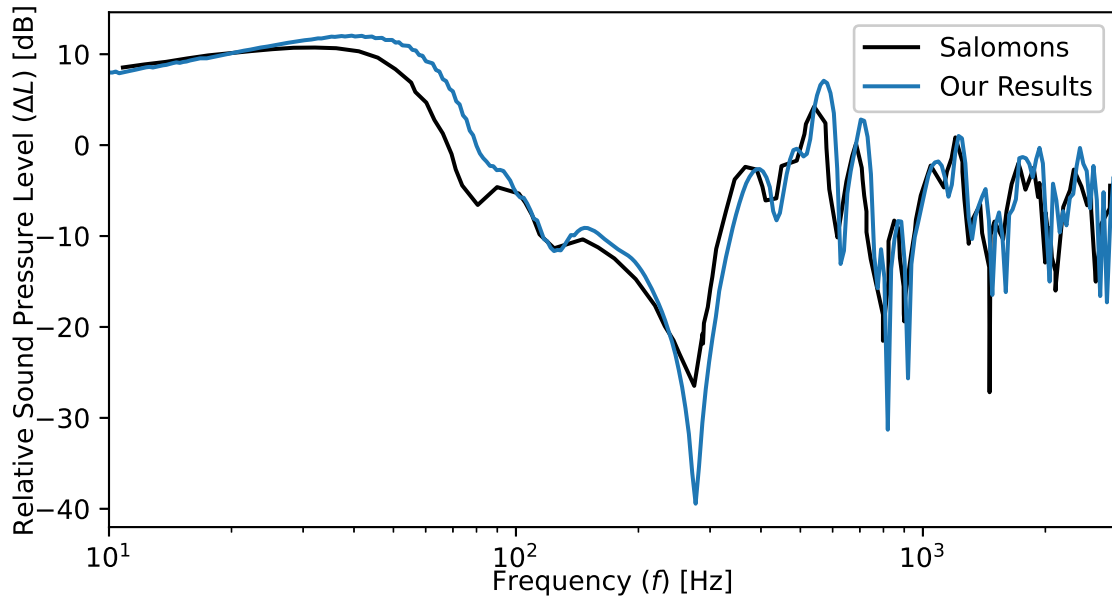


Figure 20: GFPE broadband frequency validation in downwards refracting atmosphere with an absorbing ground. The detector was at 1,000 meters downrange of the acoustic source. "Salomons" refers to the results from [19].

2.3.2.2 Upwards Refracting Atmosphere

There is only a single upward refracting atmosphere examples for comparison of our results with previous literature. Figure D.15 shows the numerical results within the vertical plane

along with agreement of our results against previous work. The defining trait of the shadow zone is very much apparent in the results. During validation, it was found that an insufficient number of absorption layers caused some inaccuracies in the shadow region of the domain due to coupling effects. These inaccuracies are reduced by increasing the depth of the absorption layer. If an inadequate amount is used, then reflected waves appear as downward sloped lines of high relative sound pressure level which penetrate the shadow region. Given a comparison of the thickness of the absorption layers used for the neutral and downwards refracting atmospheres, the larger absorption layers is necessary. As the rays are propagating upwards, the acoustic energy must somehow be removed from the numerical grid. While GFPE does zero the source field for the negative height points, naively zeroing grid points may obfuscate otherwise observable phenomenon. Hence the more practical solution is to increase the maximum atmospheric height considered and the thickness of the absorption layers.

2.3.2.3 Effect of Elevation Angle

This section focuses on some additional validation for behavior of GFPE from [13] for a downwards refracting atmosphere. This section is specifically interested in effects of the elevation angle on the accuracy of the GFPE method. Both cases studied are in downwards refracting with absorbing grounds. There is a difference in that the results shown in Figure 22 uses a linear speed of sound profile of

$$c(z) = c_0 + az$$

where $a = 0.1$ inverse seconds. The first consideration is the accuracy of the GFPE pressure for high detectors as the range changes in Figure 21. The 8th order starting field method used does rapidly converge to the analytical solution by the reported maximum elevation angle for the method. Looking at detectors located farther downrange, there are no oscillations visible but there are divergence from other reported methods. This divergence happens for GFPE and after it passes the maximum elevation method of the angle.

There were some behavior that was noted when tuning the GFPE method parameters to match the previous results. Oscillations that are noticeable still in the range sweep can be reduced by either extending the maximum height of the atmosphere or increasing the thickness of the artificial absorption layer. The oscillations that are the large period behavior occurring within 20 meters in Figure 21 are rapidly reducible by increasing the maximum height considered. The small oscillations are more reactionary to the thickness of the absorption layer. Of the two methods for dampening numerical oscillations, increasing the size of the absorption layer has a more responsive effect although too large of a layer will interfere with the source field in the domain of interest. Another interesting behavior of the oscillations is that they appear more apparent at shorter range distances, appearing in Figure 21 with a maximum range of 100 meters but not Figure 22 at a range of 200 meters. This could be caused from the second being a longer range but also potentially due to the different problem description such as the acoustic frequency and the speed of sound profile.

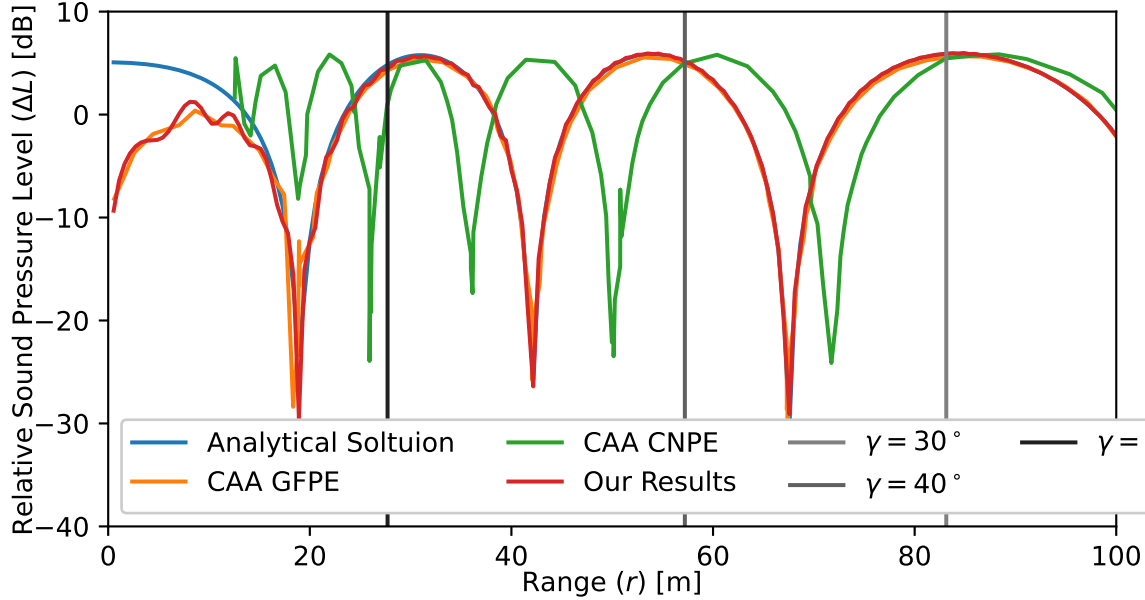


Figure 21: Effect of Range on Relative Sound Pressure Levels Accuracy in Downwards Refracting Atmosphere for GFPE. The acoustic source is at $z_s = 2$ meters and the detector at $z_d = 50$ meters over a rigid ground.

III. Results and Discussions

3.1 Theoretical Explorations

3.2 Broadband Frequency

In this section, we will explore the theoretical effects that acoustic phenomena may have on an acoustic profile. The baseline sound profile considered is a DJI-Phantom 2 sound profile obtained in an anechoic chamber [23]. This allows us to use the profile and consider it free from any acoustic phenomenon. All figures for this section are within Appendix E for convenience. For reference, the reported background noise sound pressure levels of 54 dB has been shown on all figures to indicate if the drone profile is strong enough at the frequency to be above the background level. The first effect to consider is simply the spherical spreading of the pressure wave as shown in Figure E.1. This figure shows that as the detector moves farther away from the UAV, the sound profile changes to something that only appears like combinations of pure tones above the background. The lowest peak of interest in the PSD is the reported blade pass frequency of the UAV in [23] which indicates that the blades of the UAV may be the best way to detect them. The next phenomenon is the sound attenuating in the atmosphere. Figure E.2 shows the effects of attenuation calculated using ISO 9613-1:1993(E) presented in [13] for sea level with dry air. The effects of attenuation are much more noticeably materialized at 6 kHz and above. From both the range and attenuation effects, it would seem that the maximum frequency suitable for detecting drones is 6 kHz.

The effects of attenuation and spherical spreading are easy to calculate and adjust for. However, the ground and acoustic effects must be found numerically and assumptions must

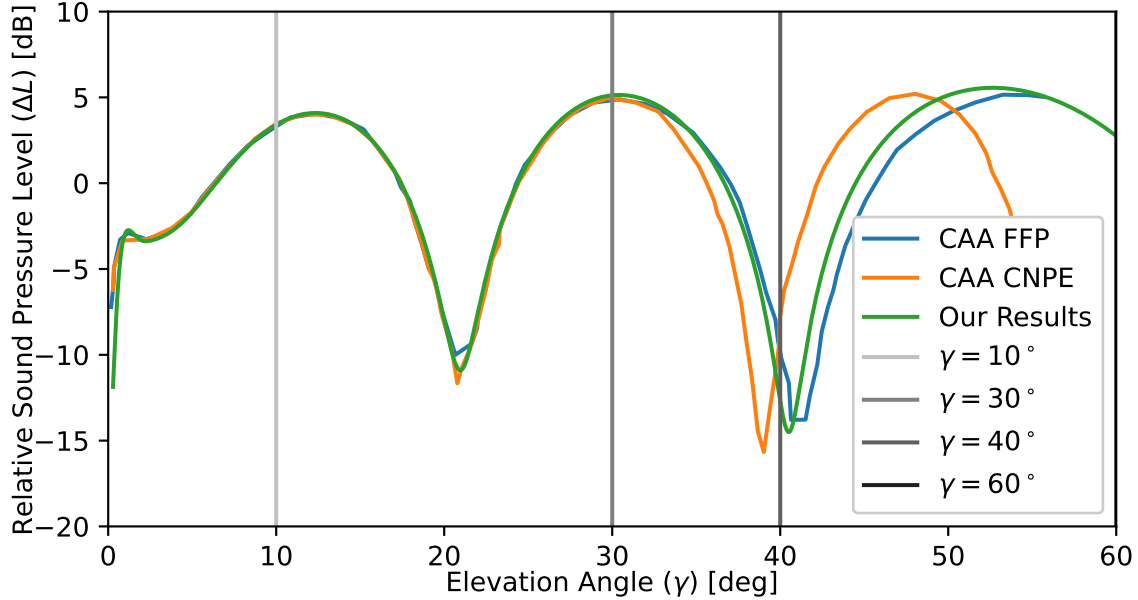


Figure 22: Effect of Elevation Angle on Relative Sound Pressure Levels Accuracy in Downwards Refracting Atmosphere for GFPE. The acoustic source is at $z_s = 2$ meters and the detector is at $r = 400$ meters.

be made about the height of the detector and UAV above a ground plane. We will consider multiple UAV heights but only a singular detector height of one meters. The numerical method used is the GFPE solver using a fourth order approximate starting field. The grid was chosen to have range and height steps of five and one-tenth wavelengths which is calculated at sea-level for a standard atmosphere. Note that a standard atmosphere does include some absorption so the reference wavenumber has a non-zero imaginary component. Additionally, the overall height of the atmosphere is 200 meters with an absorption layer that is one hundred wavelengths thick. This grid selection is well within the recommendations of [13]. The first effect that we consider is the ground reflections. For this we use the Delany-Bazely-Chessel (DBC) ground model for a rigid ground as well as various absorbing strengths. The next effects to account for are the refractions. For these cases, we use a logarithmic speed of sound profile with $b = 1$ for downwards refraction and $b = -1$ for upwards refraction but we still use the DBC with a specific flow resistance of $100 \text{ kPa} \cdot \text{s} \cdot \text{m}^{-2}$. All figures can be found in Appendix E.

It should be noted here that the numerical solving takes a long time for these broadband acoustic profiles, particularly for frequencies above 5 kHz. On a personal laptop and for the 387 frequencies from the digitized example profile, it takes roughly 12 hours per test. Additionally, there is a very high memory consumption at high frequencies. For the 20 kHz frequency, the computational grid had limited to 5 meters in range to limit memory consumption to less than 8 gigabytes so the GFPE solver had to be restarted every 5 meters. Another stumbling block for the numerical computations that we discovered was that upwards refraction has poor solutions above 1 kHz for the chosen grid parameters. This is a result of reflections from the top of the atmosphere rather than being fully absorbed in the absorption layer. There was some numerical case studies that were done on the parameters

and it was found that increasing the maximum atmosphere height to one kilometer and increasing the absorption layer to one thousand wavelengths did work for a frequency at around 2 kHz. However, this was at a cost of a higher computation time, with the computation going from roughly 20 seconds to completing to roughly 20 minutes. Thus the figures for effects will only contain up to 1 kHz for the upwards refraction cases.

Observing from Figs. E.3-E.6 in Appendix E.1, the ground effects are more noticeable as the UAV becomes closer to the ground and when the detector moves farther away. These effects tend to happen at lower frequencies, particularly below 1 kHz. Typically, the more rigid grounds have larger signals but the effects become less distinguishable. This is because the DBC model has the impedance scaling with negative powers with frequency, leading high frequencies to have similar impedance values. Similar to ground effects, the refraction effects displayed in Figs. E.15-E.18 of Appendix E.3 are more noticeable when the UAV is closer to the ground and at farther ranges. However, the effects start to distinguish themselves at around 500 Hz until about 5 kHz. After that, the attenuation of the atmosphere dominates the changed acoustic profiles.

In addition to the DBC ground model and the logarithmic ground model, we have also included other ground models discussed in Section 2.1.4.3 as well as a linear speed of sound profile within Appendix E. These model effects can be viewed in Appendix E.2 and E.4 but these models also show consistent trends of lower UAV heights and farther detector differences leading to more appreciable acoustic effects in refraction and reflection. Additionally, the larger the acoustic effects, the more the differences between the models begin to show, thus higher UAV altitudes and closer detector distances may warrant choosing simpler models.

3.3 Acoustic Parameter Estimation From Microphone Array Signals

Given the broadband frequency profile of a UAV from the previous section, one might determine a frequency band of interest for detecting the UAV. However, the long range propagation is dependent on ground and atmospheric model parameters which may be unknown at the time of recording. Some of these parameters like speed of sound profile are temperature dependent, which can vary throughout the day or over longer time intervals like seasons or years. This leads to the following question; can acoustic modeling parameters be estimated to describe the variation of pressure recorded at microphones in the environment? We attempt to use optimization to estimate model parameters of a simulated pressure field for an acoustic source located 5 meters above the ground with a signal at 100 Hz. The simulated pressure field is generated by a 2D GFPE with an Exact Starting Field method. The models used are the DBC ground model in Eq. (20) and the logarithmic speed of sound profile in Eq. (51). These two models means that the simulated pressure field is described by four parameters which may vary in the ranges given in Table 4. The specific flow resistivity σ bounds follow [13] for grasslands. The baseline speed of sound c_0 is based on temperature variations between -20 and 40 degrees Celsius with 25 degrees considered the standard temperature. The refraction parameter b bounds are slightly larger than commonly used values of ± 1 meters per second in literature. Finally, the roughness length z_0 is a nominal value for

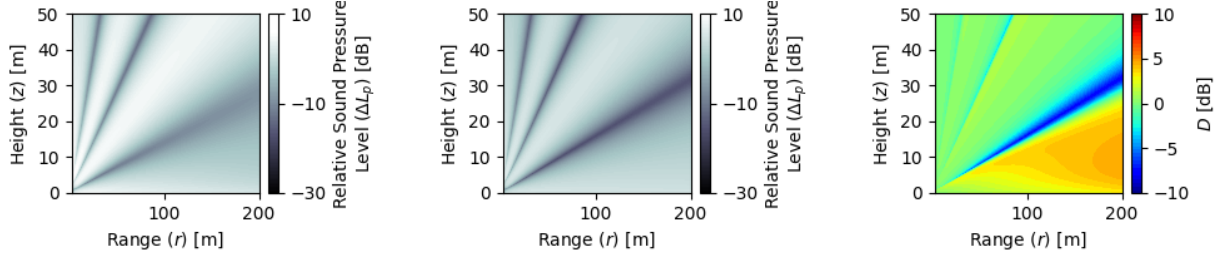


Figure 23: Relative Sound Pressure Level for Specific Flow Resistivity. Left is $\sigma = 100$ [kPa·s/m²], center is $\sigma = 300$ [kPa·s/m²], and right is the difference.

grasslands with the upper and lower bounds being an order of magnitude different from the nominal value.

Table 4: Nominal and Bounds for Parameter Values

Parameter	Lower Bound	Nominal Value	Upper Bound
σ [kPa·s/m ²]	100	150	300
c_0 [m/s]	320	340	350
b [m/s]	-1.5	0	1.5
z_0 [m]	0.01	0.1	1

We first study the effects of the parameters have on the simulated pressure field when varying a single parameter between its lower and upper value bounds to determine locations in the environment that are sensitive to the parameters. These sensitivities are shown in Figs. 23-26 by the difference in relative sound pressure level. The relative sound pressure level difference D is defined in this work as

$$D = 20 \log_{10} \left| \frac{p_{\max}}{p_{\min}} \right| \quad (53)$$

where $p_{\min}$ and $p_{\max}$ correspond to the pressure calculated with the model at lower and upper bounds for a single parameter with the other parameters being their nominal values. The only exception is the roughness length which uses a nominal b of 1 meter per second instead of 0 since the roughness length only has an effect in refracting atmospheres. From Figs. 24 and 25, the baseline speed of sound and the refraction parameter have appreciable effects on all destructive interference bands. In contrast, Figs. 23 and 26 show that specific flow resistivity and roughness length have appreciable effects on the destructive interference band closest to the ground and negligible effects on bands at angles near vertical. All parameters show a larger effect at higher ranges, particularly over 100 meters downrange. This information mean when determining model parameters from microphones, microphones should be placed farther away and close to any expected destructive interference bands. It should be noted that the number of destructive interference bands depends both on the source frequency and height above the ground, with the number increasing with both source frequency and height [13].

Given the knowledge of where to place microphones for sensitivity to modeling parameters, we construct some example cases to determine the effectiveness of using optimization for

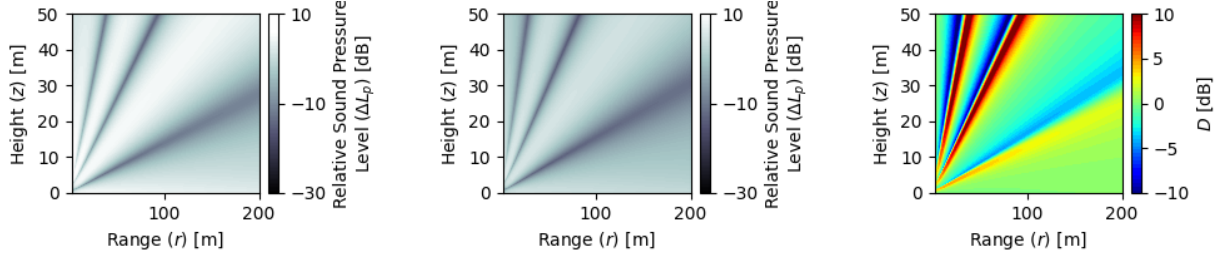


Figure 24: Relative Sound Pressure Level for Speed of Sound. Left is $c_0 = 320$ [m/s], center is $c_0 = 350$ [m/s], and right is the difference.

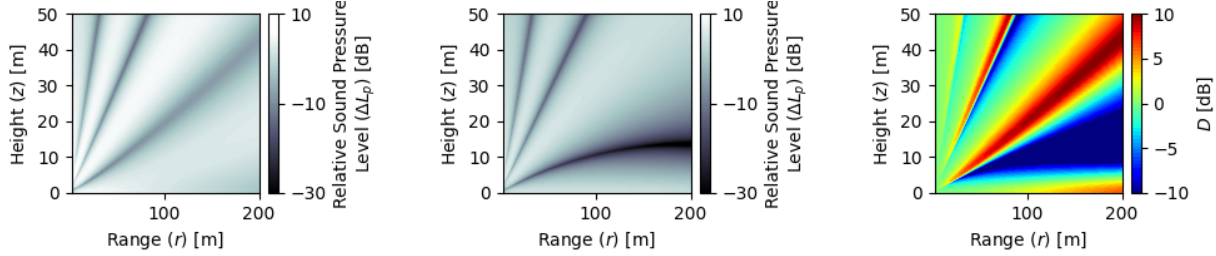


Figure 25: Relative Sound Pressure Level for Refraction Parameter. Left is $b = -1.5$ [m/s], center is $b = 1.5$ [m/s], and right is the difference.

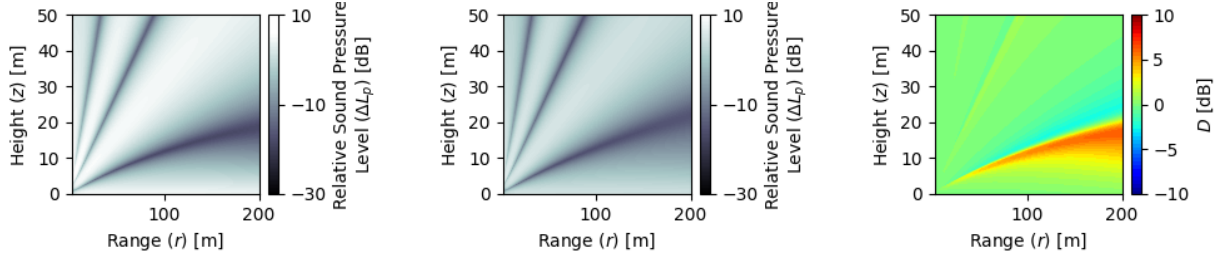


Figure 26: Relative Sound Pressure Level for Roughness Length. Left is $z_0 = 0.01$ [m], center is $z_0 = 1$ [m], and right is the difference.

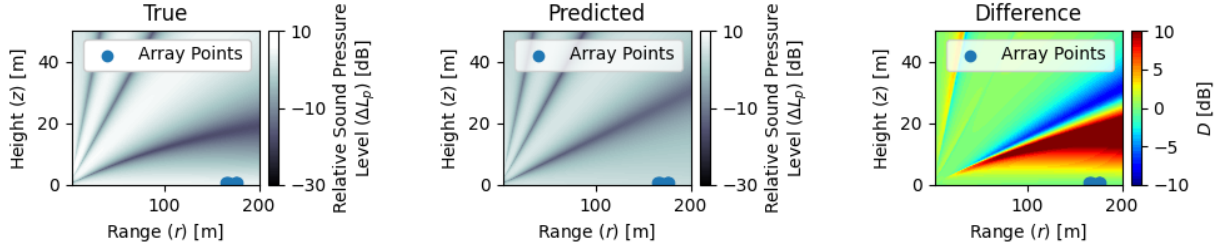
estimating unknown model parameters. We test two different microphones arrangements, a horizontal arrangement and a vertical arrangement. These arrangements are meant to mimic a six microphone array that could be deployed for detection and localization of UAVs. The microphones for the horizontal array are all at a height of 1 meter and located at distances of 165, 166, 167, 175, 176, and 177 meters downrange of the source. The microphones for the vertical array are 175 meters downrange with a heights going from 1 meter to 13 meters every 2 meters. We test both of these arrangements using the pressure field generated by values listed in Table 5. The loss function used in the optimization is

$$\mathcal{L}(\hat{\mathbf{p}}, \mathbf{p}) = \sum_{i=1}^6 (\log_{10} |\hat{p}_i| - \log_{10} |p_i|)^2 \quad (54)$$

where $\hat{\mathbf{p}}$ and $\mathbf{p}$ are the estimated and true complex pressure vectors of the six microphones. For the optimization, we show the results of a Trust Constraint solver for a local optimization

Table 5: True, Initial, and Converged Parameters for Optimization Test Cases

Parameter	σ [kPa·s/m ²]	c_0 [m/s]	b [m/s]	z_0 [m]
True Value	150	340	1	0.1
Initial Values	200	340	0.5	0.1
Trust Constraint Horizontal Array	201	332	-0.06	0.53
Trust Constraint Vertical Array	149	341	1.02	0.12
Brute Force Horizontal Array	144	340	1.17	0.56
Brute Force Vertical Array	144	337	1.17	0.67


 Figure 27: Trust Constraint estimates of $\hat{\sigma} = 201$ [kPa·s/m²], $\hat{c} = 332$ [m/s], $\hat{b} = -0.06$ [m/s], and $\hat{z}_0 = 0.53$ [m] with $\mathcal{L} = 0.359$ for a horizontal array.

method as well as a brute force search as a global search. The final convergence points for the algorithms for both the horizontal and vertical arrays are also displayed in Table 5 for reference.

To determine estimates of the modeling parameters, we test both a local optimization method via the Trust Constraint solver as well as a brute force search on a grid for both microphone array arrangements. There were two grid sizes used, both spanning the possible parameter values shown in Table 4. There was a lower resolution grid with 5 grid points per parameter (625 total points) and a higher resolution grid with 10 grid points per parameter (10,000 total points). The 5 spacing points per parameter takes roughly 12 minutes on a personal computer while doubling the grid density to 10 points per parameter takes roughly an hour. This drastic penalty for increasing the grid resolution is consistent with the exponential time and memory growth for a brute search method. In comparison, the Trust Constraint solver took roughly four minutes for both array arrangements and converged within 100 iterations and 450 pressure field simulations. The optimization results for parameter estimation using the two arrangements are shown for the Trust Constraint in Figs. 27 and 28 while the brute force results for different parameter grid spacing are shown in the following Figs 29-31. The relative sound pressure level difference is obtained using Eq. 53 but where $\hat{p}$ and p replace $p_{\max}$ and $p_{\min}$, respectively.

The solutions for a local optimization method are dependent on the array arrangement as Figs. 27 and 28 show the Trust Constraint method predicts either a downwards or upwards refracting atmosphere depending on the microphone placements. The sensitivity to microphone placement seems to be reduced by performing a global optimization, as Figs. 29 through 30 show the global optimizer gives nearly identical pressure fields for both grid sizes and microphone arrangements. The higher grid resolution led to better estimates although

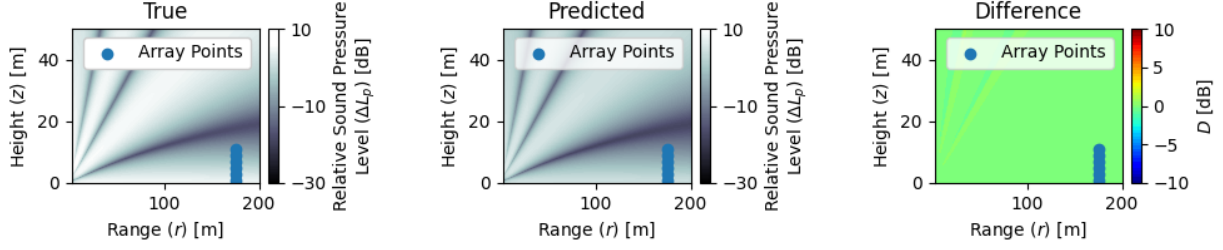


Figure 28: Trust Constraint estimates of $\hat{\sigma} = 149$ [kPa·s/m²], $\hat{c} = 341$ [m/s], $\hat{b} = 1.02$ [m/s], and $\hat{z}_0 = 0.12$ [m] with $\mathcal{L} = 02.916\text{e-}6$ for a vertical array.

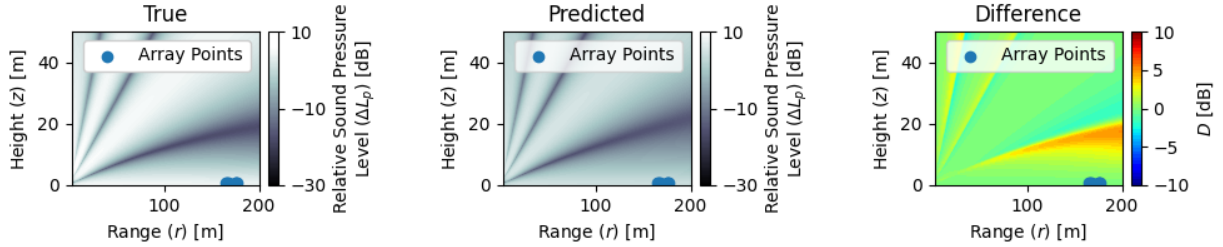


Figure 29: Brute force estimates of $\hat{\sigma} = 150$ [kPa·s/m²], $\hat{c} = 335$ [m/s], $\hat{b} = 0.75$ [m/s], and $\hat{z}_0 = 0.25$ [m] with $\mathcal{L} = 3.32\text{e-}6$ for a horizontal array with 5 grid points.

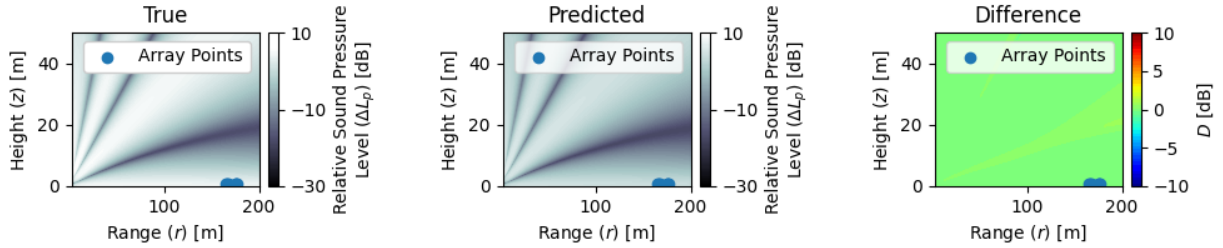


Figure 30: Brute force estimates of $\hat{\sigma} = 144$ [kPa·s/m²], $\hat{c} = 340$ [m/s], $\hat{b} = 1.17$ [m/s], and $\hat{z}_0 = 0.56$ [m] with $\mathcal{L} = 3.865\text{e-}8$ for a horizontal array with 10 grid points.

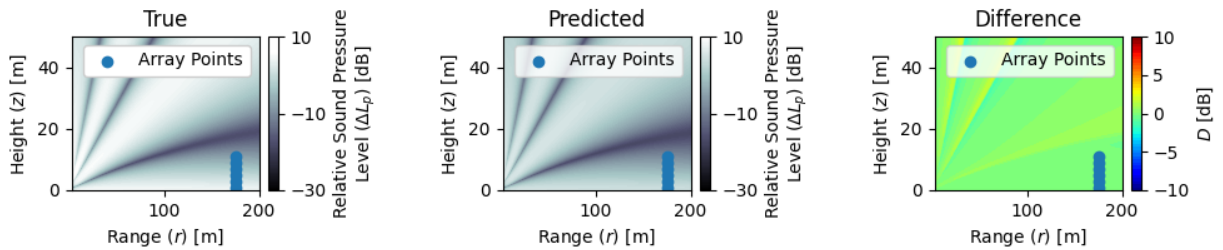


Figure 31: Brute force estimates of $\hat{\sigma} = 144$ [kPa·s/m²], $\hat{c} = 337$ [m/s], $\hat{b} = 1.17$ [m/s], and $\hat{z}_0 = 0.67$ [m] with $\mathcal{L} = 2.502\text{e-}5$ for a vertical array with 10 grid points.

the best brute force still did not outperform the best Trust Constraint result. This illustrates the trade-off that may occur when trying to estimate parameters from microphones, whether it is better to use the potentially more accurate local optimization methods or the

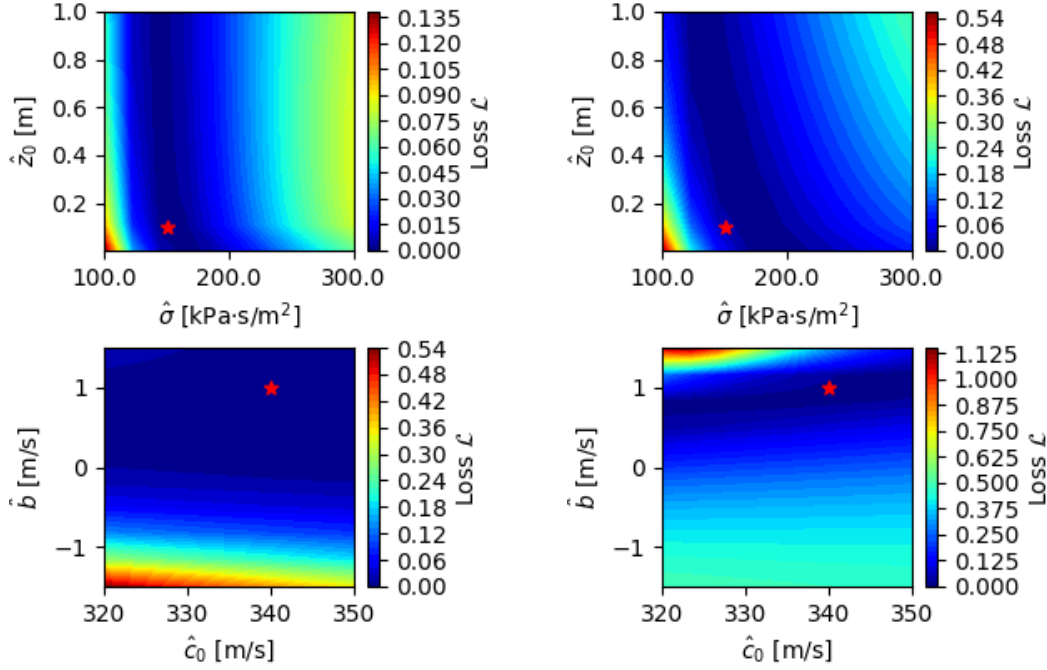


Figure 32: Loss function contours for determining optimal parameter estimates with the horizontal array on the left and vertical array on the right. Contours generated with the grid spacing of 10 points per parameter and the true parameters values are indicated by the red star.

more computationally expensive but consistent global methods.

Sets of loss function contours are shown in Fig. 32 and shows the behavior of the optimization methods when placing the microphones in the environment. The horizontal arrangement did provide a better brute force solution but the contours show that the roughness length z_0 and the refraction coefficient b are largely insensitive around the true parameter values. In contrast, the vertical microphone arrangement had measurements closer to the destructive interference band which provides more information about the atmospheric parameters. As a result, the contours for the vertical array display a larger gradient and parameter estimate coupling. Hence, we recommend the placement of microphones near potential destructive interference bands as they appear to provide the information necessary to accurately estimate the parameters.

3.4 Experimental Validations

There are multiple datasets that exist of recorded experimental acoustic data but all of the experimental analysis will need the sound pressure level of the UAVs. However, the audio recordings are in the form of Waveform Audio File Format (WAV) files, which have normalized values between -1 and 1. This is not ideal as most acoustic references deal in dimensional pressure in Pascal units. This requires us to estimate the dimensional pressure from the recorded files and then estimate other metrics such as the sound pressure level. From the specifications received from the Stevens Institute of Technology, the microphone

output level is -67 dB relative per Pascal in the ESCAPE 1 dataset so the conversion factor is

$$p(t) = 10^{67/20} p_{\text{WAV}}(t) \quad (55)$$

where p_{WAV} is simply the value in the audio file. This first order estimation is not perfect as the microphone output level tends to vary across the frequency octave bands. As such, we provide a more varied conversion factor from WAV file recordings to pressure level for ESCAPE 2. After determining the sound pressure, we find the Short Time Fourier Transforms (STFT) to get estimates of the acoustic spectrum. The discrete STFT is

$$P(f_n, t_m) = \sum_{n=-\infty}^{\infty} w(n - Rt_m) p(n) \exp(-j2\pi f_n) \quad (56)$$

where w is a window function of length N at a stride length of R . The two finite filters used in this work, the Tukey window in (57) parameterized by a shape factor a and the Hann window in (58). It is important to note that these filters are zero padded so when n is negative or greater than the window size, the window values are zero.

$$w_{\text{Tukey}}(n; a) = \begin{cases} \frac{1}{2} [1 - \cos(\frac{2\pi n}{aN})] & 0 \leq n \leq aN/2 \\ 1 & aN/2 \leq n \leq N/2 \\ w[N - n] & 0 \leq n \leq N/2 \end{cases} \quad (57)$$

$$w_{\text{Hann}}(n) = \sin^2\left(\frac{\pi n}{N}\right) \quad (58)$$

Finally, after finding the STFT coefficients, we can estimate the value of the sound pressure level. Reference [13] shows that the sound pressure level from the complex Fourier transform coefficients is

$$L_p(f) = 10 \log_{10} \left(\frac{\frac{1}{2} |P(f)|^2}{p_{\text{ref}}^2} \right) \quad (59)$$

where p_{ref} is a reference pressure taken to be 20 μPa .

After all the acoustic datasets have been transformed to sound pressure levels from the WAV files, we need to determine the level of consistency that we can achieve using our modeling methods and the obtained experimental results. If there was a microphone on-board the UAV, then we could use that as a true source signal p_s and use our methods to propagate the signal to other microphones $\hat{p}_{d_i}$ and compare the propagated signal to the experimental recording p_{d_i} using some loss function $\mathcal{L}$. Good modeling methods would have good agreement, i.e. a low loss function value, between $\hat{p}_{d_i}$ and p_{d_i} . However, we do not have this truth signal so instead we must test the consistency between the estimated source signals $\hat{p}_{s_i}$ from the microphone d_i using modeling methods. The modeling methods that we have so far discussed in previous sections require us to choose a ground model and speed of sound profile but yield an invertible transfer function from the source to any point in space. However, the ground impedance and speed of sound profile modeling often depend on environmental descriptions that we do not immediately have access to such as the wind profile and ground specific flow resistivity. Hence we have decided to use an optimization loop to include estimating these environmental parameters as well. The flow of the optimization setup is described in Fig. 33. Ideally if the modeling methods are accurate, optimizing over

various microphone sites will give a very consistent optimized parameters with the the estimated source sound pressure signals being very consistent with each other. In this case, then we can use the "average" source signal to augment the dataset by either creating artificial detector signals at different locations using the same optimal parameters or changing the optimal parameters so simulate a different environment.

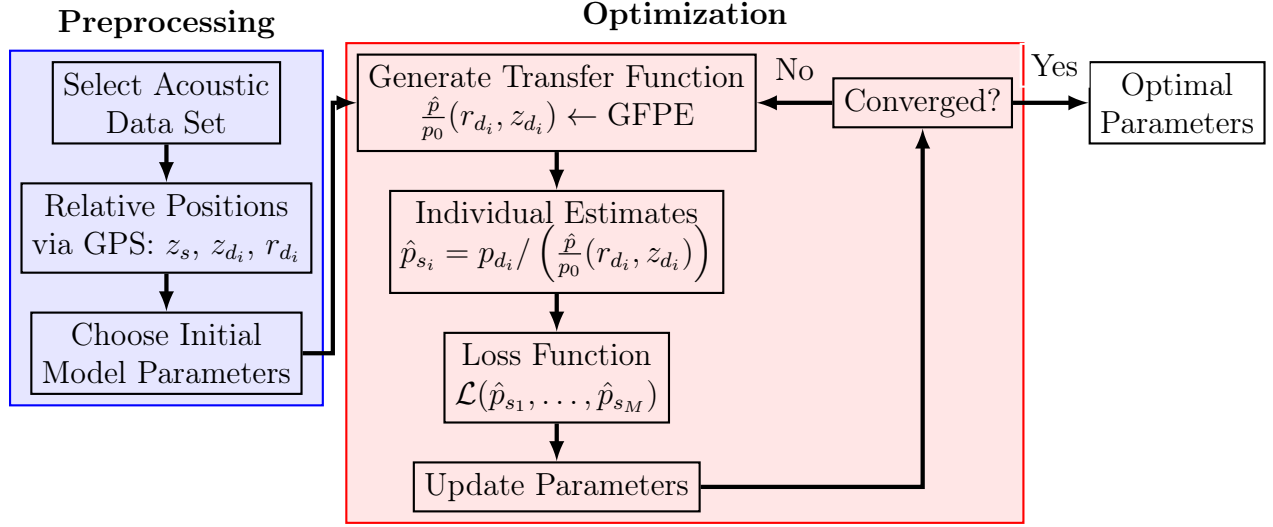


Figure 33: Flow chart for the optimization logic of determining environmental parameters

3.4.1 Anechoic Chamber

The Anechoic Chamber dataset contains the acoustic recordings of multiple small radio-controlled multirotor UAVs of different sizes and propellers. There are four microphones in chamber at different horizontal positions and elevation relative to UAV as shown in Figure 34. The UAVs were recorded doing various controls for maneuvers done multiple times, albeit the UAVs were constrained to one position. The data from microphones are in WAV files which contain the normalized pressure fluctuations. To get back the pressure in Pascals, we would need the normalizing factor or a reference calibration sound file. For this purpose we use Eq. (55).

For each WAV file available for a given UAV and maneuver, we find the spectrogram of the pressure and then average across time samples to get an average broadband acoustic frequency normalized sound pressure level recorded by that microphone during that test. After all tests are analyzed, we compare the microphone average sound pressure level against each other to get a sense of consistency between tests. The average of spectrograms for stationary processes is a reasonable procedure and we show two maneuvers are shown in Appendix F in Figures F.1 and F.2 to justify the claim of approximately stationary processes in the audio files. The half throttle maneuver was found to be consistently the least acoustically stationary maneuver across all UAVs while the full throttle maneuver was more representative of the general acoustic recordings. Most anechoic chamber recordings are fairly stationary meaning the frequency spectrum stays fairly constant.

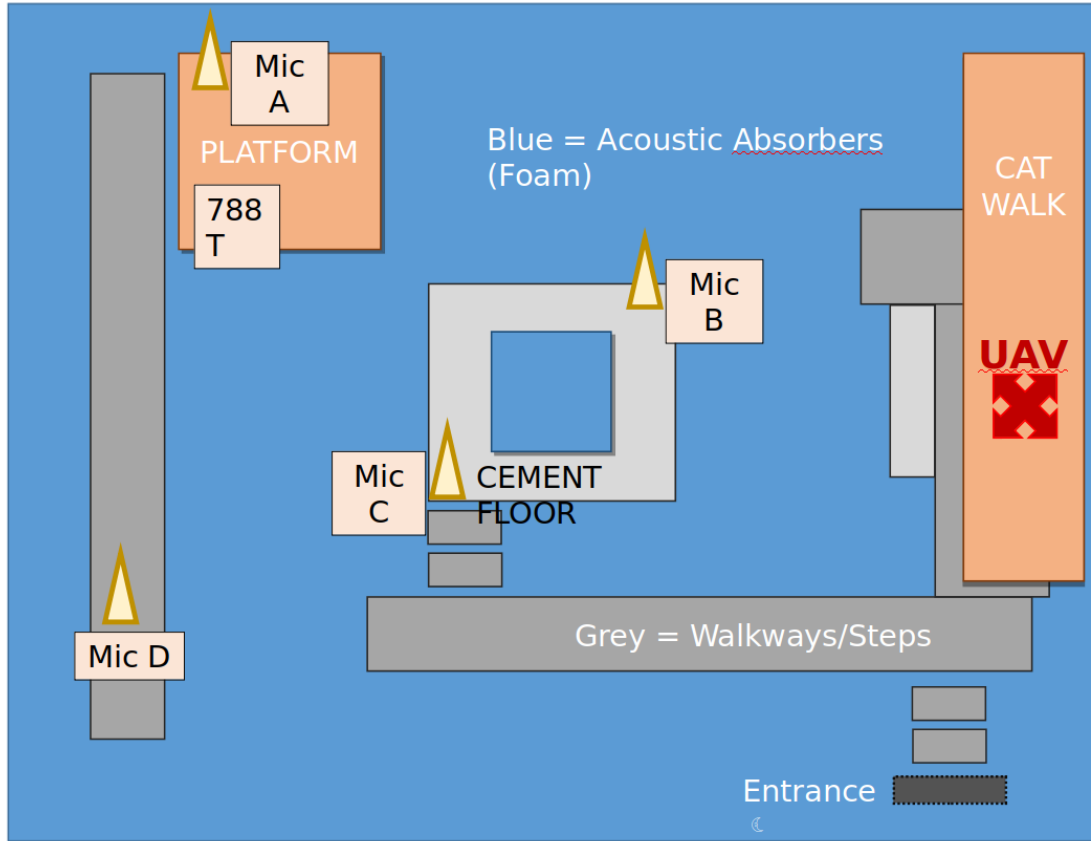


Figure 34: Representation placement of microphones and UAV during anechoic acoustic tests.

A simple microphone check for accuracy is to show the SPL corrected for spherical spreading of each microphone against each other. The spherical spreading in the anechoic chamber can be estimated by using an ideal free field monopole source shown in Eq. (60) where pressure decays inversely proportional to the distance between the UAV and the microphone. Thus, instead of plotting the SPL as defined by 59, we first multiply the Fourier Transforms by the distance as in Eq. (61) so that all microphone SPL levels should ideally be the same or at least similar levels.

$$p_c(t) = \frac{S \exp(-jk_r r)}{r} \quad (60)$$

$$L'_p(f) = 10 \log_{10} \left(\frac{\frac{1}{2} |P(f)r|^2}{p_{\text{ref}}^2} \right) \quad (61)$$

The estimated acoustic spectral density for the two maneuvers mentions are shown in Figures F.3 and F.4. Colors that are opaque are individual recording runs while the solid lines are the averages across all runs of the same maneuver. All microphones underwent a simple correction for spherical spreading to account for the different distances the microphones were from the source. This correction bring the microphones to a reasonable agreement although there is some behavior differences between the microphone. This suggest there could be

some nonuniform acoustic sound dispersion or field patterns within the anechoic chamber. While we are unable to give accurate sound power levels without knowing the normalization that went into creating the WAV files or the sound calibration file, the estimated acoustic spectral densities can be used to gain some knowledge of the drone spectrum. For example, there is consistently a noticeably locally acoustic peak at around 1 kHz for both half and full throttle for the DJI S1000 UAV examined. This feature can be used for quality checks in the later Section 3.4.2.

3.4.2 ESCAPE 1

This audio data is from ESCAPE1 which occurred in June of 2018 in an outdoor environment. The position of all microphones and UAVs used to generate this dataset are measured by GPS. We will estimate the relative geometry between the UAV and microphone sites using GPS data. The UAV location will be assumed to be static by averaging its position during the experimental test. This is a reasonable assumption as the runs considered for analysis have the UAV hovering. Similar to Section 3.4.1, the acoustic data is in the form of audio WAV files from microphones at a distance from the UAV. Multiple microphones "sites" are given which hosts arrays of condenser microphones. The sites used are S0 and S2 which have four and five channels respectively. The two sites do record at different sampling rates, with S0 recording at 25 kHz while S2 records at 48 kHz. There is no truth broadband acoustic spectrum given with this dataset. Therefore the analysis will attempt an inverse problem, estimating the quadcopter broadband acoustic spectrum from recordings taken at a distance. The UAVs used to generate this dataset are also present in the Anechoic Chamber dataset, which allows for some a priori knowledge of the acoustic spectrum. For example, the DJI S1000 UAV has noticeable SPL peaks around 1 kHz in the anechoic chamber which implies that good quality outdoor acoustic data for the DJI S1000 should have these same peaks. However, not all of the data can be used as some recordings have multiple UAVs at the same time. While it is possible to model both UAVs as the acoustics should be additive under linear acoustics, it makes the inverse problem of reconstructing UAV acoustic spectrum from microphone recordings unnecessarily difficult.

The only runs that are being considered are "s03r03-2" and "s03r04-3". The analysis will use all microphones in a site and invert their recording sound pressure level to estimate the sound pressure level at the UAV location. The idea is that if the appropriate method is used to simulate a unitary acoustic source, then inverting the microphone response gives the true UAV acoustic spectrum. It is unclear a priori if the acoustic inversions from both sites will converge to the same spectrum level as the normalization for the WAV file may be different. Rather than matching level, we wish that the inversions match the nominal shape, with perhaps a vertical shift in the logarithmic power scale.

3.4.2.1 Spherical Spreading

The monopole acoustic source in a neutral atmosphere has an acoustic spreading loss inversely proportional to the distance. Hence, the first phenomenon to account for is spherical spreading much like what was done for the anechoic chamber analysis. An aggregation of the two runs is shown in Figure 35.

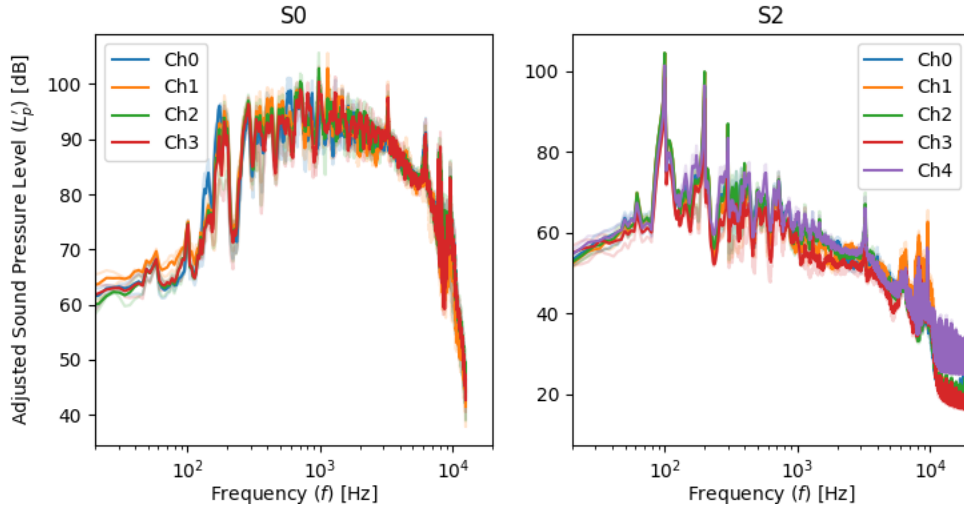


Figure 35: Sound pressure levels for all channels for sites S0 and S2 after correcting for spherical spreading. An average is shown by thick lines. The SPL has been corrected for distance.

Both sites have similar consistency across the channels and the same normalized levels. In particular, both sites share a prominent frequency response near 1 kHz as seen in the anechoic chamber. However, S0 has a higher spectral content above 300 Hz. This is likely due to the fact that it was closer to the UAV both times in the runs, being at rough distances of 30 meters versus 10 meters and 70 meters versus 40 meters. It is well known that acoustic noise attenuates as it travels through the atmosphere, with high frequency attenuating more. By modeling the attenuation, we can correct for the loss of acoustic spectrum due to the absorbing media. We hope to bring S0 and S2 into reasonable agreement with each other by introducing more acoustic phenomenon such as a reflective ground, temperature gradients, etc. using GFPE and FFP. A better agreement would indicate more accurate acoustic modeling techniques.

3.4.3 ESCAPE 2

The Escape 2 dataset also contains WAV files but we have been provided with some calibration files for determining the microphone gains. The provided calibration files contains four channels of sound from a white noise generator recorded in an anechoic chamber. All preamplifier settings and microphones are the same as used in the ESCAPE 2 dataset. The octave averaged sound pressure level of the white noise is given in Table 6 while the level of the WAV files are shown in Figs. 36 and 37.

Table 6: Octave Averaged SPL provided by AFRL

f_c	31.5 [Hz]	63 [Hz]	125 [Hz]	250 [Hz]	500 [Hz]
L_p	56 [dB]	48 [dB]	59 [dB]	60 [dB]	51 [dB]
f_c	1 [kHz]	2 [kHz]	4 [kHz]	8 [kHz]	
L_p	50 [dB]	54 [dB]	56 [dB]	47 [dB]	

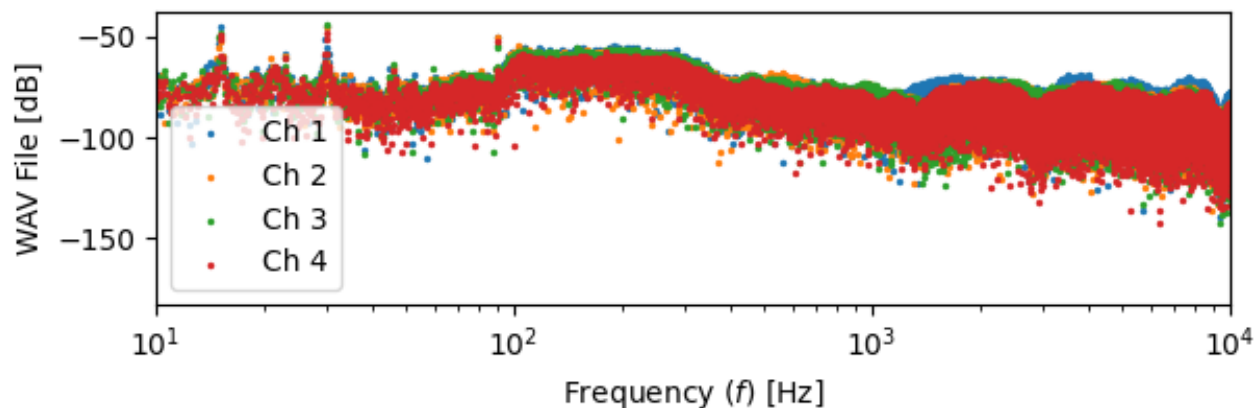


Figure 36: Fourier Transform of the Signal

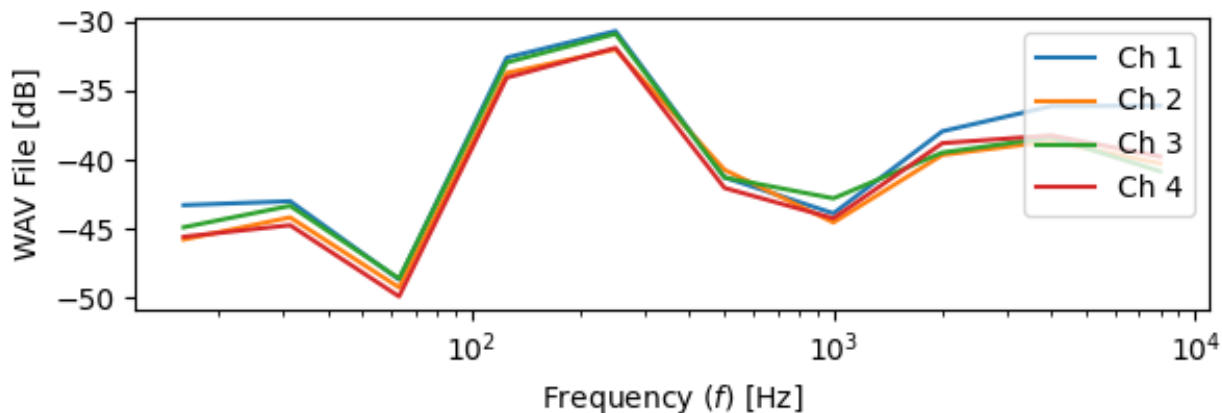


Figure 37: Octave Averaged Fourier Transform of the Signal

Figure 36 shows that the white noise generator is fairly consistent with a true white noise where we expect a flat plateau across the frequency scale. Given this, we can find the appropriate gains by determining the vertical shift in the decibel scale such that the recording levels within the WAV files after the shift match the reported sound pressure level in Table 6. This gives the octave specific microphone gains given in Fig. 38 and an average given in Table 7. The microphones are fairly consistent with each other below 500 Hz but at higher frequencies they become less consistent.

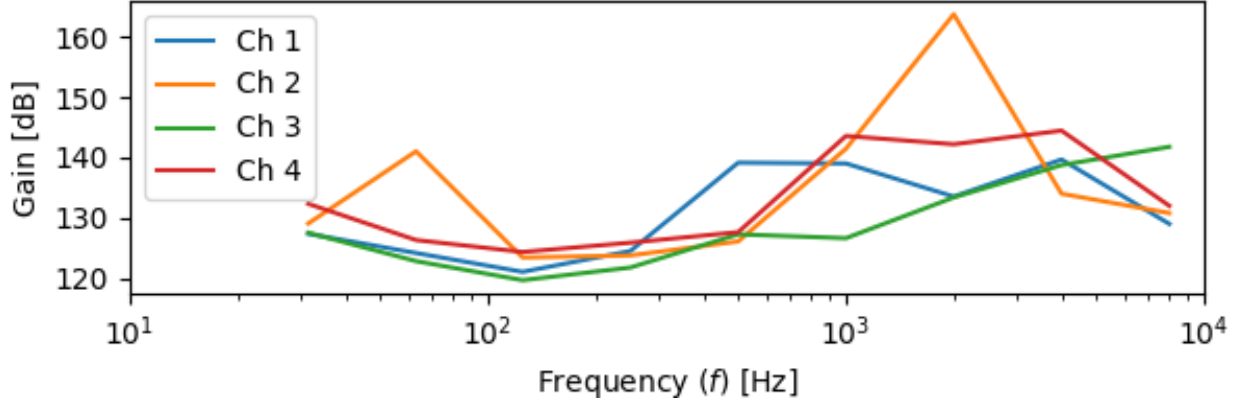


Figure 38: Microphone Gains over Octaves

Table 7: Determined Octave Gains

f_c	31.5 [Hz]	63 [Hz]	125 [Hz]	250 [Hz]	500 [Hz]
K	129.1 [dB]	128.5 [dB]	122.1 [dB]	123.9 [dB]	130.0 [dB]
f_c	1 [kHz]	2 [kHz]	4 [kHz]	8 [kHz]	
K	137.7 [dB]	143.2 [dB]	139.2 [dB]	133.4 [dB]	

IV. Conclusions

This study aimed to establish an acoustic modeling (CI) that can simulate the sound propagation in real-life environments and provide tools to generate realistic datasets for training the deep learning algorithms. To this effect, we established implementations of 2D FFP and GFPE algorithms that allow for long range sound propagation given various atmospheric profiles and ground impedance models. Additionally, we have shown two alterations to GFPE, the exact starting field and the 3-dimensional version. While relatively simple, the two dimensional FFP and GFPE implementations will allow for a baseline performance to be established for finding improvements with other methods. All of the validation tests for our developed tools indicate that the current implementation of the GFPE and FFP are consistent with results from previous works. However, all of the validation tests were against simple environments and sound models used in previous literature.

This study also examines some theoretical explorations of the effects of the acoustic modeling. We investigated how the acoustic models and parameter values affect the propagated broadband frequency spectrum for an example broadband frequency spectrum of a UAV. We determined where spatially in the environment and in what frequency bands the reflective ground and atmospheric refraction have noticeable contributions to any recorded acoustic signal. Additionally, we examined the possibility of estimating acoustic parameter values from recorded measurements using optimization. It was found that one could achieve some success estimating these parameters given vertical arrangement of microphones and sufficient downrange distance. For close downrange distance from the source, one may be able to use

nominal or reasonable parameters to then augment the dataset by modeling the long range sound propagation.

In addition to all of the theoretical validation, we have also started the analysis of modeling against real world experiments. Using data obtained from both an anechoic chamber and outdoor recordings, we have begun to develop understanding of the acoustic spectral densities of various UAVs as well as established a baseline of improvement by using our modeling methods developed. However, there were some difficulties discovered working with experimental data. Certain significant acoustic parameters are often seldom recorded and there is no "truth" recording of the UAV acoustic signal coming from onboard the UAV. Instead, one may have to estimate a "truth" signal which introduces more errors into the acoustic propagation modeling. The recommendation for future datasets involving UAV acoustic data is to include a microphone onboard the UAV to provide a baseline signal for acoustic propagation.

Continual and future comparisons of performance of the methods against experimental results will allow for identification of regions of inaccuracies or shortcomings of the methods that may be overcome with different methods, such as other frequency methods, temporal methods, or even machine learning approaches. There are some hybrid methods shown in previous work combining FFP and GFPE to overcome these angle limitations and we hope to replicate this with our current implementations of FFP and GFPE. Another method that may be advantageous to combine with the GFPE method is ray tracing. The results from validation showed that capturing the highest arcing acoustic ray is critical for accuracy of results. While there are some analytical results for simple speed of sound profiles, complicated environments may require some initial ray tracing to ensure that all relevant acoustical rays are captured. Additionally, there are some advantages to be had by incorporating more 3D effects into the developed tools. By going into three dimensions, we can start to incorporate different wind profiles such as crosswind effects in GFPE or more complex and detailed terrain maps than currently allowed by rotating rectangular domains or conformal mapping.

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Appendices

Appendix A. Derivation of the Direct Matrix Method

This appendix shows the derivation of the Direct Matrix Method presented by [18]. The derivation follows the procedure of [24] which also considers a shear wave in the acoustic medium. We start by considering the Fourier-Bessel transform of the i th layer and the corresponding wave equation.

$$\begin{bmatrix} p(r, z) \\ w(r, z_i) \end{bmatrix} = \int_0^\infty \begin{bmatrix} P(k_r, z) \\ W(k_r, z) \end{bmatrix} J_0(k_r r) k_r dk_r \quad (\text{A.1})$$

$$\frac{d^2 P}{dz^2} + (k_{\text{eff}}^2 - k_r^2) P = -2\delta(z - z_s) \quad (\text{A.2})$$

Given the layer definitions shown in Figure 11, there are three important regions: the upper boundary; the source height and its velocity discontinuity; and the bottom boundary. Looking at infinitesimal distances above and below the source will show the particle velocity discontinuity as a linear equation

$$\begin{bmatrix} P(z_s + \epsilon) \\ W(z_s + \epsilon) \end{bmatrix} = \begin{bmatrix} P(z_s - \epsilon) \\ W(z_s - \epsilon) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-2j}{\omega \rho_s} \end{bmatrix}.$$

Noting the discontinuity at the source layer, we can determine the top boundary conditions by means of a wave moving from the bottom boundary to the top boundary as well as a contribution from the source. This is done using transfer matrices of each layer,

$$\begin{aligned} \begin{bmatrix} P(z_N) \\ W(z_N) \end{bmatrix} &= d_N d_{N-1} \cdots d_{s+1} \begin{bmatrix} P(z_s + \epsilon) \\ W(z_s + \epsilon) \end{bmatrix} \\ &= d_N d_{N-1} \cdots d_{s+1} \left\{ \begin{bmatrix} P(z_s - \epsilon) \\ W(z_s - \epsilon) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-2j}{\omega \rho} \end{bmatrix} \right\} \\ &= d_N d_{N-1} \cdots d_{s+1} \left\{ d_s d_{s-1} \cdots d_1 \begin{bmatrix} P(z_0) \\ W(z_0) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-2j}{\omega \rho} \end{bmatrix} \right\} \\ &= \underbrace{d_N d_{N-1} \cdots d_1}_D \begin{bmatrix} P(z_0) \\ W(z_0) \end{bmatrix} + \underbrace{d_N d_{N-1} \cdots d_{s+1}}_{D_s} \begin{bmatrix} 0 \\ \frac{-2j}{\omega \rho} \end{bmatrix} \end{aligned}$$

Next is solving for the unknown conditions at the boundaries. Initially it appears that both the pressure and wave propagation speed are unknown but the impedance model specification will allow us to specify the boundaries in terms of an unknown wave vertical velocity and a known characteristic impedance. Thus we have two unknowns and can use a linear system of equations to determine the boundary values.

$$\frac{-2j}{\omega \rho} \begin{bmatrix} (D_s)_{12} \\ (D_s)_{22} \end{bmatrix} = W(z_N) \begin{bmatrix} \zeta(z_N) \\ 1 \end{bmatrix} - W(z_0) D \begin{bmatrix} \zeta(z_0) \\ 1 \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} \zeta(z_N) & -\{\zeta(z_0)(D)_{11} + (D)_{12}\} \\ 1 & -\{\zeta(z_0)(D)_{21} + (D)_{22}\} \end{bmatrix} \begin{bmatrix} W(z_N) \\ W(z_0) \end{bmatrix} \\
W(z_N) &= \frac{2j}{\omega \rho_s} \cdot \frac{\{\zeta(z_0)(D)_{21} + (D)_{22}\}(D_s)_{12} - \{\zeta(z_0)(D)_{11} + (D)_{12}\}(D_s)_{22}}{\{\zeta(z_0)(D)_{11} + (D)_{12}\} - \zeta(z_N)\{\zeta(z_0)(D)_{21} + (D)_{22}\}} \\
W(z_0) &= \frac{2j}{\omega \rho_s} \cdot \frac{(D)_{12} - \zeta(z_N)(D)_{22}}{\{\zeta(z_0)(D)_{11} + (D)_{12}\} - \zeta(z_N)\{\zeta(z_0)(D)_{21} + (D)_{22}\}}
\end{aligned}$$

From here, and along with the boundary conditions, we can solve for the transformed pressure at the detector height. Since we have solved for both boundary conditions, we simply need to know if the detector is above or below the source height to use the appropriate boundary. This choice allows one to avoid the discontinuity in the source layer.

$$P_i = W(z_0) [\zeta(z_0)(D_d)_{11} + (D_d)_{12}], \quad D_d = d_d d_{d-1} \cdots d_1 \text{ if } z_i \leq z_s \quad (\text{A.3})$$

$$P_i = W(z_N) [\zeta(z_N)(D_d)_{11} + (D_d)_{12}], \quad D_d = (d_N d_{N-1} \cdots d_{d+1})^{-1} \text{ otherwise} \quad (\text{A.4})$$

Appendix B. Derivation of Optimal Starting Field Approximation

For determining the starting field, we consider the cylindrical corrected pressure q where $q = p\sqrt{r}$ and also $\psi = q \exp(jk_r r)$. For convenience we let $R = \sqrt{r^2 + z^2}$ be the distance from the source, which is at the origin for simplicity of analysis. The starting field with the source at the origin is denoted by q_0 which is also equal to ψ_0 . Under the assumption that q_0 can be written as a standing wave, we can express q_0 as

$$q_0(z) = \int_{-\infty}^{\infty} S(k_z) \exp\left(jk_z z + jr\sqrt{k_{\text{eff}}^2 - k_z^2}\right) dk_z$$

From the station phase method, we know that the main contributing point to the integral determination is when the argument of the exponential has a derivative of 0. This happens at critical wavenumber k_z^* which [13] shows is $k_z^* = \frac{z}{r}k_r \approx \frac{z}{r}k_{\text{eff}}$. Hence the function q_0 can be approximated as

$$q_0(z) \approx \frac{1}{\sqrt{r}} \sqrt{\frac{2\pi k_{\text{eff}}}{j}} \left(1 - \frac{z^2}{R^2}\right)^{3/4} S(k_z^*) \exp(jk_{\text{eff}} R)$$

Matching the analytical solution for a monopole source in homogeneous atmosphere $q = \sqrt{r} \exp(jk_{\text{eff}} R)/R$ with the Stationary Phase results determines the function S to be

$$S(k_z) = \sqrt{\frac{j}{2\pi k_{\text{eff}}}} \left(1 - \frac{k_z^2}{k_{\text{eff}}^2}\right)^{-1/4}.$$

There is a singularity as $k_z^2 \rightarrow k_{\text{eff}}^2$ i.e. $\gamma \rightarrow \pm 90^\circ$ which means that the exact integral cannot be solved. However, we can use functions of the following form to approximate the singularity of S .

$$\left(1 - \frac{k_z^2}{k_{\text{eff}}^2}\right)^{-1/4} \approx \left(\sum_{n=0}^{N/2} a_{2n} \left(\frac{k_z}{k_{\text{eff}}}\right)^{2n}\right) \exp(-k_z^2/b_2 k_{\text{eff}}^2)$$

Note that the asymptotic approximation is centered at $k_z = 0$ and by symmetry only even powers are needed to match the even symmetry of the integral bounds. By increasing the order of the asymptotic approximation, we can match the S up to higher elevation angles γ_{max} . There is another reason for this form of approximation and it is about the Fourier Transform. The special relation is that the Fourier Transform of a Gaussian is a Gaussian (which approximates a Dirac delta function) and there is a useful family of integrals with a recurrence relationship:

Let $I_n(z) = \int_{-\infty}^{\infty} k^n \exp(-k^2) dk$, then

$$I_0(z) = \int_{-\infty}^{\infty} \exp(-k^2) dk = \sqrt{\pi}$$

and by recurrence

$$\begin{aligned}
I_n(z) &= \int_{-\infty}^{\infty} k^n \exp(-k^2) dk \\
&= -\frac{1}{2} k^{n-1} \exp(-k^2) \Big|_{-\infty}^{\infty} + \frac{n-1}{2} \int_{-\infty}^{\infty} k^{n-2} \exp(-k^2) dk \\
&= \frac{n-1}{2} I_{n-2} = c_n \cdot 2^{-n/2} I_0 \text{ where } c_n = \prod_{i=1}^{n/2} (2i-1)
\end{aligned}$$

As the approximation of S is comprised of only even n , this integral family is ideal for handling the Fourier Transformation for relating approximation of S to the the starting field functions. Assuming an existing approximation, we can solve for q_0 :

$$\begin{aligned}
q_0(z) &= \sqrt{\frac{j}{2\pi k_{\text{eff}}}} e^{jrk_r} \int_{-\infty}^{\infty} \left(1 - \frac{k_z^2}{k_{\text{eff}}^2}\right)^{-1/4} e^{jk_z z} dk_z \\
&\text{Let } k_z/k_{\text{eff}} \rightarrow k_z \text{ for simplicity} \\
q_0(k_{\text{eff}} z) &= \sqrt{\frac{jk_{\text{eff}}}{2\pi}} e^{jrk_r} \int_{-\infty}^{\infty} (1 - k_z^2)^{-1/4} e^{jk_z k_{\text{eff}} z} dk_z \\
q_0(k_{\text{eff}} z) &\sim \sqrt{\frac{jk_{\text{eff}}}{2\pi}} e^{jrk_r} \int_{-\infty}^{\infty} \left(\sum_{n=0}^{N/2} a_{2n} (k_z)^{2n} \right) e^{jk_z k_{\text{eff}} z - b_2 k_z^2} dk_z \\
\xi &= \sqrt{b_2} k_z - \frac{jz k_{\text{eff}}}{2\sqrt{b_2}} \\
q_0(k_{\text{eff}} z) &\sim \sqrt{\frac{jk_{\text{eff}}}{2\pi}} e^{jrk_r} \int_{-\infty}^{\infty} \left(\sum_{n=0}^{N/2} a_{2n} (k_z)^{2n} \right) e^{jk_z z - b_2 k_z^2} dk_z \\
&= \sqrt{\frac{jk_{\text{eff}}}{2\pi}} e^{jrk_r - \frac{k_{\text{eff}}^2 z^2}{4b_2}} \sum_{n=0}^{N/2} \left[a_{2n} \int_{-\infty}^{\infty} \left(\frac{\xi}{\sqrt{b_2}} + \frac{jk_{\text{eff}} z}{2b_2} \right)^{2n} e^{-\xi^2} d\xi \right]
\end{aligned}$$

From here, we can construct a starting field as in (46) with coefficients $\{A_{2n}\}$ using specified $\{a_{2n}\}$ and the recursive relationship of I_n . This is slightly non-trivial as it involves a lot of combinations due to the recurrent nature of evaluating the integrals. What is not done here is how to find the optimal approximation for S in the frequency domain but there are numerous ways to come up with the approximations. Some examples are minimizations of the average squared error or term-by-term expansions but there is no mention of this in [13]. Instead this work will simply use the reported optimal coefficients as they give desirable results.

Appendix C. FFP Theoretical Comparison Plots

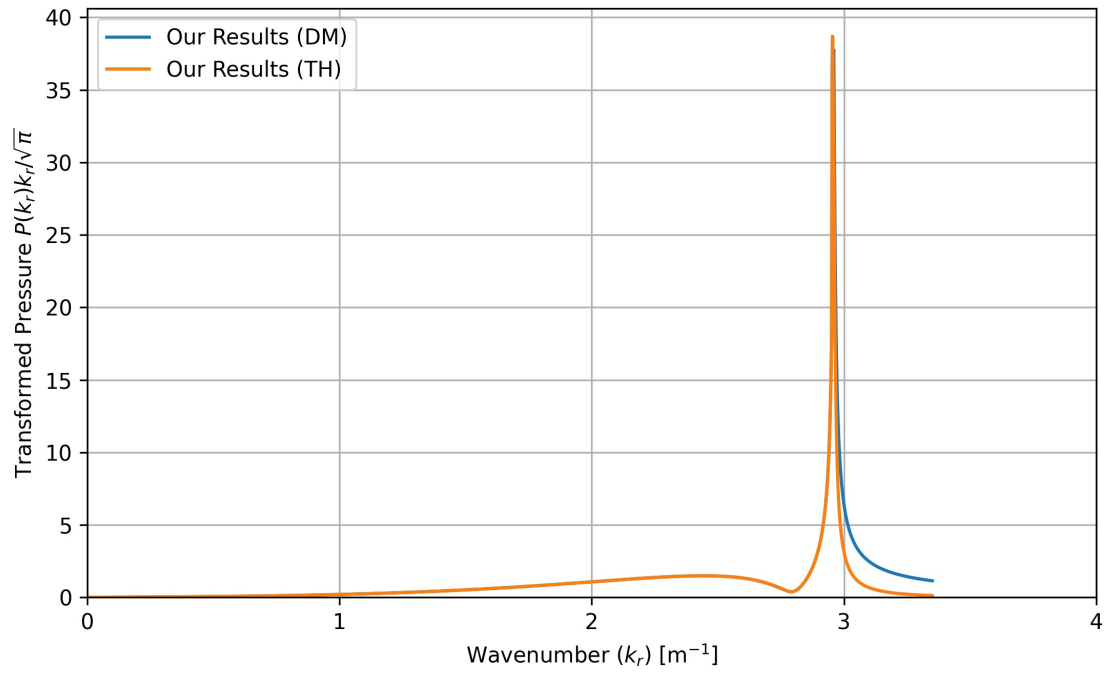


Figure C.1: Ref. [18] Case (b) Transformed Pressure Distributions

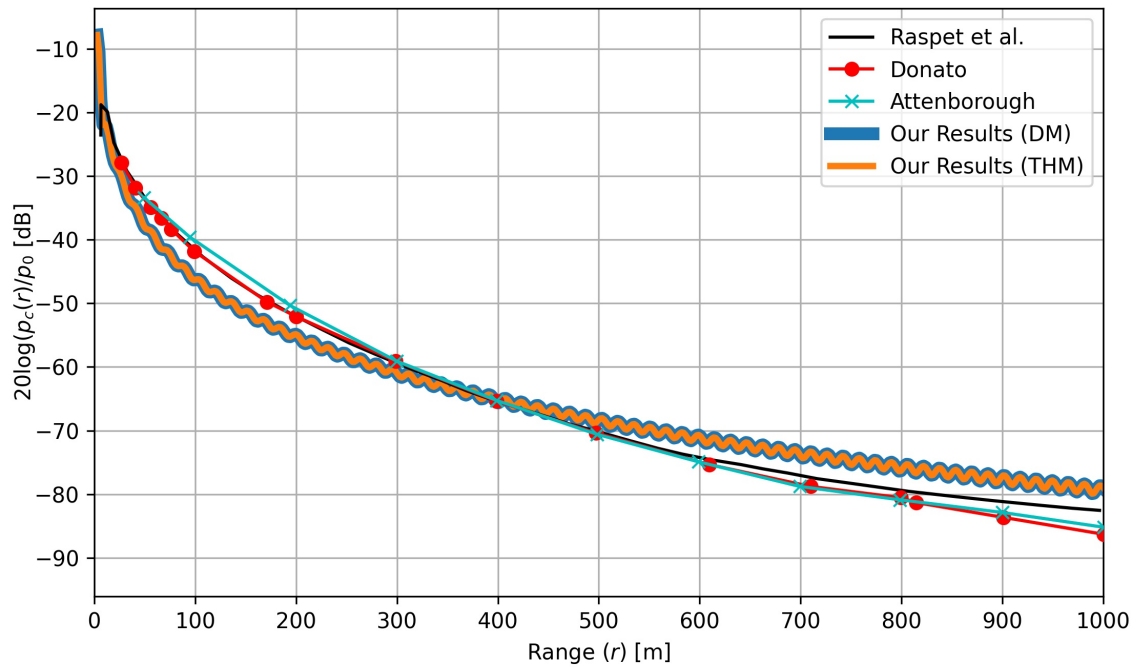


Figure C.2: Ref. [18] Case (b) Pressure Transmission Loss

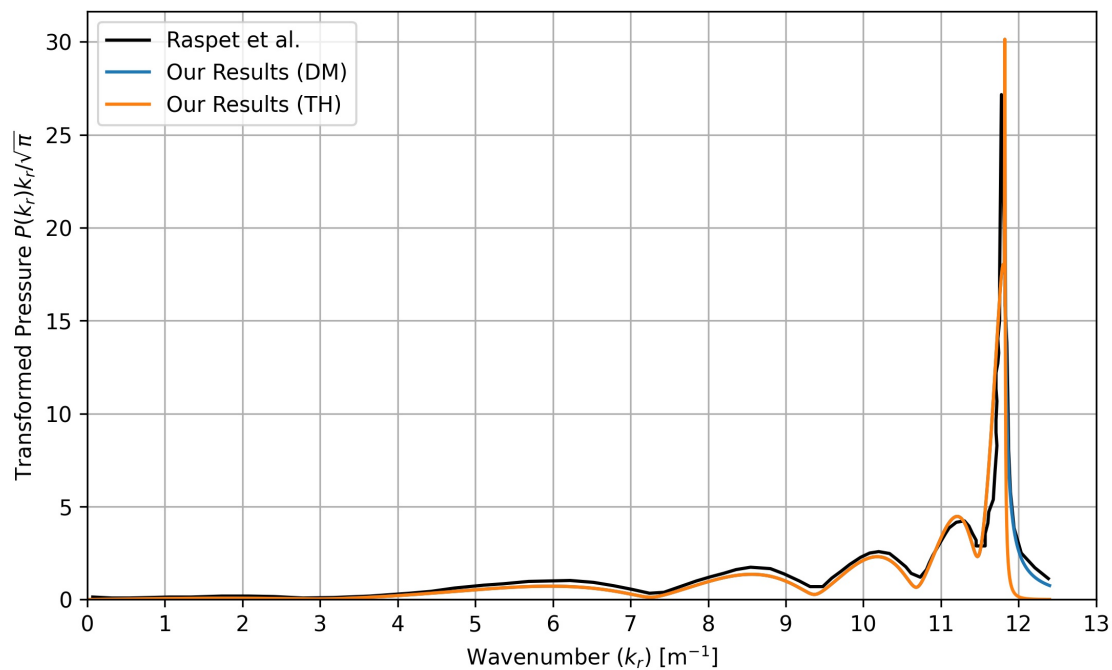


Figure C.3: Ref. [18] Case (c) Transformed Pressure Distributions

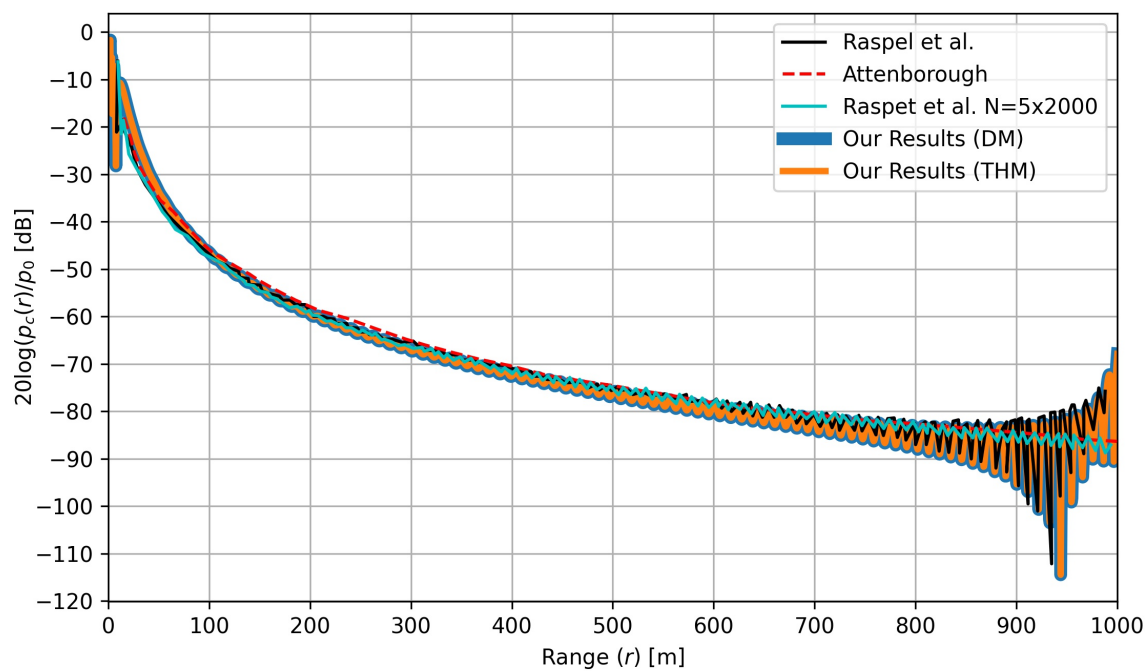


Figure C.4: Ref. [18] Case (c) Pressure Transmission Loss

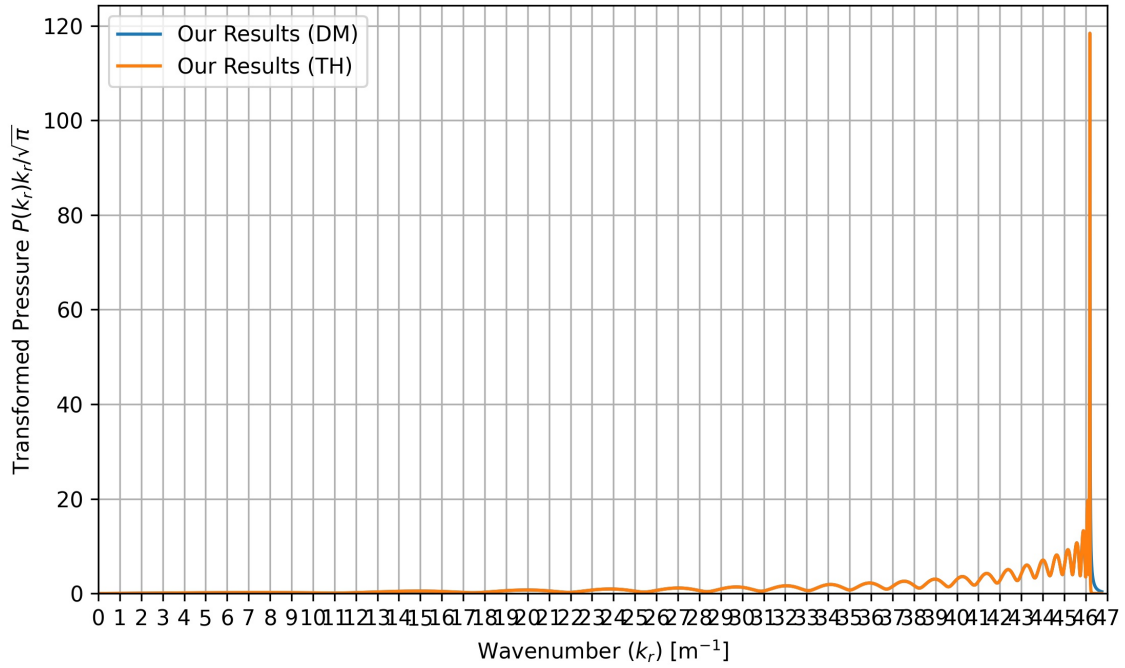


Figure C.5: Ref. [18] Case (d) Transformed Pressure Distributions

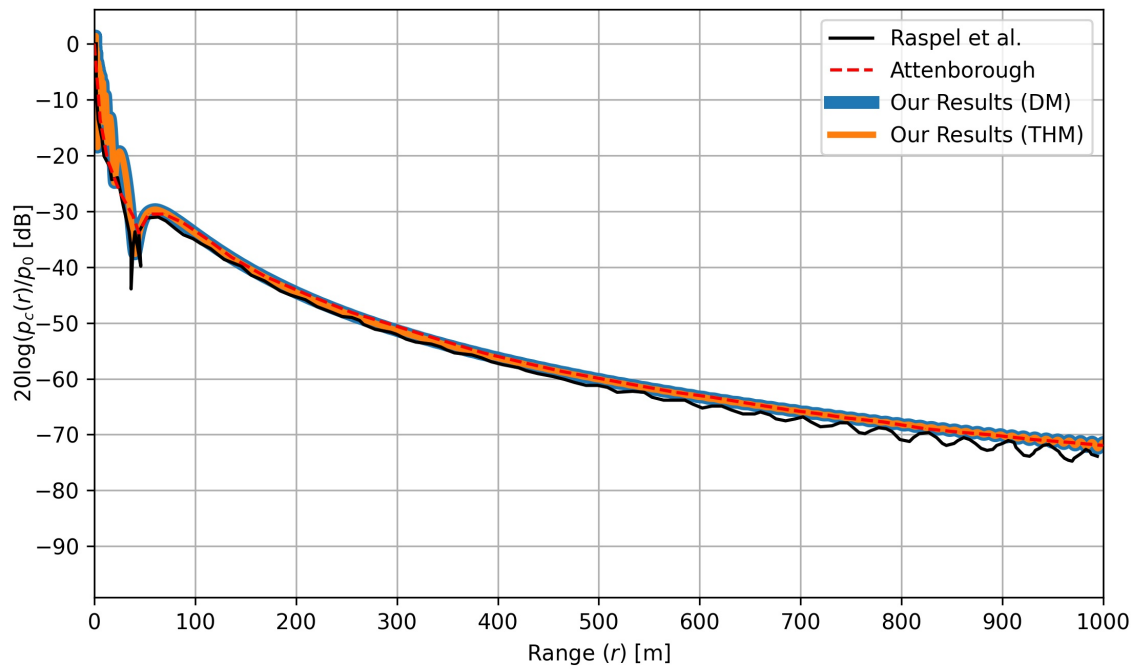


Figure C.6: Ref. [18] Case (d) Pressure Transmission Loss

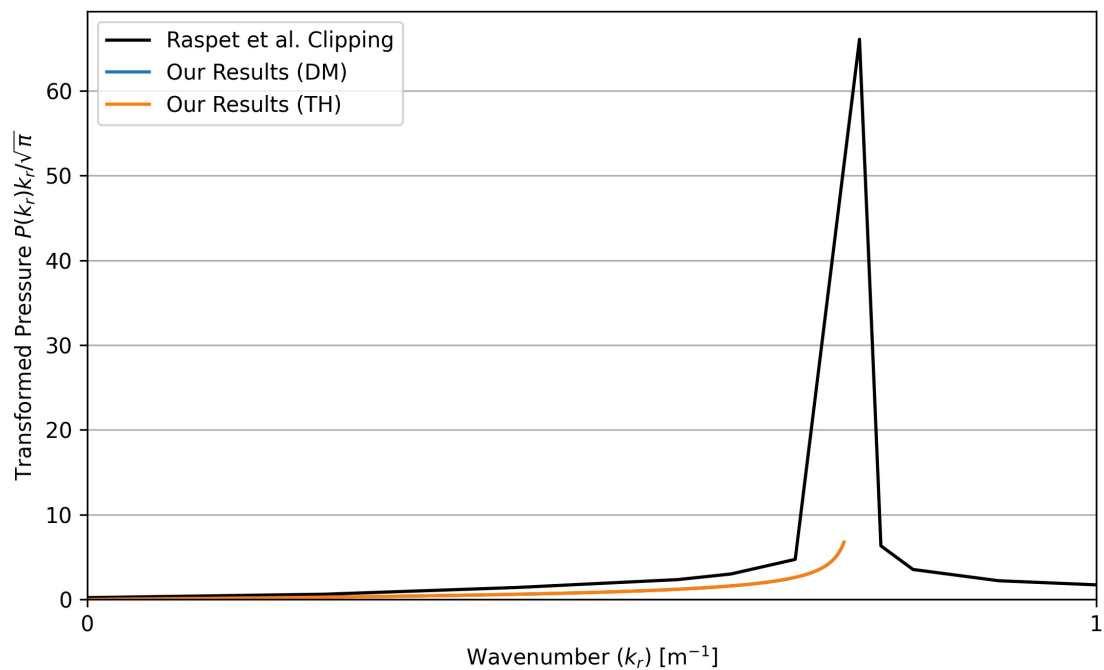


Figure C.7: Ref. [18] Case (e) Transformed Pressure Distributions

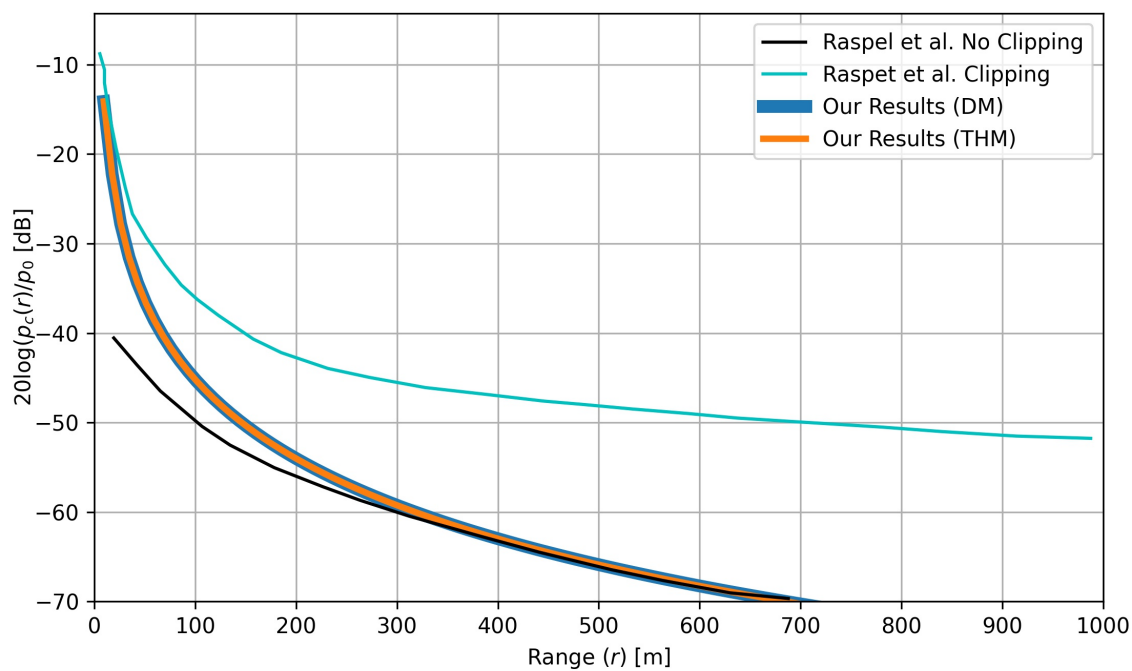


Figure C.8: Ref. [18] Case (e) Pressure Transmission Loss

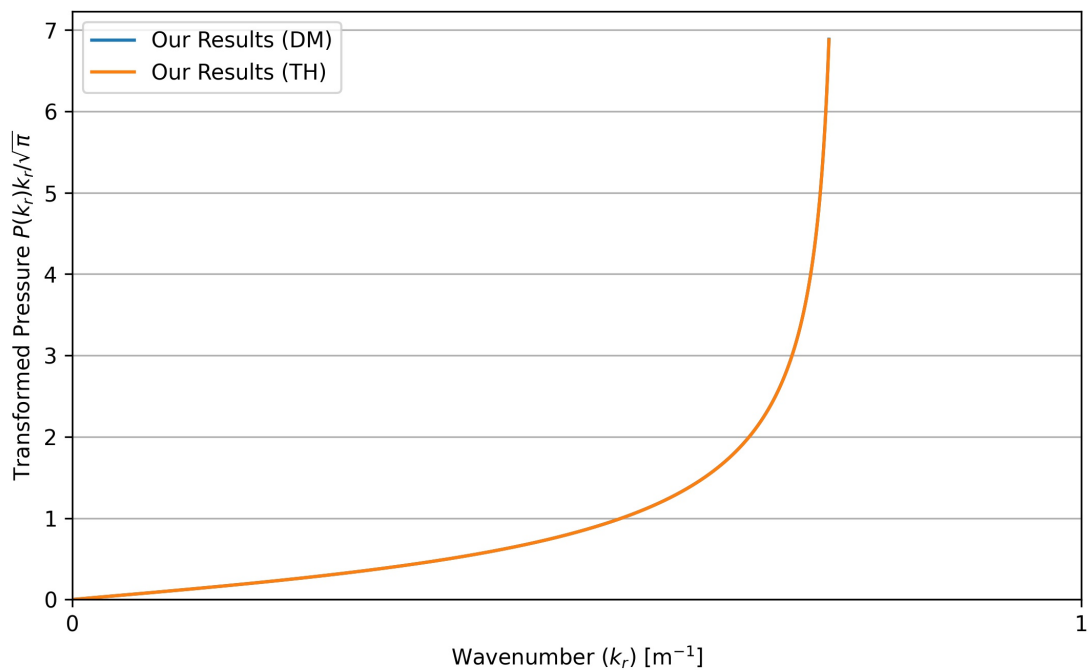


Figure C.9: Ref. [18] Case (f) Transformed Pressure Distributions

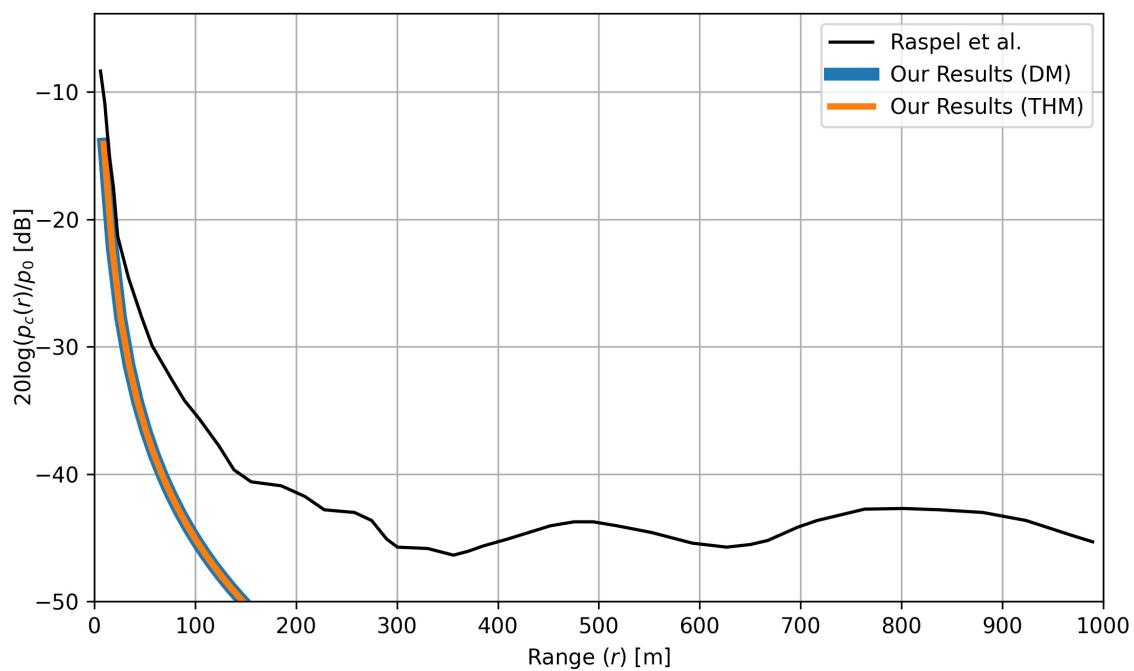


Figure C.10: Ref. [18] Case (f) Pressure Transmission Loss

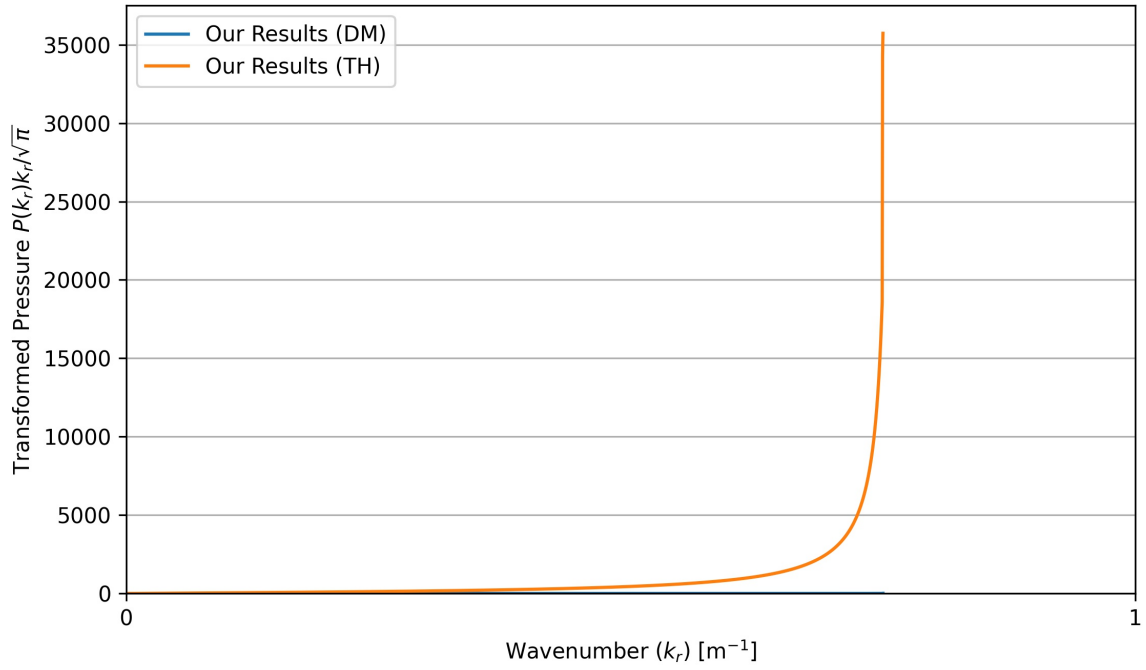


Figure C.11: Ref. [18] Case (g) Transformed Pressure Distributions

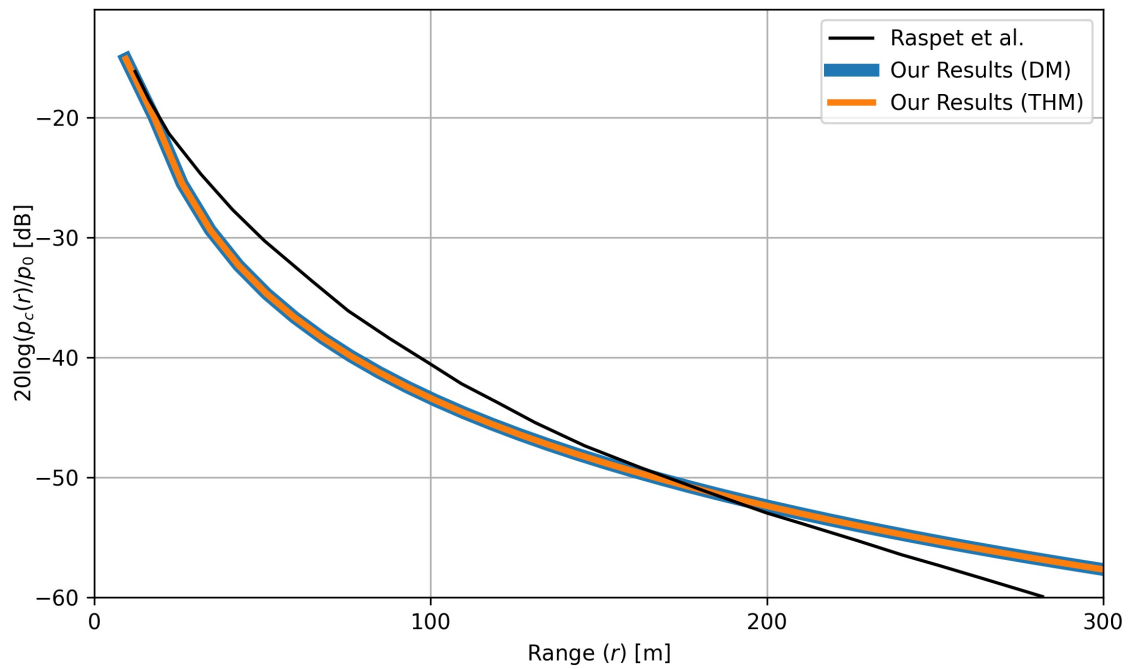


Figure C.12: Ref. [18] Case (g) Pressure Transmission Loss

Appendix D. GFPE Theoretical Comparison Plots

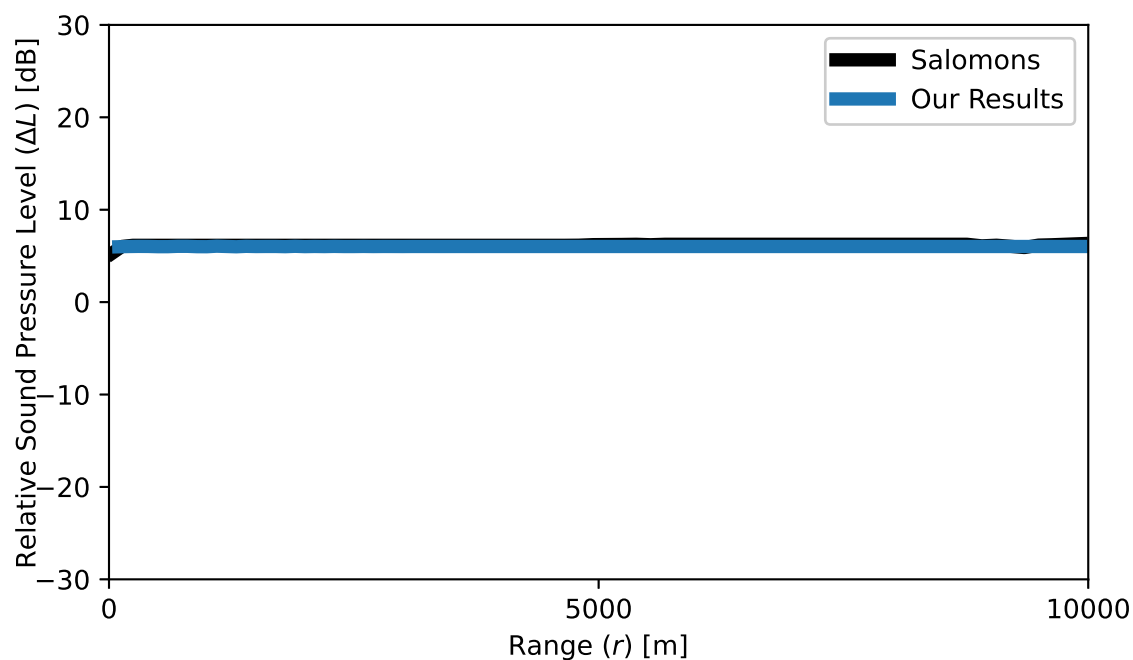


Figure D.1: Nonrefracting Atmosphere and Rigid Ground at 10 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

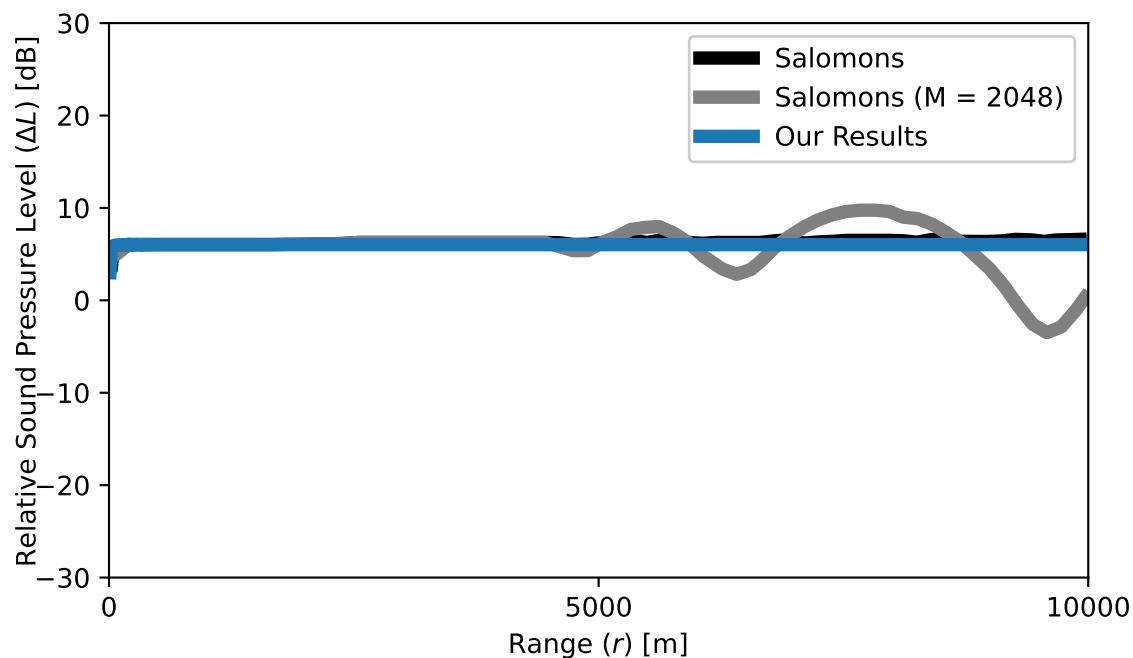


Figure D.2: Nonrefracting Atmosphere and Rigid Ground at 100 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

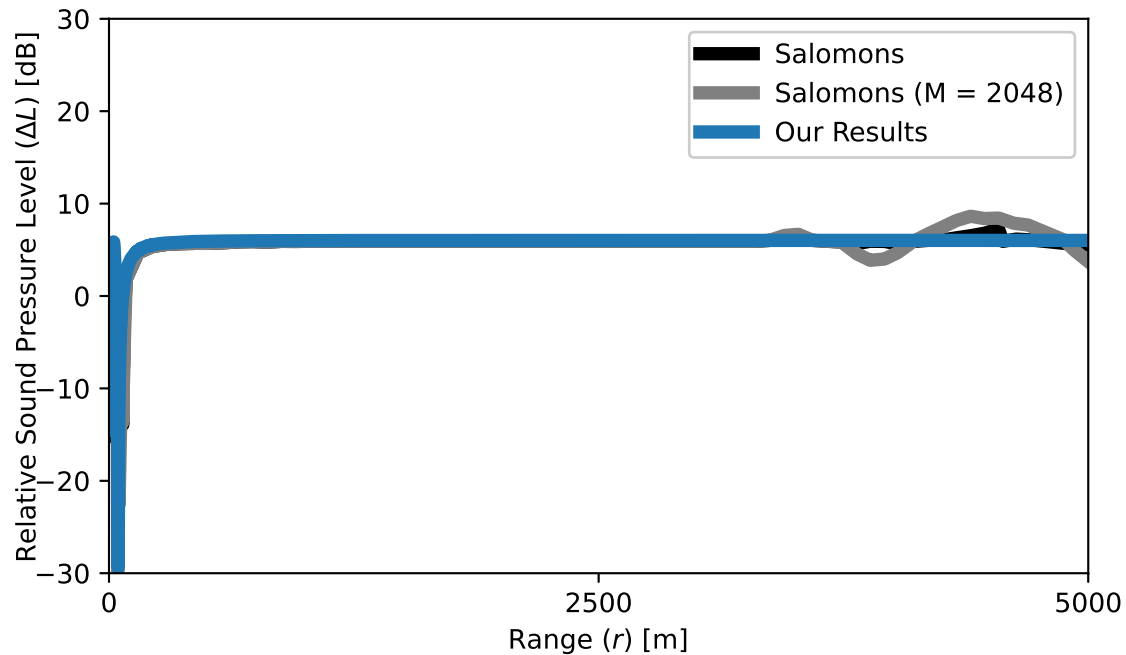


Figure D.3: Nonrefracting Atmosphere and Rigid Ground at 1,000 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

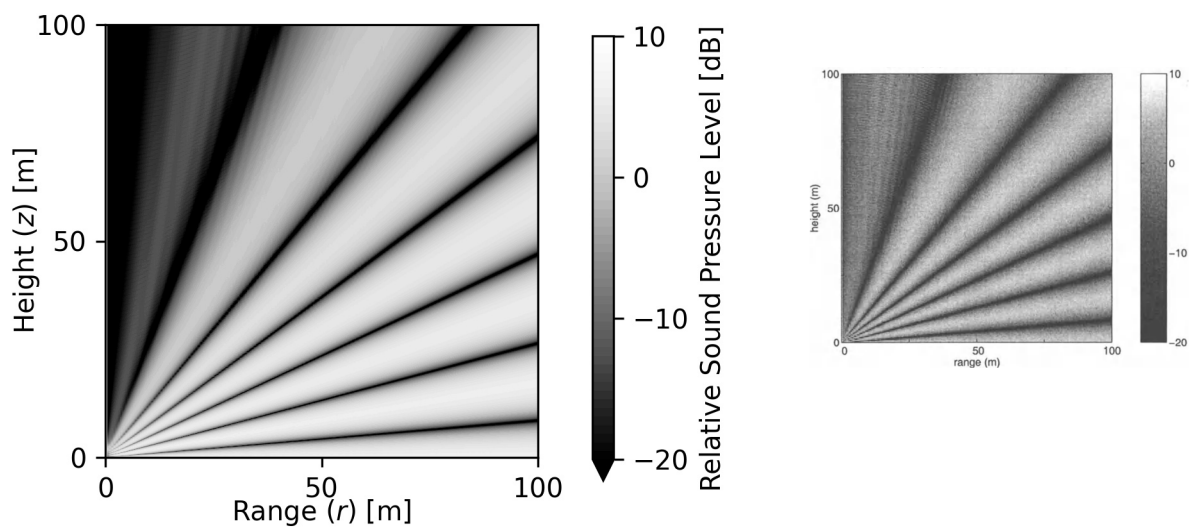


Figure D.4: 2D GFPE for Neutral Atmosphere with Rigid Ground. An acoustic source at $z = 2$ meters in a neutral atmosphere with an acoustically rigid ground. Our recreation of Figure 4.17 from [13] on left and original on the right.

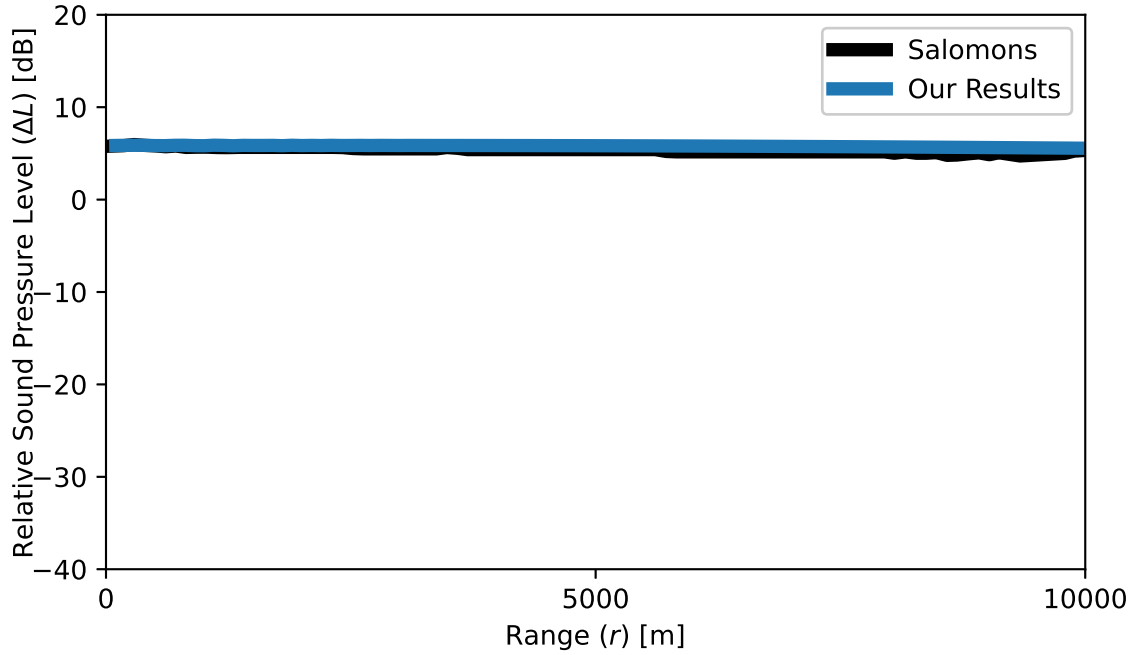


Figure D.5: Nonrefracting Atmosphere and Absorbing Ground at 10 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

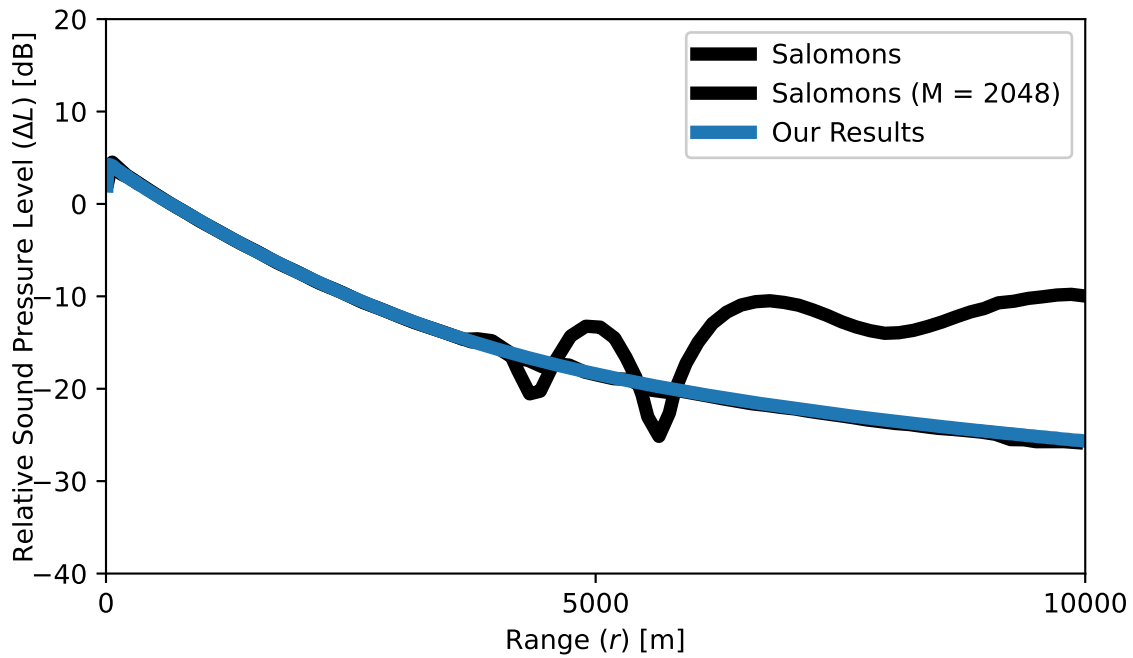


Figure D.6: Nonrefracting Atmosphere and Absorbing Ground at 100 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

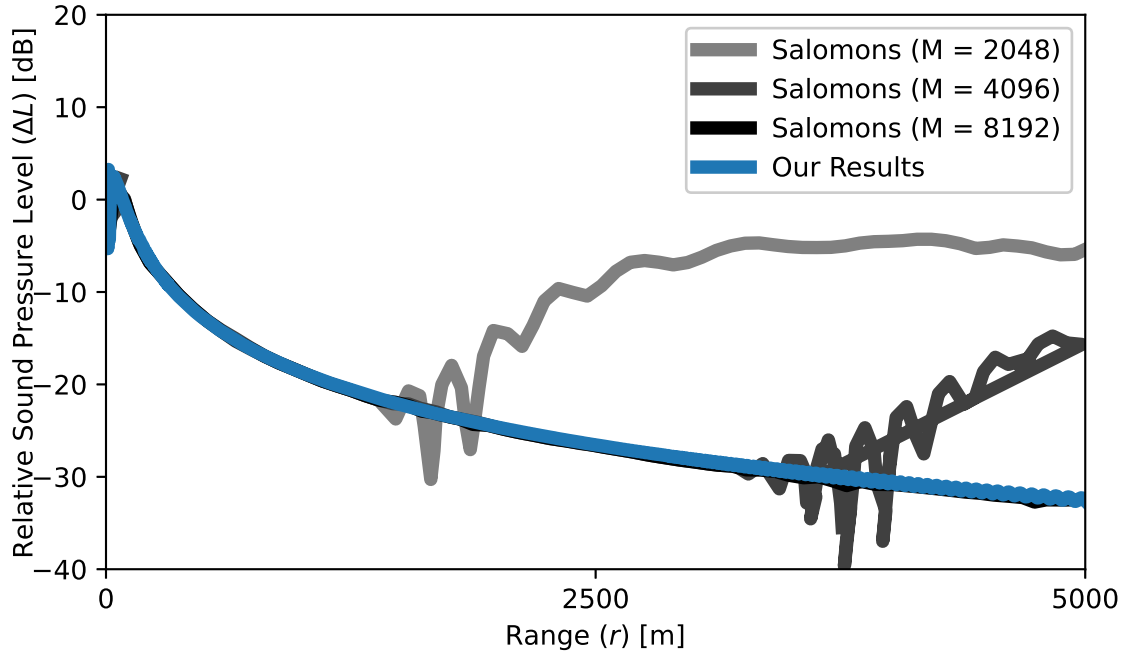


Figure D.7: Nonrefracting Atmosphere and Absorbing Ground at 1,000 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

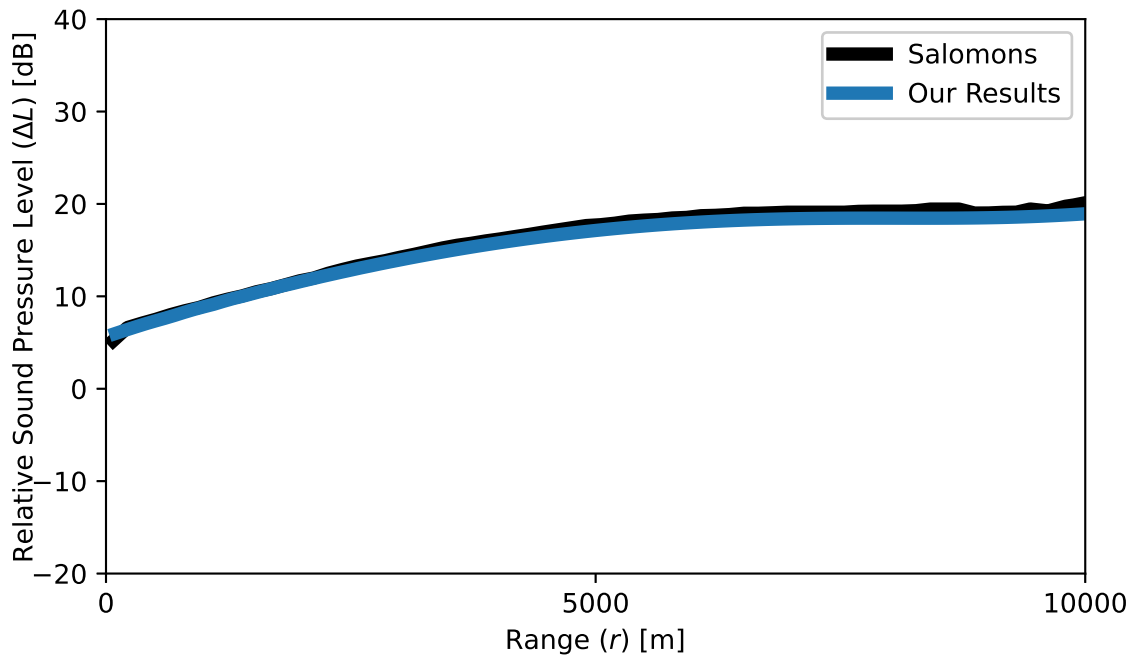


Figure D.8: Downwards Refracting Atmosphere and Rigid Ground at 10 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

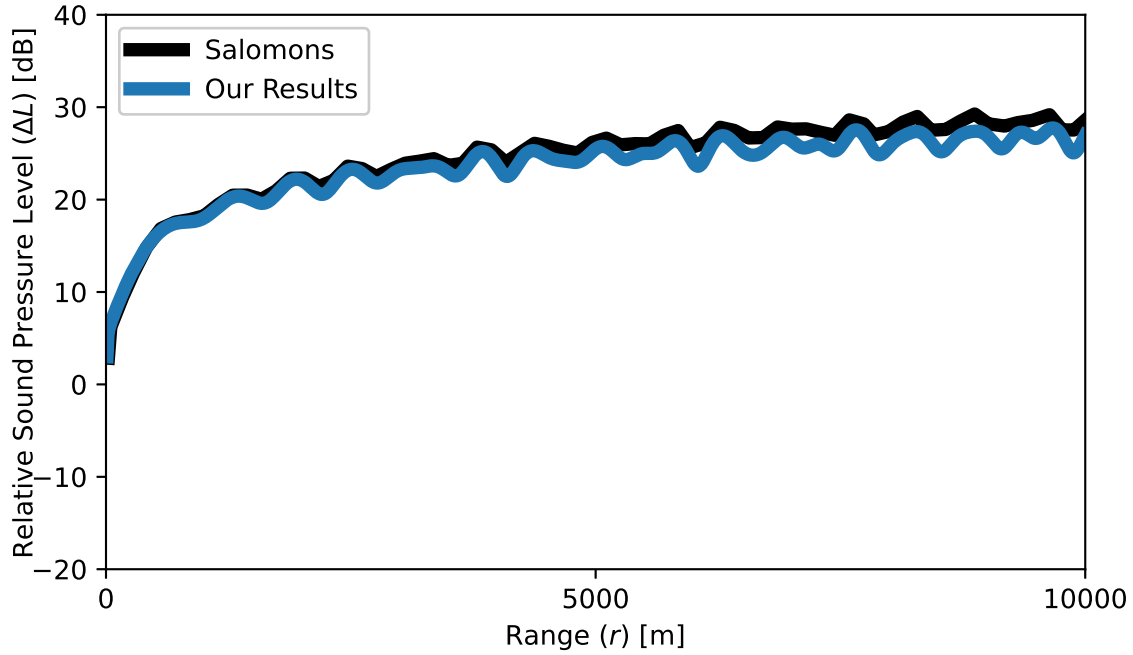


Figure D.9: Downwards Refracting Atmosphere and Rigid Ground at 100 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

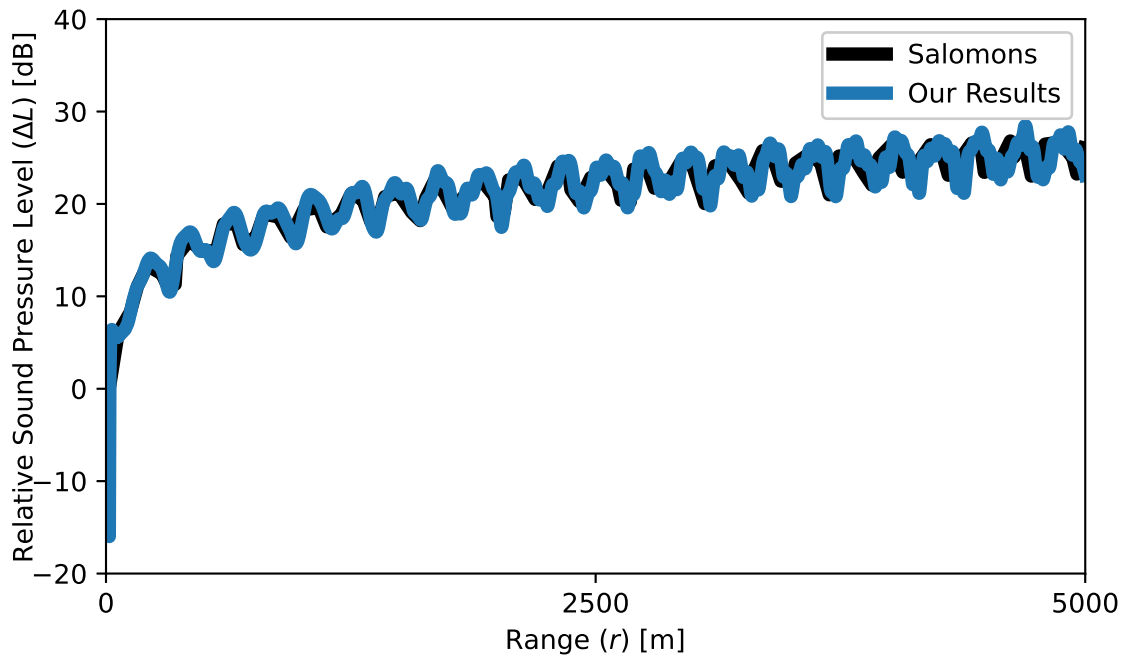


Figure D.10: Downwards Refracting Atmosphere and Rigid Ground at 1,000 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

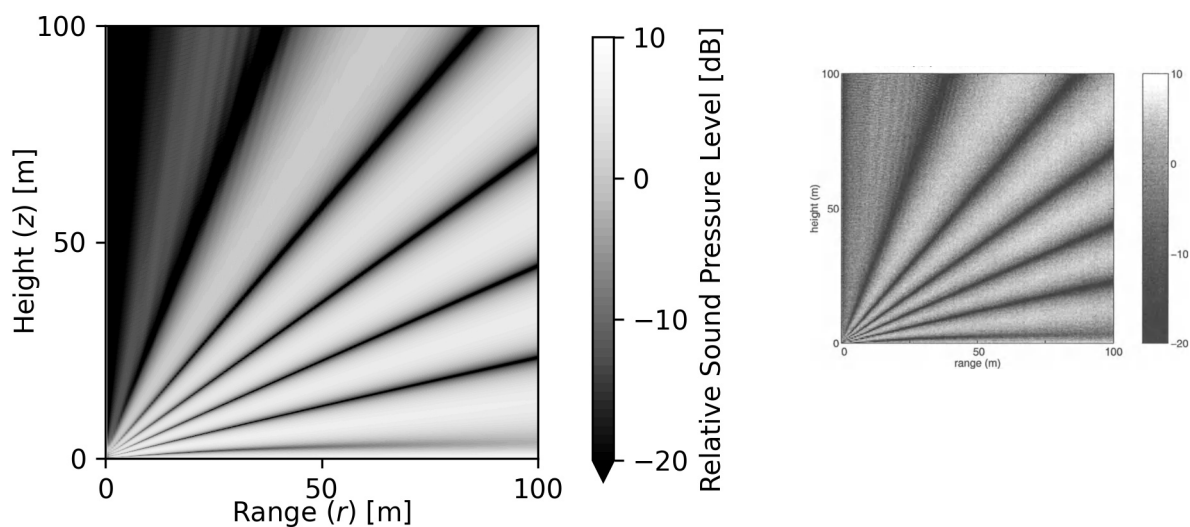


Figure D.11: 2D GFPE for Downwards Refracting Atmosphere with Rigid Ground. An acoustic source at $z = 2$ meters in a downwards refracting atmosphere with an acoustically rigid ground. Our Recreation of Figure 4.21 from [13] on the left and the original on the right.

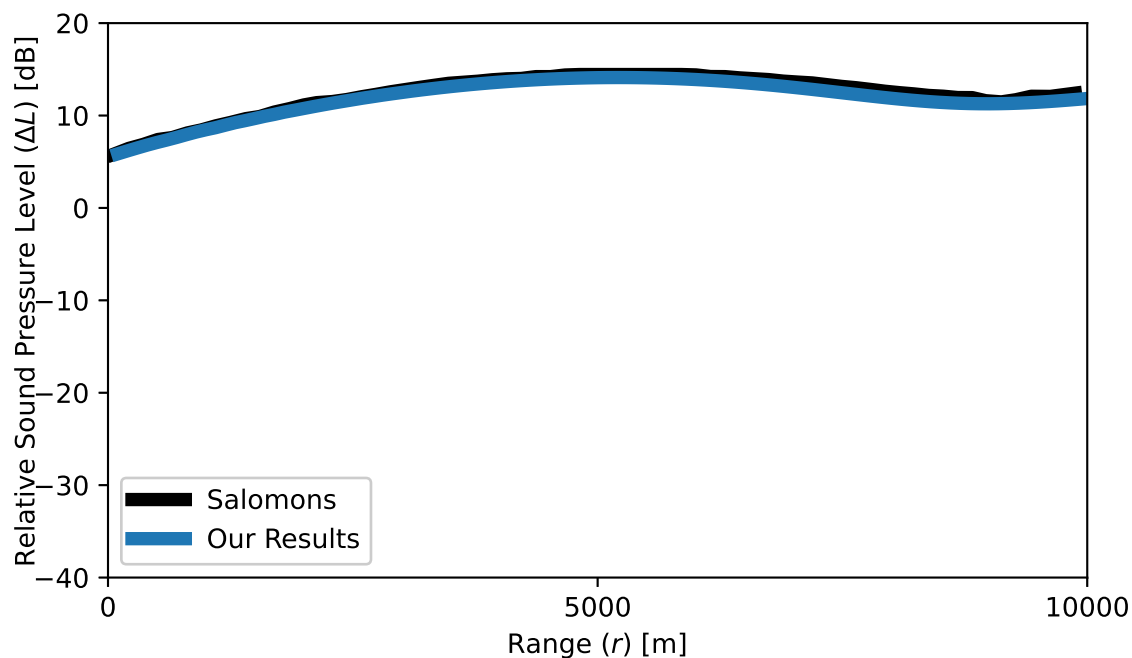


Figure D.12: Downwards Refracting Atmosphere and Absorbing Ground at 10 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

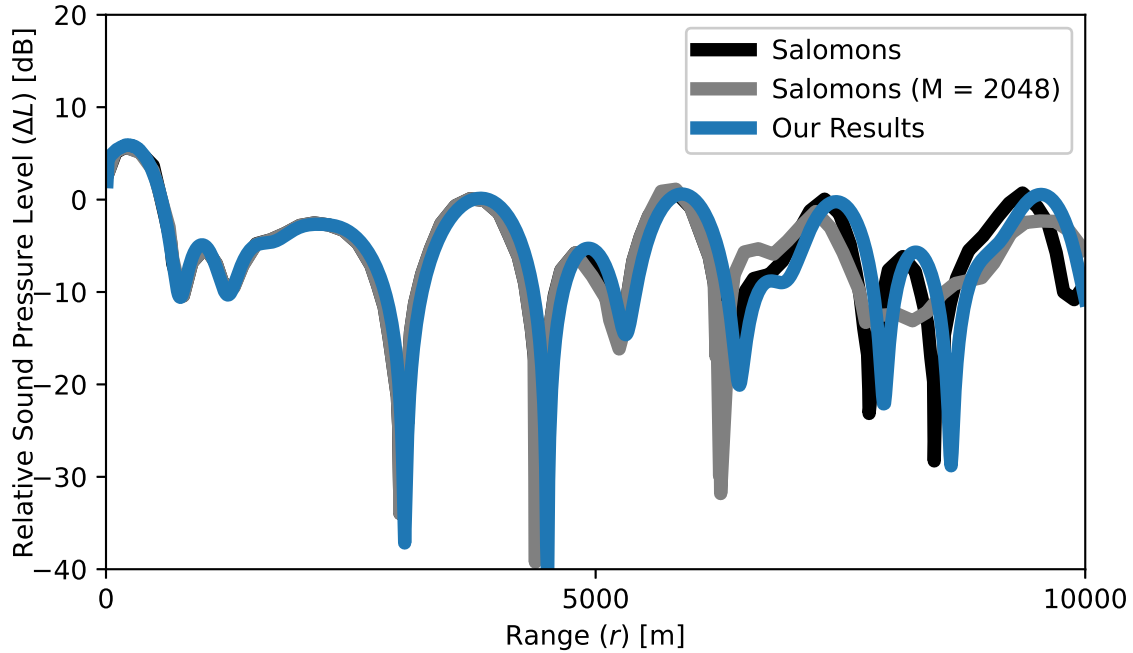


Figure D.13: Downwards Refracting Atmosphere and Absorbing Ground at 100 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

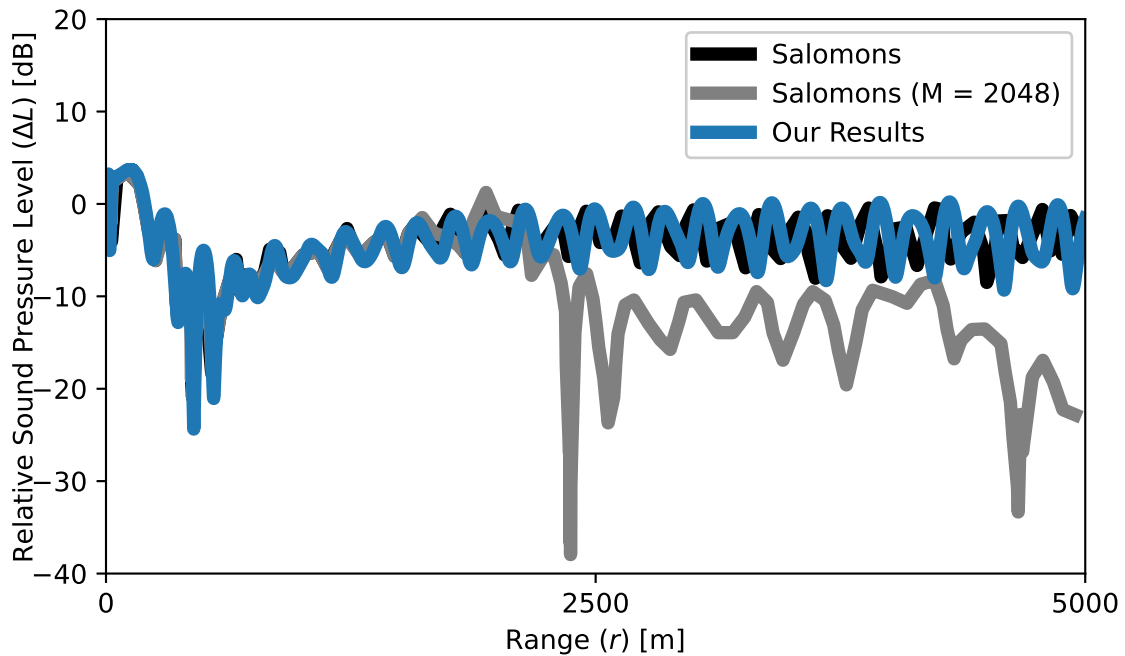


Figure D.14: Downwards Refracting Atmosphere and Absorbing Ground at 1,000 Hz using a 2D GFPE. Our results shown against results presented in Ref. [19].

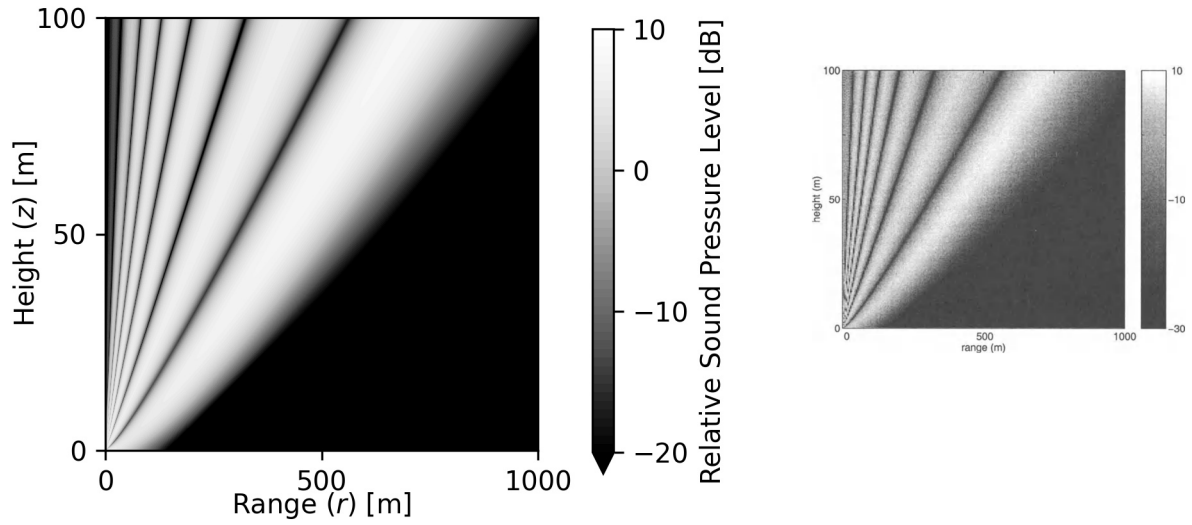


Figure D.15: 2D GFPE for an Upwards Refracting Atmosphere with Rigid Ground. An acoustic source at $z = 2$ meters in an upwards refracting atmosphere with an acoustically rigid ground. Our recreation of Figure 4.22 from [13] on the left and the original on the right.

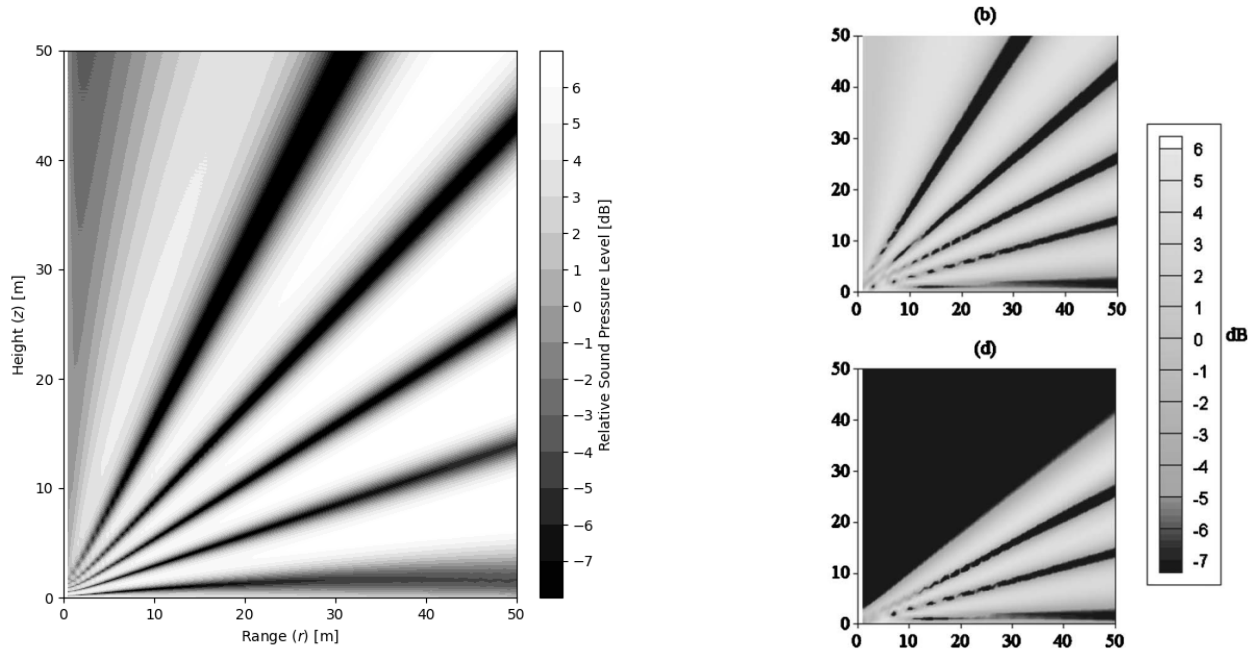


Figure D.16: Exact starting field 2D GFPE with our results (left) versus (b) in [20] for $z_s = 1.8$ and normalized ground impedance of $Z_b = 7.19the + 8.2j$ in neutral atmosphere. (d) is a lower integration window cutoff.

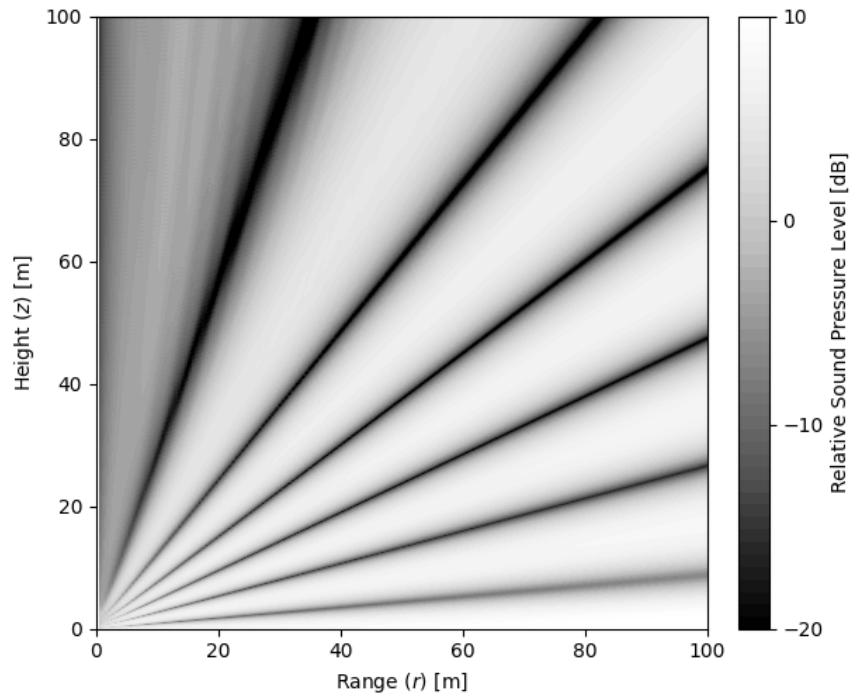


Figure D.17: Validation of the 3D GFPE implementation by recreating Figure 4-17 of [13] like in Fig. D.4

Appendix E. Acoustic Phenomenon Effects of Theoretical Acoustic Profile

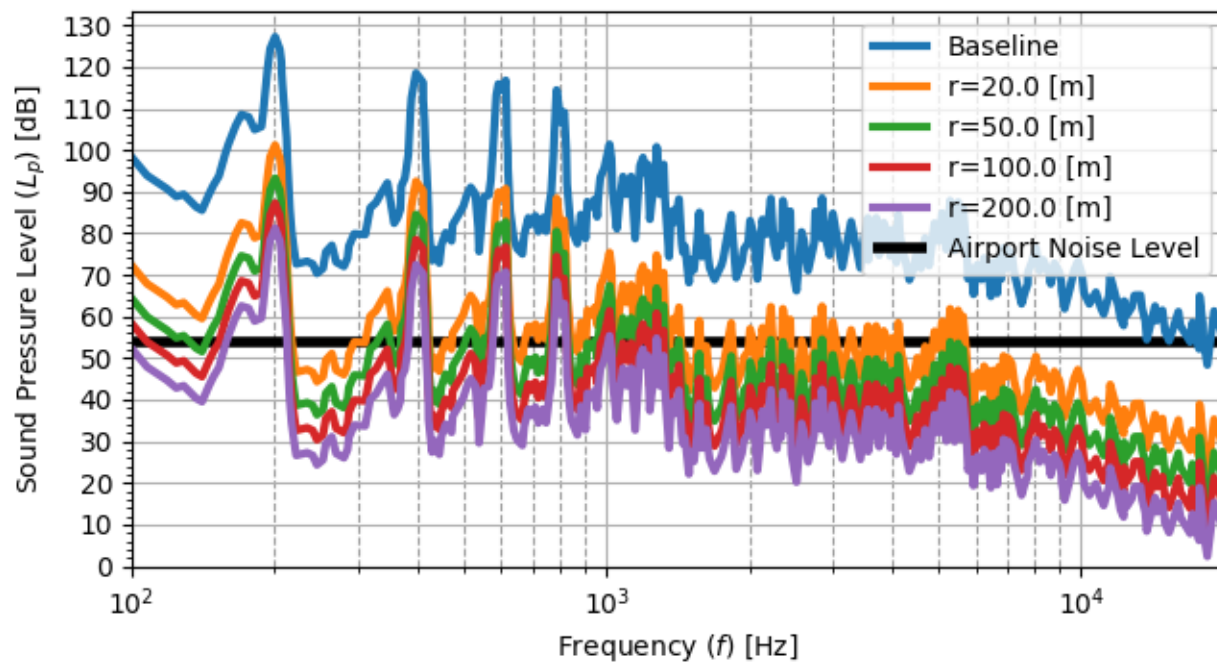


Figure E.1: Acoustic Profile due to Spherical Spreading

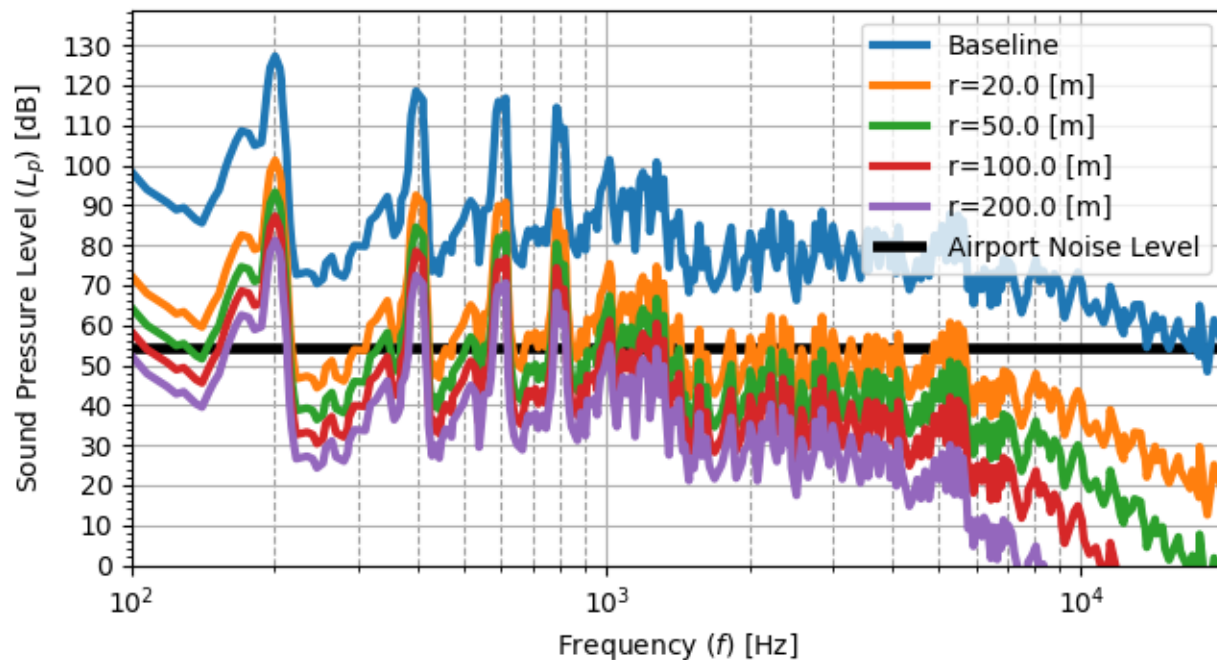


Figure E.2: Acoustic Profile Through an Absorbing Media

E.1 Inclusion of Absorbing Ground

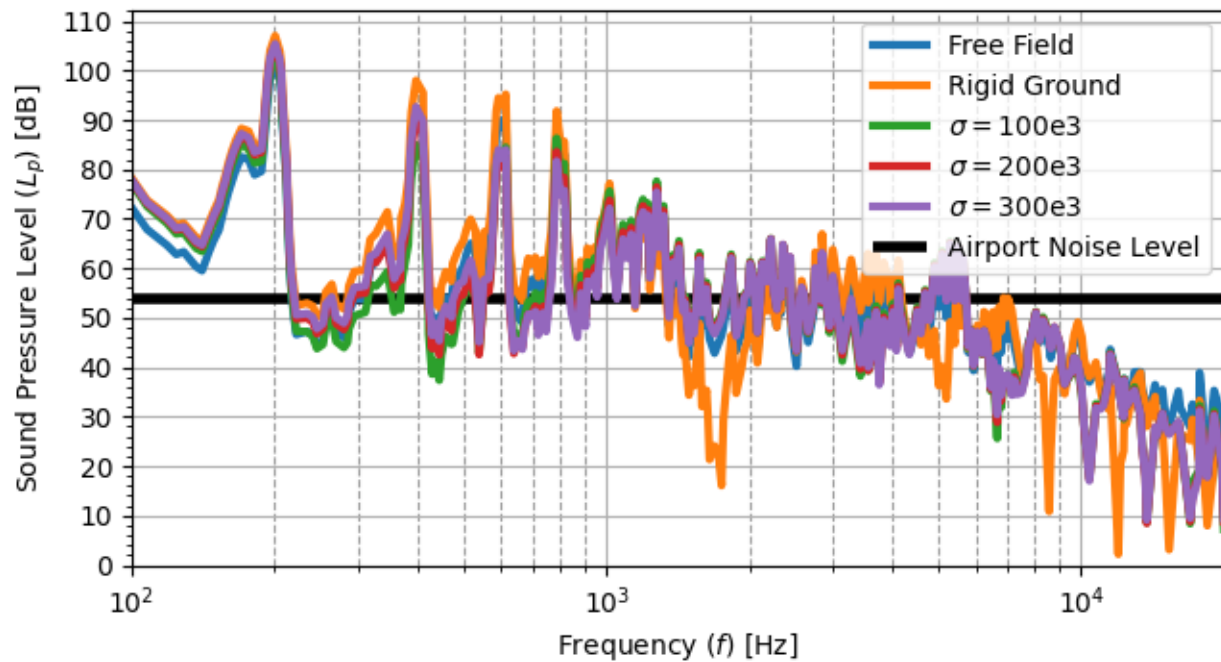


Figure E.3: Ground effects for UAV at 1 meters AGL and Detector 20 meters away.

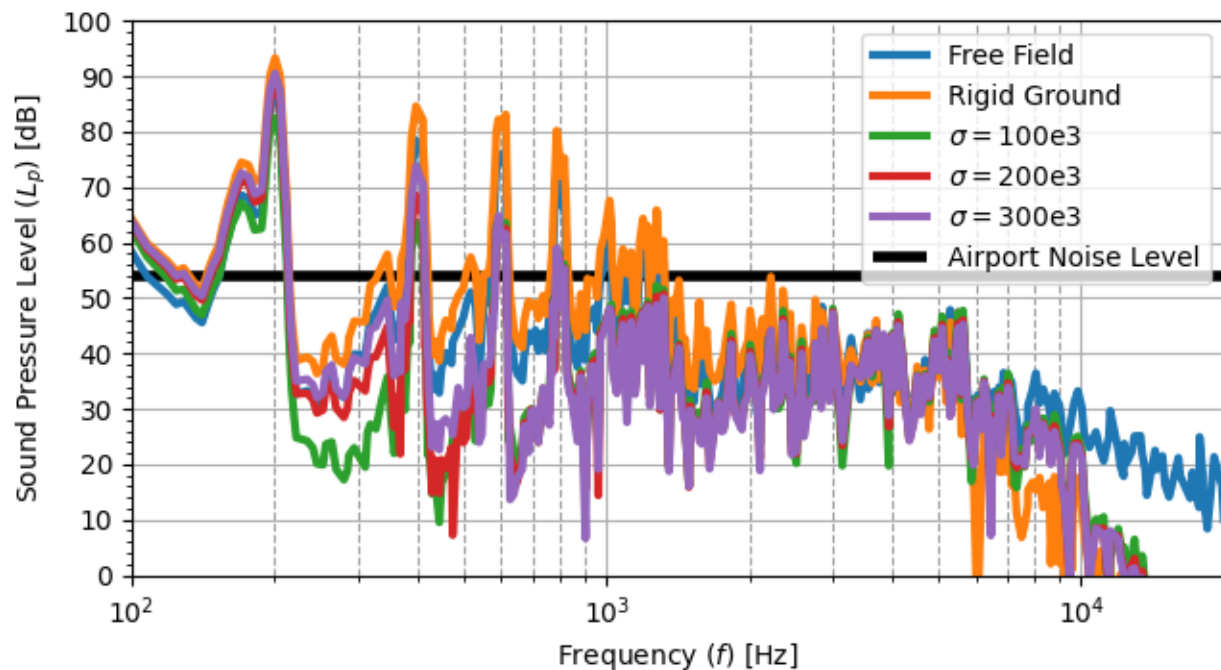


Figure E.4: Ground effects for UAV at 1 meters AGL and Detector 100 meters away.

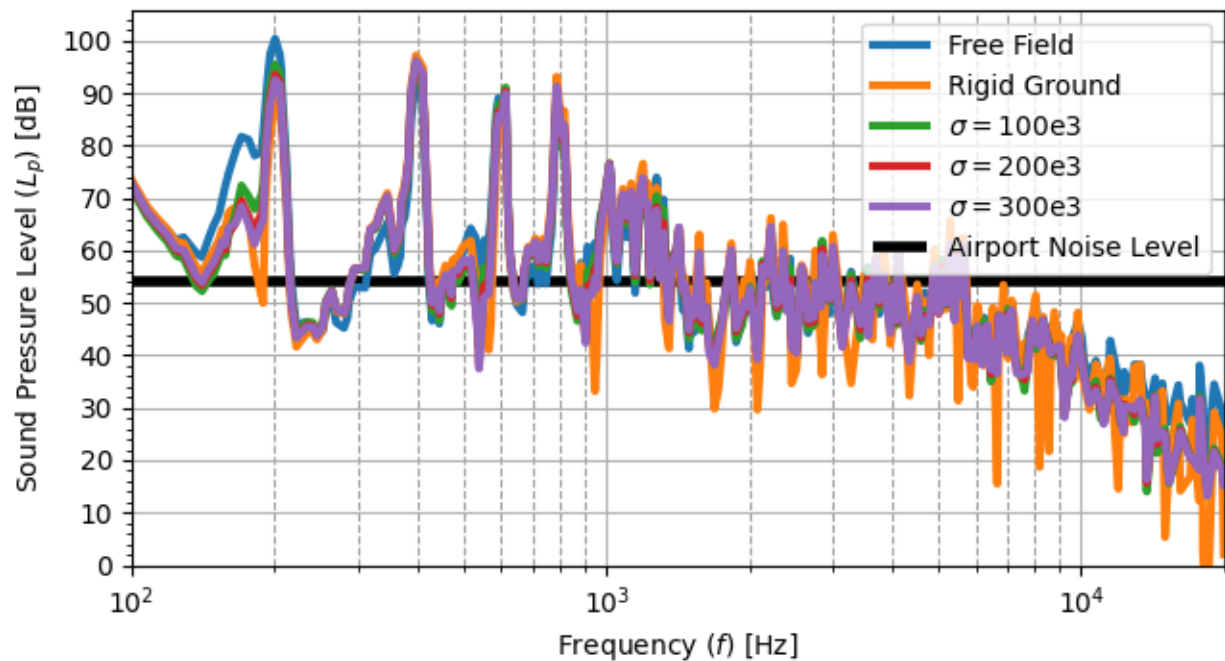


Figure E.5: Ground effects for UAV at 10 meters AGL and Detector 20 meters away.

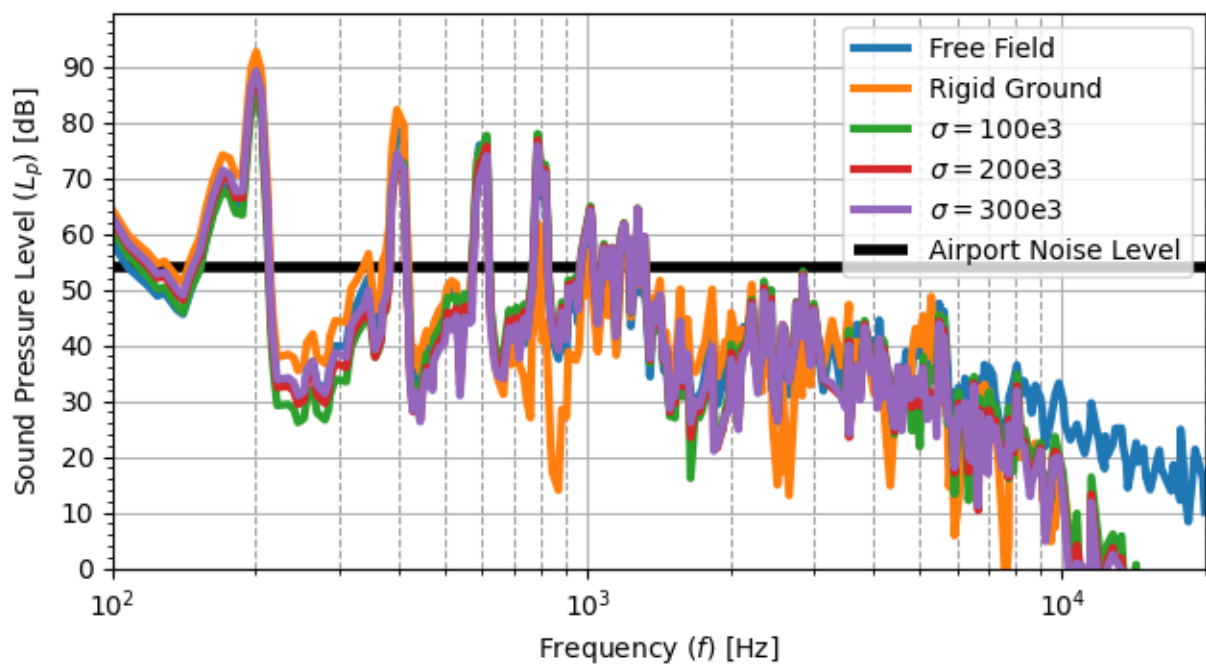


Figure E.6: Ground effects for UAV at 10 meters AGL and Detector 100 meters away.

E.2 Alternative Ground Models

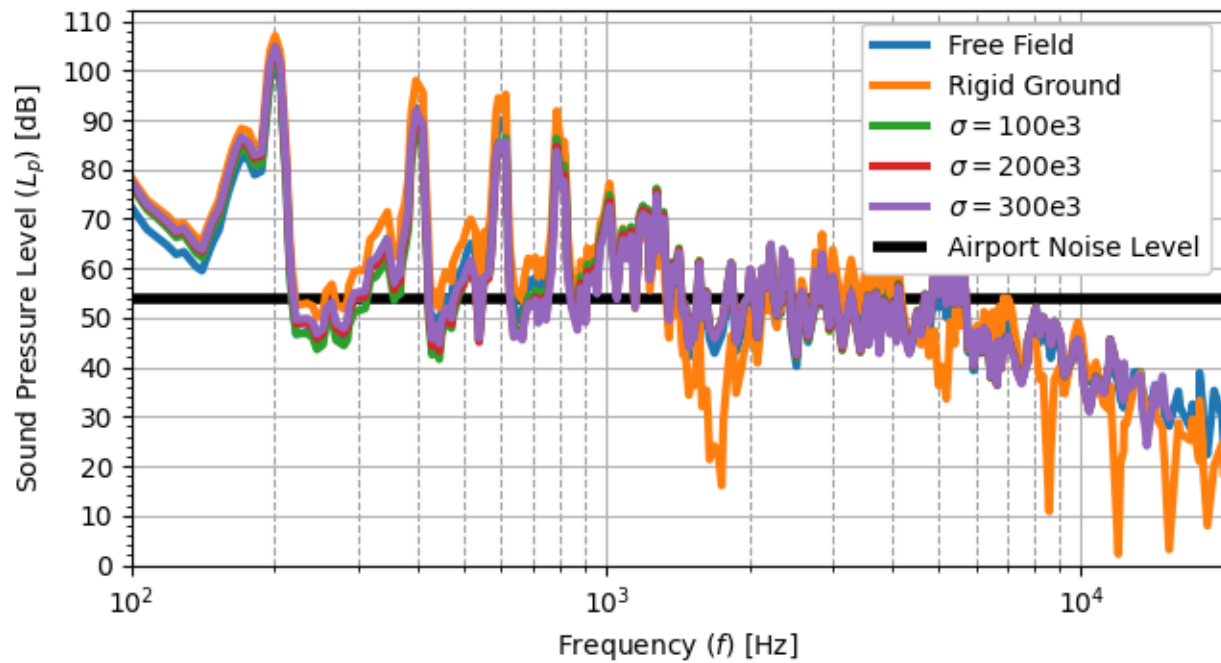


Figure E.7: Ground effects for UAV at 1 meters AGL and Detector 20 meters away using the Attenborough ground model

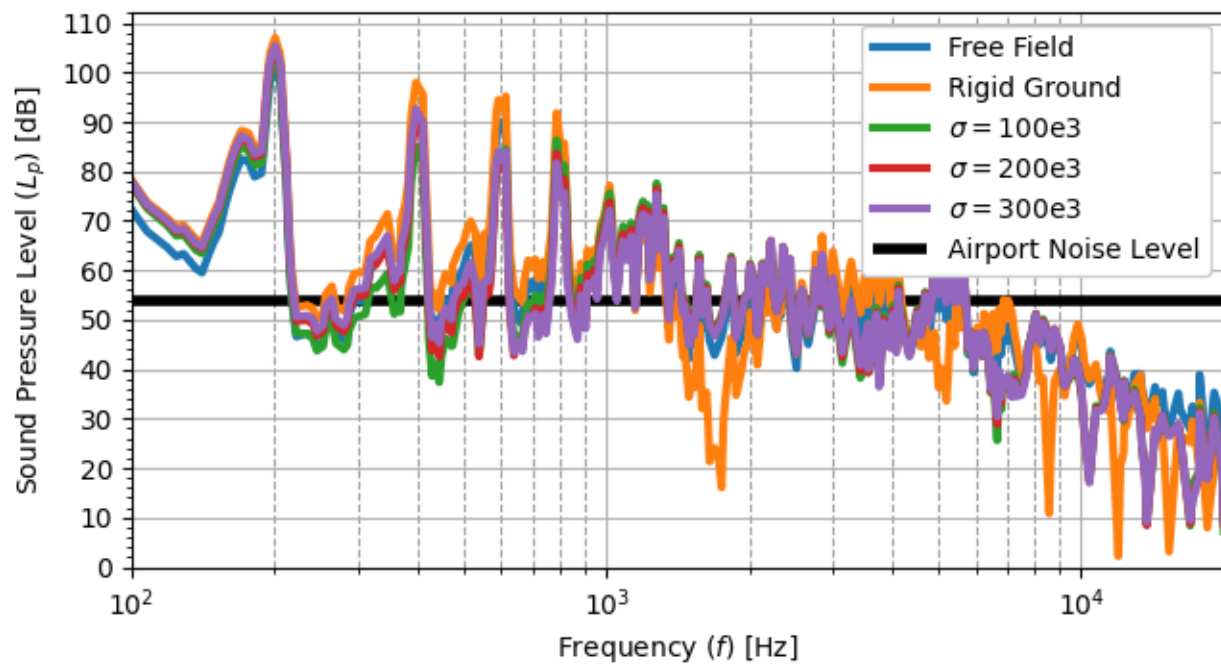


Figure E.8: Ground effects for UAV at 1 meters AGL and Detector 20 meters away using the DBC ground model

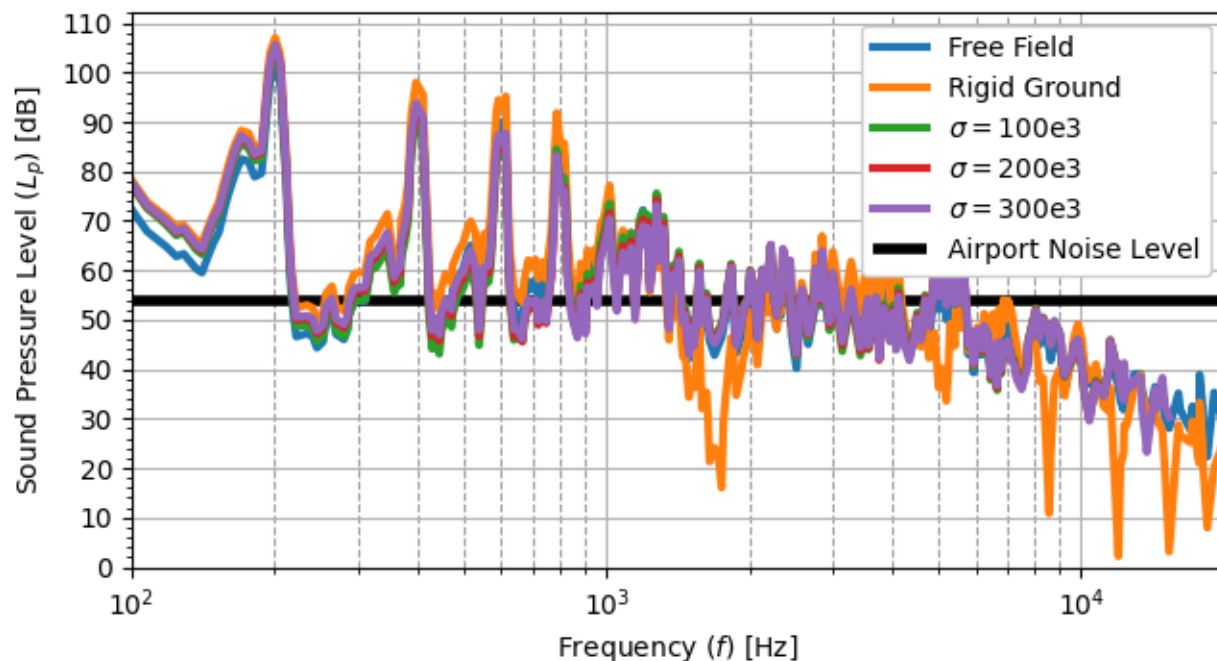


Figure E.9: Ground effects for UAV at 1 meters AGL and Detector 20 meters away using the Hamet ground model

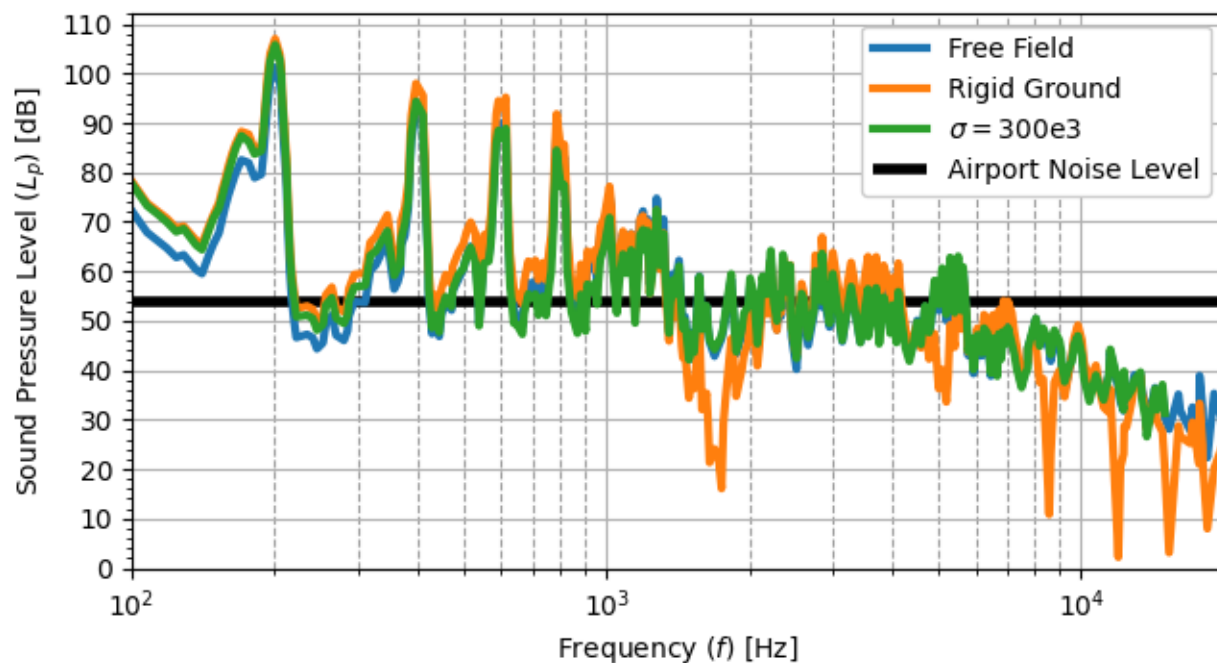


Figure E.10: Ground effects for UAV at 1 meters AGL and Detector 20 meters away using the ZK ground model

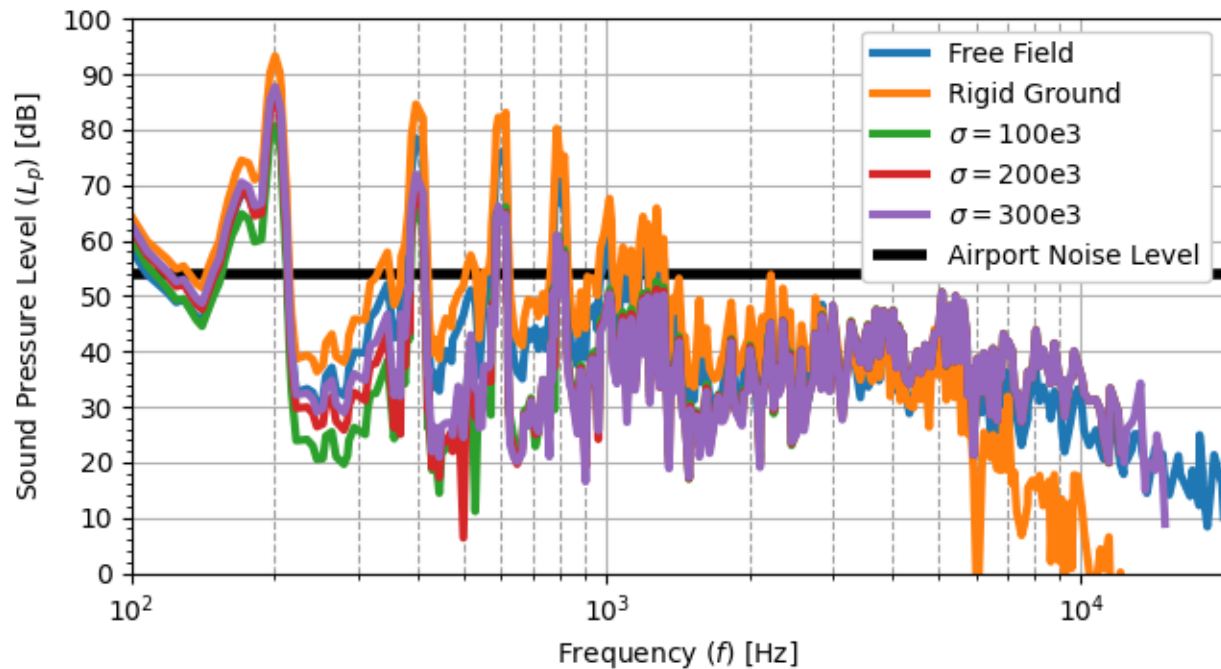


Figure E.11: Ground effects for UAV at 1 meters AGL and Detector 100 meters away using the Attenborough ground model

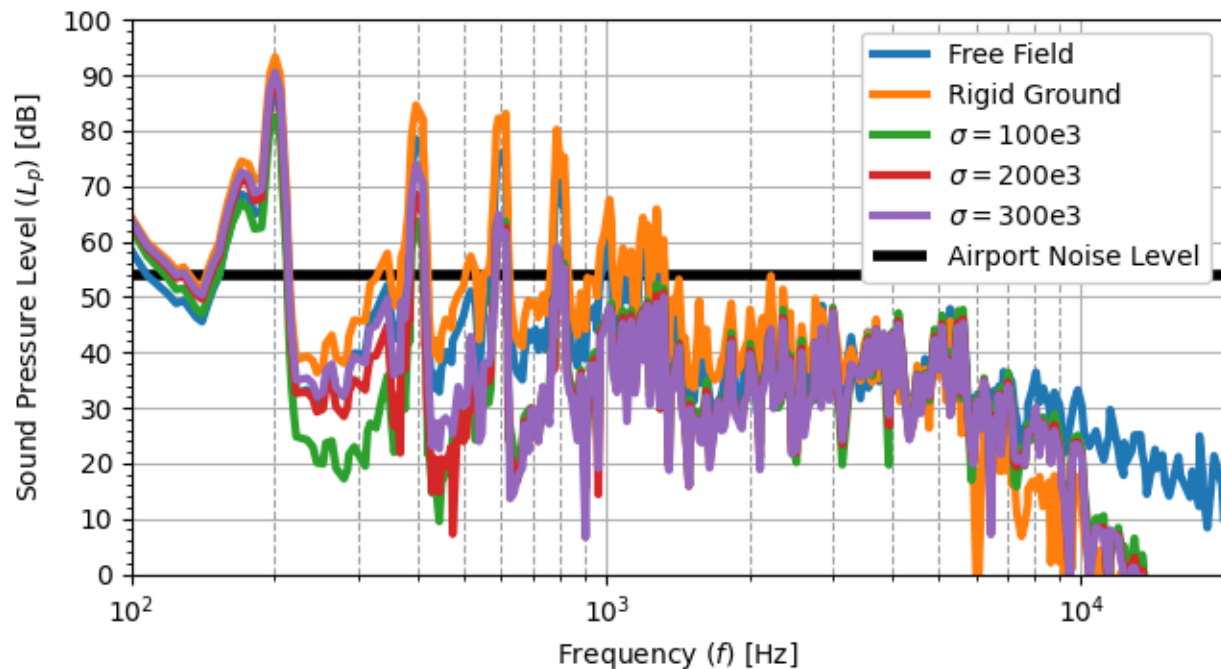


Figure E.12: Ground effects for UAV at 1 meters AGL and Detector 100 meters away using the DBC ground model

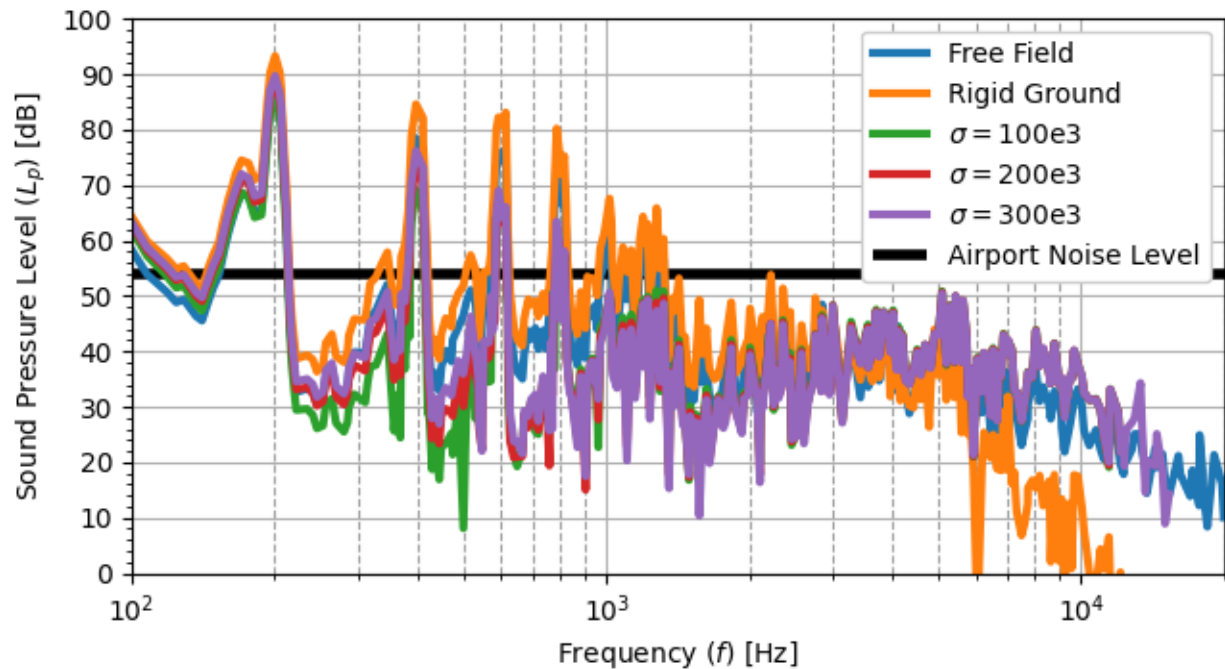


Figure E.13: Ground effects for UAV at 1 meters AGL and Detector 100 meters away using the Hamet ground model

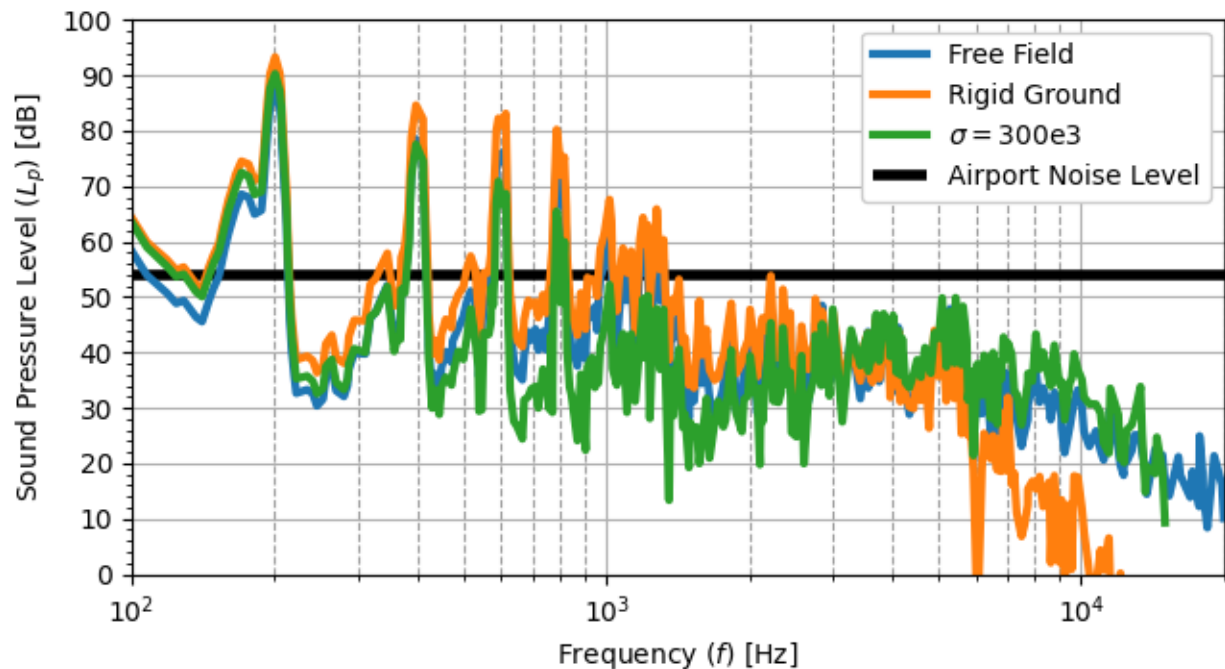


Figure E.14: Ground effects for UAV at 1 meters AGL and Detector 100 meters away using the ZK ground model

E.3 Inclusion of Refraction

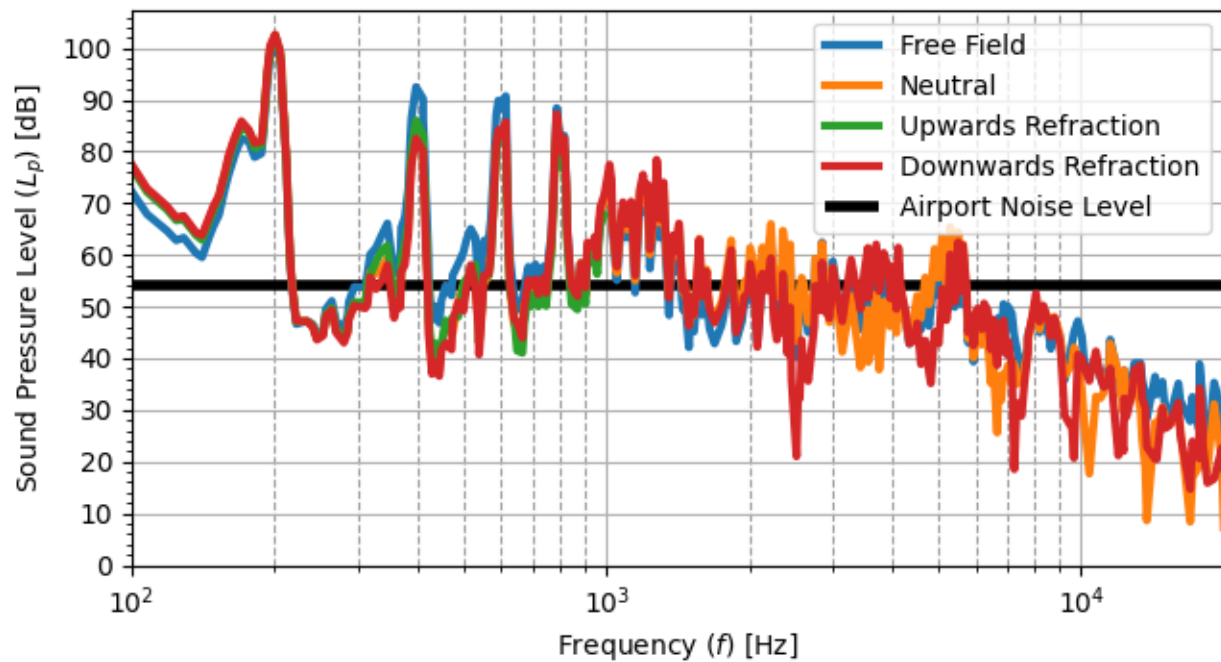


Figure E.15: Refraction effects for UAV at 1 meters AGL and Detector 20 meters away.

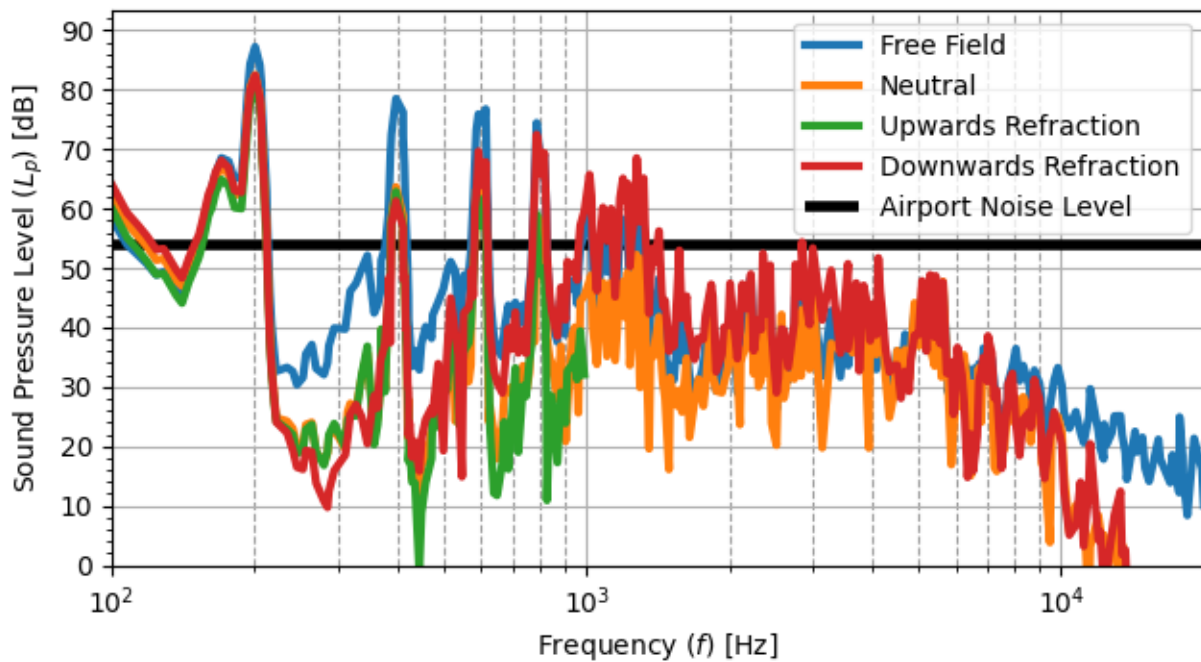


Figure E.16: Refraction effects for UAV at 1 meters AGL and Detector 100 meters away.

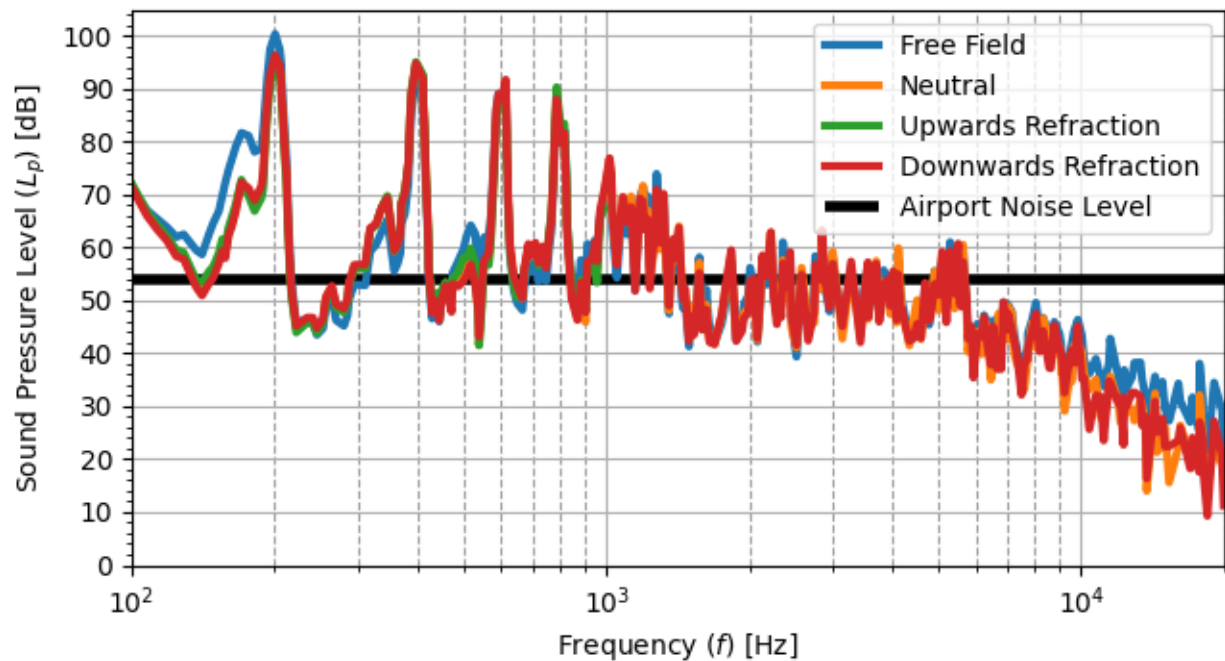


Figure E.17: Refraction effects for UAV at 10 meters AGL and Detector 20 meters away.

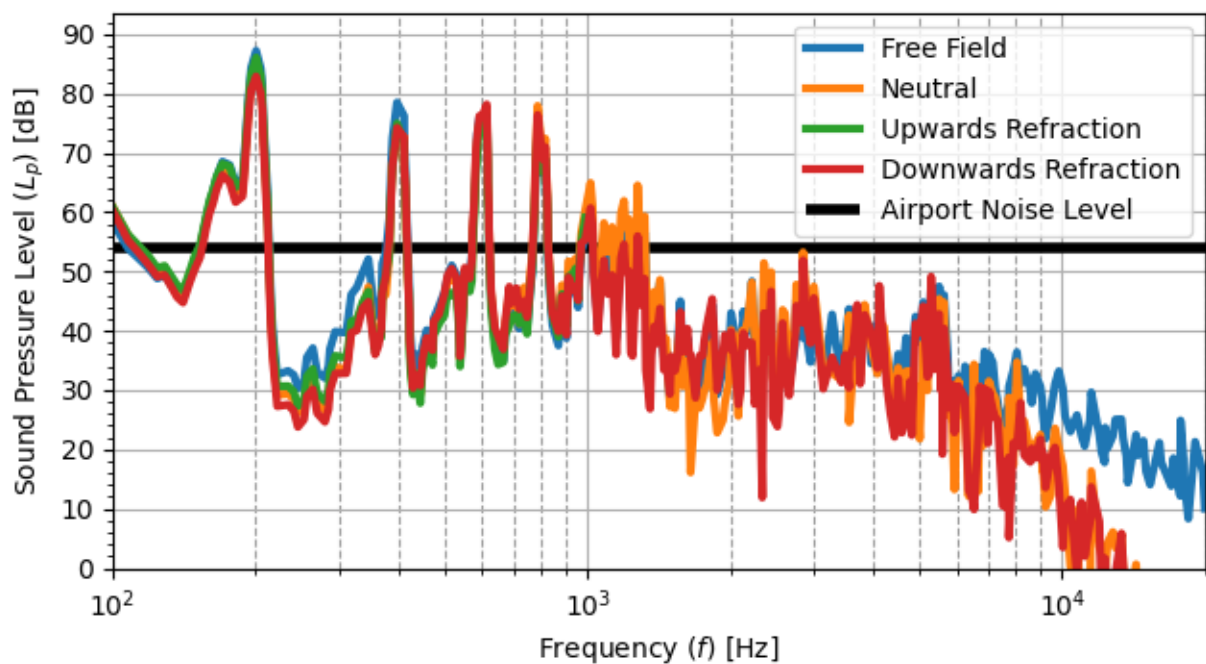


Figure E.18: Refraction effects for UAV at 10 meters AGL and Detector 100 meters away.

E.4 Alternative Speed of Sound Profiles

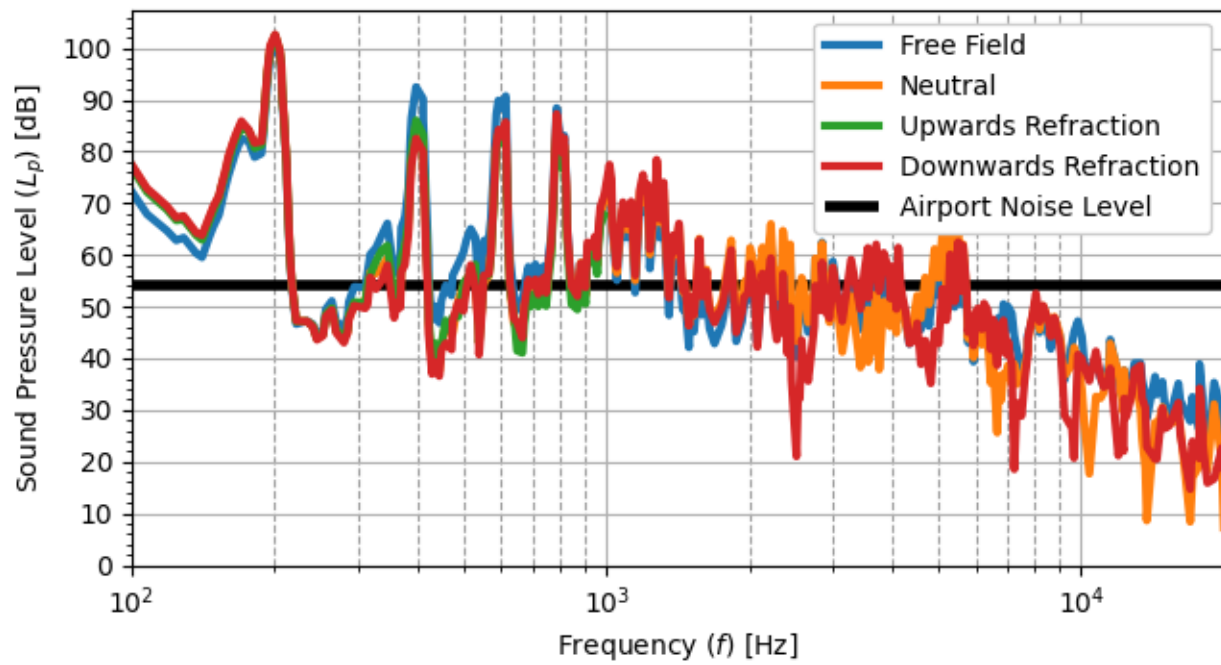


Figure E.19: Ground effects for UAV at 1 meters AGL and Detector 20 meters away using the logarithmic speed of sound profile.

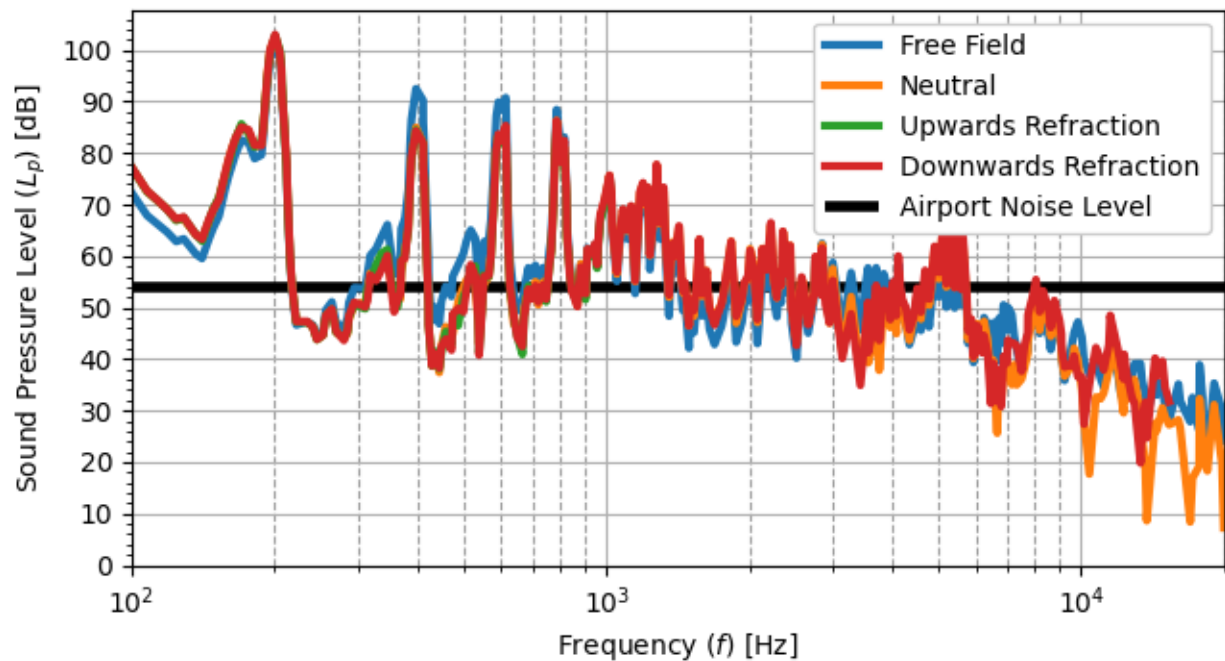


Figure E.20: Ground effects for UAV at 1 meters AGL and Detector 20 meters away using the linear speed of sound profile.

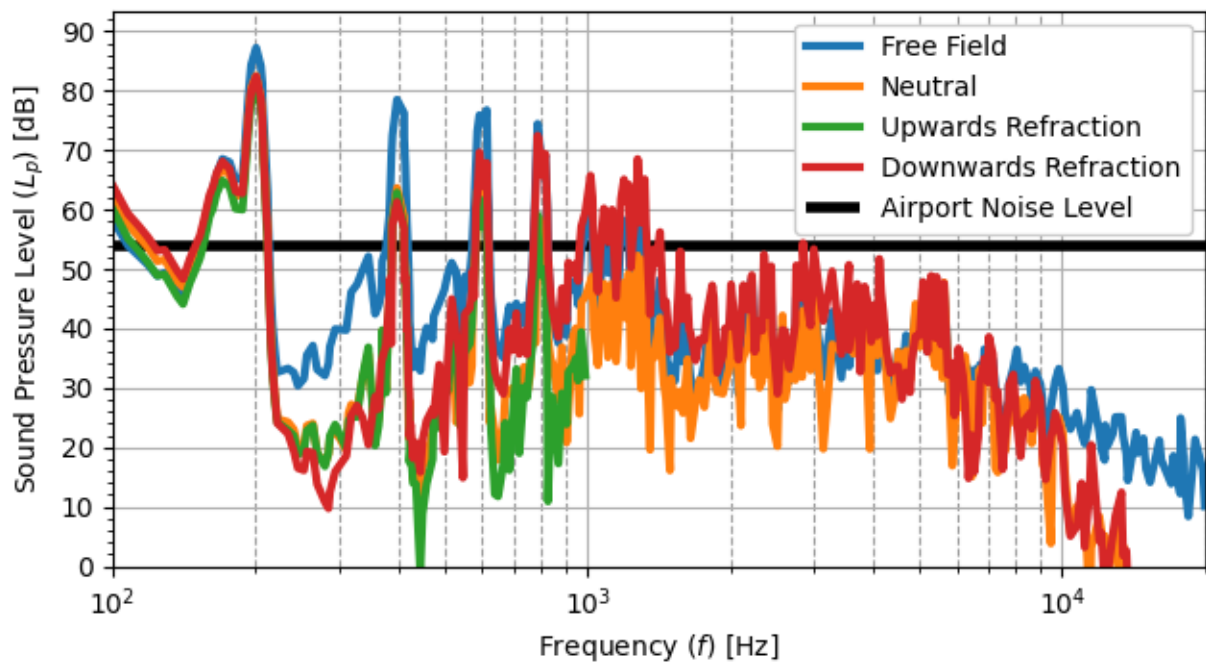


Figure E.21: Ground effects for UAV at 1 meters AGL and Detector 100 meters away using the logarithmic speed of sound profile.

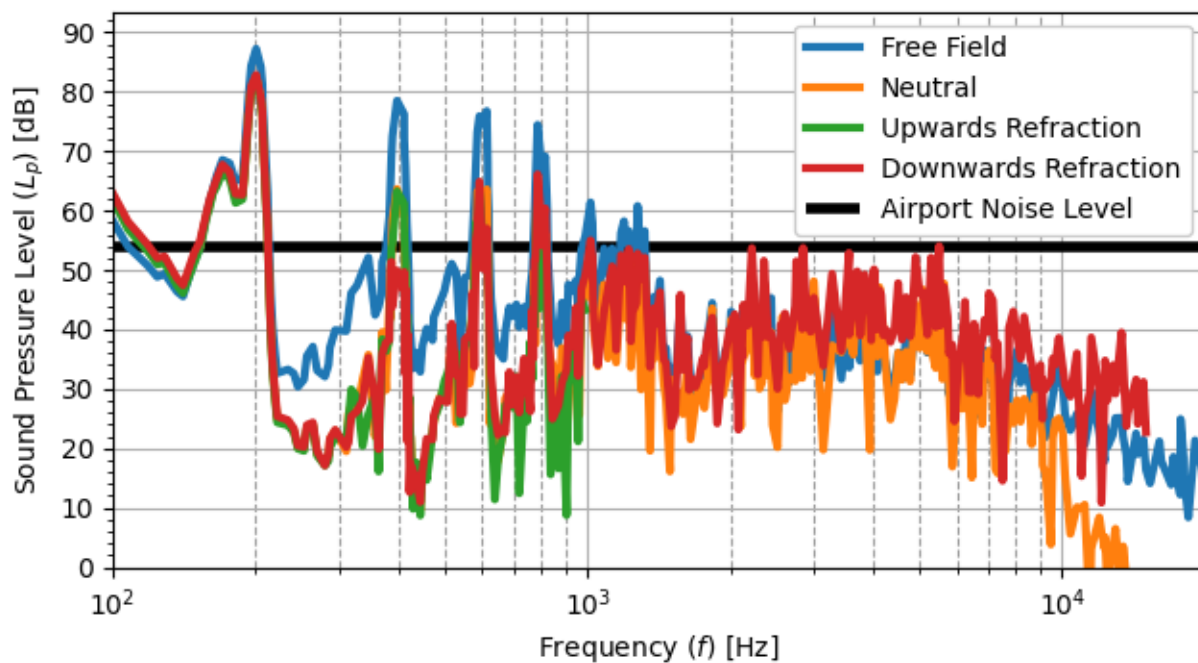


Figure E.22: Ground effects for UAV at 1 meters AGL and Detector 100 meters away using the linear speed of sound profile.

Appendix F. Anechoic Chamber

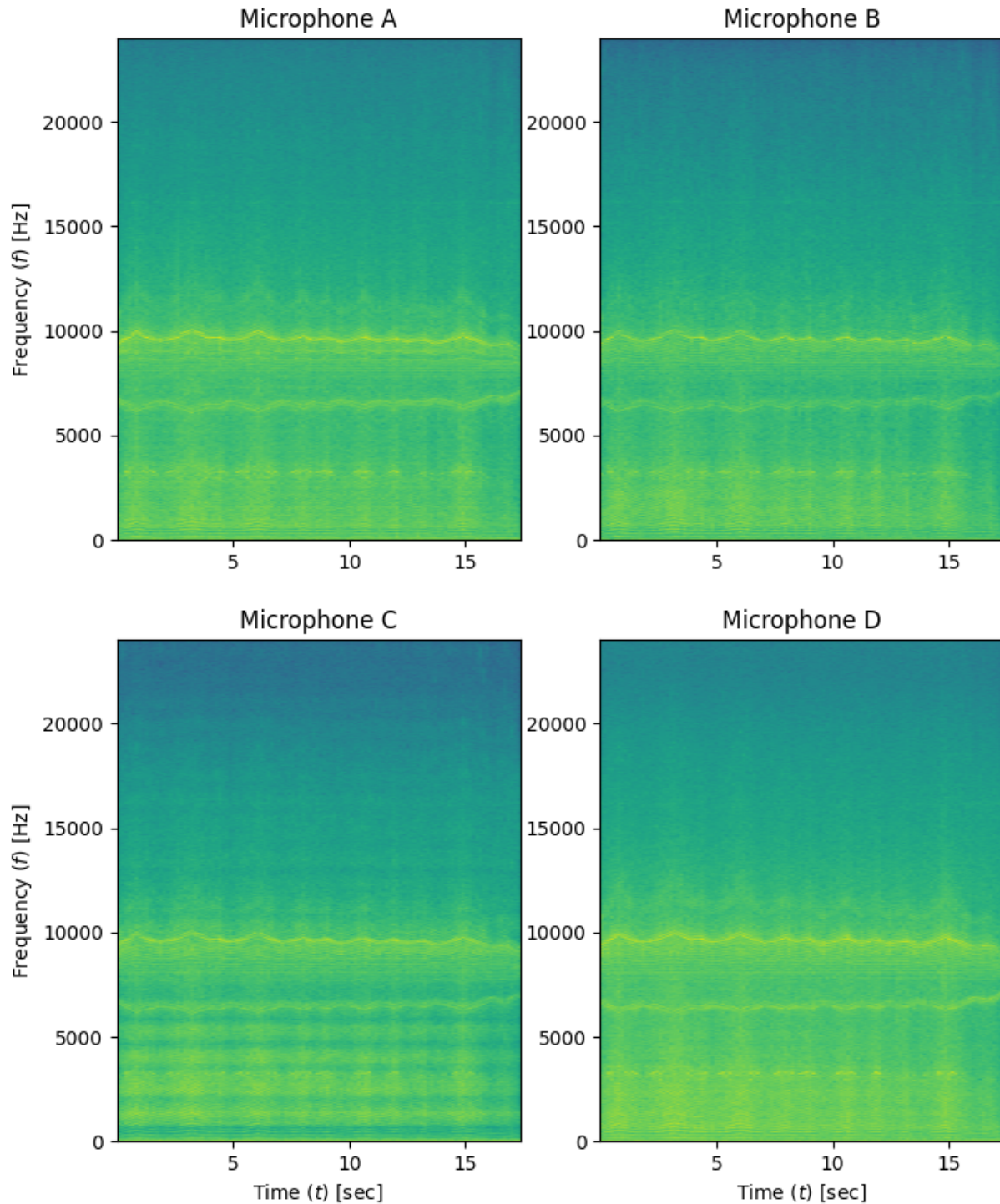


Figure F.1: Spectrogram of a DJI S1000 at Half Throttle (HT) in the Anechoic Chamber. A Hann window of 8,192 points and 128 points of overlap was used for the spectrogram.

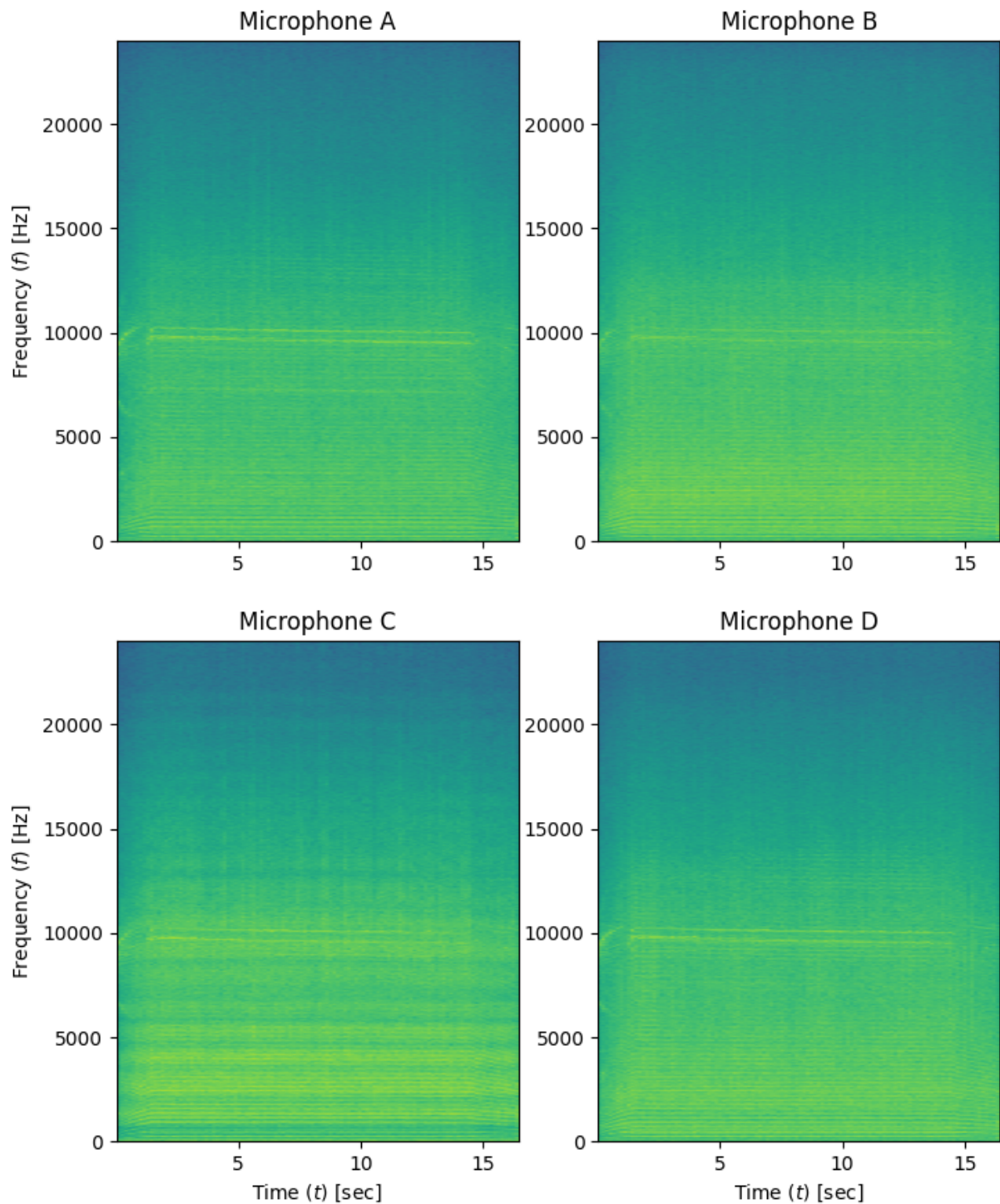


Figure F.2: Spectrogram of a DJI S1000 at Full Throttle (FT) in the Anechoic Chamber. A Hann window of 8,192 points and 128 points of overlap was used for the spectrogram.

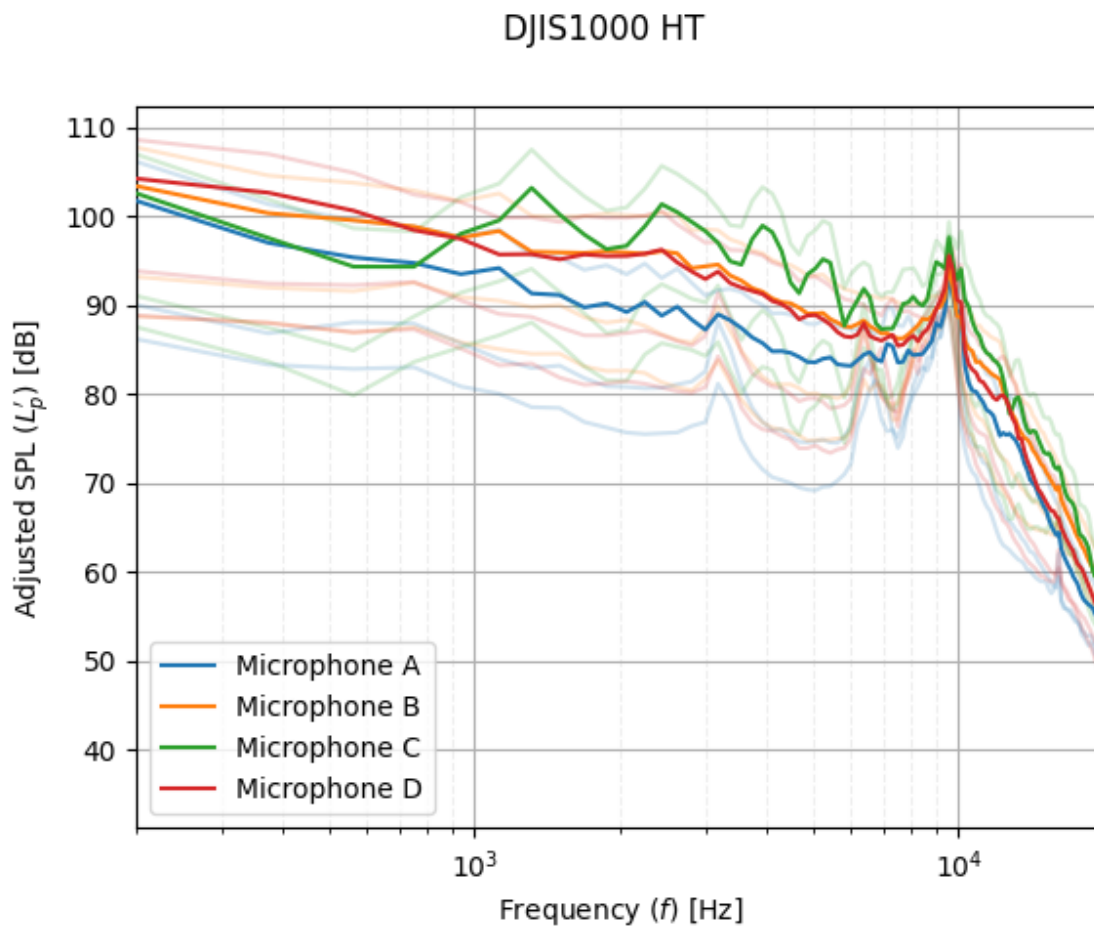


Figure F.3: Estimated Acoustic Broadband Spectrum of DJI S1000 at Half Throttle (HT). A Tukey window with a shape parameter of 1/4 was used for the STFT. The window was 8,192 points long and had 128 points of overlap.

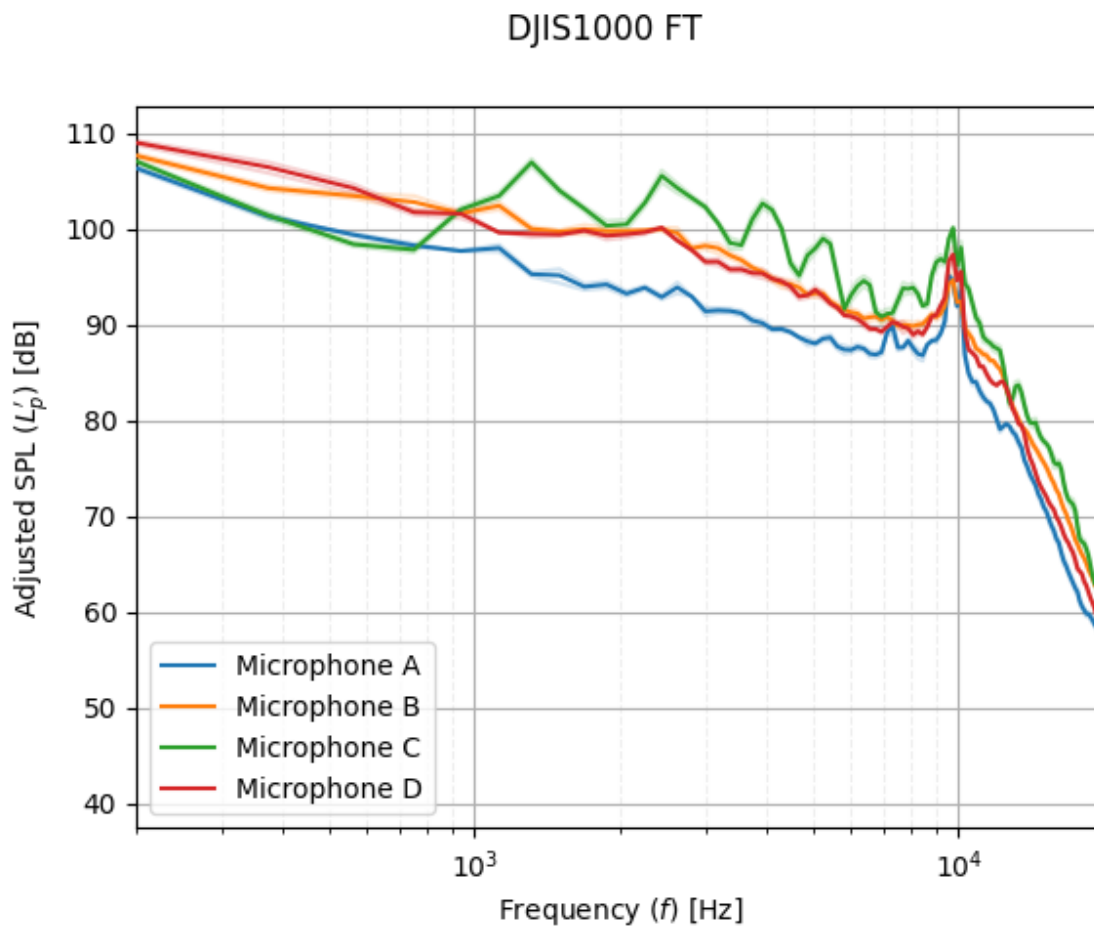


Figure F.4: Estimated Acoustic Broadband Spectrum of DJI S1000 at Full Throttle (FT). A Tukey window with a shape parameter of 1/4 was used for the STFT. The window was 8,192 points long and had 128 points of overlap.

List of Symbols, Abbreviations, and Acronyms

Symbols

Latin Symbol			
	t		Time
A	Attenuation	T	Temperature
c	Speed of sound	$\mathbf{v}$	velocity
c_s	Porous structural constant	u	Forward velocity
d	Layer transfer matrix	v	Sideways velocity
D	Transfer matrix for a set of layer	w	Vertical velocity
D	Relative sound pressure level difference	z	Height
f	Frequency	z_0	Roughness length
g	Gravity	Z	Normalized Impedance
g	Grain shape factor	Greek Symbol	
k	Wavenumber	α	Absorption coefficient
L	Obukhov length	γ	Ratio of specific heats
L_p	Sound Pressure Level	γ	Elevation angle
ΔL_p	Relative Sound Pressure Level	δ	Dirac Delta
$\mathcal{L}$	Loss function	Δ	Discrete difference
N_{pr}	Prandtl number	ζ	Dimensional impedance
p	Pressure	θ	Polar angle
q	Tortuosity	θ	Potential temperature
r	Range	μ	Attenuation coefficient
R	Specific gas constant	ρ	Density
R	Reflection coefficient	σ	Specific flow resistivity
R_c	Radius of curvature	ϕ	Azimuth angle
s_f	Pore shape factor ratio	ψ	Field variable
		ω	Angular velocity
		Ω	Porosity

λ Wavelength

Subscripts and Superscripts

a Atmosphere

av Average value

abs Absorption layer

b Bottom layer variable

c Complex variable

d Detector variable

eff Effective value

free Free atmosphere

n Normal to a surface

r Radial component

s Source variable

t Top layer variable

tol Tolerance

z Height component

θ, ϕ Angular component

Math

$\bar{\cdot}$ Averaging

$\cdot_*$ Friction quantity

$\bar{\cdot}$ Nominal quantity

$\cdot'$ Turbulent quantity

$\mathbf{x}$ Vector

$x \rightarrow X$ Transformed domain variable

Acronyms

CI	Cyber Infrastructure
CNPE	Crank-Nicholson Parabolic Equation
DOA	Direction of Arrival
DBC	Delany-Bazely-Chessel
FFP	Fast Field Program
GFPE	Green's Function Parabolic Equation
PE	Parabolic Equation
RHS	Right Hand Side
SPL	Sound Pressure Level
UAS	Unmanned Aerial System
UAV	Unmanned Aerial Vehicle
WAV	Wave Audio File Format
ZK	Zwikker and Kosten