[redacted]

SOME OPERATIONAL LIMITATIONS OF AIR-TO-AIR MISSILES AS TYPIFIED BY SPARROW I

John C. Ryon, Charles H. Dodge
and Forrest C. Titcomb

Equipment Research Branch
Radar Division

May 26, 1954

Naval Research Laboratory
Washington, D.C.

[redacted]

[redacted] UNCLASSIFIED

DECLASSIFIED

CONTENTS

DECLASSIFIED

[redacted] UNCLASSIFIED

 [redacted]

ABSTRACT

An investigation has been conducted at NRL to determine the effective launch zones for the Sparrow I weapon system against the B-45 and B-29 aircraft targets. Although many of the parameters used apply specifically to Sparrow I, the techniques employed, and to some degree the findings, are applicable to other air-to-air missile systems. Results of the investigation indicate that with available fire-control equipment a forward hemisphere launching with the Sparrow I missile against either the B-45 or B-29 aircraft types is not practical. Varying the limiting parameters (AI radar range, preparation time, and missile guidance range) changes the effective launch zone. Even with major improvements in all of these, forward hemisphere launching with the Sparrow I missile against a high performance target such as the B-45 is impractical.

A cursory study of the AI intercept problem was made to determine the differences in penetration of the enemy aircraft toward a task group for different fighter approach aspects. Findings of the investigation indicate that the penalty paid in penetration distance resulting from the use of tail attacks is not excessive when related to the problem of solving the frontal attack.

PROBLEM STATUS

This is an interim report on the problem; work is continuing.

AUTHORIZATION

NRL Problem R05-48
Project NR 505-480

Manuscript submitted April 1, 1954

 [redacted]

SOME OPERATIONAL LIMITATIONS OF AIR-TO-AIR MISSILES AS TYPIFIED BY SPARROW I

BRIEF OF INVESTIGATION

This report details a method for determining the effective launch zones for air-to-air missiles and the relative importance of certain zone limiting parameters. The guided missile system investigated uses the existing Sparrow I missile launched from a jet fighter equipped with an all-weather fire-control system currently available. Although some of the parameters used apply specifically to Sparrow I, the technique is generally applicable to other air-to-air missile systems.

The principal system elements which constitute the over-all control loop include the missile, tracking radar and computer, target aircraft, fighter aircraft, and pilot. Parameters which define the expected performance of system elements are estimated for a "normal" case and launch zone limits are established from these data. To show the effects of performance improvement upon the scope of weapon application, certain parameters (radar range, preparation time, missile guidance range) are then improved and effective launch zones are graphically determined. To permit comparison with earlier studies, the curves presented are analogous in form to those developed by a Cornell Aeronautical Laboratory report (1).

One of the assumptions of the method used for determining launch zones is that the fighter aircraft is flown along a constant true bearing course relative to the target. While the use of a constant true bearing course established launch zones beyond which it will not be practical to launch the Sparrow I missile, a study of more practical courses is necessary in order to define interceptor approach aspect limits and target penetration distances. An investigation of lead collision courses is made from which penetration distances are defined. The angles from which a lead collision course intercept may be completed verifies that the assumption of a constant true bearing course for the close-in studies did not restrict the possible launch zone, but did rather yield results which are slightly optimistic.

Inasmuch as the characteristics of several of the elements of the Sparrow I weapon system are not precisely known and the system as a whole is in a developmental state, it is emphasized that quantitative indications of many performance parameters are in fact reasonable estimates and not absolute values. For example, radar detection ranges used in this report are based in part upon incomplete measurements of effective radar reflective areas of the target.

INTRODUCTION

Design parameters for elements of an air-to-air missile system can be established practically by defining the tactical problem to be solved and from this premise determining performance requirements for an effective weapons system and its components. Although this procedure leads to an argument because the tactical situation cannot be anticipated exactly, it is more practical and economical than the more popular design method for "universal" application. The tactical problem, stated most simply, is to shoot down enemy aircraft. A weapons system designed to accomplish this under all conceivable conditions of weather, speed, altitude, and approach aspect becomes so complex that resultant system complexity and cost are prohibitive. It is preferable to establish "tolerable" parameters for system performance which give high probability of kill with minimum equipment complexity.

It is desired to have an air-to-air missile system capable of successfully firing at targets from any approach attitude. An objective of this investigation is to determine the practical feasibility of round-the-clock attacks. If it can be shown that the present state-of-the-art limits attack approaches to a particular finite area, then design of initial systems to be effective within this limited area is indicated. More "universal" specification requirements would be extravagant. This is of course only one of several complexity controlling parameters to be defined in order to specify detail system requirements.

The investigation further attempts to determine the effect of varying such parameters as radar range, preparation time, and missile guidance range. This is done for two reasons:

1. It is desired to determine upon which parameters, if any, the greatest emphasis should be laid for state-of-the-art improvement.

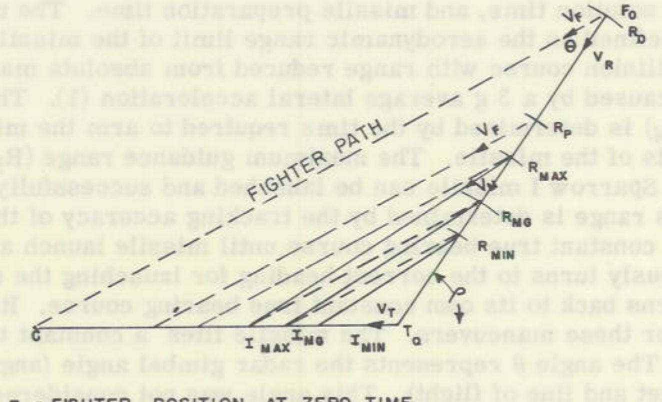
2. Performance improvement, with experience in the application of developmental equipment, is predictably certain and foreknowledge of the effects of these improvements upon system utility is important. It can reasonably be expected, for example, that achieved airborne intercept (AI) radar detection and lock-on ranges will increase and that, with training and equipment evolution, preparation time will decrease.

To appreciate some of the limitations imposed by tactical employment of an air-to-air missile carrying fighter, a study has been made of certain courses onto which the aircraft can be vectored for intercept. The distance a bomber target can penetrate toward the center of the task group before intercept varies in part as a function of the approach aspect of the fighter and this in turn with vectoring angles. When the required detection ranges are compared with the currently available AI detection ranges for various approach courses, practical limits are placed upon vectoring angles as well as fighter approach aspect. Therefore, the effect of some of these parameters upon penetration distance was investigated.

TACTICAL PROBLEM

The Sparrow I is a short-range, air-to-air, beam-rider guided missile which will be launched from Navy all-weather jet fighters against aircraft attacking fleet elements. The expected targets are bombers, either jet or propeller driven.

The scope of this report is limited to a single jet fighter attacking a single nonmaneuvering bomber. The investigation is restricted to one geometrical plane: the fighter and the target are at the same altitude. The fighter, target, and missile velocities and the aerodynamic ranges of missile were obtained from the Bureau of Aeronautics, Sperry Gyroscope Company and Cornell Aeronautical Laboratory. The effective radar reflective areas of the target were derived from data compiled by the Naval Research Laboratory and tests conducted by Hughes Aircraft Corporation. This report is concerned primarily with the attack phase.



F_0 = FIGHTER POSITION AT ZERO TIME
T_0 = TARGET POSITION AT ZERO TIME
R_D = DETECTION RANGE
V_F = FIGHTER VELOCITY
V_R = FIGHTER VELOCITY RELATIVE TO TARGET
V_T = TARGET VELOCITY
V_M = MISSILE VELOCITY
C = FIGHTER-TARGET COLLISION POINT
R_P = RANGE AT END OF PREPARATION TIME
R_{MAX} = MAX. AERODYNAMIC RANGE
R_{MG} = MAX MISSILE GUIDANCE RANGE
R_{MIN} = MIN. AERODYNAMIC RANGE
I_{MAX} = IMPACT POINT FOR R_{MAX}
I_{MG} = IMPACT POINT FOR R_{MG}
I_{MIN} = IMPACT POINT FOR R_{MIN}
θ = RADAR GIMBAL ANGLE (ANGLE BETWEEN LINE OF SIGHT TO TARGET AND LINE OF FLIGHT)
ρ = LINE-OF-SIGHT ANGLE OFF TARGET TAIL

Fig. 1 - Basic attack geometry

SPARROW I VS. B-45 AND B-29 AIRCRAFT

Standard Parameters

The attack geometry for this phase of the investigation (Fig. 1) is based upon a constant true bearing course (as if there were a trainable launcher in the fighter aircraft). This type of course was chosen because it will define launch zone limits beyond which it will not be practical to launch Sparrow I. In Fig. 1 F_O and T_O represent the fighter and target positions at the time when AI radar detection (R_D) occurs. It is assumed that at the time when radar detection occurs the fighter is flying the correct heading for collision with the target (constant true bearing course). This represents an optimum condition since no time is required for correction of errors in the heading of the fighter aircraft. For the constant true bearing course, the line of sight from the fighter to target (although not the line of flight) is the relative closure path between fighter and target. This velocity vector is designated as V_R, the fighter velocity relative to the target. It is assumed that the target is flying a straight line course and that the fighter and target are flying at constant velocities V_F and V_T respectively. R_p represents the range between the fighter and target along the line of sight at the end of the required preparation time. Preparation time, the elapsed time between detection of the target and launch of the missile, is the sum of lock-on time, computer solution time, and missile preparation time. The maximum launch range (R_{MAX}) is defined as the aerodynamic range limit of the missile while flying a perfect missile collision course with range reduced from absolute maximum by an amount equivalent to that caused by a 3 g average lateral acceleration (1). The minimum aerodynamic range (R_{MIN}) is determined by the time required to arm the missile and to damp out launching transients of the missile. The maximum guidance range (R_{MG}) is the limiting range at which the Sparrow I missile can be launched and successfully guided to destruction of the target. This range is determined by the tracking accuracy of the guidance system. The fighter flies a constant true bearing course until missile launch at which time the fighter instantaneously turns to the correct heading for launching the missile and then instantaneously turns back to its own constant true bearing course. It is assumed that no time is required for these maneuvers. The missile flies a constant true bearing course at a velocity V_M. The angle θ represents the radar gimbal angle (angle between the line of sight to the target and line of flight). This angle was not considered as a possible limiting parameter in this report. The line-of-sight angle off the target's tail is designated by ρ.

From the above discussion, it is obvious that the conditions selected favor the intercepting aircraft. The time required to correct for errors in fighter heading at the time of radar detection and the time required to maneuver to the correct heading for launching the missile will further limit the zones in which it is possible to successfully launch the Sparrow I missile.

Many of the parameters described above are common to all phases of this report. The following is a listing of these basic parameters (1):

Fighter Aircraft F3H-1

Altitudes of Interest . . . . 5000 and 30,000 ft

Fighter Speed . . . . 585 knots at 5000 ft altitude

. . . . 538 knots at 30,000 ft altitude

Sparrow I Performance at 5000-Foot Altitude

Maximum Aerodynamic Range (R_{MAX}) . . 31,800 ft

Average Velocity for R_{MAX} . . . . 1940 ft/sec

Minimum Flight Time. . . . . Ten missile time constants (which includes 5 sec arming time) plus 2 sec of unguided flight - 7 sec total

Average Velocity for R_{MIN} . . . . 2100 ft/sec

Sparrow I Performance at 30,000-foot Altitude

Maximum Aerodynamic Range (R_{MAX}). . . . 63,400 ft

Average Velocity for R_{MAX}. . . . . 1760 ft/sec

Minimum Flight Time. . . . . Ten missile time constants (which includes 5 sec arming time) plus 2 sec of unguided flight - 7 sec total

Average Velocity for R_{MIN} . . . . 2100 ft/sec

Sparrow I vs. B-45 Target

This section considers the Sparrow I missile launched from an F3H-1 fighter against a B-45 target. The B-45, a jet bomber, has approximately the same performance and reflective areas as the Russian IL-28 (taken to represent a typical expected target). The B-45 has a speed of 532 knots at a 5000-foot altitude and 489 knots at a 30,000-foot altitude (1).

The effective radar reflective areas at X-band of the B-45 for various aspect angles are given in Fig. 2. This curve was derived by smoothing test data taken by the Naval Research Laboratory (2). Measurements were not taken for the head-on and tail-on aspects. The head-on effective area was obtained by extrapolation from the nearest measured point (7.5°) and preceding points. It was assumed that the effective area from 132.5° off the nose of the target to tail-on is the same.

The 90% probability of radar detection range contour for the AN/APQ-41 radar against a B-45 target and the radar gimbal angles for various aspect angles off the tail of the target are given in Fig. 3. Gimbal angle is the angle between the line of sight from the fighter to the target and the line of flight of the fighter. For aspect angles of 50° or more off the nose of the target the fighter is approaching the target from the rear hemisphere when flying the assumed constant true bearing course. This figure and the rest of the illustrations of the report are divided into two zones of interest. The upper half of the figure is for a mission occurring at 30,000-foot altitude and the lower half for one occurring at 5000-foot altitude. The detection range contour, developed by Cornell, is based on results of tests conducted by NATC, Patuxent, using the AN/APQ-41 radar. These tests indicated 4.4 nautical miles detection range for 50% probability of detection against an F9F target (tail aspect). For these tests the AN/APQ-41 was installed in an F2H aircraft which was slowly closing on the F9F target. This performance is considered to be typical of currently available AI radars because the tests were conducted under controlled conditions.

Quantitative data describing the equivalent radar reflective area of an F9F at tail aspect is not available. Measurements have been made of the F-86 at tail aspect, however, and these have been used for the F9F case. The F-86 has an equivalent radar reflective area of 0.266 square meters at 127° off the nose (obtained from NRL). It was assumed that the reflection areas from 127° to 180° are constant. These test results and equivalent radar areas were used by Cornell to develop the range contour of Fig. 3 as described in the Cornell report (1). Table 1 lists the magnitudes of gimbal angles shown in Fig. 3 for various aspect angles off the target's tail for the assumed tactical problem.

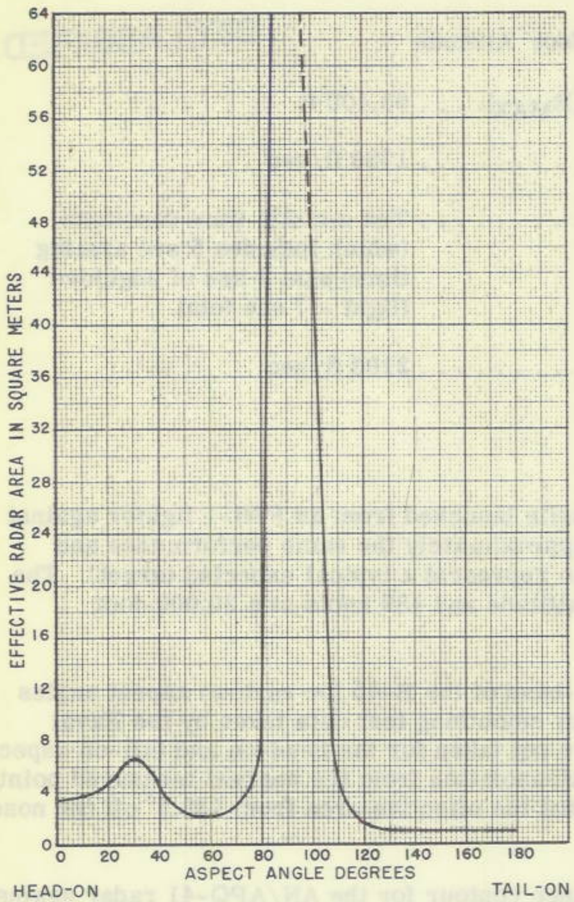


Fig. 2 - Effective radar area at X-band of the B-45 as a function of aspect angle (from Cornell Aeronautical Labs. Report No. GM-660-G-11)

Throughout this report range data for 90% probability of detection are used. The reason for this choice becomes evident when uncertainties from the time of detection to that of kill are analyzed. Consider the following equation:

$$P_S = P_D \times P_{LO} \times P_L \times P_K \quad (1)$$

where

P_S = Probability of a successful mission after fighter direction phase

P_D = Probability of AI radar detection

P_{LO} = Probability of AI radar lock-on

P_L = Probability of accurately launching missile

P_K = Probability that missile will intercept the target

Eq. (1) includes a few of the steps in the intercept mission. If each of the above probabilities is assigned a value of 90% then

$$P_S = 0.9 \times 0.9 \times 0.9 \times 0.9 \quad (2)$$

P_S = 0.656 or 65.6% probability of a successful mission after fighter direction phase.

When additional steps of the intercept mission (search, detection, scramble, vectoring, etc.) are included the over-all probability of success is further reduced. Indeed, for raids approaching saturation size a 0.9 probability requirement for each step may not be high enough.

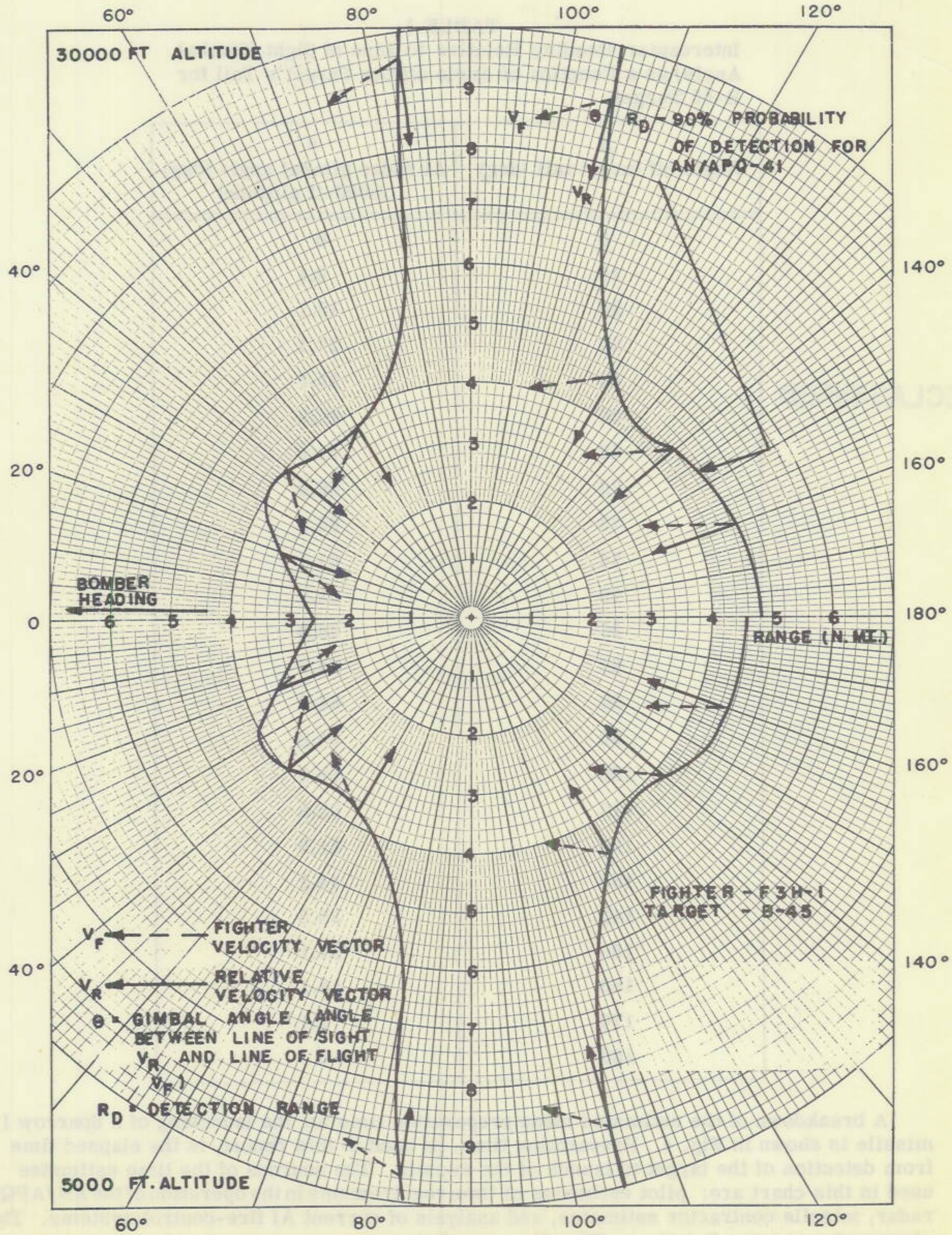


Fig. 3 - Fighter heading as a function of approach aspect against a B-45 target

TABLE 1
Interceptor Heading Relative to Line of Sight (Gimbal Angle) as a Function of Angle off the Target's Tail for B-45 Target.

ρ Angle off target tail (deg)	θ AI radar gimbal angle (deg) (5000-ft altitude)
0	0
10	9.1
20	18.1
30	27.1
40	35.8
50	44.2
60	51.9
70	58.8
73	60.5
74	61.1
75	61.5
96	64.8
97	64.7
98	64.3
99	64.0
100	63.6
110	58.8
120	51.9
130	44.2
140	35.8
150	27.1
160	18.1
170	9.1
180	0

A breakdown of the estimated mean preparation time for the launching of a Sparrow I missile is shown in Fig. 4. Preparation time, as used in this report, is the elapsed time from detection of the target to launch of the missile. The sources of the time estimates used in this chart are: pilot estimates of time requirements in the operation of the AN/APQ-41 radar, missile contractor estimates, and analysis of current AI fire-control systems. The steps and corresponding times (Fig. 4) are as follows:

1. Target Observation and Evaluation (6 seconds) - The pilot has detected the target, is observing its behavior, and is identifying it as friend or foe.

2. Antenna Positioning (5 seconds) - The pilot manually aligns the antenna in azimuth and elevation.

3. Lock-on (1 second) - The pilot brackets the target with the range gate and achieves a lock-on.

4. Antenna Transient Damping (6 seconds) - At the end of this time valid angle information is available.

5. Over-all Loop Solution (14 seconds) - Overlapping the antenna transient damping period is the over-all loop solution period. This time requirement (14 seconds) was obtained from an analysis of the over-all loop time constant of the AERO-13 system. Two loop time constants (at 7 seconds) were allowed to obtain a valid fire-control solution.

6. Pilot and Launching Delay (1 second) - The pilot can now fire the missile. One second is allowed for the pilot to observe the firing indicator, push the firing button, and launch the missile.

The time delays indicated in Fig. 4 are by no means fixed. The over-all preparation time will vary from pilot to pilot and with the conditions of the mission. It is difficult to assign and justify finite steps to the preparation time because certain events may overlap. For example, during the time that the pilot is positioning the antenna he may also move the range gate over the target. While preparation time will vary from case to case, it is believed that 27 seconds represents a reasonable estimate of the time required with present equipment.

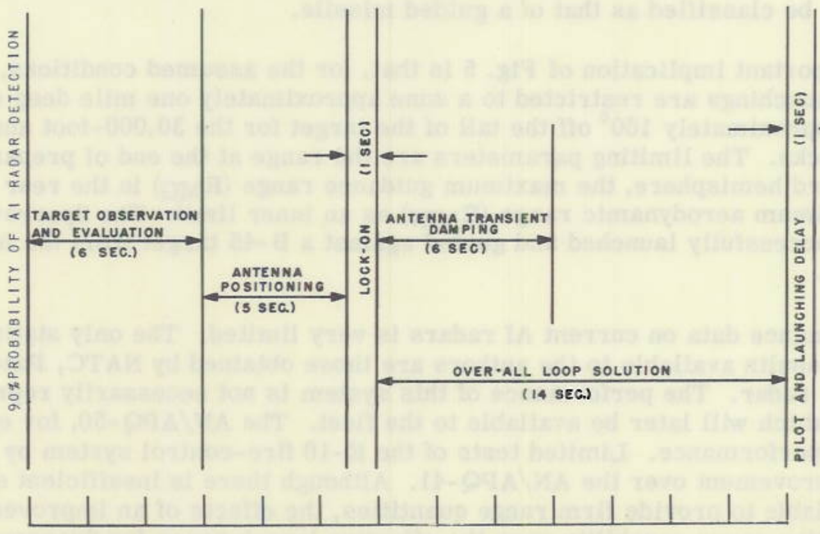


Fig. 4 - Estimated mean preparation time for Sparrow I

DECLASSIFIED

The overlay of curves of Fig. 5 define the effective launch zone for Sparrow I for "normal" conditions with 27 seconds preparation time. All curves are plotted with respect to bomber position. Normal conditions as defined in this report are:

1. AI radar detection ranges (R_D) are based on the results of the Patuxent tests of the AN/APQ-41 radar.

2. The Sparrow I maximum guidance range is limited in part by the tracking and guiding accuracy of current AI radars. Sperry Gyroscope Company, the prime contractor, assumes that the maximum range at which the missile can be effectively guided by the radar is 13,500 feet. Therefore, at the time the missile and target approach collision, there must not be more than 13,500 feet separating the target and guiding radar.

3. The average velocity for the Sparrow I missile maximum guidance range was assumed to be the average of the velocities for the Sparrow I over the maximum and minimum aerodynamic ranges.

The preparation time curve (R_P) indicates the distance between fighter and target at the end of the 27-second preparation time.

The remaining curves of Fig. 5, which are repeated in subsequent figures, are as follows:

1. the Sparrow I maximum aerodynamic range (R_{MAX}) which is limited to 31,800 feet at a 5000-foot altitude and 63,400 feet at a 30,000-foot altitude,

2. the Sparrow I minimum aerodynamic range (R_{MIN}) which is defined as ten missile time constants (including 5 seconds for arming time) plus two seconds of unguided flight. At shorter ranges the performance of the Sparrow I cannot be classified as that of a guided missile.

The important implication of Fig. 5 is that, for the assumed conditions, effective Sparrow I launchings are restricted to a zone approximately one mile deep extending from tail-on to approximately 100° off the tail of the target for the 30,000-foot and 5000-foot altitude attacks. The limiting parameters are the range at the end of preparation time (R_P) in the forward hemisphere, the maximum guidance range (R_{MG}) in the rear hemisphere, and the minimum aerodynamic range (R_{MIN}) as an inner limit. The Sparrow I missile cannot be successfully launched and guided against a B-45 target from the forward hemisphere.

Performance data on current AI radars is very limited. The only statistically valid evaluation results available to the authors are those obtained by NATC, Patuxent, for the AN/APQ-41 radar. The performance of this system is not necessarily representative of the radars which will later be available to the fleet. The AN/APQ-50, for example, should have better performance. Limited tests of the E-10 fire-control system by fleet units indicate improvement over the AN/APQ-41. Although there is insufficient statistical information available to provide firm range quantities, the effects of an improvement in AI radar detection range capability upon the effective launch zones for Sparrow I is of interest.

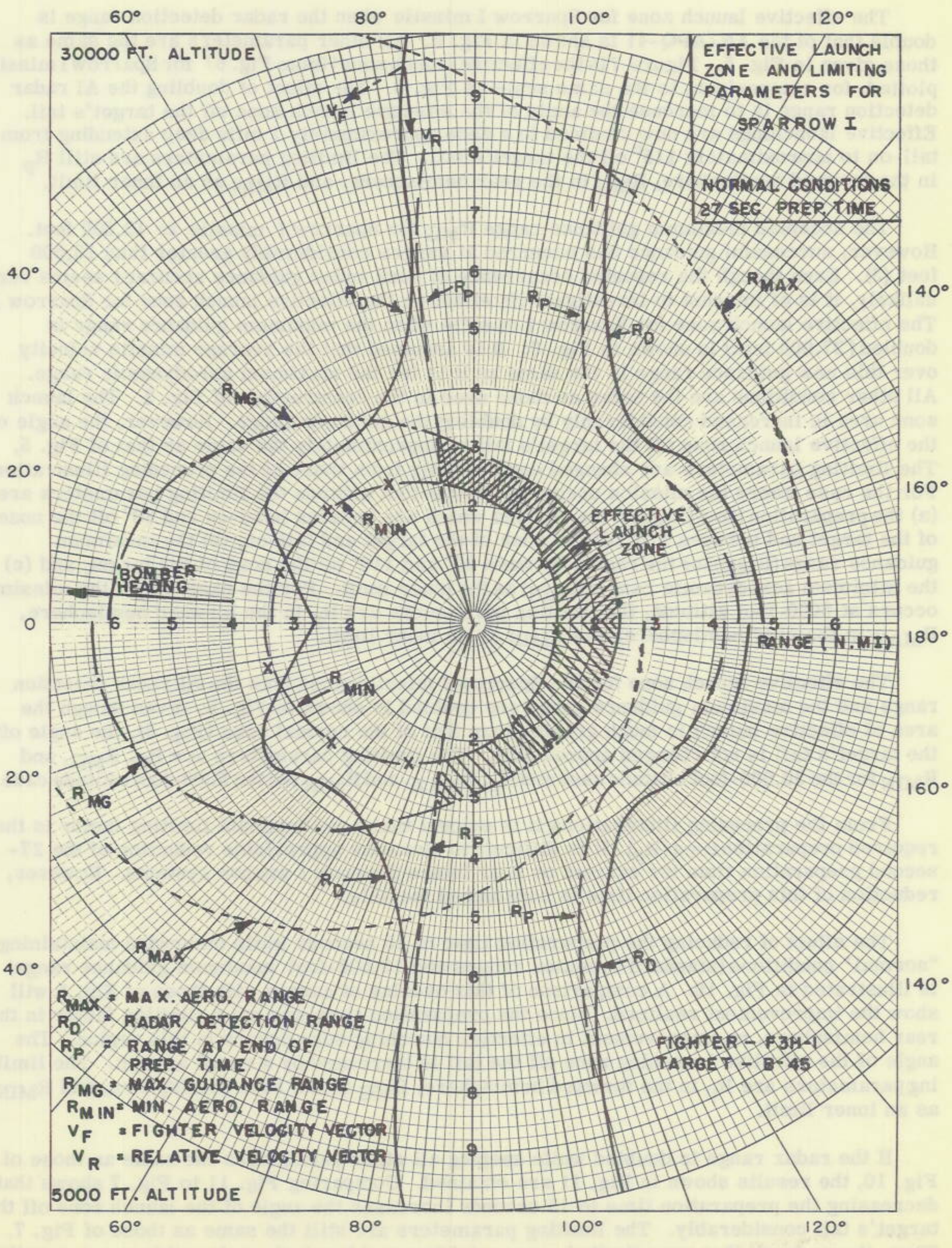

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE AND LIMITING PARAMETERS FOR SPARROW I
NORMAL CONDITIONS
27 SEC PREP. TIME
60°
80°
100°
120°
40°
140°
20°
160°
0
180°
BOMBER HEADING
EFFECTIVE LAUNCH ZONE
RANGE (N.MI)
V_F
V_R
R_D
R_P
R_{MAX}
R_{MG}
R_{MIN}
R_{MAX} = MAX. AERO. RANGE
R_D = RADAR DETECTION RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-45
5000 FT. ALTITUDE

Fig. 5

The effective launch zone for Sparrow I missile when the radar detection range is double that of the AN/APQ-41 is shown in Fig. 6. All other parameters are the same as those given in Fig. 5. Figure 7 is the effective launch zone (from Fig. 6) for Sparrow I missile plotted, for comparison, to the same scale as Fig. 5. The effect of doubling the AI radar detection range is to increase the angle of the effective launch zone off the target's tail. Effective launchings are now confined to a zone approximately 1 mile deep extending from tail-on to approximately 115° off the target's tail. The limiting parameters are still R_p in the forward hemisphere, R_{MG} in the rear hemisphere, and R_{MIN} as an inner limit.

The accepted maximum guidance range R_{MG} for Sparrow I missile is 13,500 feet. However, successful guidance has occurred at ranges considerably greater than 13,500 feet (3). Even though the repeated occurrence of such radar guidance accuracy seems very unlikely, it is of interest to investigate its effect on the effective launch zone for Sparrow I. The effective launch zone for Sparrow I missile when the maximum guidance range is doubled (27,000 feet) is shown in Fig. 8. It is assumed that the average missile velocity over this new guidance range is the same as it is for the maximum aerodynamic range. All other conditions are the same as those used in the construction of Fig. 5. The launch zone area is increased considerably by doubling the guidance range. However, the angle of the effective launch zone off the target's tail is approximately the same as that of Fig. 5. The limiting parameters are changed considerably from those of the preceding illustrations. For the case where the mission occurs at 30,000-foot altitude the limiting parameters are (a) the preparation time (R_p) for the regions which are between head-on and 80° off the nose of the target and between 110° off the nose of the target and tail-on, (b) the maximum guidance range (R_{MG}) for the region between 80° and 110° off the nose of the target, and (c) the minimum aerodynamic range (R_{MIN}) as the inner limit. For the case where the mission occurs at 5000-foot altitude, the limiting parameters are R_p in the forward hemisphere, R_{MAX} in the rear hemisphere, and R_{MIN} as an inner limit.

The effective launch zone for the Sparrow I missile when both the AI radar detection range and the maximum guidance range are doubled is shown in Fig. 9. Even though the area of effective launch is much greater than that of the normal case (Fig. 5), the angle off the target's tail is not changed appreciably. The limiting parameters are R_p, R_{MG}, and R_{MIN} for the 30,000-foot altitude case and R_p, R_{MAX}, and R_{MIN} for the 5000-foot altitude case.

From the preceding illustrations it is evident that a predominant limiting factor is the required preparation time (R_p). It is difficult to visualize appreciable reduction of the 27-second preparation time for current or near future Sparrow I weapon systems. However, reduction of this preparation time is considered feasible.

The effect of reducing the preparation time to 15 seconds while otherwise maintaining "normal" conditions (standard detection ranges and 13,500-foot maximum guidance range) is illustrated in Fig. 10. A comparison of this overlay of curves with those of Fig. 5 will show the improvement realized. Since the preparation time was not a limiting factor in the rear hemisphere, no improvement in effective launching zone occurs in this region. The angle of the effective launching zone off the tail is increased by a small amount. The limiting parameters are R_p in the forward hemisphere, R_{MG} in the rear hemisphere, and R_{MIN} as an inner limit.

If the radar range is doubled while keeping all other parameters the same as those of Fig. 10, the results shown in Fig. 11 are obtained. Comparing Fig. 11 to Fig. 7 shows that decreasing the preparation time to 15 seconds increases the angle of the launch zone off the target's tail considerably. The limiting parameters are still the same as those of Fig. 7. Thus, it was found that very limited successful forward hemisphere launchings are possible, while head-on attacks are not practical.

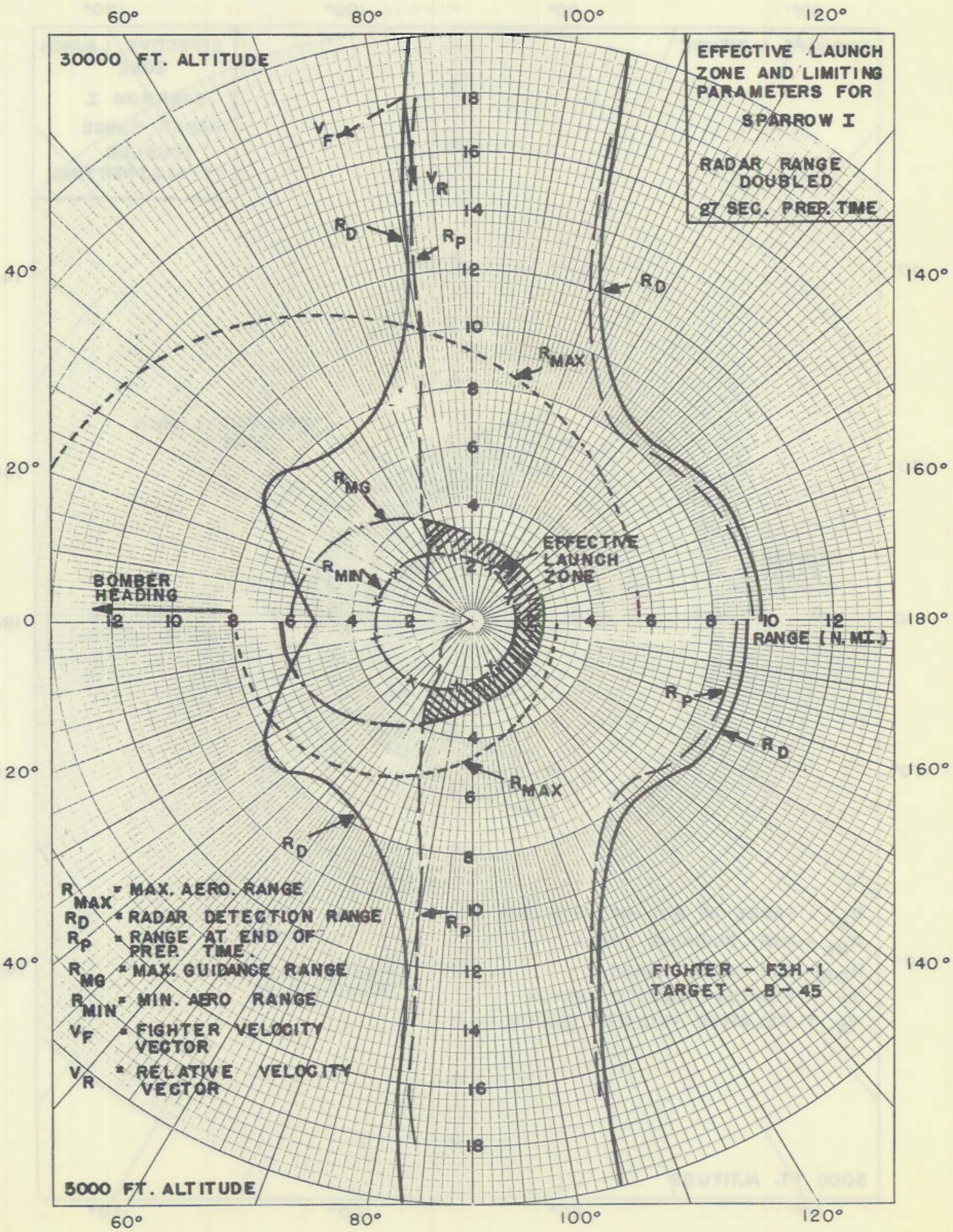

EFFECTIVE LAUNCH ZONE AND LIMITING PARAMETERS FOR SPARROW I
RADAR RANGE DOUBLED
27 SEC. PREP. TIME
30000 FT. ALTITUDE
5000 FT. ALTITUDE
BOMBER HEADING
RANGE (N. MI.)
EFFECTIVE LAUNCH ZONE
FIGHTER - F3H-1
TARGET - B-45
R MAX = MAX. AERO. RANGE
RD = RADAR DETECTION RANGE
RP = RANGE AT END OF PREP. TIME.
R MG = MAX. GUIDANCE RANGE
R MIN = MIN. AERO RANGE
VF = FIGHTER VELOCITY VECTOR
VR = RELATIVE VELOCITY VECTOR
60°
80°
100°
120°
40°
20°
0
140°
160°
180°

Fig. 6

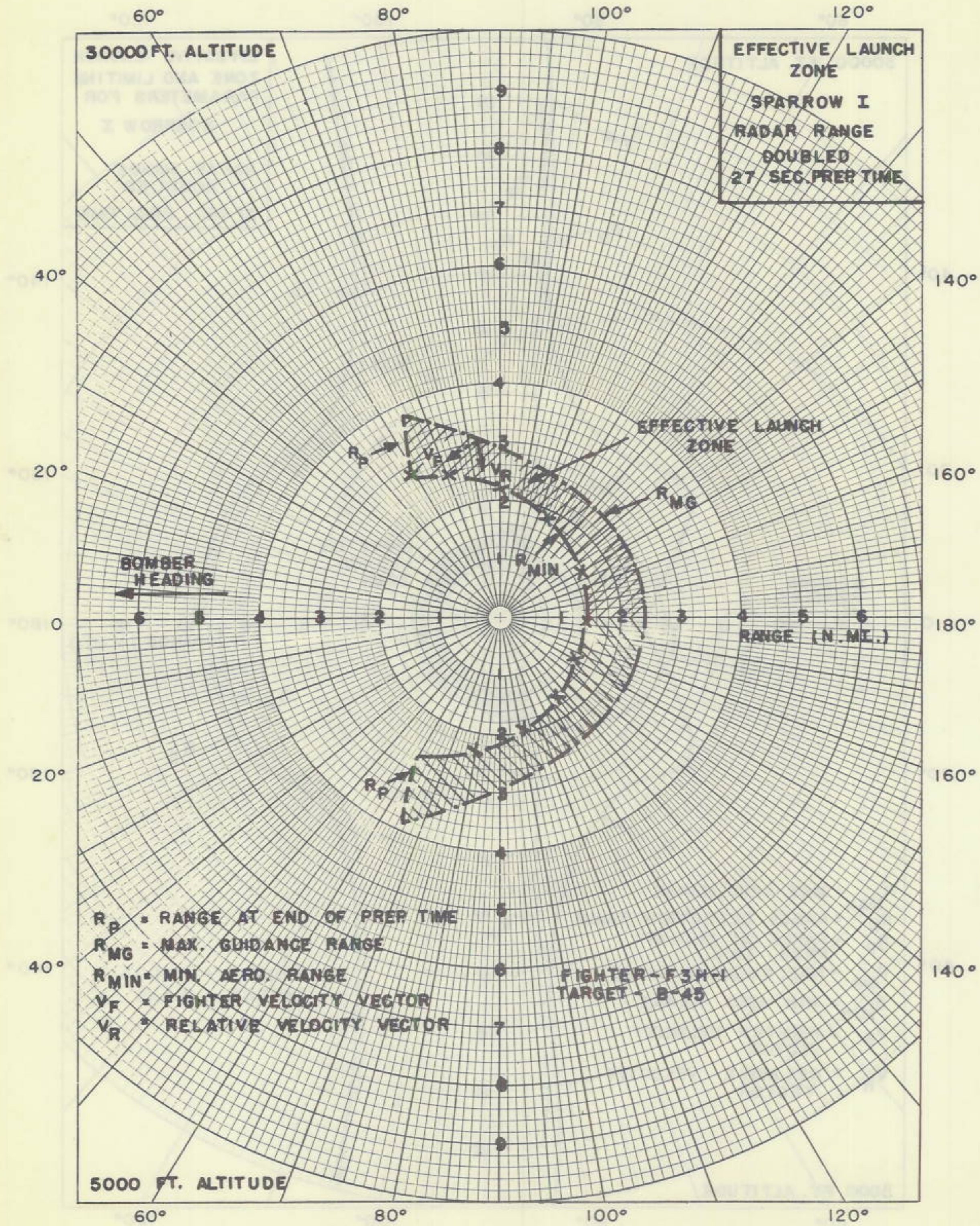

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR RANGE DOUBLED
27 SEC. PREP TIME
EFFECTIVE LAUNCH ZONE
R_P
V_F
V_R
R_MG
R_MIN
BOMBER HEADING
RANGE (N. MI.)
R_P = RANGE AT END OF PREP TIME
R_MG = MAX. GUIDANCE RANGE
R_MIN = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-45
5000 FT. ALTITUDE
60°
80°
100°
120°
40°
20°
0
140°
160°
180°

Fig. 7

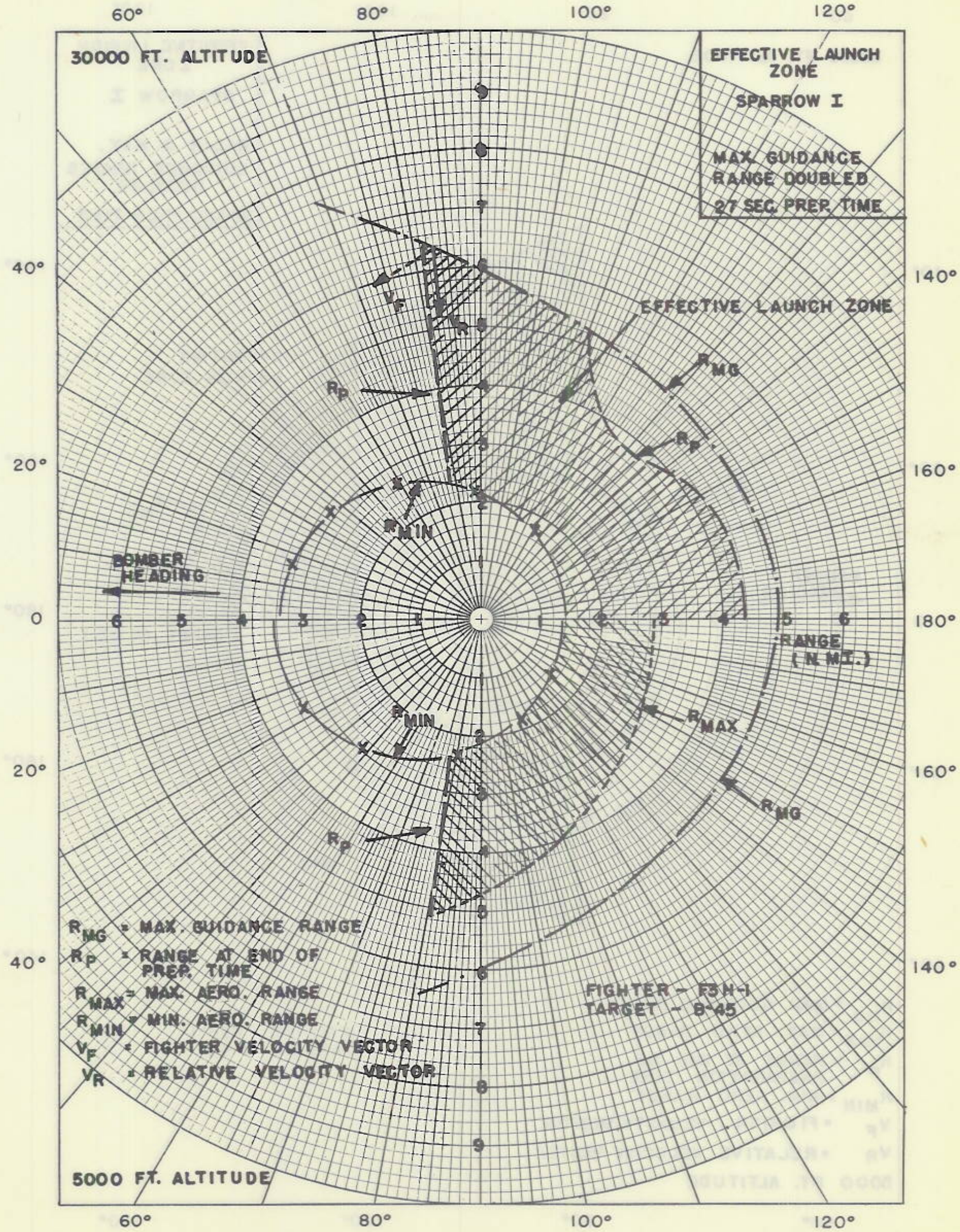
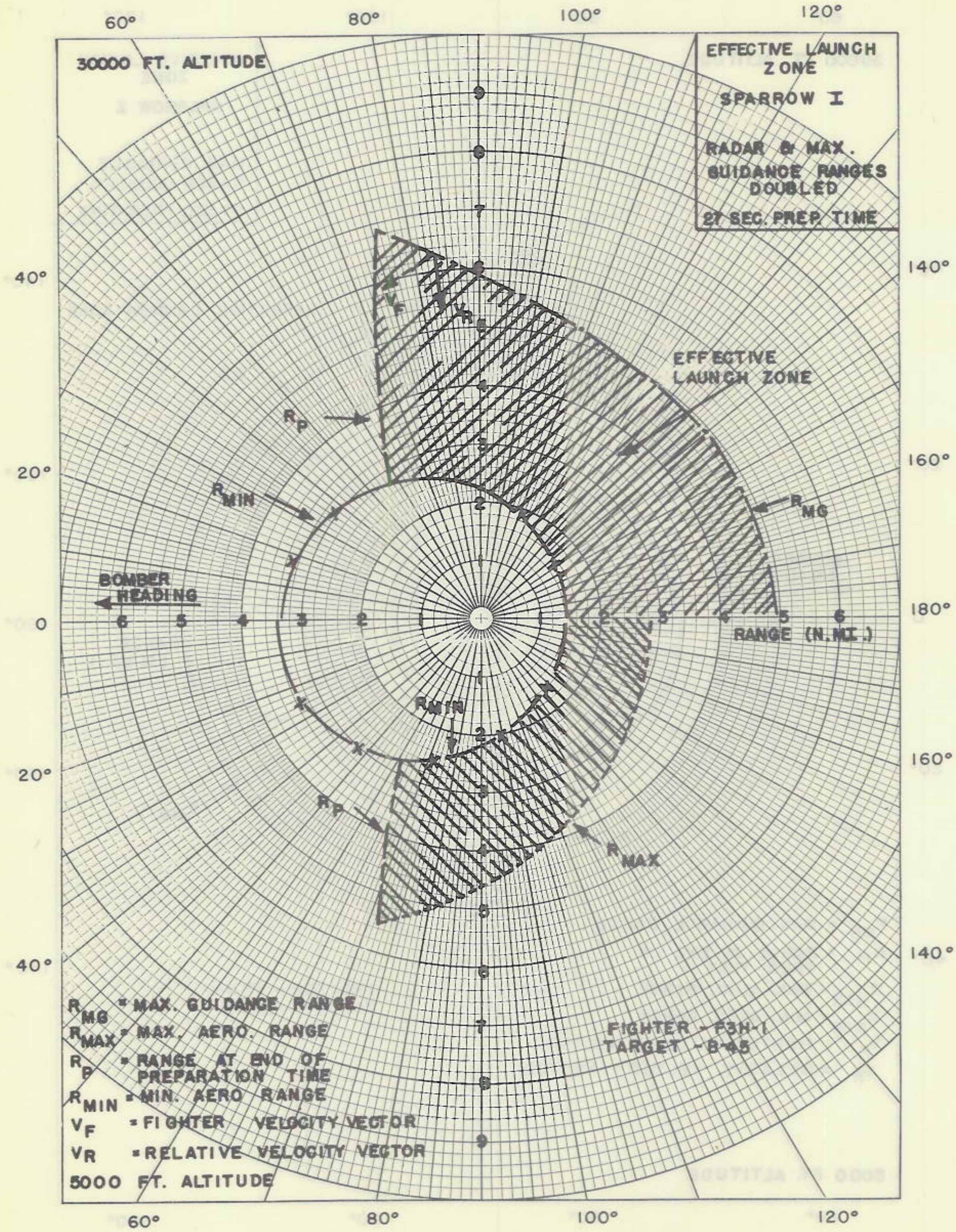

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
MAX. GUIDANCE RANGE DOUBLED
27 SEC. PREP. TIME
EFFECTIVE LAUNCH ZONE
BOMBER HEADING
RANGE (N.MI.)
R_{MG} = MAX. GUIDANCE RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MAX} = MAX. AERO. RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-45
5000 FT. ALTITUDE
60°
80°
100°
120°
40°
20°
0
140°
160°
180°

Fig. 8



30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR & MAX. GUIDANCE RANGES DOUBLED
27 SEC. PREP. TIME
EFFECTIVE LAUNCH ZONE
BOMBER HEADING
RANGE (N.MI.)
R_{MG} = MAX. GUIDANCE RANGE
R_{MAX} = MAX. AERO. RANGE
R_P = RANGE AT END OF PREPARATION TIME
R_{MIN} = MIN. AERO RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE
FIGHTER - F3H-1
TARGET - B-45

Fig. 9

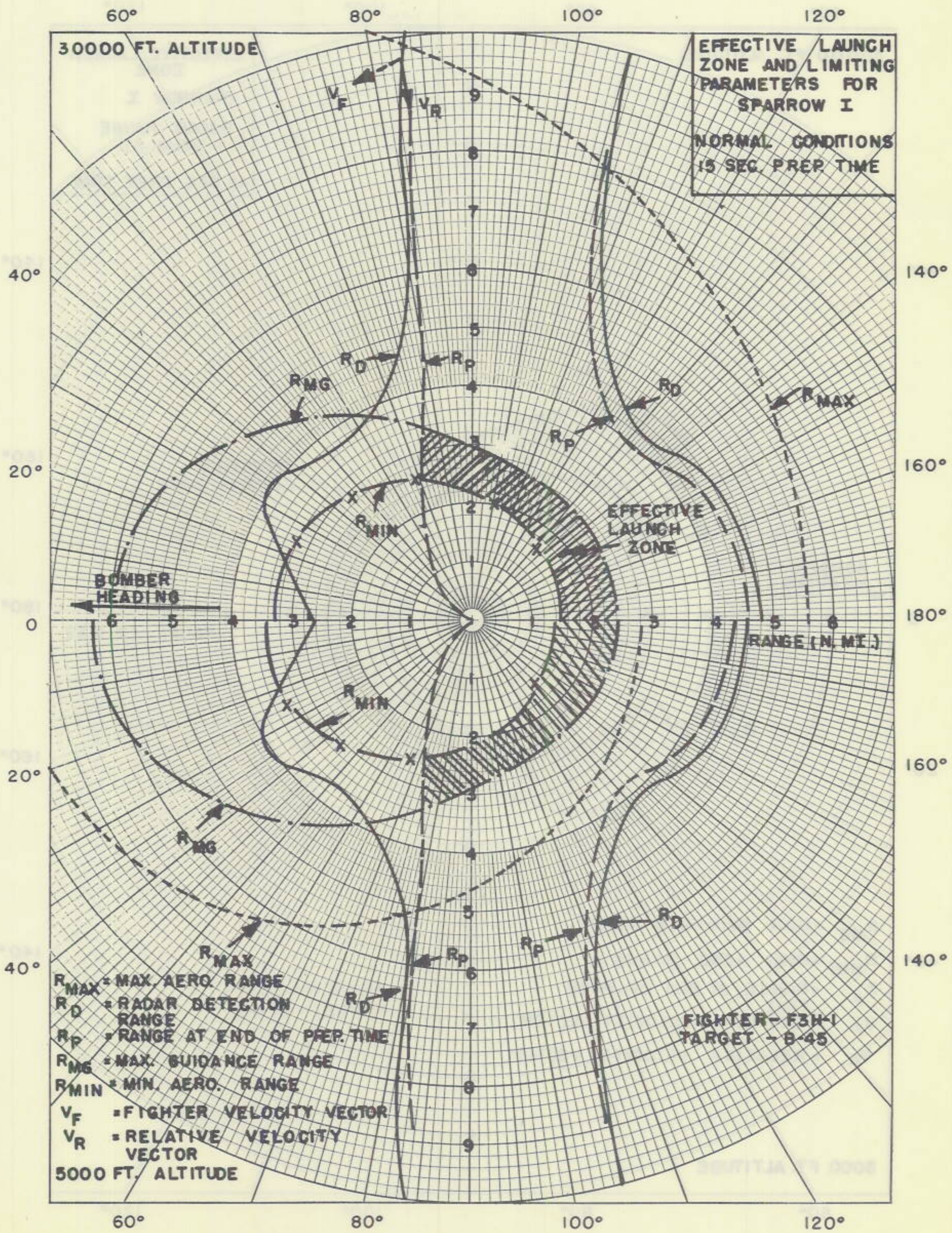
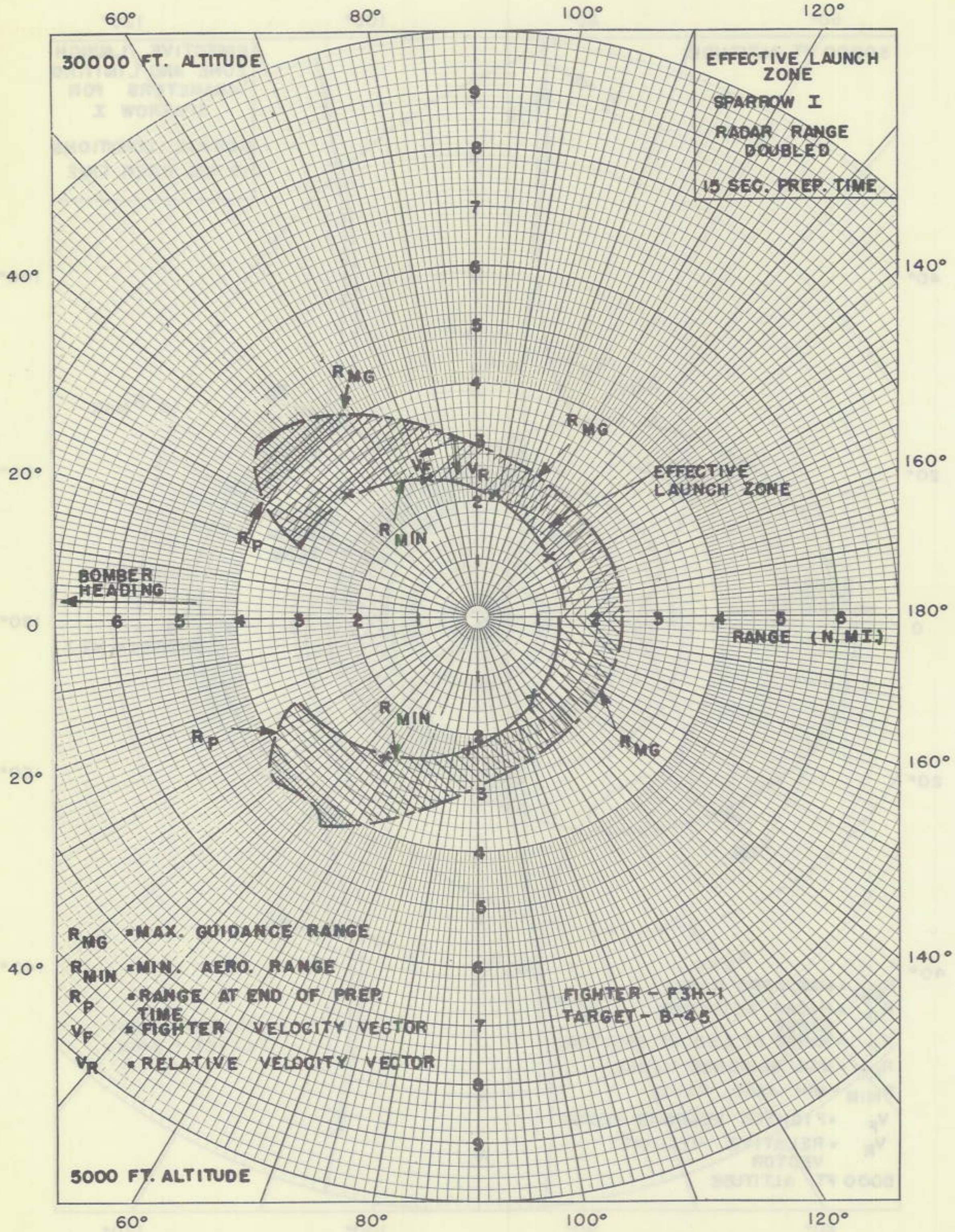

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE AND LIMITING PARAMETERS FOR SPARROW I
NORMAL CONDITIONS
15 SEC. PREP. TIME
60°
80°
100°
120°
40°
20°
0
140°
160°
180°
V_F
V_R
R_D
R_P
R_{MG}
R_{MAX}
R_{MIN}
EFFECTIVE LAUNCH ZONE
BOMBER HEADING
RANGE (N. MI.)
R_{MAX} = MAX. AERO. RANGE
R_D = RADAR DETECTION RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE
FIGHTER - F3H-1
TARGET - B-45

Fig. 10



30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR RANGE DOUBLED
15 SEC. PREP. TIME
60°
80°
100°
120°
140°
160°
180°
40°
20°
0
R_MG
V_F
V_R
R_MIN
R_P
EFFECTIVE LAUNCH ZONE
BOMBER HEADING
RANGE (N. MI.)
R_MG = MAX. GUIDANCE RANGE
R_MIN = MIN. AERO. RANGE
R_P = RANGE AT END OF PREP. TIME
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-45
5000 FT. ALTITUDE

Fig. 11

The effective launch zone for a 15-second preparation time with the maximum guidance range doubled is shown in Fig. 12. A comparison of Fig. 12 with Fig. 8 shows that an increase in the launching area for the 30,000-foot case results from reducing the preparation time. However there is only a minor improvement in the angle of the launch zone off the target's tail. Since for the 5000-foot altitude case R_p was not a limiting factor in the rear hemisphere, there is only a minor improvement in the effective launch area over that shown in Fig. 8. The limiting parameters are the same as those of Fig. 8.

The situation illustrated in Fig. 13 is considered to be optimistic. This group of curves sets the boundaries for the Sparrow I missile launch zone when the required preparation time is fifteen seconds, the radar detection range is doubled, and the maximum guidance range is doubled. Comparing Fig. 13 with Fig. 9 shows that the areas of the effective launch zones and the angles off the tail of the target for both cases (30,000 feet and 5000 feet) are appreciably improved. However, only limited forward hemisphere launchings are possible. The limiting parameters are R_p in the forward hemisphere, R_{MG} in the rear hemisphere, and R_{MIN} as an inner limit for the case of the target flying at an altitude of 30,000 feet, and for the case of the target flying at an altitude of 5000 feet, R_p in the forward hemisphere, R_{MAX} in the rear hemisphere, and R_{MIN} as an inner limit.

Sparrow I vs. B-29 Target

All of the preceding investigation has been concerned with the B-45 jet-type target. It is believed that this type of bomber will be one of the more difficult air targets that the fleet will encounter in any engagement in the near future. However, it is known that the potential enemy has a large quantity of propeller-driven bombers (B-29 type). It is of interest therefore to consider such a target.

The B-29 has a speed of 282 knots at 5000-foot altitude and a speed of 347 knots at 30,000-foot altitude (4). The effective radar area at X-band of the B-29 for various aspect angles is shown in Fig. 14. The solid portion of the curve was obtained by smoothing test data obtained from tests by the Naval Research Laboratory (5). The broken portion of the curve indicates that no test data was available for this region. Head and tail effective radar areas were obtained from tests conducted by Hughes Aircraft Corporation (6). The ratio of B-29 to B-45 areas for the head-on and tail-on aspects are approximately 17 to 1 and 66 to 1 respectively. This curve of equivalent radar areas for the B-29 will be used to develop a probability of detection curve using the same procedure as was used in the Cornell Report(1).

Figure 15 shows the probability of detection curve for the standard radar against the tail aspect of a B-45 bomber at a 50-knot closing rate. This curve and the following probability equations were obtained from the Cornell report (1). The curve was derived from the results of Patuxent tests and from measurements of effective radar areas of the F-86 and B-45 aircraft taken by the Naval Research Laboratory. The cumulative probability of detection against the B-45 target (tail aspect) when the rate of closure is 303 knots is given by Fig. 16. This 303 knots is the closure rate of the F3H-1 in a tail attack against a B-29 target at 5000-foot altitude. This curve will be used as the basis for developing the 90% probability of detection range contour for the standard radar as a function of aspect angle against the B-29 target.

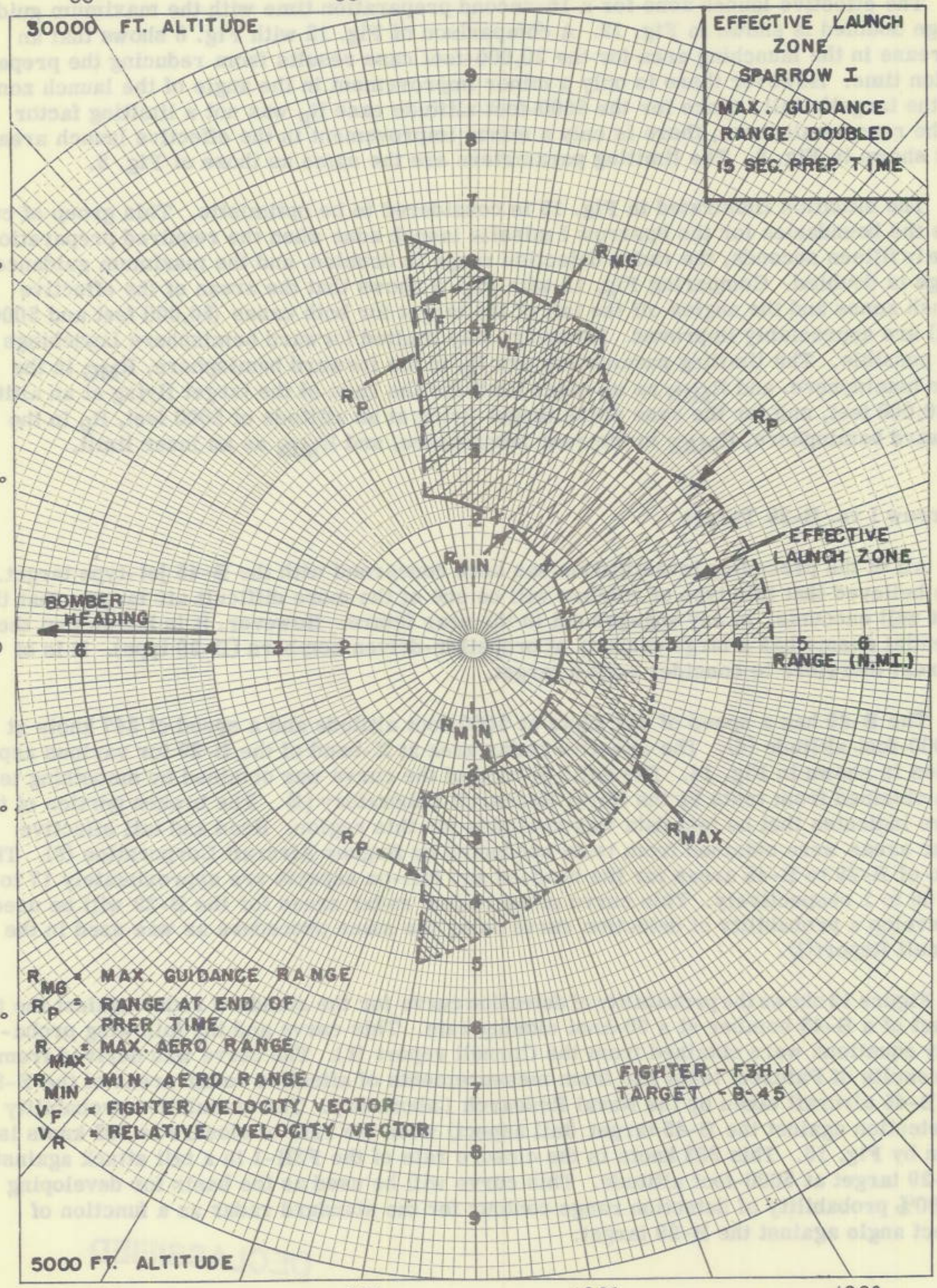

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
MAX. GUIDANCE RANGE DOUBLED
15 SEC. PREP. TIME
60°
80°
100°
120°
40°
20°
0
140°
160°
180°
R_{MG}
V_F
V_R
R_P
R_{MIN}
R_{MAX}
EFFECTIVE LAUNCH ZONE
BOMBER HEADING
RANGE (N.MI.)
R_{MG} = MAX. GUIDANCE RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MAX} = MAX. AERO RANGE
R_{MIN} = MIN. AERO RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-45
5000 FT. ALTITUDE

Fig. 12

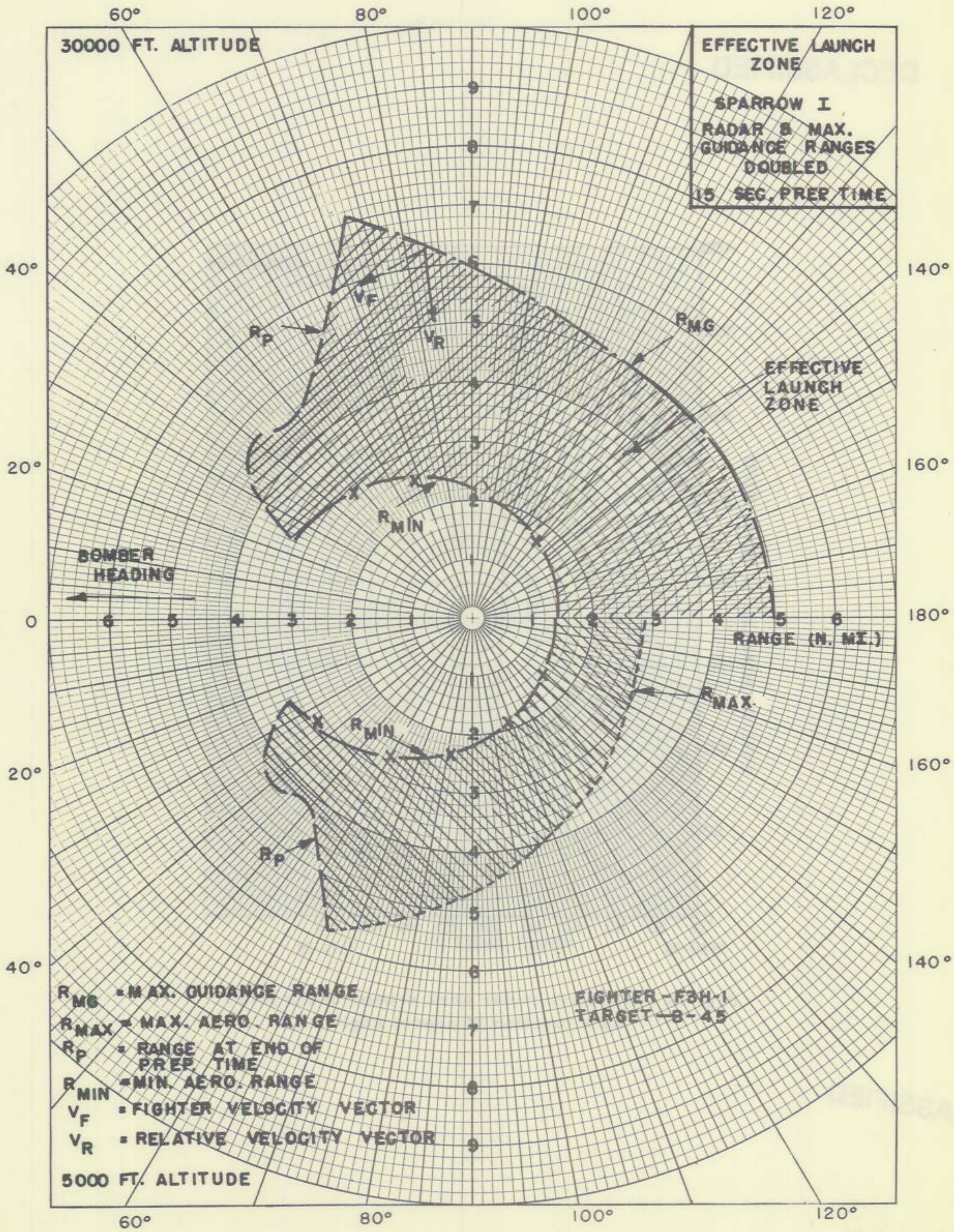

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR & MAX. GUIDANCE RANGES DOUBLED
15 SEC. PREP TIME
V_F
V_R
R_P
R_{MG}
EFFECTIVE LAUNCH ZONE
R_{MIN}
BOMBER HEADING
RANGE (N. MI.)
R_{MAX}
R_{MIN}
R_P
R_{MG} = MAX. GUIDANCE RANGE
R_{MAX} = MAX. AERO. RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-45
5000 FT. ALTITUDE
60°
80°
100°
120°
40°
20°
0
140°
160°
180°

Fig. 13

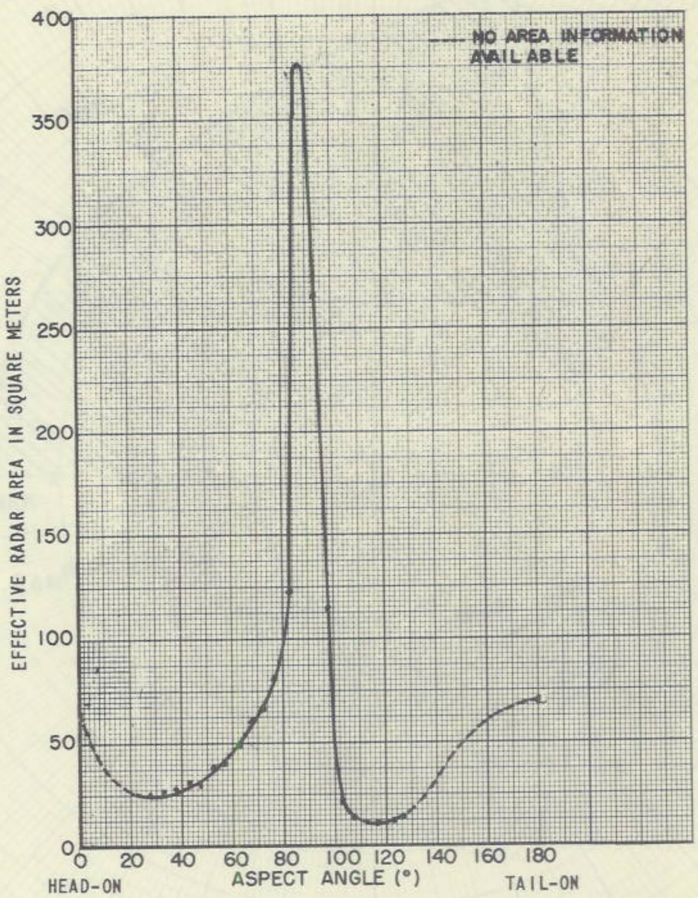


Fig. 14 - Effective radar area x-band of the B-29 as a function of aspect angle

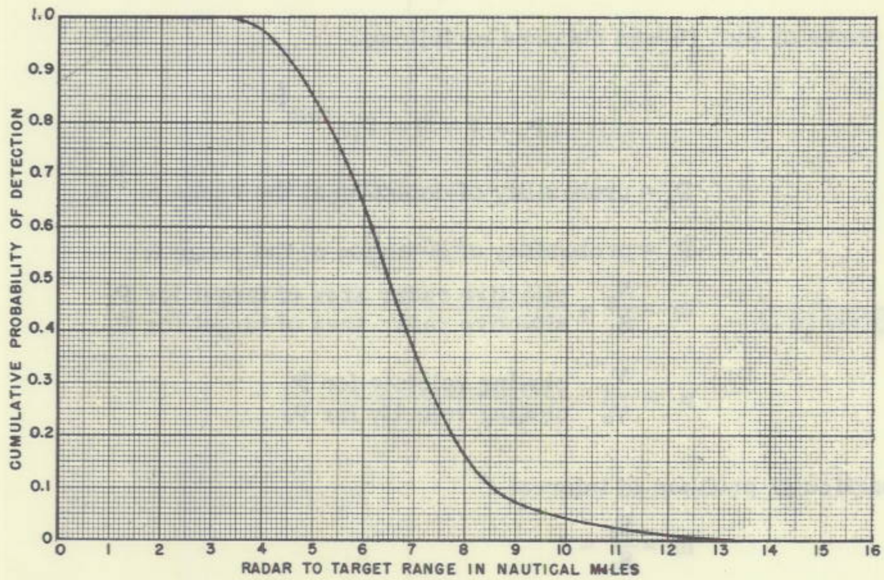


Fig. 15 - Cumulative probability of detection as a function of range for the AN/APQ-41 radar against the tail aspect of a B-45 bomber (closing rate of 50 knots)

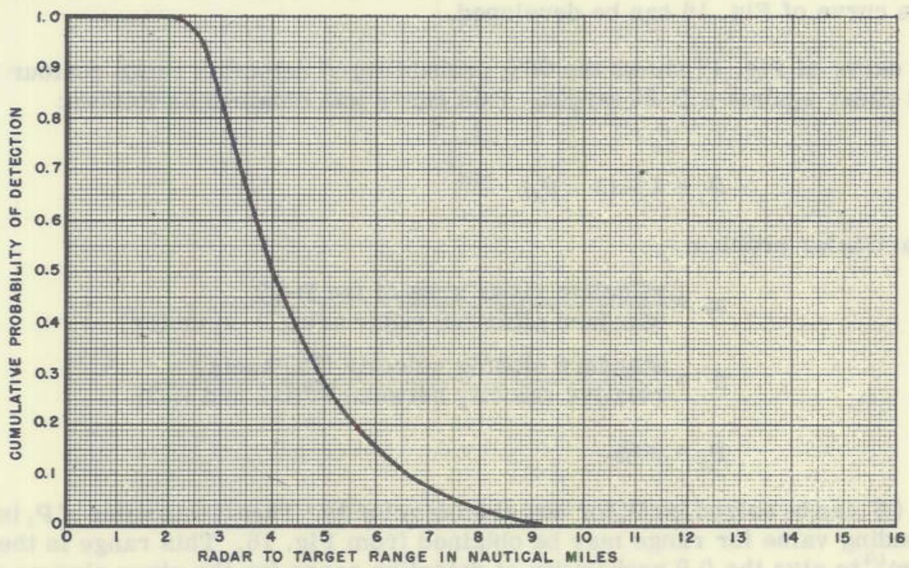


Fig. 16 - Cumulative probability of detection as a function of range for the AN/APQ-41 radar against the tail aspect of a B-45 bomber (closing rate of 303 knots)

The curve of Fig. 16 was obtained as follows:

$$P_2 = 1 - (1 - P_1)^{nm^{\frac{1}{4}}} \qquad (3)$$

where

P_2 = probability of detection at the range $m^{\frac{1}{4}} R$

P_1 = probability of detection at the range R

$$m = \frac{S_2}{S_1} = \frac{\text{effective radar area of target for } P_2}{\text{effective radar area of target for } P_1}$$

$$n = \frac{V_1}{V_2} = \frac{\text{closing velocity for } P_1}{\text{closing velocity for } P_2} .$$

In this particular problem of interest

$$m = \frac{S_2}{S_1} = 1$$

$$n = \frac{V_1}{V_2} = \frac{50}{303} = 0.165 .$$

Solving Eq. (3) for the various probabilities of detection obtained from Fig. 15 the new probabilities can be obtained. Since m = 1, the ranges will be the same as those of Fig. 15. Thus, the curve of Fig. 16 can be developed.

The curve of Fig. 17 shows the 90% probability of detection range contour for the standard radar against a B-29 target. This curve was obtained as follows:

$$P_1 = 1 - (1 - P_2)^{\frac{1}{nm^{\frac{1}{4}}}} . \qquad (4)$$

In this particular problem

$$m = \frac{\text{effective radar area of the B-29}}{\text{standard effective radar area (1 sq meter)}}$$

$$n = \frac{\text{standard closing velocity (303 knots)}}{\text{relative velocity between F3H-1 and B-29}}$$

$$P_2 = 90\% .$$

Equation (4) can be solved for P_1 for any closing velocity. Then if this value of P_1 is used, a corresponding value for range may be obtained from Fig. 16. This range is then multiplied by $m^{1/4}$ to give the 0.9 probability of detection range for the given closure rate. From these values, the range contour of Fig. 17 is then plotted.

In addition to the range contour, Fig. 17 shows the radar gimbal angle for various approach aspects for the assumed tactical problem. This gimbal angle is the angle between the line of sight (fighter-to-target) and the line of flight of the fighter. It is interesting to note that when a constant true bearing course is assumed the attack approach is from the rear hemisphere for aspect angles of 70° or more off the target's nose for the 5000-foot-altitude case and for aspect angles of 60° or more off the target's nose for the 30,000-foot-altitude case. Table 2 lists the radar gimbal angles for various approach aspects both at 5000- and 30,000-foot altitudes for the B-29 target.

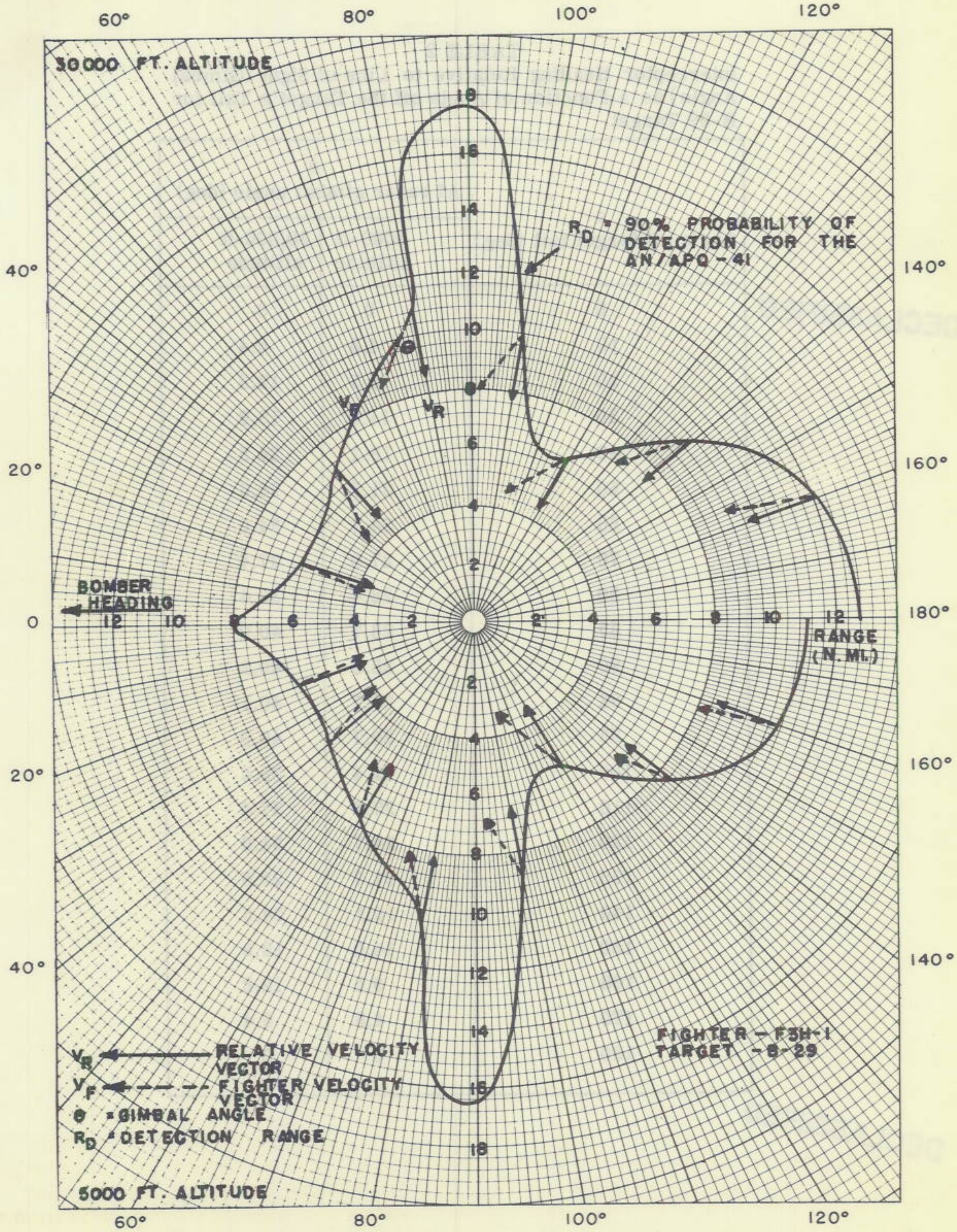


Fig. 17 - Fighter heading as a function of approach aspect against a B-29 target

TABLE 2
Interceptor Heading Relative to Line of Sight (Gimbal Angle) as a Function of Angle off the Target's Tail for B-29 Target.

ρ Angle off target tail (deg)	θ AI radar gimbal angle (deg)	
	5000-ft altitude	30,000-ft altitude
0	0	0
10	4.8	6.4
20	9.5	12.8
30	14.0	18.8
40	18.1	24.5
50	21.7	29.6
60	24.7	34.0
70	27.0	37.4
80	28.4	39.4
85	28.7	40.0
90	28.8	40.2
92	28.7	40.1
95	28.7	40.0
100	28.4	39.4
110	27.0	37.4
120	24.7	34.0
130	21.7	29.6
140	18.1	24.5
150	14.0	18.8
160	9.5	12.8
170	4.8	6.4
180	0	0

Throughout the investigation of the possible launch areas for Sparrow I against a B-29 target the same parameters are used as were used for the B-45 case except for target speed and size. Fig. 18 illustrates the effective launch zones and limiting parameters for Sparrow I against the B-29 target under normal conditions with a 27-second preparation time. In Fig. 19 the launch zone is plotted to the same scale as the preceding launch zones for the B-45 in order that a comparison can be made. When the mission occurs at 30,000-foot altitude the effective launch zone extends from tail-on to approximately 120° off the target's tail. The limiting parameters are R_P in the forward hemisphere, R_{MG} in the rear hemisphere, and R_{MIN} as an inner limit. When the mission occurs at the 5000-foot altitude the effective launch zone extends from tail-on to approximately 115° off the target's tail with a wedge removed between 50° and 80° off the target's tail. The limiting parameters are R_P for the head-on region and in the region between 50° and 80° off the target's tail, R_{MG} in the rear hemisphere, and R_{MIN} as an inner limit.

The results of doubling the radar detection ranges are illustrated in Fig. 20. The effective launch zone is replotted in Fig. 21 to the standard scale. Under conditions of double radar detection ranges, around-the-clock attacks are possible. However, one should remember that the curves are plotted for considerably more favorable conditions than normal. The limiting parameters are R_{MG} and R_{MIN} for both the 5000- and 30,000-foot-altitude cases. It is important to remember that although around-the-clock attacks are possible the time that the fighter is in a launching position is a function of approach aspect. In Fig. 21 the head-on launching area is approximately twice as deep as the tail-on launching area. However, the available time for the head-on launching is approximately one-half of the available time for the tail-on launching.

Fig. 22 shows the effective launch zone for the Sparrow I missile against a B-29 target when the maximum guidance range is doubled. All other parameters remain unchanged from those of Fig. 18. The launch zone area is appreciably increased over that of Fig. 19 for both 5000- and 30,000-foot altitudes but the angle off the target's tail is essentially unchanged. For the 5000-foot case the limiting parameters are R_P in the forward hemisphere, R_{MAX} as an outer boundary, and R_{MIN} as the inner boundary. The wedge between 50° and 80° is due to the effects of the preparation time (R_P). For the 30,000-foot case the limiting parameters are R_P in the forward hemisphere, R_{MG} as an outer boundary, and R_{MIN} as an inner boundary. A wedge is removed in the rear hemisphere due to the effects of R_P.

In Fig. 23 the results of doubling both the radar detection range and maximum guidance range are shown. All other parameters are the same as those of Fig. 18. Forward hemisphere attacks are possible. The limiting parameters are R_P from head-on to 55° off the target's nose, R_{MG} from 55° to tail-on, and R_{MIN} as the inner boundary for the 30,000-foot-altitude case and, for the 5000-foot-altitude case, R_{MAX} as the outer boundary and R_{MIN} as the inner boundary. For the 5000-foot case a small sector is removed because of R_P from 10° to 30° off the target's nose.

The following group of illustrations show the effect of reducing the preparation time to 15 seconds. Fig. 24 is an overlay of curves for a preparation time of 15 seconds with all other conditions "normal." In Fig. 25 the effective launch zone of Fig. 24 is replotted to the standard scale. Comparing Fig. 25 to Fig. 19 shows that the effect of the reduction in preparation time is to increase the angle of the effective launch zone off the target's tail. Head-on launching within a very restricted zone is possible. The wedge in the effective launch zone for the 5000-foot-altitude case is removed by reducing the preparation time to 15 seconds.

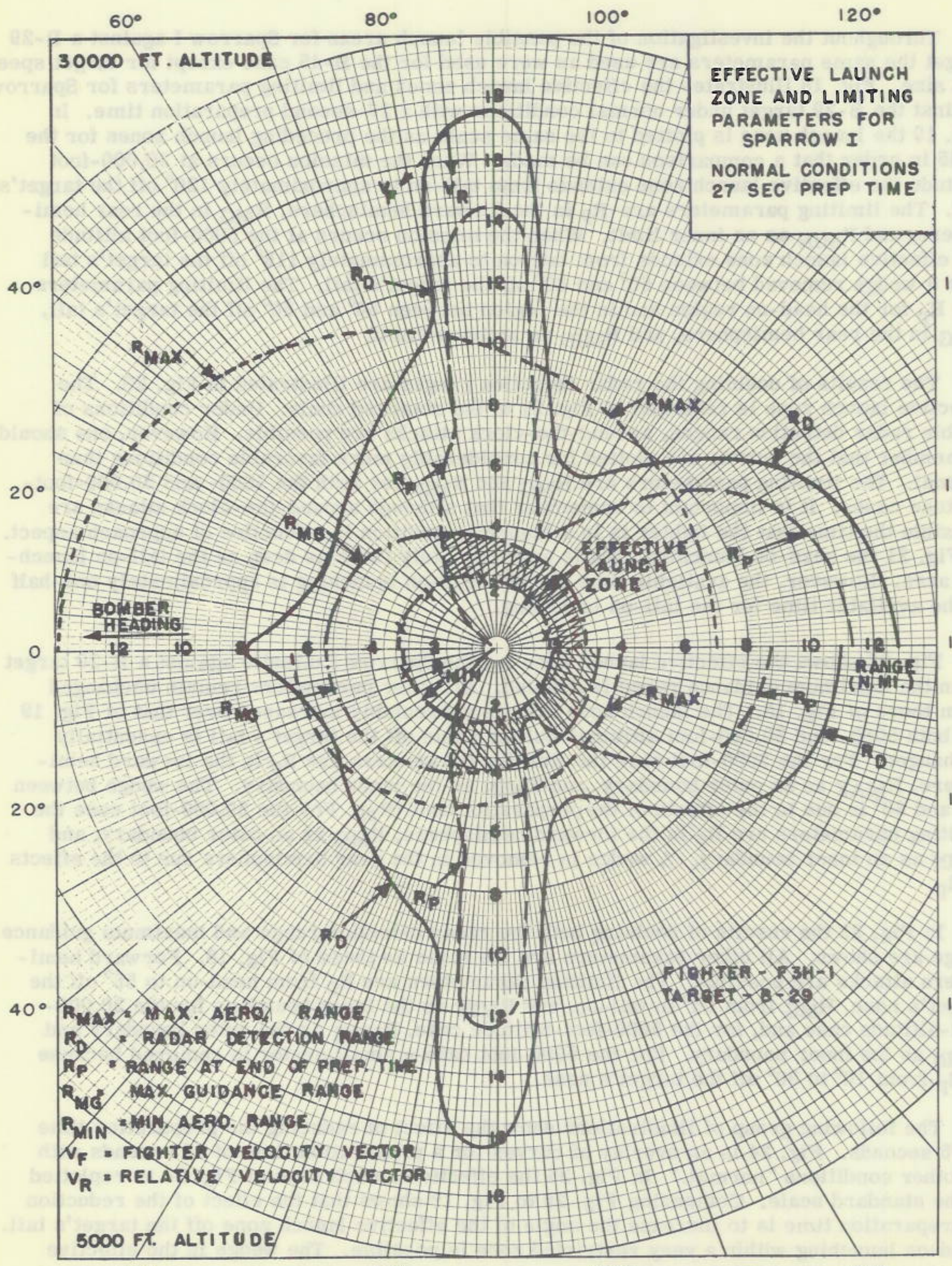

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE AND LIMITING PARAMETERS FOR SPARROW I
NORMAL CONDITIONS
27 SEC PREP TIME
60°
80°
100°
120°
40°
140°
20°
160°
0
180°
V_F
V_R
R_D
R_{MAX}
R_P
R_{MG}
R_{MIN}
EFFECTIVE LAUNCH ZONE
BOMBER HEADING
RANGE (N. MI.)
FIGHTER - F3H-1
TARGET - B-29
R_{MAX} = MAX. AERO. RANGE
R_D = RADAR DETECTION RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE

Fig. 18

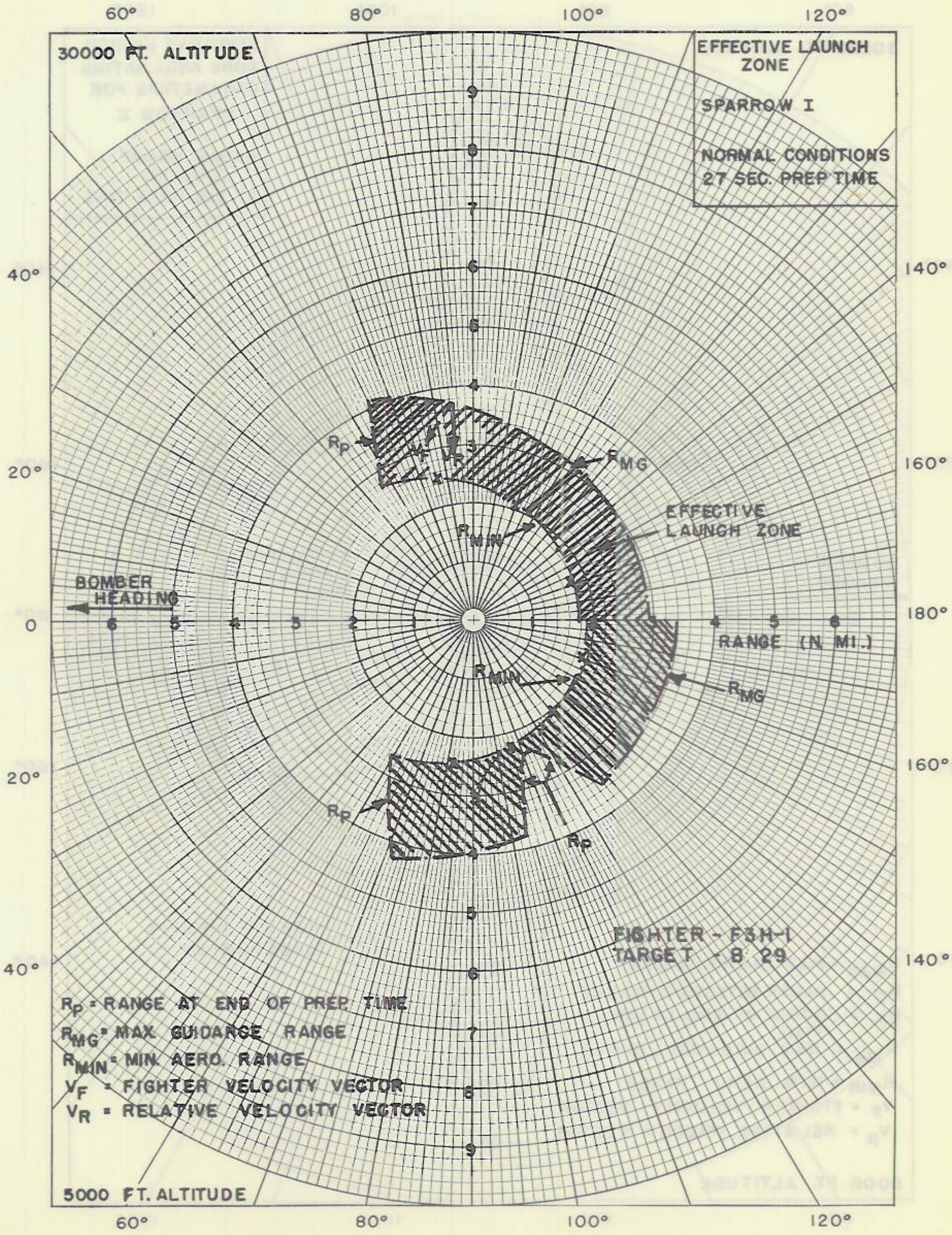

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
NORMAL CONDITIONS
27 SEC. PREP TIME
60°
80°
100°
120°
40°
20°
0
140°
160°
180°
BOMBER HEADING
RANGE (N. MI.)
R_P
V_F
V_R
X
R_{MG}
R_{MIN}
EFFECTIVE LAUNCH ZONE
FIGHTER - F3H-1
TARGET - B 29
R_P = RANGE AT END OF PREP TIME
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE

Fig. 19

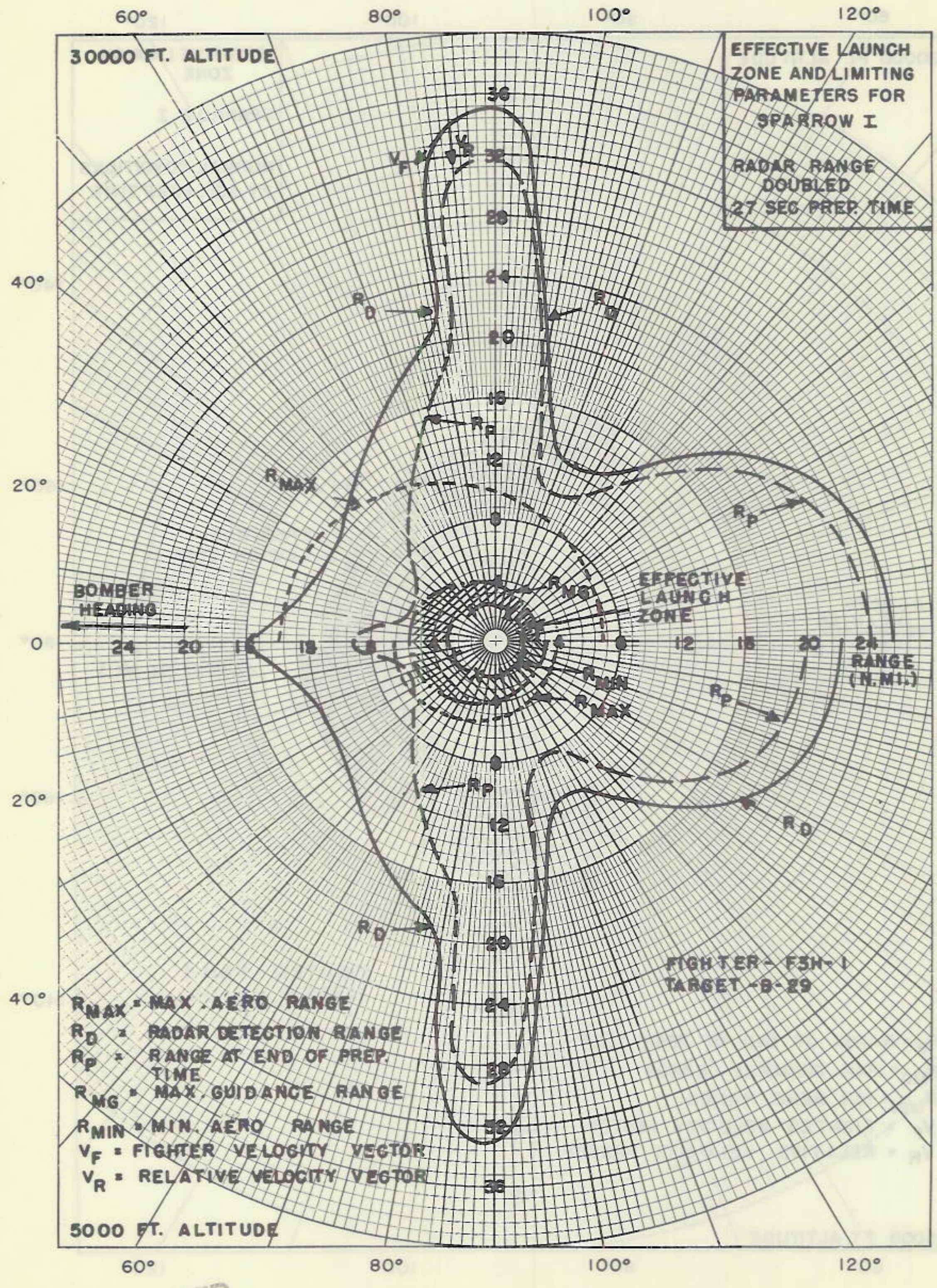

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE AND LIMITING PARAMETERS FOR SPARROW I
RADAR RANGE DOUBLED
27 SEC PREP. TIME
60°
80°
100°
120°
40°
20°
0
140°
160°
180°
BOMBER HEADING
RANGE (N.MI.)
EFFECTIVE LAUNCH ZONE
FIGHTER - F3H-1
TARGET - B-29
R_{MAX} = MAX. AERO RANGE
R_D = RADAR DETECTION RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE

Fig. 20

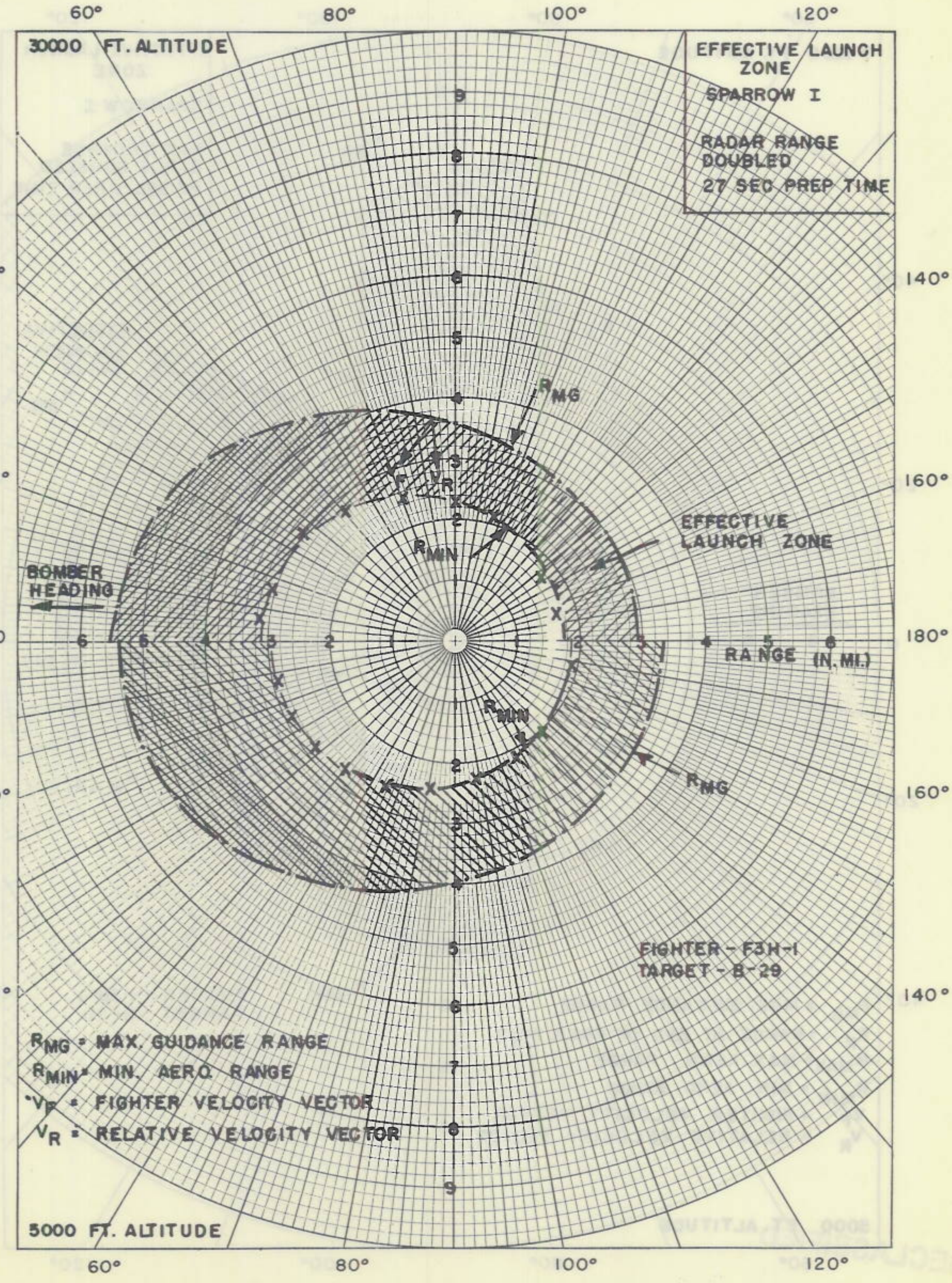

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR RANGE DOUBLED
27 SEC PREP TIME
BOMBER HEADING
EFFECTIVE LAUNCH ZONE
RMG
RMIN
RANGE (N.MI.)
FIGHTER - F3H-1
TARGET - B-29
RMG = MAX. GUIDANCE RANGE
RMIN = MIN. AERO. RANGE
VF = FIGHTER VELOCITY VECTOR
VR = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE

Fig. 21

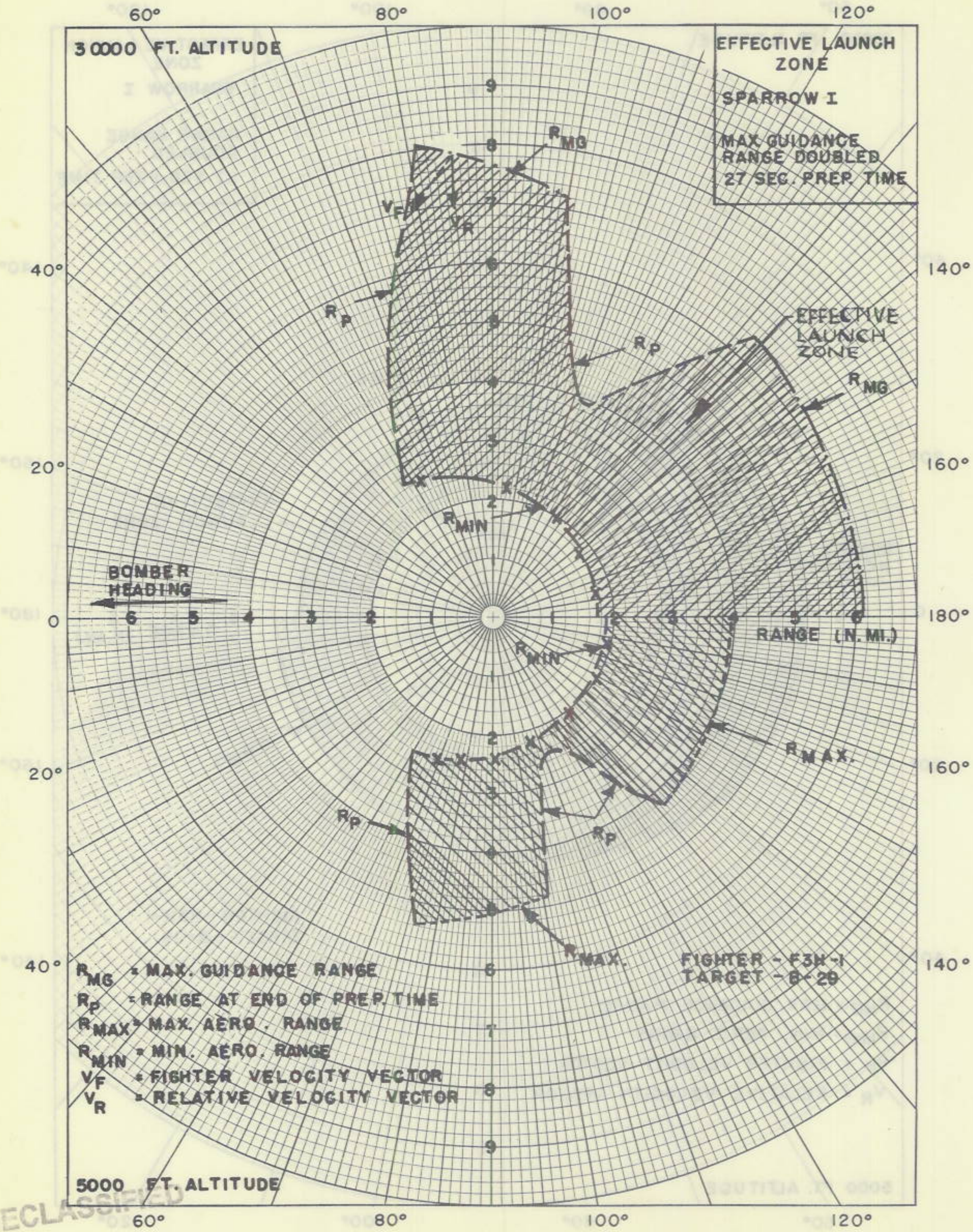
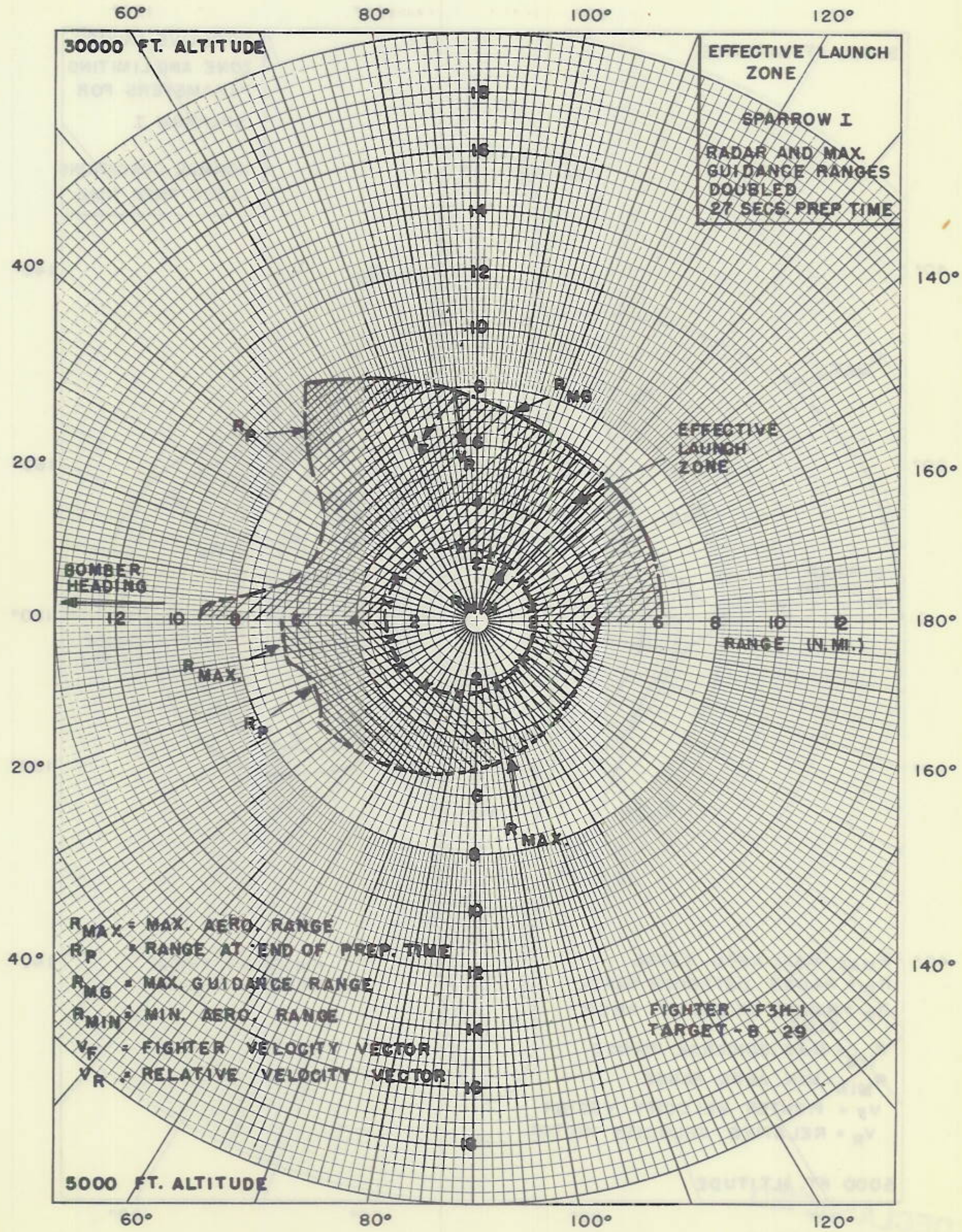
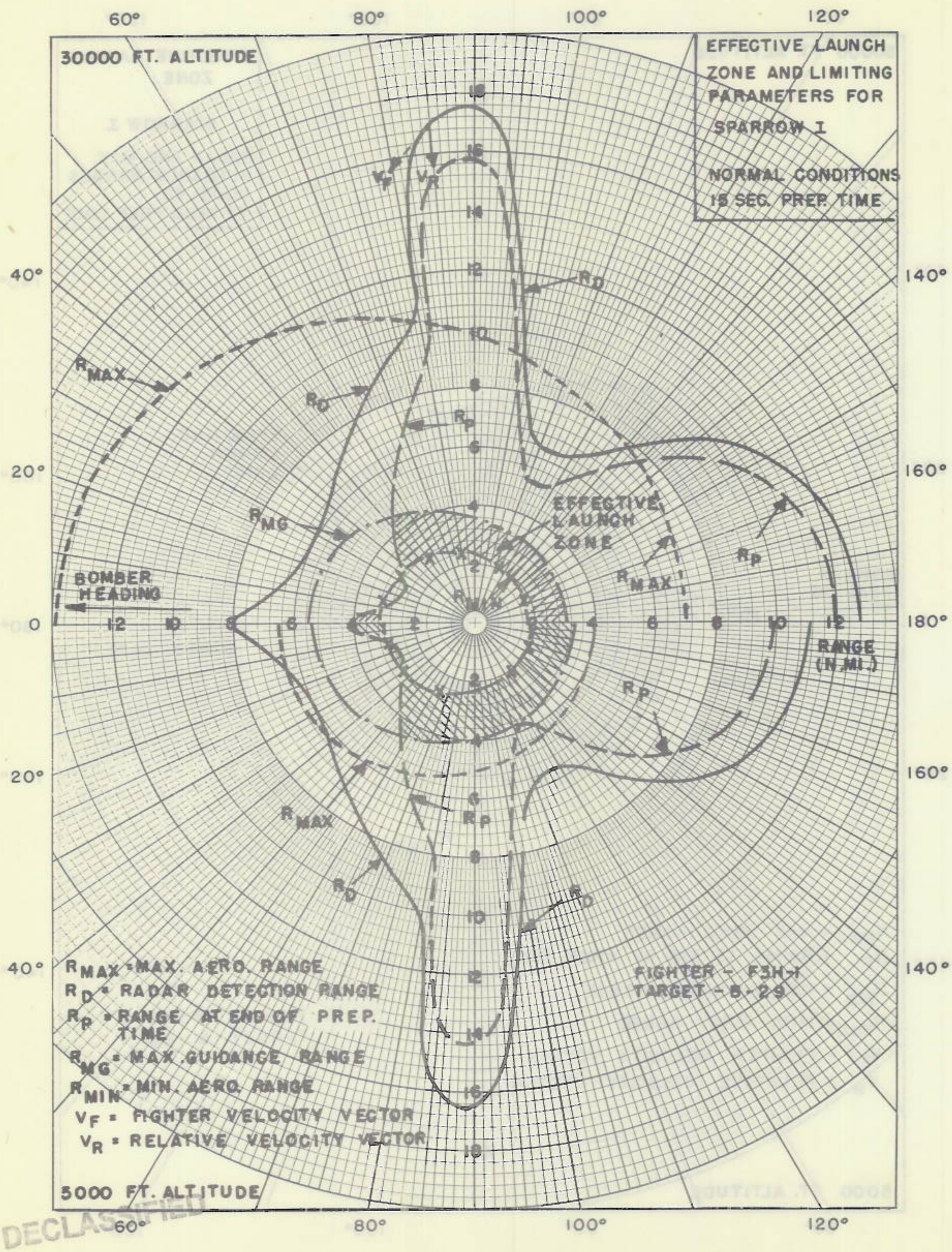

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
MAX GUIDANCE RANGE DOUBLED
27 SEC. PREP. TIME
60°
80°
100°
120°
40°
20°
0
140°
160°
180°
R_{MG}
V_F
V_R
R_P
EFFECTIVE LAUNCH ZONE
R_{MIN}
BOMBER HEADING
RANGE (N. MI.)
$R_{MAX.}$
FIGHTER - F3H-1
TARGET - B-29
R_{MG} = MAX. GUIDANCE RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MAX} = MAX. AERO. RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE

Fig. 22



60°
80°
100°
120°
30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR AND MAX. GUIDANCE RANGES DOUBLED
27 SECS. PREP TIME
40°
140°
20°
160°
R_P
R_{MG}
V_F
V_R
EFFECTIVE LAUNCH ZONE
BOMBER HEADING
0
180°
RANGE (N. MI.)
$R_{MAX.}$
R_{MIN}
R_{MAX} = MAX. AERO. RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B - 29
5000 FT. ALTITUDE

Fig. 23



60°
80°
100°
120°
30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE AND LIMITING PARAMETERS FOR SPARROW I
NORMAL CONDITIONS 15 SEC. PREP. TIME
40°
140°
20°
160°
0
180°
BOMBER HEADING
RANGE (N.MI.)
R MAX
R D
R P
R MG
R MIN
V F
V R
EFFECTIVE LAUNCH ZONE
R MAX = MAX. AERO. RANGE
R D = RADAR DETECTION RANGE
R P = RANGE AT END OF PREP. TIME
R MG = MAX. GUIDANCE RANGE
R MIN = MIN. AERO. RANGE
V F = FIGHTER VELOCITY VECTOR
V R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-29
5000 FT. ALTITUDE

Fig. 24

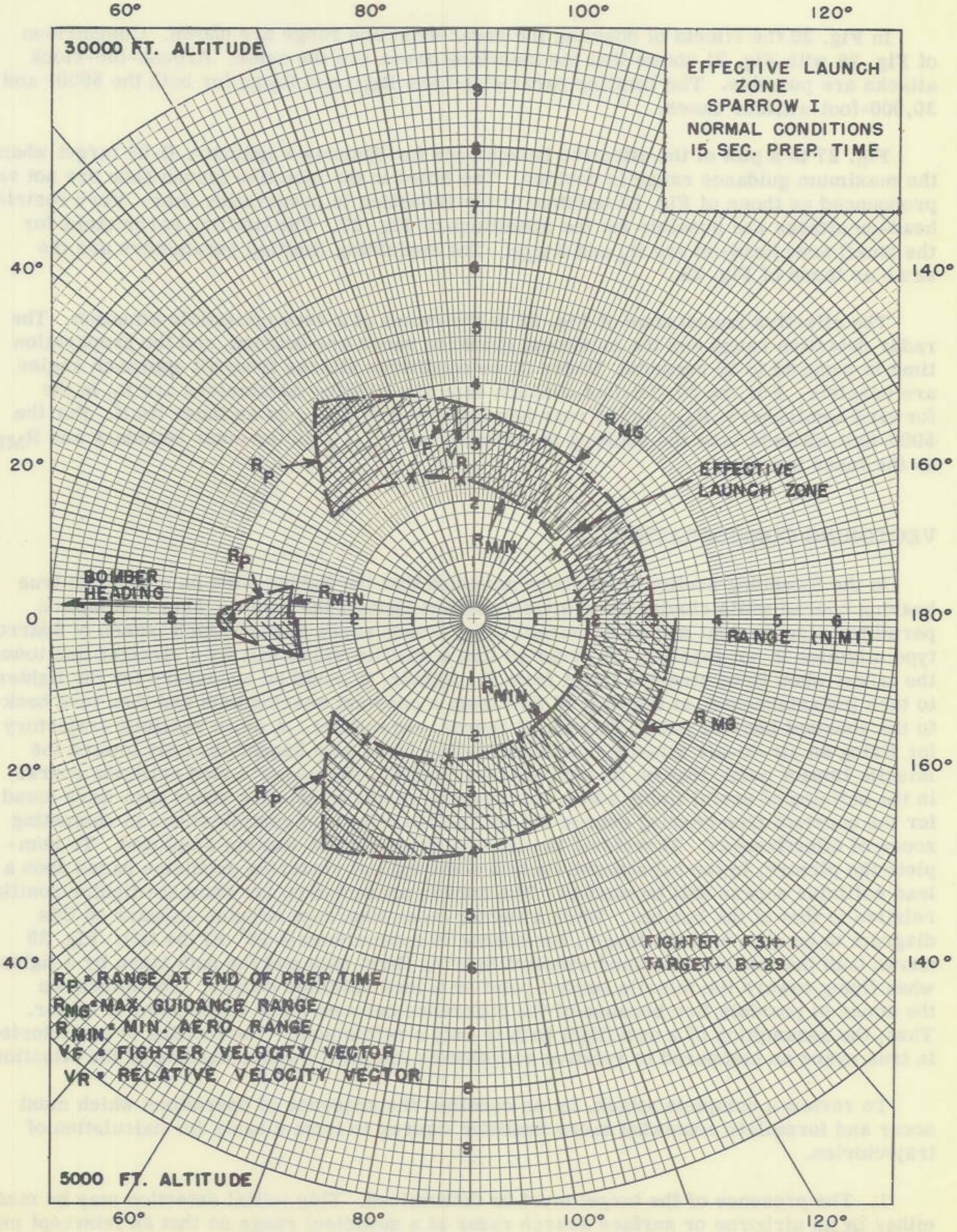

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
NORMAL CONDITIONS
15 SEC. PREP. TIME
60°
80°
100°
120°
40°
140°
20°
160°
0
180°
BOMBER HEADING
RANGE (NMI)
EFFECTIVE LAUNCH ZONE
R_{MG}
R_P
R_{MIN}
V_F
V_R
FIGHTER - F3H-1
TARGET - B-29
R_P = RANGE AT END OF PREP TIME
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE

Fig. 25

In Fig. 26 the effects of doubling the radar detection range are shown. Comparison of Fig. 26 with Fig. 21 shows that the launching areas are the same. Around-the-clock attacks are possible. The limiting parameters are R_{MG} and R_{MIN} for both the 5000- and 30,000-foot-altitude cases.

Fig. 27 is a plot of the effective launch zone for Sparrow I against a B-29 target when the maximum guidance range is doubled. The holes in the effective launch zone are not as pronounced as those of Fig. 22 because of the reduction in preparation time. Very restricted head-on attacks are possible for the conditions of Fig. 27. The limiting parameters for the small zone head-on are R_p and R_{MIN}. The remaining limiting parameters are the same as those of Fig. 22.

The effective launch zone of Fig. 28 is the result of a very optimistic situation. The radar detection range and the maximum guidance range are doubled, and the preparation time is reduced to 15 seconds. Under these conditions attacks from all approach angles are possible. For the 30,000-foot-altitude case the limiting parameters are R_p in the forward hemisphere, R_{MG} in the rear hemisphere, and R_{MIN} as an inner limit. For the 5000-foot-altitude case the limiting parameters are R_{MAX} as the outer boundary and R_{MIN} as the inner boundary.

VECTORING CONSIDERATIONS

In the foregoing section it has been assumed that the fighter flies on a constant true bearing course while closing to an attack position against the target. This assumption permitted a convenient means for establishing launch zones beyond which launch of Sparrow I type missiles is not feasible. In order to fire a missile which will fly a straight line toward the target when fired straight ahead from the fighter, it would be necessary for the fighter to turn instantaneously at the time of fire from a constant bearing path and then turn back to the constant bearing path. This is, of course, unrealistic. A more realistic trajectory for firing an air-to-air missile is a straight-line flight path headed directly toward the missile impact point; that is, a lead collision course. The use of a lead collision course in the defining of useful firing zones not only will result in smaller zones than were found for the constant true bearing course but will lead to increased complications in computing zones of effectiveness. Therefore, the lead collision course was not employed. To complete the picture, in the subsequent section some typical vectoring problems based upon a lead collision course will be studied. The curves presented so far show the fighter position relative to that of the bomber. When a fighter on an intercept mission appears on this diagram to be abeam of the target, his course is approximately 30° off the tail. Fig. 29 shows both the initially assumed constant bearing flight path and the path taken by a fighter when flying toward the missile impact point on a lead collision course. The effects on the zones of possible fire because of the difference between these flight paths is minor. Thus, the assumption of a new flight path is only a refinement. The plotting of trajectories in true space as compared to relative coordinates simplifies understanding of the situation.

To review a complete attack, let us consider the sequence of operations which must occur and formulate somewhat more realistic figures to form a basis for calculation of trajectories.

1. The presence of the target must be determined. This initial detection may be made either by an airborne or surface search radar at a sufficient range so that an intercept may be accomplished. It is generally conceded that the World War II detection ranges of from 50 to 100 miles are inadequate and that a minimum of 150 miles is necessary. For the

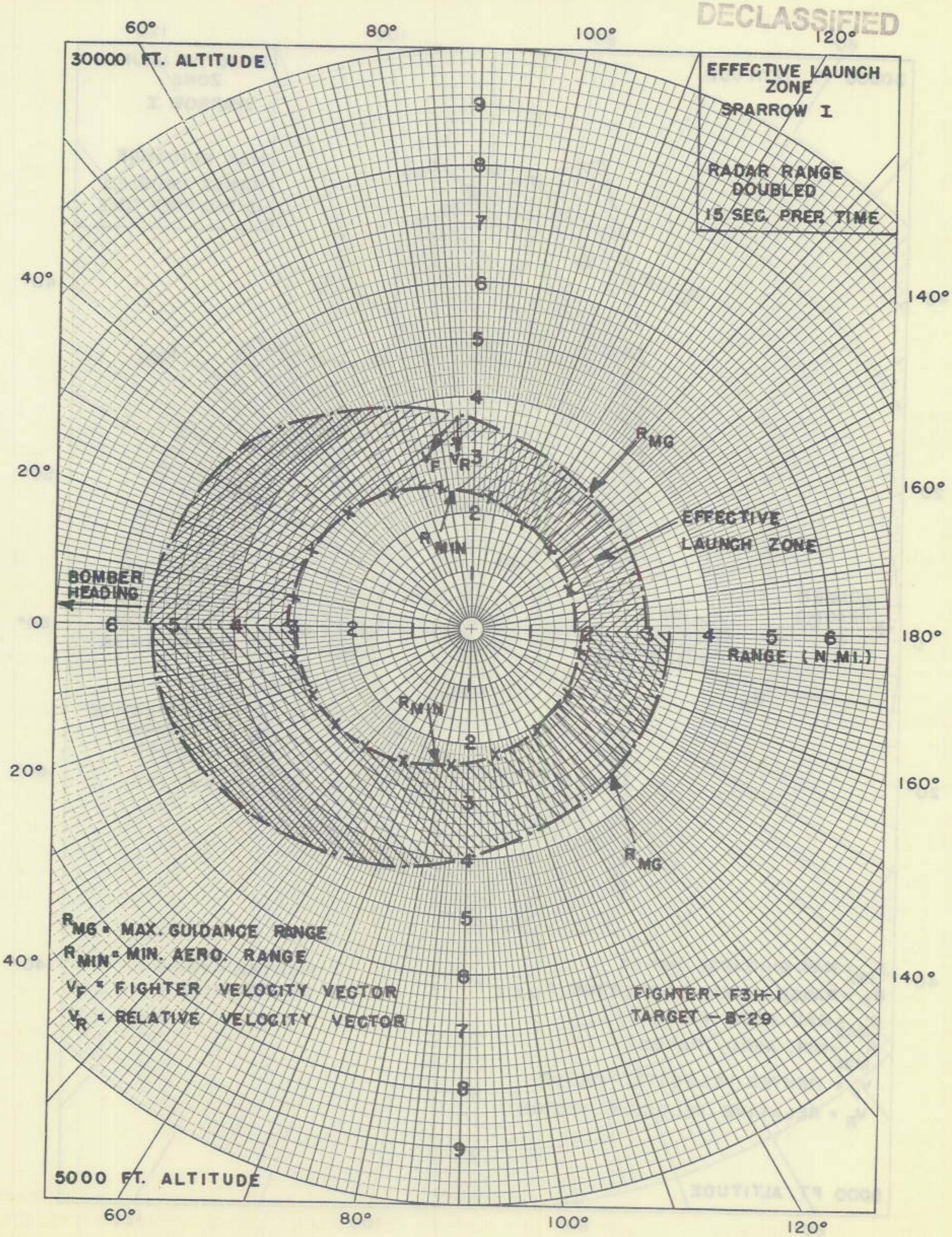
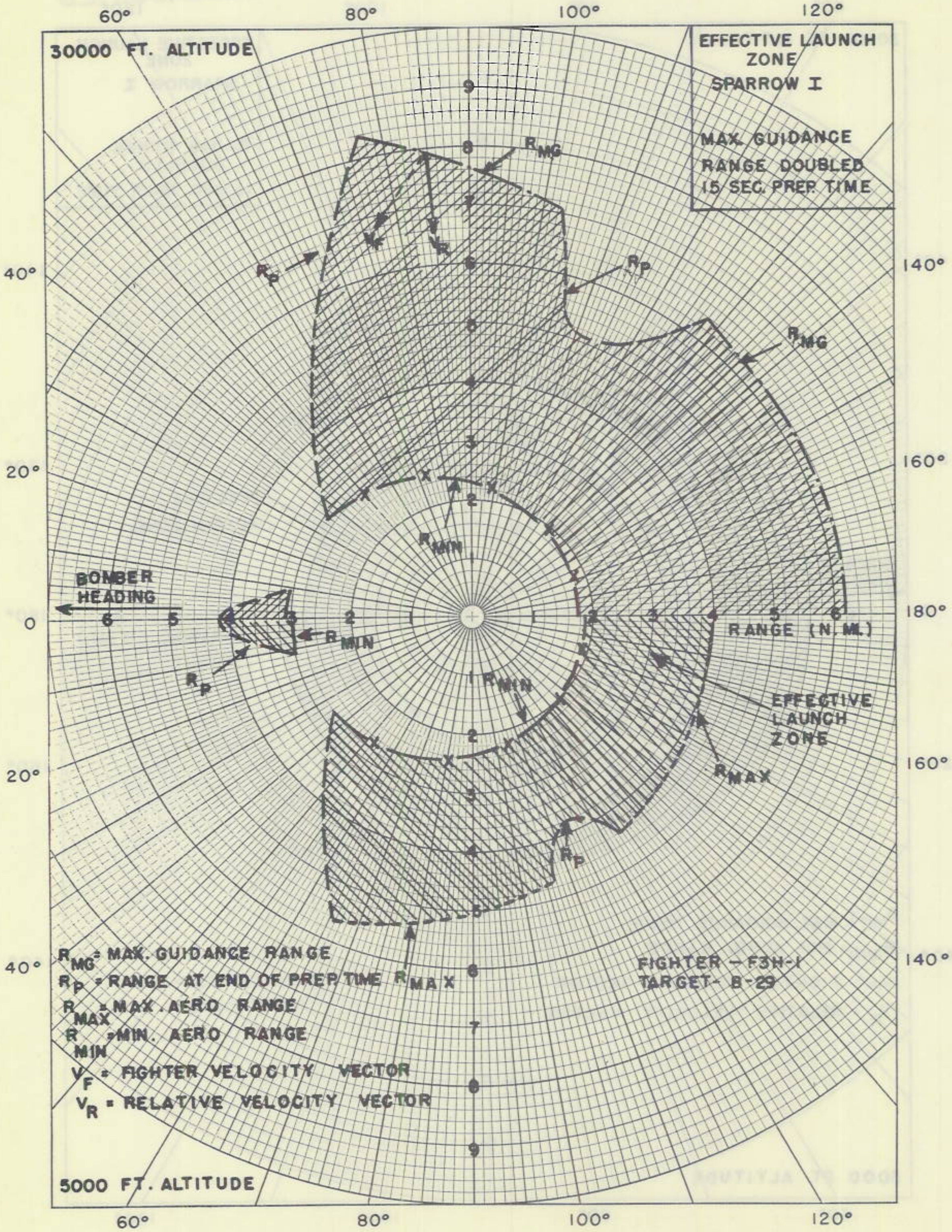

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR RANGE DOUBLED
15 SEC. PREP. TIME
BOMBER HEADING
R_{MG}
R_{MIN}
V_F
V_R
EFFECTIVE LAUNCH ZONE
RANGE (N.MI.)
R_{MG} = MAX. GUIDANCE RANGE
R_{MIN} = MIN. AERO. RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
FIGHTER - F3H-1
TARGET - B-29
5000 FT. ALTITUDE

Fig. 26



30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
MAX. GUIDANCE RANGE DOUBLED
15 SEC. PREP. TIME
60°
80°
100°
120°
40°
20°
0
140°
160°
180°
R MG
R P
V F
V R
R MIN
R MAX
BOMBER HEADING
RANGE (N. MI.)
EFFECTIVE LAUNCH ZONE
R MG = MAX. GUIDANCE RANGE
R P = RANGE AT END OF PREP TIME
R MAX = MAX. AERO RANGE
R MIN = MIN. AERO RANGE
V F = FIGHTER VELOCITY VECTOR
V R = RELATIVE VELOCITY VECTOR
FIGHTER – F3H-1
TARGET – B-29
5000 FT. ALTITUDE

Fig. 27

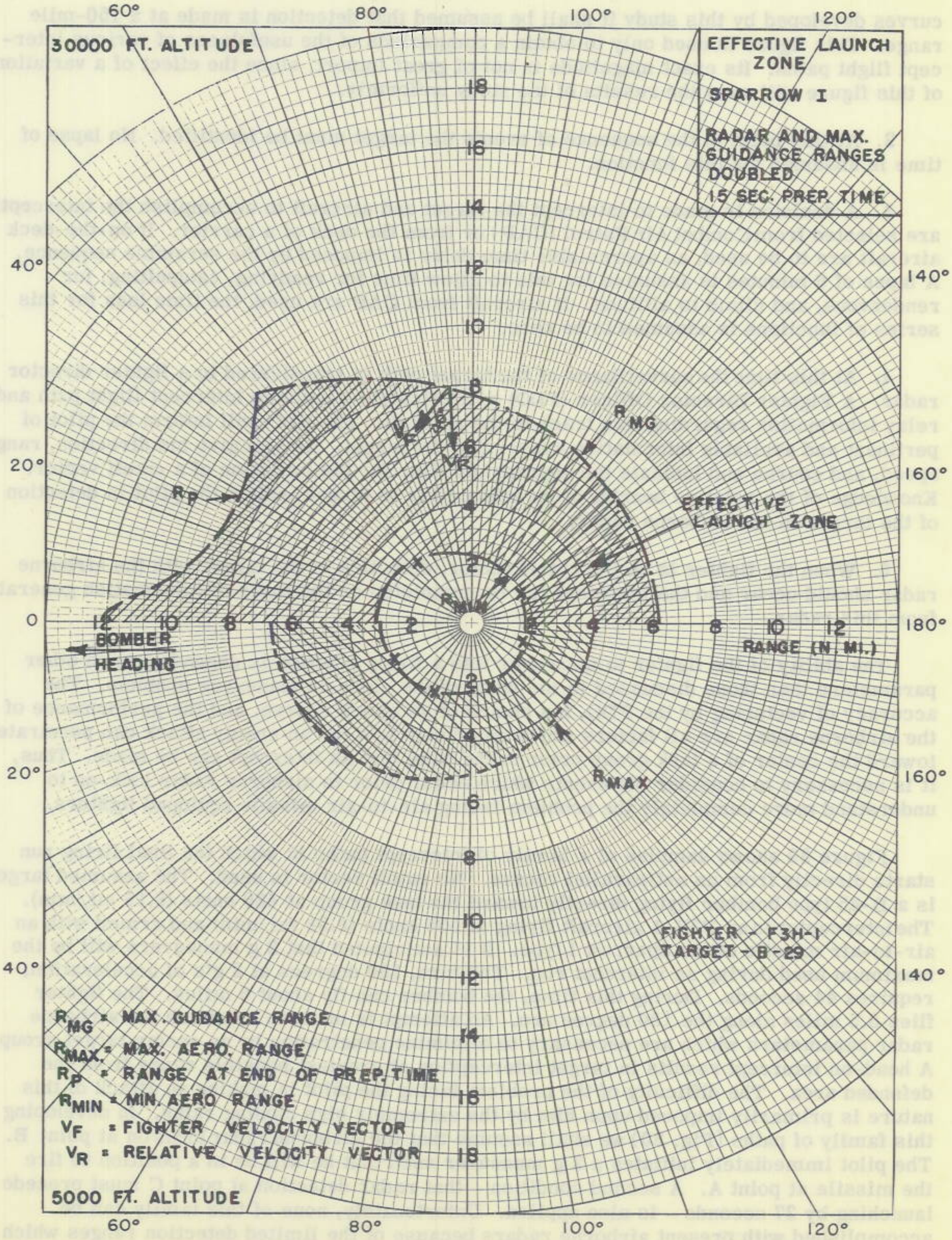

30000 FT. ALTITUDE
EFFECTIVE LAUNCH ZONE
SPARROW I
RADAR AND MAX. GUIDANCE RANGES DOUBLED
15 SEC. PREP. TIME
60°
80°
100°
120°
40°
140°
20°
160°
0
180°
R_{MG}
V_F
V_R
R_P
EFFECTIVE LAUNCH ZONE
R_{MIN}
BOMBER HEADING
RANGE (N. MI.)
R_{MAX}
FIGHTER - F3H-1
TARGET - B-29
R_{MG} = MAX. GUIDANCE RANGE
R_{MAX} = MAX. AERO. RANGE
R_P = RANGE AT END OF PREP. TIME
R_{MIN} = MIN. AERO RANGE
V_F = FIGHTER VELOCITY VECTOR
V_R = RELATIVE VELOCITY VECTOR
5000 FT. ALTITUDE

Fig. 28

curves developed by this study it shall be assumed that detection is made at a 150-mile range. This figure is used only to obtain a comparison of the usefulness of various intercept flight paths. Its exact magnitude is not of great import, since the effect of a variation of this figure will vary the results of the study uniformly.

2. At this point in the sequence of events the target must be identified. No lapse of time is assumed for this function.

3. A decision is made to intercept the target and aircraft to accomplish the interception are selected from Combat Air Patrol (CAP) or from the deck of a carrier. If on-the-deck aircraft are to be used, an appreciable time delay is imposed by the scramble sequence. A lapse of 6 minutes is assumed for this to allow time for readying, catapulting, for rendezvous, and climb to altitude. If aircraft from CAP are used, the time loss for this series of functions is assumed to be zero.

4. At this time the surveillance of the target path is transferred to a fighter director radar. A Fighter Director Officer (FDO) will calculate a suitable intercept flight path and relay information regarding this track to the aircraft. The FDO will inform the pilot of pertinent and available information concerning the target. He will give his direction, range, speed, and course and indicate the appropriate time for the initiation of a cutoff vector. Knowledge of the relative bearing from interceptor to target assists the pilot in detection of the target on his airborne radar.

5. When the fighter is on the cutoff vector, detection of the target with the airborne radar should occur and the intercept can be completed on the basis of information generated from this radar.

The ability of the fighter to maneuver into a firing position is dependent upon other parameters than those discussed in the study of the close-in maneuver problem. The accuracy of vectoring by the FDO, his choice of an attack course, and the performance of the airborne radar are all factors which determine how far an enemy attack can penetrate toward the center of a task group prior to the time that an intercept can be made. Thus, it is necessary to consider the events which occur prior to airborne radar lock-on to understand more completely the problem facing air-to-air missile equipped fighters.

Figure 30 shows samples of a family of intercept paths in which the final firing run starts directly from an antiparallel course. No cutoff vector is used. The assumed target is a B-45 type bomber flying directly toward the task group at 489 knots (8.15 mi/min). The interceptor is an F3H-1 aircraft flying at 538 knots (8.98 mi/min) and armed with an air-to-air missile of the Sparrow I type. It is anticipated that 2 g maneuvers will be the maximum used during an intercept run. The turn, 180 degrees at 2 g's of acceleration, requires 44 seconds. During this time, the bomber can fly about 6 miles. The fighter flies 6.5 miles along the 180 degree arc. An attempt is made to establish the airborne radar parameters which are necessary to minimize penetration by an enemy target group. A head-on intercept results in target interception at greatest possible range from the defended area. The difficulty of the problem facing the pilot in making an attack of this nature is primarily imposed upon him by the extremely high closing rates. In developing this family of paths (Fig. 30) we shall assume that the airborne radar locks on at point B. The pilot immediately initiates a 2 g maneuver such that he will be in a position to fire the missile at point A. A second condition – that radar detection at point C must precede launching by 27 seconds – is also applied. Unfortunately, none of this family can be accomplished with present airborne radars because of the limited detection ranges which are now available. Since radar lock-on must occur when the closing rates are high, prior

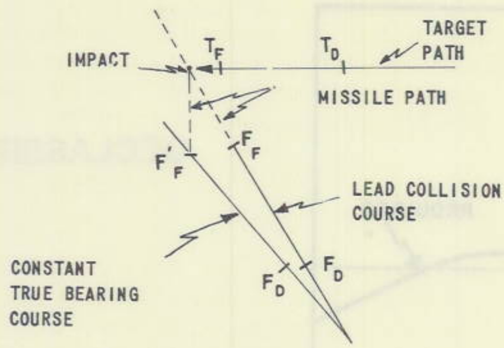


Fig. 29 - Comparison of constant true bearing and lead collision approaches

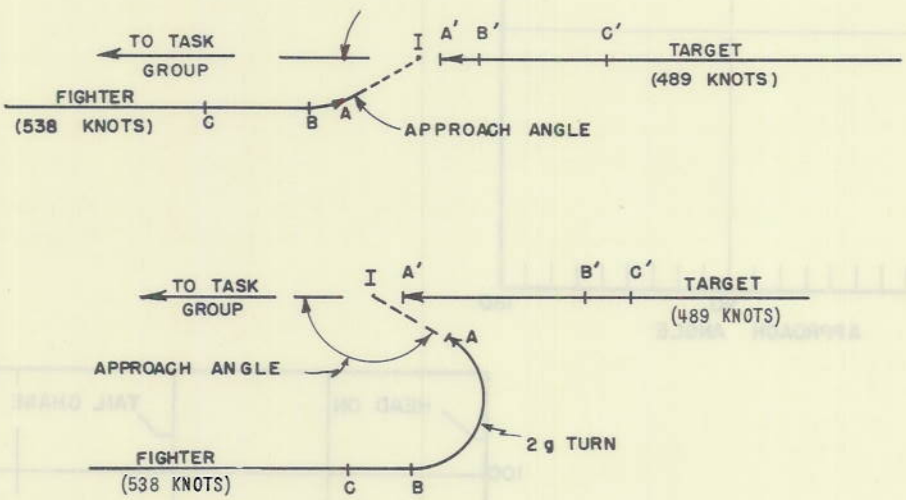


Fig. 30 - Approach with no cutoff vector

to initiation of the 2 g turn, and detection must precede a lock-on by 12 seconds, the required range at the time the target aircraft is detected is high. The required detection range as a function of approach aspect is shown in Fig. 31. Detection ranges required are from 9 to 11 miles, far in excess of those currently available. If the primary search radar had detected this target at the 150-mile range, the intercept would have been completed at ranges shown in Fig. 32. The target at the time of intercept would have penetrated to 81 miles for a head-on attack when fighter aircraft from CAP were used. If the interceptor had turned through 180 degrees and fired directly from the tail of the bomber, the target penetration would have been only 4 miles greater (77 miles).

The FDO currently uses an attack procedure which causes the interceptor to approach its target from other than the head-on aspect. The lower closing rates associated with such angles of approach are compatible with short detection ranges which are currently available from airborne radars. Higher closing rates could be used if larger detection ranges were possible.

DECLASSIFIED

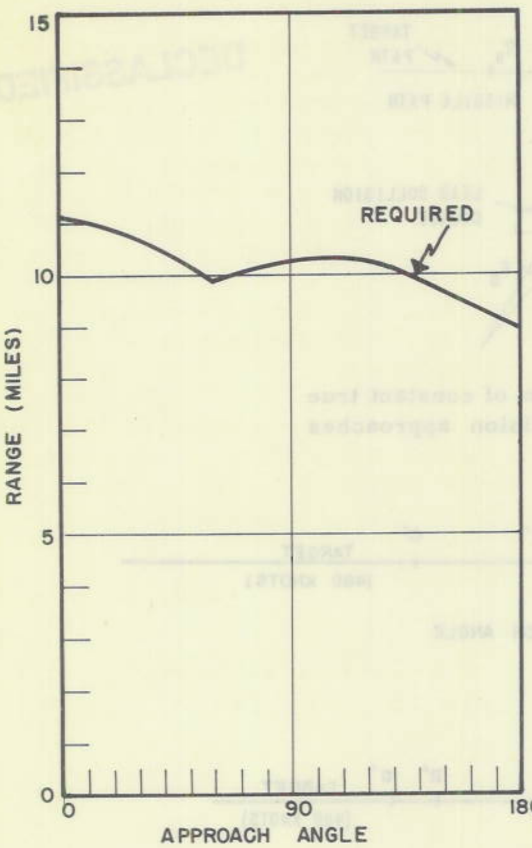


Fig. 31 - Required AI detection range where the fighter approaches the target with no final cutoff vector

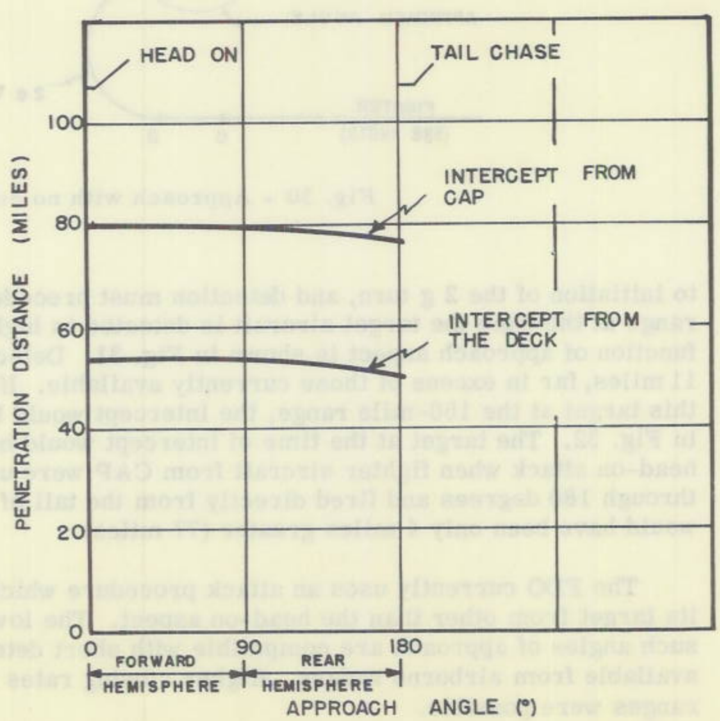


Fig. 32 - Penetration toward the task group where the fighter approaches the target with no final cutoff vector

DECLASSIFIED

The FDO generates first a vector to place the fighter in the vicinity of the target on an antiparellel course, then turns it onto a new vector. This new vector is toward the rear of the bomber at such an angle that the closing rates are compatible with conditions for detection at useful ranges. Two samples of a family of such courses are shown in Fig. 33. The missile would intercept the target at point I after having been launched at point A. Twenty-seven seconds prior to the missile launch, detection occurred at point B. A straight-line portion of flight on the final cutoff vector is allowed so that the FDO can make corrections for errors generated during the turn onto the cutoff vector. It was assumed that points C and B are separated by 30 seconds time of flight because the surface fighter director radar illuminates the fighter and target only five or ten times per minute. Thus, for this family, 57 seconds of flight time in a straight line on the final cutoff vector prior to launch of an air-to-air missile was assumed to be necessary.

Fig. 34 shows the penetration ranges of the bomber toward the center of the task group as a function of the angle of the final cutoff vector. In the case of a head-on approach, no turn is required, thus it is the same as the case studied before. Penetration distance is 81 miles when fighters from CAP are used and 55 miles when it is necessary to use aircraft from the deck. To vector the aircraft onto a final approach directly from the tail of the target will allow the target to penetrate an additional 13 miles.

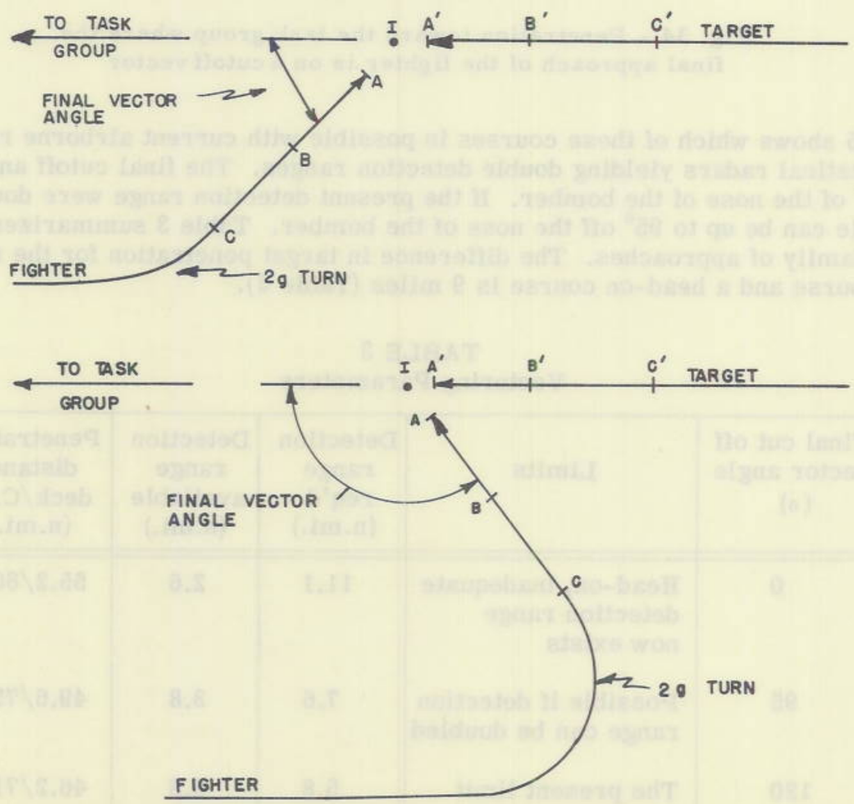


Fig. 33 - Approach with cutoff vector

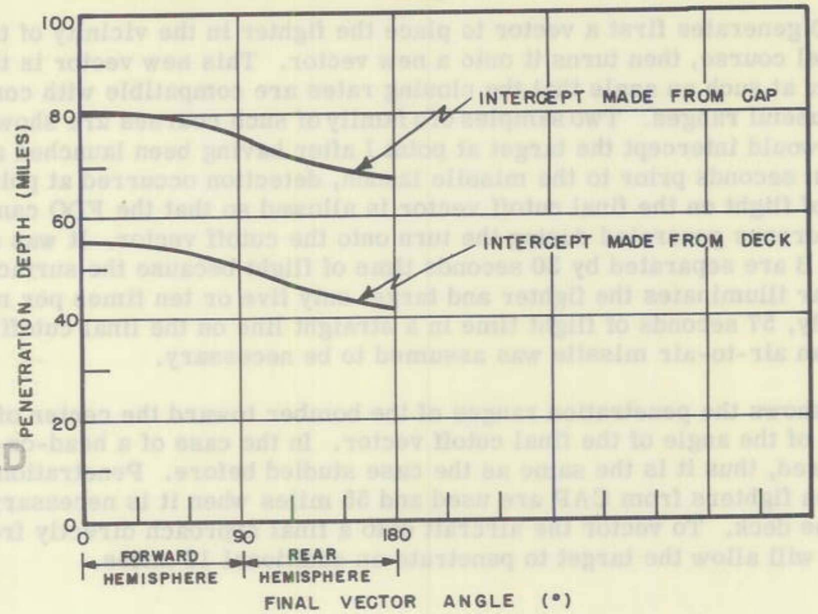


Fig. 34 - Penetration toward the task group where the final approach of the fighter is on a cutoff vector

Fig. 35 shows which of these courses is possible with current airborne radars and with hypothetical radars yielding double detection ranges. The final cutoff angle must not be within 120° of the nose of the bomber. If the present detection range were doubled, the vector angle can be up to 95° off the nose of the bomber. Table 3 summarizes the results from this family of approaches. The difference in target penetration for the most forward possible course and a head-on course is 9 miles (Table 3).

TABLE 3
Vectoring Parameters

Final cut off vector angle (°)	Limits	Detection range req'd (n.mi.)	Detection range available (n.mi.)	Penetration distance* deck/CAP (n.mi.)
0	Head-on, inadequate detection range now exists	11.1	2.6	55.2/80.8
95	Possible if detection range can be doubled	7.6	3.8	49.6/75.2
120	The present limit	5.8	5.8	46.2/71.8
180	Tail approach	1.9	4.6	41.8/67.4

*Initial detection occurs when the target is 150 nautical miles from the task group center.

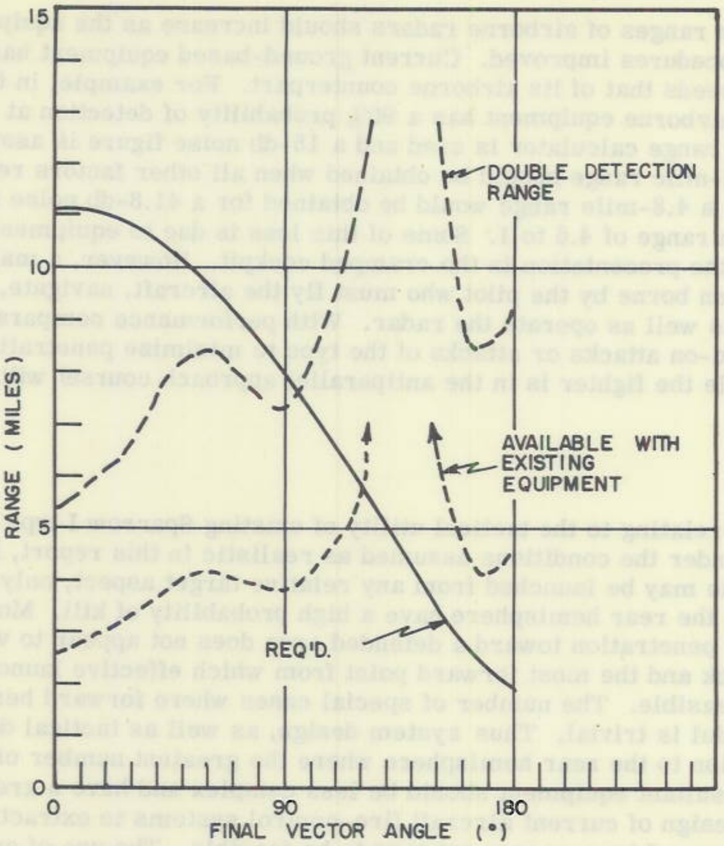


Fig. 35 Required AI detection range where the final approach of the fighter is on a cutoff vector

The FDO on shipboard is limited by the accuracy of the fighter director radar data in attempting to give an accurate cutoff vector to the fighter pilot. Because of this uncertainty he is obliged to use the marginal forward hemisphere courses only as a last resort. As possible airborne detection ranges increase, the results of this inaccuracy become less important because the uncertainty of position represents a smaller arc. For short detection ranges, large angles of search by the airborne radar must be utilized. Further, as the search angle is increased the probability of detection at a finite range is reduced. Hence this problem compounds itself. Accurate relative bearing data transmitted from the ground to the fighter would minimize the difficulty. The AN/SPS-8 fighter director radar is estimated to be able to locate a target bearing relative to the ship with a standard error of 1.2 miles at an 80-mile range. The standard error in altitude is 0.4 mile. In range, the resolution of the PPI limits the standard error to about 0.05 mile. Information rates are low because 6 or 12 seconds exists between successive radar sweeps. The FDO must establish a vector between two such areas of uncertainty. The actual standard error which results is a function of vectoring angle, closing aspect, closing speed, and range, and is dependent upon the radar resolution characteristic and scan rate. Improvement in the accuracy of information from the fighter director radar delivered to the pilot will result in greater airborne radar detection ranges. This can be done by improvement in the search radar equipment, or in more adequate use of the data currently available by improved computation methods or improved data handling.

The detection ranges of airborne radars should increase as the equipment is refined and operating procedures improved. Current ground-based equipment has performance which greatly exceeds that of its airborne counterpart. For example, in the rear of a B-45, the AN/APQ-41 airborne equipment has a 90% probability of detection at 4.8 miles. If a standard radar range calculator is used and a 15-db noise figure is assumed, calculation shows that a 22.2-mile range should be obtained when all other factors remain the same. On the calculator a 4.8-mile range would be obtained for a 41.8-db noise figure. This is a deterioration in range of 4.6 to 1. Some of this loss is due to equipment design problems and some due to the presentation in the cramped cockpit. However, a major part is due to the great burden borne by the pilot who must fly the aircraft, navigate, and serve as radio operator, as well as operate the radar. With performance comparable to the shipboard radar, head-on attacks or attacks of the type to minimize penetration (where lock-on can occur while the fighter is in the antiparallel approach course) will be possible.

CONCLUSIONS

Conclusions relating to the tactical utility of existing Sparrow I type air-to-air missiles, when employed under the conditions assumed as realistic in this report, indicate that although a missile may be launched from any relative target aspect, only those launched in a limited zone in the rear hemisphere have a high probability of kill. Moreover, it is indicated that enemy penetration toward a defended area does not appear to vary excessively between tail attack and the most forward point from which effective launching of the Sparrow I type missile is feasible. The number of special cases where forward hemisphere attacks could be successful is trivial. Thus system design, as well as tactical doctrine, should restrict application to the rear hemisphere where the greatest number of effective attacks are feasible. Resultant equipment should be less complex and have a greater real utility. Optimizing the design of current aircraft fire-control systems to extract full tactical utility of the Sparrow I type weapon appears to be feasible. The use of an air-to-air missile will increase the effective firing range of the fighter aircraft over that possible with 20 mm guns and rockets. No consideration is given in this report to the relative damage potential of missiles, rockets, and guns. The following conclusions have been evolved from this study:

1. It is not practical under the stated "normal" conditions (page 10) to attack medium jet bombers (i.e., B-45's) from the forward hemisphere using air-to-air missile systems of the Sparrow I type.

When the AI radar detection range, preparation time, and missile guidance range are improved on the order of twice "normal," the effective launch zone against the B-45 is still theoretically limited to an area extending approximately ±150° with respect to the target's tail.

2. Round-the-clock launchings are not practical under "normal" conditions against a propeller-driven medium bomber (B-29) with an air-to air missile system of the Sparrow I type.

For the hypothetical case where the AI radar detection range is doubled, the effective launch zone of the Sparrow I is not restricted angle-wise relative to the propeller-driven bomber (B-29).

3. The three parameters which place the greatest restriction on forward hemisphere launching of air-to-air missiles, such as Sparrow I, are AI detection range, preparation time, and relative speed.

4. The operational utility of theoretically possible forward zones of fire is made problematical by the accurate timing requirements imposed by the high closing rates.

5. The potential penetration of a high speed bomber toward a defended area is dependent upon the approach angle of the intercepting fighter aircraft. A head-on fighter approach angle permits the least penetration, while a tail-on approach permits the greatest. The difference in penetration between the head-on and tail-on approaches, for the assumed conditions describing a practical vectoring situation, is 14 miles.

6. For the assumed conditions the penetration toward the task group by a high performance bomber varies only 4 miles from a tail-on intercept approach to the most forward practical approach. The air-to-air missile-equipped fighter is limited by detection capabilities and preparation time of current airborne fire-control systems to a final lead collision vector angle within 60 degrees of the bomber flight path.

7. No single parameter is responsible for the inability to make round-the-clock attacks. The controlling factors which affect the over-all problem are: initial target detection, fighter direction, AI radar acquisition, preparation time, maximum and minimum missile aerodynamic ranges, maximum missile guidance range, etc.

8. The tactical utilization of the missile is highly dependent upon accurate CIC information. Accurate vectoring information can increase the probability of the fighter aircraft entering the effective attack zone with the required heading. Accurate vectoring information will also reduce the required search angle and increase the AI detection range.

9. Because of the extreme limitations existing for attacks from other than the rear hemisphere, the value of designing more costly and more complex equipment to allow round-the-clock use of existing Sparrow I type missiles is questionable.

ACKNOWLEDGMENT

The authors wish to thank Peter Waterman, William C. Hodgson, Arthur S. Locke, Laurence F. Gilchrist, and John P. Barry for their contributions to this report. They also wish to express their appreciation to individuals of the Bureau of Aeronautics who, in discussions of air-to-air missile problems, contributed information of considerable value in the preparation of this report.

* * *

REFERENCES

(1) Stieber, A., and Wolf, R. A., "A Comparison of Navy Air-to-Air Guided Missiles," Cornell Aeronautical Laboratory Report No. GM-660-G-11, Secret, April 25, 1953

(2) Ament, W. S., MacDonald, F. C., and Passerini, H. J., "Quantitative Measurements of Radar Echoes from Aircraft VIII, B-45," NRL Memorandum Report No. 116, Confidential, January 28, 1953

(3) "Sparrow I Weapon System," Sperry Gyroscope Company Report No. 14-2A, Confidential, January 1953

(4) "Standard Aircraft Characteristics," Wright Air Development Center, Secret looseleaf data book

(5) Ament, W. S., MacDonald, F. C., and Passerini, H. J., "Quantitative Measurements of Radar Echoes from Aircraft XI, B-29," NRL Memorandum Report No. 164, Confidential, May 25, 1953

(6) Siegel, K. M., Crispin, J.W., and Kleinman, R. E., "Studies in Radar Cross-Sections - VII," Willow Run Research Center, Engineering Research Institute, University of Michigan, UMM-108, Secret, November 1, 1952

* * *