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THE SHIELDED TRANSMISSION LINE METHOD OF GENERATING STANDARD FIELDS

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ABSTRACT

One of the methods extensively used for the calibration of equipments using loop antennas is the shielded transmission-line method, wherein a wire is erected between two end walls of a shielded room. One end of the conductor is connected to an end wall through a resistance equal to the characteristic impedance of the line, and the output of a signal generator is connected between the other end of the line and the wall. Thus, the wire and shielded room form a shielded transmission line terminated in its characteristic impedance.

The loop antenna to be calibrated is placed below and in the vertical plane of the wire. The true field strength at the loop, produced by a known current in the line, can be calculated by applying the Biot-Savart law to the line and all of the images of the line in the ceiling, floor, and walls. When only a few images are considered, as is common practice, it was found that the error in the calculated field can be 20 percent or more. For some typical installations a 5 percent error was obtained when as many as 96 images were considered.

PROBLEM STATUS

This is a final report on one phase of this problem; work is continuing on other phases.

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THE SHIELDED TRANSMISSION-LINE METHOD OF GENERATING STANDARD FIELDS

INTRODUCTION

There are several methods of generating standard electromagnetic fields for the calibration of field strength measuring equipments using loop antennas. One of these which is used extensively, although not originally intended for this purpose, is the shielded transmission-line method. In this method, a shielded room is required and this shield together with a wire stretched between two opposite walls form a transmission line. For purposes of simplicity, however, the term transmission line (or line as used hereafter) will refer to the wire stretched between the two end walls of the room, midway between the side walls, and parallel to the ceiling. This line is connected to one end wall through a resistance equal to the characteristic impedance of the line. The other end is insulated from the wall, and the output of a signal generator is connected between that end and the wall. The loop antenna of the equipment to be calibrated is normally placed below and in the vertical plane of the line. Figure 1 shows a sketch of a typical installation. The field embraced by the loop can then be calculated from the Biot-Savart law by applying the method of images if the current in the line and the dimensions of the installation are known.

The transmission-line method of generating standard fields was originally intended to be used for determining the sensitivity of direction finder equipments, for which great accuracy is not required. Discussions of this method, particularly for this application, along with formulas for calculating the field strength can be found in several publications(1, 2, 3, 4). In all these the method of images is used to calculate the field strength at the loop to be calibrated. If we assume that the walls of the shielded room give perfect reflection, an infinite number of images must theoretically be considered in order to calculate the field embraced by the loop. Of the four publications cited above, only one considers as many as three images. D. S. Bond(1) considers only the field from the line and its image in the ceiling. P. C. Sandretto(2) and K. O. Hornberg(3) considered the line, its ceiling and floor image. F. E. Terman and J. M. Pettit(4) use the line, its ceiling and floor image, and the image of the ceiling image in the floor. Technicians, employing this method of loop calibration and using the formulas given in the above literature, reported that errors, as great as 20 or 30 percent, were encountered in some instances when comparing the results with other calibration methods. In 1951 and 1952, the shielded transmission-line method, along with other methods, was thoroughly investigated at the Naval Research Laboratory.

At the 1954 spring meeting of the International Scientific Radio Union (URSI), held in Washington, D. C., Mr. Fred Haber of the University of Pennsylvania presented a paper on this subject, entitled "Generation of Standard Fields in Shielded Enclosures." In this paper the author gave a rather complete theoretical treatment of the problem, and developed a formula in a closed form which he stated has, for most applications for the

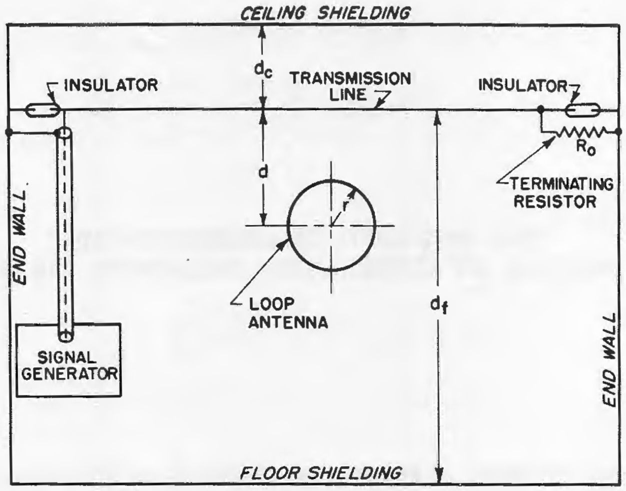

Fig. 1 - Installation for generating standard field by the shielded transmission-line method

calculation of the field strength, an error well within 1 percent. In the present report, the more practical problems involved in using this method of generating standard fields are discussed and formulas, including Haber's, necessary for calculating the field strength are presented.

INSTALLATION

The line should be parallel to the ceiling, floor, and side walls, and, for most shielded rooms, approximately 12 to 18 inches from the ceiling (Fig. 1). To simplify the calculation of the field strength, the line should be midway between the side walls, so that the side images are symmetrical about the vertical plane of the line. All equations given in this report assume this to be so. Leakage from the signal generator may be a problem; if so, the signal generator should be mounted outside the shielded room, and its output cable fed in through a hole in the room at the sending end of the line.

The current in the line must be uniform throughout its length. To assure this, the line must be terminated in its characteristic impedance. The characteristic impedance (Z_o) of the line may be calculated approximately from the equation

$$Z_o = 138 \log_{10} \frac{4 d_c}{a} \tag{1}$$

where d_c is the distance from the line to the ceiling, and a is the diameter of the line in the same units as d_c. However, greater accuracy may be obtained by radio-frequency bridge measurements. The impedance of the line is measured when short-circuited (Z_{sc}) and again when open-circuited (Z_{oc}). The characteristic impedance is then the square root of the product of these two measured impedances. For the installation used for this investigation at the Laboratory, the characteristic impedance was calculated to be 424 ohms by Eq. (1), and bridge measurements gave a value of 376 ohms. The latter value was checked by other means and found to be the more accurate.

The field strength at any point below the line is directly proportional to the current (I_L) in the line. Most signal generators do not indicate output current, but instead indicate output voltage across a particular value of impedance. Therefore, the current in the line is usually calculated by dividing the voltage applied to the line by the terminating resistance (R_o) which should be equal to the characteristic impedance of the line. Because of impedance mismatching, the voltage applied to the line (E_L) will probably not be the signal-generator output voltage (E_g). The line voltage can be calculated from the expression

$$E_L = \frac{E_g R_o}{R_s + R_o} \tag{2}$$

where R_s is the output impedance of the signal generator. The current (I_L) in the line is therefore

$$I_L = \frac{E_g}{R_s + R_o}. \tag{3}$$

METHOD OF IMAGES

The field strength at any point in the shielded room produced by a current flowing in the line can be calculated by the summation of the fields due to the line and its images in the reflecting surfaces of the shielded room. The images can be thought of much in the same way as images of an object in front of a mirror. The image is considered to be carrying the same current as that flowing in the line. If the line were near only one flat conducting plane it would have but one image; however, when it is completely enclosed by conducting surfaces, such as a shielded room, it has its primary image in each of the side walls, the ceiling and the floor. Each of these images, moreover, has its own image in the opposite reflecting surface; thus there is an infinite number of images.

Figure 2 shows an end view of a typical installation and some of the images. Letters have been assigned to designate the various images, and the symbols show the differences in polarity. It should be noted that when the line is anywhere other than midway between the ceiling and floor, the images are situated more or less in pairs. The closer the line is to the ceiling the closer the images in each pair are together. Throughout this report reference will be made to image pairs, and for convenience the line (L) and its ceiling image (C), as shown in Fig. 2, will be considered as a pair. Also, in the tables, the line is sometimes referred to as an image when stating the number of images used in a calculation. All equations presented in this report assume that the line is midway between the side walls. Thus, the terms involving side images are multiplied by 2 and thereby the corresponding images on the other side of the room are taken into account. In designating the side images the numeral 2 may precede the letter symbol: 2 (CS) or 2 (S, CS, etc.). This means that the corresponding images on both sides of the room are considered.

In calculating the field at any point in the room, the field contributions of the line and each image are added together taking into account both the polarity and the phase of each. Relative polarities of the images are shown in Fig. 2. For any point below the line, all images below the floor will be opposite in phase from the line and from all images above the floor. In most installations, the line is relatively close to the ceiling (this is recommended), and, as a result, the images in each pair are relatively close together. The individual field strengths produced at some point in the vertical plane of the line by the images in any one pair have opposite signs, and the closer the images in a pair come to each other, the more they tend to cancel each other. In calculating the field strength by the method of images, it is important, therefore, that the images be considered in pairs.

P. C. Sandretto(2) and K. O. Hornberg(3) in their treatment of this subject consider only the line together with the C and F images in calculating the field strength. Except for

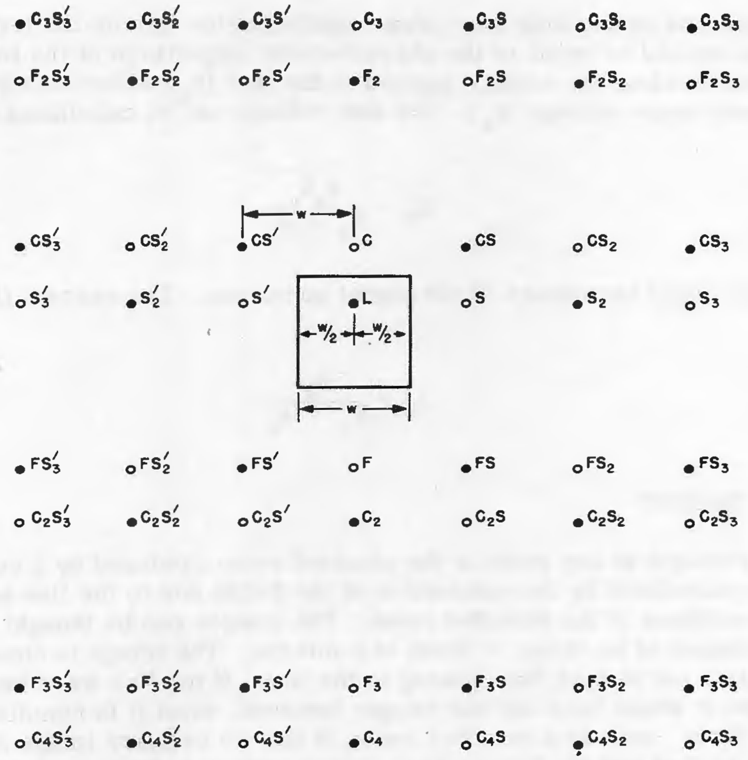

Fig. 2 - End view of shielded room showing transmission line and some images. Solid and open circles indicate differences in polarity

an installation with a wide room and the loop very close to the line, this may not be nearly enough images for accurate work, and might result in even larger errors if the loop were placed anywhere but close to the line. At greater distances from the line, where the floor images have a greater effect, much of the effect of the F image would be cancelled by the C_2 image, depending on the distance of the line from the ceiling. F. E. Terman and J. M. Pettit(4) use the line together with images C, F, and C_2 in calculating the field strength. This gives greater accuracy, but for a particular installation the error may be greater than 10 percent because of the insufficient number of images. It will be shown later how greatly the calculated value of field strength depends upon the number of images used in the calculation.

FIELD STRENGTH AT A POINT

By applying the Biot-Savart law, the magnetic field strength, H, at a point of distance D from a line of current I can be found from the relation

$$H = \frac{I}{2\pi D}. \tag{4}$$

The electric field strength $\mathcal{E}$ can be determined by multiplying the magnetic field strength H by the impedance of free space. Thus,

$$\mathcal{E} = 120\pi H. \tag{5}$$

The field strength at any point in the shielded room is the vector sum of all the fields calculated by applying this equation to the line and all its images.

This report is primarily concerned with the application of the shielded transmission-line method of generating standard fields for use with loop antennas wherein the loop is normally placed below and in the vertical plane of the line. The loop should be mounted approximately midway between the end walls so as to eliminate the end effects. This is particularly important if the shielded room is relatively small. Throughout this report it will be assumed that the loop is so arranged with respect to the line and walls. A loop antenna has a "figure 8" pattern, and, when the plane of the loop lies in the plane of a line in which a current is flowing, maximum voltage is induced in the loop. Therefore, when a loop antenna is mounted as described above, it is oriented for maximum pickup from the line and all of its ceiling and floor images (the images in the vertical plane of the line). However, this is not true for the side images. For the calibration of loop antennas, it is desired, therefore, to calculate the vertical component of the electric field embraced by the loop antenna. If the angle between the plane of the loop and the direction of propagation of the wave from the image or line being considered is θ, then Eq. (4) becomes

$$H = \frac{I}{2\pi D}\cos\theta \tag{6}$$

and Eq. (5) becomes

$$\mathcal{E} = \frac{60\,I}{D}\cos\theta. \tag{7}$$

Hence all expressions for field strength refer to the total vertical component of the electric field existing below and in the vertical plane of the line.

Applying Eq. (7) to the line and each image, the resultant, vertical component of the total field at a point below the line can be calculated. This is the resultant field that would be embraced by a very small loop whose center is at the point and whose vertical dimension is very small compared to the distance from the line. Referring to Figs. 1 and 2 for the symbol designation, and measuring all distances in inches and the current I_L in microamperes, the resultant field strength $\mathcal{E}$ in microvolts per meter, considering only a few images is

$$\mathcal{E} = 2362\, I_L \left[\frac{1}{d} - \frac{1}{2d_c+d} + \frac{1}{2d_f-d} - \frac{1}{2d_f+2d_c-d} \cdots - \frac{2d}{d^2+w^2} + \frac{2(2d_c+d)}{(2d_c+d)^2+w^2} - \frac{2(2d_f-d)}{(2d_f-d)^2+w^2} + \frac{2(2d_f+2d_c-d)}{(2d_f+2d_c-d)^2+w^2} \cdots \right] \tag{8}$$

where w is the width of the room in inches. Since the current in the line and in all images is the same, only the distance varies for each image. The expression for each side image also includes a $\cos\theta$ factor. The quantities within the brackets represent these terms, and the images considered in order are the vertical images L and C; F and C_2, and the side images 2 (S, CS, FS, and C_2S). This group normally comprises the six pairs of images closest to the point at which the field is to be determined. The greater the accuracy desired, the more pairs of images must be considered. However, the images to be considered should be selected in symmetrical groupings about the enclosure.

Haber, in his work on this subject, developed two equations in closed form which give the upper and lower bounds for the resultant field. These equations are:

$$\mathcal{E}_{\text{upper}} = 14842 \frac{I_L}{w} \left\{ \left[\frac{\sinh \alpha}{\cosh \beta - \cosh \alpha} - \frac{\sinh 2\alpha}{\cosh 2\beta - \cosh 2\alpha} \right] + \frac{2 \tanh \alpha \cosh \alpha}{e^{2\gamma} - 1} \left[\frac{1}{e^{\beta} - (2e^{-2\gamma} \cosh \alpha)} + \frac{1}{e^{-\beta} - (2e^{-2\gamma} \cosh \alpha)} \right] - \frac{2 \tanh 2\alpha \cosh 2\alpha}{e^{4\gamma} - 1} \left[\frac{1}{e^{2\beta}} + \frac{1}{e^{-2\beta}} \right] \right\}, \quad (9)$$

$$\mathcal{E}_{\text{lower}} = 14842 \frac{I_L}{w} \left\{ \left[\frac{\sinh \alpha}{\cosh \beta - \cosh \alpha} - \frac{\sinh 2\alpha}{\cosh 2\beta - \cosh 2\alpha} \right] + \frac{2 \tanh \alpha \cosh \alpha}{e^{2\gamma} - 1} \left[\frac{1}{e^{\beta}} + \frac{1}{e^{-\beta}} \right] - \frac{2 \tanh 2\alpha \cosh 2\alpha}{e^{4\gamma} - 1} \left[\frac{1}{e^{2\beta} - (2e^{-4\gamma} \cosh 2\alpha)} + \frac{1}{e^{-2\beta} - (2e^{-4\gamma} \cosh 2\alpha)} \right] \right\}, \quad (10)$$

where

$$\alpha = \frac{\pi d_c}{w}$$

$$\beta = \frac{\pi(d + d_c)}{w}$$

$$\gamma = \frac{\pi(d_c + d_f)}{w}$$

and all units are the same as in Eq. (8). For most installations, the difference between the bounds is much less than 5 percent. Haber, therefore, formulated an equation that gives an approximate solution which has a value between the two bounds:

$$\mathcal{E} = 14842 \frac{I_L}{w} \left[\frac{\sinh \alpha}{\cosh \beta - \cosh \alpha} - \frac{\sinh 2\alpha}{\cosh 2\beta - \cosh 2\alpha} + \frac{4 \sinh \alpha \cosh \beta}{e^{2\gamma} - 1} - \frac{4 \sinh 2\alpha \cosh 2\beta}{e^{4\gamma} - 1} \right]. \quad (11)$$

The last term of this equation contributes very little to the total result and therefore can be ignored with little effect on the accuracy. Also, for most installations, the approximation $\frac{1}{(e^{2\gamma} - 1)} = e^{-2\gamma}$ is satisfactory, and Eq. (11) can be written in the more approximate form

$$\mathcal{E} = 14842 \frac{I_L}{w} \left[\frac{\sinh \alpha}{\cosh \beta - \cosh \alpha} - \frac{\sinh 2\alpha}{\cosh 2\beta - \cosh 2\alpha} + 4e^{-2\gamma} \sinh \alpha \cosh \beta \right]. \quad (12)$$

Calculating the resultant field strength by the summation of the individual images is very laborious and time-consuming because, to ensure accuracies greater than 5 percent, a large number of images must be used. However, it must be pointed out that for some installations where the loop is very close to the line, the error may be small when only the line and its ceiling image are considered. A general method of determining the number of images necessary to give a predicted accuracy could not be found since the number depends upon the individual installation dimensions, and the number of variables becomes unwieldy. Attempts were made to develop a graphical method of determining the resultant field strength but this was not successful for the same reasons. The closed-form equations developed by Haber require rather extensive tables of hyperbolic functions, and all transcendental quantities should be carried out to five or more significant figures to ensure good accuracy.

In calculating the resultant field strength by the summation of the individual images it can be seen that, as more images are incorporated in a symmetrical manner about the enclosure, the total oscillates about a value which is approximately the value obtained by considering an infinite number of images. It was found that considering only the close-in six pairs of images (L, C, F, C_2 and 2 (S, CS, FS, C_2S)) and taking the average of the summations of the field strengths obtained by successively incorporating another of these images, produced results with better than 5 percent accuracy for several installations. By taking the average of the summations of the field strengths is meant that the field strengths obtained by considering the line alone, then the line and image C, and then the line and images C and F, and so on, until the close-in six pairs are all considered, are averaged. The corresponding side images on either side of the room are added simultaneously. Obviously, if more images are used in this average of the summations greater accuracy will be obtained. However, if more images are used, they must be selected in symmetrical groupings about the enclosure. The resultant field strength calculated by simply considering the close-in six pairs of images collectively gives accurate results for some installations, but taking the average of the summations gave more consistent results for several installations.

The resultant field strength at two points was calculated for three different installations using various equations and numbers of images. The results of these calculations are presented in Tables 1 and 2 to illustrate the magnitude of the errors involved when a sufficient number of images is not used. Also shown are the results obtained by using the average of the summations method as described above. This method is designated by the symbols "Avg. Σ 6 Pairs." It can be seen that for the particular installation considered in Table 1, when the distance d from the line to the point equals 40 inches, the error in calculation is almost 5 percent when as many as 48 pairs of images were considered. For installation C in Table 2, the error is negligible when less than 48 pairs of images are considered. The installation considered in Table 1 is the one used for the experimental investigation of this work. These data also show the results (and errors as great as 58 percent) that would be obtained by calculating the field strength according to information in some of the published literature. Bond(1) used the line L and image C. Sandretto(2) and Hornberg(3) used L, C, and F and Terman and Pettit(4), L, C, F, and C_2.

AVERAGE FIELD STRENGTH OVER A RECTANGULAR LOOP

The resultant field strength below the transmission line is roughly a function of the reciprocal of the distance from the line. If a small loop is employed at a large distance from the line, the average resultant field strength over the area of the loop is very nearly the resultant field strength calculated at the center of the loop, and the equations for the resultant field strength at a point can be used with very little error. However, since the field is not uniform if the loop is placed near the line, or if a large loop is used, the average resultant field strength over the loop must be calculated. This requires that the equations for the field strength at a point be integrated over the area of the loop. The installation and loop are arranged as in Fig. 1, except that the loop, being rectangular, has vertical sides of length 2h. The distance d from the line is measured to the center of

TABLE 1
Effect of Contributions of Various Images on the Calculated Field Strength

Field Strength (μv/m) d = 20 in.	d = 40 in.	Images Used
--	107.05	All (Eq. 10, Lower limit)
--	107.84	All (Eq. 9, Upper limit)
244.0	107.5	All (Eq. 11)
244.4	109.8	All (Eq. 12, Approx. Solution)
314.5	157.3	L (Line only)
221.2	85.3	L, C
263.8	134.7	L, C, F
231.5	98.7	L, C, F, C_2
258.3	123.4	All foregoing and F_2
236.0	102.6	All foregoing and C_3
253.4	120.9	All foregoing and F_3
238.1	104.8	All foregoing and C_4
231.6	92.2	All foregoing and S
251.3	116.1	All foregoing and CS
220.5	86.8	All foregoing and FS
252.6	118.6	All foregoing and C_2S
221.0	87.6	All foregoing and F_2S
251.1	116.9	All foregoing and C_3S
224.2	89.2	All foregoing and F_3S
249.2	115.0	All foregoing and C_4S
246.2	112.0	48 Pairs of images

Installation Constants: d_c=23.7 in.; d_f=83.7 in.; w=196 in.; E_L=1000 μv; R_o=376 ohms

the loop. The horizontal dimension of the loop does not enter into the calculations because the field strength is constant along a line parallel to the transmission line, except when near the end walls. The average resultant field strength considering the images individually as in Eq. (8) is:

$$\mathcal{E}_{average} = 2362\frac{I_L}{2h}\ln\left\{\frac{(d+h)(2d_c+d-h)(2d_f-d+h)(2d_f+2d_c-d-h)}{(d-h)(2d_c+d+h)(2d_f-d-h)(2d_f+2d_c-d+h)}\cdots\right.$$

$$\left.\frac{[(d-h)^2+w^2][(2d_c+d+h)^2+w^2][(2d_f-d-h)^2+w^2][(2d_f+2d_c-d+h)^2+w^2]}{[(d+h)^2+w^2][(2d_c+d-h)^2+w^2][(2d_f-d+h)^2+w^2][(2d_f+2d_c-d-h)^2+w^2]}\cdots\right\} \quad (13)$$

where h is in inches and all other units are the same as before. The same images are considered and in the same order as in Eq. (8).

For most installations it is necessary to integrate only the fields from the near images; the fields due to the more distant images may be calculated at the center of the loop and added in with the integrated fields linearly. Haber states that in his closed form equations (11 and 12) for the resultant field strength at a point the first two terms represent the closer images and the other terms the far removed images. Therefore, to

TABLE 2
Field Strength at a Point Calculated Using Various Images
and Installation Dimensions

Installation*	Images Used	Field Strength‡ (μv/m)		Error§ (%)	
		d = 20 in.	d = 40 in.	d = 20 in.	d = 40 in.
A	All (Eq. 11)	91.7	40.4	Ref.	Ref.
A	All (Eq. 12)	91.9	41.3	+ 0.2	+ 2.2
A	L, C	83.1	32.1	− 9.4	−20.5
A	L, C, F	99.1	50.6	+ 8.1	+25.2
A	L, C, F, C_2	87.1	37.1	− 5.0	− 8.2
A	6 Pairs†	92.5	42.3	+ 0.9	+ 4.7
A	Avg. Σ 6 Pairs†	92.2	40.7	+ 0.5	+ 0.7
		d = 20 in.	d = 40 in.	d = 20 in.	d = 40 in.
B	All (Eq. 11)	82.1	34.2	Ref.	Ref.
B	All (Eq. 12)	82.4	34.7	+ 0.4	+ 1.5
B	L, C	75.8	27.9	− 7.7	−18.4
B	L, C, F	90.8	44.9	+10.6	+31.3
B	L, C, F, C_2	78.6	31.4	− 4.3	− 8.2
B	6 Pairs†	82.9	35.4	+ 1.0	+ 3.5
B	Avg. Σ 6 Pairs†	84.5	35.5	+ 2.9	+ 3.8
		d=23.75 in.	d = 50 in.	d=23.75 in.	d = 50 in.
C	All (Eq. 11)	65.1	25.2	Ref.	Ref.
C	All (Eq. 12)	65.2	25.6	+ 0.2	+ 1.6
C	L, C	57.2	18.5	−12.0	−26.6
C	L, C, F	74.5	40.0	+14.4	+58.7
C	L, C, F, C_2	60.4	23.3	− 7.2	− 7.5
C	6 Pairs†	67.0	27.6	+ 2.9	+ 9.5
C	Avg. Σ 6 Pairs†	66.0	25.3	+ 1.4	+ 0.4

*Installation Dimensions: A; d_c = 23.7 in.; d_f = 83.7 in.; w = 196 in.
B; d_c = 17.9 in.; d_f = 89.5 in.; w = 196 in.
C; d_c = 16.0 in.; d_f = 80.0 in.; w = 122 in.

‡Current in the line is 1.0 microamperes.

§Errors are given in percent above or below the value obtained using Eq. (11).

†See text.

obtain the average resultant field strength, only the first two terms are integrated. His equation for the average resultant field strength over a rectangular loop is, therefore,

$$\mathcal{E}_{average} = 1181 \frac{I_L}{h} \ln\left\{\frac{\sinh^2[(d+h)\pi/2w]}{\sinh^2[(d-h)\pi/2w]} \cdot \frac{\sinh^2[(d+2d_c-h)\pi/2w]}{\sinh^2[(d+2d_c+h)\pi/2w]}\right.$$

$$\left. \cdot \frac{\sinh[(d-h)\pi/w]}{\sinh[(d+h)\pi/w]} \cdot \frac{\sinh[(d+2d_c+h)\pi/w]}{\sinh[(d+2d_c-h)\pi/w]}\right\} + \frac{4I_L}{w} e^{-2\gamma} \sinh \alpha \cosh \beta. \quad (14)$$

AVERAGE FIELD STRENGTH OVER A CIRCULAR LOOP

In practice, many loops encountered are circular. Integrating the point field-strength equations over a circle is much more difficult and cumbersome. The equation for the average resultant field strength over a circular loop obtained by integrating Eq. (8) is

$$\mathcal{E}_{average} = 4724 \frac{I_L}{r^2} \Bigg[(d-\sqrt{d^2-r^2}) - (2d_c+d-\sqrt{(2d_c+d)^2-r^2})$$

$$+ (2d_f-d-\sqrt{(2d_f-d)^2-r^2}) - (2d_f+2d_c-d-\sqrt{(2d_f+2d_c-d)^2-r^2})\ldots$$

$$- 2\left(d-\sqrt[4]{(w^2+r^2-d^2)^2+4w^2d^2}\cdot\sin\left\{\frac{1}{2}\tan^{-1}\left[\frac{2wd}{w^2+r^2-d^2}\right]\right\}\right)$$

$$+ 2\left(2d_c+d-\sqrt[4]{[w^2+r^2-(2d_c+d)^2]^2+4w^2(2d_c+d)^2} \cdot \sin\left\{\frac{1}{2}\tan^{-1}\left[\frac{2w(2d_c+d)}{w^2+r^2-(2d_c+d)^2}\right]\right\}\right)$$

$$- 2\left(2d_f-d-\sqrt[4]{[w^2+r^2-(2d_f-d)^2]^2+4w^2(2d_f-d)^2} \cdot \sin\left\{\frac{1}{2}\tan^{-1}\left[\frac{2w(2d_f-d)}{w^2+r^2-(2d_f-d)^2}\right]\right\}\right)$$

$$+ 2\left(2d_f+2d_c-d-\sqrt[4]{[w^2+r^2-(2d_f+2d_c-d)^2]^2+4w^2(2d_f+2d_c-d)^2}\right.$$

$$\left.\cdot \sin\left\{\frac{1}{2}\tan^{-1}\left[\frac{2w(2d_f+2d_c-d)}{w^2+r^2-(2d_f+2d_c-d)^2}\right]\right\}\right)\ldots\Bigg] \quad (15)$$

where the radius of the loop r and all other lineal dimensions are in inches. For most installations it is necessary to integrate only the fields from the near images as discussed for the rectangular loop; the fields at the center of the loop due to the other images are added in linearly. This equation, however, is very cumbersome and Haber does not give an equation for the average field over a circular loop. Several mathematicians at this Laboratory have attempted without success to integrate Haber's point equations over a circular loop.

To avoid using the circular-loop equation, Bond(1) suggests a method of finding the bounds of the average resultant field strength and then taking the average of these bounds. In this method, the average resultant field strength is first calculated by assuming that the loop is equal in size to a circumscribed square and then equal to an inscribed square. A simpler method of calculating the average resultant field strength over a circular loop was investigated at the Laboratory and found to yield very accurate results. The circular loop is assumed to be a square loop of area equal to that of the circular loop, and the equations for the rectangular loop are applied. The length of one side of the equal-area square loop is $2h$, and can be found from the relation

$$2h = \sqrt{\pi}\, r \tag{16}$$

where r is the radius of the circular loop. Considering only the line or any one image, the percent of error in the calculated field strength resulting from assuming the circular loop to be a square loop or equal area can be found from the equation

$$\%\ \text{Error} = \left\{ \left[\frac{1}{2\sqrt{\pi}\,(C-\sqrt{C^2-1})} \ \ln \frac{C+\frac{\sqrt{\pi}}{2}}{C-\frac{\sqrt{\pi}}{2}} \right] - 1 \right\} 100 \tag{17}$$

where $C = D/r$, the ratio of the distance from the line or image to the center of the loop, to the radius of the loop. This equation is shown in the form of a graph in Fig. 3. It can be seen that for values of C greater than 1.22 the error is less than 0.3 percent. Considering the field due to the line only, this means that the error would be less than one percent for almost any practical positioning of the loop below the line. The error in the field strength calculated for any image would be negligible since the ratio C would be greater than 2 for almost any installation. And, since the fields produced by the various images do not all have the same sign, the error in the calculation of the average resultant field strength would be less than one percent for almost any installation if the ratio d/r is greater than 1.2. To verify this, calculations were made of the average, resultant field strength over a circular loop and over a square loop of equal area. For these calculations

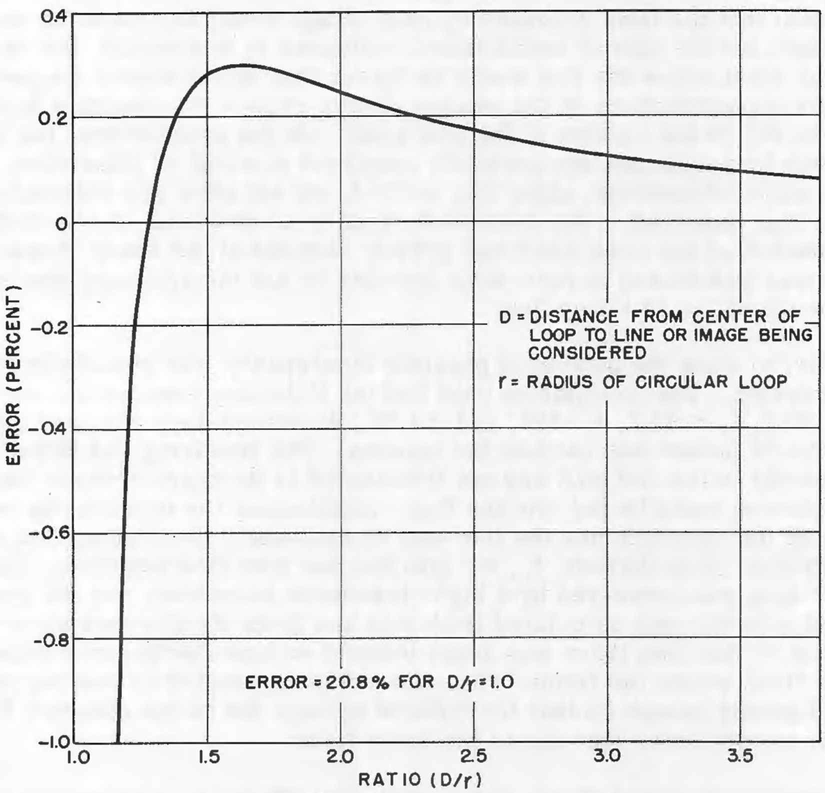

Fig. 3 - Percent error of the calculated average field strength from line or any image individually, assuming circular loop to be a square loop of equal area

the closest nine pairs of images were considered. The parameters expressed in inches were: $d_c = 23.7$, $d_f = 83.7$, $w = 196$, $d = 20$, $r = 15.13$, and $2h = 26.82$. In this the ratio $d/r = 1.32$. The average, resultant field strength calculated for the square loop of equal area was in error by +0.096 percent as compared with that calculated for the circular loop. When these results were checked experimentally by using two single-turn loops, the error was imperceptible.

FREQUENCY CONSIDERATIONS

The usable frequency range of the shielded transmission-line method of generating standard fields is dependent upon the installation. Haber analyzes the high-frequency case from a consideration of the effects of the propagation time on the field strength. He assumes that the line is properly terminated so that the standing-wave ratio is unity, and derives an equation for the error involved depending upon the frequency used. For a particular installation he shows that the error increases with frequency, and at 25 megacycles is -5.0 percent. During the investigation at the Laboratory, it was found experimentally that it is impossible to terminate the transmission line properly with resistance only. This is because of the inductance in the end wall of the room. Therefore, additional errors are introduced since standing waves are present on the line. Also, at frequencies above about 15 megacycles the "body effects" of personnel or of large objects in the room affect the accuracy. These factors would vary with the particular installation.

Since the method of generating standard fields depends upon a shielded room to confine the field, it might be assumed that the accuracy at very low frequencies would be poor since the attenuation of most rooms greatly decreases at these frequencies. It would then seem that the field produced by each image would be somewhat reduced. This would mean that, for the type of installation considered in this report, the resultant field strength at any point below the line would be higher than at the higher frequencies because the field contributions of the images always reduce the resultant field from that which would be due to the current in the line alone. At the present time the lowest frequency at which loop antennas are normally employed is about 15 kilocycles. Work performed at such frequencies, using this method, did not show any noticeable increase in error over that observed at the medium frequencies. However, it was definitely known that the attenuation of the room used was greatly reduced at the lower frequencies. An investigation was undertaken to determine whether or not the accuracy was reduced at frequencies much below 15 kilocycles.

In an effort to show the maximum possible discrepancy, the investigation was performed at 60 cycles. The installation used had the following dimensions, with all units in inches: $d_c = 23.7$, $d_f = 83.7$, $w = 196$, and $d = 20$. An eleven-turn circular loop with a radius, r, of 15.13 inches was used as the antenna. The receiving end of the line was connected directly to the end wall and not terminated in its characteristic impedance so that a large current could be fed into the line. Eliminating the terminating resistance had no effect on the results since the line was so extremely short compared to a wavelength at 60 cycles. The current, I_L, fed into the line was 10.0 amperes. The voltage induced in the loop was measured by a high-impedance voltmeter, and the generated resultant field strength was calculated from this and from the dimensions of the loop. With no current in the line, there was some induced voltage due to stray fields from nearby power lines within the room. This effect was eliminated by making the generated standard field strong enough so that the induced voltage due to the standard field was approximately twenty times that due to the stray field.

For the conditions listed above, and considering 100-percent reflection from the room surfaces, the average resultant field strength over the loop would be 1169 volts per meter. If the effects of the images were reduced to zero, the average field strength over the loop would be produced by the line alone and would be 1428 volts per meter. The measured average field strength was 1139 volts per meter. A comparison of the measured and calculated values (considering 100-percent contribution from the images) shows an error of -2.6 percent. The measurement accuracy is no better than this, and it is significant that the measured value was less than the calculated value instead of being greater, which would be true if the image fields were reduced. This investigation shows that the accuracy of this standard field method is not reduced even at the extremely low frequencies if a reasonably good shielded room is employed.

A review of the wave theory of shielding gives a possible explanation of why the method of images, which assumes 100-percent reflection from the shielding surfaces, is valid at the very low frequencies. The shielding material of the room employed for this investigation was sheet copper with an average thickness of 4.5 mils (trade name, Sisalkraft). It can be determined(5) that for the type of wave and the distance involved at 60 cycles, the initial reflection loss of the energy impinging on the ceiling was 14 decibels. The initial reflection loss from the walls and floor was even greater, because of the greater distances involved. This loss means that 96 percent of the energy that reached the ceiling was reflected back into the room, and that an even greater percent was reflected by the walls and floor. Therefore, the over-all error of the calculated resultant field strength at frequencies as low as 60 cycles was less than 4 percent for this particular installation. If some other shielding material, such as copper window screening, were used, the accuracy at the very low frequencies would have been somewhat poorer.

PRACTICAL CONSIDERATIONS

The number of pairs of images required to calculate the resultant field strength to some particular accuracy is reduced by placing the line nearer the ceiling. This brings the images in each pair closer together, and because they have opposite polarities, the expression for field strength converges rapidly. However, placing the line very close to the ceiling also has some drawbacks. The closer the line is to the ceiling, the lower the resultant field strength is at the same distance from the line. Also the line should not be very close to the ceiling unless the ceiling is a flat, smooth surface. Some shielded rooms are so constructed that the shielding surface is not flat. To assure a uniform line impedance, the irregularities in the surface should be very small compared to the spacing of the line from the ceiling.

When employing the shielded transmission-line method at frequencies where the length of the line is very short compared to a wavelength, the line does not have to be terminated in its characteristic impedance. It can be terminated to the end wall through any impedance (or even directly connected), a situation which is often useful for experimental purposes when it is desired to produce a very strong field.

This method of generating a standard field is not recommended above about 15 megacycles if good accuracy is required. If it is used above that frequency, it is recommended that the field be checked by some other method of generating a standard field.

Measurements made of fields produced by the shielded transmission-line method indicated that in general the least error attainable was 5 percent while other methods consistently gave accuracies better than 5 percent. The measurements were made under well-controlled conditions by comparing various methods of standard field generation. Also, as is shown in this report, the calculation of the field strength is very laborious. Other methods recommended are the two-loop method(6) and, for most shielded loops, the shield-injection method(7).

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