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NRL Report 4469

IN-PILE PERFORMANCE OF NRL-1 THIMBLE AT MTR

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Statement of the Problem

A preliminary description of the NRL-1 experiment has been given in a previous report[1]. Briefly, the experiment will be performed in a circulating water loop constructed principally of carbon steel. Operation with a portion of the loop, called the in-pile thimble, inserted in the HB-1 beam hole of the MTR is planned to determine the feasibility of using carbon steel as a principal construction material in high temperature water cooled nuclear reactors.

Absorption of gamma radiation and thermal neutrons in the walls of the in-pile thimble introduces thermal stresses which are additional to the stresses normally encountered in unfired pressure vessel operation. This report is concerned with a detailed examination of the performance of the in-pile thimble under these conditions of elevated temperature, high pressure, "gamma" heating, and irradiation. The discussion is confined to the portion of the thimble extending into the reactor tank, since only that portion is subjected to significant thermal stress. The remainder of the in-pile thimble has been designed in accordance with the applicable unfired pressure vessel codes.

A somewhat similar problem has been discussed by Fromm[2] for the ANL-2 experiment at MTR. The principal differences between the two are summarized in Table 1.

TABLE 1

Comparison of ANL-2 and NRL-1 Characteristics

	ANL-2	NRL-1
Thimble Material	AISI-316 stainless steel	A-201-B carbon steel
Design Pressure	1650 psi	2500 psi
Operating Temp.	500°F	600°F

Operating Characteristics

The characteristics of the system which are pertinent to this discussion are given in Table 2.

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[1] NRL Report No. 4366
[2] ANL Reactor Engr.Div., Tech. Memo. 8

TABLE 2

NRL-1 System Characteristics

Bulk Water Temp.,	inlet	600° F
	outlet	610° F
System Pressure,	design	2500 psi
	operating	2200 psi
Flow, maximum		135 gpm

The reactor is assumed to operate at 30 MW and to produce an effective gamma heating in the HB-1 beam hole in beryllium as given in drawing AED-R-1049[3]. This heating is assumed to be 1.25 times greater in iron than in beryllium, to be uniform in the thimble wall in a plane ⊥ to the thimble axis, and to decrease along the axis as shown in AED-R-1049, a decrease which may be described analytically as

$$q = 8.3\ e^{-.095z}\ \text{watts/gm}$$

where z is the distance in cm along the thimble axis. ($z = 0$ at the in-pile end of the thimble at 600°F.)

No allowance has been taken for possible flux depression in the vicinity of the thimble.

Description of Thimble

The portion of the thimble exposed to significant gamma heating is a single unit hot deep drawn from a 7/8" plate of A-201 Grade B silicon-killed carbon steel, normalized by heat treatment after drawing. This unit is joined to the remainder of the thimble by a butt weld approximately 16" from the end, at which point the gamma heating is negligible. The end of the drawing is machined as shown in Figure 1 to reduce the thermal stress induced by gamma heating. The configuration shown was chosen to provide minimum combined thermal and membrane tangential stress in the vicinity of the intersection of the spherical end and the cylinder, and to provide substantially uniform tangential thermal stress in the region $z = 0$ to $z = 10$ cm. Care was taken to avoid a configuration which might give rise to points of stress concentration or abrupt changes in dimensions.

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[3] ORNL Report No. 963

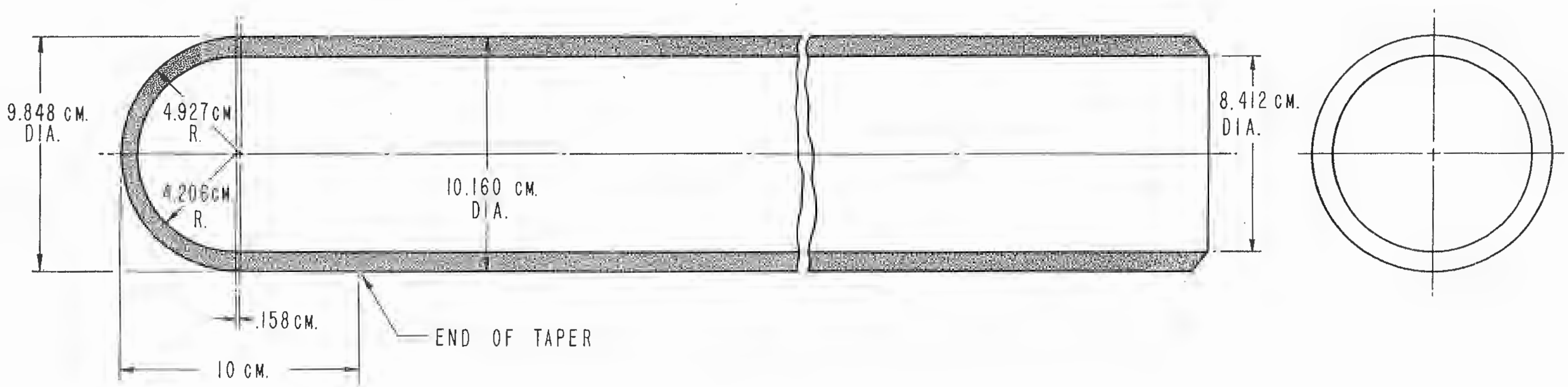


Fig. 1 NRL-1 In-Pile Pressure Thimble

Physical Properties of Thimble Material

Ten thimble ends were drawn from one lot of A201-B with identical treatment. These units were machined to the dimensions shown in Figure 2. Several of these were used for the tests described in this section. The chemical analysis of the material used is given in Table 3.

TABLE 3

Chemical Analysis of ASTM-A-201 Grade B

Used in In-Pile Thimble

C	Mn	Si	S	P
.19	.65	.20	.034	.026

A detailed examination at room and elevated temperatures has been made of the physical properties of specimens cut from one of the thimble ends (see Appendix B for details). The location of the specimens is shown in Figure 3. The results of the tests so far completed are given in Table 4. Additional stress rupture tests at lower stresses are in progress. A stress rupture test at 30,000 psi and 600°F will be continued for at least two months to determine a creep rate which may be reliably extrapolated to one year or longer. Representative creep rates which have been previously found for similar materials are shown in Table 5.

TABLE 5

Creep Rates for Typical Carbon Steel

Rate	Stress	Temp.
$1\%/10^5$ hr	23,000 psi	650°F
$1\%/10^5$ hr	20,000 psi	700°F
$1\%/10^5$ hr	18,000 psi	750°F

The values of physical properties required for the stress calculations are given for 300°C in Table 6. These values

TABLE 4

PHYSICAL PROPERTIES OF A-201B STEEL USED IN IN-PILE THIMBLE

Tensile Tests

		Temp.	Yield(psi)	Ultimate(psi)	Elongation	Reduction of Area
1.	Plate	∿ 70°F	41,460	68,930	29.0% in 2"	48.9%
2.	Cut from Thimble	∿ 70°F	37,000	67,700	31.2% in 4"	62.3
	(Fig.3)		37,500	66,800	30.5% in 4"	61.4
3.	Same as 2	600°F	28,400	78,600	35.0% in 1"	59.7
			28,900	79,200	33.0% in 1"	56.0
4.	Same as 2	650°F	26,900	74,800	37.0% in 1"	64.6
5.	Same as 2	700°F	25,100	72,000	41.0% in 1"	67.8

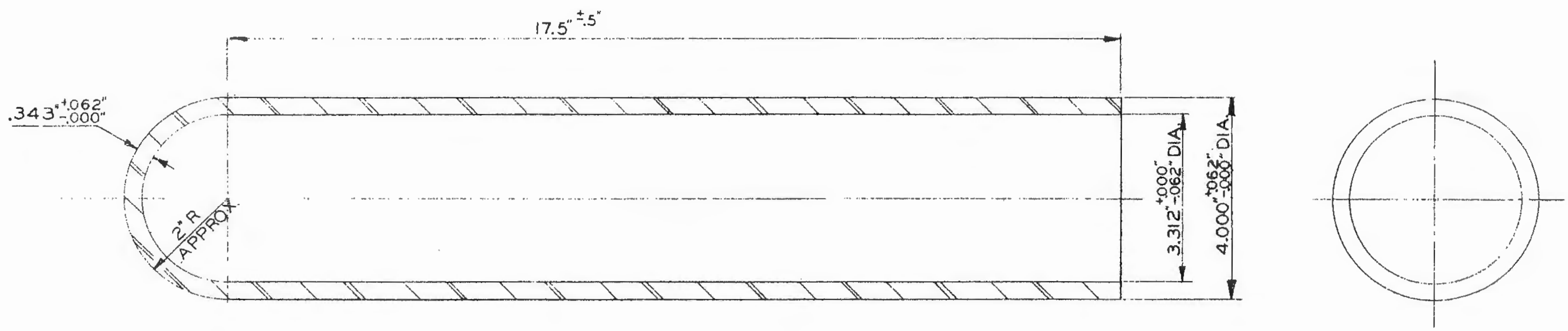
Notch-Bar Impact Test (..82°F)

	Notch	Energy Absorption	% Granular
1.	Inside	104 ft.lbs.	20
2.	Inside	68	50
3.	Outside	96	40
4.	Outside	112	20

Stress Rupture Tests (600°F)

Applied stress (psi)	80,000	75,000	73,000	70,000	30,000
Time to rupture	immed.	9 hr.	∿ 9 d.	in progress	in progress

Rockwell Hardness 75 B ave.
Microhardness (VHN, 1kg load) 138-163 VHN 153 VHN ave. no marked trend in hardness from exterior to interior surface
Flattening Test. (ASTM-A210-46 Spec):Very fine hairline cracks seen with 20X microscope when OD is reduced to 2.45". First clearly visible crack at OD 1.31". Specimen did not break when flattened so that opposite interior surfaces were in contact.
Grain Size - ASTM-7.



MANUFACTURED BY CUP AND DRAW METHOD
MACHINE ALL OVER 250 FINISH
HYDROSTATIC TEST — 3000 P.S.I.
HEAT TREAT — NORMALIZE TO 60,000 P.S.I. MIN. TENSILE
INSPECTION — GOVERNMENT FURNISHED

ESTIMATED WEIGHT — 28LBS.

MATERIAL
STEEL – ASTM-A-285-GRADE "C" OR EQUAL

REFERENCE
NAVAL RESEARCH LAB. DWG. D26-007

Fig. 2 In-Pile Pressure Thimble before Finished Machining were obtained from steels having closely similar composition to the A-201 used in the thimble. From the range of values given in the literature it is concluded that these parameters are rather insensitive to small changes in composition.

TABLE 6

Values of Physical Properties Required in Stress Calculations

Parameter	Value	Reference
E (Young's Modulus)	25×10^6 psi	4,5
k (Thermal Conductivity)	.460 watt/cm°C	6
α (Coeff. Linear thermal expans.)	1.3×10^{-5}/°C	6
ν (Poisson's Ratio)	.28	6
ρ (Density)	.786 g/cc	6

Irradiation Effects on Physical Properties

The available evidence on the effects of irradiation is summarized in a report by D. O. Leeser[8]. The reported work has been done at lower flux levels, lower total flux, and lower temperatures than contemplated for the NRL-1 thimble. The results indicate a general tendency to saturation of effect with increasing total flux, and a reduction of effect with increasing temperature. In general it has been found at approx. 100°F and 5×10^{19} NVT that the hardness increases slightly, the tensile strength increases ∿ 10-20%, the yield point increases ∿ 25-50%, and the elongation decreases ∿ 25-50%

Irradiation increases the transition temperature for brittle fracture. This effect appears to decrease with increasing temperature and to reach a higher limit with

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[4] M.H.Roberts and J.Nortcliffe, J.Iron and Steel Inst.157:345,1941
[5] G.C.Seager and F.C.Thompson, J.Iron and Steel Inst.147:103,1943
[6] Metals Handbook, 1948
[7] ANL-4792

increasing total flux. The increase in transition temperature appears to be reduced with finer grain steels. Increases of ∿ 20-30°F are reported at temperatures of 500°F and 2×10^{19} NVT.

Evidence on creep is very scanty but indicates only slight effects.

The most nearly applicable results are summarized in Table 7.

TABLE 7

Selected Results of Effects of Irradiation on Carbon Steel

Property-Mat'l	Temp. °F	Flux (NVT slow)	Results Pre	Post
Hardness-SAE-1018	450-500	1×10^{19}	R_B81-83	R_B85-87
Tensile Strength -SA-212	70-140	2×10^{19}	74,000psi ult. 42,000 yield 19.2% elongation	82,000 psi 70,000 14.5%
Creep-SA-194 (15,000 psi load)	450-500	10^{19}	0.2% increase in overall length	
Impact Strength-0.34C Al-killed (transition temp.)	425	8×10^{18}	15-40°F	35-60°F

As a part of the NRL-2 test program at MTR (an experiment in which simple specimens may be irradiated at elevated temperatures in a reflector piece) tensile and notch impact specimens have been prepared from thimble material and are awaiting irradiation. Operating temperature of 600°F and total flux of 10^{20} NVT are planned. A comparison will be made of the behavior of the irradiated specimens with unirradiated controls.

Stress Calculations

The analysis of the stresses in the in-pile thimble has been made in accordance with the assumptions and method of Timoshenko[8]. The details of the calculations are given in Appendix A. The stresses obtained, assuming an adiabatic outer surface, are given in Table 8. The details of the calculations

[8] Timoshenko and Goodier,"Theory of Elasticity",2nd ed.,p.408 et seq.

are given in Tables 9 and 10. Figure 4 shows the variation of stress across the wall at the point of maximum tangential stress. Figure 4 also shows the effect on the stress of removing various fractions β of the heat generated in the wall by external cooling.

Thimble Tests

One thimble end machined as shown in Figure 3 has been subjected to increasing hydrostatic pressure at room temperature until it burst at 14,000 psi, at which pressure the tangential stress was estimated to be $\sim$ 98,000 psi. No increase was noted in cylindrical diameter at 5,000 psi. The result of this test is shown in Figures 5 and 6. A second bursting test is being prepared for 600°F. Measurements of stress and expansion will be included.

The thimble end intended for in-pile test will be subjected to a hydrostatic test at 3500 psi and inspection by magnaflux and zyglo techniques. Thermal gradients obtained in-pile will be monitored on the first thimble inserted. The present plan is to remove the first thimble at the end of the first test ($\sim$ 60 days) and to perform comparison tests on it to determine deformation and changes in physical properties.

Conformity to ASME Boiler and Pressure Vessel Code[9]

The maximum allowable tensile stress for A-201-B carbon steel is given in Table UCS-23, Section VII, as S = 15,000 psi in the temperature range -20 to 650°F for material with minimum room temperature ultimate strength of 60,000 psi. The criteria for determining the allowable stress values are varied and not well defined. They are given in Appendix P of Section VII and may be briefly summarized as:

(1) 25% of min. tensile strength at room temp.
(2) 25% of min. tensile strength at operating temp.
(3) 62 1/2% of min. yield strength at operating temp.
(4) 100% of stress to produce creep rate of $1\%/10^5$ hr.

It should be kept in mind that these criteria are for stresses produced by pressure only, and allow for safe operation for a range of material properties as encountered in commercial production, for the presence of unknown discontinuity stresses, and for service over an extended period of time. By any of these criteria, the data of Table 4 show that the maximum pressure stress of 16,000 psi obtained in this thimble is within code allowances.

The code provides no specific allowances or criteria for determining maximum allowable thermal stress. Of the specific allowances made in the code, the following three are most nearly comparable to the in-pile case:

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[9] ASME Boiler and Pressure Vessel Code, 1952

TABLE 8

COMBINED STRESSES

z (cm)	Inner Surface Tangential (psi)	Inner Surface Longitudinal (psi)	Outer Surface Tangential (psi)	Outer Surface Longitudinal (psi)
0*	16,940	16,940	4,660	4,660
1*	16,700	16,700	4,120	4,120
2*	16,450	16,450	3,480	3,480
3	16,830	18,450	2,760	1,460
4	16,410	18,210	2,290	830
5	23,090	13,050	9,510	4,060
6	22,040	11,180	8,750	5,130
7	21,800	11,870	8,370	4,020
8*	21,720	13,280	8,320	2,470
9*	20,920	12,750	8,010	2,380
10*	20,460	12,480	7,690	2,270
12*	19,180	11,200	8,260	2,840
14*	18,080	10,100	8,740	3,320
16*	17,360	9,380	9,060	3,640
25*	15,050	7,070	10,090	4,670

* Calculations of the discontinuity stress at these points were not made. The only significant change would be to increase the inner surface longitudinal stress for z = 0, 1, and 2 cm by $\sim$ 1000 psi or less.

Note 1 - the intersection of the sphere and cylinder occurs at z = 4.769 cm.

Note 2 - the maximum discontinuity stress occurs at z = 3.8 and 5.9 cm. The combined stresses at these points are less than the maximum combined stresses shown in this table.

TABLE 9

VALUES FOR STRESS CALCULATION PARAMETERS

z cm	b cm	t cm	g watts/cc	$\frac{\alpha E g}{k(1-\upsilon)}$ (10^4psi/cm^2)
0	4.769	.563	65.3	6.41
1	4.802	.596	59.4	5.83
2	4.835	.629	54.2	5.32
3	4.867	.661	49.0	4.81
4	4.900	.694	44.4	4.36
4.769	4.924	.718	41.5	4.07
5	4.931	.725	40.5	3.98
6	4.961	.755	36.6	3.59
7	4.991	.785	34.0	3.34
8	5.020	.814	31.3	3.07
9	5.050	.844	27.8	2.73
10	5.080	.874	25.5	2.50
12	5.080	.874	20.9	2.05
14	5.080	.874	17.0	1.67
16	5.080	.874	14.4	1.41
25	5.080	.874	6.0	.59

a = 4.206 cm

COMPONENT STRESSES (psi)

z (cm)	Inner Surface Tangential Thermal	Inner Surface Tangential Pressure	Inner Surface Tangential Discon-tinuity	Inner Surface Longitudinal Thermal	Inner Surface Longitudinal Pressure	Inner Surface Longitudinal Discon-tinuity	Outer Surface Tangential Thermal	Outer Surface Tangential Pressure	Outer Surface Tangential Discon-tinuity	Outer Surface Longitudinal Thermal	Outer Surface Longitudinal Pressure	Outer Surface Longitudinal Discon-tinuity
0	7,600	9,340		7,600	9,340		-3,430	8,090		-3,430	8,090	
1	7,850	8,850		7,850	8,850		-3,500	7,620		-3,500	7,620	
2	8,050	8,400		8,050	8,400		-3,690	7,170		-3,690	7,170	
3	8,200	8,000	630	8,200	8,000	2,250	-3,480	6,750	-510	-3,480	6,750	-1,810
4	8,060	7,650	700	8,060	7,650	2,500	-3,540	6,390	-560	-3,540	6,390	-2,020
5	7,560	15,850	-320	7,560	6,640	-1,150	-3,510	13,280	260	-3,510	6,640	930
6	7,480	15,300	-740	7,480	6,360	-2,660	-3,360	12,710	600	-3,360	6,360	2,130
7	7,480	14,800	-476	7,480	6,090	-1,700	-3,430	12,180	380	-3,430	6,090	1,360
8	7,420	14,300		7,420	5,860		-3,390	11,710		-3,390	5,860	
9	7,120	13,800		7,120	5,630		-3,250	11,260		-3,250	5,630	
10	7,060	13,400		7,060	5,420		-3,150	10,840		-3,150	5,420	
12	5,780	13,400		5,780	5,420		-2,580	10,840		-2,580	5,420	
14	4,680	13,400		4,680	5,420		-2,100	10,840		-2,100	5,420	
16	3,960	13,400		3,960	5,420		-1,780	10,840		-1,780	5,420	
25	1,650	13,400		1,650	5,420		- 750	10,840		- 750	5,420	

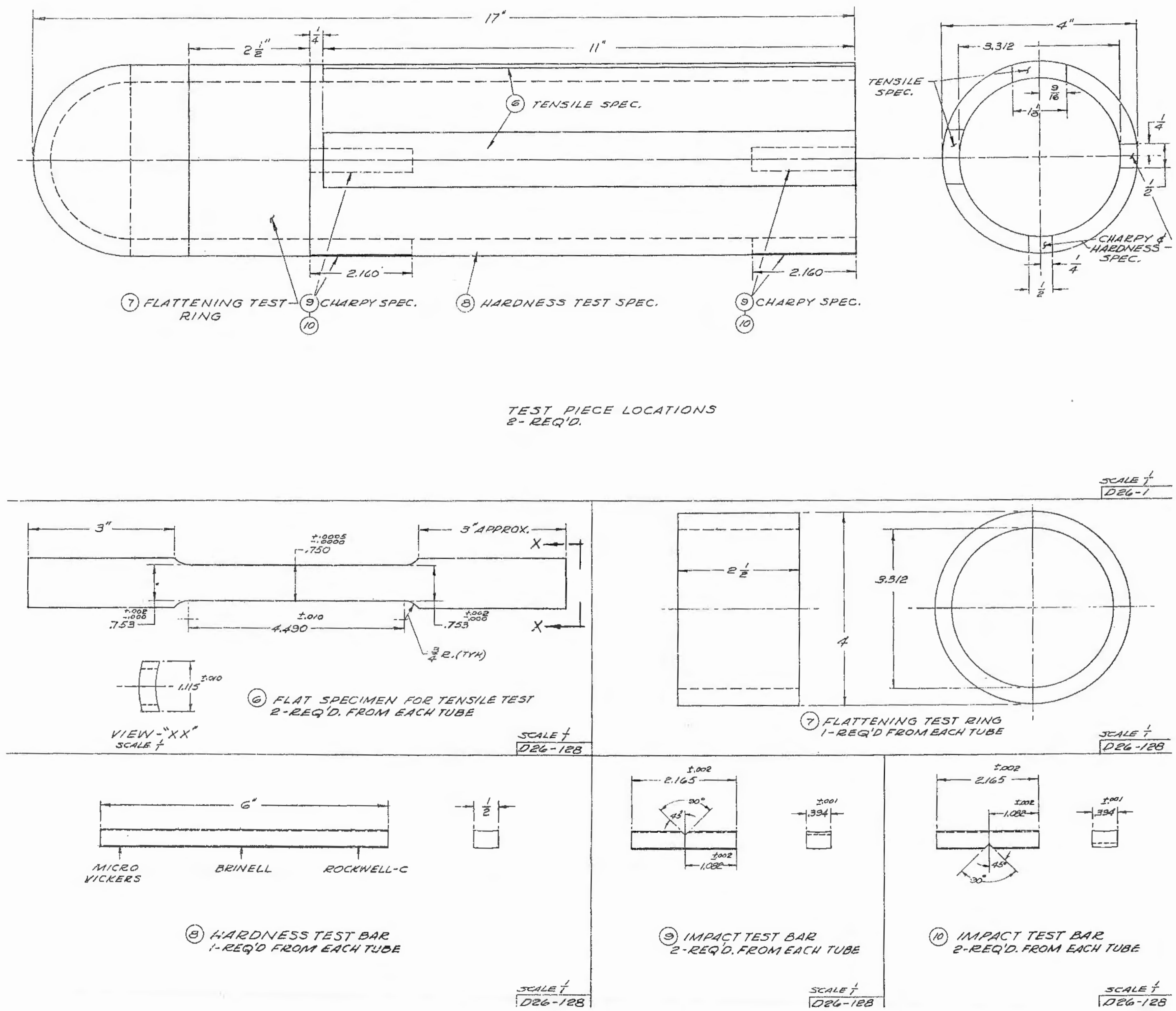

Fig. 3 Location of Physical Test Specimen

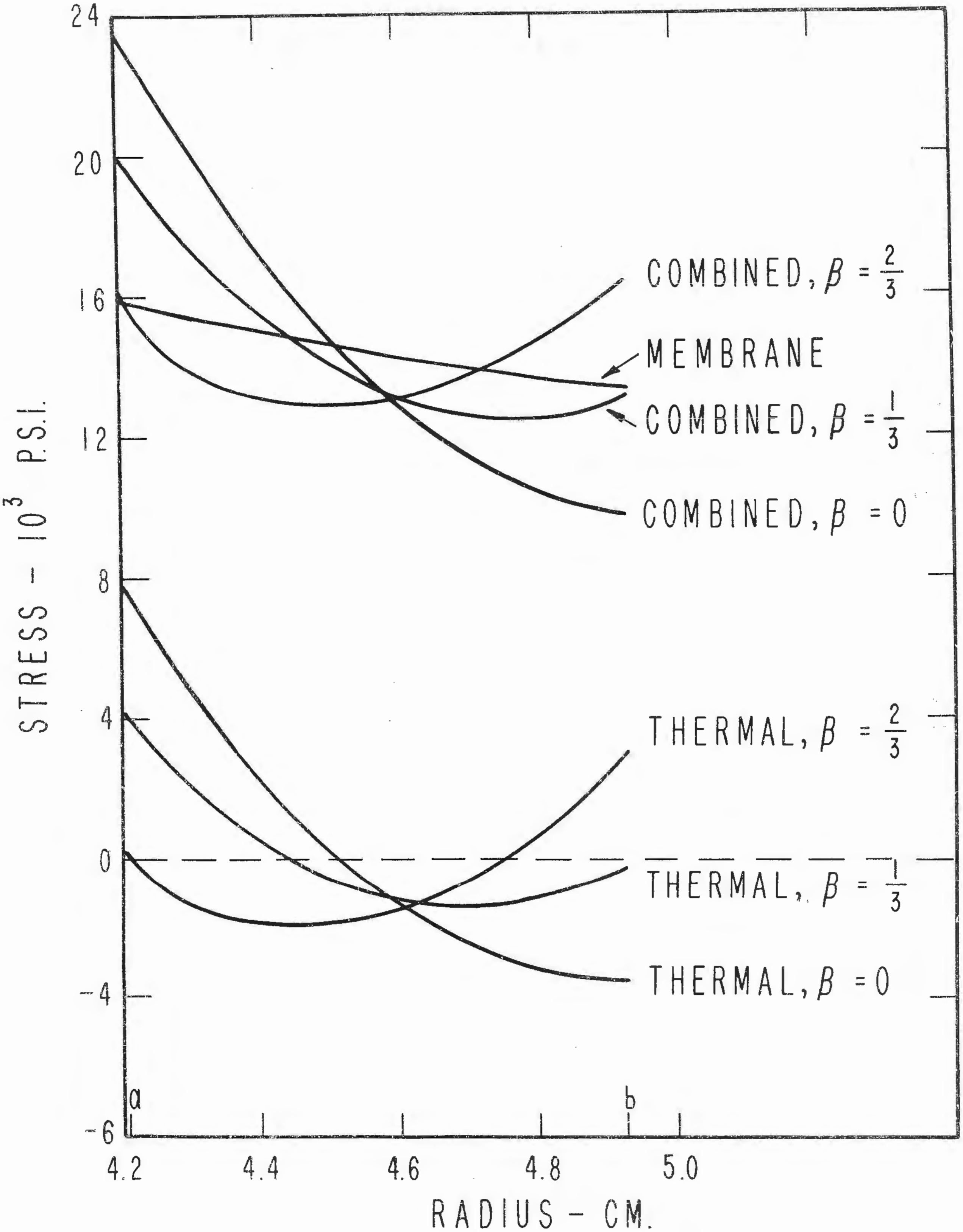

Fig. 4 Variation of Thermal and Membrane Stresses with Radius as a Function of β (β = Fraction of Heat Generated in the Wall which is removed by external cooling.)

THIMBLE BURST TEST

DEEP DRAWN FROM A-201-B PLATE
HEAT TREATED & NORMALIZED
YD PT = 37,000 ULT = 67,000 ELONG = 30%
START - O.D. = 4."045 I.D. = 3."274
5,000 PSI — NO CHANGE
14,000 PSI — THIMBLE BURST O.D. ≈ 4."8
I.D. 4."2 STRESS ≈ 98,000 PSI.

Fig. 5 Result of Cold Hydrostatic Burst Test

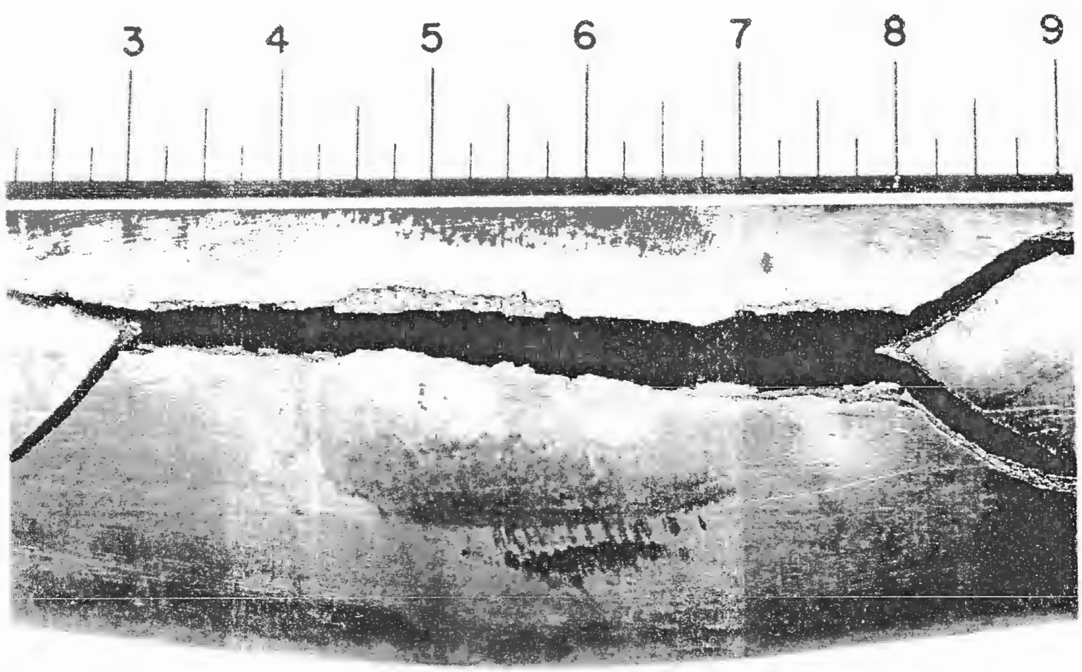

Fig. 6 Detail of Cold Hydrostatic Bursting

(1) (a) For pressure piping ASA B 31.1-1951 para.620 allows combined bending and pressure stresses computed by the formula

$$S_c = .75(S + S_1)$$

where

S_c = combined stress

S = allowed working stress at operating temp.

S_1 = allowed working stress at room temp.

Using $S_1 = 15{,}000$ $S_c = 22{,}500$

$S_1 = 16{,}000$ $S_c = 24{,}000$

(b) This criteria is sometimes expressed as $S_c = \frac{.75}{5}(u + u_1)$

where u = ultimate strength at operating temp.

u_1 = ultimate strength at room temp.

using the data of Table 4 this gives $S_c = 21{,}900$

(2) For superheaters, economizers, and boiler generator tubes, the power boiler code, section I, para.P-21, gives the following formula for the maximum working pressure

$$p = \frac{S(\frac{t}{b} - .027)}{1 - \frac{t}{b} + .027}$$

where t = wall thickness

b = outer tube radius

using the data of Table 9, the following values are obtained:

S(psi)	b(cm)	p(psi)
15,000	5.080	2542
15,000	4.924	2025
16,000	5.080	2712
16,000	4.924	2160

(3) For steam pipes, Section I, para.P-23 gives for the maximum working pressure

$$p = \frac{S}{\frac{b}{t} - y}$$ where y is a constant = 0.4

this gives the following values

S(psi	b(cm)	p(psi)
15,000	5.080	2790
15,000	4.924	2250
16,000	5.080	2980
16,000	4.924	2400

These figures show that the vessel would be safely designed according to the code (under criteria(2) and (3) which are for more severe service than anticipated here) if a uniform diameter tube using the outer radius 5.080 cm were employed. This radius is used in the NRL-1 vessel except for a 5 cm length in the vicinity of the intersection of spherical end and cylinder. In this length the stresses obtained with the current NRL-1 design are less than those obtained with a uniform wall thickness, and therefore the design is assumed to be safer than a code designed vessel. It may be seen from Figure 4 and Table 8 that the maximum allowance for combined stress by criteria (1) is exceeded, if all (depending on the criterion chosen), by less than 1000 psi. This "excess" furthermore exists for $\sim$1 cm in length and $\sim$5 mm in depth in the wall

No corrosion allowance has been made in wall thickness in the thimble tip. On the basis of out-of-pile loop experience, the maximum anticipated corrosion rate is approx. 0.5 mils per year. It is likely that the actual rate will be less by a factor of 2 to 5. The corrosion rate will be carefully monitored during operation. Operation of a thimble for more than two years is not anticipated, so that no corrosion allowance is required.

Conclusions

From these considerations it is concluded that the vessel is safely designed within the applicable ASME codes and to acceptably conservative ciriteria for those points which are not covered by ASME codes.

The effects of irradiation are not such as to affect the foregoing conclusion. Irradiation will tend to improve some qualities of the steel to compensate for degradation in others.

From the thermal gradient measurements of the ANL-2 experiment[1] it is concluded that the actual gamma heating may be lower than that used in this report by as much as a factor 2. Accordingly, it is likely that the design is safer than the calculations of this report indicate.

Acknowledgments

Acknowledgment is gratefully made for the advice of Dr. O. T. Marzke; for discussions of the thermal stress calculations with Messrs. J. P. Walsh and R. E. Blake; for discussions and measurements of the physical properties of the material by Messrs. W. S. Pellini and R. H. Raring; for assistance with the calculations by Dr. J. W. Teener and Mr. B. L. Hoffman, and for rupture tests performed by Mr. G. E. MacVeigh.

Appendix A

Stress Calculations

The assumptions and method of analysis for thermal stress are those of Timoshenko[6]. The stresses due to internal pressure are calculated from the usual formulas[7]. The discontinuity stresses are calculated by Roark's method[8].

1. Stresses due to internal pressure.

Cylinder

At a point in a long circular cylinder closed at the ends with a concentric circular hole and internal pressure only:

At all points $\sigma_\theta = \frac{a^2 p}{b^2-a^2}\left(1 + \frac{b^2}{r^2}\right)$

$$\sigma_z = \frac{pa^2}{b^2-a^2}$$

At inside surface $\sigma_\theta = p\left(\frac{b^2+a^2}{b^2-a^2}\right)$

At outside surface $\sigma_\theta = \frac{2pa^2}{b^2-a^2}$

Sphere

At a point in a sphere with a concentric spherical hole and internal pressure only:

At all points $\sigma_t = \frac{pa^3}{2r^3} \cdot \frac{2r^3+b^3}{b^3-a^3}$

At inside surface $\sigma_t = \frac{p}{2} \cdot \frac{2a^3+b^3}{b^3 - a^3}$

At outside surface $\sigma_t = \frac{3pa^3}{2(b^3-a^3)}$

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[6]Timoshenko and Goodier,"Theory of Elasticity",2nd ed.,p.408,et.seq.
[7]Timoshenko and Goodier,"Theory of Elasticity",2nd ed.,p.58 and 359
[8]Roark, "Formulas for Stress and Strain", 3rd ed., p.275

where σ_θ = circumferential stress in cylinder

σ_z = longitudinal stress in cylinder

σ_t = tangential stress in sphere

p = internal pressure

r = radius at the point

b = outside radius

a = inside radius

2. Stresses due to "gamma" heating in the walls of the vessel.

Because of the mathematical complications, the analysis is simplified by the assumption that the thermal stresses at a point may be approximated by calculating the stress obtained in a cylinder or sphere uniformly heated by gamma heat of the magnitude existing at the point. The validity of this approximation is improved by varying the thickness as has been done in this vessel, so as to maintain essentially uniform thermal gradient in all planes ⊥ to the axis.

Cylinder

It is assumed that the heating is uniform in a plane ⊥ to the axis of the cylinder. Then from Timoshenko, the stresses are:

$$\sigma_\theta = \frac{\alpha E}{(1-\upsilon)r^2}\left[\frac{r^2+a^2}{b^2-a^2}\int_a^b Trdr + \int_a^r Trdr - Tr^2\right]$$

$$\sigma_z = \frac{\alpha E}{(1-\upsilon)}\left[\frac{2}{b^2-a^2}\int_a^b Trdr - T\right]$$

The temperature at a point in the cylinder is given by:

$$\nabla^2 T = \frac{1}{r}\frac{d}{dr}\left(\frac{rdT}{dr}\right) = -\frac{g}{k}$$

$$T = -\frac{gr^2}{4k} + C_1 \ln r + C_2$$

The constants are evaluated for the general case in which a fraction β of the heat generated in the cylinder is transmitted outward through the external surface. The case $\beta = 0$ represents the case for internal cooling only; $\beta = 1$ external cooling only. From the resulting equations, the effects of external cooling may be determined. The boundary conditions for this case are:

$$T = T_o \qquad \text{for } r = a$$

$$\frac{dT}{dr} = -\beta \frac{g}{k} \frac{(b^2-a^2)}{2b} \qquad \text{for } r = b$$

Leading to:

$$T = \frac{g(a^2-r^2)}{4k} + \left[\frac{gb^2}{2k} (1-\beta) + \frac{ga^2}{2k} \beta \right] \ln \frac{r}{a} + T_o$$

Sphere

It is assumed that the heating is uniform along a radius to facilitate the mathematical calculation. The value taken in the actual computation is that at the intersection of the radius and the outer surface so that the stress obtained is somewhat higher than the actual stress. The deviation is significant only for radii near the axis of the cylinder.

$$\sigma_t = \frac{2\alpha E}{1-\upsilon} \left[\frac{2r^3+a^3}{2(b^3-a^3)r^3} \int_a^b Tr^2 dr + \frac{1}{2r^3} \int_a^r Tr^2 dr - \frac{T}{2} \right]$$

The temperature at a point in the sphere is given by

$$\nabla^2 T = \frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = -\frac{g}{k}$$

$$T = -\frac{gr^2}{6k} - \frac{C_1}{r} + C_2$$

The boundary conditions, as before, are

$$T = T_o \qquad \text{for} \qquad r = a$$

$$\frac{dT}{dr} = -\frac{\beta g}{k} \frac{b^3-a^3}{3b^2} \qquad \text{for } r = b$$

which gives

$$T = \frac{g}{3k}\left(\frac{a^2-r^2}{2} + \frac{b^3}{a} - \frac{b^3}{r}\right) + \frac{\beta g}{3k}(b^3-a^3)\left(\frac{1}{r} - \frac{1}{a}\right) + T_o$$

Integration of the stress equations then leads to the following results:

Cylinder

r=r $$\sigma_\theta = \frac{\alpha E g}{16k(1-\upsilon)}\left\{\frac{4b^4}{r^2}\frac{r^2+a^2}{b^2-a^2}\ln\frac{b}{a} - 4b^2\ln\frac{r}{a} - a^2 - 5b^2 - \frac{a^2b^2}{r^2} + 3r^2 + \frac{2\beta}{r^2}\left[(r^2+a^2)\left(-2b^2\ln\frac{b}{a} + b^2 - a^2\right) + (b^2-a^2)\left(2r^2\ln\frac{r}{a} + r^2-a^2\right)\right]\right\}$$

r=r $$\sigma_z = \frac{\alpha E g}{8k(1-\upsilon)}\left\{\frac{4b^4}{b^2-a^2}\ln\frac{b}{a} - 3b^2-a^2 + 2r^2-4b^2\ln\frac{r}{a} + \beta\left[-4b^2\ln\frac{b}{a} + 2b^2 - 2a^2 - 4(a^2-b^2)\ln\frac{r}{a}\right]\right\}$$

r=a $$\sigma_z = \sigma_\theta = \frac{\alpha E g}{8k(1-\upsilon)}\left[\frac{4b^4}{b^2-a^2}\ln\frac{b}{a} + a^2-3b^2 + 2\beta\left(-2b^2\ln\frac{b}{a} + b^2-a^2\right)\right]$$

r=b $$\sigma_z = \sigma_\theta = \frac{\alpha E g}{8k(1-\upsilon)}\left[\frac{4a^2b^2}{b^2-a^2}\ln\frac{b}{a} - a^2 - b^2 + 2\beta\left(-2a^2\ln\frac{b}{a} + b^2 - a^2\right)\right]$$

Sphere

r=r $$\sigma_t = \frac{\alpha E g}{3k(1-\upsilon)}\left\{\frac{2r^3+a^3}{r^3(b^3-a^3)}\left[\frac{a^2b^3}{3} + \frac{b^6}{3a} - \frac{3b^5}{5} - \frac{a^5}{15}\right] + \frac{1}{r^3}\left[\frac{a^2r^3}{6} - \frac{r^5}{10} + \frac{b^3r^3}{3a} - \frac{b^3r^2}{2} + \frac{a^2b^3}{6} - \frac{a^5}{15}\right]\right.$$

$$+ \frac{r^2-a^2}{2} + b^3 \quad (\frac{1}{r} - \frac{1}{a}) + \beta \left[\frac{2r^3+a^3}{r^3} (\frac{b^2}{2} - \frac{b^3}{3a} - \frac{a^2}{6}) \right.$$

$$\left. + \frac{b^3-a^3}{r^3} (\frac{r^2}{2} - \frac{r^3}{3a} - \frac{a^2}{6}) - (b^3-a^3)(\frac{1}{r} - \frac{1}{a}) \right] \Bigg\}$$

$r=a$
$$\sigma_t = \frac{\alpha E g}{k(1-\upsilon)} \left[\frac{1}{b^3-a^3} (\frac{a^2b^3}{3} + \frac{b^6}{3a} - \frac{3b^5}{5} - \frac{a^5}{15}) + \beta(\frac{b^2}{a} - \frac{b^3}{3a} - \frac{a^2}{6}) \right]$$

$r=b$
$$\sigma_t = \frac{\alpha E g}{3k(1-\upsilon)} \left\{ \frac{2b^3+a^3}{b^3(b^3 1 a^3)} \left[\frac{a^2b^3}{3} + \frac{b^6}{3a} - \frac{3b^5}{5} - \frac{a^5}{15} \right] \right.$$

$$\left. + \frac{9b^2}{10} - \frac{a^2}{6} - \frac{2b^3}{3a} - \frac{a^5}{15b^3} + \beta(\frac{b^2}{2} - \frac{3a^2}{2} + \frac{a^3}{b}) \right.$$

These equations may be simplified for the accuracy required in this problem by introducing the wall thickness $t = b-a$ and calculating the series expansion of the preceding equations. The simplified equations which follow are accurate to approximately 2% for $t/a < 0.2$.

Cylinder

$r=a$
$$\sigma_z = \sigma_\theta = \frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{3} \left[(1 + \frac{t}{2a}) - \beta(\frac{3}{2} + \frac{t}{2a}) \right]$$

$r=b$
$$\sigma_z = \sigma_\theta = -\frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{6} \left[1 - \beta(3 - \frac{t}{a}) \right]$$

Sphere

$r=a$
$$\sigma_t = \frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{3} \left[(1 + \frac{t}{a}) - \beta(\frac{3}{2} + \frac{t}{a}) \right]$$

$r=b$
$$\sigma_t = -\frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{6} \left[1 - \beta(3 - \frac{2t}{a}) \right]$$

3. Stresses due to the unequal "unjoined" displacements of the spherical end and the cylinder, so-called discontinuity stresses.

The stresses arise from a shear force V_o and a bending moment M_o which restrain the difference in displacement. M_o for a uniform thickness in both sphere and cylinder walls is zero. For the present case in which the thickness varies quite slowly, M_o is found to be less than 2% of V_o and may be neglected except in the immediate vicinity of the junction between sphere and cylinder. In this region, however, the contribution of the discontinuity stress to the total stress is very small and may be neglected.

The "unjoined" displacements are given by the following formulas:

Cylinder

$$\Delta a_c = \frac{pa}{E}\left[\frac{b^2+a^2}{b^2-a^2} - \upsilon\left(\frac{a^2}{b^2-a^2} - 1\right)\right]$$

$$\Delta b_c = \frac{pb}{E}\left[\frac{a^2}{b^2-a^2}(2-\upsilon)\right]$$

Sphere

$$\Delta a_s = \frac{pa}{E}\left[\frac{b^3+2a^3}{2(b^3-a^3)}(1-\upsilon) + \upsilon\right]$$

$$\Delta b_s = \frac{pb}{E}\left[\frac{3a^3}{2(b^3-a^3)}(1-\upsilon)\right]$$

Let

$\Delta a'$ = the portion of $\Delta a_c - \Delta a_s$ taken up by the cylinder

$\Delta a''$ = the portion of $\Delta a_c - \Delta a_s$ taken up by the sphere

and similarly for the outer surface.

Neglecting M_o as discussed above and taking as the average value of the wall thickness that at the intersection of sphere and cylinder (the result obtained is within 2% of the result obtained by considering the thickness variation in detail)

$$\Delta a'' = \Delta a' = \frac{\epsilon V_{oa}}{E} \left(\frac{b+a}{b-a}\right)$$

where $\epsilon = \left[\frac{3}{4}(1-\upsilon^2)\left(\frac{b+a}{b-a}\right)^2 \right]^{1/4}$

$$V_{oa} = \left(\frac{b-a}{b+a}\right) \frac{E}{2\epsilon} (\Delta a' + \Delta a'') = \left(\frac{b-a}{b+a}\right) \frac{E}{2\epsilon} (\Delta a_c - \Delta a_s)$$

similarly $V_{ob} = \left(\frac{b-a}{b+a}\right) \frac{E}{2\epsilon} (\Delta b_c - \Delta b_s)$

The shear force V_o exerts a moment $M_{z'}$ given by

$$M_{z'} = \frac{e^{-\lambda z'}}{\lambda} V_o \sin \lambda z'$$

where $\lambda = \frac{2\epsilon}{b+a}$

z' is the distance along the axis of the cylinder,

$z'=0$ at the intersection of cylinder and sphere.

This expression is valid for all values of z' along the cylinder, and is applicable within 2% accuracy to $z' \sim 1$ cm on the sphere. Beyond 1 cm, the actual moment decreases at a rate slightly less than that given by this expression.

The stresses are calculated from the moment by the relations

$$\sigma_z = \frac{6M_{z'}}{(b-a)^2}$$

$$\sigma_t = \upsilon \sigma_z$$

In these calculations the following symbols and units are used which are not explained in the text:

E = Young's Modulus (psi)

α = Coefficient of linear thermal expansion (/°C)

υ = Poisson's ratio

k = Thermal conductivity (watts/cm°C)

g = volumetric rate of heat generation (watts/cc)

p = pressure (psi)

T = temperature (°C)

Appendix B

Details of Physical Property Tests

Tensile Test

The room temperature tests (item 2 of Table 4) were made with standard specimens in accordance with Federal Specification QQ-M-151a, Section IV, Part 5, Figure 5A. The elevated temperature tests (items 3-5 of Table 4) were made on longitudinal specimens, 0.250" diameter threaded end, cylindrical, 1 inch gage length bars. An external extensomenter which measured crosshead displacement was used to find the yield point.

Notch Bar Impact Test

The tests were made on a standard 240 ft. lb. capacity Charpy Impact Machine at 82° F. Inside notch refers to a notch on the inside surface of the tube, etc. Results from these non-standard specimens can be compared only with other results from similar specimens. This test will be used to compare with irradiated specimens from the first in-pile thimble.

Flattening Test

The specimen was tested in accordance with ASTM specification A210-46 which is a specification for seamless steel boiler tubes. The specification requires that no visible (naked eye) cracks shall appear with the outer diameter of a tube of this dimension reduced to 2.45 in.

Appendix A

Stress Calculations

The assumptions and method of analysis for thermal stress are those of Timoshenko[6]. The stresses due to internal pressure are calculated from the usual formulas[7]. The discontinuity stresses are calculated by Roark's method[8].

1. Stresses due to internal pressure.

Cylinder

At a point in a long circular cylinder closed at the ends with a concentric circular hole and internal pressure only:

At all points $\sigma_\theta = \frac{a^2 p}{b^2-a^2}(1 + \frac{b^2}{r^2})$

$$\sigma_z = \frac{pa^2}{b^2-a^2}$$

At inside surface $\sigma_\theta = p(\frac{b^2+a^2}{b^2-a^2})$

At outside surface $\sigma_\theta = \frac{2pa^2}{b^2-a^2}$

Sphere

At a point in a sphere with a concentric spherical hole and internal pressure only:

At all points $\sigma_t = \frac{pa^3}{2r^3} \cdot \frac{2r^3+b^3}{b^3-a^3}$

At inside surface $\sigma_t = \frac{p}{2} \cdot \frac{2a^3+b^3}{b^3 - a^3}$

At outside surface $\sigma_t = \frac{3pa^3}{2(b^3-a^3)}$

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[6]Timoshenko and Goodier, "Theory of Elasticity", 2nd ed., p.408, et.seq.
[7]Timoshenko and Goodier, "Theory of Elasticity", 2nd ed., p.58 and 359
[8]Roark, "Formulas for Stress and Strain", 3rd ed., p.275

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where σ_θ = circumferential stress in cylinder

σ_z = longitudinal stress in cylinder

σ_t = tangential stress in sphere

p = internal pressure

r = radius at the point

b = outside radius

a = inside radius

2. Stresses due to "gamma" heating in the walls of the vessel.

Because of the mathematical complications, the analysis is simplified by the assumption that the thermal stresses at a point may be approximated by calculating the stress obtained in a cylinder or sphere uniformly heated by gamma heat of the magnitude existing at the point. The validity of this approximation is improved by varying the thickness as has been done in this vessel, so as to maintain essentially uniform thermal gradient in all planes $\perp$ to the axis.

<u>Cylinder</u>

It is assumed that the heating is uniform in a plane $\perp$ to the axis of the cylinder. Then from Timoshenko, the stresses are:

$$\sigma_\theta = \frac{\alpha E}{(1-\upsilon)r^2}\left[\frac{r^2+a^2}{b^2-a^2}\int_a^b Trdr + \int_a^r Trdr - Tr^2\right]$$

$$\sigma_z = \frac{\alpha E}{(1-\upsilon)}\left[\frac{2}{b^2-a^2}\int_a^b Trdr - T\right]$$

The temperature at a point in the cylinder is given by:

$$\nabla^2 T = \frac{1}{r}\frac{d}{dr}\left(\frac{rdT}{dr}\right) = -\frac{g}{k}$$

$$T = -\frac{gr^2}{4k} + C_1 \ln r + C_2$$

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The constants are evaluated for the general case in which a fraction β of the heat generated in the cylinder is transmitted outward through the external surface. The case $\beta = 0$ represents the case for internal cooling only; $\beta = 1$ external cooling only. From the resulting equations, the effects of external cooling may be determined. The boundary conditions for this case are:

$$T = T_o \qquad \text{for } r = a$$

$$\frac{dT}{dr} = -\beta \frac{g}{k} \frac{(b^2-a^2)}{2b} \qquad \text{for } r = b$$

Leading to:

$$T = \frac{g(a^2-r^2)}{4k} + \left[\frac{gb^2}{2k} (1-\beta) + \frac{ga^2}{2k} \beta \right] \ln \frac{r}{a} + T_o$$

Sphere

It is assumed that the heating is uniform along a radius to facilitate the mathematical calculation. The value taken in the actual computation is that at the intersection of the radius and the outer surface so that the stress obtained is somewhat higher than the actual stress. The deviation is significant only for radii near the axis of the cylinder.

$$\sigma_t = \frac{2\alpha E}{1-\upsilon} \left[\frac{2r^3+a^3}{2(b^3-a^3)r^3} \int_a^b Tr^2 dr + \frac{1}{2r^3} \int_a^r Tr^2 dr - \frac{T}{2} \right]$$

The temperature at a point in the sphere is given by

$$\nabla^2 T = \frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = -\frac{g}{k}$$

$$T = -\frac{gr^2}{6k} - \frac{C_1}{r} + C_2$$

The boundary conditions, as before, are

$$T = T_o \qquad \text{for} \qquad r = a$$

$$\frac{dT}{dr} = -\frac{\beta g}{k} \frac{b^3-a^3}{3b^2} \qquad \text{for } r = b$$

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which gives

$$T = \frac{g}{3k}\left(\frac{a^2-r^2}{2} + \frac{b^3}{a} - \frac{b^3}{r}\right) + \frac{\beta g}{3k}(b^3-a^3)\left(\frac{1}{r} - \frac{1}{a}\right) + T_o$$

Integration of the stress equations then leads to the following results:

Cylinder

r=r $$\sigma_\theta = \frac{\alpha E g}{16k(1-\upsilon)}\left\{ \frac{4b^4}{r^2}\frac{r^2+a^2}{b^2-a^2}\ln\frac{b}{a} - 4b^2\ln\frac{r}{a} - a^2 \right.$$

$$-5b^2 - \frac{a^2b^2}{r^2} + 3r^2 + \frac{2\beta}{r^2}\left[(r^2+a^2)\left(-2b^2\ln\frac{b}{a} + b^2 - a^2\right)\right.$$

$$\left.\left. +(b^2-a^2)\left(2r^2\ln\frac{r}{a} + r^2-a^2\right)\right]\right\}$$

r=r $$\sigma_z = \frac{\alpha E g}{8k(1-\upsilon)}\left\{\frac{4b^4}{b^2-a^2}\ln\frac{b}{a} - 3b^2-a^2 + 2r^2-4b^2\ln\frac{r}{a}\right.$$

$$\left. + \beta\left[-4b^2\ln\frac{b}{a} + 2b^2 - 2a^2 - 4(a^2-b^2)\ln\frac{r}{a}\right]\right\}$$

r=a $$\sigma_z = \sigma_\theta = \frac{\alpha E g}{8k(1-\upsilon)}\left[\frac{4b^4}{b^2-a^2}\ln\frac{b}{a} + a^2-3b^2\right.$$

$$\left. +2\beta\left(-2b^2\ln\frac{b}{a} + b^2-a^2\right)\right]$$

r=b $$\sigma_z = \sigma_\theta = \frac{\alpha E g}{8k(1-\upsilon)}\left[\frac{4a^2b^2}{b^2-a^2}\ln\frac{b}{a} - a^2 - b^2\right.$$

$$\left. +2\beta\left(-2a^2\ln\frac{b}{a} + b^2 - a^2\right)\right]$$

Sphere

r=r $$\sigma_t = \frac{\alpha E g}{3k(1-\upsilon)}\left\{\frac{2r^3+a^3}{r^3(b^3-a^3)}\left[\frac{a^2b^3}{3} + \frac{b^6}{3a} - \frac{3b^5}{5} - \frac{a^5}{15}\right]\right.$$

$$+ \frac{1}{r^3}\left[\frac{a^2r^3}{6} - \frac{r^5}{10} + \frac{b^3r^3}{3a} - \frac{b^3r^2}{2} + \frac{a^2b^3}{6} - \frac{a^5}{15}\right]$$

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$$+ \frac{r^2-a^2}{2} + b^3 \left(\frac{1}{r} - \frac{1}{a}\right) + \beta \left[\frac{2r^3+a^3}{r^3} \left(\frac{b^2}{2} - \frac{b^3}{3a} - \frac{a^2}{6}\right) \right.$$

$$\left.\left. + \frac{b^3-a^3}{r^3} \left(\frac{r^2}{2} - \frac{r^3}{3a} - \frac{a^2}{6}\right) - (b^3-a^3)\left(\frac{1}{r}-\frac{1}{a}\right)\right]\right\}$$

r=a $$\sigma_t = \frac{\alpha E g}{k(1-\upsilon)} \left[\frac{1}{b^3-a^3} \left(\frac{a^2b^3}{3} + \frac{b^6}{3a} - \frac{3b^5}{5} - \frac{a^5}{15}\right)+\beta\left(\frac{b^2}{a} - \frac{b^3}{3a} - \frac{a^2}{6}\right)\right]$$

r=b $$\sigma_t = \frac{\alpha E g}{3k(1-\upsilon)} \left\{\frac{2b^3+a^3}{b^3(b^31a^3)} \left[\frac{a^2b^3}{3} + \frac{b^6}{3a} - \frac{3b^5}{5} - \frac{a^5}{15} \right]\right.$$

$$+ \frac{9b^2}{10} - \frac{a^2}{6} - \frac{2b^3}{3a} - \frac{a^5}{15b^3} + \beta\left(\frac{b^2}{2} - \frac{3a^2}{2} + \frac{a^3}{b}\right)$$

These equations may be simplified for the accuracy required in this problem by introducing the wall thickness $t = b-a$ and calculating the series expansion of the preceding equations. The simplified equations which follow are accurate to approximately 2% for $t/a < 0.2$.

Cylinder

r=a $$\sigma_z = \sigma_\theta = \frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{3}\left[\left(1 + \frac{t}{2a}\right) - \beta\left(\frac{3}{2} + \frac{t}{2a}\right)\right]$$

r=b $$\sigma_z = \sigma_\theta = -\frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{6} \left[1-\beta\left(3 - \frac{t}{a}\right)\right]$$

Sphere

r=a $$\sigma_t = \frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{3} \left[\left(1 + \frac{t}{a}\right) - \beta\left(\frac{3}{2} + \frac{t}{a}\right)\right]$$

r=b $$\sigma_t = -\frac{\alpha E g}{k(1-\upsilon)} \frac{t^2}{6} \left[1 - \beta\left(3 - \frac{2t}{a}\right)\right]$$

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3. Stresses due to the unequal "unjoined" displacements of the spherical end and the cylinder, so-called discontinuity stresses.

The stresses arise from a shear force V_o and a bending moment M_o which restrain the difference in displacement. M_o for a uniform thickness in both sphere and cylinder walls is zero. For the present case in which the thickness varies quite slowly, M_o is found to be less than 2% of V_o and may be neglected except in the immediate vicinity of the junction between sphere and cylinder. In this region, however, the contribution of the discontinuity stress to the total stress is very small and may be neglected.

The "unjoined" displacements are given by the following formulas:

Cylinder

$$\Delta a_c = \frac{pa}{E}\left[\frac{b^2+a^2}{b^2-a^2} - \upsilon\left(\frac{a^2}{b^2-a^2} - 1\right)\right]$$

$$\Delta b_c = \frac{pb}{E}\left[\frac{a^2}{b^2-a^2}(2-\upsilon)\right]$$

Sphere

$$\Delta a_s = \frac{pa}{E}\left[\frac{b^3+2a^3}{2(b^3-a^3)}(1-\upsilon) + \upsilon\right]$$

$$\Delta b_s = \frac{pb}{E}\left[\frac{3a^3}{2(b^3-a^3)}(1-\upsilon)\right]$$

Let

$\Delta a'$ = the portion of $\Delta a_c - \Delta a_s$ taken up by the cylinder

$\Delta a''$ = the portion of $\Delta a_c - \Delta a_s$ taken up by the sphere

and similarly for the outer surface.

Neglecting M_o as discussed above and taking as the average value of the wall thickness that at the intersection of sphere and cylinder (the result obtained is within 2% of the result obtained by considering the thickness variation in detail)

$$\Delta a'' = \Delta a' = \frac{\epsilon V_{oa}}{E} \left(\frac{b+a}{b-a}\right)$$

$$\text{where } \epsilon = \left[\frac{3}{4}(1-\upsilon^2)\left(\frac{b+a}{b-a}\right)^2 \right]^{1/4}$$

$$V_{oa} = \left(\frac{b-a}{b+a}\right) \frac{E}{2\epsilon} (\Delta a' + \Delta a'') = \left(\frac{b-a}{b+a}\right) \frac{E}{2\epsilon} (\Delta a_c - \Delta a_s)$$

$$\text{similarly } V_{ob} = \left(\frac{b-a}{b+a}\right) \frac{E}{2\epsilon} (\Delta b_c - \Delta b_s)$$

The shear force V_o exerts a moment $M_{z'}$ given by

$$M_{z'} = \frac{e^{-\lambda z'}}{\lambda} V_o \sin \lambda z'$$

$$\text{where } \lambda = \frac{2\epsilon}{b+a}$$

z' is the distance along the axis of the cylinder,

$z'=0$ at the intersection of cylinder and sphere.

This expression is valid for all values of z' along the cylinder, and is applicable within 2% accuracy to $z' \sim 1$ cm on the sphere. Beyond 1 cm, the actual moment decreases at a rate slightly less than that given by this expression.

The stresses are calculated from the moment by the relations

$$\sigma_z = \frac{6M_{z'}}{(b-a)^2}$$

$$\sigma_t = \upsilon \sigma_z$$

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In these calculations the following symbols and units are used which are not explained in the text:

E = Young's Modulus (psi)

α = Coefficient of linear thermal expansion ($/^{\circ}C$)

ν = Poisson's ratio

k = Thermal conductivity ($watts/cm^{\circ}C$)

g = volumetric rate of heat generation (watts/cc)

p = pressure (psi)

T = temperature ($^{\circ}C$)

Appendix B

Details of Physical Property Tests

Tensile Test

The room temperature tests (item 2 of Table 4) were made with standard specimens in accordance with Federal Specification QQ-M-151a, Section IV, Part 5, Figure 5A. The elevated temperature tests (items 3-5 of Table 4) were made on longitudinal specimens, 0.250" diameter threaded end, cylindrical, 1 inch gage length bars. An external extensomenter which measured crosshead displacement was used to find the yield point.

Notch Bar Impact Test

The tests were made on a standard 240 ft. lb. capacity Charpy Impact Machine at 82° F. Inside notch refers to a notch on the inside surface of the tube, etc. Results from these non-standard specimens can be compared only with other results from similar specimens. This test will be used to compare with irradiated specimens from the first in-pile thimble.

Flattening Test

The specimen was tested in accordance with ASTM specification A210-46 which is a specification for seamless steel boiler tubes. The specification requires that no visible (naked eye) cracks shall appear with the outer diameter of a tube of this dimension reduced to 2.45 in.