



# **NAVAL POSTGRADUATE SCHOOL**

**MONTEREY, CALIFORNIA**

## **THESIS**

**A DSP APPROACH TO INTERPOLATE FAST MANEUVER  
PROFILES OF THE NASA LUNAR RECONNAISSANCE  
ORBITER**

by

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September 2022

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THE NASA LUNAR RECONNAISSANCE ORBITER**

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## ABSTRACT

The NASA Lunar Reconnaissance Orbiter (LRO) began using two star trackers as a primary means of attitude determination following the imminent failure of its inertial measurement unit in 2018. New types of maneuver trajectories that steer the star trackers around the keep-out regions are needed to ensure stability of the attitude determination filter. The new mode of operation necessitates the need for the LRO conventional quaternion error feedback controller to operate as an attitude tracking system opposed to a traditional attitude regulator. However, the LRO is not equipped to support the transition because its command buffers are not large enough to store the time-varying attitude commands. This thesis explores and recommends a solution for NASA to install a uniquely designed digital signal processing (DSP) based interpolation filter that enables the time-varying attitude command data to be compressed to a tolerable size. The recommended filter is a practical implementation of a spline interpolator that leverages functionality of the existing flight software. A Kalman Filter is used to optimize the filter inputs to minimize interpolation error of the command trajectories. High-fidelity simulations were conducted on the recommended filter to illustrate the overall performance and 50:1 data compression (or more) can be achieved. The results of this thesis are currently being implemented by NASA to facilitate a transition of FastMan to a fully operational status.

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## LIST OF ACRONYMS AND ABBREVIATIONS

|         |                                                   |
|---------|---------------------------------------------------|
| AAS     | American Astronautical Society                    |
| ACS     | attitude control system                           |
| AttMan  | attitude maneuver planning utility                |
| CIC     | cascaded integrator-comb                          |
| CRaTER  | cosmic ray telescope for the effects of radiation |
| DFII    | direct form II                                    |
| DLRE    | diviner lunar radiometer                          |
| DSP     | digital signal processing                         |
| ECI     | Earth-centered inertial                           |
| FastMan | Lunar Reconnaissance Orbiter fast maneuver        |
| FT      | Fourier transform                                 |
| Hi-Fi   | high fidelity                                     |
| IFT     | inverse Fourier transform                         |
| LAMP    | Lyman alpha mapping project                       |
| LEND    | lunar exploration neutron detector                |
| LOLA    | lunar orbiter laser altimeter                     |
| LRO     | Lunar Reconnaissance Orbiter                      |
| LROC    | Lunar Reconnaissance Orbiter camera               |
| MATLAB  | matrix laboratory                                 |
| MIMU    | miniature inertial measurement unit               |
| Mini-RF | miniature radio frequency                         |
| NASA    | National Aeronautics and Space Administration     |
| OCS     | orbit coordinate system                           |
| TRACE   | Transition Region and Coronal Explorer            |
| ZOH     | zero order hold                                   |

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# I. INTRODUCTION

## A. LRO MISSION

NASA launched the Lunar Reconnaissance Orbiter (LRO) on June 18, 2009, to explore a deeper understanding of the moon. The satellite is equipped with seven scientific instruments that enable a variety of research capabilities (Figure 1). The following descriptions of the instruments are from the NASA webpage for the LRO [1]. The Cosmic Ray Telescope for the Effects of Radiation (CRaTER) investigates the atmospheric radiation environment of the moon to discover its effects on humans as well as aid in the development of emerging technologies. The Diviner Lunar Radiometer (DLRE) conducts thermal mapping of the lunar surface and has previously identified cold traps where ice deposits could reside. The Lyman Alpha Mapping Project (LAMP) conducts ultraviolet mapping of the lunar surface and has provided imaging of the permanently shadowed regions of the moon. The Lunar Exploration Neutron Detector (LEND) maps out hydrogen distribution zones and takes radiation measurements to support future manned missions.

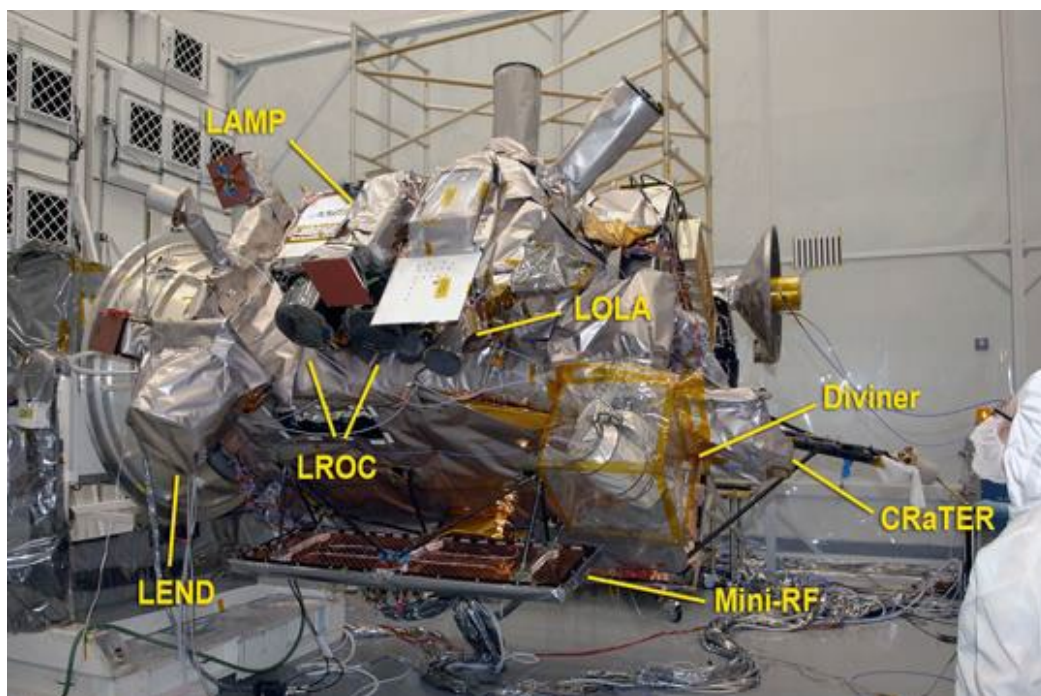
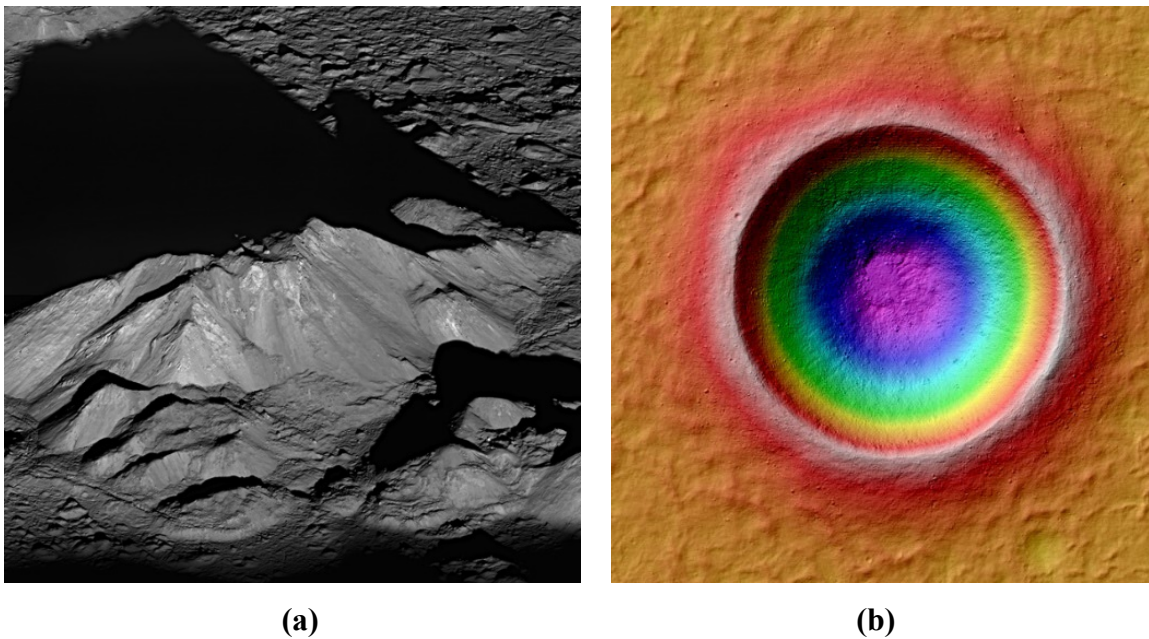


Figure 1. Instrument suite of the Lunar Reconnaissance Orbiter. Source: [2].

The Lunar Orbiter Laser Altimeter (LOLA) is capable of high-resolution 3D mapping and has produced an accurate lunar surface model to include permanently illuminated and permanently shadowed regions of the moon. The Lunar Reconnaissance Orbiter Camera (LROC) can capture one meter resolution panchromatic images of the lunar surface to identify potential resources, hazards, and landing site selections. The cameras are also capable of imaging in color and ultraviolet. Previous images are shown in Figure 2. Many of the media files are available on the LRO webpage. Lastly, the Miniature Radio Frequency (Mini-RF) instrument is a synthetic aperture radar that can image shadowed regions and probe surface and sub-surface rock content for ice deposits.



(a) Tycho crater during sunrise on June 10, 2011; (b) topographic image of the 2.2 km diameter Linné crater.

Figure 2. Images collected from the LROC instrument. Source: [1].

The LRO has achieved historic scientific results that have broadened our understanding of the moon. Some of the more significant findings of the LRO include measuring the coldest temperature in our solar system, discovering recent lunar impact craters, improving lunar age dating, discovering potential ice deposits, and creating a high-resolution 3D map of the moon [1]. The LRO continues to produce important data that may be used for future human and robotic missions to the moon and possibly other planets.

## B. MANEUVERING IN SPACE WITH STAR TRACKERS

The LRO was originally designed to conduct frequent maneuvers off the lunar nadir<sup>1</sup>, which is its normal pointing attitude. Its attitude control system (ACS) used a miniature inertial measurement unit (MIMU), which is a sensor that measures the satellite rotational rate during maneuvers. In 2018, the LRO MIMU began to show signs of imminent failure. Due to its decline, NASA decided to use the MIMU for contingencies. Consequently, the LRO shifted to using its two star trackers as a means of measuring the satellite rotational rate [3]. The star trackers, mounted on top of the LRO, consist of cameras that image and measure star fields in deep space to determine the attitude. These measurements can be differentiated over time to estimate the spacecraft rotational rate. The two star trackers are currently the primary means of feedback attitude control for the LRO.

To maintain an accurate and clear image of the stars, the star trackers must avoid occultations caused by the earth, moon, and sun [4]. In addition, the light reflected from the earth and moon can saturate the star trackers and prevent them from viewing faint stars in deep space. Therefore, the earth and moon limb must be avoided by  $15^\circ$  from the fields of view of the star trackers. Due to the intensity of light of the sun, it must be avoided by  $25^\circ$  from the fields of view of the star trackers. The seven instruments on the LRO must also avoid the sun to protect their sensors, especially the LRO camera. For the instruments to sufficiently avoid the sun, the positive z-axis must continuously be at an angle greater than  $63^\circ$  from the sun, even while performing maneuvers. Occultation avoidance is a major consideration when planning LRO maneuvers.

When the LRO was operating with the MIMU, the only occultation restriction was to ensure an angle greater than  $63^\circ$  between the positive z-axis and the sun. But now that the star trackers are primarily used, there are more stringent constraints placed on maneuvers. Traditionally, satellites perform maneuvers about an eigen-axis, which represents a direct, shortest angular distance from the current orientation to the target orientation. This type of maneuver is often no longer feasible for the LRO due to the star tracker occultation avoidance requirements. Alternatively, a new type of maneuver was

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<sup>1</sup> The direction pointing from the spacecraft toward the center of the moon.

designed for the LRO that incorporates occultation avoidance with shortest-time maneuvers.

Shortest-time maneuvers were first employed on the NASA Transition Region and Coronal Explorer (TRACE) satellite in August 2010 [5]. These maneuvers rely on algorithms that compute a time-optimal trajectory to the final attitude based on the spacecraft inertia and the available control authority. Although these maneuvers require more distance to traverse than traditional, eigen-axis slews, they can decrease the time it takes to maneuver by up to 30 to 40 percent [4].

The shortest-time maneuvers specific to the LRO are referred to as Fast Maneuvers (FastMan). The maneuver design algorithm computes the time-optimal path to include star tracker occultation avoidance constraints for the LRO.

The LRO ACS was originally designed as a regulator control system, which is a feedback control system that works to zero the error to a given attitude target. However, FastMan maneuvers must use a tracking control system to follow a particular attitude path. This mode of operation requires thousands of input commands to complete a maneuver. The LRO has a limited command buffer data storage, which impedes on its ability to conduct FastMan maneuvers. The data buffers do not have the capacity to store thousands of commands. The goal of this thesis is to alleviate the data storage limitations of the LRO so it can conduct frequent FastMan maneuvers in support of various science objectives.

### **C. THESIS OBJECTIVES AND SCOPE**

This research consists of a series of phases in which the ultimate goal is to implement an interpolation filter within the LRO flight software to reduce the number of input commands required for FastMan. Figure 3 is a roadmap to the final interpolation filter installation.

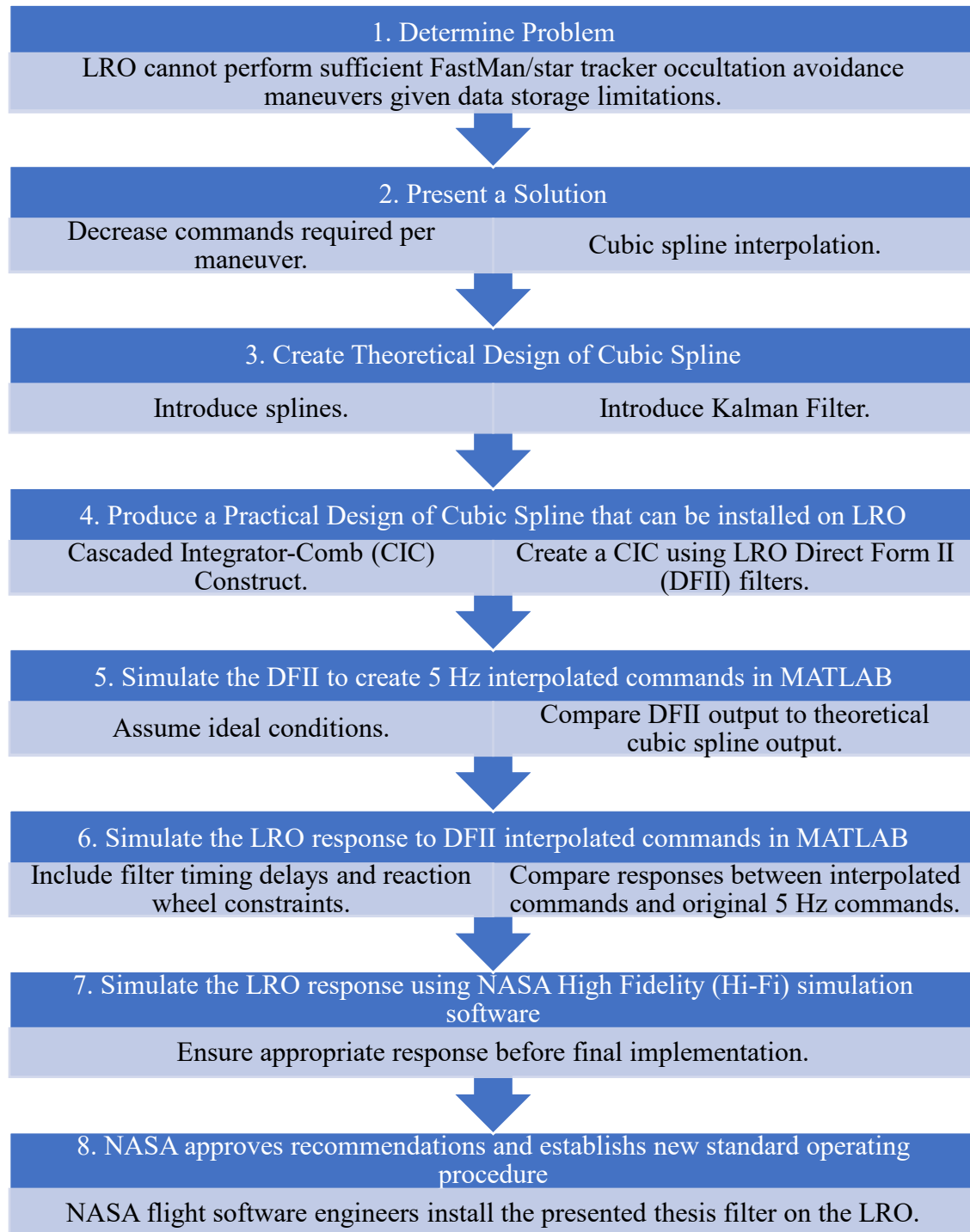


Figure 3. Roadmap of the thesis process from the identification of the problem to the final implementation and NASA approval

Since the LRO data buffers do not have sufficient storage capacity for FastMan, there have been several lost opportunities for science collection missions. Some missions could still proceed if there was no star tracker occultations for the given maneuver. However, many missions do have star tracker constraints that must be accounted for, and thus necessitate FastMan type maneuvers.

This thesis explores multi-rate DSP filter solutions that interpolate ACS input data to reduce the required number of commands for FastMan maneuvers. The reduction in data needed for maneuvering will enable the LRO to conduct FastMan at an appropriate operational cadence. This will increase science productivity from the only U.S. lunar orbiting satellite.

#### **D. PUBLICATIONS BASED ON THIS WORK**

Portions of this thesis was published by the American Astronautical Society (AAS) as the paper “A Practical Interpolating Pre-Filter for Closed-Loop Attitude Guidance” [6], which was written concurrently by the same author. The research contained herein was also presented at the AAS/AIAA Astrodynamics Specialist Conference on August 11, 2022, in Charlotte, NC. One patent [7] regarding the use of a practical cubic spline interpolation filter to alleviate spacecraft storage limitations for tracking-based commands has been submitted for this research and is listed in the references. A second patent regarding the optimization of the filter coefficients using the Kalman Filter will be submitted shortly prior to the publication of this thesis.

## II. LRO OPERATIONAL CAPACITY

The capabilities and limitations of the LRO ACS are described in this chapter. This chapter also introduces maneuver planning methods and discusses how the ACS currently processes quaternion commands. The background information for the ACS is important to understand because this thesis seeks to develop a solution to compress FastMan data files with minimal modifications to the ACS software.

### A. LIMITING FACTORS FOR MANEUVERABILITY

The ACS on the Lunar Reconnaissance Orbiter is not operating at its fullest capacity. Since the degradation of its MIMU in 2018, the LRO has been limited in its maneuvers due to occultation constraints and the relatively small capacity of its command storage buffers, which limits the use of FastMan [8]. When the MIMU was fully operational, the ACS acted as an error regulating control system, where the input was the target orientation for the spacecraft. The slews conducted before 2018 only required a few input commands to orient the spacecraft towards the target and then transition back to nadir after the science collect. With Fastman, the LRO ACS must be operated as an attitude tracking system that regulates to a series of closely spaced attitude waypoints [8]. But, now that the LRO only uses star trackers for attitude determination, there are many slews that cannot be completed in the traditional manner [9]. FastMan maneuvers must be conducted instead because they compensate for the star tracker constraints. However, the LRO does not have the storage capacity to hold many FastMan data files.

As mentioned in Chapter I, FastMan is the shortest-time maneuver specific to the LRO. Karpenko et al. [11] demonstrated how the relationship between inertial and control authority can be used to reorient a spacecraft in a time-optimal manner. This concept leverages an agility envelope (Figure 4) that expresses the achievable angular rotation accelerations and rates of a spacecraft in all directions [10]. The LRO FastMan algorithm considers the spacecraft dynamics and control constraints, occultation avoidance constraints, and maneuver boundary conditions [2]. The implementation of FastMan is a significant benefit for the LRO (as well as other spacecraft) because it can reduce the

maneuver time by up to 50% [5], which also lowers the power consumption of the ACS and improves performance with respect to thermal and power constraints.

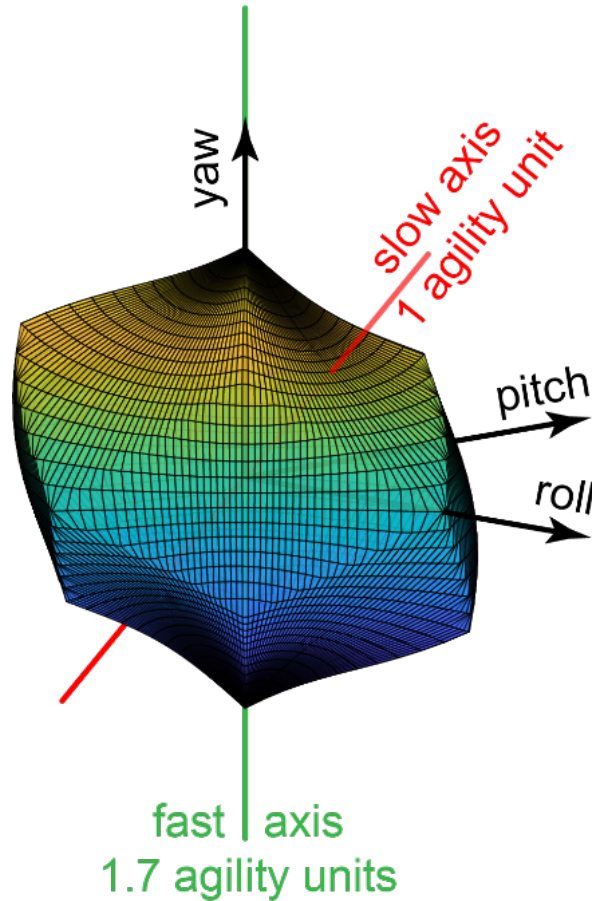


Figure 4. Example of the LRO agility envelope, which represents the maximum achievable maneuver acceleration and rate in all directions based on the spacecraft inertia and reaction wheels. Source: [10].

LRO maneuvers are calculated on the ground by the NASA Attitude Maneuver Planning Utility (AttMan) [9]. The software will process slew requests (Figure 5a) and output a series of times with associated command keywords and desired orientations (four quaternion values) while accounting for the constraints of the LRO. A modified version of AttMan is used to process FastMan profiles [2]. The first several commands of a typical FastMan slew are shown in Figure 5b.

```

1  <begin_event>
2  event = roll_slew,
3  time = 2022119.122852,
4  angle = -53.576,
5  duration = 109,
6  settle = 120,
7  <end_event>
8
9  <begin_event>
10 event = FastMan_return,
11 duration = 5,
12 settle = 0,
13 fastman_file = 2022119_122852_return_LROC_slew.txt,
14 <end_event>

```

(a)

```

25 2022-063-10:38:56, roll_continue, OFF, -0.59451058, -0.20882170, 0.16263683, 0.75927592
26 2022-063-10:38:58, roll_continue, OFF, -0.59419600, -0.20881997, 0.16264777, 0.75952027
27 2022-063-10:39:00, roll_continue, OFF, -0.59356609, -0.20881480, 0.16266177, 0.76001107
28 2022-063-10:39:02, roll_continue, OFF, -0.59261847, -0.20880517, 0.16268331, 0.76074825
29 2022-063-10:39:04, roll_continue, OFF, -0.59136237, -0.20879651, 0.16270127, 0.76172361
30 2022-063-10:39:06, roll_continue, OFF, -0.58983723, -0.20881257, 0.16268408, 0.76290448
31 2022-063-10:39:08, roll_continue, OFF, -0.58810593, -0.20889155, 0.16258677, 0.76423902
32 2022-063-10:39:10, roll_continue, OFF, -0.58623255, -0.20907139, 0.16235829, 0.76567639
33 2022-063-10:39:12, roll_continue, OFF, -0.58426477, -0.20937893, 0.16195350, 0.76718068
34 2022-063-10:39:14, roll_continue, OFF, -0.58222912, -0.20982902, 0.16134871, 0.76873118
35 2022-063-10:39:16, roll_continue, OFF, -0.58013572, -0.21043053, 0.16055338, 0.77031432
36 2022-063-10:39:18, roll_continue, OFF, -0.57798602, -0.21119317, 0.15961164, 0.77191562
37 2022-063-10:39:20, roll_continue, OFF, -0.57577891, -0.21213104, 0.15859290, 0.77351623

```

(b)

(a) Slew requests to be processed by AttMan; (b) partial list of timestamped commands and quaternion values output by AttMan to fulfill slew request.

Figure 5. Typical maneuver created by AttMan.

For a satellite to perform a fast maneuver, its ACS must operate as a tracking system where the input varies with time [8]. These maneuvers require relatively large data files because the ACS takes commands at a rate of five Hz, or every 0.2 s. An average LRO science collection maneuver, which takes longer than 1000 s to complete, requires over 5000 commands. Although modern technology can store and process giga-bits of data, the LRO cannot. It was launched in 2009 and took several years to build. The technology onboard is now almost two decades old and is restricted in size and weight due to the

constraints imposed by the space environment. Due to the limited data processing and storage capacity of the LRO, it can only store about three FastMan maneuver files, which is not sufficient for day-to-day operations.

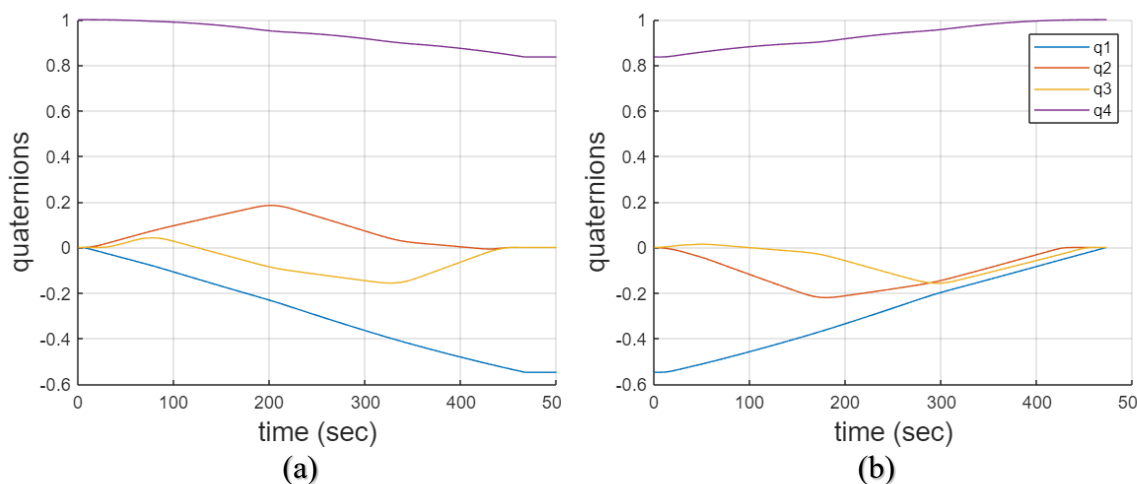
Interpolation is one solution to reduce the data required for FastMan maneuvers. Later chapters will introduce interpolation methods that can compress the amount of data to a comparatively small set of commands with tolerable error, which is much more palatable for the LRO processing system.

## **B. SPACECRAFT REFERENCE FRAMES**

The LRO attitude quaternions represent the orientation of the spacecraft with respect to an appropriate frame. In normal operating mode, the spacecraft is pointing in the lunar nadir direction, or pointing towards the center of the moon. The ACS coordinate systems reference documentation [12] provides an overview of four coordinate systems that the LRO establishes to include an earth-centered inertial (ECI) frame, an orbit frame, a body frame, and a mechanical frame. The inertial frame aligns with the earth coordinate system where the +z-axis is directed from the center of the earth to the north pole, the +x-axis is on the equatorial plane directed toward the first point of Aries, and the +y-axis is found by the right-hand rule for an orthogonal system. The Orbit Coordinate System (OCS) frame represents the main mission attitude and is defined by the orbital plane. In the OCS frame the +z-axis is directed from the orbit plane to the center of the moon, the +y-axis is in the direction of the normal to the orbit plane, and the +x-axis is found using the right-hand rule. The OCS orientation is continuously measured by the ACS through transformation of the inertial frame by a given rotation matrix. The body and mechanical frames are coordinate systems that align with the spacecraft structure. These frames change in conjunction with spacecraft maneuvers and their axes are measured by the offset angles between the spacecraft orientation and the OCS.

This thesis will only reference the OCS frame, given that it is the primary mission coordinate system. The quaternion values measured in the OCS frame may be referred to as offset quaternions, because they represent the “offset” angle that the spacecraft is oriented away from lunar nadir as it orbits the moon.

An example offset quaternion set for a FastMan slew of the LRO is shown in Figure 6. The four curves are the offset quaternion values that represent the spacecraft orientation. There are five discrete points for each second of the maneuver to match the five Hz update rate of the LRO attitude controller. The angular rate of the LRO is limited to a maximum of  $0.13^\circ$  per s [8], which is why the quaternion curves are relatively smooth, and also why maneuvers are very time consuming to perform. Smoothness of the quaternion profiles is important to note because interpolation is most accurate for curves with a relatively constant acceleration.



(a) Slew to target; (b) return slew to nadir.

Figure 6. Example of FastMan offset quaternions for the LRO.

### C. ATTITUDE CONTROL SYSTEM

The LRO ACS operates as a second order controller. Karpenko et al. [8] discusses a spacecraft quaternion feedback ACS, shown in Figure 7. Like Figure 7, the controller on the LRO was designed as a zero-error regulator where a target angle was input, and the system drove the error between the current and the target orientations to zero. Here, the term  $J_e$  represents the spacecraft inertia about the rotational eigenaxis. Traditional LRO ACS commands have a constant input signal,  $\Phi_{cmd}(t)$ , which represents quaternion values of the target orientation. The input command passes through a zero-order hold (ZOH) filter

at a rate of  $\delta_T = 0.2s$ . The ZOH feeds the closed loop system to drive the spacecraft true position to the command angle,  $\Phi(t)$ , where  $\omega(t)$  is the angular rate of spacecraft.

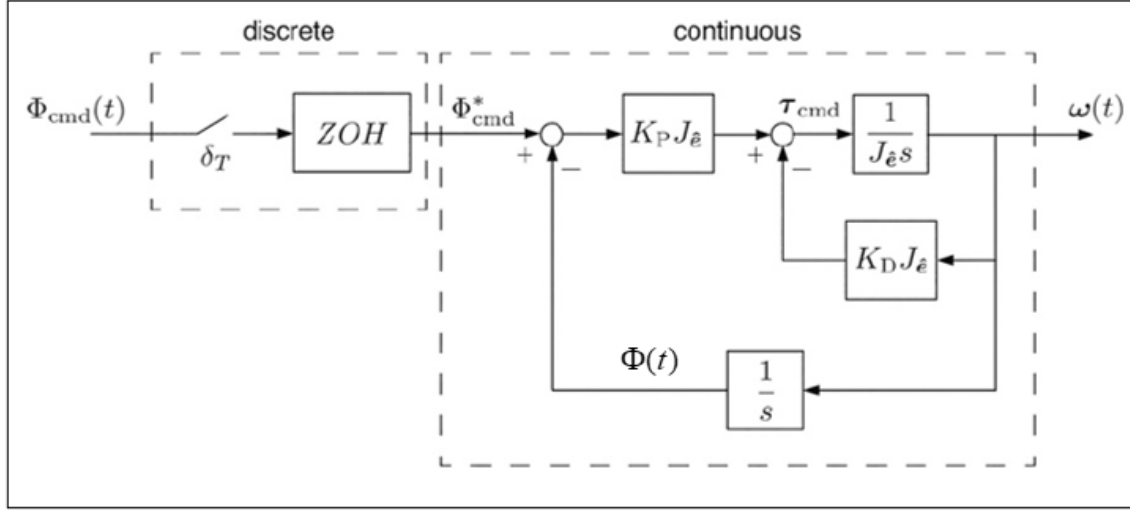


Figure 7. Block diagram of a second order, discrete-continuous time attitude control system with quaternion feedback. Adapted from [8].

The ZOH filter will continuously hold the last input command at the controller input, so theoretically, only one command is required to slew the spacecraft to a desired position. However, as mentioned previously, a FastMan slew generally requires thousands of commands to implement, where each command takes up 20 B of storage [13]. The ACS has two data memory buffers to store the commands that are created by AttMan. Figure 8 depicts a graphic of how the buffers interoperate.

Each buffer can store 100 kB [13] of data that may include one or more different slews, or just a fraction of one slew. For example, buffer 1 may store the first two thirds of the commands for a slew while buffer two stores the last third. When buffer 1 has exhausted its cache of commands, buffer 2 will then begin feeding commands to the ACS. Only one buffer feeds at a time. While buffer 2 feeds, buffer 1 is cleared and updated with a new set of commands. Likewise, when buffer 2 exhausts its commands, it will clear and update as buffer 1 takes over again. Practically, however, the buffers cannot be continually updated because the LRO needs to have line of sight with the ground stations and the LRO needs

to have priority over other transmissions that take place concurrently. Thus, there is a limited amount of time per day where maneuver data files can be uploaded to the LRO.

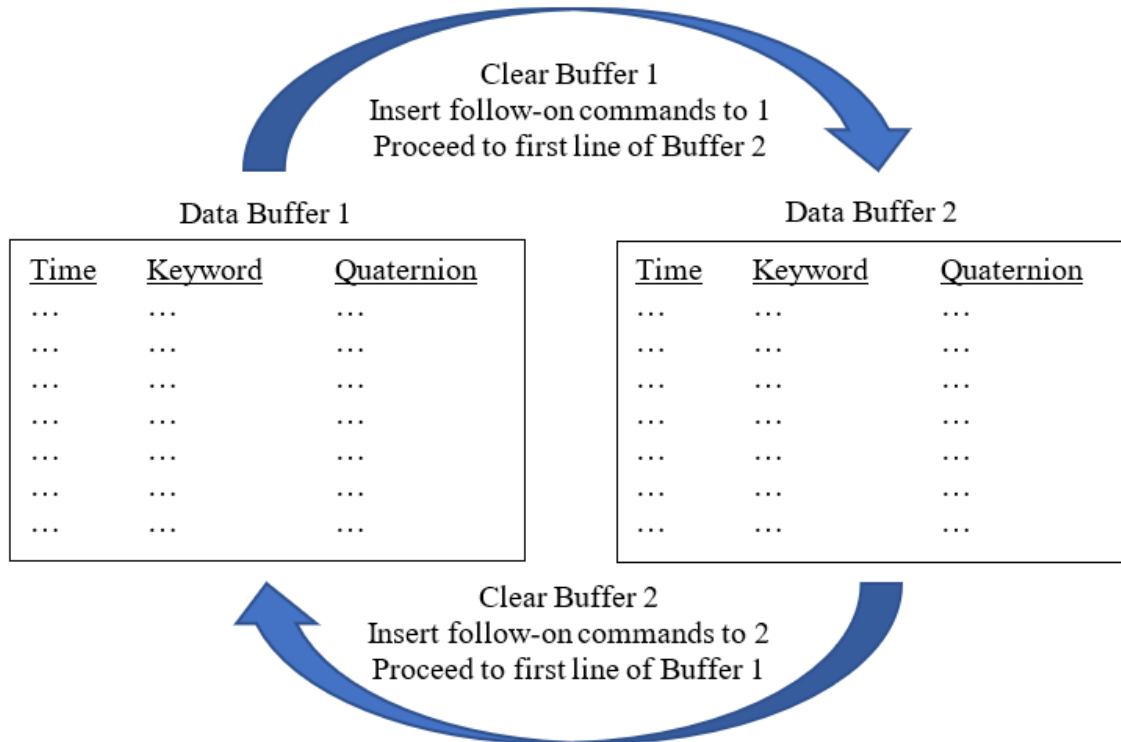


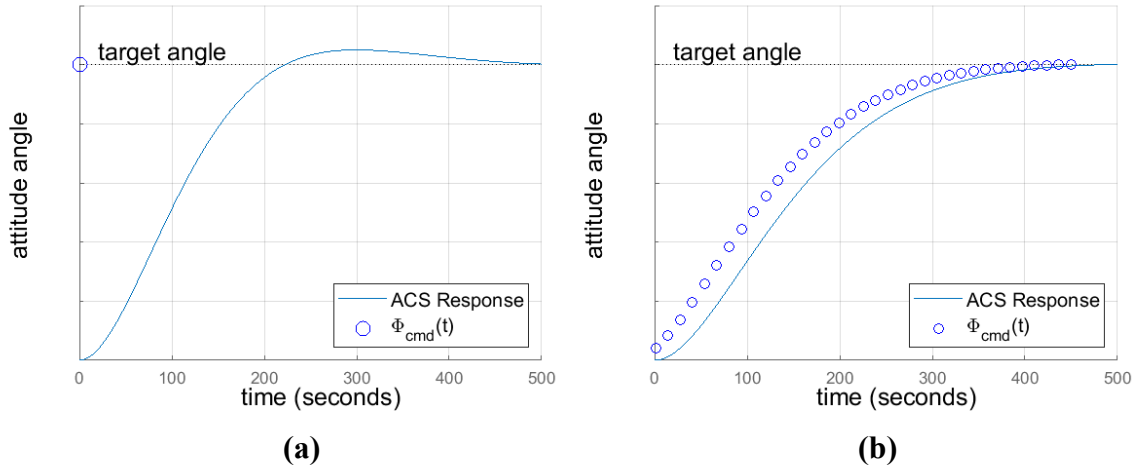
Figure 8. Graphical representation of the LRO ACS command data storage buffers. The buffers alternate feeding commands to the ACS. Each buffer can store 100 kB of data

Ideally, NASA would like to store a week of maneuvers while conducting seven maneuvers per day, which comes out to about 49 maneuvers. Assume a given FastMan slew requires 3000 commands to complete where each command consists of 20 B of data. This maneuver alone would consume 60 kB of the 200 kB combined storage. The LRO buffers can only hold three of these maneuvers, which is drastically short of the 49-maneuver goal. The interpolation techniques presented later reduce the data required per maneuver so that there is plenty of storage to meet the NASA goal. An added advantage of interpolation is that it also reduces the transmission time required for uploading maneuver files, which is important given the short transmission windows available to the LRO. The

reduced transmission time also yields more time for the transmitting ground antennas to focus on other priorities.

#### D. REGULATING VS TRACKING CONTROL SYSTEMS

When a command is issued to a second order system, such as the one described in the previous section, the system will have a transient response with a certain percentage of overshoot. Traditionally, the desired target offset angle would be the command issued to the ACS. The new approach to facilitate FastMan and star tracker occultations is a tracking-based approach (Figure 9b), meaning several step commands are issued to achieve the target offset angle.

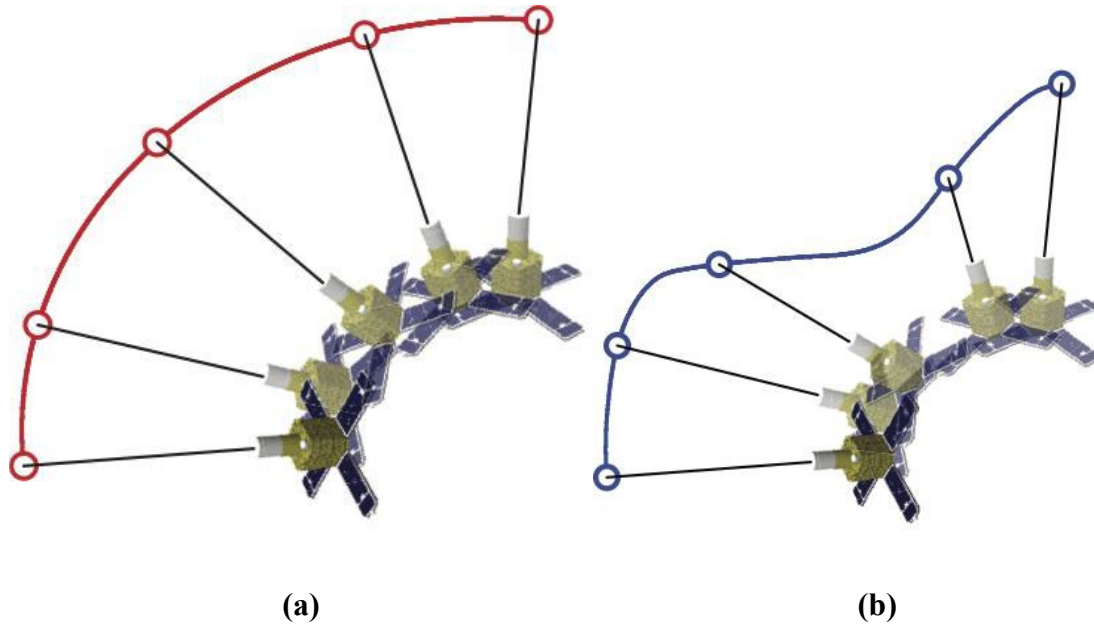


(a) Step command for a regulator system; (b) tracking-based approach using several step commands.

Figure 9. Input responses of a second order system.

Figure 9 represents a second order system response to a step command for plot (a) and a tracking approach for plot (b). Referring to Figure 7, a step command consists of inputting a single command to the system where the ZOH filter holds the value indefinitely and the feedback loop corrects the error between the spacecraft attitude and the command angle. There is generally some percentage of overshoot that results from a step command, though it can be regulated by varying the gain. It is important to note that a spacecraft

trajectory cannot be altered for a step command because the ACS response is unique to the system configuration. A step command is not a viable option for the LRO due to required star tracker occultation avoidance. Since the transitory path of a step command cannot be altered, the path may expose an obstacle to the field of view of the star trackers, which is unacceptable. Figure 10 provides an illustration of traditional and optimal-time maneuvers.



(a) Traditional slew using a regulator; (b) shortest-time slew using a tracking-based or path approach.

Figure 10. Traditional maneuver vs. optimal-time maneuver. Source: [2].

A tracking-based approach consists of issuing periodic commands to gradually bring the spacecraft to the desired attitude. Figure 9 plot (b) shows an example of this. The advantage of a tracking-based approach is that an operator can specifically choose how the spacecraft maneuvers toward the target, which is essential for star tracker occultation avoidance and the FastMan algorithm. For example, the shortest-time slew in Figure 10b is not achievable by a single step command (Figure 10a), but it can be achieved using a tracking-based approach. Since the LRO ACS operates at 5 Hz, a tracking-based approach would require five commands per second, which coincides with the data storage buffer

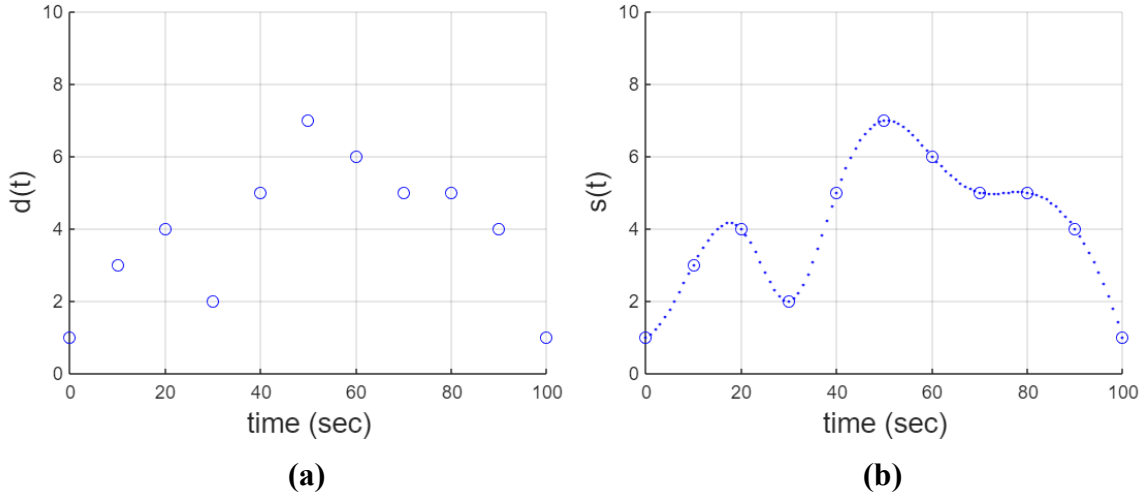
issue mentioned in the previous section. Interpolation can reduce the number of commands required for a path approach.

Due to previous ACS degradations, the LRO is not currently operating at its maximum capacity and many science activities cannot be performed. The LRO can only conduct a minimal number of maneuvers due to star tracker constraints, its heritage data storage and processing power, and the large data files required to conduct FastMan maneuvers. These limitations have resulted in missed science collection opportunities. One solution to increase the LRO operational capacity is to implement an attitude command interpolation filter. Interpolation can reduce the required data storage per command, reduce the maneuver processing power, and reduce the costs required per maneuver. The next chapter will introduce an interpolation solution that will improve the LRO capabilities to support future lunar science.

### III. SPLINE INTERPOLATION

#### A. POLYNOMIAL SPLINES

This thesis explores solutions for efficient data compression of the LRO attitude commands. One such solution is to use the method of splines. Splines are piecewise polynomials of varying degrees that can be used to smoothly connect a set of data points, which is known as interpolation. Interpolation is used to estimate the values of points within the ‘gaps’ of a set of input data. This process can significantly reduce the amount of input data required to perform a FastMan maneuver for the LRO (as well as other satellites). Figure 11, for example, shows an arbitrary input sequence spaced by a 10 s interval that is passed through a cubic spline filter. The output consists of interpolated data for each second of the event.



(a) Data input; (b) cubic spline filter output.

Figure 11. Example of Cubic spline interpolation given an input signal.

Although there are several techniques that can be used to interpolate data, splines are especially attractive because they limit the amount of contiguous data needed for interpolation. Other interpolation methods, including Chebyshev interpolation, require processing of all input data for each interpolated point. The cubic spline method (explained

in Section III.C) only requires four sequential data points to process each interpolated point, which lowers the processing required for interpolation.

This chapter focuses on cubic splines because they yield constant jerk motion and are the most efficient spline for interpolating the LRO quaternion curves. The following sections refer at length to the article by Unser [14] to discuss spline derivations and how to use them to interpolate data.

## B. SPLINE DERIVATION

As stated above, splines are piecewise polynomials of varying degrees. The most basic spline is the zero-order spline given by Equation (1) and is a rectangular pulse.

$$\beta^0(t) = \text{rect}(t) \quad (1)$$

Each successive order spline is formed by convolving the zero-order spline  $n+1$  times given by Equation (2). For example, a third-order spline consists of the convolution of four zero-order splines.

$$\beta^n(t) = \underbrace{\beta^0 * \beta^0 * \dots * \beta^0(t)}_{(n+1) \text{ times}} \quad (2)$$

Figure 12 shows a graphical representation of an individual spline basis function up to the third order where each spline is formed using Equation (2). Eventually, coefficients will be introduced, and it will be shown how they are convolved with the spline basis functions to calculate the interpolated signal. The convolution of the basis function and the coefficient data will produce a series of “weighted” basis functions at intervals consistent with the sampled data. This process is described in detail in Section III.D.

The easiest way to derive a mathematical formula for splines of degree  $n$  is to take the Fourier Transform (FT) of Equation (2). Begin by taking the FT of Equation (1) to get:

$$\beta^0(f) = FT\{\beta^0(t)\} = FT\{\text{rect}(t)\} = \text{sinc}(f) = \frac{e^{j\pi f} - e^{-j\pi f}}{2j\pi f}. \quad (3)$$

The result is raised to the  $(n+1)$  power to solve for the FT of Equation (2). Then, the binomial theorem may be applied to yield the following:

$$\beta^n(f) = \left( \frac{e^{j\pi f} - e^{-j\pi f}}{2j\pi f} \right)^{n+1} = \left( \sum_{k=0}^{n+1} \binom{n+1}{k} e^{j\pi f(n+1-k)} (-1)^k e^{-j\pi f k} \right) \left( \frac{1}{(j2\pi f)^{n+1}} \right). \quad (4)$$

Lastly, to convert back to the time domain, the Inverse Fourier Transform (IFT) of Equation (4) is solved to yield:

$$\beta^n(t) = IFT\{\beta^n(f)\} = \frac{1}{n!} \sum_{k=0}^{n+1} \binom{n+1}{k} (-1)^k \left( t - k + \frac{n+1}{2} \right)^n \mu \left( t - k + \frac{n+1}{2} \right), \quad (5)$$

where  $\mu$  is the unit step function. Equation (5) is a concrete mathematical equation that can be used to determine the transfer functions of spline filters, which is used to solve for the spline coefficients. Plugging in a value of  $n$  will yield the various basis functions shown in Figure 12.

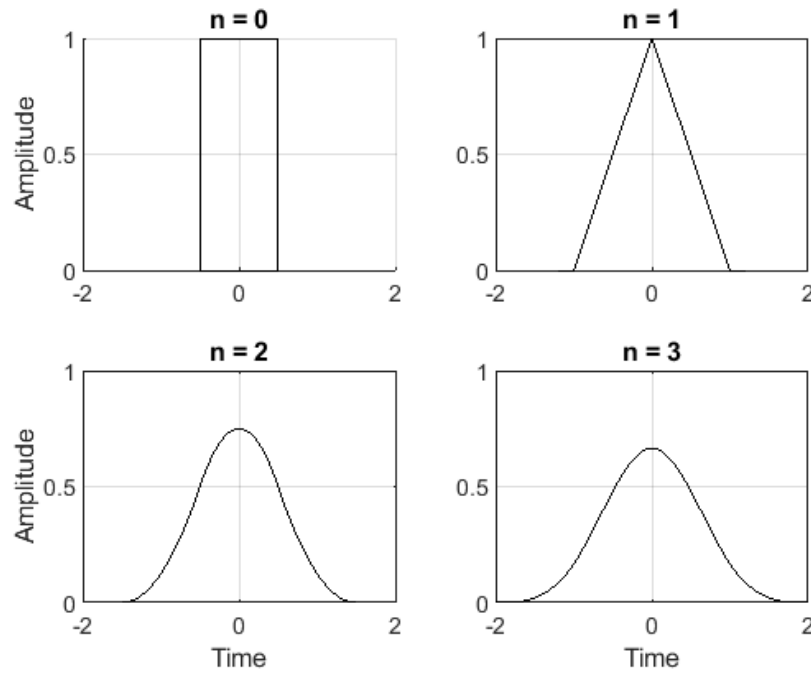


Figure 12. Basis functions for splines of degrees  $n = 0$  to  $n = 3$ .

### C. CUBIC SPLINE

Cubic splines, where  $n = 3$ , tend to be the most popular choice of spline in various applications. This is mostly true because it exhibits the minimum curvature property [15]. In other words, the cubic spline produces the smoothest non-linear trajectory from point to point. The smooth characteristic, among other principles later discussed, of the cubic spline makes it especially appealing for maneuvers conducted by the LRO. For that reason, this chapter focuses primarily on the cubic spline transfer function.

The output of a spline,  $s(t)$ , can be characterized by Equation (6), where  $c(k)$  are the polynomial coefficients, and  $\beta^n$  is the spline function of degree  $n$ .

$$s(t) = \sum_{k \in \mathbb{Z}} c(k) \beta^n(t - k) \quad (6)$$

The output is the sum of the coefficients multiplied by each index of the spline basis functions. Figure 14 shows the graphical solution to Equation (6) for the arbitrary data presented in Figure 13. The data is passed through spline transfer functions of varying degrees. It can be noted that the cubic spline produces the most attractive output due to its minimum curvature aspect. In this case, the eleven sampled data points are the coefficients that are injected into the spline transfer function.

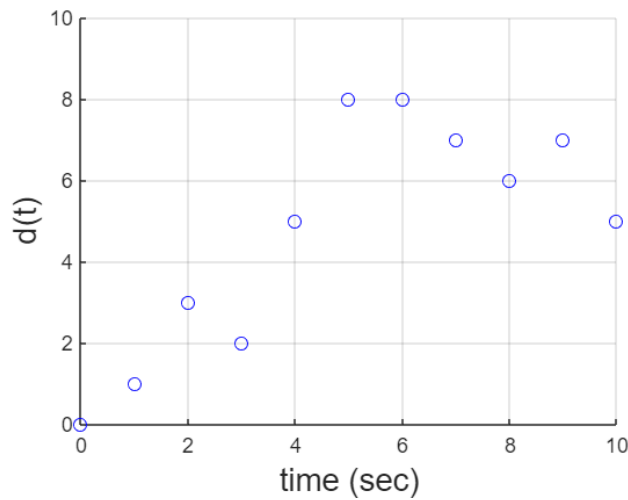
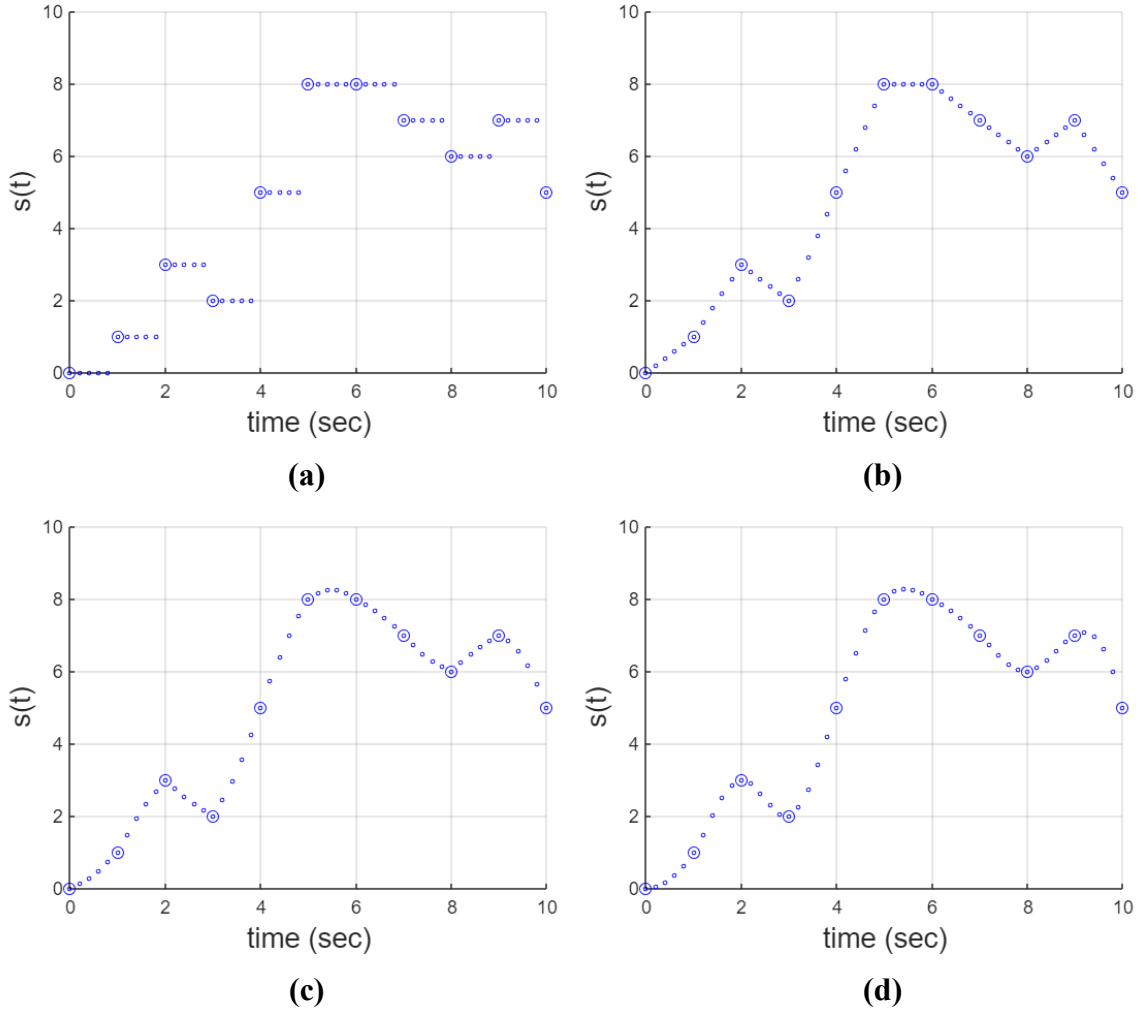


Figure 13. Sampled input data to the spline filters in Figure 14



(a)  $n = 0$ , sample and hold; (b)  $n = 1$ , linear interpolation; (c)  $n = 2$ , quadratic interpolation; (d)  $n = 3$ , cubic interpolation.

Figure 14. Spline outputs of the sampled data in Figure 13.

#### D. CUBIC SPLINE EVALUATION

In accordance with Equation (6), the output of a spline is evaluated by convolving the coefficients with its basis functions. This section discusses how to go about the spline evaluation process for a cubic spline.

## 1. Cubic Spline Transfer Function

The first step in performing cubic spline interpolation is to derive the transfer function of a cubic spline. The discrete output of a spline,  $s[l]$ , is characterized by the following:

$$c[l] \rightarrow \boxed{\beta^n(z)} \rightarrow s[l], \quad (7)$$

where  $c[l]$  contains the coefficients and  $\beta^n(z)$  is the z-transform of Equation (5). Discretizing Equation (5) and plugging in  $n = 3$  yields the following:

$$\beta^3[l] = \frac{1}{6} \sum_{k=0}^4 \binom{4}{k} (-1)^k (l-k+2)^3 \mu(l-k+2), \quad (8)$$

which simplifies to:

$$\beta^3[l] = \frac{1}{6} \begin{bmatrix} (l+2)^3 \mu(l+2) - 4(l+1)^3 \mu(l+1) + \dots \\ \dots 6(l)^3 \mu(l) - 4(l-1)^3 \mu(l-1) + (l-2)^3 \mu(l-2) \end{bmatrix}. \quad (9)$$

Equation (9) results in zero except for the cases where  $l = -1, 0, 1$ , as shown below:

$$\begin{aligned} l &= [\dots -3, -2, -1, 0, 1, 2, 3, \dots] \\ \beta^3[l] &= [\dots \quad 0, \quad 0, \quad \frac{1}{6}, \frac{4}{6}, \frac{1}{6}, \quad 0, \quad 0, \dots]. \end{aligned} \quad (10)$$

The non-zero values of the discrete cubic spline are the coefficients of its z-transform, which is also the cubic spline transfer function in Equation (7).

$$\beta^3(z) = Z\{\beta^3[l]\} = \frac{1}{6} (z + 4 + z^{-1}) = \frac{z^2 + 4z + 1}{6z} \quad (11)$$

The coefficients may now be solved using this z-transform.

## 2. Spline Coefficients

Since this thesis focuses on interpolating data that is already produced, it can be assumed that  $s[l]$  is given information. Additionally, the cubic spline z-transform was solved in Equation (11). Looking back at Equation (7), two out of the three parameters are

known and can be used to solve for  $c[l]$ , the coefficients. Equation (7) may be reversed by inputting  $s[l]$  into the inverse of the cubic spline transfer function as shown below:

$$s[l] \rightarrow \boxed{\frac{1}{\beta^3(z)}} \rightarrow c[l] \quad \equiv \quad s[l] \rightarrow \boxed{\frac{6z}{z^2 + 4z + 1}} \rightarrow c[l]. \quad (12)$$

The transfer function in Equation (12) has one pole within the unit circle and one pole outside the unit circle, which means it exhibits both causal and noncausal properties. Thus, the transfer function must be broken down into its causal and noncausal components via partial fraction expansion. By separating the terms, Equation (12) may then be re-written as:

$$s[l] \rightarrow \boxed{\frac{6z}{z-p}} \rightarrow v[l] \rightarrow \boxed{\frac{1}{z-1/p}} \rightarrow c[l]. \quad (13)$$

Causal                      Noncausal

In Equation (13),  $v[l]$  is the intermediate state that is used to solve for the coefficients. The values  $p$  and  $1/p$  represent the pole values where  $p = -2 + \sqrt{3}$ . Equation (13) can be broken down into two state space equations by solving the Linear Difference Equation for each filter, where  $z^{-n}$  represents a time delay of  $n$  samples.

$$v(z) = \left( \frac{6z}{z-p} \right) s(z) \quad (14)$$

$$v(z)(1 - pz^{-1}) = 6 \cdot s(z) \quad (15)$$

$$v[l] = p \cdot v[l-1] + 6 \cdot s[l] \quad (16)$$

Similarly, the noncausal filter may be broken down into a state space equation. The noncausal filter cannot be solved by past values, so  $c[l-1]$  is solved for vice  $c[l]$ .

$$c[l-1] = p \cdot c[l] - p \cdot v[l-1] \quad (17)$$

Enough information is now available to solve for the coefficients. Applying the given input,  $s[l]$ , to Equation (16) and applying the result,  $v[l]$ , to Equation (17) will

compute the coefficients. The final step is to evaluate Equation (6) at a desired interpolation rate.

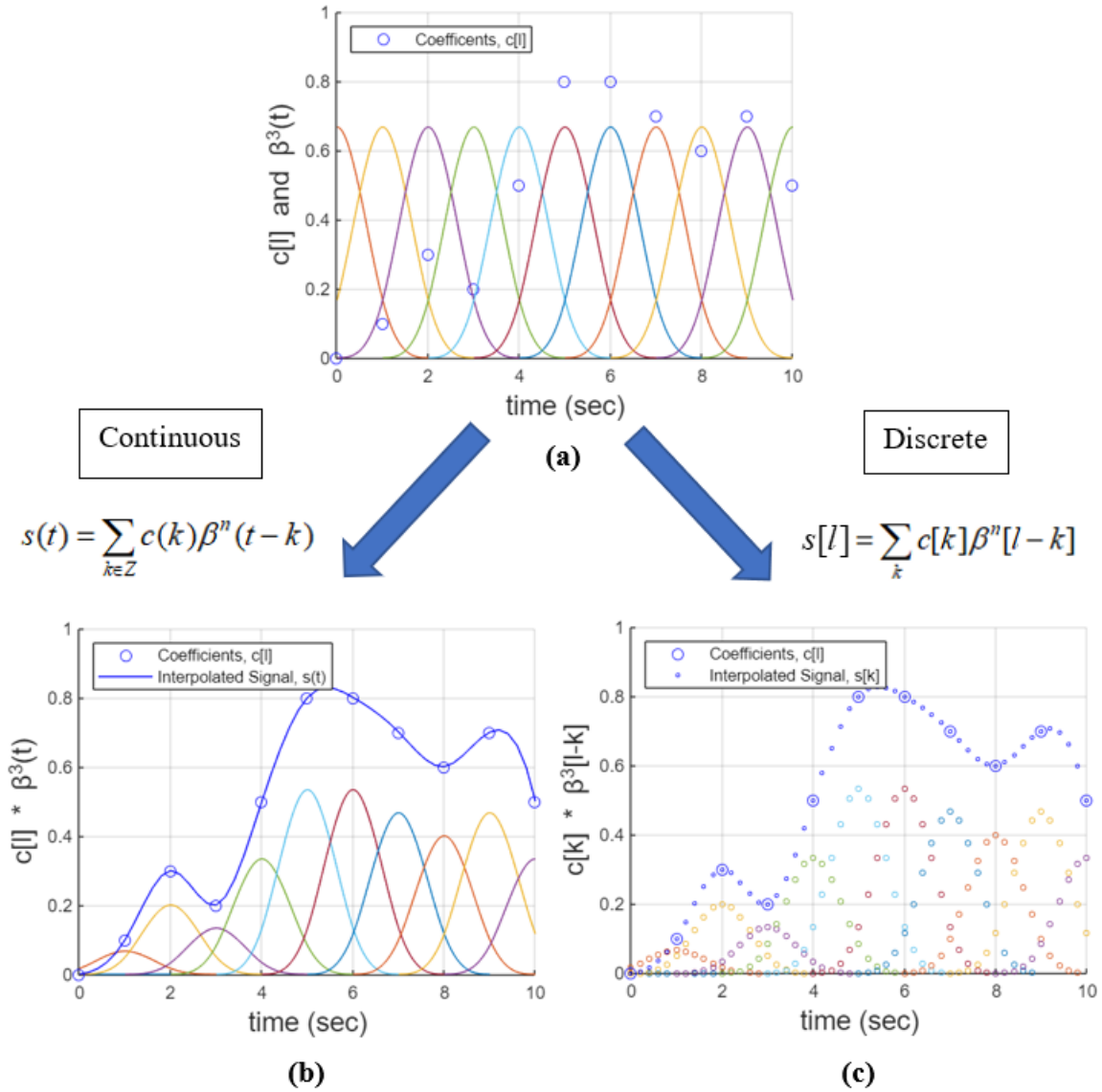
### 3. Interpolation

Figure 11 shows input data spaced at a 10 s interval being interpolated at a one s interval. In this section, the larger input interval will be represented as index  $l$  and the smaller interpolated interval will be represented as index  $k$ . Equation (6) can therefore be discretized to:

$$s[k] = \sum_k c[k] \beta^n[l - k], \quad (18)$$

where  $M$  is the number of samples per spline basis function. In this equation, there is one spline basis function paired with each coefficient. An easier way to approach Equation (18) is to think of multiple spline basis functions spaced by an equivalent interval of the input data and weighted by (or convolved with) the coefficients. Figure 15 shows a graphical representation of Equations (6) and (18) by interpolating data from Figure 13.

The plots in Figure 15 illustrate how a cubic spline interpolates data. Plot (a) shows the incoming coefficients with equivalently spaced and standardized cubic spline basis functions. Plots (b) and (c) show the weighted basis functions, which are formed by convolving the individual coefficients with their respective basis function. Plots (b) and (c) also convey the output of the spline filter in the form of a cubic polynomial. The continuous output in (b) is the sum of all the weighted basis functions at any given point in time. The discrete output in (c) is simply the sum of the weighted basis functions at 0.2 s increments. There is a caveat, however. The output experiences a transient response from time  $0 \leq t \leq 1$  before the spline filter reaches steady state. In a practical design, the transient response is unwanted, but there are tricks to negate the transient period. These methods will be explored when discussing the practical implementation of the spline.



(a) Coefficients coupled with standardized cubic spline basis functions; (b) weighted basis functions with continuous interpolation; (c) discrete interpolation.

Figure 15. Cubic spline interpolation in the continuous and discrete time domain.

In Figure 15, the incoming coefficients are sampled at one Hz while the output is displaced at five Hz. The result is a compression ratio of 5:1. One final note on the output is that each interpolated point only requires a maximum of four local coefficients to calculate, differing from other interpolation methods, like Chebyshev Interpolation, that require the use of global coefficients to process each interpolated point. Interpolating at the

local level is a significant advantage of splines because it reduces the amount of processing needed, which is a rare commodity in space.

#### **E. APPLICATION TO THE LRO**

The LRO executes attitude guidance commands at a rate of five Hz or every 0.2 s. A routine maneuver that lasts 600 s requires 3000 commands. However, using the cubic spline method, the trajectory can be down-sampled at a uniform interval and be input into the spline filter to reproduce a highly detailed curve. This research assesses the outcome of interpolating LRO five Hz maneuvers when data points are sampled at 10 s intervals. Doing so will require only 60 commands for a 600 s maneuver, which yields a data compression ratio of 50:1. The significant reduction of input commands will result in a practicable workload for LRO activities and increase lunar science collection opportunities for NASA.

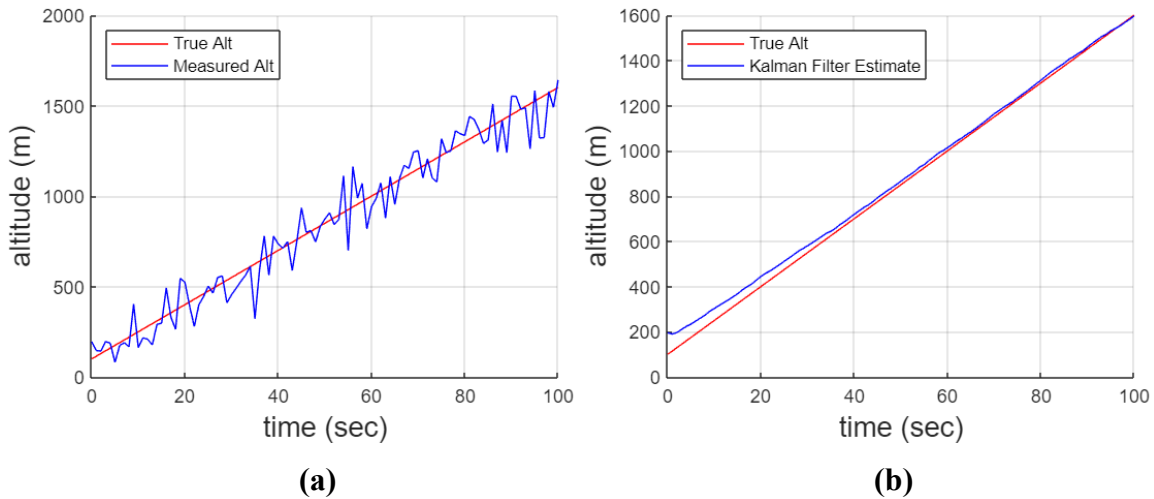
Splines are an effective, efficient, and relatively easy to implement method of interpolation. Cubic splines are advantageous because they exhibit the minimum curvature property, which is an important quality for satellite maneuver curves. Cubic splines also have an advantage over other interpolation methods because they allow processing at the local level opposed to the global level, which reduces processing needed. For the reasons above, the cubic spline is used for the practical implementation of LRO FastMan maneuvers in this thesis.

## IV. INTRODUCTION TO THE KALMAN FILTER

### A. WHAT IS THE KALMAN FILTER?

This chapter pulls mostly from the class notes of Naval Postgraduate School professors, Roberto Cristi [16] and Xiaoping Yun [17]. The Kalman Filter is a recursive filter meaning it operates through a continuous loop of prediction and correction. It is widely used for sensor measurements induced with random error. The Kalman Filter is effective at removing the processing noise from the measurement to receive an accurate estimate of the truth. The Kalman Filter becomes more accurate with time because the more measurements it takes, the better it estimates the associated error covariance.

Consider Figure 16 where an airplane altimeter displays altitude throughout takeoff. Assume the airplane is ascending at a rate of exactly 15 m/s. Due to system and sensor noise, the altimeter readings are scattered (plot (a)). The Kalman Filter may be used to continuously sample the altimeter measurements, estimate the induced noise, and output an altitude much closer to the truth. The output of the Kalman Filter (plot (b)) shows a much more accurate estimation of the truth compared to the noisy altimeter readings. Note that the Kalman Filter converges to get more accurate with time.



(a) Altimeter measurements; (b) Kalman Filter output.

Figure 16. Example of Kalman Filter estimation.

The Kalman Filter is applied in this thesis to produce an alternate method for finding the set of coefficients for the cubic spline filter. In the previous chapter, the coefficients were computed by imposing zero error at the given sampled points. That method resulted in an interpolation error of zero at the sampled points, but consequently higher error between samples. With the Kalman Filter, the interpolated signal error is “spread” uniformly over the whole trajectory (including at the sampled points). This method can therefore be used to reduce the maximum magnitude of the signal error. The practical implementation of the Kalman Filter as it relates to this thesis will be discussed in Chapter VI, but this chapter introduces the Kalman Filter equations that are used later.

## B. STATE SPACE EQUATIONS

This section will discuss a generic set of state space equations that may be used in concert with the Kalman Filter. Upon deriving state space equations of any particular system and sensor, the Kalman Filter simply becomes an iterative “plug and play” technique.

Consider the linear stochastic dynamic model in Figure 17. There is a discrete input,  $u[n]$ , to an arbitrary system. The state that is to be measured,  $x[n]$ , is the true measurement induced with some noise,  $v[n]$ . The sensor is used to estimate  $x[n]$ , but it has its own noise,  $w[n]$ . The output,  $y[n]$ , is thus a noisy measurement that may or may not be an accurate representation of the measured state. The Kalman filter may be applied to estimate  $x[n]$ , but first the state space equations must be determined.

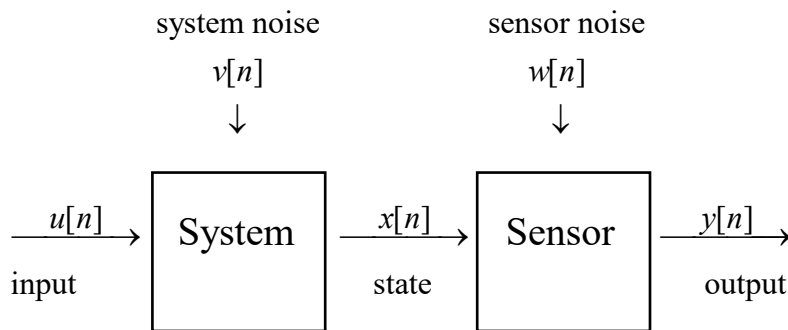


Figure 17. Block diagram of a linear stochastic dynamic model

The generic discrete state space equations [16] for a system and its sensor are shown by Equation (19) where the matrices  $A$ ,  $B$ , and  $C$  are known.

$$\begin{aligned} x[n+1] &= Ax[n] + Bu[n] + v[n] && \text{System} \\ y[n] &= Cx[n] + w[n] && \text{Sensor} \end{aligned} \quad (19)$$

For this chapter, it will be assumed that the noises are both bandlimited white noise with zero mean. Thus, their autocorrelations and cross correlation may be defined as:

$$E\{w[n]w[n]^T\} = R \quad (20)$$

$$E\{v[n]v[n]^T\} = Q \quad (21)$$

$$E\{w[n]v[n]^T\} = 0. \quad (22)$$

From the equations above,  $R$  is the covariance of the sensor noise and  $Q$  is the covariance of the system noise. The cross correlation of white noise is assumed to be zero and the algorithm can be easily modified to account for this cross correlation. The parameters from Equations (19) through (22) will be used to process the Kalman Filter.

### C. KALMAN FILTER EQUATIONS

This section will not derive the Kalman Filter equations. Instead, it will provide a broad overview of how to manipulate the equations given the state space equations in the previous section.

The Kalman Filter operates by predicting the next state,  $x[n+1]$ , and then correcting the prediction when the measurement is taken. The estimator considers the current state, the current input, and the current measurement weighted by a time varying gain,  $K[n]$ . The gain is a function of the estimated covariance,  $P[n]$ , which gets more accurate with each measurement. The Kalman Filter may be summarized by the equations below where  $\hat{x}[n]$  is the current, corrected state, and  $\hat{x}[n+1]$  is the next estimated state.

$$K[n] = AP[n]C^T (CP[n]C^T + R)^{-1} \quad (23)$$

$$\hat{x}(n+1) = (A - K[n]C)\hat{x}[n] + Bu[n] + K[n]y[n] \quad (24)$$

$$P[n+1] = AP[n]A^T - K[n]CP[n]A^T + Q \quad (25)$$

Equations (23) through (25) can be rewritten in the form of a recursive filter, where the future state is predicted, and then the prediction is corrected when a measurement becomes available. The predictor and corrector equations [16] are as follows:

Predictor:

$$\hat{x}_-[n+1] = A\hat{x}_+[n] + Bu[n]$$

$$P_-[n+1] = AP_+[n]A^T + Q$$

Corrector:

$$K[n] = P_-[n]C^T (CP_-[n]C^T + R)^{-1}$$

$$\hat{x}_+[n] = \hat{x}_-[n] + K[n](y[n] - C\hat{x}_-[n])$$

$$P_+[n] = P_-[n] - K[n]CP_-[n].$$

(26)

Since  $A$ ,  $B$ ,  $C$ ,  $R$ , and  $Q$  are given, the only variables needed for the calculation are  $u[n]$  and  $y[n]$ . The Kalman Filter may be shown in block diagram form with  $u[n]$  and  $y[n]$  representing the input and measurement, respectively, from Figure 18.

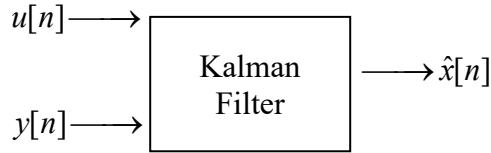


Figure 18. Block diagram of Kalman Filter outputting the estimated state.

Note that the equations in this section are in matrix form. Recall from Chapter III that a cubic spline requires a minimum of four coefficients at any given time to interpolate a signal. Chapter VI will show how the Kalman Filter is used to calculate the coefficients, in which  $\hat{x}[n]$  will be a  $4 \times 1$  vector representing the four coefficients used for the current state.

The Kalman Filter is commonly used as an error optimization filter. In this thesis, the Kalman Filter is used to optimize the spline coefficients to minimize the interpolation error over the duration of a maneuver. The interpolated signal created from the Kalman

Filter coefficients may result in a lower magnitude of error when compared to the coefficients that are created from the spline transfer function. The practical implementation of the Kalman Filter process is described in Chapter VI.

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## V. CUBIC SPLINE IMPLEMENTATION FOR LRO MANEUVERS

This chapter discusses how to practically implement the theoretical spline interpolation filter from Chapter III. Simulations reveal that a data compression rate of 50:1 can be achieved while still maintaining the desired signal resolution. Thus, with the interpolation filter installed on the LRO, a maneuver data file of 3000 commands (60 kB) can be compressed to 60 commands (1.2 kB). The command data buffers, which have a total storage of 200 kB, can store over 150 of these maneuvers, which far exceeds the NASA desired storage of 49 maneuvers (discussed in Chapter II).

### A. GENERATING THE COEFFICIENTS

The coefficient calculations are conducted on the ground, so there are no limiting factors of the LRO (i.e., legacy technology, data rate limitations, etc.) that hinder this process. The coefficient generation will be ultimately installed as an update to AttMan. This section walks through a simple coefficient generation process that may be easily programmed.

The coefficients that are generated for each command are designed to pass through a cubic spline filter to replicate the original quaternion curves. This thesis focuses on a compression ratio of 50:1, so there will be one coefficient for every 50 quaternion points (i.e., 10 s spacing of the quaternion samples). Also, since a quaternion consists of four separate values, there will be four sets of coefficients for each maneuver.

First, begin with the causal and non-causal filters shown in Equation (13). Let  $s[l]$  be a  $4 \times N_s$  matrix of the “sampled” quaternion curves, where each row contains one of the four sampled quaternion values of length  $N_s$ . The down sampling rate may be varied, but here, a value of 50 is used. Therefore, if the original quaternion data is a  $4 \times 3000$  matrix, then  $s[l]$  will be a  $4 \times 60$  matrix. Equations (16) and (17) may then be evaluated using  $s[l]$  to solve for  $v[l]$  and  $c[l]$ , respectively. For the first iteration of each equation,  $v[l-1]$  and  $c[l]$  may both be set to zero. Note that Equation (16) represents a causal system, so the indexing for  $l$  will proceed forward in time from the first value to the last. Equation (17),

on the other hand, is a non-causal system, so the indexing for  $l$  will proceed backward in time from the last value to the first. The coefficients are represented by the matrix,  $c[l]$ , which will be the same size as  $s[l]$ .

### 1. Transient Response Error

When passing the coefficients through the spline filter, there will be an initial transient response. Figure 19 shows a simulation where the scalar of a sampled quaternion curve is passed through a cubic spline filter. There is an oscillatory transient response for the first few cycles of the output while the latter portion of the output drops toward zero. The reason for the deviations is as follows.

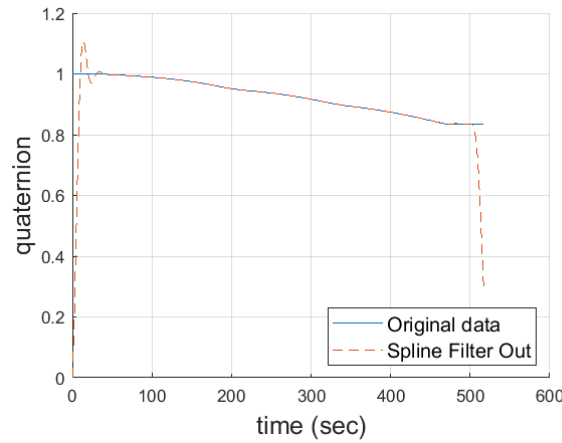


Figure 19. Spline interpolation for a single quaternion with transient response at the beginning and at the end

Throughout the cubic spline filtering process, the interpolated curve exactly matches the original data at each sample (discussed in Chapter III). It is the interpolated data between sample points that has variable error. In this case, the interpolated data will have zero error at every 10 s interval.

In Figure 19, there are no initial conditions set for the filter coefficients, so the interpolated output starts at zero. Since the coefficients solved from Equations (16) and (17) are designed to ensure zero error at the sample points, there is an intersection between

the interpolated data and the original data at every 10 s interval. Since the filter output started at a value of zero, the first few coefficients experience a series of overshoots and undershoots to achieve the property of zero error at every 10 s interval. This is what causes the initial oscillations in the filter output. The resulting oscillations concur with the cubic spline minimum curvature property, which prevents drastic changes in the derivative of the output to produce a smooth curve.

The error at the end of the curve in Figure 19 can be explained by observing how the coefficients were created. Recall two key concepts. First, the coefficients were calculated by passing the original data through a causal and non-causal filter. Second, only four local coefficients are required at any given time during cubic spline interpolation. Interpolation can be achieved by using two causal (past or present) coefficients and two non-causal (future) coefficients. These concepts allow Equation (18) to be rewritten as:

$$s[k] = c[k-1]\beta^3[l-k-1] + c[k]\beta^3[l-k] + c[k+1]\beta^3[l-k+1] + c[k+2]\beta^3[l-k+2]. \quad (27)$$

The last two cycles (each cycle being 10 s) of interpolation drop off toward zero because there are no longer any future coefficient values, which are represented by  $c[k+1]$  and  $c[k+2]$  from Equation (27). Those coefficients are then set to zero. Although the interpolation drops to zero in this simulation, the true filter output will be held constant at the last sample point. Referring to Figure 7, the ZOH filter will hold constant the last command that was given, which will be the final sample point. It is beneficial that the system will hold the last value given because the science collect at the end of the maneuver requires a stationary OCS attitude, which is achieved by the ZOH filter.

The beginning oscillations are not desired and need to be addressed. The next subsection will discuss a simple method that can be used to calculate initial conditions for the coefficients and consequently eradicate the oscillations.

## 2. Correcting the Transient Response Error

To correct the transient response error, initial conditions must be determined for the filter. One way to calculate the initial conditions is to mirror and vertically flip the incoming data at the start time and then sample the whole signal as normal. An example of this is

shown in Figure 20. The purpose of vertically flipping the data is to ensure there are no discontinuities of the derivative at the start of the maneuver. The latter portion of the curve does not need to be mirrored, but rather the final coefficient may be repeated as many times as necessary to simulate the ZOH filter of the LRO second order ACS. Using the “mirror and flip” method, Equation (16) may be rewritten as:

$$v[l] = p \cdot v[l-1] + 6 \cdot s'[l], \quad (28)$$

where  $s'[l]$  is the extended sampled data. The cubic spline output may then be solved using the same method used to produce the curve in Figure 19, except using the extended curve and by also repeating the last coefficient for two cycles to simulate the ZOH filter. The resulting curve is shown in Figure 21 where (a) represents the full interpolation and (b) represents a truncated version denoting the actual time of the maneuver.

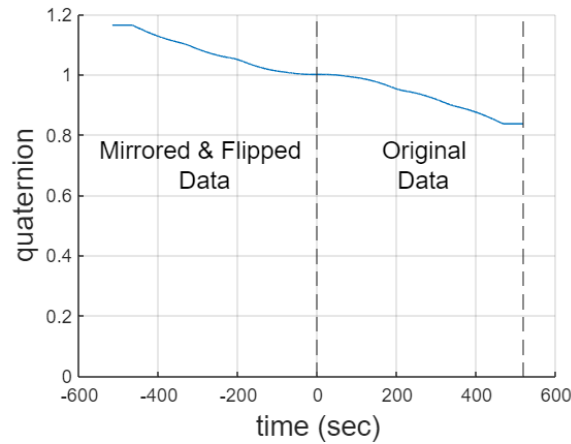
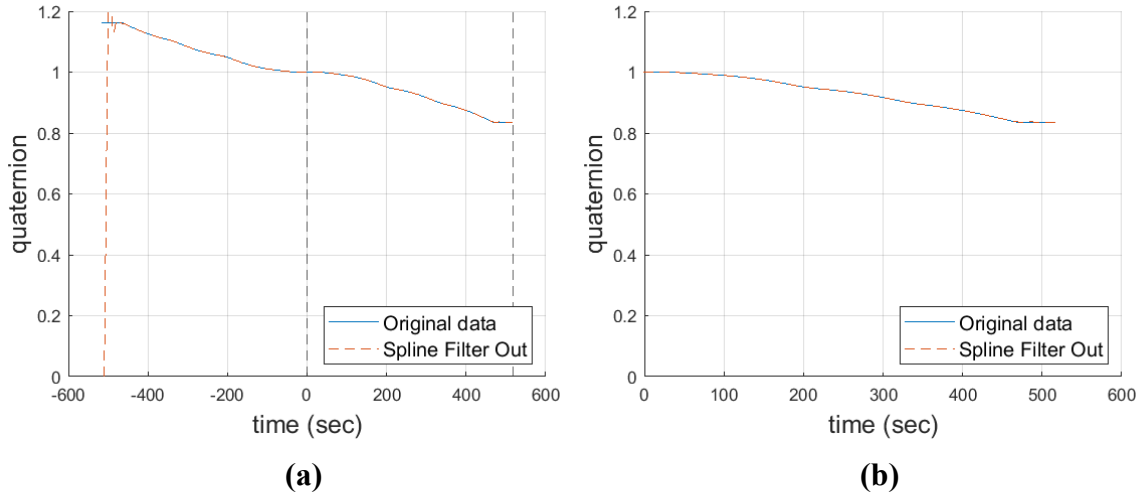


Figure 20. Quaternion curve that is extended at the start to calculate the initial conditions for the coefficients



(a) Interpolation of the extended samples,  $s'[I]$ , with initial oscillations; (b) interpolation of the original samples,  $s[I]$ , without oscillations by truncating the plot in (a).

Figure 21. Cubic spline output of the data in Figure 20.

Figure 21a has the initial oscillations starting at roughly  $t = -500s$ , which gives the cubic spline filter plenty of time to stabilize before the maneuver start time at  $t = 0s$ . This simulation may be implemented on the LRO in two different ways. One way is to physically start the LRO maneuver before  $t = 0s$ , so that the ACS reaches steady state before beginning the maneuver. The issue with starting the maneuver early, however, is that it would add a few additional commands. The second approach is to use this simulation to obtain the steady state coefficients at the start of the maneuver, and then set the filter initial conditions to avoid a transient response. Figure 21b shows the result of initializing the filter with the initial coefficients to avoid a transient response.

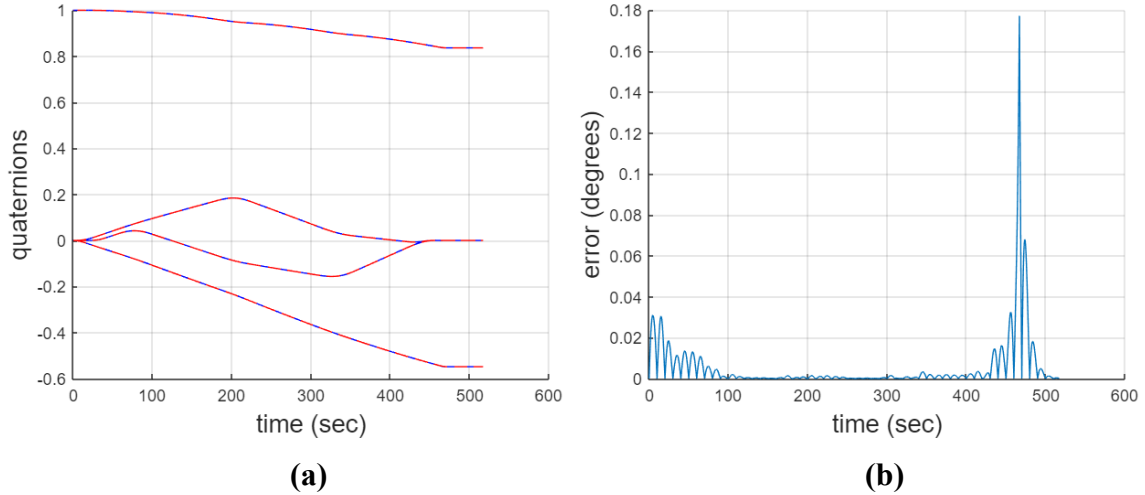
The spline process used to produce the interpolated signal in Figure 21 can be applied to each quaternion. Figure 22 overlays the cubic spline interpolated signals with original quaternions and shows the associated interpolation error plot. Note that the error is the cone error and is solved by finding the minimum angular distance between the original quaternion attitude and the interpolated quaternion attitude as shown by Equations (29) through (31). The error in plot (b) is the theoretical error that results from cubic spline

interpolation for a particular maneuver. This plot is later compared to error results from LRO simulations.

$$\text{original quaternion} = \begin{bmatrix} q_0(1) \\ q_0(2) \\ q_0(3) \\ q_0(4) \end{bmatrix} \quad \text{interpolated quaternion} = \begin{bmatrix} q_i(1) \\ q_i(2) \\ q_i(3) \\ q_i(4) \end{bmatrix} \quad (29)$$

$$q_e = \begin{bmatrix} (q_o(4))(-q_i(1)) + (q_o(3))(-q_i(2)) - (q_o(2))(-q_i(3)) + (q_o(1))(q_i(4)) \\ (-q_o(3))(-q_i(1)) + (q_o(4))(-q_i(2)) + (q_o(1))(-q_i(3)) + (q_o(2))(q_i(4)) \\ (q_o(2))(-q_i(1)) - (q_o(1))(-q_i(2)) + (q_o(4))(-q_i(3)) + (q_o(3))(q_i(4)) \\ (-q_o(1))(-q_i(1)) - (q_o(2))(-q_i(2)) - (q_o(3))(-q_i(3)) + (q_o(4))(q_i(4)) \end{bmatrix} \quad (30)$$

$$\text{Interpolation Error (degrees)} = 2 \tan^{-1} \frac{(q_e(1)^2 + q_e(2)^2 + q_e(3)^2)^{\frac{1}{2}}}{q_e(4)} \quad (31)$$



(a) Spline interpolation of each quaternion; (b) error in degrees between original signal and interpolated signal.

Figure 22. Spline interpolation of quaternion command trajectories.

Now that an effective method has been established to generate the coefficients, the next step is to determine how to practically implement the cubic spline filter on the LRO.

## B. CUBIC SPLINE FILTER ON THE LRO

Ideally, the cubic spline transfer function from Equation (12) could be patched into the flight software to evaluate the coefficients and perform the interpolation. But in a practical sense, the transfer function cannot be patched for two reasons. First, the transfer function has a non-causal property that is impossible to implement because the ACS cannot measure future values. Second, there may be high risk involved with altering the software of the LRO operating system because unexpected errors could arise and potentially damage the spacecraft. Thus, the labor cost to patch the cubic spline filter directly would be too high. The most feasible option is to implement the cubic spline filter by using the software and filters that are already available within the flight software.

### 1. Direct Form II Filter

The LRO already has a Direct Form II (DFII) filter routine that is used for closed-loop attitude control. The DFII is a second order filter that can be called in series multiple times. The block diagram of a DFII filter is shown in Figure 23 and its associated state space equations are represented by Equations (32) and (33). As shown in Figure 23, the DFII has five coefficients represented by  $a_1, a_2, b_0, b_1$ , and  $b_2$ . Each coefficient may be defined separately even if several filters are called in series. So, the first filter in the series may have different parameters than the second filter in the series. If the filters are set in a series, then the output of the first filter,  $y(n)$ , will be the input to the second filter,  $x(n)$ . The DFII has two time delays represented by  $z^{-1}$ . Lastly, there are three intermediate states represented by  $v(n), v(n-1)$ , and  $v(n-2)$  that are each separated by a time delay.

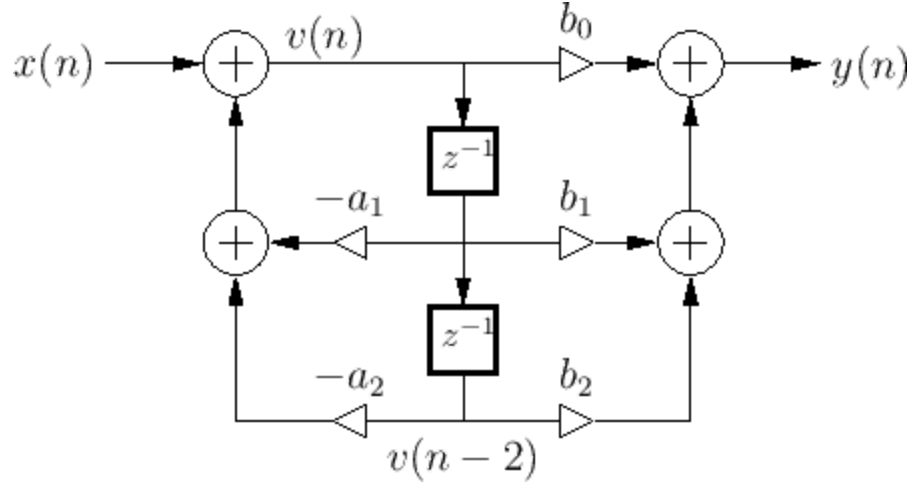


Figure 23. Block diagram of a DFII filter. Source: [18].

$$v(n) = x(n) - a_1 v(n-1) - a_2 v(n-2) \quad (32)$$

$$y(n) = b_0 v(n) + b_1 v(n-1) + b_2 v(n-2) \quad (33)$$

The DFII filter is a convenient tool to use for cubic spline interpolation because it is already installed on the LRO. The next section will discuss how to apply this filter to achieve interpolation.

## 2. Cascaded Integrator-Comb Filter

The discrete transfer function of the cubic spline must be derived to implement a practical version of the filter. Start by solving for the discrete frequency domain of the cubic spline by plugging in  $n = 3$  to Equation (5) and taking the z-transform, which yields:

$$\beta^3(z) = \left( \frac{1 - z^{-M}}{1 - z^{-1}} \right)^4. \quad (34)$$

Equation (34) can be used as a discrete cubic spline interpolator.

Equation (34) may also be derived by multiplying four rectangular pulses together in the z-domain. Figure 24 shows the plot of a discrete rectangular pulse with  $M$  samples. One pulse is essentially the addition between a positive step function at sample zero and a negative step function at sample  $M$ . The addition of the two step functions is shown as:

$$\frac{1}{1-z^{-1}} + \frac{-z^{-M}}{1-z^{-1}} = \frac{1-z^{-M}}{1-z^{-1}}, \quad (35)$$

where the first term is the positive step at sample zero, the second term is the negative step delayed to sample  $M$ , and the solution is the z-transform of a rectangular pulse of  $M$  samples. Equation (35) can be multiplied by itself four times since it is in the frequency domain (same as convolution of four pulses in the time domain) to yield Equation (34).

The discrete cubic spline interpolator block diagram in Figure 25 is derived from Equation (34). Here, an up-sampler is added to increase the sample rate of the coefficients to the desired system rate and a gain is added to normalize the output after it is convolved with four rectangular pulses. Figure 25 may be manipulated into a practical design by converting it to a Cascaded Integrator-Comb (CIC) construct. A CIC consists of a series of filters and an up-sampler that are configured to conduct interpolation. The cubic spline configuration of the CIC is shown in Figure 26 where  $c[l]$  represents the incoming coefficients at a rate of 0.1 Hz and  $\hat{s}[n]$  represents the interpolated output data at the ACS operating rate of 5 Hz. This CIC configuration consists of four differentiator filters, an up-sampler, four integrator filters, and a scaling factor.

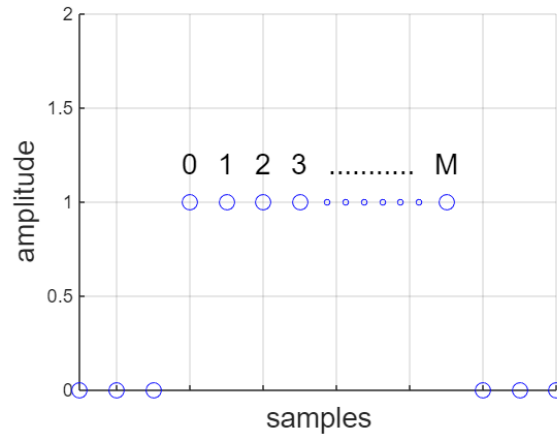


Figure 24. Discrete rectangular pulse of  $M$  samples in the z-domain

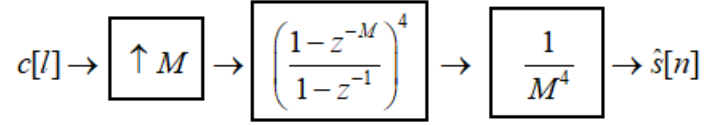


Figure 25. Discrete cubic spline interpolator with an adjustable up-sampler and gain

The CIC configuration in Figure 26 is achievable by means of the Nobel Identity [19]. The Nobel Identity enables the transfer function numerator and the up-sampler,  $\uparrow M$ , to be interchanged if  $(1-z^{-M})^4$  is replaced with  $(1-z^{-1})^4$  since both configurations represent the same amount of time shift in actual time.

Recall from Chapter III the spline derivation. An  $n$ -order spline basis function is produced by convolving rectangular pulses  $n+1$  times as shown by Equation (2). A cubic spline, where  $n$  is three, requires the convolution of four rectangular pulses. Since convolution is a form of integration, the first half of each basis function may be viewed as the integral of the previous basis function. For example, the integration of a rectangular pulse is a ramp, which is the first half of the triangular first-order basis function. Since the cubic spline consists of convolving four rectangular pulses, it can be derived by integrating a signal four times. The CIC uses this spline integration principle to interpolate a signal.

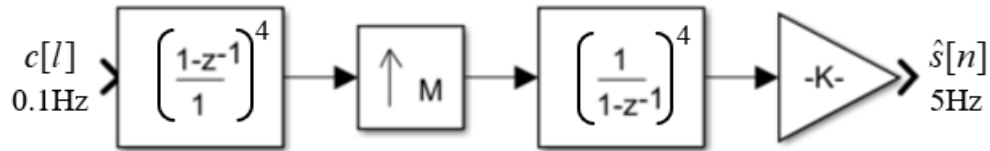


Figure 26. CIC implementation of a cubic spline interpolator

The CIC configuration in Figure 26 has four integrators to replicate the four rectangular convolutions of a cubic spline. To ensure the incoming signal translates to the output, the integrators are offset by the four preceding differentiators. So, if there was no up-sampler between the differentiators and the integrators, then  $\hat{s}[n] = \hat{s}[l] = Kc[l]$ . This

CIC configuration essentially “breaks down” the incoming signal by differentiating it four times, and then builds it back up at a rate of  $M$  by integrating it four times. The reconstruction of the signal by the integrators replicates interpolation by filling in the gaps between the incoming coefficients at a rate of  $M$  times the coefficient rate.

Recall that the coefficients previously generated had a compression ratio of 50:1. The up-sampler has to be configured to return the coefficient sampling rate of 0.1 Hz back to the ACS operating rate of 5 Hz. To achieve the proper expansion rate,  $M$  is equal to 50. Note that  $M$  may be adjusted, however, a larger  $M$  value will result in more interpolation error and a lower  $M$  value will result in less data compression. An  $M$  value of 50 was chosen for this thesis because it appears to be the around the “sweet” spot for optimum performance. The gain value,  $K$ , can be solved by the following equation:

$$K = \frac{1}{M^4}, \quad (36)$$

which cancels out the total gain due to the four rectangular convolutions. The gain may be placed either at the beginning or the end of the CIC, but it may not be placed anywhere in the middle. The pole of Equation (34) is located to the right of the imaginary axis, which makes it an unstable pole. Since the zero is overlapping with the pole, shown in Figure 27, the system is stable only when the pole and zero are exactly the same. Thus, fixed point numbers must be used in the filter calculations. Placing the gain in the middle of the filter bank may necessitate the need for floating point numbers, which will cause instabilities in the interpolation algorithm.

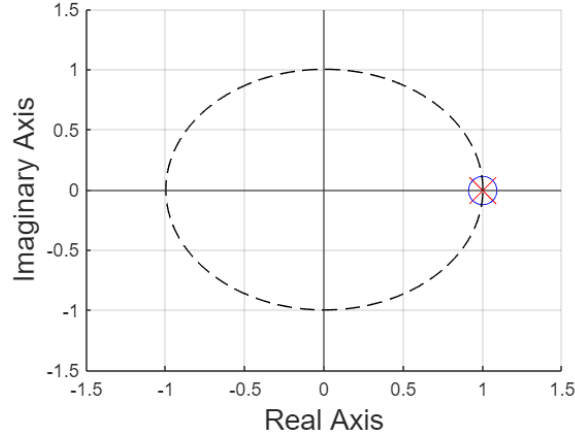


Figure 27. Pole-Zero mapping for the z-transform in Equation (34)

The up-sampler in Figure 26 increases the sampling rate by using “zeros” as its interpolating function, meaning each incoming coefficient will be followed by  $M - 1$  zeros. In practical applications, it is more advantageous to using a ZOH up-sampler that will repeat each coefficient  $M - 1$  times instead of inserting zeros. The first advantage is that the only dependence on the parameter  $M$  is the “hold” operation of the up-sampler, which makes it easy to adjust the sample rate. The second advantage is that the CIC consists of only summation and difference operations to perform interpolation, which simplifies the CIC computations. This more practical configuration of a CIC cubic spline interpolator is shown in Figure 28.



Figure 28. CIC cubic spline interpolator using the Zero-Order Hold up-sampler

The more practical CIC structure in Figure 28 has only three pairs of differentiators and integrators in concert with a ZOH up-sampler. This configuration is achievable because

the ZOH up-sampler produces a rectangular pulse in the same manner as a zero-order spline interpolator. Since the ZOH up-sampler acts as a zero-order spline, one of the four integrator pairs from Figure 26 must be removed. Since there are now three pairs of integrators, the gain must be adjusted to:

$$K = \frac{1}{M^3}. \quad (37)$$

The resulting CIC filter bank yields a simple and efficient implementation of the cubic spline interpolating filter. The final step is to implement the CIC using the DFII filters already installed within the LRO flight software.

### **3. Configuring the DFII into a CIC Construct**

The CIC construct in Figure 28 consists of two third-order systems. Therefore, a total of four DFII filters will be needed (two for each third order element). The DFII filter bank configuration is shown in Figure 29. This filter construct is specifically designed for an  $M$  value of 50 because the incoming coefficients are sampled at 0.1 Hz. A gain value equal to Equation (37) must also be inserted anywhere in the DFII construct as a scaling factor. The coefficient values are also listed in Figure 29 and are designed to establish two third-order systems.

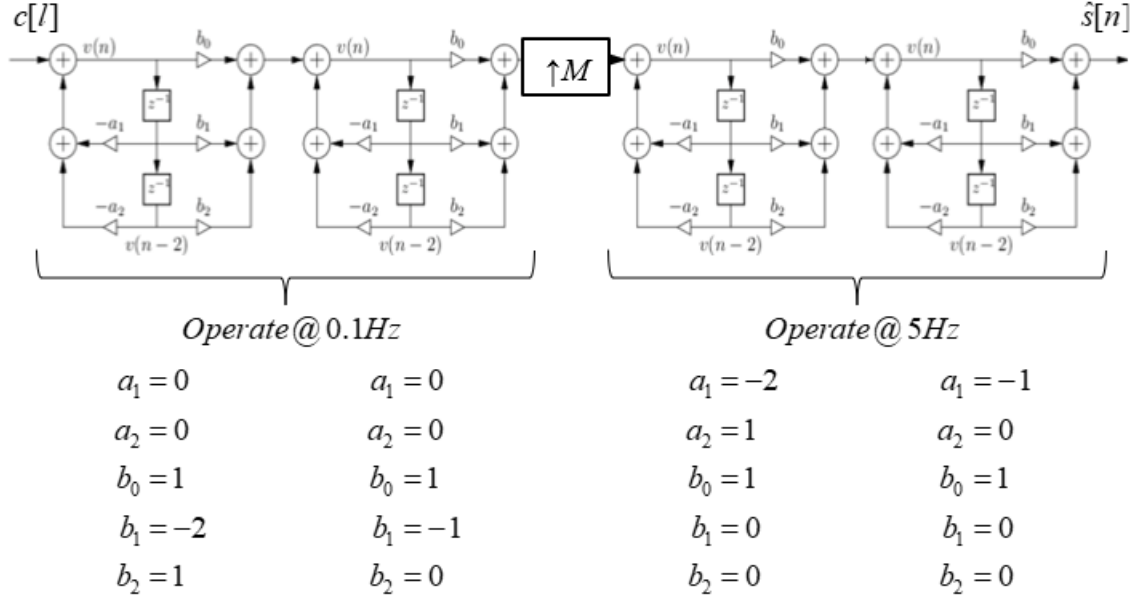


Figure 29. DFII filter bank configured to replicate the CIC cubic spline interpolator

The DFII construct in Figure 29 enables the LRO ACS to replicate a cubic spline interpolation filter by re-using elements of the existing flight software, making this construct a simple and practical approach to reduce the quaternion command rate needed to drive the ACS.

The DFII filter coefficients may be solved by first establishing the discrete transfer function of an individual filter. The filter state,  $V(z)$ , is related to the input,  $X(z)$ , from the following equation:

$$V(z) = X(z) - a_1 z^{-1} V(z) - a_2 z^{-2} V(z), \quad (38)$$

which can be rewritten as:

$$V(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}} X(z). \quad (39)$$

Likewise, the filter output,  $Y(z)$ , is related to the filter state as:

$$Y(z) = (b_0 + b_1 z^{-1} + b_2 z^{-2}) V(z). \quad (40)$$

Equation (39) can be substituted in for  $V(z)$  to yield:

$$\frac{Y(z)}{X(z)} = H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}. \quad (41)$$

The first half of CIC construct consists of three differentiators. Since the DFII is a second order filter, two of them are required to achieve third order differentiation. The first filter is designed as a second order differentiator by solving for its z-transform and comparing it to the transfer function in Equation (41). Its z-transform is simply:

$$(1 - z^{-1})(1 - z^{-1}) = 1 - 2z^{-1} + z^{-2}. \quad (42)$$

To achieve the second order differentiator with the DFII, the coefficients must be set as:  $b_0 = 1$ ,  $b_1 = -2$ ,  $b_2 = 1$ , and  $a_1 = a_2 = 0$ . The coefficients for the second filter are solved by equating the filter transfer function to a first order differentiator,  $(1 - z^{-1})$ .

Similarly, the third order integrators on the second half of the CIC are designed using two DFII filters. The first filter is equivalent to a second order integrator:

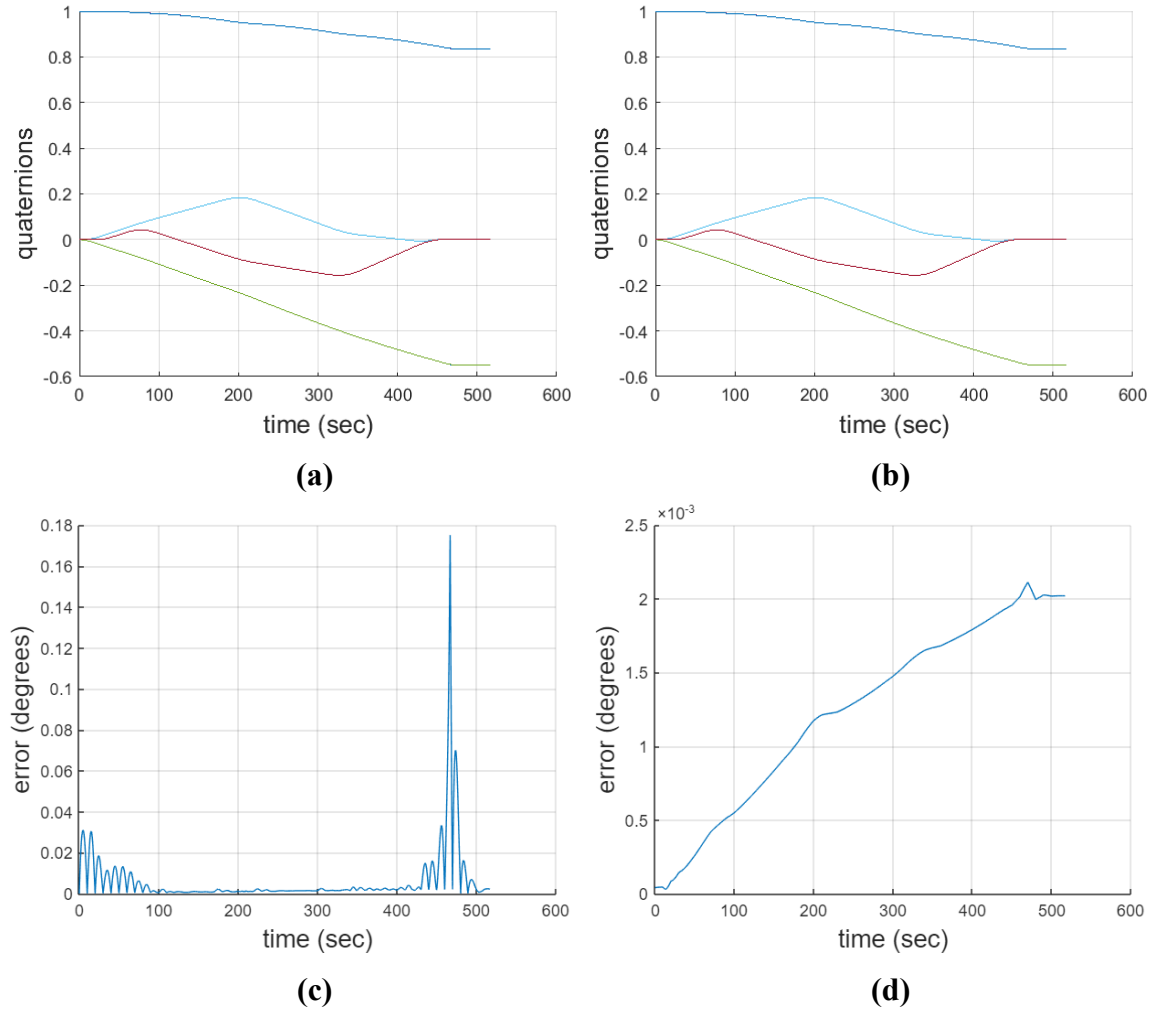
$$\frac{1}{(1 - z^{-1})(1 - z^{-1})} = \frac{1}{1 - 2z^{-1} + z^{-2}}, \quad (43)$$

where  $b_0 = 1$ ,  $b_1 = b_2 = 0$ ,  $a_1 = -2$ , and  $a_2 = 1$ . The second filter is equivalent to a single integrator,  $1/(1 - z^{-1})$ . This process is how the DFII filter coefficients in Figure 29 were solved to achieve a third order differentiator followed by a third order integrator.

### C. SIMULATION OF THE DFII IMPLEMENTATION

A MATLAB simulation was conducted by using the state space equations from (32) and (33) to ensure the proper interpolation of the DFII filter bank. The full process for the quaternion interpolation was conducted in this simulation. The coefficients were generated from the cubic spline transfer function to include the initial conditions described in Section V.B.2 to prevent an oscillatory transient response. The coefficients were then processed through the DFII filter bank where the output was analyzed and compared to the original data and the output of the theoretical cubic spline interpolation filter. Figure 30

shows the results. Note that the error is calculated from Equations (29) through (31). An error in degrees provides a more conceptual and practical measure of interpolation effectiveness, especially because the star tracker occultation constraints are measured in degrees.



(a) Dashed lines are the original quaternion data from FastMan and the solid lines are the DFII output; (b) dashed lines are the cubic spline filter output and the solid lines are the DFII output; (c) quaternion angle difference between original data and DFII output; (d) quaternion angle difference between theoretical cubic spline interpolation and the DFII output.

Figure 30. Simulation results from MATLAB comparing the DFII filter bank output to the original quaternion signal and the output of the theoretical cubic spline filter

The results from the DFII simulation (Figure 30) are very promising. Plot (a) shows the original quaternion data produced from FastMan overlaid by the DFII interpolated output. The difference is so slight that the two sets of curves are not easily discernable. Plot (c) shows the angular difference between the two sets of quaternions in degrees, and it is very similar to the ideal cubic spline output error in Figure 22b. Overall, the error is low with a maximum peak of about  $0.175^\circ$ . The peak error occurs when the original quaternion curves reach a terminal steady state value toward the end of the maneuver. Figure 31 shows a close-up view of quaternion four at the peak error to visualize the oscillatory filter response. There is a sharp “turn” in the desired data (dashed line), which causes the sharp peak in error. Afterwards, the error dissipates as the system reaches a steady state target attitude.

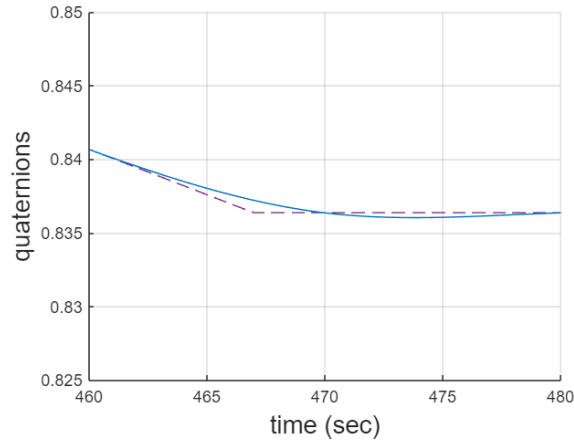


Figure 31. Zoomed in view of quaternion four in Figure 30a

Plot (b) in Figure 30 overlays the DFII output on top of the discrete cubic spline filter output. Since the cubic spline filter cannot be used in practical systems due to its non-causal features, the purpose of this plot is to depict the accuracy of the DFII filter bank compared to an ideal cubic spline filter. Plot (d) shows the angle difference (specified as error) in degrees between the two sets of curves. The error is on the order of  $10^{-3}^\circ$ , which results in a negligible difference between DFII interpolation and cubic spline interpolation. The small error is most likely due to a slight timing difference between the two filter

responses. These results prove that the DFII construct operates as desired and may be used as a practical interpolation filter for the LRO ACS.

This chapter discussed how to generate cubic spline coefficients from LRO maneuvers and interpolate those coefficients using a DFII filter bank to reconstruct the original signal. The coefficients that are generated for each quaternion curve are the command inputs to the DFII filter bank. For this thesis, the incoming coefficients operate at a rate of 0.1 Hz. An up-sampler with a rate increase of 50 will convert the 0.1 Hz coefficient rate to the 5 Hz ACS update rate. The DFII filter bank was derived from the frequency domain of the discrete cubic spline. The cubic spline transfer function was manipulated using the Nobel Identity to form a CIC construct, which is easily constructed by means of a series of DFII filters. The DFII construct is a simple and practical way to perform cubic spline interpolation within the LRO ACS.

The final simulation revealed that an original 3000 command signal produced by AttMan can be down-sampled to approximately 60 commands that are interpolated within the DFII filter bank to reconstruct the original signal with a tolerable error. The significant reduction in commands enables the practical application of FastMan maneuvers onboard the LRO.

The next chapter will discuss how to use a Kalman Filter as an alternative method to generate the coefficients. The results will be analyzed to determine the advantages and disadvantages the Kalman Filter method.

## **VI. GENERATING COEFFICIENTS WITH THE KALMAN FILTER**

### **A. A NEW METHOD TO GENERATE COEFFICIENTS**

The coefficients generated in the previous section were calculated by passing the sampled quaternion data through the causal and non-causal filters of the cubic spline. These coefficients result in zero-error at each sample point with variable error at the interpolated points between samples. The plot in Figure 30c shows how the error oscillates up to a local maximum and then back down to zero every 10 s, or between every sample point.

This chapter experiments with using the Kalman Filter to generate the cubic spline input coefficients. The Kalman Filter is used because it functions to optimize a system by minimizing the system error over a given time interval. Theoretically, coefficients created with the Kalman Filter will minimize the running error of the cubic spline interpolation. This method produces coefficients that do not necessary result in zero error at the sample points, but rather, the interpolation error is “spread” throughout the curve to minimize the error peaks. An advantage to using the Kalman Filter to generate coefficients is that the peak error may be reduced. A disadvantage is that the interpolated mean error may be increased. Depending on the tolerable error for the star tracker occultation avoidance, a larger mean error may be worth it if the maximum error is reduced.

### **B. DERIVING THE KALMAN FILTER COEFFICIENT GENERATOR**

#### **1. Constructing the Cubic Spline State Space Equations**

The Kalman Filter equations in Chapter IV are designed from an *ABCD* state space model. So, the first step is to create an *ABCD* state space model for the cubic spline. Start by taking a zero-order spline (rectangular pulse) with  $M$  samples. Four of the zero-order splines are convolved to produce a cubic spline, which results in  $4M - 4$  samples. The resulting cubic spline curve may be normalized to a maximum value of one, which is represented by Equation (44) and Figure 32.

$$h[n] = \frac{\beta^3[n]}{\max(\beta^3[n])}, \quad n = 0, \dots, 4M - 4 \quad (44)$$

The discrete cubic spline in the time domain may then be written as:

$$\hat{s}[n] = \sum_{k=0}^{+\infty} c[k]h[n - kM] \quad (45)$$

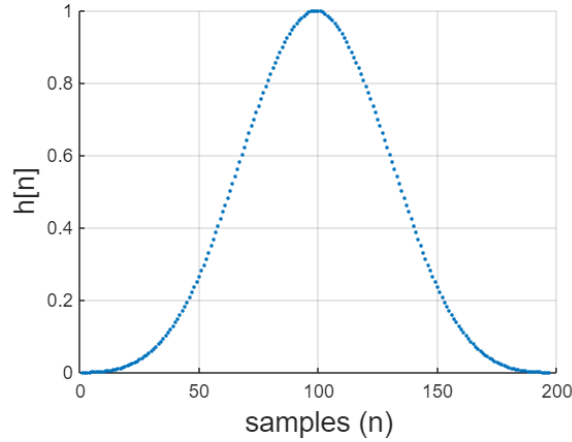


Figure 32. Plot of normalized cubic spline curve with  $M = 50$ .

By changing the index to  $n = mM + \ell$ , Equation (45) may be manipulated to the following:

$$\begin{aligned} \hat{s}[mM + \ell] &= \sum_k c[k]h[mM + \ell - kM] \\ &= \sum_k c[k]h_\ell[m - k], \text{ where } h_\ell[m] = h[mM + \ell]. \end{aligned} \quad (46)$$

Using the convolution Commutative Property, Equation (46) can be rewritten for a cubic spline:

$$\hat{s}[mM + \ell] = \sum_{k=0}^3 h_\ell[k]c[m - k], \text{ for } 0 \leq \ell \leq M - 1. \quad (47)$$

Equation (47) may be written in block diagram form (Figure 33) with an  $M$  value of 50 to match the sample rate used in the previous chapter.

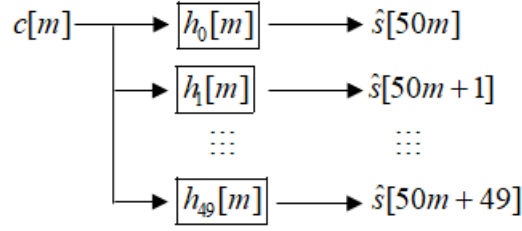


Figure 33. Block diagram of cubic spline configuration from Equation (47) using  $M = 50$ .

In Figure 33, each coefficient is passed through fifty sequential filters, each one of low order (three at most) that, when combined, result in a cubic spline output. The purpose of configuring the filters in this manner is to produce equations that can then be manipulated to derive a Kalman Filter expression. The fifty equations from Figure 33 can be broken down into fifty equations that are derived from the properties of Equation (48).

$$c[m] * h_\ell[m] = \sum_{k=0}^3 c[k] h_\ell[m-k] = \sum_{k=0}^3 h_\ell[k] c[m-k] \quad (48)$$

$$\begin{aligned} \hat{s}[50m] &= h_0[0]c[m] + h_0[1]c[m-1] + h_0[2]c[m-2] + h_0[3]c[m-3] \\ \hat{s}[50m+1] &= h_1[0]c[m] + h_1[1]c[m-1] + h_1[2]c[m-2] + h_1[3]c[m-3] \\ \vdots &= \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \\ \hat{s}[50m+49] &= h_{49}[0]c[m] + h_{49}[1]c[m-1] + h_{49}[2]c[m-2] + h_{49}[3]c[m-3] \end{aligned} \quad (49)$$

The equations in (49) each have similar state equations:

$$x[m] = \begin{bmatrix} c[m] \\ c[m-1] \\ c[m-2] \\ c[m-3] \end{bmatrix}, \quad x[m+1] = \begin{bmatrix} c[m+1] \\ c[m] \\ c[m-1] \\ c[m-2] \end{bmatrix}. \quad (50)$$

The final equation in (49) can be rewritten as:

$$c[m+1] = \frac{1}{h_{49}[0]} \left[ -h_{49}[1]c[m] - h_{49}[2]c[m-1] - h_{49}[3]c[m-2] + \hat{s}[50(m+1)+49] \right], \quad (51)$$

which facilitates the creation of the  $A$  matrix in the discrete  $ABCD$  state space equations below, where  $x[m]$  is the observed state of the coefficients and  $y[m]$  is the state of the

cubic spline output. Note that  $x[m]$  can be solved by only examining the last equation in (49) because the coefficients are the same for each of the spline sequences, whereas  $y[m]$  relies on the combination of the other sequences.

$$x[m+1] = Ax[m] + Bu[m] + v[m] \quad (52)$$

$$y[m] = Cx[m] + w[m] \quad (53)$$

$$\begin{aligned} \begin{bmatrix} c[m+1] \\ c[m] \\ c[m-1] \\ c[m-2] \end{bmatrix} &= \begin{bmatrix} -\frac{h_{49}[1]}{h_{49}[0]} & -\frac{h_{49}[2]}{h_{49}[0]} & -\frac{h_{49}[3]}{h_{49}[0]} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c[m] \\ c[m-1] \\ c[m-2] \\ c[m-3] \end{bmatrix} + \dots \\ x[m+1] & \quad \quad \quad A \quad \quad \quad x[m] \\ \dots & \begin{bmatrix} 1 \\ \frac{1}{h_{49}[0]} \\ 0 \\ 0 \\ 0 \end{bmatrix} s[50(m+1)+49] + \begin{bmatrix} 1 \\ \frac{1}{h_{49}[0]} \\ 0 \\ 0 \\ 0 \end{bmatrix} \varepsilon_{49}[50(m+1)+49] \\ & \quad \quad \quad B \quad \quad \quad u[m] \quad \quad \quad v[m] \end{aligned} \quad (54)$$

$$\begin{aligned} \begin{bmatrix} s[50m] \\ s[50m+1] \\ \vdots \\ s[50m+48] \end{bmatrix} &= \begin{bmatrix} h_0[0] & h_0[1] & h_0[2] & h_0[3] \\ h_1[0] & h_1[1] & h_1[2] & h_1[3] \\ \vdots & \vdots & \vdots & \vdots \\ h_{48}[0] & h_{48}[1] & h_{48}[2] & h_{48}[3] \end{bmatrix} \begin{bmatrix} c[m] \\ c[m-1] \\ c[m-2] \\ c[m-3] \end{bmatrix} + I_{48 \times 48} \begin{bmatrix} \varepsilon_0[m] \\ \varepsilon_1[m] \\ \vdots \\ \varepsilon_{48}[m] \end{bmatrix} \\ y[m] & \quad \quad \quad C \quad \quad \quad x[m] \quad \quad \quad w[m] \end{aligned} \quad (55)$$

The variables  $v[m]$  and  $w[m]$  represent the “noise” in the system and sensor, respectively. In this case, the noise is characterized as the interpolation error, which should be minimized by the Kalman Filter. Thus, the noise may be written as:

$$\varepsilon_\ell[m] = s_\ell[m] - \hat{s}_\ell[m] \quad (56)$$

where  $\hat{s}_\ell[m]$  is the interpolated output and  $s_\ell[m]$  is the original quaternion signal. The error for each state space equation can be further defined as:

$$\varepsilon_{49}[50(m+1)+49] = \varepsilon_{sensor}, \quad (57)$$

$$\begin{bmatrix} \varepsilon_0[m] \\ \varepsilon_1[m] \\ \vdots \\ \varepsilon_{48}[m] \end{bmatrix} = \varepsilon_{sys}, \quad (58)$$

to represent the distinction between sensor error and system error. Now that the cubic spline has been derived into its discrete  $ABCD$  state space equations, the Kalman Filter may be applied.

## 2. Applying the Kalman Filter

Recall from Equations (20) and (21) that  $R$  and  $Q$  are the sensor noise and the system noise, respectfully. These two values represent the autocorrelations of the interpolation error. The true variance of the interpolation error is not known, so arbitrary variances must be assigned. But, the most important aspect is to ensure that the system noise,  $Q$ , is much greater than the sensor noise,  $R$ . Therefore, it will be assumed that the system noise variance is 100 and the sensor noise variance is 1.0. The autocorrelations may then be solved as:

$$R = E\{w[m]w[m]^T\} = E\{I_{48 \times 48} \varepsilon_{sensor} \varepsilon_{sensor}^T I_{48 \times 48}^T\} = \sigma_{sensor}^2 I_{48 \times 48}, \quad (59)$$

$$Q = E\{v[m]v[m]^T\} = E\{B \varepsilon_{sys} \varepsilon_{sys}^T B^T\} = \sigma_{sys}^2 B B^T, \quad (60)$$

where  $\sigma_{sensor}^2 = 1.0$  and  $\sigma_{sys}^2 = 100$ .

The error covariance matrix of the interpolated signal,  $P$ , is solved for iteratively throughout the Kalman Filter equations, but an initial value must be defined. The initial value will be assumed to be very high because the Kalman Filter will estimate it with precision after a few iterations. The initial condition of the error covariance matrix is defined as:

$$P[0] = E\{\hat{s}[0]\hat{s}[0]^T\} = 1000 I_{4 \times 4} \quad (61)$$

where 1,000 is an intentional overestimate of the error covariance matrix.

Lastly, the initial “guess” of the filter must be defined to be processed through the filter. An arbitrary value of zero can be assigned to the initial value of the coefficients:

$$\hat{x}[0] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (62)$$

where  $\hat{x}[n]$  is the estimated state of the coefficients. Assigning the initial guess to zero will result in a transient response of the Kalman Filter, however, the transient response will not cause any issues if the data is initially extended like it was in Section V.B.2.

Now that the Kalman Filter constants have been calculated, the equations in (26) may be applied. The coefficient estimation process using the Kalman Filter is shown below.

$$u[m] = s[50(m+1) + 49] \quad (63)$$

$$y[m] = \begin{bmatrix} s[50m] \\ s[50m+1] \\ \vdots \\ s[50m+48] \end{bmatrix} \quad (64)$$

$$\hat{x}_-[m+1] = A\hat{x}_+[m] + Bu[m] \quad (65)$$

$$P_-[m+1] = AP_+[m]A^T + Q \quad (66)$$

$$K[m] = P_-[m]C^T (CP_-[m]C^T + R)^{-1} \quad (67)$$

$$\hat{x}_+[m] = \hat{x}_-[m] + K[m](y[m] - C\hat{x}_-[m]) \quad (68)$$

$$P_+[m] = P_-[m] - K[m]CP_-[m] \quad (69)$$

Recall that  $y[m]$  includes the mirror and flipped quaternion data. Note that  $\hat{x}[m]$  is a 4x1 vector and each element changes after each iteration. The last element of  $\hat{x}[m]$  is the most accurate, optimal coefficient. So, the vector of coefficients that are derived from the Kalman Filter should be obtained by taking the last element of  $\hat{x}[m]$  after each iteration.

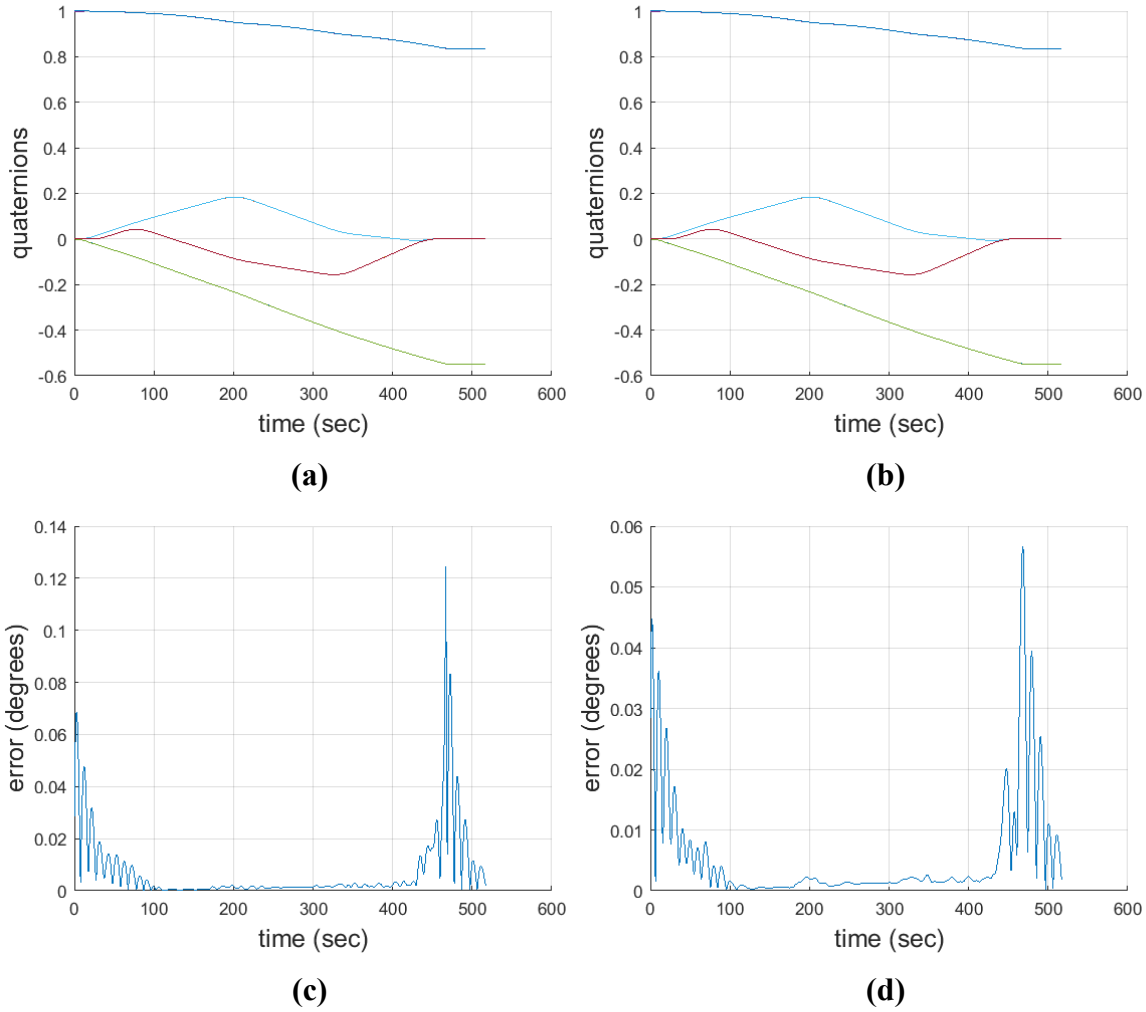
Equations (63) through (69) are used for each individual quaternion, so each quaternion will have a separate set of coefficients.

### **C. KALMAN FILTER RESULTS**

The Kalman Filter results are analyzed like the spline interpolation results from the previous chapter. Here, equations (63) through (69) are used to generate coefficients for the same set of quaternion curves that were used in Figure 30. These coefficients were then processed through the DFII filter bank from Figure 29. The results below compare the output of the Kalman Filter coefficients to the original quaternion curves as well as the output of the cubic spline coefficients.

Plots (a) and (b) in Figure 34 contain two sets of four quaternion curves that are so similar that the difference can only be visualized by zooming in on the data. The similarities are good because they mean that the interpolated signal is accurate. The error plots in (c) and (d) yield more detailed information. The error plots in (c) and (d) were calculated from Equations (29) through (31).

There are two subtle differences between the outputs of the cubic spline coefficients and the Kalman Filter coefficients. These differences can be observed by analyzing the plots from Figure 30c and Figure 34c. First, as previously noted, the spline coefficients attain zero error at every sample point, or every 10 s. The Kalman Filter coefficients do not quite reach zero error at every sample point, but they do slightly spread the error which results in a lower maximum error. The error peaks at  $0.175^\circ$  using the spline and  $0.125^\circ$  using the Kalman Filter. Second, the Kalman Filter has a slightly higher mean error than the spline output due to the error spreading. Even though the Kalman Filter coefficients produce a higher mean error, they still seem like a more attractive option than the spline coefficients because they result in a lower peak error, which is most important when considering the star tracker occultation avoidance constraints.



(a) Dashed lines are the original quaternion data from FastMan and the solid lines are the Kalman Filter coefficients output; (b) dashed lines are the cubic spline coefficients output and the solid lines are the Kalman Filter coefficients output; (c) quaternion angle difference in degrees from plot (a); (d) quaternion angle difference in degrees from plot (b).

Figure 34. Simulation results from MATLAB using the DFII filter bank comparing the Kalman Filter coefficients to the cubic spline coefficients

Plot (d) in Figure 34 does not have a significant impact, but rather, it just shows that there is a noticeable difference between the Kalman Filter coefficients output and the cubic spline coefficients output. The two main differences are at the beginning and the end of the maneuver because these sections have the sharpest “turns” in the original data, which may cause partial discontinuities. The two methods address discontinuities uniquely, which is why the difference between their outputs spike at these locations.

Overall, the resulting data from the Kalman Filter coefficients was favorable, as expected. The output of the Kalman Filter coefficients produced an optimized curve that lowered the maximum error by about 28.5%. It must be noted, however, that the results from this chapter and the previous chapter are produced from simulations with ideal conditions. The next chapter will provide results from more precise simulations that include filter timing delays and random thermal noise within the ACS.

This chapter explained how to convert the theoretical cubic spline filter into state space equations so that the Kalman Filter could be used to optimize the spline coefficients. The Kalman Filter in this chapter was designed to produce an optimized set of coefficients given a set of quaternion data. A simulation was conducted to assess the output of these coefficients and to compare their output to that of the cubic spline coefficients. The results concluded that the Kalman Filter coefficients produced an output with a higher mean error but a lower maximum error, which is favorable for ensuring the star tracker occultation avoidance constraints are met. The final step of this thesis is to run “real-world” LRO simulations that include timing delays and thermal noise. These simulations are conducted in the next chapter.

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## **VII. LRO SIMULATION RESULTS**

### **A. REQUIRED SIMULATIONS**

So far, phases one through five from Figure 3 (in Chapter I) have been accomplished. Now that there are two reliable methods of generating coefficients and processing them through a DFII simulation, this chapter will compare them in practical LRO ACS response simulations to complete phases six and seven.

Phase six consists of simulating the LRO attitude control system in MATLAB. This simulation includes filter timing delays and LRO reaction wheel constraints for angular rates and torques. Phase seven consists of a similar procedure but using the NASA high fidelity (Hi-Fi) simulation software, which includes the aforementioned constraints as well as system noise and uncertainty. Successful results from the NASA simulation are required before the interpolation filter may be installed on the LRO.

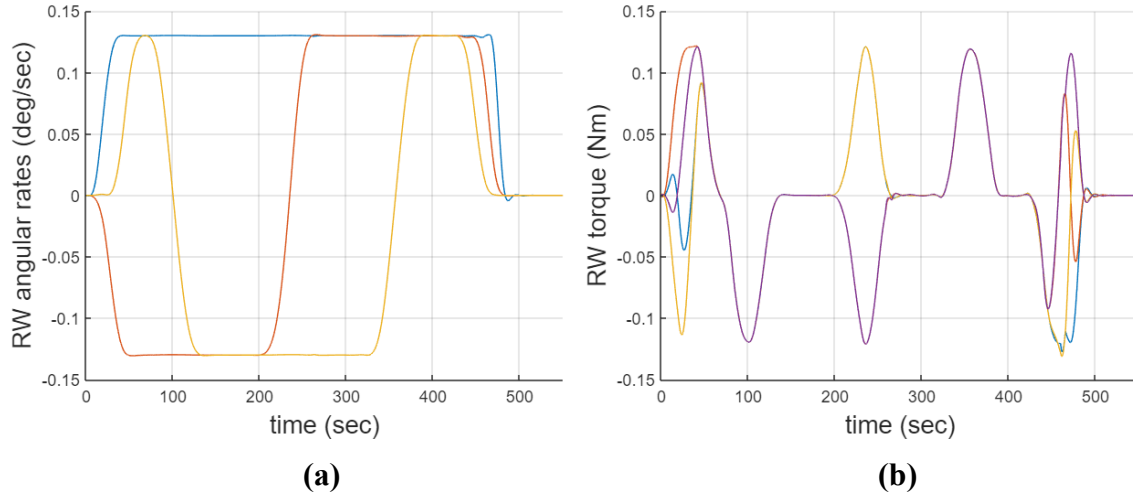
### **B. REACTION WHEEL CONSTRAINTS**

The spacecraft body rate is limited to  $0.13^\circ/\text{s}$  to avoid excessive gyroscopic torque, which can overpower the reaction wheels [8]. The maximum torque the reaction wheels can tolerate is  $0.2 \text{ N} \cdot \text{m}$ . Figure 35 shows a typical plot of the angular rate and torque curves for an LRO fast maneuver.

The quaternion commands for FastMan are designed to maximize the agility of the spacecraft. Therefore, the spacecraft travels at its maximum angular rate for most of the maneuver as seen in Figure 35a. In each of the simulations to follow, the reaction wheel angular rates are observed to not violate any constraints. This ensures that no functional issues arise when the interpolation filter is used for real maneuvers.

If the LRO conducts a slew at its maximum angular rate, then it will take approximately 7.7 s to rotate one degree. Since the spline interpolation filter can take samples every 10 s, then maneuvers can be conducted at a rate of less than one command for every degree of motion.

Plot (b) in Figure 35 shows the reaction wheel torques that are applied throughout the maneuver. The torque,  $\tau_{cmd}$ , is measured in the ACS. The following simulations do not exceed the torque constraints of  $0.2 \text{ N} \cdot \text{m}$ .



(a) Angular rates; (b) torques.

Figure 35. Reaction wheel parameters for a FastMan simulation using cubic spline interpolation.

The reaction wheel plots will not be shown for the following simulations. Figure 35 is provided as an example to the reader to better understand constraints acting on the LRO ACS throughout maneuvers.

### C. ACS ARCHITECTURE

The DFII filter bank that is recommended by this thesis is designed as an ideal method to conduct operational FastMan slews. Figure 36 shows the recommendation to NASA on how the interpolation filter may be inserted into the ACS architecture. The DFII filter bank will be designed to have an IF statement to conditionally turn on the filter. LRO maneuvers may be planned as the traditional single command maneuvers when star tracker constraints are not an issue, in which case the DFII filter bank is not needed. But many maneuvers require star tracker occultation avoidance measures and must use the DFII filter bank to interpolate and compress the number of commands required.

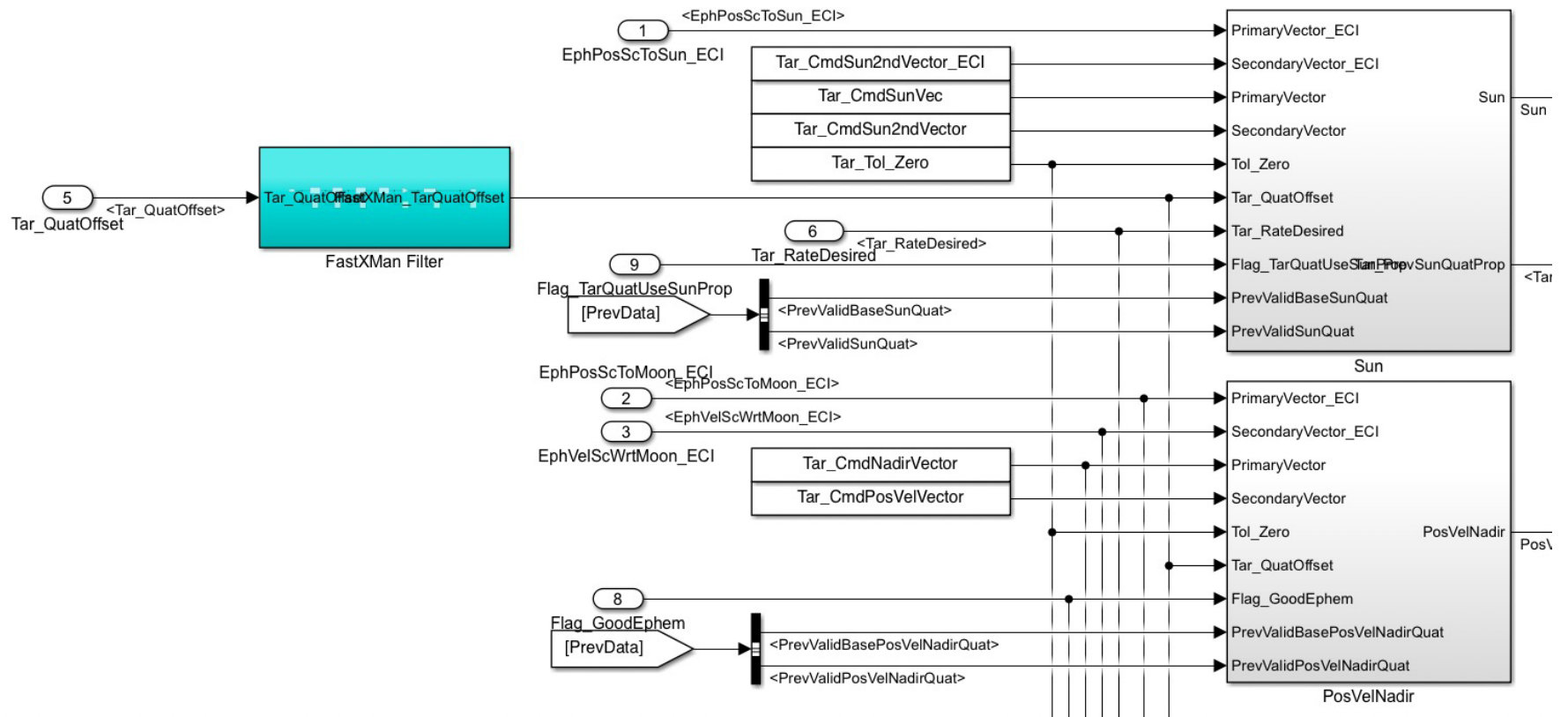


Figure 36. Diagram of the LRO ACS architecture where the blue box is the recommended DFII filter bank that may be used or bypassed. Source: [20].

#### D. ERROR ESTIMATIONS

There are three different error calculations to consider in each simulation to measure the performance of the interpolated signals. The first is the interpolation error. The interpolation error is the difference between the original 5 Hz commands for FastMan and the interpolated commands produced by the DFII filter bank given the 0.1 Hz spaced coefficients. The second error is the controller tracking error, which is inherent to how the system is designed to operate. The controller tracking error is the difference between the system input and the system output and is regulated by the gain values of the system. The final and most important error to consider is the system response error. This error consists of the difference between how the system would operate if it was fed the original 5 Hz commands and how it would operate if it was fed interpolated commands derived from the 0.1 Hz space coefficients. Figure 37 is a simplified graphic showing where each performance metric is measured in the LRO second order control system.

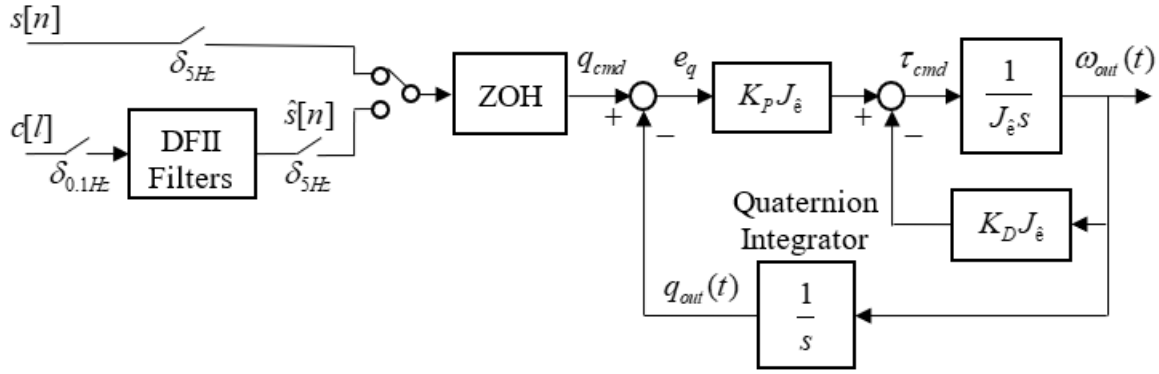


Figure 37. Simplified block diagram of the LRO second-order ACS with the option of using the DFII interpolation filter to generate 5 Hz commands.  $J_{\hat{e}}$  is the inertia about the rotational axis. Adapted from [8].

From the diagram in Figure 37,  $s[n]$  represents the four original 5 Hz quaternion curves created for FastMan. The four estimated 5 Hz quaternion curves,  $\hat{s}[n]$ , are created by passing the 0.1 Hz spaced coefficients,  $c[l]$ , through the DFII filter bank from Figure 29. The operators will have the option to either use the  $s[n]$  or  $\hat{s}[n]$  to produce the

commands,  $q_{cmd}$ , for the ACS. The angular rate,  $\omega_{out}(t)$ , is the output of the spacecraft. When angular rate is integrated by a quaternion integrator, the spacecraft output quaternion values,  $q_{out}$ , are produced.

The three, error performance metrics mentioned previously are easy to identify by observing Figure 37. The interpolation error is the minimum angular difference between  $s[n]$  and  $\hat{s}[n]$ , where each set of quaternions are first converted to degrees off the eigenaxis. The controller tracking error (Figure 38), represented by  $e_q$ , is the difference between the system output,  $q_{out}(t)$ , and the system input,  $q_{cmd}$ . The system response error is the difference between the system output,  $q_{out}(t)$ , if  $s[n]$  is used compared to if  $\hat{s}[n]$  is used. The goal in the following simulations is to illustrate the system response error when using the interpolation filter.

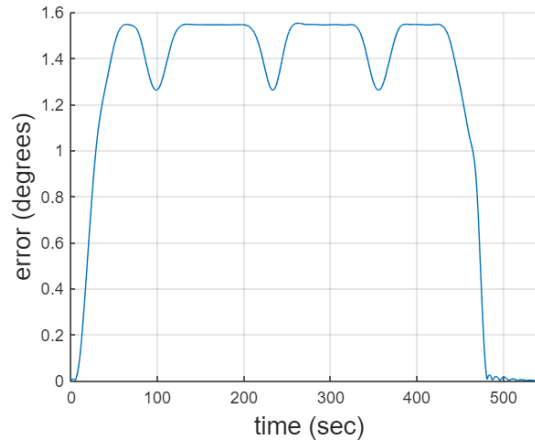


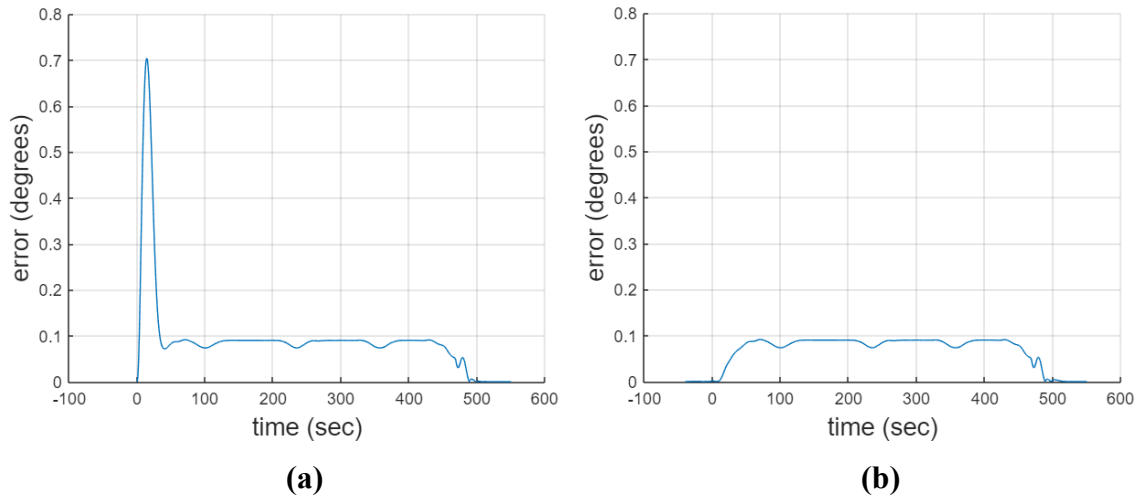
Figure 38. Example of controller tracking error in degrees for a typical maneuver

The system response error is the most important error to monitor because it measures how well the interpolated commands (50:1 compression ratio) operate compared to the ideal scenario of running all the original 5 Hz commands. The controller tracking error is the least significant error to monitor because it is inherent to the system and is essentially the definition of a feedback control system. The system technically needs some

magnitude of error as it uses a PD controller (type 0), which has a non-zero tracking error. An example of the controller tracking error for a typical maneuver is shown in Figure 38. The following simulation results do not display the controller tracking error for the reasons mentioned above, and because each plot is typically the same for all maneuvers.

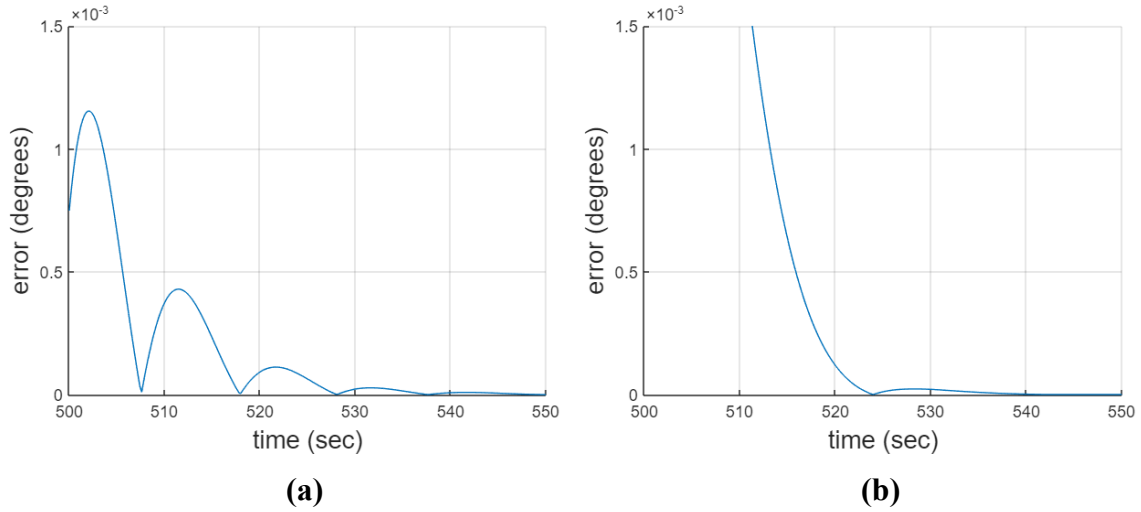
#### E. MINIMUM NUMBER OF COEFFICIENTS PER MANEUVER

The coefficients used in the following simulations are designed to achieve a compression ratio of 50:1. However, there is a minor caveat. Four additional coefficients are needed at the beginning of the maneuver to minimize the transient response and one extra coefficient is needed at the end of the maneuver to achieve the precise target attitude. Figure 39 shows the difference between a maneuver with and without initial stabilizing coefficients. Note that there are four coefficients fed 40 s before the maneuver begins. The four coefficients are each  $c = [0 \ 0 \ 0 \ 1]$ , which results in the normal operating nadir-pointing attitude of the LRO to provide a smooth transition into the interpolated slew.



(a) No initial conditions are used; (b) four nadir-pointing coefficients are used to minimize the transient response.

Figure 39. Difference in system response error for an arbitrary maneuver with and without initial conditions.



(a) Coefficients are unchanged to cause oscillations; (b) last coefficient is equal to the target attitude resulting in zero error.

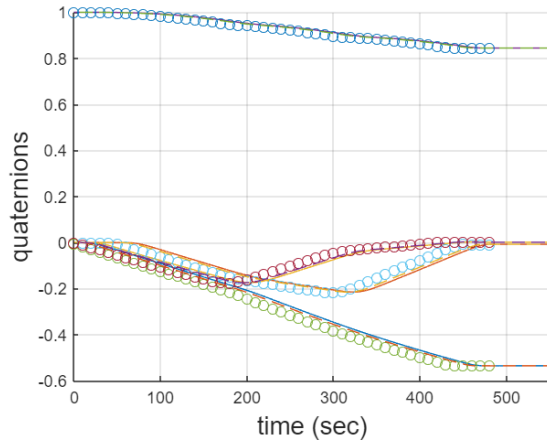
Figure 40. Magnified view of the last 50 s of the plots in Figure 39.

The error difference at the beginning of the maneuver is easy to identify in Figure 39. The transient response error reaches  $0.7^\circ$  in plot (a), whereas the maximum error in plot (b) is less than  $0.1^\circ$ . The end of the maneuver also has a difference in error. A coefficient equal to the target quaternion attitude is added to the end of the coefficients in plot (b). The DFII filters naturally have a memory of the last command given. Therefore, the last coefficient fed to the system will repeat indefinitely. By making the last coefficient equal to the target attitude, the LRO will stabilize on the target. However, if the last coefficient is not changed, then the LRO will experience small oscillations at the end of the maneuver, which may interfere with science collection. Figure 40 shows a magnified view of the two plots in Figure 39 to show the effect of setting the last coefficient equal to the target attitude.

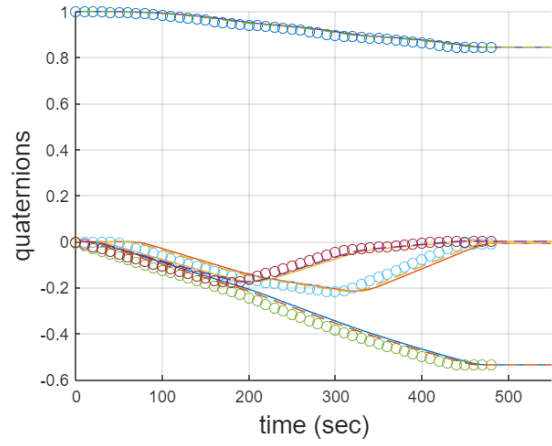
## F. MATLAB SIMULATION RESULTS

A MATLAB simulation code was produced for this section to replicate the LRO ACS. Historic quaternion data from past LRO maneuvers (Figure 41 through Figure 44) was used as input to the code to simulate and analyze the LRO response if it were to be fed the 0.1 Hz spaced coefficients. The maneuvers were hand selected to ensure the simulations run through a wide variety of slews. The left column of each figure displays the results of

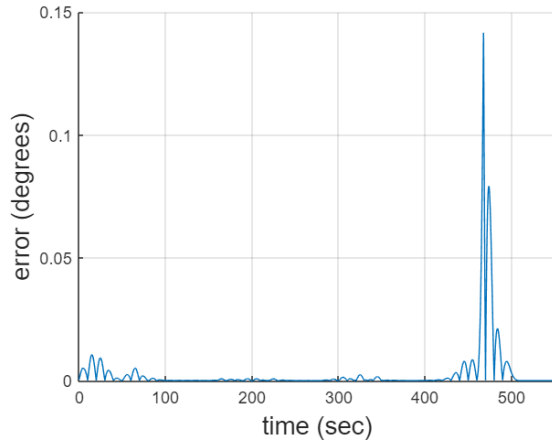
the spline coefficients, and the right column displays the results of the Kalman Filter coefficients.



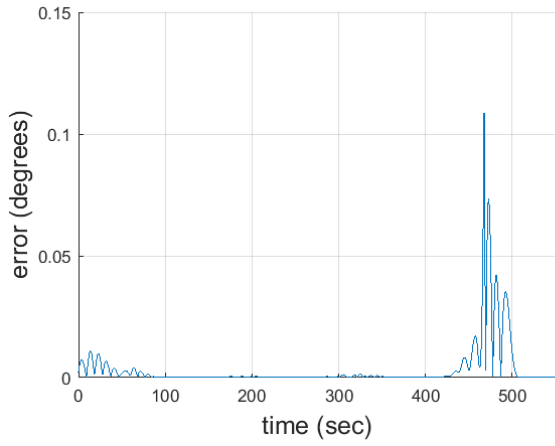
**(a)**



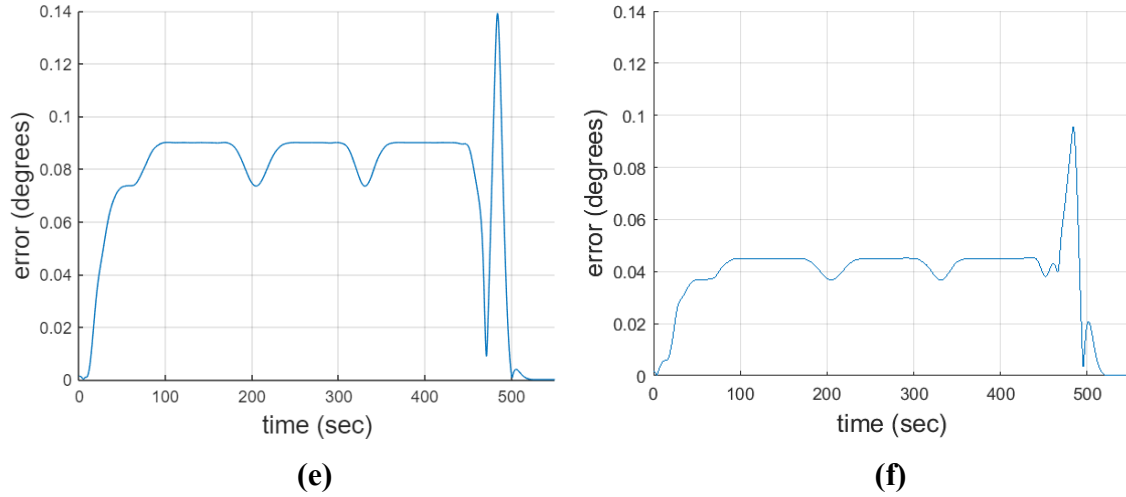
**(b)**



**(c)**

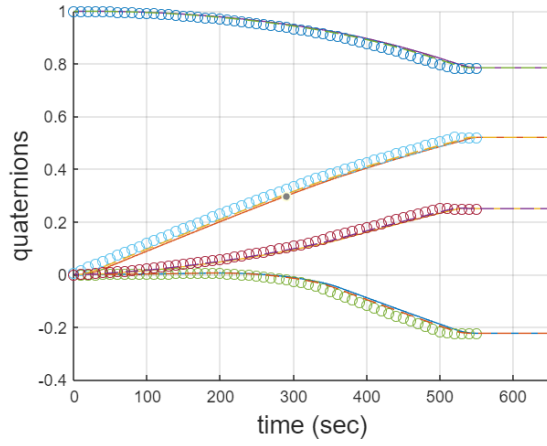


**(d)**

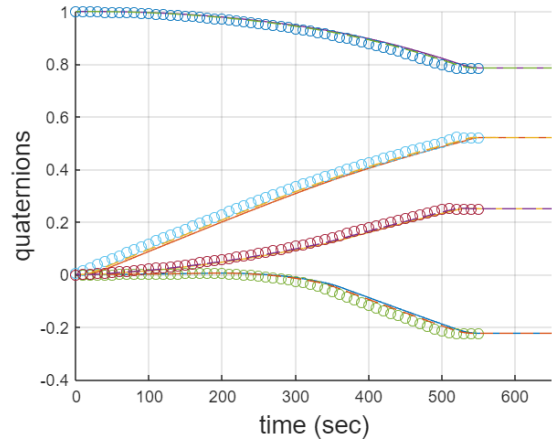


(a) Circles are input spline coefficients, dashed lines are the interpolated commands, and solid lines represent the system response attitude; (b) circles are input Kalman Filter coefficients, dashed lines are the interpolated commands, and solid lines represent the system response attitude; (c) spline interpolation error of the original quaternions; (d) Kalman Filter interpolation error of the original quaternions; (e) system response error between output of the 0.1 Hz spline coefficients and output of the original 5 Hz quaternions; (f) system response error between output of the 0.1 Hz Kalman Filter coefficients and output of the original 5 Hz quaternions.

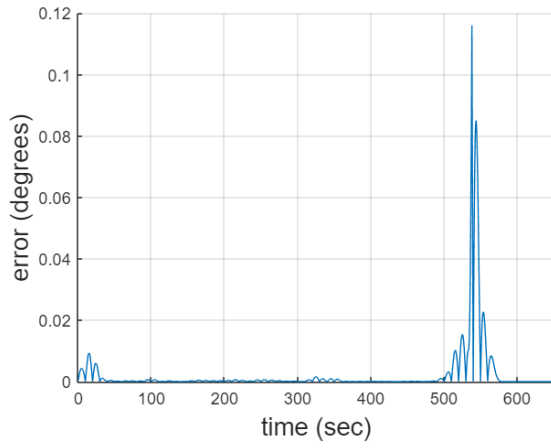
Figure 41. FastMan trajectory originally implemented on August 22, 2021, for a  $64.8^\circ$  slew in support of a LROC mission



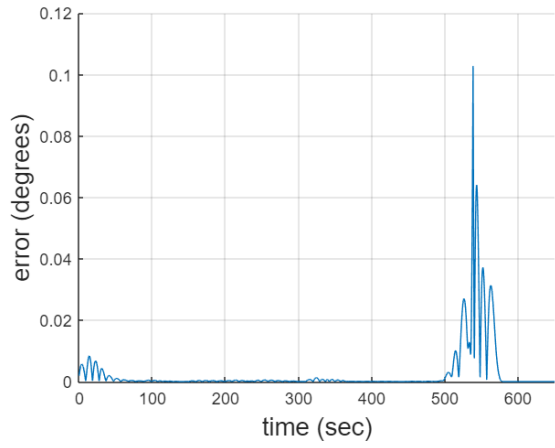
**(a)**



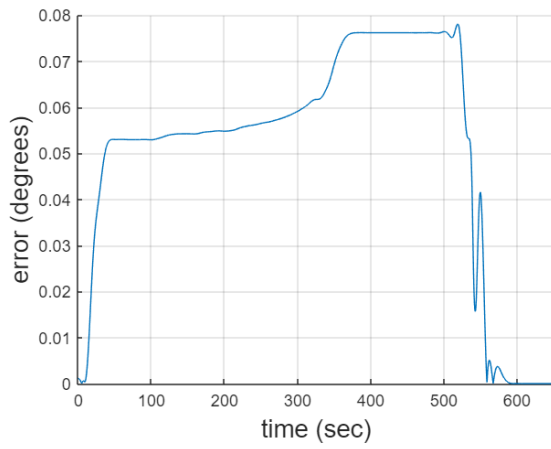
**(b)**



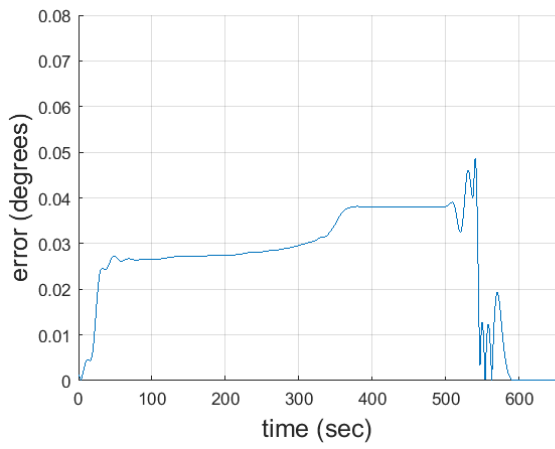
**(c)**



**(d)**



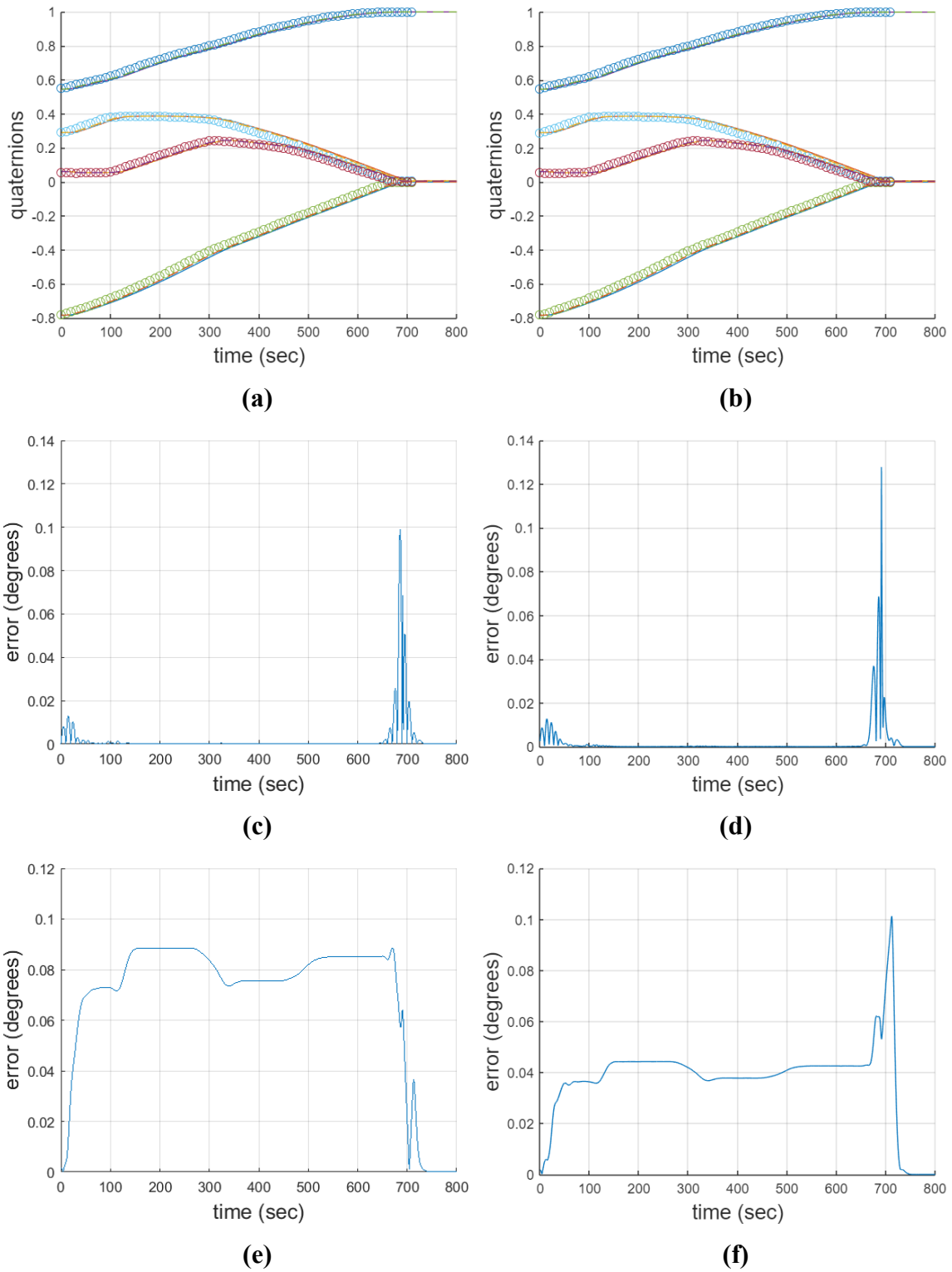
**(e)**



**(f)**

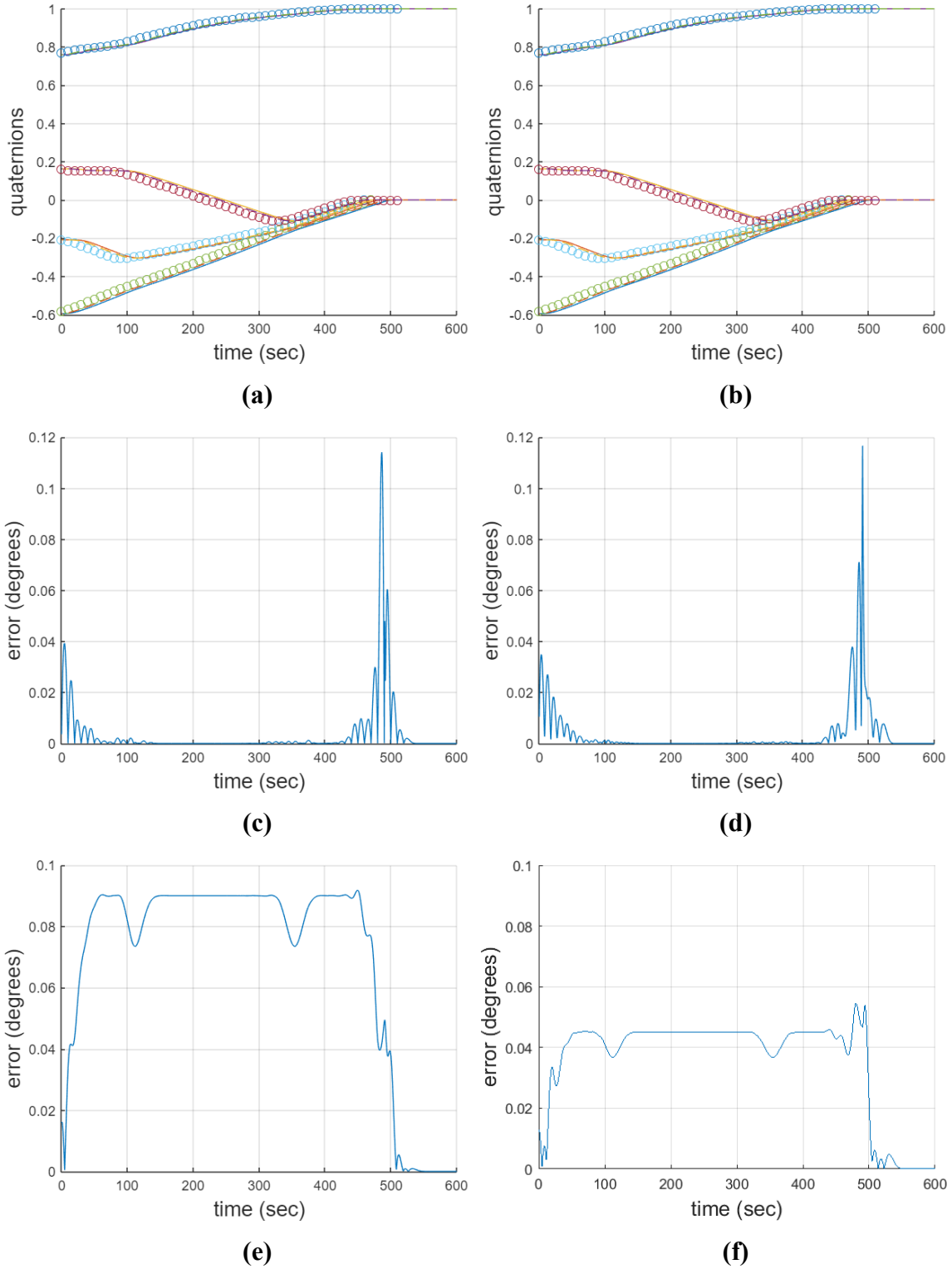
(a) – (f) refer to Figure 41.

Figure 42. FastMan trajectory implemented on July 6, 2021, for a  $76.5^\circ$  slew in support of a Mini-RF mission.



(a) – (f) refer to Figure 41.

Figure 43. FastMan trajectory implemented on August 6, 2021, for a  $113.6^\circ$  slew in support of a LOLA mission.



(a) – (f) refer to Figure 41.

Figure 44. FastMan trajectory implemented on March 4, 2022, for an  $81.2^\circ$  slew in support of a LAMP mission.

The (a) and (b) plots from Figure 41 through Figure 44 represent the LRO response to spline coefficients and Kalman Filter coefficients, respectively. The circles represent the series of coefficients, the dashed lines represent the DFII filter bank interpolated output, or the interpolated command quaternions, and the solid lines represent the true quaternion values of the LRO, or the system response. Figure 45 shows a magnified view of one of these plots to better visualize the difference between each component. Note that there is a 20.4 s delay between the insertion of the coefficients and the interpolated output due to the two 10 s delays from the DFII filters operating discretely at 0.1 Hz and the two 0.2 s delays from the DFII filters operating at 5 Hz. The coefficient commands can only be timestamped at integers values of seconds, so they are advanced by 20 s to minimize the filter delay error. The resulting 0.4 s delay that cannot be accounted for is the reason for the steady state system response error. The delay between the interpolated commands and the system response is inherent to the system as it is a function of the gains of the ACS and the inertia about the axis of rotation.

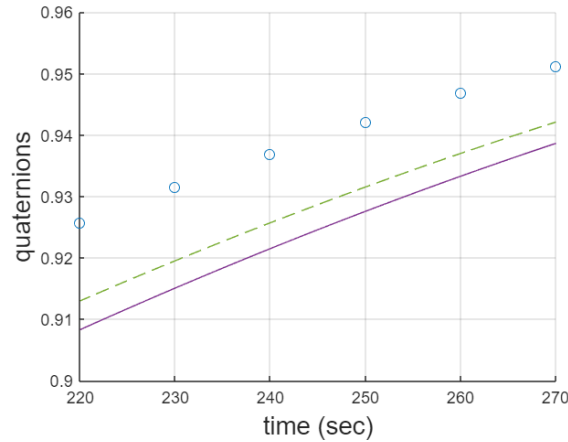


Figure 45. Magnified view of Figure 44a

The (c) and (d) plots from Figure 41 through Figure 44 represent the interpolation error between the DFII interpolated output and the original 5 Hz commands. Recall from Chapter V that the spline interpolation error dropped to zero at every coefficient cycle (every 10 s). In these simulations, the interpolation error also drops to zero at each cycle,

however, there is a filter delay that causes the system response error to have a steady state error. Recall from Chapter VI that the Kalman Filter optimizes the error by ‘spreading’ it out. The spreading of the error is why these error results do not drop to zero at each cycle. The Kalman Filter coefficients optimize the system response by minimizing the error.

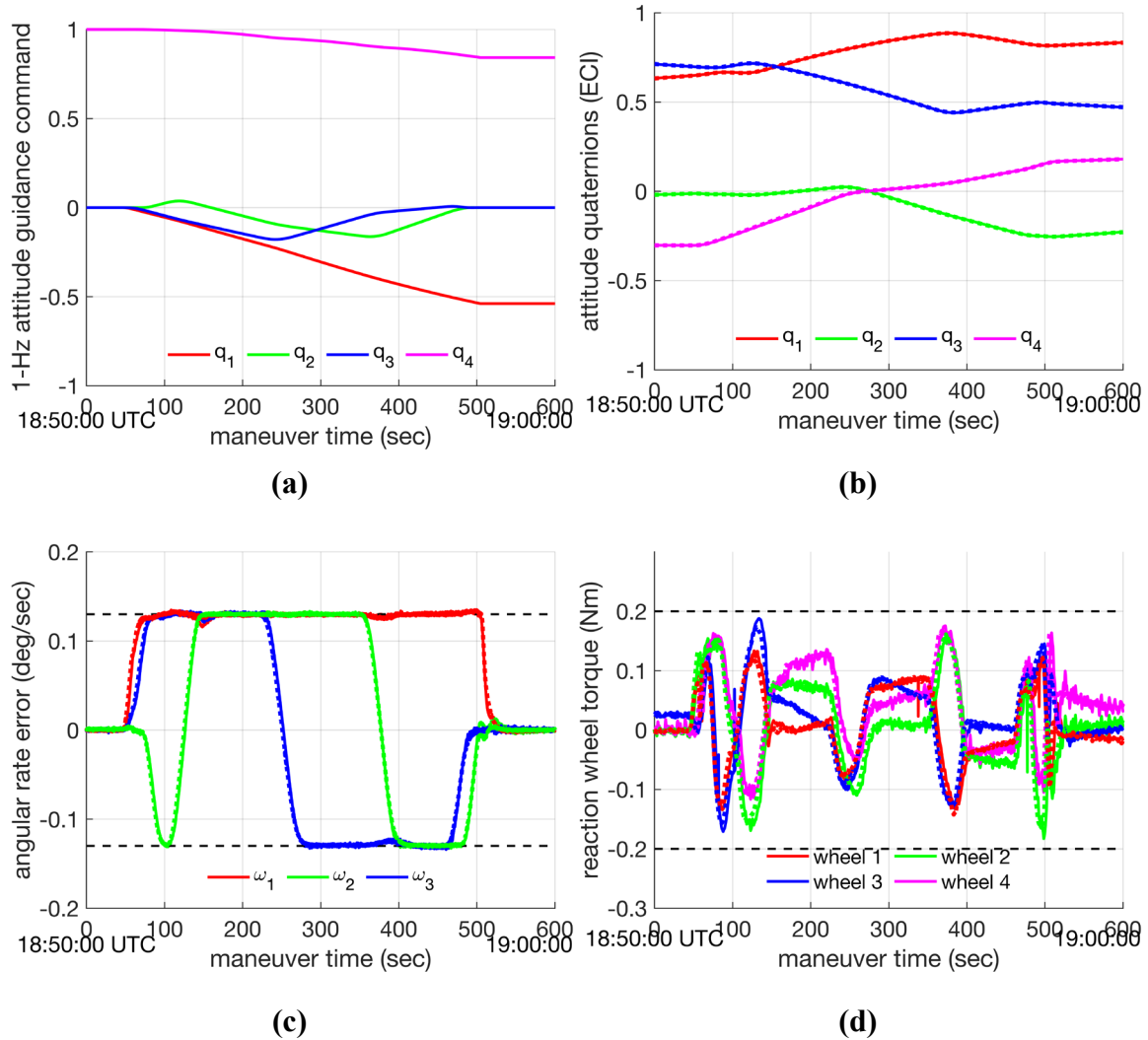
The (e) and (f) plots from Figure 41 through Figure 44 display the system response error, which is the difference between the ACS output when the DFII interpolated commands are used and the ACS output when the original commands are used. The system response error maintains a steady value, which is mostly due to the 0.4 s delay from the 5 Hz DFII filters. There is also a sharp peak in error towards the end of the maneuver because of the sharp change in the acceleration of the original curves where the system reaches its target attitude (discussed in Section V.D). Overall, the system response error was better than expected.

The system response error of the spline coefficients and the Kalman Filter coefficients were both excellent, but the Kalman Filter coefficients proved to be better. The response error using the Kalman Filter coefficients had a mean error less than  $0.05^\circ$  and a max peak error of about  $0.1^\circ$ . These maneuvers that would originally take thousands of commands to process, would now only require less than 100 commands. Additionally, the resulting system response error is small enough to avoid violating star tracker occultation avoidance constraints, so the process is safe for flight operations. These results show that the command buffer data limitations mentioned in Section II.D can be resolved by using the interpolation methods presented in this thesis.

## **G. NASA HI-FI SIMULATION RESULTS**

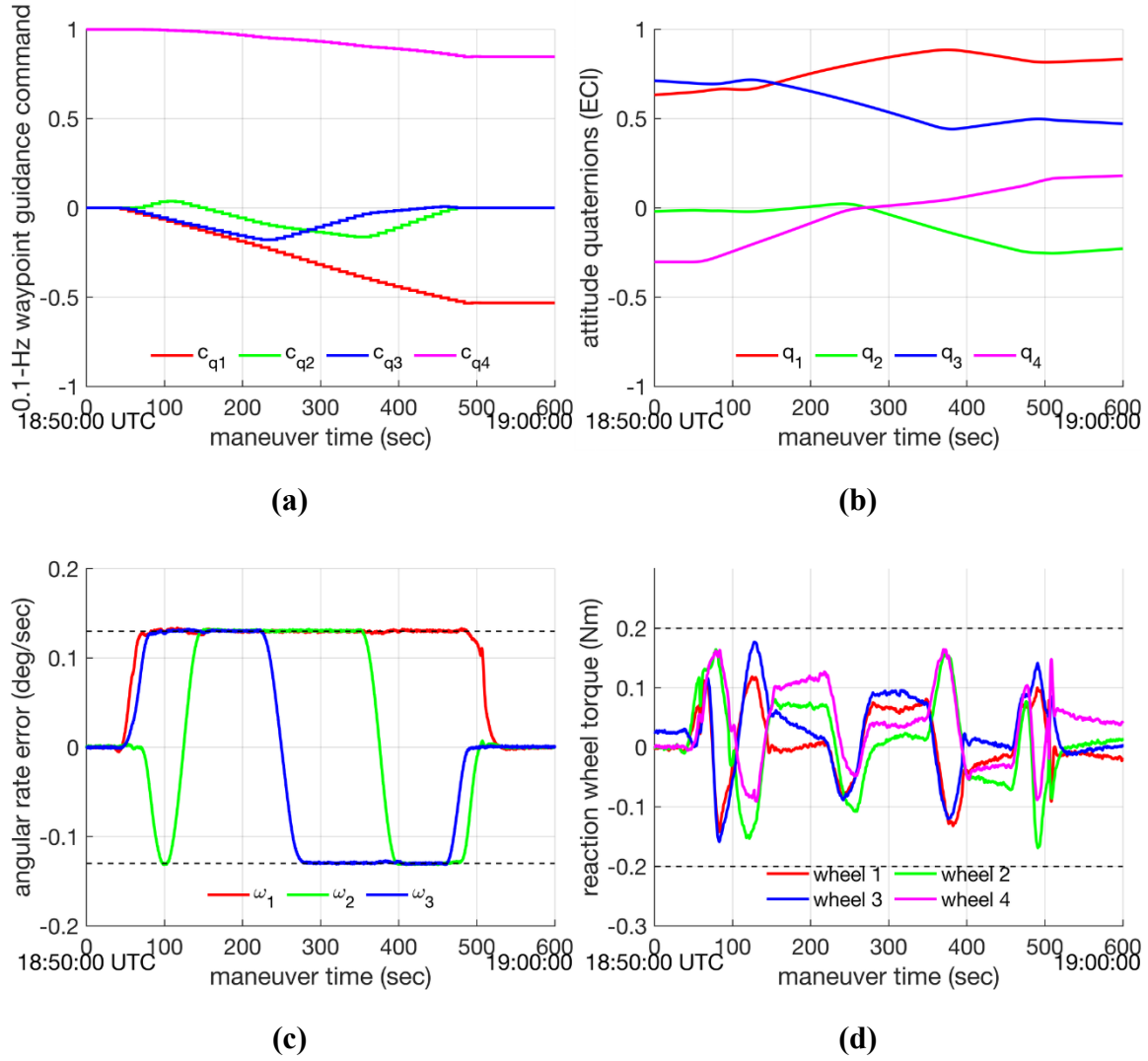
Hi-Fi produces the most accurate simulation results as it replicates the LRO control system and includes various practical noise sources. However, the Hi-Fi output is more difficult to analyze than the MATLAB output because the numerical data cannot be easily manipulated. So, instead of calculating the error from the Hi-Fi plots, the results may be visually compared to the telemetry data of a previous maneuver. Therefore, the purpose of these Hi-Fi simulations is to ensure there are no glaring issues with the interpolation filter. For numerical error results, the previous MATLAB simulations are the best source.

The first simulation tests the interpolation filter using cubic spline coefficients for a FastMan slew conducted on August 25, 2021, to image lunar volcanic landforms. The telemetry data and the filter simulation results are shown in Figure 46 and Figure 47, respectively.



(a) ACS commands at 1 Hz; (b) LRO attitude in ECI frame; (c) reaction wheel angular rates; (d) reaction wheel torques.

Figure 46. Data from a FastMan slew conducted on August 25, 2021. Solid lines represent the telemetry data and dotted lines represent the predictions from Hi-Fi.

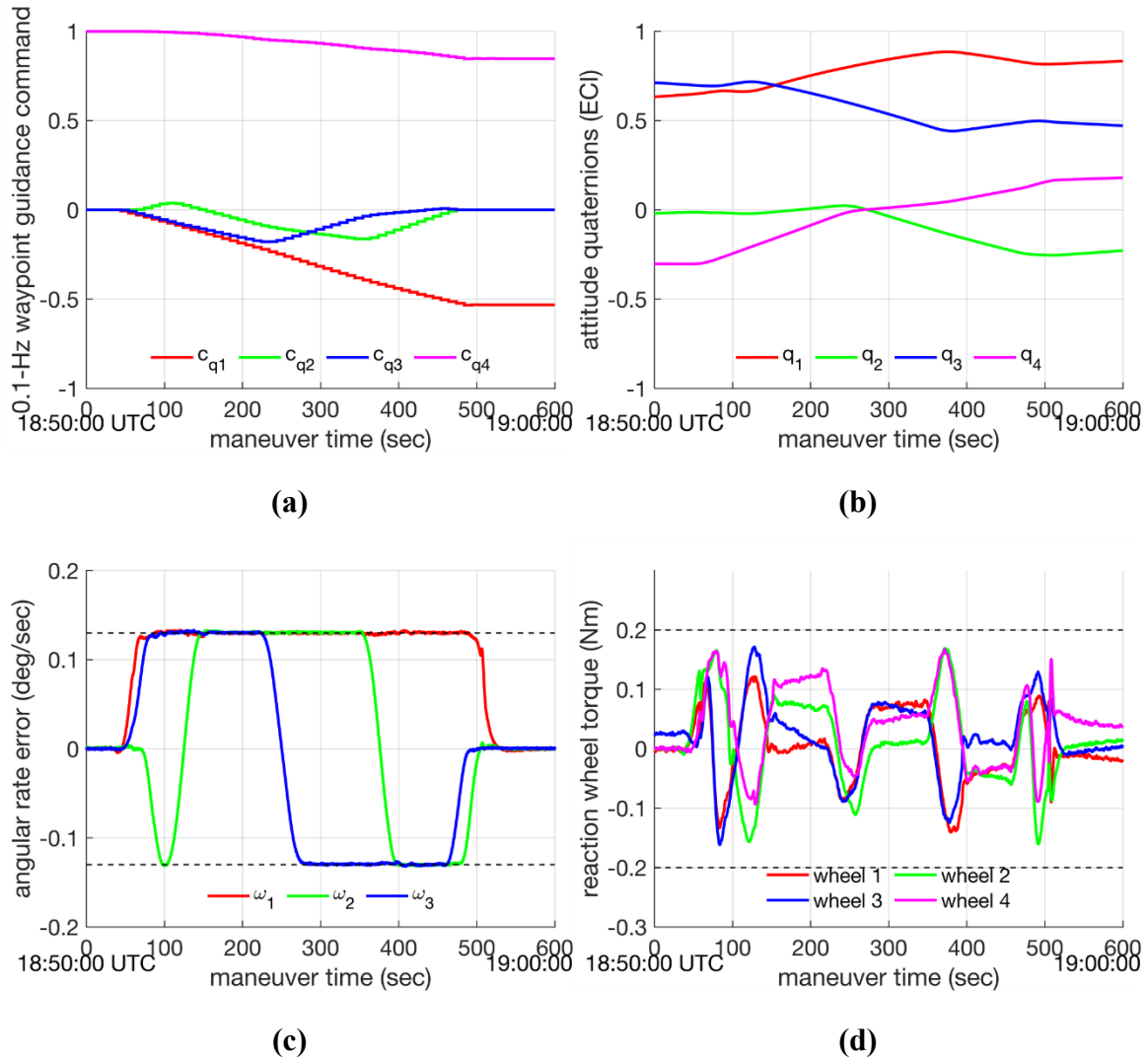


(a) Cubic spline coefficient commands at 0.1 Hz; (b) LRO attitude in ECI frame; (c) reaction wheel angular rates; (d) reaction wheel torques.

Figure 47. Hi-Fi simulation results using the interpolation filter with the cubic spline coefficients for the same maneuver in Figure 46.

Plot (a) in Figure 47 shows the coefficient commands that are executed every 10 s. This plot is in the OCS coordinate frame. Plot (b) shows the LRO attitude response in the inertial coordinate (ECI) frame. The two plots do not have a similar shape because they are in different coordinate frames. Plots (c) and (d) in Figure 47 show that the rotational rate and the torque constraints are not violated during this maneuver. Since the numerical results and the coordinate frames from Hi-Fi cannot be easily manipulated, these results

can be visually compared to the telemetry data in Figure 46. All the plots between the two figures look almost identical, which shows that the interpolation filter operates as expected when using the cubic spline coefficients. The second simulation tests the Kalman Filter coefficients for the same maneuver. The results are shown in Figure 48.



(a) Kalman Filter coefficient commands at 0.1 Hz; (b) LRO attitude in ECI frame; (c) reaction wheel angular rates; (d) reaction wheel torques.

Figure 48. Hi-Fi simulation results using the interpolation filter with the Kalman Filter coefficients for the same maneuver in Figure 46.

Again, all the plots within Figure 46 and Figure 48 are nearly identical. Since there are no distinct differences between the interpolation filter output and the telemetry data, there is high confidence that the interpolation filter will work as expected when it is implemented on the spacecraft. Although there are no visual differences in the Hi-Fi output between the spline coefficients and the Kalman Filter coefficients, the MATLAB simulations demonstrated that the Kalman Filter method had lower interpolation error overall. Therefore, calculating the coefficients using the Kalman Filter is the recommended method of choice. The image in Figure 49 is an example of what LROC can collect when using FastMan slews.

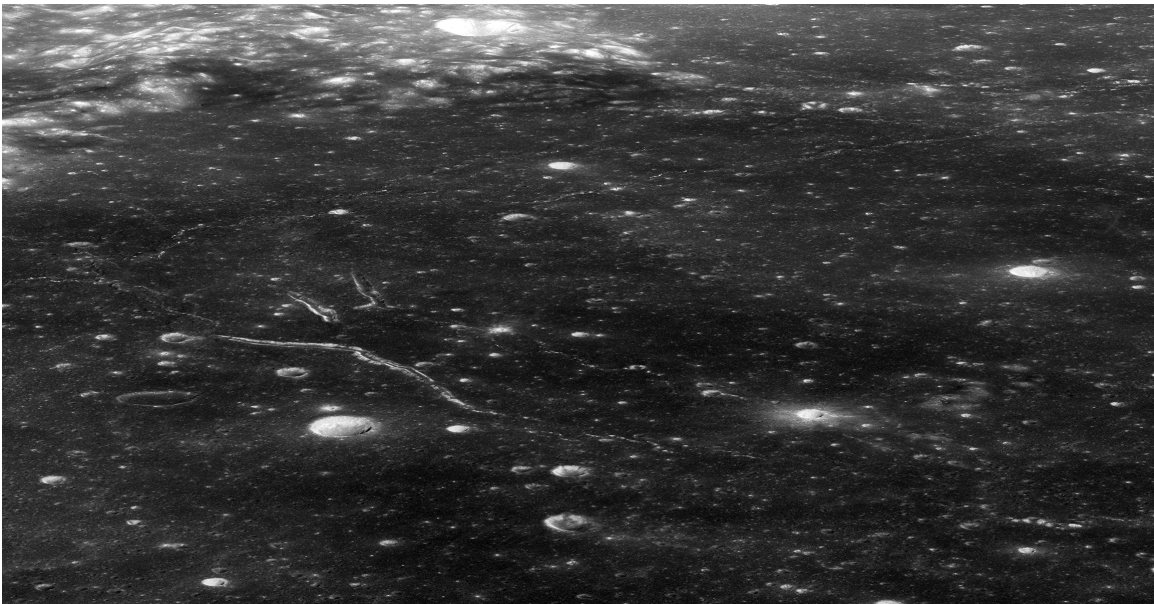


Figure 49. Volcanic landforms near Mare Serenitatis imaged by the LRO on August 25, 2021. This image was captured following the FastMan maneuver profile in Figure 46. (Courtesy of NASA/GSFC/Arizona State University. Image number: M1384557239LR).

Simulations were needed to ensure the designed DFII filter bank operated as expected. A comprehensive MATLAB code was created to replicate the control system of the LRO. Four simulations of various maneuver profiles previously conducted on the LRO were used in this MATLAB code. These simulations provided the LRO output attitude given various factors to include filter timing delays, error feedback delays, rotational rate

constraints, and torque constraints. The results of the cubic spline coefficients and the Kalman Filter coefficients were run side by side for each maneuver. Though the error was practically insignificant for each method, the Kalman Filter coefficients proved to be slightly better. Of the four simulations, the lowest steady state error for the spline coefficient method was about  $0.08^\circ$  while the max peak error was about  $0.14^\circ$ . The lowest steady state error for the Kalman Filter coefficient method was about  $0.04^\circ$  while the max peak error was about  $0.1^\circ$ . These results show that the filter can operate with a compression ratio of 50:1 without sacrificing the signal resolution. Thus, according to these simulations, this filter can be operated with confidence that the LRO star tracker constraints will not be violated.

The final experiment was to simulate the interpolation filter using the NASA Hi-Fi simulation program. The results of two simulations are presented to show the attitude response of the interpolation filter compared to the telemetry data of previous LRO FastMan maneuvers. Both simulations were nearly identical to the telemetry data, which means that there is high confidence that the filter will operate as expected on the spacecraft.

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## VIII. CONCLUSION

In 2018, the LRO inertial measurement unit degraded, so the spacecraft began using its two star trackers for rotational rate measurements. Due to occultation avoidance constraints of these star trackers, there are certain orientations that are unachievable for the LRO. Thus, there are many times when conventional maneuvers are not an option during science collection missions. The LRO has lost several science collection opportunities due to the star tracker constraints. The FastMan trajectory accounts for the star tracker constraints and computes the shortest-time path to the target orientation. Therefore, the use of FastMan is required to enable more science collection missions.

The Lunar Reconnaissance Orbiter is not currently equipped to perform time-optimal maneuvers as part of its daily operation. FastMan trajectories, which are the time-optimal maneuvers specific to the LRO, require a tracking-based approach when they are implemented using a classical control system. A single FastMan slew typically requires thousands of tracking-based commands, which quickly saturates the LRO command storage buffers. Consequently, only about three FastMan slews can be stored on the LRO. To meet the needs of daily operations, the goal is to store a week of maneuvers (seven per day or 50 maneuvers per week).

This thesis explored the development of a cubic spline interpolation filter to compress the data required for FastMan maneuvers without sacrificing signal resolution. An interpolation filter was designed to replicate the function of a cubic spline interpolator. The B-spline interpolation method was chosen for two distinct reasons. First, the cubic spline exhibits the minimum curvature property, which is convenient to use for maneuver trajectories. Second, the cubic spline requires only four sequential data points to interpolate any given point, whereas other interpolation methods may require the global set of data samples to interpolate each point.

It was found that the practical application of the cubic spline filter can be done with minimal modifications to the flight software to ensure a cost-effective approach. The design consisted of using a series connection of already-installed DFII filters. The filters

were placed in a Cascaded Integrator Comb construct to replicate the function of a cubic spline filter. The interpolation process was further improved by using the Kalman Filter to optimize the down-sampled coefficients of the cubic spline.

The simulations conducted using MATLAB demonstrated that the Kalman Filter coefficients yielded the best results. Several previous LRO maneuver trajectories were simulated and analyzed with the designed interpolation filter. It was found that the interpolated curves yielded an output error that peaked at about  $0.1^\circ$ . The error proved to be satisfactory because it ensures the star tracker constraints will not be violated when using this filter design. The NASA Hi-Fi simulation program was also tested using this interpolation filter. The Hi-Fi simulation produced curves that closely matched the results of the MATLAB simulations, which means that this filter will operate as expected when it is installed within the LRO flight software.

The interpolation filter designed in this thesis alleviates the command buffer storage limitations of the LRO. When this filter is implemented, the LRO will have the capacity to store and conduct maneuvers that were not previously viable, and thus there will be several more science collection opportunities for the LRO in the future.

NASA has reviewed the design and results of the interpolation filter presented in this thesis, and the LRO project team decided to install the filter on the LRO under work order CCR-139. The contribution of this thesis will therefore play a key role in advancing the FastMan concept towards a fully operational status.

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