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RPPR Final Report
as of 12-Jan-2023

Agency Code: 21XD

Proposal Number: 71882TE **Agreement Number: W911NF-18-1-0003**
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Report Date: 30-Nov-2022 Date Received: 12-Jan-2023
Final Report for Period Beginning 01-Nov-2017 and Ending 31-Aug-2022
Title: Nonlinear and Linear Elastodynamics Transformation Cloaking
Begin Performance Period: 01-Nov-2017 **End Performance Period:** 31-Aug-2022
Report Term: 0-Other
Submitted By: Arash Yavari Email: arash.yavari@ce.gatech.edu
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STEM Degrees: 2 **STEM Participants:**

Major Goals: The main goal of this project was to lay the mathematical and physical foundations of transformation elasticity and elastodynamic cloaking. Understanding the transformation properties of the governing equations of nonlinear and linearized elasticity for different types of materials is a crucial task for systematic design of elastic cloaks. This study was done in the settings of both linear and nonlinear elasticity. In elastic cloaking one would like to hide an object (a hole, inhomogeneity, inclusion, etc.) from elastic waves.

Accomplishments: We formulated transformation cloaking as a bijective map between the boundary-value problems of the physical body and a virtual body that is homogenous and has no holes/inhomogeneities. We carefully studied this problems for nonlinear elasticity, classical linear elasticity, small-on-large theory, gradient solids, and (generalized) Cosserat solids. We have proved several no-go theorems for transformation cloaking. It turns out that the balance of angular momentum is the obstruction to exact elastic cloaking. However, we have found an example of a cylindrical cloak that can exactly cloak a cylindrical cloak under any in-plate excitations. We also studied the problem of cloaking for elastic plates. It turns out that exact transformation cloaking is not possible for elastic plates either.

Our final conclusions are:
i) Exact transformation cloaking is not possible in elastic materials.
ii) The existing works in the literature in the past fifteen years have some fundamental flaws.
iii) The path forward for cloaking applications is approximate cloaking.

The last year of the project was focused on formulating cloaking as an optimal design problem. Some of our recent results can be seen here: https://arxiv.org/abs/2212.11668

The following journal papers have acknowledged this grant.

1) F. Sozio, M.F. Shojaei and A. Yavari, Optimal elastostatic cloaks, https://arxiv.org/abs/2212.11668

2) F. Sozio and A. Yavari, A geometric field theory of dislocation mechanics, under review.

3) A. Yavari and S. P. Pradhan, Accretion mechanics of nonlinear elastic circular cylindrical bars under finite torsion, Journal of Elasticity, DOI: 10.1007/s10659-022-09957-6.

4) A. Yavari and F. Sozio, On the direct and reverse multiplicative decompositions of deformation gradient in

nonlinear anisotropic anelasticity, Journal of the Mechanics and Physics of Solids 170, 2023, 105101.

5) A. Yavari and A. Goriely, Universality in anisotropic linear anelasticity, Journal of Elasticity, DOI: 10.1007 /s10659-022-09910-7.

6) A. Yavari and A. Goriely, The universal program of nonlinear hyperelasticity, Journal of Elasticity, DOI: 10.1007 /s10659-022-09906-3.

7) A. Yavari and A. Goriely, The universal program of linear elasticity, Mathematics and Mechanics of Solids 28(1), 2023, 251–268.

8) A. Yavari, Universal deformations in inhomogeneous isotropic nonlinear elastic solids, Proceedings of the Royal Society A 477, 2021, 20210547.

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13) F. Sozio, A. Golgoon, and A. Yavari, Elastodynamic transformation cloaking for non-centrosymmetric gradient solids, Zeitschrift für Angewandte Mathematik und Physik (ZAMP) 72(3), 2021, 123.

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15) C. Goodbrake, A. Yavari, and A. Goriely, The Anelastic Ericksen problem: Universal deformations and universal eigenstrains in incompressible nonlinear anelasticity, Journal of Elasticity 142, 2020, pp. 291–381.

16) A. Golgoon and A. Yavari, Transformation cloaking in elastic plates, Journal of Nonlinear Science 31: 17, 2021.

17) F. Sozio, M.F. Shojaei, S. Sadik, and A. Yavari, Nonlinear mechanics of thermoelastic accretion, Zeitschrift für Angewandte Mathematik und Physik (ZAMP) 71(3), 2020, 87.

18) A. Yavari, C. Goodbrake, and A. Goriely, Universal displacements in linear elasticity, Journal of the Mechanics and Physics of Solids 135, 2020, 103782.

19) F. Sozio and A. Yavari, Riemannian and Euclidean material structures in anelasticty, Mathematics and Mechanics of Solids 25(6), 2020, 1267-1293.

20) M.F. Shojaei and A. Yavari, Compatible-strain mixed finite element methods for 3D compressible and incompressible nonlinear elasticity, Computer Methods in Applied Mechanics and Engineering 357, 2019, 112610.

21) A. Yavari and A. Golgoon, Nonlinear and linear elastodynamics transformation cloaking, Archive for Rational Mechanics and Analysis 234(1), 2019, 211-316.

Invited Seminars:

The PI has given twelve incited seminars related to this project. He also gave a seminar at the U.S. Army Research Laboratory in Aberdeen in 2019.

RPPR Final Report
as of 12-Jan-2023

News release:

https://www.ce.gatech.edu/news/research-lays-foundation-partial-cloaking-holes-elastic-plates-stress-waves

https://www.ce.gatech.edu/news/dashing-dream-ideal-invisibility-cloaks-stress-waves

Training Opportunities: This grant supported one graduate fully and one partially (the last year of the project).

Results Dissemination: Several journal papers acknowledge this grant and have already been published. Two journal papers under review acknowledge this grant.

Honors and Awards: The PI was elected a Fellow of the Society of Engineering Science (SES) in 2009.

The PhD student who was fully supported by this grant (Dr. Ashkan Golgoon) received GA Tech CEE's Best PhD Thesis award in 2021.

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: Graduate Student (research assistant)
Participant: Ashkan Golgoon
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Graduate Student (research assistant)
Participant: Fabio Sozio
Person Months Worked: 6.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: PD/PI
Participant: Arash Yavari
Person Months Worked: 2.00 **Funding Support:**
Project Contribution:
National Academy Member: N

RPPR Final Report
as of 12-Jan-2023

Partners

,

I certify that the information in the report is complete and accurate:
Signature: Arash Yavari
Signature Date: 1/12/23 12:00PM

Final Report for

Nonlinear and Linear Elastodynamics Transformation Cloaking

Contract Number: W911NF1810003

PI: Dr. Arash Yavari

Program Manager: Dr. Daniel P. Cole

Goal: The main goal of this project was to lay the mathematical and physical foundations of transformation elasticity and elastodynamic cloaking. Understanding the transformation properties of the governing equations of nonlinear and linearized elasticity for different types of materials is a crucial task for systematic design of elastic cloaks. This study was done in the settings of both linear and nonlinear elasticity. In elastic cloaking one would like to hide an object (a hole, inhomogeneity, inclusion, etc.) from elastic waves.

Accomplished under Goals: We formulated transformation cloaking as a bijective map between the boundary-value problems of the physical body and a virtual body that is homogenous and has no holes/inhomogeneities. We carefully studied this problems for nonlinear elasticity, classical linear elasticity, small-on-large theory, gradient solids, and (generalized) Cosserat solids. We have proved several no-go theorems for transformation cloaking. It turns out that the balance of angular momentum is the obstruction to exact elastic cloaking. However, we have found an example of a cylindrical cloak that can exactly cloak a cylindrical cloak under any in-plate excitations. We also studied the problem of cloaking for elastic plates. It turns out that exact transformation cloaking is not possible for elastic plates either.

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