REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
14-11-2022	Final Report	15-Feb-2019 - 14-Aug-2022

4. TITLE AND SUBTITLE
Final Report: Analysis and Control of Dynamical Systems on Complex Networks of Unbounded Size

5a. CONTRACT NUMBER
W911NF-19-1-0110

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

6. AUTHORS

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
McGill University
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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
72893-MI.19

12. DISTRIBUTION AVAILIBILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Peter Caines
UU	UU	UU	UU		19b. TELEPHONE NUMBER
					151-474-6605

RPPR Final Report
as of 16-Nov-2022

Agency Code: 21XD

Proposal Number: 72893MI **Agreement Number: W911NF-19-1-0110**
INVESTIGATOR(S):

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DUNS Number: 205667090 EIN: 986001153
Report Date: 14-Nov-2022 Date Received: 14-Nov-2022
Final Report for Period Beginning 15-Feb-2019 and Ending 14-Aug-2022
Title: Analysis and Control of Dynamical Systems on Complex Networks of Unbounded Size
Begin Performance Period: 15-Feb-2019 **End Performance Period:** 14-Aug-2022
Report Term: 0-Other
Submitted By: Peter Caines Email: peterc@cim.mcgill.ca
Phone: (151) 474-66059
Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: **STEM Participants:**

Major Goals: Complex networks of dynamical systems arise in many applications such as the Internet of Things (IoT), 5G communications, grid networks, social interactions, epidemic networks and biological neuronal networks, and there is an obvious need to analyze
and control such networked systems. The study of the control of systems on complex networks typically involves such notions as controllability, control energy input node selection, the low-complexity control synthesis problems with simplified control (e.g. consensus or synchronization , pinning control, ensemble control, low-rank (e.g. mean field) coupling, and patterned coupling.

The first major goal of the project is to place these fundamental notions in a completely new setting when the systems under consideration are distributed over unbounded networks.

In a development which was entirely separate from systems and control, the mathematical theory of graphons was created
to model large networks and their infinite limits and, furthermore, it was applied to the analysis of purely dynamical models such as the heat equation and coupled oscillator models. This theory has been fully utilized to achieve the objectives the project.

The second major gaol of the Principal Investigator and co-workers was to
initiate graphon systems and control theory to develop a control theory and methodology of large-scale network-coupled linear dynamical systems, where a particular feature of this theory is that it permits the generation of low-complexity approximate solutions to problems on large networks which would otherwise be intractable.

Linear regulator graphon systems and control theory is based upon both the sophisticated theory of graphons and on the well established infinite dimensional linear system theory presented. We observe that this work follows the spirit of mean field game theory and, indeed, in another project a theory of so-called Graphon Mean Field Games was developed and was interlinked with this project.

It is an important fact that the solutions proposed in graphon systems and control theory can be implemented in a distributed, or decentralized, manner.

This program of research within the overall framework of graphon control and graphon mean field games is supported by the ARO and ARL and was initiated in February 2019.
At this time of completion of the project it can be fairly stated that its major objectives ion developing the theory of

control over arbitrarily large networks and to generate numerical methods which efficiently implement the resulting control theory have been achieved.

Accomplishments: (See uploaded PDF.)

Graphon Control:

The graphon control methodology was developed in (Gao,CainesTAC18,Gao,CainesCDC19(a)-(b)) to study control problems for arbitrary-size networks of linear dynamical systems. While the complete treatment of problems involving well-posedness, controllability, approximation errors for state-to-state control and regulations was established in (Gao,Caines,TAC20). Moreover, scalable and tractable solution methods were established for a class of graphon control problems in (Gao,Caines,CDC19(a)-(b)).

Graphon control formulations were extended to more general cases where the state of each individual agent on the network is multi-dimensional in (Gao, Caines,TCNS20). Exploiting this, we provide a solution method generalizing our previous formulation with an application to large-scale networks of coupled harmonic oscillators (see Gao, Caines,TCNS20)

During the reporting period from February 15th, 2019 to August 14th, 2022, results in these directions have been reported in the IEEE Transactions on Control of Network Systems (2021) and IEEE Transactions on Automatic Control (2020), and the IEEE Conference on Decision and Control (2019-2021).

Graphon (Mean) Field Games:

Graphon mean field games (GMFG) were first formulated and then investigated in the non-linear and LQG settings in (Caines, Huang,2019, Caines,Huang,SIAM21, Gao,Foguen Tchuendom,Caines CIS21, Gao,Caines,Huang, CDC21, Gao,Caines,Huang, TAC22).

The fundamental properties of the existence and uniqueness of solutions are established by two principal methods: (i) Riccati equation decomposition (for LQG-GMFG systems) and (ii) fixed point contraction theorem methods (applying in the general nonlinear case). The invariant subspace properties of graphons are further explored and exploited in computing tractable solutions for LQG graphon mean field games problems in (Gao,Caines,Huang, CDC21, Gao,Caines,Huang, TAC22). A class of linear quadratic graphon field problems has been analyzed and the corresponding epsilon-Nash result for applications to large-scale game problems on networks established in (Gao,Foguen Tchuendom,Caines, CIS21).

Critical nodes and stationary points for LQG graphon mean field games, that is to say system nodes where the Nash value function has a stationary value with respect to position, are investigated in (Foguen-Tchuendom, Caines, Huang, CDC21, Foguen-Tchuendom, Gao, CDC22, Foguen-Tchuendom, Gao, Caines, MTNS22). In particular, explicit Nash values for infinite horizon LQG graphon mean field games are established in (Foguen-Tchuendom, Gao, Caines, Huang CDC 22, Foguen-Tchuendom, Gao, Caines, MTNS 22). The notion of embedded graphons which enables the analysis of graphs and graph limits in metric space is proposed and developed for studying graphon mean field games with spatial features in (Caines CDC 22). In particular this formulation permits the application of the differential calculus to the Nash value functions of GMFG problems.

During the period from February 15th, 2020 to August 14th, 2022, our results in this direction have been reported in the SIAM Journal on Control and Optimization (2021), the IEEE Transactions on Automatic Control (conditionally accepted in 2022), Communications in Information and Systems (2021), and IEEE Conference on Decision and Control (2021, 2022), and the International Symposium on Mathematical Theory of Networks (2022).

Network Analysis:

A family of centralities (called fixed-point centrality) is proposed and analyzed employing the fixed-point characterization of equivariant mappings. This centrality family unifies the standard centralities such as PageRank centrality, Katz centrality, eigencentrality, and new centralities such as graphon centrality, and Nash value centrality for LQG graphon mean field games (see Foguen-Tcheundom,Gao,Caines, CDC22), and it provides a unified analysis for the centrality variation with respect to graph structure variations. The notion of centrality is applied to opinion dynamics on social networks in (Gao, CDC21) and its important role in network partitions following

opinion states is analyzed based on real-world network data. Results in this direction have been reported to IEEE Conference on Decision and Control (2021, 2022).

This work advances the theory of centralities for dynamical systems on networks and its associated applications. In (Gao,CDC2021) a family of centralities characterized by fixed-points of networks is identified which includes the standard centralities (e.g. PageRank, Katz, eigen-centralities) and centralities (e.g. Nash values of LQG GMFG). Variation bounds of centralities with respect to the network variations are established.

Epidemics Modeling and Control on Large Networks:

The initial investigation of applying graphon theory to the control of epidemic spread is first proposed in (Gao, Caines, CDC19a) where scalable solutions based on spectral approximation and linearization are established for controlling virus spreads on networks following a type of controlled network SIS model. Analysis of the optimal lockdown efforts based on one type of network SIR model is carried out in (Salii, 2021). A computational package is developed and traffic flow data are gathered for the numerical simulations in (Salii 2021).

Results in this direction have been reported to the International Conference on Complex Networks and Their Applications (2021) and the IEEE Conferences on Decisions and Control (2019).

Training Opportunities: Nothing to Report

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Results Dissemination: A. Journal Publications

Graphon Control:

S. Gao and P. E. Caines, “Subspace decomposition for graphon LQR: Applications to VLSNs of harmonic oscillators,” IEEE Transactions on Control of Network Systems vol. 8, no.2, pp. 576-586, 2021.

S. Gao and P. E. Caines, “Graphon control of large-scale networks of linear systems,” IEEE Transactions on Automatic Control, vol. 65, no. 10, pp. 4090–4105, 2020.

Graphon (Mean) Field Games

S. Gao, P. E. Caines, and M. Huang, “LQG graphon mean field games: Analysis via graphon invariant subpaces,” IEEE Transactions on Automatic Control (Conditionally Accepted), 2022, arXiv preprint arXiv:2004.00679.

S. Gao, R. Foguen Tchuendom, and P. E. Caines, “Linear quadratic graphon field games,” Com- munications in Information and Systems, vol. 21, no. 3, pp. 341–369, 2021.

B. Peer-Reviewed Conference Publications

1) Graphon Control

S. Gao and P. E. Caines, “Spectral representations of graphons in very large network systems control,” in Proceedings of the 58th IEEE Conference on Decision and Control (CDC), Nice, France, December 2019, pp. 5068–5075.

S. Gao and P. E. Caines, “Optimal and approximate solutions to linear quadratic regulation of a class of graphon dynamical systems,” in Proceedings of the 58th IEEE Conference on Decision and Control (CDC), Nice, France, December 2019, pp. 8359–8365.

2) Graphon (Mean) Field Games

R. Foguen Tchuendom, P. E. Caines and M.Y. Huang, ”On the Master Equation for Linear Quadratic Graphon Mean Field Games”, Proceedings of the 59th IEEE Conference on Decision and Control, Jeju Island, Republic of Korea, December, 2020, pp. 1026-10

S. Gao, P. E. Caines, and M. Huang, “LQG graphon mean field games: Graphon invariant sub- spaces,” Proceedings of the 60th IEEE Conference on Decision and Control, Austin, Texas, USA, December 2021, pp. 5253–5260.

R. F. Tchuendom, S. Gao, P. E. Caines, and M. Huang, “Optimal network location in infinite horizon LQG graphon mean field games,” Accepted for presentation at the 61st IEEE Conference on Decision and Control (CDC), 2022.

R. Foguen Tchuendom, S, Gao and P. E. Caines, “Stationary Cost Nodes in Infinite Horizon LQG- GMFGs”, Presented at the 25th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2022):, Bayreuth, Germany, 12-16 September 2022

P. E. Caines, “Embedded vertexon-graphons and embedded GMFG systems,” Accepted for presen- tation at the 61th IEEE Conference on Decision and Control (CDC), 2022.

R. F. Tchuendom, S. Gao, and P. E. Caines, “Stationary cost nodes in infinite horizon LQG GM- FGs,” In Proceedings of the 25th International Symposium on Mathematical Theory of Networks and Systems (MTNS), 2022, pp. 663–668.

3) Network Analysis

S. Gao, “Fixed-point centrality for networks,” Accepted for presentation at the 61st IEEE Confer- ence on Decision and Control (CDC), 2022.

S. Gao, “Centrality-weighted opinion dynamics: Disagreement and social network partition,” in Proceedings of the 60th IEEE Conference on Decision and Control, Austin, Texas, USA, December 2021, pp. 5496–5501.

4) Epidemics Spread on Networks and Control

S. Gao and P. E. Caines, “Spectral representations of graphons in very large network systems control,” in Proceedings of the 58th IEEE Conference on Decision and Control (CDC), Nice, France, December 2019, pp. 5068–5075. (see also in graphon control section)

Y. V. Salii, “Benchmarking optimal control for network dynamic systems with plausible epidemic models,” in International Conference on Complex Networks and Their Applications. Springer, 2021, pp. 194–206.

C. Talks: February 15th, 2019 to September 29th, 2022 (invitations received before August 14th, 2022).

P. E. Caines, “Graphon Controal and Graphon Mean Field Games”, Department of Applied Mathe- maics, Hong Kong Polythenic Universty, Hong Kong), 28th June, 2019 (Distinguished Lecturer).

P. E. Caines, “Graphon Mean Field Games: A Dynamical Equilibrium Theory for a Networked World”, University of Saint Petersburg, Russia, 6th September, 2019 (Distinguished Speaker).

P. E. Caines, “Graphon Mean Field Games: A Dynamical Equilibrium Theory for Networked Sys- tems”, Invited Speaker at the ”Conference on Mean Field Games and Related Topics - 5”. 9th - 13th September, 2019

P. E. Caines, “Graphon Mean Field Games: A Dynamical Equilibrium Theory for Large Populations on Complex Networks”, Canadian Applied and Industrial Mathematics Society (CAIMS), June 23rd, 2021 (Plenary talk).

P. E. Caines, “Optimal Execution Problems in Single and Networked Markets: a Mean Field Game Formulation”, the Financial Mathematics/Engineering Seminar Series, Hong Kong Polytechnic Uni- versity, June 16th 2021 (Invited talk).

P. E. Caines, “Static and Time Varying Graphons in Economic and Pandemic GMFG Models” SIAM Financial Mathematics and Engineering (FM21), June 1, 2021 (Invited talk).

S. Gao, “Subspace Decompositions in Graphon Control and Graphon Mean Field Games”, the Informal Systems Seminar, Centre for Intelligent Machines, McGill University, Groupe d’Etudes et de Recherche en Analyse des Decisions, Montreal, CA, Dec. 11, 2020 (Invited talk).

P. E. Caines, ”Optimal Execution Problems in Single and Networked Markets: a Mean Field Game Formulation”, (Invited talk) 29th August, 2022, in the Conference on Many Player Games and Applications, Humbolt University Berlin, 29 - 31 August 2022.

P. E. Caines, ”Vertexons and Embedded Graphon Mean Field Games”, the Simons Institute for the Theory of Computing, UC Berkeley, CA, Sept.29, 2022 (Invited talk) in the ”Graph Limits, Nonparametric Models, and Estimation Workshop” 26 - 30 September, 2022.

D. Workshop Organization: February 15th, 2019 to August 14th, 2022.

Name: Mean Field Games on Networks Workshop
Location: Vancouver, BC, Canada (Virtual Meeting)
Dates: October 26 - 29, 2021

Organizers: Peter E. Caines, Shuang Gao, Minyi Huang, Rinel F-Tchuendom
This workshop [19] brought together researchers in applied mathematics, mean field games, network science, network games, and systems and control theory to exchange ideas and to work on the extensions of mean field game theory to dynamic game problems on heterogeneous large-scale networks. The workshop offered a platform to current results and stimulate discussions on the open challenges in this emerging field, on mathematical theory, computational algorithms, and the applications methodologies of MFG on general networks.

Sponsors: PIMS, GERAD, and the Field Institutes.
Participants: Peter E. Caines (PI) and PI’s Postdoctoral Researchers: Rinel Foguen Tchuendom, Shuang Gao, and Yaroslav Salii, plus 50 other international participants.

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Honors and Awards: Elected Fellow: International Federation of Automatic Control (IFAC) (13.09.2019) Citation: "For contributions to system identification and adaptive control, and the creation of mean field game theory"

Continuing: FRSC, FIEEE (Life Fellow), FIMA (UK), FSIAM, FCIAR

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Shuang Gao
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Rinel Foguen Tchuendom
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: Postdoctoral (scholar, fellow or other postdoctoral position)
Participant: Yaroslav Salii
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
National Academy Member: N

Participant Type: PD/PI
Participant: Peter E. Caines
Person Months Worked: 15.00 **Funding Support:**
Project Contribution:
National Academy Member: N

ARTICLES:

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Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: IEEE Transactions on Automatic Control
Publication Identifier Type: DOI Publication Identifier: 10.1109/TAC.2019.2955976
Volume: 65 Issue: 10 First Page #: 4090
Date Submitted: 8/31/21 12:00AM Date Published: 10/1/20 8:00AM
Publication Location:
Article Title: Graphon Control of Large-Scale Networks of Linear Systems
Authors: Shuang Gao, Peter E. Caines
Keywords: Complex networks, graphon control, graphons, infinite dimensional systems, large networks.
Abstract: To achieve control objectives for extremely large-scale complex networks using standard methods is essentially intractable. In this article, a theory of the ap- proximate control of complex network systems is proposed and developed by the use of graphon theory and the theory of infinite dimensional systems. First, graphon dynamical system models are formulated in an appropriate infinite dimensional space in order to represent arbitrary-size net- works of linear dynamical systems, and to define the con- vergence of sequences of network systems with limits in the space. Exact controllability and approximate controllability of graphon dynamical systems are then investigated. Second, the minimum energy state-to-state control problem and the linear quadratic regulator problem for systems on complex networks are considered. The control problem for graphon limit systems is solved in each case and approx- imations are defined which yield control laws for the origi- nal control problems.
Distribution Statement: 2-Distribution Limited to U.S. Government agencies only; report contains proprietary info
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: Communications in Information and Systems
Publication Identifier Type: DOI Publication Identifier: 10.4310/CIS.2021.v21.n3.a2
Volume: 21 Issue: 3 First Page #: 341
Date Submitted: 8/31/21 12:00AM Date Published: 6/4/21 4:00AM
Publication Location:
Article Title: Linear quadratic graphon field games
Authors: Shuang Gao, Rinel Foguen Tchuendom, Peter E. Caines
Keywords: graphon mean field games, linear quadratic optimal control
Abstract: Linear quadratic graphon field games (LQ-GFGs) are defined to be linear quadratic games which involve a large number of agents that are weakly coupled via a weighted undirected graph on which each node represents an agent. The links of the graph correspond to cou- plings between the agents' dynamics, as well as between the indi- vidual cost functions, which each agent attempts to minimize. We formulate limit LQ-GFG problems based on the assumption that these graphs lie in a sequence which converges to a limit graphon. First, under a finite-rank assumption on the limit graphon, the exis- tence and uniqueness of solutions to the formulated limit LQ-GFG problem is established. Second, based upon the solutions to the limit LQ-GFG problem, ?-Nash equilibria are constructed for the corresponding game problems with a very large but finite number of players. This result is then generalized to the case with ran- dom initial conditions.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

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as of 16-Nov-2022

Publication Type: Journal Article Peer Reviewed: Y **Publication Status:** 1-Published
Journal: IEEE Transactions on Control of Network Systems
Publication Identifier Type: DOI Publication Identifier: 10.1109/TCNS.2021.3058923
Volume: 8 Issue: 2 First Page #: 576
Date Submitted: 8/31/21 12:00AM Date Published: 6/1/21 4:00AM
Publication Location:
Article Title: Subspace Decomposition for Graphon LQR: Applications to VLSNs of Harmonic Oscillators
Authors: Shuang Gao, Peter E. Caines
Keywords: Complex networks, graphon, graphon con- trol, large-scale systems, optimal control, very large-scale networks (VLSNs).
Abstract: Graphon control has been proposed and de- veloped in [1]–[3] to approximately solve control problems for very large-scale networks of linear dynamical systems based on graphon limits. This article provides a solution method based on invariant subspace decompositions for a class of graphon linear quadratic regulation (LQR) prob- lems, where the local dynamics share homogeneous pa- rameters but the graphon couplings may be heterogeneous among the coupled agents. Graphon couplings in this ar- ticle appear in states, controls, and costs, and these cou- plings may be represented by different graphons. By ex- ploiting a common invariant subspace of the couplings, the original problem is decomposed into a network coupled LQR problem of finite dimension and a decoupled infinite dimensional LQR problem. A centralized optimal control solution, and a nodal collaborative optimal control solution, where each agent computes its part of the optimal solution locally, are established.
Distribution Statement: 1-Approved for public release; distribution is unlimited.
Acknowledged Federal Support: **Y**

Publication Type: Journal Article Peer Reviewed: **Publication Status:** 4-Under Review
Journal: IEEE Transactions on Automatic Control
Publication Identifier Type: Publication Identifier:
Volume: Issue: First Page #:
Date Submitted: Date Published:
Publication Location:
Article Title: LQG graphon mean field games: Analysis via graphon invariant subpaces
Authors: Shuang Gao, Peter E. Caines, Mini Huang,
Keywords: Large-scale networks, mean field games, com- plex networks, graphon control, infinite dimensional systems
Abstract: This paper studies approximate solutions to large- scale linear quadratic stochastic games with homogeneous nodal dynamics parameters and heterogeneous network couplings within the graphon mean field game framework in [2]–[4]. A graphon time-varying dynamical system model is first formulated to study the finite and then limit problems of linear quadratic Gaussian graphon mean field games (LQG-GMFG). The Nash equilibrium of the limit problem is then characterized by two coupled graphon time-varying dynamical systems. Sufficient con- ditions are established for the existence of a unique solution to the limit LQG-GMFG problem. For the computation of LQG-GMFG solutions two methods are established and employed where one is based on fixed point iterations and the other on a decoupling operator Riccati equation; furthermore, two corresponding sets of solutions are established based on spectral decompositions. Finally, a set of numerical simulations on networks associated with different types
Distribution Statement: 3-Distribution authorized to U.S. Government Agencies and their contractors
Acknowledged Federal Support: **Y**

CONFERENCE PAPERS:

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Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 58th IEEE Control Systems Society Conference on Decision and Control
Date Received: 31-Aug-2021 Conference Date: 12-Dec-2019 Date Published: 12-Dec-2019
Conference Location: Nice, France
Paper Title: Spectral Representations of Graphons in Very Large Network Systems Control
Authors: Shuang, Gao and Peter, Caines
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 58th IEEE Control Systems Society Conference on Decision and Control
Date Received: 19-Sep-2019 Conference Date: 12-Dec-2019 Date Published: 12-Dec-2019
Conference Location: Nice, France
Paper Title: Optimal and Approximate Solutions to Linear Quadratic Regulation of a Class of Graphon Dynamical Systems
Authors: Shuang, Gao and Peter, Caines
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2021 60th IEEE Conference on Decision and Control (CDC)
Date Received: 13-Nov-2022 Conference Date: 15-Dec-2021 Date Published:
Conference Location: Austin, TX, USA
Paper Title: Centrality-Weighted Opinion Dynamics: Disagreement and Social Network Partition
Authors: Shuang, Gao
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2021 60th IEEE Conference on Decision and Control (CDC)
Date Received: 13-Nov-2022 Conference Date: 14-Dec-2021 Date Published:
Conference Location: Austin, TX, USA
Paper Title: LQG Graphon Mean Field Games: Graphon Invariant Subspaces
Authors: Shuang, Gao and Peter E., Caines and Minyi, Huang
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2021 60th IEEE Conference on Decision and Control (CDC)
Date Received: 13-Nov-2022 Conference Date: 14-Dec-2021 Date Published:
Conference Location: Austin, TX, USA
Paper Title: Critical Nodes in Graphon Mean Field Games
Authors: Rinel, Foguen Tchuendom and Peter E., Caines and Minyi, Huang
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 2-Awaiting Publicat
Conference Name: 25th International Symposium on Mathematical Theory of Networks and Systems
Date Received: 13-Nov-2022 Conference Date: 12-Sep-2022 Date Published:
Conference Location: Bayreuth, Germany
Paper Title: Stationary Cost Nodes in Infinite Horizon LQG-GMFGs
Authors: Rinel, Foguen Tchuendom and Shuang, Gao and Peter E., Caines
Acknowledged Federal Support: **Y**

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as of 16-Nov-2022

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 62nd IEEE Conference on Decision and Control
Date Received: 13-Nov-2022 Conference Date: 07-Dec-2022 Date Published:
Conference Location: Cancun, Mexico
Paper Title: Optimal Network Location in Infinite Horizon LQG Graphon Mean Field Games
Authors: Rinel, Foguen-Tchuendom and Shuang, Gao and Minyi, Huang and Peter E., Caines
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 3-Accepted
Conference Name: 61st IEEE Conference on Decision and Control
Date Received: 13-Nov-2022 Conference Date: 07-Dec-2022 Date Published:
Conference Location: Cancun, Mexico
Paper Title: Fixed-Point Centrality for Networks
Authors: Shuang, Gao
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: International Conference on Complex Networks and their Applications
Date Received: 13-Nov-2022 Conference Date: 30-Nov-2021 Date Published:
Conference Location: Madrid, Spain
Paper Title: Benchmarking Optimal Control for Network Dynamic Systems with Plausible Epidemic Models
Authors: Yaroslav, V. Salii
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 3-Accepted
Conference Name: 61st IEEE Conference on Decision and Control
Date Received: 13-Nov-2022 Conference Date: 07-Dec-2022 Date Published:
Conference Location: Cancun, Mexico
Paper Title: Embedded Vertexon-Graphons and Embedded GMFG Systems
Authors: Peter E.,Caines
Acknowledged Federal Support: **Y**

Partners

,

I certify that the information in the report is complete and accurate:
Signature: Peter E. Caines
Signature Date: 11/14/22 11:47PM

Accomplishments

This section describes the progress and accomplishments made during the funding period February 15th, 2019 to August 14th, 2022, on the funded research program entitled "*The Analysis and Control of Dynamical Systems on Complex Networks of Unbounded Size*" under grant number W911NF1910110

Progress and accomplishmets under this grant included the following:

(a) **Graphon Control**: The graphon control methodology is developed in [1], [2], [3] to study control problems for arbitrary-size networks of linear dynamical systems. The complete treatment of problems involving well-posedness, controllability, approximation errors for state-to-state control and regulations is established in [1]. Scalable and tractable solution methods are established for a class of graphon control problem in [2], [3]. The graphon control formulations are extended to more general cases where the state of each individual agent on the network is multi-dimensional [4]. Exploiting this, we provide a solution method generalizing our previous formulation with an application to large-scale networks of coupled harmonic oscillators [4]. During the reporting period from February 15th, 2019 to August 14th, 2022, results in this direction have been reported in the IEEE Transactions on Control of Network Systems (2021) and IEEE Transactions on Automatic Control (2020), and IEEE Conference on Decision and Control (2019).

(b) **Graphon (Mean) Field Games:** Graphon mean field games (GMFG) were first formulated and then investigated in the non-linear and LQS settings in [5], [6], [5], [7], [8], [9]. The fundamental properties of the existence and uniqueness of solutions are established by two principal methods: (i) Riccati equation decomposition (for LQG-GMFG systems) and (ii) fixed point contraction theorem methods (applying in the general nonlinear case). The invariant subspace properties of graphons are further explored and exploited in computing tractable solutions for LQG graphon mean field games problems in [8], [9]. Critical nodes and stationary points for LQG graphon mean field games, that is to say system nodes where the Nash value function has a stationary value with respect to position, are investigated in [10], [11], [12]. In particular, explicit Nash values for infinite horizon LQG graphon mean field games are established in [11], [12]. The notion of embedded graphons which enables the analysis of graphs and graph limits in metric space is proposed and developed for studying graphon mean field games with spatial features in [13]. In particular this formulation permits the application of the differential calculus to the Nash value functions of GMFG problems. During the period from February 15th, 2020 to August 14th, 2022, results in this direction have been reported in the SIAM Journal on Control and Optimization (2021), the IEEE Transactions on Automatic Control (conditionally accepted in 2022), Communications in Information and Systems (2021), and IEEE Conference on Decision and Control (2021, 2022), and the International Symposium on Mathematical Theory of Networks (2022).

(c) **Network Analysis:** A family of centralities (called fixed-point centrality) is proposed and analyzed employing the fixed-point characterization of equivariant mappings. This centrality family unifies the standard centralities such as PageRank centrality, Katz centrality, eigencentrality, and new centralities such as graphon centrality, and Nash value centrality for LQG graphon mean field games [11], and provide a unified analysis for the centrality variation with respect to graph structure variations. The notion of centrality is applied to opinion dynamics on social networks in [14] and its important role in network partitions following opinion states is analyzed based on real-world network data. Results in this direction have been reported to IEEE Conference on Decision and Control (2021, 2022).

(d) **Epidemics Modeling and Control on Large Networks:** The initial investigation of applying graphon theory to the control of epidemic spread is first proposed in [2] where scalable solutions

based on spectral approximation and linearization are established for controlling virus spreads on networks following a type of controlled network SIS model. Analysis of the optimal lockdown efforts based on one type of network SIR model is carried out in [15]. A computational package is developed and traffic flow data are gather for the numerical simulations in [15]. Results in this direction have been reported to the International Conference on Complex Networks and Their Applications (2021) and the IEEE Conferences on Decisions and Control (2019).

I. Publication List (February 15th, 2019 to August 14th, 2022)

A. Journal Publication (5 Publications)

(a) **Graphon Control:**

S. Gao and P. E. Caines, "Subspace decomposition for graphon LQR: Applications to VLSNs of harmonic oscillators," IEEE Transactions on Control of Network Systems vol. 8, no.2, pp. 576-586, 2021. [4]

S. Gao and P. E. Caines, "Graphon control of large-scale networks of linear systems," IEEE Transactions on Automatic Control, vol. 65, no. 10, pp. 4090–4105, 2020. [1]

(b) **Graphon (Mean) Field Games**

P. E. Caines and M.Y. Huang," Graphon Mean Field Games and Their Equations", *SIAM Journal on Control and Optimization,* 2021; 59(6):4373-99

S. Gao, P. E. Caines, and M. Huang, "LQG graphon mean field games: Analysis via graphon invariant subpaces," IEEE Transactions on Automatic Control (Conditionally Accepted), 2022, arXiv preprint arXiv:2004.00679. [9]

S. Gao, R. Foguen Tchuendom, and P. E. Caines, "Linear quadratic graphon field games," Communications in Information and Systems, vol. 21, no. 3, pp. 341–369, 2021. [7]

B. Peer-Reviewed Conference Publications (14 Publications)

1) **Graphon Control**

S. Gao and P. E. Caines, "Spectral representations of graphons in very large network systems control," in Proceedings of the 58th IEEE Conference on Decision and Control (CDC), Nice, France, December 2019, pp. 5068–5075. [2]

S. Gao and P. E. Caines, "Optimal and approximate solutions to linear quadratic regulation of a class of graphon dynamical systems," in Proceedings of the 58th IEEE Conference on Decision and Control (CDC), Nice, France, December 2019, pp. 8359–8365. [3]

2) **Graphon (Mean) Field Games**

P. E. Caines and M.Y. Huang, " Graphon Mean Field Games and the GMFG Equations." Proceedings of the 57th IEEE Conference on Decision and Control, Miami Beach, FL, USA, December, 2018, pp 4129-4134 [16]

P. E. Caines and M.Y. Huang, "Graphon Mean Field Games and the GMFG Equations: ϵ-Nash Equilibria". Proceedings of the 58th IEEE Conference on Decision and Control, Nice, France, December, 2019 pp. 286-292. (Published as Cahier du GERAD: G-2019-81, November, 2019) [5]

R. Foguen Tchuendom, P. E. Caines and M.Y. Huang, "On the Master Equation for Linear Quadratic Graphon Mean Field Games", Proceedings of the 59th IEEE Conference on Decision and Control, Jeju Island, Republic of Korea, December, 2020, pp. 1026-10 [17]

S. Gao, P. E. Caines, and M. Huang, "LQG graphon mean field games: Graphon invariant subspaces," Proceedings of the 60th IEEE Conference on Decision and Control, Austin, Texas, USA, December 2021, pp. 5253–5260. [8]

R. F. Tchuendom, S. Gao, P. E. Caines, and M. Huang, "Optimal network location in infinite horizon LQG graphon mean field games," Accepted for presentation at the 61st IEEE Conference on Decision and Control (CDC), 2022. [11]

R. Foguen Tchuendom, S, Gao and P. E. Caines, "Stationary Cost Nodes in Infinite Horizon LQG-GMFGs", Presented at the 25th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2022):, Bayreuth, Germany, 12-16 September 2022

P. E. Caines, "Embedded vertexon-graphons and embedded GMFG systems," Accepted for presentation at the 61th IEEE Conference on Decision and Control (CDC), 2022. [13]

R. F. Tchuendom, S. Gao, and P. E. Caines, "Stationary cost nodes in infinite horizon LQG GMFGs," In Proceedings of the 25th International Symposium on Mathematical Theory of Networks and Systems (MTNS), 2022, pp. 663–668. [12]

3) **Network Analysis**
S. Gao, "Fixed-point centrality for networks," Accepted for presentation at the 61st IEEE Conference on Decision and Control (CDC), 2022. [18]

S. Gao, "Centrality-weighted opinion dynamics: Disagreement and social network partition," in Proceedings of the 60th IEEE Conference on Decision and Control, Austin, Texas, USA, December 2021, pp. 5496–5501. [14]

4) **Epidemics Spread on Networks and Control**
S. Gao and P. E. Caines, "Spectral representations of graphons in very large network systems control," in Proceedings of the 58th IEEE Conference on Decision and Control (CDC), Nice, France, December 2019, pp. 5068–5075. [2] (see also in graphon control section)

Y. V. Salii, "Benchmarking optimal control for network dynamic systems with plausible epidemic models," in International Conference on Complex Networks and Their Applications. Springer, 2021, pp. 194–206. [15]

C. Talks: February 15th, 2019 to September 29th, 2022 (invitations received before August 14th, 2022).

1. P. E. Caines, "Graphon Controal and Graphon Mean Field Games", Department of Applied Mathemaics, Hong Kong Polythenic Universty, Hong Kong), 28th June, 2019 (Distinguished Lecturer).
2. P. E. Caines, "Graphon Mean Field Games: A Dynamical Equilibrium Theory for a Networked World", University of Saint Petersburg, Russia, 6th September, 2019 (Distinguished Speaker).
3. P. E. Caines, "Graphon Mean Field Games: A Dynamical Equilibrium Theory for Networked Systems", Invited Speaker at the "Conference on Mean Field Games and Related Topics - 5". 9th - 13th September, 2019
4. P. E. Caines, "Graphon Mean Field Games: A Dynamical Equilibrium Theory for Large Populations on Complex Networks", Canadian Applied and Industrial Mathematics Society (CAIMS), June 23rd, 2021 (Plenary talk).
5. P. E. Caines, "Optimal Execution Problems in Single and Networked Markets: a Mean Field Game Formulation", the Financial Mathematics/Engineering Seminar Series, Hong Kong Polytechnic University, June 16th 2021 (Invited talk).

6. P. E. Caines, “Static and Time Varying Graphons in Economic and Pandemic GMFG Models” SIAM Financial Mathematics and Engineering (FM21), June 1, 2021 (Invited talk).
7. S. Gao, “Subspace Decompositions in Graphon Control and Graphon Mean Field Games”, the Informal Systems Seminar, Centre for Intelligent Machines, McGill University, Groupe d’Etudes et de Recherche en Analyse des Decisions, Montreal, CA, Dec. 11, 2020 (Invited talk).
8. P. E. Caines, ”Optimal Execution Problems in Single and Networked Markets: a Mean Field Game Formulation”, (Invited talk) 29th August, 2022, in the Conference on Many Player Games and Applications, Humbolt University Berlin, 29 - 31 August 2022.
9. P. E. Caines, ”Vertexons and Embedded Graphon Mean Field Games”, the Simons Institute for the Theory of Computing, UC Berkeley, CA, Sept.29, 2022 (Invited talk) in the ”Graph Limits, Nonparametric Models, and Estimation Workshop” 26 - 30 September, 2022.

D. Workshop Organization: February 15th, 2019 to August 14th, 2022.

Name: **Mean Field Games on Networks Workshop**

Location: Vancouver, BC, Canada (Virtual Meeting)

Dates: October 26 - 29, 2021

Organizers: Peter E. Caines, Shuang Gao, Minyi Huang, Rinel F-Tchuendom

This workshop [19] brought together researchers in applied mathematics, mean field games, network science, network games, and systems and control theory to exchange ideas and to work on the extensions of mean field game theory to dynamic game problems on heterogeneous large-scale networks. The workshop offered a platform to current results and stimulate discussions on the open challenges in this emerging field, on mathematical theory, computational algorithms, and the applications methodologies of MFG on general networks.

Sponsors: PIMS, GERAD, and the Field Institutes.

Participants: Peter E. Caines (PI) and PI’s Postdoctoral Researchers: Rinel Foguen Tchuendom, Shuang Gao, and Yaroslav Salii, plus 50 other international participants.

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[3] S. Gao and P. E. Caines, “Optimal and approximate solutions to linear quadratic regulation of a class of graphon dynamical systems,” in *Proceedings of the 58th IEEE Conference on Decision and Control (CDC)*, Nice, France, December 2019, pp. 8359–8365.

[4] S. Gao and P. E. Caines, “Subspace decomposition for graphon LQR: Applications to VLSNs of harmonic oscillators,” *IEEE Transactions on Control of Network Systems*, vol. 8, no. 2, pp. 576–586, 2021.

[5] P. E. Caines and M. Huang, “Graphon mean field games and the gmfg equations: ε-nash equilibria,” in *2019 IEEE 58th conference on decision and control (CDC)*. IEEE, 2019, pp. 286–292.

[6] P. E. Caines and M. Huang, “Graphon mean field games and their equations,” *SIAM Journal on Control and Optimization*, vol. 59, no. 6, pp. 4373–4399, 2021.

[7] S. Gao, R. Foguen Tchuendom, and P. E. Caines, “Linear quadratic graphon field games,” *Communications in Information and Systems*, vol. 21, no. 3, pp. 341–369, 2021.

[8] S. Gao, P. E. Caines, and M. Huang, “LQG graphon mean field games: Graphon invariant subspaces,” in *Proceedings of the 60th IEEE Conference on Decision and Control*, Austin, Texas, USA, December 2021, pp. 5253–5260.

[9] S. Gao, P. E. Caines, and M. Huang, “LQG graphon mean field games: Analysis via graphon invariant subpaces,” Conditionally accepted by *IEEE Transactions on Automatic Control*, 2022, arXiv preprint arXiv:2004.00679.

[10] R. F. Tchuendom, P. E. Caines, and M. Huang, “Critical nodes in graphon mean field games,” *Proceedings of the 60th IEEE Conference on Decision and Control (CDC)*, pp. 166–170, 2021.

[11] R. Foguen Tchuendom, S. Gao, M. Huang, and P. E. Caines, "Optimal network location in infinite horizon LQG graphon mean field games," Accepted for presentation at *the 61st IEEE Conference on Decision and Control (CDC)*, 2022.
[12] R. Foguen Tchuendom, S. Gao, and P. E. Caines, "Stationary cost nodes in infinite horizon LQG-GMFGs," *Proceedings of the 25th International Symposium on Mathematical Theory of Networks and Systems*, pp. 663–668, 2022.
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