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INFORMAL REPORT

SOME MATHEMATICAL ASPECTS OF UNDERWATER EXPLOSIONS CAUSED BY EXPLODING WIRES

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ABSTRACT

The correction factor for very fast, small amplitude explosions is calculated. The formal basis for defining the explosion time constant for EWP events is established. The electrical energy of the capacitor is related to the equivalent weight of explosive (TNT); circuit losses and wire heating requirements are accounted for.

This report has been reviewed and is approved as an UNCLASSIFIED Informal Report.

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SOME MATHEMATICAL ASPECTS OF UNDERWATER EXPLOSIONS CAUSED BY EXPLODING WIRES

INTRODUCTION

Recently, the peak pressure and reduced time constant parameters of underwater exploding wires (EWP) were compared to underwater chemical explosions (CUE) (reference 1). The purpose of that study was to compare two rather diverse phenomena. The circuit for EWP experiment is shown in Figure 1.

The purpose of this report is to treat several topics not completely developed in reference 1. These topics are: the calculation of a correction factor for peak pressure measurements of very fast, small amplitude explosions, the formal basis for defining the explosion time constant for EWP events underwater, and the equivalent weight of TNT for stored electrical energy.

PRESSURE CORRECTION FACTOR

In this section, the method of calculating the pressure correction factor is treated by means of a mathematical model representing the circumstances presented to the detection system.

Two assumptions for this case are made: first, the response of the gauge and amplifier may be combined and expressed as the amplifier response; and second, the form of the pressure wave is assumed to be a decaying exponential. Hence, the gauge and amplifier are assumed to behave as one system (A') whose response to a unit step function, u(t), is A(t). The response of the system (A') to an arbitrary function, p(t), is h(t) as shown in the diagram below. The model of a more detailed system is outlined in Appendix A.

The physical situation at the detector (crystal gauge - amplifier - oscilloscope) is schematically:

explosion wave

here A' is the combined gauge-amplifier system.

If a step function u(t) is imposed, A' presents this on the CRT as A(t) where A(t) has the analytic form:

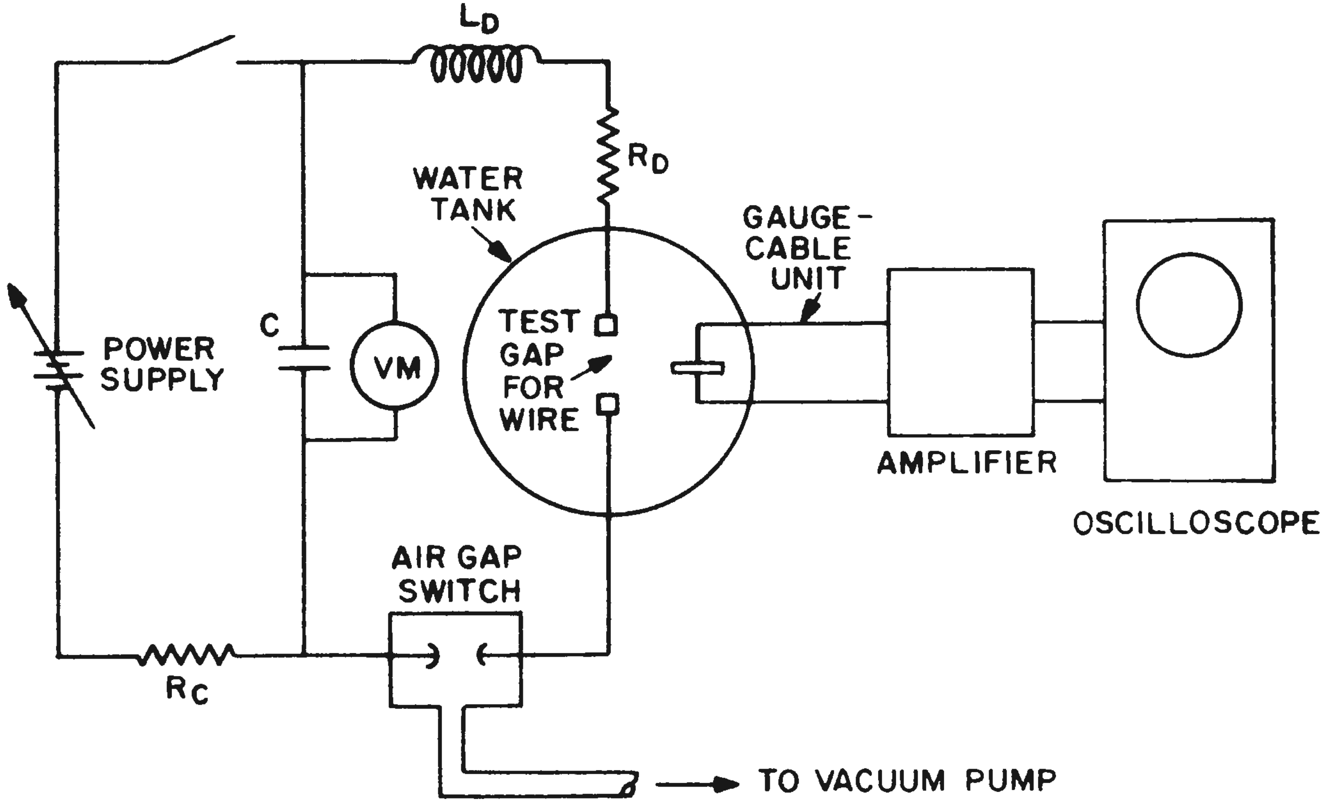


FIGURE 1. SCHEMATIC DIAGRAM OF THE EXPERIMENTAL ARRANGEMENT

$$A(t) = A_\infty(1 - \exp[-t/\tau_a])$$

where τ_a is a characteristic time constant for the combined system. A(t) is the amplifier response to the step function u(t). If an explosion wave, p(t), is imposed, A' presents this on the CRT as h(t); where h(t) is the amplifier response to the explosion wave, p(t) has the analytic expression:

$$p(t) = p_m \exp(-t/\tau_e)$$

where τ_e is a characteristic time constant of the explosion wave. These analytic quantities are defined for the following regions:

$$u(t) = \begin{cases} 0 \text{ for } t < 0 \\ 1 \text{ for } t \geqslant 1 \end{cases}$$

$$A(t) = \begin{cases} 0 \text{ for } t < 0 \\ A_\infty (1 - \exp(-t/\tau_a)) \text{ for } t \geqslant 0 \end{cases}$$

$$p(t) = \begin{cases} 0 \text{ for } t < 0 \\ p_m \exp(-t/\tau_e) \text{ for } t \geqslant 0 \end{cases}$$

$$h(t) = \begin{cases} 0 \text{ for } t < 0 \\ p(o)\, A(t) + \int_{\lambda=o}^{\lambda=t} p'(\lambda)\, A(t-\lambda)\, d\lambda \text{ for } t \geqslant 0 \end{cases}$$

Evaluation of h(t) is accomplished by the superposition theorem (references 2 and 3) also known as Duhamels Theorem. This theorem expresses the output voltage as a function of the input voltage and the response of the network to a step function; hence:

$$h(t) = p(o)\, A(t) + \int_{\lambda=o}^{\lambda=t} p'(\lambda)\, A(t-\lambda)\, d\lambda$$

where:

$$p(\lambda) = p_m \exp(-\lambda/\tau_e) \qquad A(t) = A_\infty(1 - \exp(-t/\tau_a))$$

$$p'(\lambda) = -(1/\tau_e)\, p_m \exp(-\lambda/\tau_e) \qquad A(t-\lambda) = A_\infty(1 - \exp((-t+\lambda)/\tau_a))$$

$$p(o) = p_m$$

then:

$$h(t) = p(o)\,A(t) + \int_{\lambda = o}^{\lambda = t} p_m A_\infty \left[(-1/\tau_e) \exp(-\lambda/\tau_e)\right]\left[1 - \exp(-t + \lambda/\tau_a)\right] d\lambda$$

$$h(t) = p_m A_\infty \left(\frac{\tau_e}{\tau_e - \tau_a}\right)\left[\exp(-t/\tau_e) - \exp(-t/\tau_a)\right]$$

Now we seek the value of t when h(t) is maximum.

Then: $$\frac{1}{\tau_a} \exp(-t/\tau_a) = \frac{1}{\tau_e} \exp(-t/\tau_e)$$

At h(t): max

$$t_{max} = \left(\frac{\tau_a \tau_e}{\tau_e - \tau_a}\right) \ln\left(\frac{\tau_e}{\tau_a}\right)$$

Now substitute this expression for t at h(t) equal to its maximum value into the expression for h(t0, yielding:

$$h(t)_{max} = p_m A_\infty \left(\frac{\tau_e}{\tau_e - \tau_a}\right)\left[\exp\left(\frac{-\tau_a}{\tau_e - \tau_a}\right) \ln\left(\frac{\tau_e}{\tau_a}\right) - \exp\left(\frac{\tau_e}{\tau_e - \tau_a}\right) \ln\left(\frac{\tau_e}{\tau_a}\right)\right]$$

Solving this for p_m we obtain:

$$\frac{p_m^*}{\left(\frac{\tau_e}{\tau_e - \tau_a}\right)\left[\left\{\exp\left(\frac{-\tau_a}{\tau_e - \tau_a}\right)\right\}\left\{\ln\left(\frac{\tau_e}{\tau_a}\right)\right\} - \left\{\exp\left(\frac{-\tau_e}{\tau_e - \tau_a}\right)\right\}\left\{\ln\left(\frac{\tau_e}{\tau_a}\right)\right\}\right]}$$

We define $h(t)_{max}/A_\infty \triangleq p_m^*$, where p_m^* is the uncorrected (observed) peak pressure and the corrected (real) pressure is p_m. The correction factor is the denominator of the last equation.

<u>Definition/Calculation of EWP Time Constant</u>

The pressure-time curve for CUE events varies slowly when compared to the transit time (of the shockwave) across the pressure gauge (see Figure 2); or

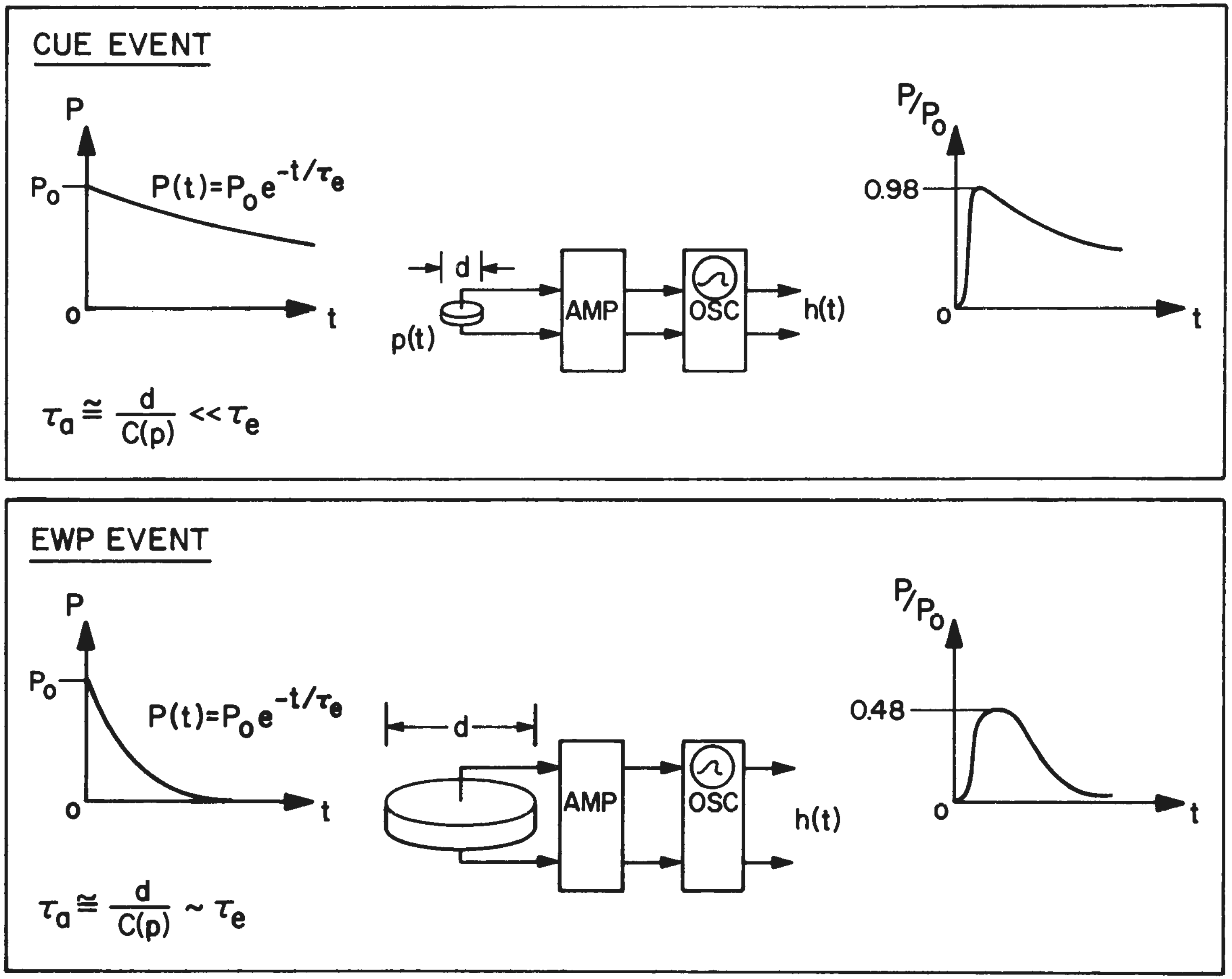


FIGURE 2. DISTINCTION BETWEEN CUE AND EWP EVENTS

equivalently, the characteristic (explosion) time constant for CUE events is much larger than the transit time. Such pressure-time data is photographically recorded from the CRT of an oscilloscope. A typical CUE oscillogram appears (at arbitrary time zero) to increase to peak pressure value in negligible time and then slowly decrease in value. A semilog plot of pressure vs. time yields a straight line with a negative slope. The time required for the pressure amplitude to decrease from maximum value to e^{-1} of its maximum value is defined as the explosion or characteristic time constant. For most CUE data the explosion time constant is of the order of milliseconds.

In the case of underwater exploding wires, however, the explosion time constant (τ_e) and the combined amplifier time constant (τ_a) are of the same magnitude. This presents a problem. The problem relates to the definition of the explosion time constant for EWP. Since the EWP event nominally is over in 30μ seconds, the explosion time constant is clearly of that order of magnitude. Since the pressure-time history of the EWP event does not give a straight line initially from peak pressure, we need a new definition for such rapid events. The explosion time constant is defined in the following paragraph.

The time at which h(t) becomes maximum is given as:

$$t_{max} = \left(\frac{\tau_a \tau_e}{\tau_e - \tau_a}\right) \ln \left(\frac{\tau_e}{\tau_a}\right)$$

Now t_{max} can be experimentally determined and τ_a is partly known from the manufacturer's data and partly calculated. Consequently τ_e can be obtained, $\tau_e \simeq \tau_a + \Delta$ where we expect Δ to be $1/2\ \tau_a < \Delta < \tau_a$. Upon substitution in the above equation, we obtain:

$$t_{max} = \frac{\tau_a \tau_e}{\tau_a + \Delta - \tau_a} \ln \left(\frac{\tau_a + \Delta}{\tau_a}\right) = \frac{\tau_a \tau_e}{\Delta} \ln \left(1 + \frac{\Delta}{\tau_a}\right)$$

$$t_{max} = \frac{\tau_a \tau_e}{\Delta} \left[\frac{\Delta}{\tau_a} - \frac{1}{2}\left(\frac{\Delta}{\tau_a}\right)^2 + \frac{1}{3}\left(\frac{\Delta}{\tau_a}\right) \text{---}\right]$$

Then:

$$t_{max} \simeq (\tau_a + \Delta)\left(1 - \frac{1}{2}\frac{\Delta}{\tau_a}\right)$$

$$\simeq \tau_a + \frac{1}{2}\Delta$$

$$= \tau_a + \frac{1}{2}(\tau_e - \tau_a)$$

$$t_{max} = \frac{1}{2}(\tau_a + \tau_e)$$

$$\tau_e = 2\,t_{max} - \tau_a$$

And this, by definition, is the explosion time constant. An equivalent method for defining τ_e can be used starting with:

$$t_{max} = \frac{\tau_a \tau_e}{\tau_e - \tau_a} \ln\left(\frac{\tau_e}{\tau_a}\right) \text{ and let } \alpha = \frac{\tau_a}{\tau_e}$$

$$t_{max} = \tau_e \left(\frac{\alpha}{1 - \alpha}\right) \ln\left(\frac{1}{\alpha}\right)$$

$$t_{max} = \tau_e \left(\frac{\alpha}{\alpha - 1}\right) 2 \left(\frac{\alpha - 1}{\alpha + 1}\right)$$

$$\tau_e = \left(\frac{1}{2} + \frac{\tau_e}{\tau_a}\right) t_{max}$$

$$\tau_e = \frac{\tau_a\, t_{max}}{2\tau_a - t_{max}}$$

$$\tau_e = 2\,t_{max} - \tau_a \quad \text{when } \tau_a \simeq \tau_e$$

The value of τ_a is made up of two contributions: The rise time of the amplifier (0.20μ sec) and the calculated transit time of the shockwave across the gauge (3.21μ sec). The combined value of τ_a is 3.41μ seconds, or:

$$\tau_a = \frac{d\,(1 - e^{-1})}{c} + t_{rise}$$

where d is the diameter of the circular gauge and c is the speed of sound which is not significantly different from the shock wave propagation velocity for small amplitude explosions.

The Relationship Between Capacitor Energy and Its Equivalent Weight of TNT

The heat of detonation (H_D) is usually expressed in terms of energy (E_D) per weight (W) of charge, or:

$$H_D = \frac{E_D}{W}$$

The energy stored in the capacitor of the experimental arrangement is $1/2\ CV_o^2$ due to the loss at the spark gap switch, k_1 is the constant (0.65) indicating the fraction of stored electrical energy that can be delivered to the wire. The vaporization energy is E_v. The corrected electrical energy available for the explosion process is, therefore:

$$E_e = k_1\ (\frac{1}{2}\ CV_o^2) - E_v$$

$$E_e = 0.325\ CV_o^2 - E_v$$

To relate these energies, we require that

$$E_D = E_e$$

or

$$WH_D = 0.325\ CV_o^2 - E_v$$

Finally

$$W = 525 \left(\frac{0.325\, CV_o^2 - E_v}{H_D}\right) \quad (\mu lb)$$

For example, if

$C = 0.5 \times 10^{-6}$ farad

$V_o = 20{,}000$ volts

$E_v = 10$ joules

$H_D = 1000$ cal/gm

Then

$$W = 525 \frac{0.324\,(0.5)\,10^{-6}\,(4)\,10^{8} - 10}{1000}$$

or

$$W \cong 29\ \mu lb \text{ of TNT at } p = 1.59 \text{ gm/cc}$$

The same technique is used for unconnected energies (i.e., 100% $1/2\, CV_o^2$)

$$E_D = E_e$$

where

$$E_D = W\, H_D$$

and

$$E_D = 1/2\, CV_o^2$$

Then

$$W = 0.239 \left(\frac{\frac{1}{2} CV_o^2}{H_D}\right) \text{(gm)}$$

or

$$W = 0.239 \left(\frac{\frac{1}{2} CV_o^2}{H_D}\right) (.0022) \quad \text{(lb)}$$

$$= 525 \times 10^{-6} \left(\frac{\frac{1}{2} CV_o^2}{H_D}\right) \text{(lb)}$$

Finally

$$W = 263 \left(\frac{CV_o^2}{H_D}\right) (\mu\text{lb})$$

For example, if

$$C = 0.5 \times 10^{-6} \text{ farad}$$

$$V = 20{,}000 \text{ volts}$$

$$H_D = 1000 \text{ cal/gm}$$

Then

$$W = \frac{263\ (0.5)\ 10^{-6}\ (4)\ 10^{8}}{1000}$$

$$W \cong 53 \mu\text{lb of TNT at } p = 1.59 \text{ gm/cc.}$$

The calculation of the radius of this equivalent charge of TNT proceeds as follows:

$$p = \frac{m}{v} = \frac{W/g}{v} = \frac{W/g}{\frac{4\pi}{3} a_o^3}$$

For a TNT density of 1.59 gm/cc we obtain

$$a_o = \left[\frac{W}{\frac{4\pi}{3} (1.59)\ (980)}\right]^{1/3} \text{cm}$$

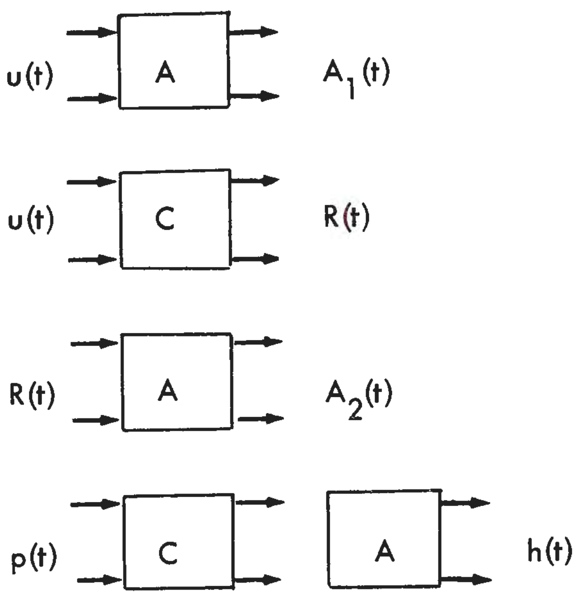
$$a_o = 0.41\ W^{1/3} \text{ in centimeters}$$

where W is the weight of the equivalent TNT charge in pounds.

APPENDIX A

In this section the detection system consisting of the gauge, amplifier, and oscilloscope are treated in greater detail.

With reference to the diagram below, the following quantities are identified: (A) refers to the preamplifier and the oscilloscope, (C) refers to the tourmaline gauge, $A_1(t)$ is the response of (A) to the step function u(t), R(t) is the response of the circular gauge to the step function u(t), and $A_2(t)$ is the response of (A) to the input function R(t). As shown, if the initial input is p(t), (a pressure wave), the output of (A) is h(t) instead of $A_2(t)$.



The functions $A_1(t)$, u(t), and R(t) are defined as follows:

$$A_1(t) = \begin{cases} 0 & \text{for } t < o \\ 1 - \exp(-t/\tau_a) & \text{for } t > o \end{cases}$$

$$u(t) = \begin{cases} 0 & \text{for } t < o \\ 1 & \text{for } t > o \end{cases}$$

$$R(t) = \begin{cases} 0 & \text{for } t < o \\ \dfrac{(\varphi - \sin\varphi \cos\varphi)}{\pi} & \text{for } 0 < t < 2a/c \\ 1 & \text{for } t > 2\,a/c \end{cases}$$

where τ_a is the rise time of the preamplifier and oscilloscope and φ is the arc cosine of $(1 - ct/a)$.

Considering a circular gauge (reference 4) may be approached by using Duhamel's theorem to solve for $A_2(t)$ and then h(t); specifically

$$A_2(t) = R(o)\, A_1(t) + \int_o^t R'(\lambda)\, A_1(t-\lambda)d\lambda$$

and

$$h(t) = p(o)\, A_2(t) + \int_o^t p'(\lambda)\, A_2(t-\lambda)d\lambda$$

Application of LaPlace transforms to these equations yields:

$$A_2(s) = s\, R(S)\, A_1(s)$$

and

$$H(s) = s\, p(s)\, A_2(s)$$

or $$H(s) = s^2\, p(s)\, R(s)\, A_1(s)$$

then $$p(s) = \frac{H(s)}{s^2\, R(s)\, A_1(s)}$$

In theory, if one knows H(s), R(s), and $A_1(s)$, then p(t) can be obtained.

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14 KEY WORDS	LINK A		LINK B		LINK C	
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