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<b>14. ABSTRACT</b> <p>In high-speed flight, increased viscous heating past the onset of transition to turbulence is a critical limiting factor in the design of hypersonic vehicles. Acoustic exponential instabilities, which are ineffective at low Mach numbers, amplify and open up additional avenues for breakdown to turbulence in the compressible boundary layer. The seeding of the instabilities is generated during the receptivity stage which governs the initiation of a disturbance field inside the shear, for instance by external disturbances. The present study seeks to rigorously analyze the receptivity process of the high-speed flow over a cone geometry by leveraging the structural sensitivity information provided by the adjoint linearized governing equations. The approach enables the direct identification of the free-stream conditions which are most effective at triggering perturbation growth in the boundary layer. A second key element of the study is the direct identification of strategies for delaying transition to turbulence in high-speed flows. It is well known that for instance surface porosity can weaken the amplification of instabilities, in particular the Mack second mode. The properties such as spacing and depth of the surface porosity which most effectively dampen the amplification of instabilities have however not been thoroughly established. As part of our study, we thus seek to rigorously identify the properties of optimal surface porosity using adjoint-based sensitivity analysis. A further element of the study, which is currently pursued, is the first identification of nonlinear optimal perturbations in high-speed flows. Nonlinear optimal perturbations can be seen as minimal seeds which induce transition to turbulence as efficiently as possible, and they can thus be interpreted as worst-case conditions for breakdown to turbulence. The present work for the first time introduces a framework for evaluating general nonlinear optimal disturbances in high-speed flows involving complex geometries.</p>					
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# Final Technical Report

Global stability and sensitivity analysis of a hypersonic slender cone

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## **Section I: Project Summary**

### **1. Overview of Project**

**Abstract:** In high-speed flight, increased viscous heating past the onset of transition to turbulence is a critical limiting factor in the design of hypersonic vehicles. Acoustic exponential instabilities, which are ineffective at low Mach numbers, amplify and open up additional avenues for breakdown to turbulence in the compressible boundary layer. The seeding of the instabilities is generated during the receptivity stage which governs the initiation of a disturbance field inside the shear, for instance by external disturbances. The present study seeks to rigorously analyze the receptivity process of the high-speed flow over a cone geometry by leveraging the structural sensitivity information provided by the adjoint linearized governing equations. The approach enables the direct identification of the free-stream conditions which are most effective at triggering perturbation growth in the boundary layer. A second key element of the study is the direct identification of strategies for delaying transition to turbulence in high-speed flows. It is well known that for instance surface porosity can weaken the amplification of instabilities, in particular the Mack second mode. The properties such as spacing and depth of the surface porosity which most effectively dampen the amplification of instabilities have however not been thoroughly established. As part of our study, we thus seek to rigorously identify the properties of optimal surface porosity using adjoint-based sensitivity analysis. A further element of the study, which is currently pursued, is the first identification of nonlinear optimal perturbations in high-speed flows. Nonlinear optimal perturbations can be seen as minimal seeds which induce transition to turbulence as efficiently as possible, and they can thus be interpreted as worst-case conditions for breakdown to turbulence. The present work for the first time introduces a framework for evaluating general nonlinear optimal disturbances in high-speed flows involving complex geometries.

**Objective:** The principal objectives of the study are to advance the insight into the physics driving transition in high-speed flows involving complex geometries of practical relevance such as cones and shock waves and to devise well-founded strategies for delaying the breakdown to turbulence in these settings. Specifically, an adjoint formulation of the linearized governing equations will for the first time allow the rigorous analysis of the receptivity stage of transition to turbulence in the flow over a cone. Relating external disturbances to perturbations inside the boundary layer is a critical prerequisite to the development of computational models. The adjoint methodology is also applied in the formulation of strategies for delaying transition to turbulence by weakening the amplification of instabilities such as the Mack second mode. Lastly, a new framework based on the adjoint of the full nonlinear Navier-Stokes equations will allow insights into so-called minimal seeds, or kernels, specific perturbations which trigger transition to turbulence more effectively than the sequence of linear mechanisms which is commonly thought to cause breakdown to turbulence.

**Introduction:** Laminar-turbulent transition describes the sequence of disturbance receptivity and the amplification of perturbations through interactions with mean shear. The receptivity stage governs the initiation of a perturbation field inside regions of shear such as boundary layers, e.g. due to external disturbances or surface roughness. It has long remained elusive and difficult to grasp, although the arrival of adjoint-based methods during the past decade or so has led to notable

advancements for incompressible flows. In contrast, receptivity studies in the hypersonic regime have largely relied on empirical parameter studies which reported the response of the flow for different upstream conditions. The present study is the first to make use of the structural receptivity information provided by the adjoint governing equations to rigorously evaluate and study the receptivity process. In addition, our study also seeks to rigorously identify strategies for delaying transition to turbulence. It has been established that porous surface coatings can delay the amplification of instabilities and as such also the onset of transition. However, a rigorous analysis of the optimal properties of porous surfaces is missing from the literature. As part of the present study, we thus aim to analyze the structural sensitivity of porous walls with the aim of identifying the specific parameters which most effectively delay instability growth. An additional element of fundamental novelty is the analysis of nonlinear optimal disturbances in compressible flow.

Background: The stability analysis is facilitated through a newly developed computational framework. The spatial discretization of the governing equations is based on fourth-order central finite differences. Time integration is facilitated through a fourth-order Runge-Kutta scheme. Complex geometries are treated via a curvilinear formulation invoking the chain rule to evaluate the Cartesian derivatives. The grid metrics are computed to discretely satisfy geometric conservation, ensuring that a uniform flow field is not affected by the grid topology down to machine precision. Skew-symmetrically split convective terms are used to reduce aliasing errors and spurious contributions to the kinetic energy from the nonlinear terms. Boundary conditions are imposed via penalty terms in the governing equations using the simultaneous approximation term (SAT) method. Far-field boundaries are enforced with inviscid characteristics to reduce reflections. The penalties for both far-field and wall boundaries are chosen such that, when used with summation-by-parts (SBP) operators, the code is provably linearly stable. In the case of axisymmetric geometry with a coordinate singularity, the axis is treated by shifting grid points to avoid evaluating the governing equations on the axis with differentiation performed through the pole as though it was an internal point. Shocks are captured using artificial bulk viscosity, which ensures minimal impact on vortical structures passing through shocks. In the present setting, the nonlinear flow solver is mainly used to compute the base flow used in the linear stability and sensitivity analyses. The numerical solution of the direct and adjoint linearized NSE is based on the same spatial and temporal discretization as the nonlinear solver. The boundary conditions are imposed using the SAT method with Dirichlet conditions on all boundaries. Owing to the linear nature of the equations, shock capturing is not required. The linearized NSE are implemented in operator form. Once the direct operators are formed, the adjoint operators are effortlessly implemented by suitably recombining the elements of the direct operator. The eigenvalue problems arising from the linearized equations are solved using the P\_ARPACK package, which is based on the implicitly restarted Arnoldi method. In the case when the base state is homogeneous in one dimension (e.g., for an axisymmetric problem), the size of the eigenvalue problem can be significantly reduced by Fourier transforming in that direction.

## **2. Activities and Accomplishments**

We have run the first global stability and sensitivity analyses in the flow over a blunt cone, as set out in the original proposal of the project. In addition, we have also successfully completed two studies in fields related to hypersonic flows. The first work investigates the most effective delay of flow separation, an issue that is also of concern in high-speed applications. The second study describes a method for identifying nonlinear optimal disturbances in compressible flows, using a formulation that is compatible for instance with the presence of shocks. Both studies have been recently published in the Journal of Fluid Mechanics.

## Receptivity analysis of high-speed cone

A key element of our project is the integration of the curved bow shock of the blunt cone in our computational domain. Following the validation of the core formulation of our newly developed computational framework, as described in last year's report, we identified possible inaccuracies in our solutions, specifically related to curved shock waves. Further analysis of the problem showed that it was related to the implemented shock-capturing method which follows the current state of the research (Kawai et al. 2010, JCP), but nonetheless introduced small oscillations in the streamwise vorticity component as visualized in figure 1. We note that the colormap in this figure is highly saturated at both ends and chosen to specifically visualize the small perturbations. For a linear colormap that captures both the minimum and maximum in the flow field, these perturbations are invisible. The eigensolutions computed in the linear analysis are nonetheless highly sensitive to the vorticity in the vicinity of the shock, and show visible oscillations for this base flow.

We identified small artifacts arising from the evaluation of the bulk viscosity, which is added in the shock capturing scheme to smear the shock over a few grid points and thus enables the computational treatment of the discontinuity, as the main cause for these distortions in the vorticity field. Specifically, the current approach relies on high-order derivatives of the flow field to identify regions of strong gradients that represent shocks. However, in the case of curvilinear grids, this approach can lose robustness. We have developed a solution for the problem by applying a Gaussian filter to the gradient of the density field that is used in the computation of the relevant length scale of the bulk viscosity computation. A second key change was the consideration of the divergence of the velocity field in determining the magnitude of the added bulk viscosity, instead of the fourth power of the Laplacian of the divergence, as recommended in the literature on artificial bulk viscosity. Together with an optimization of the computational grid in the vicinity of the shock, these modifications allowed us to very accurately capture the curved shock on our grid, as visualized in figure 1.

The effect of the base flow on the computed eigenfunctions is presented in figure 2. The left panel shows an eigenfunction that was computed for a base flow without adjustments to the shock capturing scheme. The eigenfunction displays oscillations in a region below the shock, but also on a considerably finer scale in the immediate vicinity of the shock. An additional effect of the oscillatory base flow was a reduction in the convergence rate of the eigenvalue solver which complicated the solution of the stability problem. The eigenfunction for the base flow computed with the improved shock-capturing scheme on the optimized grid is presented in the right panel of figure 2. The modification of the base flow has eliminated both types of oscillations in the eigenfunction which now represents the actual physics of the flow.

At this stage, our newly developed computational framework is thus able to reliably solve the stability and sensitivity problem in high-speed flows with shock for geometries of moderate size. However, the comparatively long transition length in realistic cone settings requires a considerable extension of the size of the computational domain compared to the test cases

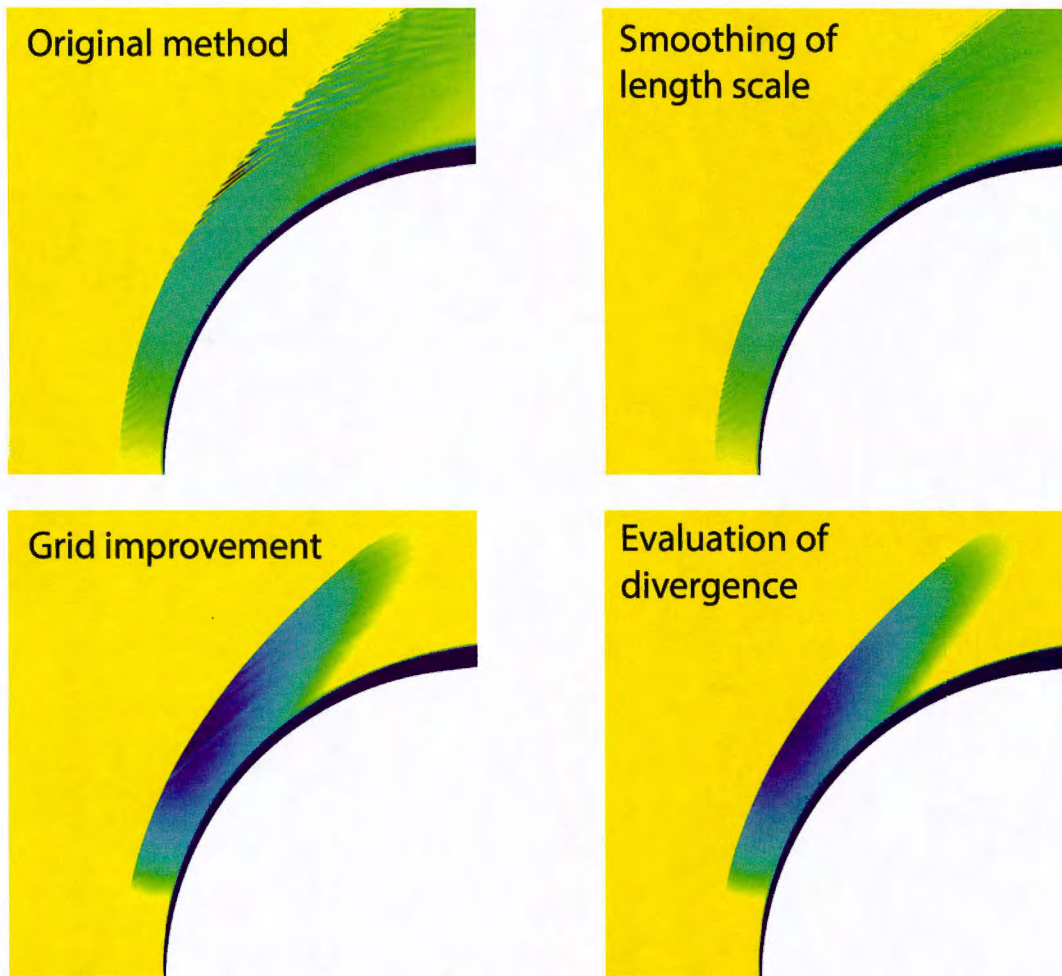


Figure 1. Streamwise vorticity at the curved bow shock. Top left, original method. Top right, after the smoothing of the length scale. Bottom left, after additionally adjusting the grid. Bottom right, after additionally basing the magnitude of the bulk viscosity on the divergence of the velocity field. The colormaps are saturated with ranges chosen to optimally visualize the distortions in the vorticity.

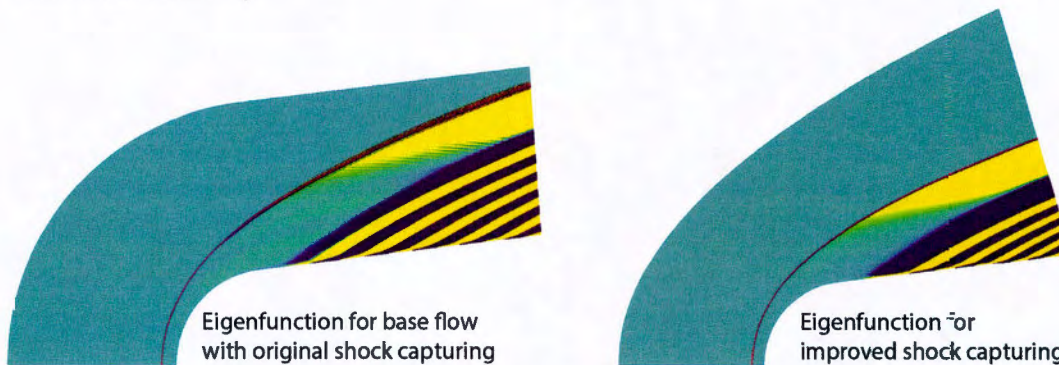


Figure 2. Streamwise velocity component of eigenfunctions computed for before (left) and after (right) improving the shock capturing method. The original shock capturing method introduced visible artifacts into the eigenfunctions, both in a region under the shock, but also in band along the shock.

considered so far. In terms of the computational framework, this setting required changes to the parallelization of our code. The code was already fully parallel based on the MPI standard, both in the computation of the base flow, and the solution of the stability problem. The parallelization was nonetheless limited to the azimuthal and radial (wall-normal) dimensions. To facilitate the effective treatment of an extension of the axial length of the cone, we have extended the parallelization to the third dimension.

The computational grid of the analysis is presented in figure 3. For visualization purposes, only every 24th grid point is shown. The geometry follows the experiments by Stetson et al. 1984 and has a half-angle of 7 degrees with a length of the geometry of 300 nose radii. The grid captures the full length of the bow shock within the computational domain.

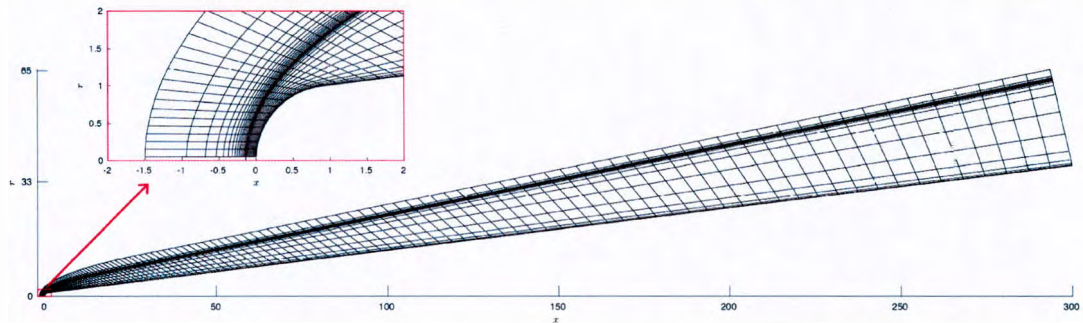


Figure 3. Computational grid of the full-length cone to be considered. The streamwise extent of the geometry is 300 nose radii. The full extent of the bow shock is captured within the computational domain. Only every 24th grid point is shown.

Contours of the Mach number of the base flow for a free-stream velocity corresponding to  $Ma=6$  are presented in figure 4. The dashed line indicates the location of the curved shock, and the solid line approximates the position of the boundary layer thickness based on the Mach number attaining 99% percent of its free-stream value. The inset magnifies the region near the nose of the cone, where the boundary layer is thinnest.

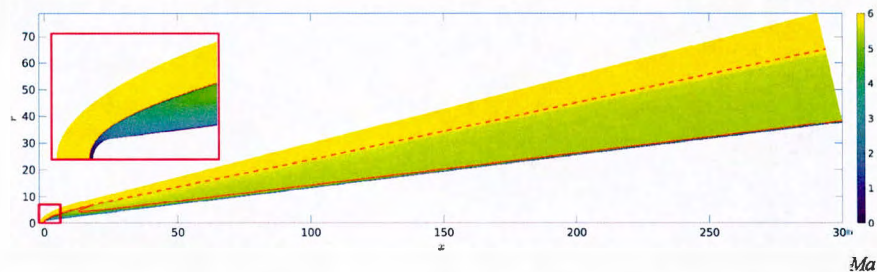


Figure 4. Base flow of the analysis of flow over a blunt cone. Contours of the Mach number. The inset magnifies the region near the nose of the cone.

The global stability analysis conducted as part of this project is based on the solution of the global stability eigenvalue problem

$$i\omega \mathbf{q} = \mathbf{L}\mathbf{q}$$

where  $\omega$  is the complex frequency,  $\mathbf{q}$  is the disturbance state vector and the operator  $\mathbf{L}$  represents the linearized compressible Navier-Stokes equations for the base flow presented in figure 4. Eigensolutions to the stability problem determine the amplification of disturbances during long time horizons. The first mode computed for the cone at  $Ma=6$  is presented in figure 5. The result shows a disturbance that is localized within the boundary and amplifies in the streamwise direction beyond a distance to the leading edge that is representative of Reynolds numbers  $Re_x > 3.6 \times 10^6$ . The velocity, pressure and temperature profiles of the global mode at  $x=240$  bear resemblance to the local eigenfunctions of the Mack second mode, see figure 6.

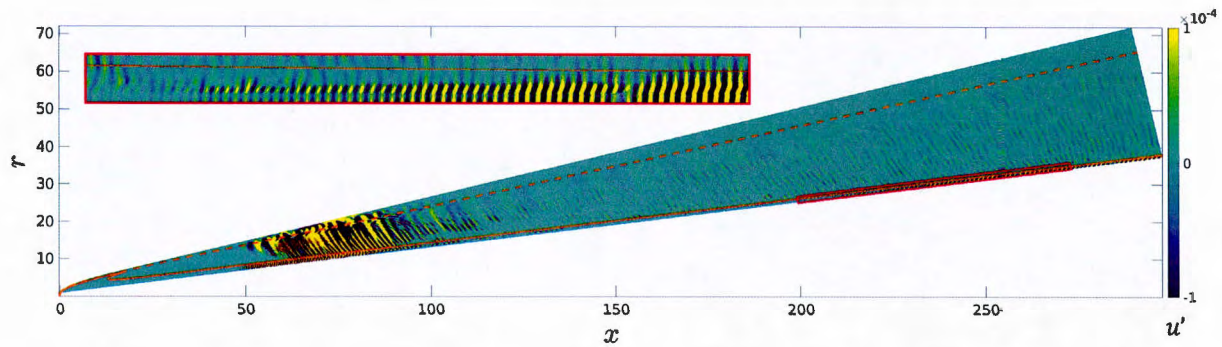
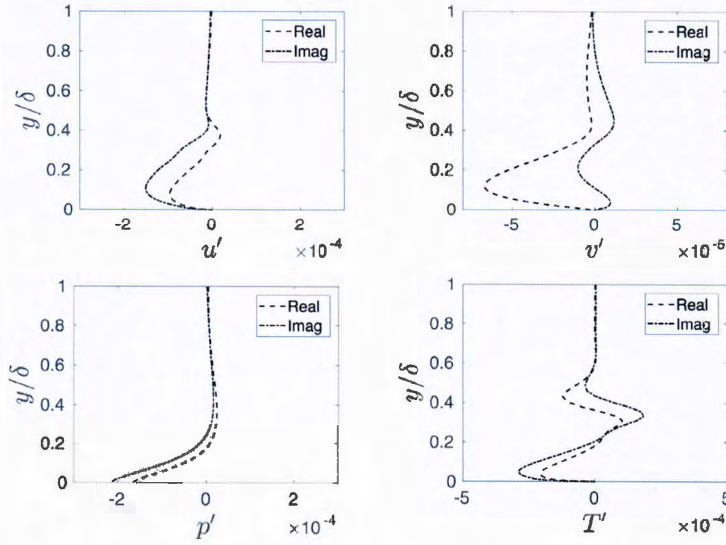


Figure 5. First eigenfunction of the linearized governing equations. Contours of the streamwise perturbation velocity component. The inset magnifies the boundary layer in the region of spatial perturbation growth.



The adjoint eigenvalue problem is described by

$$i\omega^* \mathbf{q}^\dagger = \mathbf{L}^\dagger \mathbf{q}^\dagger.$$

Figure 6. Profiles of the global eigenfunction within the boundary layer at  $x=240$ . Streamwise velocity disturbance (top left), wall-normal velocity disturbance (top right), pressure disturbance (bottom left) and temperature disturbance (bottom right).

Here,  $\omega^*$  is the adjoint eigenvalue,  $\mathbf{q}^\dagger$  is the adjoint state vector and the operator  $\mathbf{L}^\dagger$  represents the adjoint of the linearized governing equations. For each eigenvalue  $\omega$  of the forward stability problem, there exists an adjoint eigenvalue  $\omega^*$  which is the complex conjugate of the former. In mathematical terms, the related adjoint eigenfunction  $\mathbf{q}^\dagger$  represents the sensitivity of the instability mode described by the eigenfunction  $\mathbf{q}$  of the stability problem. Physically, the adjoint eigenfunctions represent the receptivity process of the flow, i. e. they indicate where and how the spatially amplifying perturbations represented by the direct eigenfunctions are triggered most effectively.

The adjoint eigenfunction corresponding to the mode shown in figure 5 is presented in figure 7. The eigenfunction attains its highest amplitude in the upstream part of the computational domain, indicating that the instability is triggered most effectively close to the nose and during the early stages of the development of the boundary layer. The region of highest receptivity is upstream of the normal part of the bow shock near the nose of the cone. The eigensolution also indicates that the flow is receptive to free-stream forcing of a preferred wavelength in regions away from the nose.

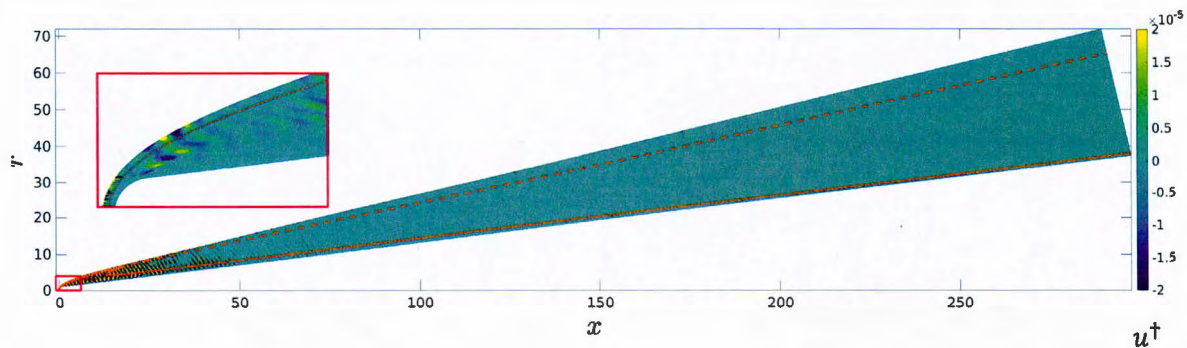


Figure 7. First eigenfunction of the adjoint of the linearized governing equations. Contours of the adjoint streamwise velocity component. The inset magnifies the region of highest sensitivity near the cone tip.

Solving eigenvalue problems of the size shown in figure 3 require significant computational resources and convergence stalls in certain situations making it difficult to confidently identify eigenfunctions. A more efficient way to solve the eigenvalue problem is to use a shift-invert method where a modified form of the eigenvalue problem is solved

$$\mu \mathbf{q} = (\mathbf{L} - \sigma \mathbf{I})^{-1} \mathbf{q}$$

where  $\sigma$  is the shift parameter controlling where in the spectrum is amplified and  $\mu$  is the modified eigenvalue defined by

$$\mu = \frac{1}{i\omega - \sigma}$$

The shifted eigenvalue,  $\mu$ , has large magnitude when  $\sigma$  is close to  $i\omega$  which makes the eigenvalues close to  $-i\sigma$  easy to find. The shift-invert method also allows the solver to target specific regions of the eigenvalue spectrum as controlled by the user, a functionality that was not present with the previous, time advancing, method and can be used to investigate various physical mechanisms present in this setting.

Solving the shift-invert problem requires the explicit construction and solution of a large sparse system of equations in a highly parallel setting. In general, the solution of such large systems is a difficult problem and is made possible by the sparsity of the system resulting from our choice to use a finite difference discretization. We use the MUMPS software package to solve the system of equations quickly and accurately. With the shift-invert method, we have now successfully solved eigenvalue problems on the mesh shown in figure 3 in a fraction of the time that was required by the original method and with significantly improved accuracy.

We are now focusing on methods to eliminate numerical eigenfunctions (i.e. those arising from our numerical discretization rather than the underlying physics of the problem) from the region of interest in the eigenvalue spectrum. These numerical eigenfunctions make it difficult to isolate the modes of interest by contaminating the spectrum. As shown in figure 8, a large number of

eigenvalues that were found are not physical (the open symbols) and mean that we must converge many modes in order to find a small number of physical modes. The application of a high-order compact filter effectively dampens the numerical modes, shifting them to a part of the spectrum that we are not interested in and allowing more physical modes to be found (blue symbols in figure 8). The discrepancy between the purple and blue physical modes is thought to be related to insufficient grid resolution. A high-order compact filter is a very controlled method of dampening modes that have grid scale oscillations while leaving well resolved modes relatively unmodified when compared to other methods such as adding a hyperviscosity term or using a dissipative numerical scheme which are prevalent in the current literature.

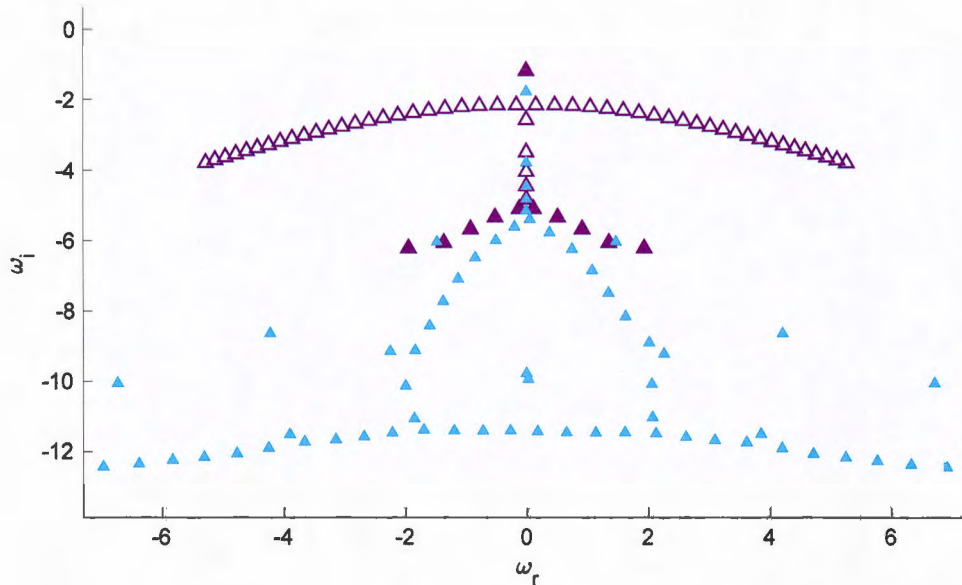


Figure 8. Eigenvalue spectrum of the flow over a short blunt cone. Purple symbols are computed without a high-order filter; blue symbols are computed with a high-order compact filter; and the open symbols denote numerical eigenvalues that correspond to highly oscillatory eigenfunctions.

### Nonlinear optimal perturbations in compressible flows

While the main focus of our project is the analysis of the linear mechanisms leading to disturbance growth, nonlinear mechanisms can also play an important role in the transition to turbulence. Specifically, the disturbances identified in classical linear stability analysis which describe exponentially or transiently amplifying solutions are not necessarily the most effective in triggering transition to turbulence, which is an inherently nonlinear process. More accurately, the linear solutions are not even necessarily most effective at amplifying at finite amplitude. We have thus developed a unique computational framework, which for the first time allows the identification of nonlinear optimal disturbances in compressible flows. The formulation is compatible with the presence of shocks and moderately hypersonic Mach numbers, consistent with the conditions considered within our cone project. This work has been published in the *Journal of Fluid Mechanics*, and we refer to the paper for the full details. In the present context, we nonetheless note

that this unique capability prepares the ground for future research, for instance related to the full nonlinear interaction of perturbations with shock waves, which eludes current linear analyses.

### **Optimal delay of laminar separation bubbles**

Flow separation is known to be a critical issue, not only at low speeds, but particularly also in the hypersonic flow regime where configurations such as ramps, or changes in the angle of cones can trigger sustained flow separation. As part of this study, we have investigated the particular upstream conditions, which most effectively delay the onset of a separation bubble in a laminar boundary layer. The original study was recently published in the *Journal of Fluid Mechanics* and considered a flat-plate boundary layer. However, we have since also applied the underlying concept to more realistic geometries, including a NACA 0012 airfoil. The project was also presented as part of the start-up meeting of the NATO STO ET-190 Hypersonic Transition workgroup, and we believe that an application to high-speed flows, where flow separation is a particularly critical issue as it can increase the heat flux towards the walls, could have significant potential.

## **3. Findings and Conclusions**

### **Structural receptivity analysis**

Global stability analysis for a blunt cone at Mach 6 was conducted. The geometry of the cone describes a 7 degree half-angle and a length of 300 nose radii and follows experimental setups such as the one by Stetson et al. (1984). The computed eigenfunctions amplify spatially within the boundary layer along the cone. The wall-normal profile of the modes is related to the Mack second mode as identified in the literature. The modes are initially stable but begin to amplify past a critical Reynolds number based on the distance to the leading edge of approximately  $Re_x > 3.6 \times 10^6$ . The corresponding adjoint eigenfunctions show that the region of highest sensitivity, which governs the physical receptivity process is near the tip of the cone.

### **Optimal suppression of laminar separation**

*Abstract of the published journal paper (DOI 10.1017/jfm.2020.157):*

By means of nonlinear optimization, we seek the velocity disturbances at a given upstream position that suppress a laminar separation bubble as effectively as possible. Both steady and unsteady disturbances are examined and compared. For steady disturbances, an informed guess based on linear analysis of transient perturbation growth leads to significant delay of separation and serves as a starting point for the nonlinear optimization algorithm. It is found that the linear analysis largely captures the suppression of the separation bubble attained by the nonlinear optimal perturbations. The mechanism of separation delay is the generation of a mean flow distortion by nonlinear interactions during the perturbation growth. The mean flow distortion enhances the momentum close to the wall, counteracting the deceleration of the flow in that region. An examination of the effect of the disturbance spanwise wavenumber reveals that perturbations maximizing the mean flow distortion also approximately maximize the peak wall pressure, which is beneficial for lowering form drag. The optimal spanwise wavenumber leading to maximal peak wall pressure is significantly larger than the one maximizing the shift in separation onset. For unsteady disturbances, the mechanism of separation delay relies on enhancing wall-normal momentum transfer by triggering instabilities of the separated shear layer. It is found that Tollmien-Schlichting waves obtained from linear stability theory provide accurate estimates of the nonlinearly optimal disturbances. Comparison of optimal steady and unsteady perturbations reveals that the latter are able to obtain a higher time-averaged peak wall pressure.

## **Nonlinear optimal disturbances**

*Abstract of the published journal paper (DOI 10.1017/jfm.2020.189):*

A variational framework for the identification and analysis of general nonlinear optimal disturbances in compressible flows is derived. The formulation is based on the compressible Navier–Stokes equations in conserved variables for an ideal gas with temperature-dependent viscosity. A discretely consistent implementation based on generalized coordinates allows the accurate analysis of a wide range of settings. An application in the identification of the optimal disturbances which experience the highest amplification in kinetic energy in pipe flow is presented. At low Mach numbers and moderate initial amplitude, the disturbances undergo a sequence of Orr mechanism, oblique nonlinear interaction and lift-up mechanism, and the energy amplification is consistent with results reported for incompressible flow (Pringle & Kerswell, *Phys. Rev. Lett.*, vol. 105, 2010, 154502). When the Mach number is increased, the gain in perturbation kinetic energy grows appreciably, and the initial disturbance field becomes increasingly localized. Nonlinear optimal disturbances which are rescaled to higher initial kinetic energy than prescribed in the optimization procedure are demonstrated to evolve into a chaotic state. For a constant time horizon, the initial perturbation energy to reach a high-energy state decreases monotonically with Mach number.

### **4. Plans and Upcoming Events**

Upcoming milestones:

- Finalize and publish the results from the adjoint-based receptivity analysis of the flow over a full-length cone in a peer-reviewed journal paper

### **5. Transitions and Impacts**

Not applicable.

### **6. Collaborations**

Not applicable.

### **7. Personnel**

Parviz Moin, Principal investigator, 1 month effort, National Academy Member  
Philipp Hack, Co-PI, 12 months worked

### **8. Students**

One doctoral student and two postdocs have been working on the project.

### **9. Technology Transfer**

Not applicable.

### **10. Products, Publications, Patents, License Agreements, etc.**

Publications resulting from this project:

Archival Publications (publication reference information (article title, authors, journal, date, volume, issue) can be automatically entered using a DOI)

- a. Title: Extreme events in wall turbulence
  - b. Journal: Journal of Fluid Mechanics
  - c. Authors: M. J. Philipp Hack, Oliver T. Schmidt
  - d. Keywords: intermittency, turbulent boundary layers
  - e. Distribution Statement: N/A
  - f. Publication Status: Published
  - g. Publication Identifier Type: DOI
  - h. Publication Identifier: 10.1017/jfm.2020.798
  - i. Publication Date: November 2020
  - j. Volume: 907
  - k. Issue
  - l. First Page Number: A
  - m. Publication Location: Cambridge, United Kingdom
  - n. Acknowledgement of Federal Support? Yes
  - o. Peer Reviewed? Yes
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- a. Title: A variational framework for computing nonlinear optimal disturbances in compressible flows
  - b. Journal: Journal of Fluid Mechanics
  - c. Authors: Zhu Huang, M. J. Philipp Hack
  - d. Keywords: nonlinear instability, variational methods
  - e. Distribution Statement: N/A
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  - n. Acknowledgement of Federal Support? Yes
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- a. Title: Optimal suppression of a separation bubble in a laminar boundary layer
  - b. Journal: Journal of Fluid Mechanics
  - c. Authors: Michael Karp, M. J. Philipp Hack
  - d. Keywords: boundary layer separation, flow control
  - e. Distribution Statement: N/A
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b. Journal: Journal of Fluid Mechanics  
c. Authors: M. J. Philipp Hack, Parviz Moin  
d. Keywords: boundary layers, instability, turbulent flows  
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k. Issue  
l. First Page Number: 917–955  
m. Publication Location: Cambridge, United Kingdom  
n. Acknowledgement of Federal Support? Yes  
o. Peer Reviewed? Yes

a. Title: Transition to turbulence over convex surfaces  
b. Journal: Journal of Fluid Mechanics  
c. Authors: Michael Karp, M. J. Philipp Hack  
d. Keywords: boundary layers, boundary layer stability, transition to turbulence  
e. Distribution Statement: N/A  
f. Publication Status: Published  
g. Publication Identifier Type: DOI  
h. Publication Identifier: doi:10.1017/jfm.2018.690  
i. Publication Date: November 2018  
j. Volume: 855  
k. Issue  
l. First Page Number: 1208–1237  
m. Publication Location: Cambridge, United Kingdom  
n. Acknowledgement of Federal Support? Yes  
o. Peer Reviewed? Yes

a. Article Title: Modeling boundary-layer transition in direct and large-eddy simulations using parabolized stability equations  
b. Journal: Physical Review Fluids  
c. Authors: Adrian Lozano-Duran, M. J. Philipp Hack, Parviz Moin  
d. Keywords: boundary layers, flow instability, transition to turbulence, fluid dynamics  
e. Distribution Statement: Approved for public release: distribution unlimited  
f. Publication Status: published  
g. Publication Identifier Type: DOI  
h. Publication Identifier: 10.1103/PhysRevFluids.3.023901  
i. Publication Date: 21 February 2018  
j. Volume: 3  
k. Issue: 2  
l. First Page Number: 023901  
m. Publication Location:

- n. Acknowledgement of Federal Support? Yes
- o. Peer Reviewed? Yes

### Conference Papers

- a. Title: Use of instabilities for optimal laminar separation delay
- b. Authors: Michael Karp, M. J. Philipp Hack
- c. Conference Name: IUTAM Symposium on Laminar-Turbulent Transition
- d. Conference Date: September 2019
- e. Conference Location: London
- f. Publication Status: Accepted
- g. Publication Date:
- h. Publication Identifier Type:
- i. Publication Identifier:
- j. Acknowledgement of Federal Support? Yes

- a. Title: Nonlinear optimal disturbances in compressible shear flows
- b. Authors: M. J. Philipp Hack, Tim Flint, Zhu Huang
- c. Conference Name: IUTAM Symposium on Laminar-Turbulent Transition
- d. Conference Date: September 2019
- e. Conference Location: London
- f. Publication Status: Accepted
- g. Publication Date:
- h. Publication Identifier Type: DOI
- i. Publication Identifier: 10.1017/jfm.2020.189
- j. Acknowledgement of Federal Support? Yes

- a. Title: Global stability analysis of supersonic jets
- b. Authors: Michael Karp, Tim Flint, M. J. Philipp Hack
- c. Conference Name: AIAA SciTech
- d. Conference Date: January 2020
- e. Conference Location: Orlando
- f. Publication Status: Published
- g. Publication Date: January 2020
- h. Publication Identifier Type: DOI
- i. Publication Identifier: 10.2514/6.2020-1248
- j. Acknowledgement of Federal Support? Yes

- a. Title: Separation delay by optimal streaks
- b. Authors: Michael Karp, M. J. Philipp Hack
- c. Conference Name: AIAA Aviation
- d. Conference Date: June 2019
- e. Conference Location: Dallas
- f. Publication Status: Published
- g. Publication Date: June 2019
- h. Publication Identifier Type: DOI
- i. Publication Identifier: 10.2514/6.2019-2960
- j. Acknowledgement of Federal Support? Yes

- a. Title: Flows over convex surfaces undergoing transient growth
- b. Authors: Michael Karp, M. J. Philipp Hack
- c. Conference Name: AIAA Aviation
- d. Conference Date: June 2018
- e. Conference Location: Atlanta
- f. Publication Status: Published
- g. Publication Date: June 2018
- h. Publication Identifier Type: DOI
- i. Publication Identifier: 10.2514/6.2018-3385
- j. Acknowledgement of Federal Support? Yes

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## **Section II: Project Metrics**

**Grant or Contract Number:** N0004-17-1-2341

**Date Prepared:** 5/25/2021

**[Project Title]:** Global stability and sensitivity analysis of a hypersonic slender cone

**Annual Summary Report:** FY21

**PI:** Parviz Moin, (650) 723-9713, moin@stanford.edu

Leland Stanford Junior University

### **Metrics**

Number of faculty supported under this project during this reporting period: 1

Number of post-doctoral researchers supported under this project during this period: 2

Number of graduate students supported under this project during this reporting period: 1

Number of undergraduate students supported under this project during this period: 0

Number of scientists / engineers / technicians supported under this project during this reporting period: 1

Number of refereed publications during this reporting period for which at least 1/3 of the work was done under this effort: 6

Number of publications (all) during this reporting period: 11

Number of patents during this reporting period: 0

Number of M.S. students graduated during this reporting period: 0

Number of Ph.D. students graduated during this reporting period: 0

Awards received during this reporting period: N/A

Invited talks given: 1

Conferences at which presentations were given (not including invited talks above):

American Physical Society/Division of Fluid Dynamics (4 talks)

American Institute of Aeronautics and Astronautics/Aviation Conference (2 talks)

**1. Financial information**

<b>FY 2019</b>	<b>Total Budget</b>	<b>Obligated This Period</b>	<b>Obligated Cumulative</b>	<b>Expended This Period</b>	<b>Expended Cumulative</b>	<b>Grant/Contract Period of Performance</b>
<b>6.1 (Basic Research Funding)</b>	<b>\$1,242,084</b>	<b>\$426,592</b>		<b>\$426,592</b>		<b>04/01/2019 – 12/31/2020</b>
<b>6.2 (Applied Research Funding)</b>						
<b>Total (if both 6.1 and 6.2 funding was used)</b>	<b>\$1,242,084</b>	<b>\$426,592</b>		<b>\$426,592</b>		<b>04/01/2019 – 12/31/2020</b>

**2. Administrative notes and other items of interest**

Not Applicable