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A SIMPLE P/T AEROMETER

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ABSTRACT

There are numerous situations in which a knowledge of the density of ambient air is more useful than a simple measurement of barometric pressure. By application of the gas laws a commercial diaphragm-type absolute pressure indicator can be converted into an instrument giving a direct indication of air density. This can be done by simply filling the normally evacuated chamber with dry air and sealing it under conditions such that the pointer indicates the density of ambient air at the time of sealing. The range of densities covered in a given instrument would depend on the dimensions of the chamber and the mechanical properties of the diaphragm. The principles were put into practice using a commercial instrument, and the converted instrument gives accurate, reproducible results with a response time comparable to that of a mercury-in-glass thermometer.

PROBLEM STATUS

This is a final report. Unless otherwise notified by the Bureau, the Laboratory will close this problem one month after the issuance of this report.

AUTHORIZATION

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INTRODUCTION

There are several situations in routine fleet operations for which a knowledge of the density of the atmosphere is more useful than a simple measurement of barometric pressure. Whenever such a situation arises, it is customary to divide the ratio of barometric pressure to absolute temperature by the ratio of 760 mm Hg to 273°K, which gives the density of ambient air at standard conditions. If $\rho = 760/273$ and ρ_x is the ratio $B/(T + 273)$, then the relative density is

$$\frac{\rho_x}{\rho} = \frac{B/(273 + T)}{760/273} = \frac{B}{760} \times \frac{273}{T + 273}.$$

Some of the situations in which a knowledge of the density of ambient air is useful or actually essential are:

1. Combustion processes
2. Power calculations for all types of engines
3. Lift and drag of airfoils
4. Flight of projectiles and missiles
5. Meteorological observations
6. Altimetry.

These situations arise so frequently that there appears to be a real need for an instrument capable of giving a direct indication of the density of ambient air. That is, the instrument must be able to indicate with a reasonable accuracy and with as little lag as possible the ratio of barometric pressure to absolute temperature when both pressure and temperature are varying. Such an instrument is possible and it has been found that a standard commercial diaphragm-type absolute pressure indicator can be converted into a P/T indicator simply by filling the normally evacuated diaphragm capsule with dry air to a certain pressure. Filling the capsule with air enables the free end to move in response to changes in ambient temperature as well as barometric pressure with the result that the ratio P/T can be indicated on a calibrated scale.

FUNDAMENTAL PRINCIPLES

The action of a body of dry air confined in a flexible capsule to indicate density of the ambient air depends entirely on the elementary gas laws. Knowledge of the density of ambient air can be obtained from measurements of atmospheric pressure and temperature and these varying external conditions can cause changes in the body of gas confined in such a capsule thereby producing movements of the diaphragm. It is easy to show that a sealed capsule containing dry air can be made to operate a pointer over a scale in direct proportion to the density of the ambient air, and it is possible in this manner to obtain an empirical calibration of density relative to 1.00 at 0°C and 760 mm Hg.

The only reference in the literature that has been found relating to the use of a gas-filled flexible vessel to operate a pointer over a scale is U. S. Patent 2,000,308. In this patent the inventor uses a sealed bellows to indicate "specific volume" which is the reciprocal of density, but he gives no calculations or examples of the application of the gas laws to the behavior of the sealed system. There are also several patents in which the effects of temperature on the measurement of absolute pressure by means of a diaphragm capsule are compensated by leaving a little air in the capsule. In all the patents examined there were no actual quantitative calculations of the effects of temperature and pressure on the movement of the flexible member, and the whole procedure was empirical.

However, if it is desired to indicate density of the air over a certain range with a certain mechanism, it is necessary to select a diaphragm of known internal volume V_o which will give full-scale reading with the maximum pressure differential which will be encountered. Assuming the pressure differential and the deflection are zero at the lowest density (low barometric pressure and high temperature) the capsule is sealed at this point when it contains a volume V_o of dry air at temperature T_o and pressure B_o giving a density $d_o = B_o/T_o$.

As the ambient temperature diminishes and the barometric pressure increases the diaphragm moves in accordance with the relation

$$X = K_1 (B - p) \tag{1}$$

where X is the deflection, B is the barometric pressure, p is the pressure inside the sealed capsule, and K_1 is a diaphragm constant for the linear portion of the movement.

If B_oV_o is the product of pressure and volume at zero deflection and pV is the product under any other conditions, we have, according to the simple gas laws

$$pV = B_oV_o \left(\frac{273 + T}{273 + T_o}\right) \tag{2}$$

from which

$$p = \frac{B_oV_o}{V}\left(\frac{273 + T}{273 + T_o}\right). \tag{3}$$

If (3) is put into (1), we have

$$X = K_1 \left[B - \frac{B_oV_o}{V}\left(\frac{273 + T}{273 + T_o}\right)\right]. \tag{4}$$

In order to calculate X it is necessary to know V in terms of X, and from the dimensions of the capsule it can be calculated that

$$V = V_o - K_2X. \tag{5}$$

Substituting (5) into (4), we obtain

$$X = K_1 \left[B - \frac{B_oV_o}{V_o - K_2X}\left(\frac{273 + T}{273 + T_o}\right)\right]. \tag{6}$$

From (6) is obtained the second-degree equation

$$\frac{K_2}{K_1} X^2 - \left(\frac{V_o}{K_1} + K_2B\right) X + \left[V_oB - V_oB_o\left(\frac{273 + T}{273 + T_o}\right)\right] = 0. \tag{7}$$

ILLUSTRATIVE EXAMPLE

It is now possible to calculate X for any value of B and T, and in this manner a relation between deflection X and density ρ can be obtained. Suppose a capsule 2 inches in diameter gives a linear deflection of 0.043 inch for a pressure differential of 100 mm Hg. Then the value of K_1 in Eq. (1) will be 0.00043.

The value of k_2 can be calculated from Eq. (5) by assuming that as the diaphragm is deflected inwards it forms a cone the volume of which is the area of the diaphragm times 1/3 the deflection X. If this 2-inch-diameter capsule is 0.25 inch thick, the internal volume V_o will be 0.7854 cubic inch. We find then that $K_2 = 1.047$, when the movement of both sides of the capsule is considered.

If Eq. (7) is solved for X, we find

$$X = \frac{\left(\frac{V_o}{K_1} + K_2B\right) - \sqrt{\left(\frac{V_o}{K_1} + K_2B\right)^2 - 4\frac{K_2}{K_1}V_o\left[B - B_o\left(\frac{273 + T}{273 + T_o}\right)\right]}}{2\ K_2/K_1} \qquad (8)$$

If the values for K_1 and K_2 are placed in (8), we have as the final numerical equation

$$X = \frac{1826.5 + 1.047B - \sqrt{(1826.5 + 1.047B)^2 - 7649.8B\left(1 - \frac{\rho_o}{\rho_x}\right)}}{4870} \qquad (9)$$

in which ρ_o is the lowest density to be indicated and ρ_x is any other density.

In order to get the value of X from Eq. (9), it is necessary to assume values for B_o and T_o which are the pressure and temperature at ρ_o the lowest density indicated on the scale (zero deflection). After the assumed value for ρ_o is placed in the equation, X can be found for any combination of B and T over the range of air densities it is desired to operate the instrument.

Table 1 shows values for relative density for several pressures and temperatures, and if it is desired to start our scale at the relative density 0.800, it can be seen from the table that such a value for relative density is obtained when the ambient temperature is 40°C and the barometric pressure is 697 mm Hg. That is

$$\frac{697}{760} \times \frac{273}{313} = 0.800.$$

Figure 1 shows how X calculated from Eq. (9) varies with the ratio of $B/760$ to $273/(273 + T)$ for the given diaphragm. Even though the graph covers quite a wide spread in density, points calculated from (9) fall close to the curve except for the conditions of high pressure and temperature as well as for low pressure and temperature. Since both these conditions are rather extreme, it would be necessary to employ special scales for instruments to be used in the tropics, in arctic regions or at elevated positions where the temperature is low.

TABLE 1
Density of Ambient Air at Several Temperatures
and Pressures Relative to 1.00 at 0°C and 760 mm Hg

Pressure B (mm Hg)	Absolute Temperature (°K)						
	253	263	273	283	293	303	313
800	1.1358	1.0926	1.0526	1.0154	0.9807	0.9484	0.9181
790	1.1216	1.0790	1.0395	1.0027	0.9685	0.9365	0.9066
780	1.1074	1.0653	1.0263	0.9900	0.9562	0.9247	0.8951
770	1.0932	1.0516	1.0131	0.9773	0.9439	0.9128	0.8836
760	1.0790	1.0380	1.0000	0.9646	0.9317	0.9009	0.8722
750	1.0648	1.0243	0.9868	0.9519	0.9194	0.8891	0.8607
740	1.0506	1.0107	0.9737	0.9392	0.9072	0.8772	0.8492
730	1.0364	0.9970	0.9605	0.9266	0.8949	0.8654	0.8377
720	1.0222	0.9834	0.9474	0.9139	0.8827	0.8535	0.8263
710	1.0080	0.9697	0.9342	0.9012	0.8704	0.8417	0.8148
700	0.9938	0.9560	0.9210	0.8885	0.8581	0.8298	0.8033
690	0.9796	0.9424	0.9079	0.8758	0.8459	0.8180	0.7918
680	0.9654	0.9287	0.8947	0.8631	0.8336	0.8061	0.7804

ERROR DUE TO MOISTURE

Since the molecular weight of water vapor is only 18 compared to 28.95 for dry air, the presence of moisture in the ambient air will introduce an error in the indicated density when the instrument is calibrated on the basis of dry air. The magnitude of the error introduced is not excessive except at high temperatures and low pressures. It is a simple matter to calibrate an instrument on the basis of 50% relative humidity at the mean barometric pressure for the locality where the instrument is to be used, and in that case the error is entirely negligible. Table 2 shows the error which will be introduced when a dry air calibration is used and a condition of 50% relative humidity exists at 760 mm Hg over a temperature range 0-40°C. Even at the high temperature the error is not excessive, but this error can be practically eliminated by calibrating for 50% RH at the mean barometric pressure.

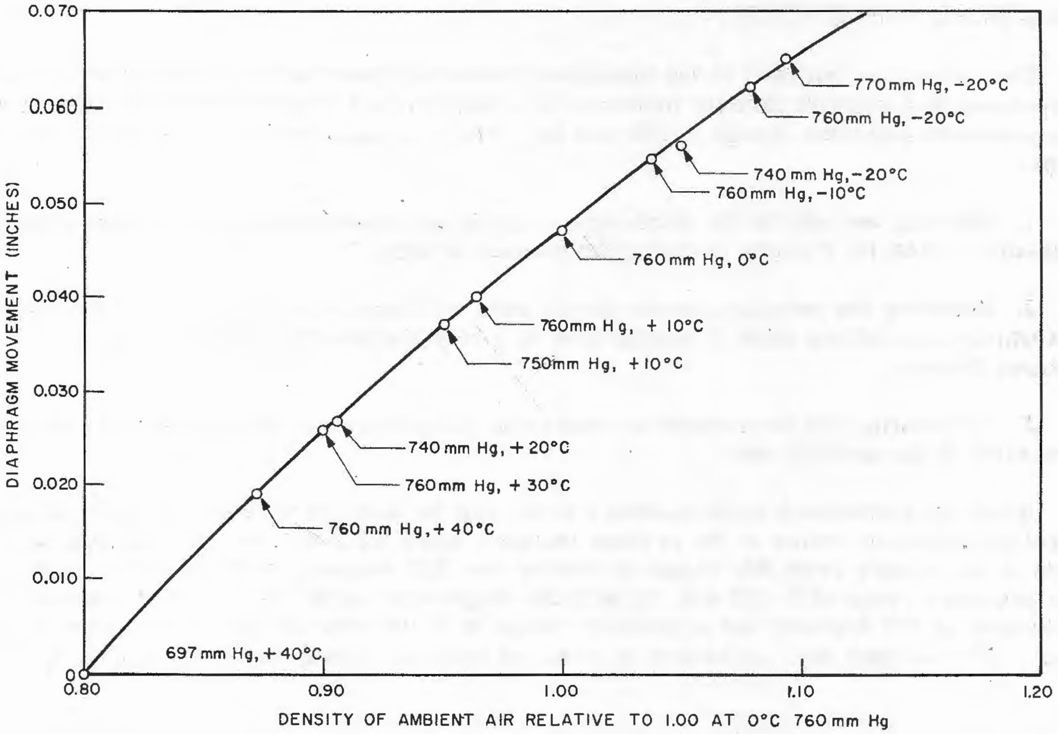

Fig. 1 - Relation between movement of gas-filled diaphragm capsule and density of ambient air

TABLE 2
Showing the Error Introduced by Moisture When Instrument is Calibrated for Dry Air at 760 mm Hg

Temperature (°C)	Saturation Pressure p of H_2O (mm Hg)	$\frac{p}{760}$	p for 50% RH	Density* Relative to Dry Air at 0°C and 760 mm Hg	Density of Dry Air	Error (%)
0	4.58	0.00602	0.00301	0.9988	1.0000	0.12
10	9.21	0.01211	0.00605	0.9624	0.9647	0.23
20	17.55	0.0230	0.01154	0.9276	0.9318	0.44
30	31.86	0.04192	0.02096	0.8938	0.9010	0.80
40	55.35	0.07288	0.03644	0.8602	0.8722	1.39

*Calculated by means of the relation $D = \frac{28.95\ (1-x)\ +\ 18x}{28.95}\left(\frac{273}{273 + T}\right)$

where D is the density of moist air relative to dry air and X is the percent water vapor by volume

PRACTICAL APPLICATION

The principles outlined in the foregoing discussion have been put into practice in the conversion to a relative density indicator of a Wallace and Tiernan Type FA-160122 absolute pressure indicator, range 0-100 mm Hg. The procedure consisted in the following steps:

1. Opening the tube to the diaphragm capsule and allowing dry air at atmospheric pressure to flow in, thereby bringing the pointer to zero;

2. Reducing the pressure inside the capsule until the pointer takes a position near that which calculations show it should have at a relative density corresponding to the ambient density;

3. Calibrating the instrument by observing the pointer position for known conditions of density of the ambient air.

After the instrument is calibrated a scale can be laid out between the desired limits of relative density, which in the present instance were 0.8 and 1.10. The angular movement of the pointer over this range of density was 328 degrees, corresponding to an absolute pressure range of 0-120 mm Hg with the diaphragm used, instead of an angular movement of 283 degrees and a pressure range of 0-100 mm for the unconverted instrument. The six-inch dial calibrated in terms of relative density is shown in Fig. 2.

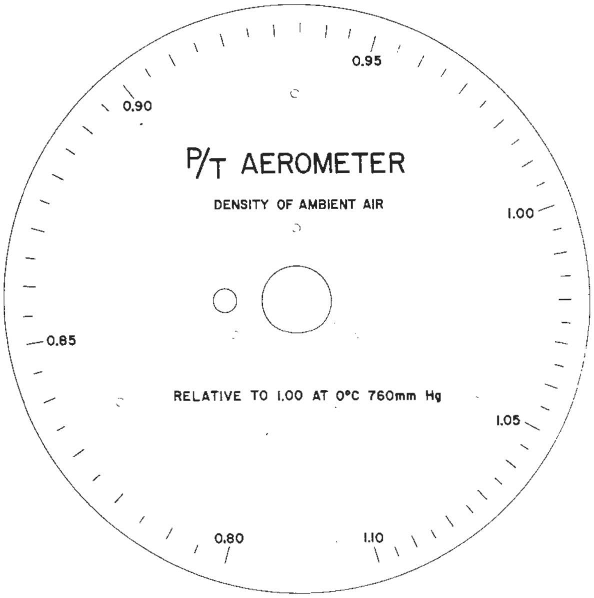

Fig. 2 - Dial of aerometer

The only other change made in the original instrument was to replace the heavy-walled vacuum-tight case with a lighter case having spaced ventilation holes to permit circulation of ambient air around the diaphragm capsule. For field use it would be necessary to surround the mechanism with a small dustproof inner case to protect the jewelled bearings and the gear train.

The performance of this instrument has been very satisfactory as regards accuracy and reproducibility, and the time required to respond to a sudden change in ambient temperature is less than that of a mercury-in-glass thermometer. The design of the mechanism, jewelled bearings, and the lightness of the moving parts result in a precision instrument of remarkable ruggedness.

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