# NAVAL POSTGRADUATE SCHOOL

**MONTEREY, CALIFORNIA**

# THESIS

**GAME-THEORETIC APPROACHES TO NAVY BUDGET ALLOCATION**

by

Merril D. Kline II

September 2023



| | |
|---|---|
| Thesis Advisor: | Johnathan C. Mun |
| Co-Advisor: | Kyle Y. Lin |
| Second Reader: | Thomas J. Housel |



**Approved for public release. Distribution is unlimited.**

*This project was funded in part by the NPS Naval Research Program.*

THIS PAGE INTENTIONALLY LEFT BLANK

| REPORT DOCUMENTATION PAGE | *Form Approved OMB No. 0704-0188* |
|---|---|



| **1. AGENCY USE ONLY** ***(Leave blank)*** | **2. REPORT DATE** September 2023 | **3. REPORT TYPE AND DATES COVERED** Master's thesis |
|---|---|---|

| | |
|---|---|
| **4. TITLE AND SUBTITLE** GAME-THEORETIC APPROACHES TO NAVY BUDGET ALLOCATION | **5. FUNDING NUMBERS** NPS-23-N202-_ |
| **6. AUTHOR(S)** Merril D. Kline II | |
| **7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)** Naval Postgraduate School Monterey, CA 93943-5000 | **8. PERFORMING ORGANIZATION REPORT NUMBER** |
| **9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)** N/A | **10. SPONSORING / MONITORING AGENCY REPORT NUMBER** |

**11. SUPPLEMENTARY NOTES** The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. This project was funded in part by the NPS Naval Research Program.

| **12a. DISTRIBUTION / AVAILABILITY STATEMENT** Approved for public release. Distribution is unlimited. | **12b. DISTRIBUTION CODE** A |
|---|---|

**13. ABSTRACT (maximum 200 words)**

Many agencies within the Navy are competing for a portion of the Navy budget to ensure their agency receives the maximum amount of funding possible. With a budget of $255.8 billion for FY24, it is imperative the Navy does everything in its power to make the most informed investment decisions.

Consider a department that allocates funds to several subdivisions each fiscal year. We use mechanical design theory to create an investment decision–based game that motivates each subdivision to report their annual budget prediction truthfully. Specifically, the department penalizes subdivisions for inaccurate budget predictions by giving the subdivision a smaller budget next year, and offers a bonus for subdivisions that predict their budgets accurately. We use Monte Carlo simulation to demonstrate our approach with a set of assumptions on how each subdivision behaves. The output of these simulation experiments provides a confidence interval for the budget each subdivision receives. Finally, the department can run stochastic optimization to determine the final budget allocation to each of its subdivisions to maximize the department's overall performance.

| | |
|---|---|
| **14. SUBJECT TERMS** game theory, Monte Carlo simulation, capital budgeting, risk analysis, decision analysis, stochastic optimization | **15. NUMBER OF PAGES** 73 |
| | **16. PRICE CODE** |

| **17. SECURITY CLASSIFICATION OF REPORT** | **18. SECURITY CLASSIFICATION OF THIS PAGE** | **19. SECURITY CLASSIFICATION OF ABSTRACT** | **20. LIMITATION OF ABSTRACT** |
|---|---|---|---|
| Unclassified | Unclassified | Unclassified | UU |

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89) Prescribed by ANSI Std. 239-18

THIS PAGE INTENTIONALLY LEFT BLANK

**Approved for public release. Distribution is unlimited.**

# GAME-THEORETIC APPROACHES TO NAVY BUDGET ALLOCATION

Merril D. Kline II
Lieutenant, United States Navy
BS, State University of New York, Oswego, 2011

Submitted in partial fulfillment of the
requirements for the degree of

**MASTER OF SCIENCE IN OPERATIONS RESEARCH**

from the

**NAVAL POSTGRADUATE SCHOOL**
**September 2023**

Approved by: Johnathan C. Mun
Advisor

Kyle Y. Lin
Co-Advisor

Thomas J. Housel
Second Reader

W. Matthew Carlyle
Chair, Department of Operations Research

THIS PAGE INTENTIONALLY LEFT BLANK

# ABSTRACT

Many agencies within the Navy are competing for a portion of the Navy budget to ensure their agency receives the maximum amount of funding possible. With a budget of $255.8 billion for FY24, it is imperative the Navy does everything in its power to make the most informed investment decisions.

Consider a department that allocates funds to several subdivisions each fiscal year. We use mechanical design theory to create an investment decision–based game that motivates each subdivision to report their annual budget prediction truthfully. Specifically, the department penalizes subdivisions for inaccurate budget predictions by giving the subdivision a smaller budget next year, and offers a bonus for subdivisions that predict their budgets accurately. We use Monte Carlo simulation to demonstrate our approach with a set of assumptions on how each subdivision behaves. The output of these simulation experiments provides a confidence interval for the budget each subdivision receives. Finally, the department can run stochastic optimization to determine the final budget allocation to each of its subdivisions to maximize the department's overall performance.

THIS PAGE INTENTIONALLY LEFT BLANK

## TABLE OF CONTENTS

# LIST OF FIGURES

THIS PAGE INTENTIONALLY LEFT BLANK

# LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

# EXECUTIVE SUMMARY

The United States Navy has a very large budget, $255.8 billion for FY 2024. When working on a scale this large, there are bound to be misallocation of funds, as well as a few program managers taking advantage of the system. This thesis aims to provide a game-theoretic approach that allocates the funds where they are most needed, while penalizing inaccurate budget predictions that include the problem of program managers taking advantage of the budgeting system.

Consider a department within the U.S. Navy that has several sub-divisions that receive funding from the department. Using game theory, a game is proposed, using the Nash equilibria to create an optimal strategy for the players to solve investment decisions. The model proposed in this thesis goes in reverse, since we know the Nash equilibrium that is desired is that all divisions within the department provide an accurate budget prediction. A theory known as mechanism design theory allows us to design a game that has the Nash equilibrium as a solution, and we do not need to consider alternative games since the theory states that they will at best be equivalent to the game designed (Royal Swedish Academy of Sciences, 2007).

The game sequence has three phases in the first year: proposed, planned and actual. For subsequent years the proposed stage is removed, leaving only the planned and actual stages. During the proposed stage, divisions decide their requested budget and the number of projects they can complete within that budget.

For the first-year planned stage, the department explains the new system being used for budget allocation involving penalties for inaccurate budget, tolerances, optional bonus, and budget multiplier equations. Departments then provide adjusted budget numbers based on previously used budget allocation methods. In other words, the new budgeting system has not been in place long enough, so the previous method needs to be used for one more year. Then, divisions adjust their number of projects for the new budget numbers. An efficiency ratio of the number of projects to proposed budget is then calculated using this number of projects and the result becomes the basis for future calculations.

For the actual stage, the actual budget consumed, and the number of projects completed are determined for the divisions. The actual efficiency ratio is then computed. The department must then determine the penalty, tolerance, and optional bonus parameters at this point. A budget multiplier for each division is computed, using the efficiency ratio from the planned stage and actual stage for each division, as well as the penalty parameters, tolerance, and bonus from the department. This budget multiplier is then multiplied by the previous planned budget number for each division to determine the upcoming year's new planned budget numbers.

The next year starts with the planned stage and divisions determining the number of projects they can complete with their new planned budgets. The game sequence then mirrors the first year from this point with the actual stage to follow. This process then repeats for subsequent year.

The game sequence, as designed, may be too difficult to implement in a real-world scenario. Thus, an alternate way to include a form of the game in budget allocation was introduced in the current study. Instead of divisions making decisions in the game sequence, certain assumptions can be made using historical data that can be used to determine the budget and number of projects for the divisions. Additionally, the department decisions for penalty parameters, bonus and tolerance are assigned probability distributions to cover the range of values that they could take on. Monte Carlo simulations are run using these inputs. Each division is simulated separately with the output of each simulation being a 95% confidence interval for the budget multiplier for each division.

Using the confidence interval as the range of budget multipliers and the division's requested budgets, a lower and upper limit can be placed on the final budget allocation for each division. Traditionally an optimized budget allocation can be produced using return and risk values for each division. However, in a military context, it makes more sense to use military value in place of return and military downsides in place of risk. What determines these values depends on the decision-maker and is not explored in this thesis. However, a probability distribution is assigned to each of these parameter values. The ratio between military value to military downsides that is comparable to the investment finance, Sharpe's ratio, is determined. A stochastic optimization is then used to determine the best budget allocation for each of the parameters.

The findings of the thesis are that game theory can be a valuable tool in improving the budget allocation process. The model presented incentivizes the divisions to be accurate with budget predictions. Additionally, the introduction of the new efficiency ratio is an important new ratio to determining the budget multiplier. Thus, divisions can return a portion of their budget to maintain this ratio and receive a better budget multiplier for future years. This allows money to be saved by the department and the Navy in general.

Using the simulation confidence intervals as the bounds in the stochastic optimization allows departments a fairer allocation of the budget as opposed to traditional deterministic approach. The examples shown in this thesis show that divisions can have their requested budgets reduced unnecessarily more harshly in the deterministic approach as opposed to the game theory approach.

All in all, game theory can be useful to the U.S. Navy budget allocation process and the game theory model presented in this model should be considered for future budget allocation processes.

**References**

Royal Swedish Academy of Sciences. (2007). *Mechanism design theory*. https://www.nobelprize.org/uploads/2018/06/advanced-economicsciences2007.pdf

THIS PAGE INTENTIONALLY LEFT BLANK

## ACKNOWLEDGMENTS

First, I would like to thank my wife Lauren for being so loving and supportive of pursuing my degree and allowing me the time needed complete my work at NPS. To my daughter Cora, thank you for being the most awesome child a parent could ask for. It is an honor to be your father.

Second, I would like to thank my advisors. Dr. Mun, thank you for walking me through the risk simulator software and providing great examples while allowing me to take everything and make it my own. Dr. Lin, thank you for your knowledge, patience, and attention to detail. I don’t believe there was any feedback that you provided that I disagreed with or did not incorporate into my thesis. I don’t think I would have made it through the process without you.

Lastly, I would like to thank my awesome cohort. You all have made my time here in Monterey enjoyable and unforgettable. It’s a shame it has to come to an end. I hope to serve with you all as I head back into the fleet.

THIS PAGE INTENTIONALLY LEFT BLANK

# I. INTRODUCTION

The United States Navy commands a substantial budget of $255.8 billion allocated for FY 2024. Operating at such a large scale, the potential for budgetary misallocation and the exploitation of the system by certain program managers cannot be overlooked. This thesis endeavors to introduce a game-theoretic framework to ensure the judicious allocation of funds to areas of genuine need, concurrently deterring any imprudent practices, including those by program managers seeking to exploit the budgetary process.

Game theory serves as a fundamental analytical tool for investigating decision-making processes amid competition among individuals or entities. It establishes a structured framework to scrutinize the choices made by multiple participants, termed players, in competitive scenarios, aiming to develop strategies that yield the most favorable outcomes. In the context of capital budgeting, wherein critical determinations are made regarding project selection and investment allocation, diverse players with varying interests compete to obtain a share of the allocated funds. Unfortunately, conventional capital budgeting methods, such as discounted and non-discounted cash flow techniques (e.g., net present value, internal rate of return, payback period, profitability index), often fail to account for the preferences and interactions of these competing interests. Consequently, integrating game theory becomes crucial to formulate more comprehensive strategies that holistically consider the intricate dynamics of competitive decision-making in capital budgeting scenarios, thereby optimizing project selection and investment allocation.

## A. RESEARCH QUESTIONS AND METHODOLOGY

(1) What is the best game theoretic approach to budget allocation in the U.S. Navy?

Using the mechanism design theory, we are afforded the ability to design a game with the desired Nash equilibrium of a perfectly accurate budget prediction from each division. Chapter III explores a game sequence model that is designed with this Nash equilibrium.

(2) What is the best way to implement this model into real-world budget allocation?

One way to implement this model is a table-top game and or wargame. However, due to resource and time constraints, this thesis does not explore this method.

The second way to implement this model is to make assumptions about the decisions the divisions and department make in the game sequence. Using these assumptions, Monte Carlo simulations are run with the game sequence. Using the output of these simulations, a traditional budget portfolio stochastic optimization method is used within the parameters from the simulations. Chapter IV explores this method in detail.

## B. LIMITATIONS

The primary constraint of this study stemmed from the lack of historical data suitable for employment in the Monte Carlo simulations and stochastic optimization conducted in Chapter IV. Instead, we resorted to using stylized data, which, regrettably, significantly reduced our capacity to perform in-depth analyses of the data outcomes. While online sources provide easy access to Navy budget figures, the intricate details of the budget allocation process remain elusive, with only the final numbers at the conclusion of the process being accessible. Additionally, crucial information essential for ascertaining the quantity of projects or establishing criteria for military value and downsides evaluation remains unavailable.

Additionally, not having the time and resources to complete a table-top exercise with experts in the field left a potential application method unexplored.

## C. THESIS ORGANIZATION

The rest of this thesis is organized as follows: Chapter II is a literature review, Chapter III is the budget game sequence, Chapter IV is Monte Carlo simulation and stochastic optimization of budget game model, and Chapter V are the key findings and conclusion.

# II. LITERATURE REVIEW

## A. GAME THEORY LITERATURE

The inception of game theory as a branch of applied mathematics is attributed to John von Neumann and Oskar Morgenstern's seminal work, *Theory of Games and Economic Behavior* (Princeton University Press, 1944). Initially confined to economic applications, the concept gradually gained prominence and found relevance in diverse fields, including political science, biology, economics, and finance. While capital budgeting primarily falls under the domain of finance, its potential integration with game theory was not immediately apparent. However, there were early indications of its future significance in business applications, as evidenced by Bennion's remarks in 1956, acknowledging the evolving prospects of game theory (p. 123).

Subsequently, the paper under consideration explores the application of game theory in capital budgeting, exemplified through a two-player zero-sum game involving a businessman and the business cycle. This foundational setup provides insights into strategies for capital budgeting within the Department of Defense (DOD). Nevertheless, as Bennion points out, this approach may prove inadequate in scenarios with multiple players or when the gains of the winner and the losses of the loser do not offset each other, presenting challenges that warrant further examination in the realm of game theory application (Bennion, 1956, p. 123).

As observed by Ng in 1968, the increasing potential of large-scale computing machinery promised significant applications for game theory, including its relevance as a tool for capital budgeting. Over the 12 years since the Bennion article, the notion of utilizing game theory in capital budgeting had been steadily gaining importance. However, the development of more complex game models encountered a roadblock due to the limitations of early-stage computers, which were still in their nascent phase (Ng, 1968, p. 11).

Ng's assessment highlighted the prospect of overcoming these challenges as computer capabilities continued to advance. The paper explored a case study known as the

Shell model or code name STRATCOM, investigating a multi-person non-zero-sum game within a continuous variable space. Unlike the previous Bennion game, this model addressed the limitations of considering more than two players and incorporating non-zero-sum game dynamics (Ng, 1968, p. 103).

While not delving into the specific details of the model, the STRATCOM study demonstrated an advancement in tackling complex competitive decision problems through game theory. Ng acknowledged that the Shell model represented one of the initial concrete efforts in yielding meaningful results for management contemplating game theory's application in intricate competitive decision scenarios (Ng, 1968, p. 117).

According to Thakor's analysis in 1991, game theory saw limited adoption in the economic realm until the 1970s. During this period, economists increasingly recognized the significance of individuals' information limitations in comprehending economic behavior, leading to behavioral adjustments. This evolution further established the relevance of game theory in economics and, by extension, in the domain of capital budgeting (Thakor, 1991, p. 71).

The paper delves into various equilibrium concepts, including Nash equilibrium, subgame perfection, sequential equilibrium, the Cho-Kreps intuitive criterion, and the Banks-Sobel divinity and universal divinity refinements. These equilibrium concepts offer a multitude of possibilities for further exploration and application in the context of capital budgeting within the Department of Defense (DOD). By utilizing these concepts, the DOD can gain valuable insights into complex competitive decision-making scenarios, thereby enhancing its capital budgeting strategies (Thakor, 1991, pp. 91–92).

Looking forward, Boquist et al. (1998) proposes a significant advancement in capital budgeting practices by advocating the use of game theory and probabilistic scenarios to estimate the likelihood of competition. They emphasize the necessity of determining a probability distribution for competitive entry and its impact on project cash flows. Integrating this information into the capital budgeting process allows for a more comprehensive assessment of the uncertain and stochastic elements arising from competition (Boquist et al., 1998, p. 62).

This extension to the multi-person game framework introduces the incorporation of probability distributions, acknowledging the inherent uncertainty in competitive decision-making. By embracing this insight, companies, including the Department of Defense (DOD), can effectively enhance their capital budgeting strategies, accounting for and managing uncertainty arising from competitive scenarios. The utilization of game theory in conjunction with probabilistic analysis empowers decision-makers to make more informed choices, thereby increasing the potential for successful project outcomes in dynamic and competitive environments (Boquist et al., 1998, p. 62).

According to Huisman et al. (2003), the study of the effects of strategic interactions on the option value of waiting associated with investments under uncertainty remains relatively limited in the present. The author attributes this constraint to the less developed application of game theory under continuous-time models. However, there is optimism expressed that this situation would change, as the importance of the topic becomes increasingly recognized, leading to more publications in the future (Huisman et al., 2003, p. 1).

In their paper, Huisman et al. delve into an analysis of identical firms operating in a duopoly setting, which has potential application to capital budgeting within the Department of Defense (DOD). Exploring such a scenario can shed light on strategic decision-making within competitive environments, allowing the DOD to gain valuable insights into the implications of strategic interactions on investment decisions made under uncertainty. As the application of game theory and continuous-time models develops further, more sophisticated methodologies can aid in optimizing capital budgeting strategies, ensuring the DOD's decisions align with its long-term objectives (Huisman et. Al, 2003, pp. 2–18)

The integration of game theory into capital budgeting is often acknowledged as one of today's sophisticated tools in the field (Kengatharan, 2016, p. 7). However, its specific application to capital budgeting remains relatively new and not yet fully explored (Verbeeten, 2006, p. 117; Siziba & Hall, 2021, p. 2). Smit further supports this notion, noting that only recently have strategic considerations about imperfect competition been formally addressed in the economics literature (Smit, 2003, p. 29).

Throughout the history of the subject, a recurring theme emerges while game theory is recognized as a sophisticated tool for capital budgeting, a true breakthrough remains on the horizon without complete realization. As highlighted by Li (2014), some critics argue that these techniques can be challenging to implement and may offer only marginal contributions to enhancing decision-making processes. This aspect could potentially pose difficulties moving forward in the widespread adoption of game theory in capital budgeting (Li, 2014, p. 14-15).

Despite these challenges, researchers and practitioners continue to explore the potential of game theory in capital budgeting, with optimism that further advancements and refinements will lead to more impactful and practical applications in strategic decision-making.

The presence of uncertainty combined with competition among multiple players creates an ideal environment for the application of game theory (Nichols, 1994, p. 88; Boquist et al., 1998, p. 62; Thijssen et al., 2004, p. 1; Araújo, 2015, p. 1). While alternative theories, such as real option theory, might suggest waiting for more information before making decisions, game theory drastically reduces the viability of this option due to the concern of competition pre-emption (Smit, 2003, p. 29; Zhu & Weyant, 2003, p. 645; Verbeeten, 2006, p. 106).

A viable solution for investment strategies in the context of competition can be achieved within a Nash-Cournot framework. Grenadier further notes that the exercise strategies in the Nash equilibrium closely resemble those obtained in an "artificial" perfectly competitive equilibrium, with only minor adjustments to the demand function. Therefore, game theory demonstrates its necessity in this scenario, as it allows for the identification of equilibria within the Nash-Cournot framework. Moreover, these equilibria are applicable in real-world settings when making slight modifications to the demand function compared to the perfectly competitive model. By incorporating game theory, decision-makers can effectively navigate competitive landscapes and develop robust investment strategies aligned with real-world conditions (Grenadier, 2002, p. 691).

Game theory not only emphasizes the significance of basing budgets on accurate and realistic reporting but also sheds light on the potential risks of inflated reporting. Jiang and Gong (2019) illustrate that the optimal payoff arises from a fiscal management organization that approves a real budget, underscoring the importance of transparent financial reporting (Jiang and Gong, 2019, pp. 196–198).

Furthermore, Anton (2019) highlights that game theory validates the possibility of managerial opportunistic behavior, where managers may misrepresent financial statements for personal gain under certain circumstances. Game theory serves as a valuable tool in comprehending the challenges of implementing new accounting policies and procedures, particularly when management faces low incentives or payoffs. Anton also asserts that game theory can aid in designing appropriate accounting standards that foster cooperation among all stakeholders (Anton, 2019, pp. iii–v).

By applying game theory, it becomes evident that while individual managers may be tempted to manipulate financial statements, a more cooperative approach can benefit everyone involved without resorting to misrepresentation. In the context of the Department of Defense (DOD), embracing this cooperative approach can help curtail artificial overspending, ultimately saving resources that can be utilized more effectively elsewhere. Thus, game theory offers valuable insights for sound financial decision-making and resource allocation in various settings, including the DOD (Anton, 2019, pp. iii–v).

In the realm of economics, Gibbons (1992) offers a comprehensive primer on game theory, targeting those who will engage with game-theoretic models in applied economic fields. The book covers various games, encompassing static games of complete information, dynamic games of complete information, static games of incomplete information, and dynamic games of incomplete information, providing a fundamental baseline for referencing other games explored in different sources (Gibbons, 1992).

Denault (2001) examines risk capital allocation, acknowledging game theory as an excellent framework for addressing the allocation problem and facilitating insightful discussions. The paper explores multiple coherent allocation principle approaches, such as the Shapley and Aumann-Shapley values, which hold relevance for capital budgeting in the Department of Defense (DOD) (Denault, 2001, p. 1).

Murto and Keppo (2002) develop a model that merges game theory and irreversible investment under uncertainty, positing that values follow geometric Brownian motion. The study characterizes the resulting Nash equilibrium under different assumptions about firms' information on each other's valuations for the project. This insight has potential applications in capital budgeting and its implications for the DOD (Murto & Keppo, 2002, p. 127)

Thijssen et al. (2004) primarily focus on capital budgeting, highlighting the potential benefits of postponing investments with initially low positive returns, as they may yield higher returns later. Additionally, the study outlines a method to measure error, assessing the accuracy of capital budgeting rules in a stochastic environment. This aspect could prove valuable when investigating capital budgeting strategies within the DOD. The combined insights from these studies contribute to advancing the understanding and application of game theory in various economic and financial decision-making contexts, including capital budgeting within the DOD (Thijssen et al., 2004, p. 1).

Zhang et al. (2005) focus on a game theoretic model for infrastructure. However, it does introduce an agent-based simulation solution structure to solve flow equilibrium and, more importantly, optimal budget allocation problem for the infrastructure model. This model could prove to be useful when describing capital budgeting and the DOD (Zhang et al., 2005, p. 147).

Various studies have explored the application of game theory in capital budgeting and its relevance to the Department of Defense (DOD). Verbeeten (2006) investigates uncertainties and sophisticated budgeting practices in 189 Dutch organizations, revealing game theory as one of several techniques used, alongside Monte Carlo simulation, certainty equivalents, and real option reasoning—approaches closely related to game theory and applicable to DOD capital budgeting (Verbeeten, 2006, p. 106).

Ferreira et al. (2009) focus on competition in capital-intensive industries, emphasizing the value of option games in high-stakes decisions involving substantial capital investments, a situation relevant to DOD's budgeting process, where internal funding and external contractor competition create clear competition dynamics (p. 101).

Shapiro (2011) provides a first-hand account of game theory's application to budget negotiations in the White House, demonstrating how it is currently used in national budgeting, a significant portion of which affects DOD funding.

Allen and Morris (2014) describes game theory models in finance, emphasizing their essential role in understanding investor, financial intermediary, and corporate manager behaviors and highlighting their capability to incorporate asymmetric information and strategic interaction, with potential implications for capital budgeting in the DOD (p. 17).

Araújo (2015) explores capital budgeting in telecommunications economies, employing strategic game theory and extending into Monte Carlo simulation with complex statistical distributions to evaluate project risk, along with advanced capital budgeting algorithms and real options. These techniques can be applied beyond telecommunications to describe capital budgeting in the DOD (p. 1).

Jiang and Gong (2019) analyze game theory issues in budgeting, depicting the process as a static and asymmetric game with incomplete information. They acknowledge the challenge of evaluating budget authenticity in budget performance management and conclude that approving a real budget leads to the best payoff, advocating against inflating reports for greater funds, a lesson applicable to DOD capital budgeting (p. 193).

Sikalo et al. (2022) describe a game theory-based model for efficient asset allocation, challenging the widely-known Markowitz mean-variance model and proposing a minimax model based on maximum loss as a risk measure. This insight could prove valuable in the DOD's capital budgeting process, especially when returns deviate from a normal distribution. The combined findings from these studies demonstrate the significance of game theory in strategic decision-making and resource allocation in capital budgeting, presenting opportunities for improved efficiency and optimized resource utilization within the DOD (Sikalo et al., 2022, p. 1).

Tian (2013) provides a rigorous foundation for basic topics in game theory such as the strategic and normal forms of a game, prisoner's dilemma, iterated elimination of dominated strategies, Nash equilibria, pure and mixed strategies, and duopolies.

## B. BUDGET ALLOCATION LITERATURE

### 1. Introduction to Asymmetric Information and Mechanism Design

In many negotiations, interactions between parties, or players in game theoretic terms, involve some players possessing information that others lack. Exploiting this advantage is incentivized by game theory, leading to actions like reporting a larger budget or more employees than necessary for a project. Such decisions are often made by individuals less familiar with the specific project, resulting in information asymmetry.

Asymmetric information has garnered significant attention in economics and game theory, being linked to multiple Nobel Prizes. James A. Mirrlees and William Vickrey won the 1996 Nobel Prize for their contributions to the economic theory of incentives under asymmetric information, while George A. Akerlof, A. Michael Spence, and Joseph E. Stiglitz received the 2001 Nobel Prize for analyzing markets with asymmetric information. This underscores the importance of asymmetric information in these fields (Nobel, 1996; 2001).

The 2007 Nobel Prize in Economics was awarded to Leonid Hurwicz, Eric S. Maskin, and Roger B. Myerson for laying the foundations of Mechanism Design Theory. This theory, seen as the inverse of game theory, focuses on designing allocation mechanisms to minimize economic losses caused by private information. By starting with the desired Nash Equilibrium (NE) and determining the corresponding game, mechanism design theory addresses the asymmetric information problem and offers a system for fair and balanced budget allocation as the desired NE (Royal Swedish Academy of Sciences, 2007).

The Revelation Principle theorem, developed by R. Myerson, centers on searching for mechanisms or games that are direct and truth-telling. The principle suggests discarding mechanisms that do not meet these criteria, as there is no loss in payoff by doing so (Royal Swedish Academy of Sciences, 2007).

Regarding the budget allocation problem, the designed mechanism must adhere to the Revelation Principle. Program managers, therefore, benefit from truthfully reporting their spending and the number of projects executed, contributing to an optimal solution for budget allocation (Royal Swedish Academy of Sciences, 2007).

### 2. Mechanism Design and the Budget Problem

The literature review reveals various incentive mechanisms designed to encourage program managers to report accurately, with most mechanisms employing monetary incentives rather than other metrics. These incentives vary significantly, encompassing fixed and variable mechanisms, as well as linear and complex approaches. Some mechanisms not only promote accurate reporting but also allow program managers to excel and exceed their initial reports.

One illustrative example of a bonus ($\Pi_0$) for production ($\phi$) that encourages accurate reporting is represented by (2.1):

$$\Pi_0 = \kappa_1\phi - \kappa_2\left|\phi_f - \phi\right| \tag{2.1}$$

In (2.1), $\kappa_1$ and $\kappa_2$ are constants, $\phi$ is the completed production, and $\phi_f$ is the forecasted production (Young 1985, Ashton and Ashton 1995). The second term introduces a penalty for any deviation from the forecasted production in the actual production. Thus, no penalty is incurred only when there is no deviation from the forecast.

Another example is presented in Pindyck and Rubinfeld's (1995) microeconomics textbook. In this scenario, program managers are tasked with forecasting the production of a certain unit and possess more information than the principal, who is the key decision-maker, regarding the production capacity with normal effort.

In Figure 1, an illustration of this mechanism depicts annual output plotted against bonus payout. Three functions are displayed for three initial forecast values: 10,000 units, 20,000 units, and 30,000 units. Each function comprises two different linear functions intersecting at the initial forecast value. One linear function is used for values below the forecast, and the other for values above it. The figure demonstrates that for the true output of 20,000 units, the highest bonus payout is achieved among the three cases. As a result, program managers are incentivized to be accurate in their reporting, as it leads to the highest bonus rewards.

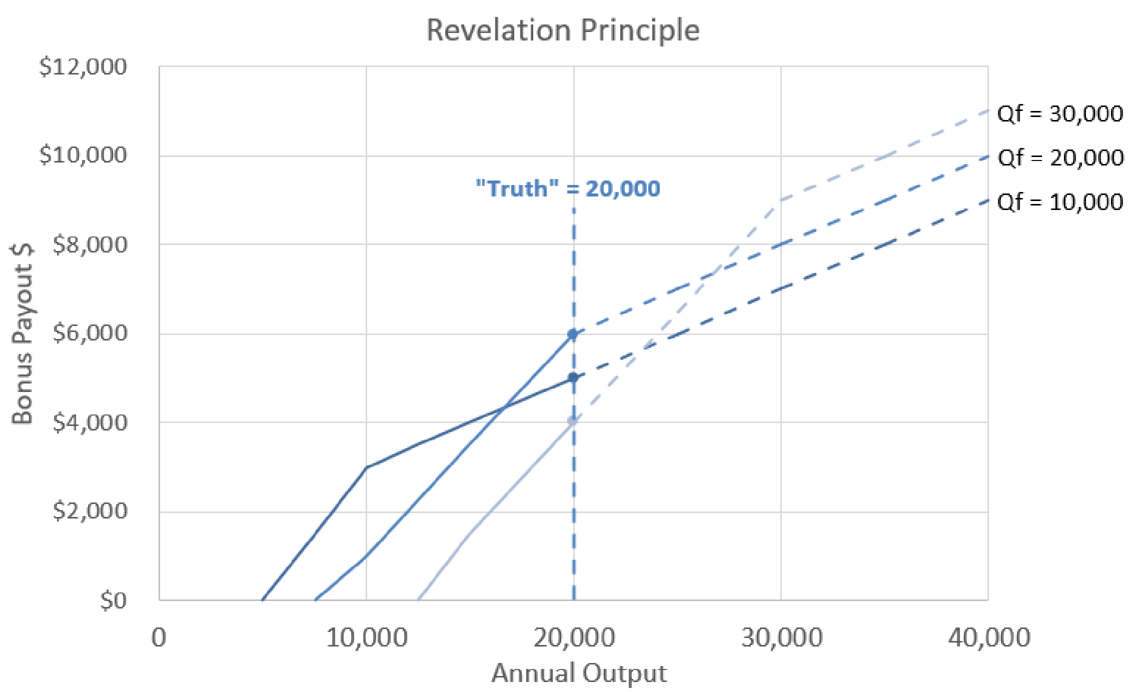


Figure 1. Linear Model Incentive: Highest Bonus Results from Telling the Truth (20,000 Production Units after). Adapted from Pindyck and Rubinfeld (1995).

However, the DOD does not offer bonuses to incentivize its government officials or staff. Therefore, the proposed approach instead relies on using the budget for the next period as an incentive. Despite being nonlinear, the proposed model is expected to have an inflection point with the new information disclosed.

# III. GAME THEORY MODEL IN BUDGET ALLOCATION

## A. THE PROPOSED MODEL

### 1. Problem

The U.S. Navy budget for FY2022 is approximately $212B, according to The President's Budget (2022). The budget is typically divided into the following:

Infrastructure: $3B is allocated to base realigning and closure, family housing, and military construction.

Research & Development: $22.6B is allocated to basic research, advanced technology development, applied research, software pilots, management support, operations systems development, system development and demonstration, and advanced component development.

Military Personnel: $56.5B is allocated to a permanent change of station, special and hazard pay, subsistence, health, reserve personnel, retired pay and thrift savings plan, and housing, as well as pay and allowances.

Operation and Maintenance: $71.2B is allocated to environmental protection, reserves, mobilization, training and education, service-wide support, combat weapons support, base operations, ship operations and prepositioning, aviation operations, and expeditionary.

Procurement: $52.8B is allocated to ammunition, Marine Corps, weapons, ships, aircraft, and other ancillary procurements.

There are various subcategories within the larger category of ships, such as operations, maintenance, and upgrades. For example, within the upgrade's subcategory, there may be programs like the Technology Insertion program. This program allocates a certain budget to upgrade the hardware and software components of different classes of Navy ships such as upgrades to MH60R Sikorsky Seahawk, NIFC-CA Naval Integrated Fire Control – Counter Air, Anti Surface Weapons, SPQ-9B/AN X-Band Pulse Doppler Radar for Littorals, CIWS-CEC Close-In Weapon System – Cooperative Engagement, RDDL Radar Designated Capability Decoy Launch, SM-2 BLK II & SM-6 BLK I Interceptors and Anti Surface Missiles, etc. One or more program managers are assigned to each of these programs and the acquisition, development, deployment, testing, and follow-up of their programs is their responsibility.

The program managers request a budget for their projects, but they possess additional information regarding their division's capacity to execute those projects within the given budget. This creates an information imbalance between the program managers and the key decision-makers at the Pentagon who allocate the budget, such as the Office of the Chief of Naval Operations, which will be referred to as "headquarters" for simplicity.

### 2. Model Purpose

The purpose of the model is to encourage program managers to provide accurate information about their division's ability to complete projects within the allocated budget. This is crucial because if each program manager provides honest and transparent forecasts about production outcomes, budget requirements, schedule and cost risks, and other relevant program metrics, the budget allocation process can be fair and efficient. The objective is to eliminate any inefficient distribution of resources where there is manipulation, leading to certain programs receiving more than necessary while delivering inadequate results. This undermines the potential benefits that could have been achieved by reallocating the surplus budget to other areas that would yield optimal outcomes for the U.S. Navy, DOD, and the nation's overall security.

Given that the analysis pertains to government divisions, and since the government does not offer performance bonuses, the performance incentive mechanism will rely on budget allocation for the upcoming year. Program managers will be informed that their department's budget allocation for the next year will be reduced if they engage in dishonest behavior. Conversely, if they comply with regulations, they will receive the requested budget allocation for the following year. This reward structure will greatly diminish any dishonesty in these strategic games.

The model does not consider the portion of the budget designated for mandatory expenses such as salaries and benefits, health, subsistence, hazard pay, and pre-existing contracts. Instead, the focus is on discretionary programs such as new initiatives, employee training, optional enhancements, and specialized procurement.

### 3. Measures of Performance

Asking program managers to request the budget and ensure their compliance with it may not be suitable since they could potentially spend funds on frivolous items simply to avoid spending less than what was requested. Therefore, it appears more rational to commence the budget negotiations with the divisions in the first year by inquiring about their capabilities with the given budget.

Therefore, to execute the discretionary budget, it is necessary to have some form of performance measurement. Typically, organizations have a means of assessing managerial performance. It is assumed that in this case, the measure of performance is based on "units of projects done" or "units of work done," although any quantitative measure of performance that the organization is comfortable using can be utilized. For example, metrics can include operational and logistics such as Inherent Availability (IA), Effective Availability (EA), Mission Reliability (MR), Operational Dependability (OD), Mean Down Time (MDT), Mean Maintenance Time (MMT), Logistics Delay Time (LDT), Achieved Availability (AA), Operational Availability (OA), Mission Availability (MA), and others. Financial and economic metrics can, of course, be similarly used, such as net present value (NPV), internal rate of return (IRR), modified internal rate of return (MIRR), return on investment (ROI), profitability index (PI), payback period (PP), and discounted payback period (DPP) (Mun, 2020, pp. 84–85).

In this model, it is assumed that after negotiating the budget for each division in the first year, each program manager will then provide a report indicating the number of project units that can be completed within this budget. This report will result in the program manager's unit budget forecast.

## B. MODEL VARIABLES AND PARAMETERS

The model described in this chapter follows the mechanism design theory described in Chapter II. The desired Nash equilibrium is that all divisions provide an accurate and truthful prediction. Mechanism design theory then affords the ability to go in reverse to design a game with this as the Nash equilibrium.

In this model, it is assumed that the department is at the top level and is responsible for allocating the budget to all of the divisions where $I$ represents the set of all divisions in the department where $i \in I$ denotes a particular division. The time period is denoted by $j$ and is an integer starting with 1. Typically, budgets are done yearly, but the model can be generalized to cover any period of time, as long as this period remains fixed throughout the rest of the game.

There are three stages of the game sequence, proposal (*Prop*), planning (*Plan*), and actual (*Act*). Two variables are determined during each of these stages for each division during each time period. The only exception is that the proposal stage is only completed during the first time period. The first variable is the number of completed projects ($n_{i,j}$), for the time period and the second is the budget ($B_{i,j}$ ) for the time period. More details on these two variables and the stages of the game are discussed in the *Game Sequence* section.

Using these two variables, an efficiency ratio ($r_{i,j}$), is determined during each stage, for each division during each time period. These efficiency ratios serve as the basis for applying penalties to the divisions and are discussed in detail in the *Basic Model Equations* section.

Lastly, there are some additional department parameters. The first is the entire department budget ($\beta_j$). Next are two penalty values, the first ($x_j$) is the penalty value for underpredicting performance during that time period and the second ($y_j$) is the penalty value for over-predicting performance during the time period. Using these penalty values and the efficiency ratios for each division, budget multipliers ($b_{i,j}$) are determined for each division for the time period. The last parameters are tolerance ($t_j$) and bonus ($u_j$) which are optional and can be used to adjust the budget multipliers for divisions that are within the tolerance.

Table 1 summarizes the model variables and parameters outlined in this section.

Table 1. Game Variables and Parameters

| | |
|---|---|
| **Sets** | |
| $I$ | Set of all divisions within the department |
| **Indices** | |
| $i \in I$ | Corresponds to each individual division |
| $j$ | Corresponds to a time period in the game |
| **Game Sequence Stages** | |
| $Prop$ | Corresponds to values proposed by divisions in Round 1 |
| $Plan$ | Corresponds to planned values at beginning of time period |
| $Act$ | Corresponds to actual value at end of time period |
| **Division Variables** | |
| $n_{i,j}$ | Number of completed projects for division $i$ for time period $j$ |
| $B_{i,j}$ | Budget for division $i$ for time period $j$ |
| $r_{i,j}$ | Efficiency ratio for division $i$ for time period $j$ |
| **Department Variables** | |
| $\beta_j$ | Total allocated budget for time period $j$ |
| $b_{i,j}$ | Budget multiplier for division $i$ for time period $j$ |
| $x_j$ | Penalty value for under predicting performance for time period $j$ |
| $y_j$ | Penalty value for over predicting performance for time period $j$ |
| $t_j$ | Tolerance allowed for accurate reporting for time period $j$ (Optional) |
| $u_j$ | Bonus for accurate reporting for time period $j$ (Optional) |

## C. BASIC MODEL EQUATIONS

During the planning stage of the game, $B_{i,j\ Plan}$, needs to be chosen such that (4.1) is satisfied. More information on this will be discussed in the *Game Sequence* section.

$$\beta_j \geq \sum_{i \in I} B_{i,j\ Plan} \tag{4.1}$$

Additionally, (4.2) is used to compute the value of the efficiency ratios during each of the game sequence stages.

$$r_{i,j} = \frac{n_{i,j}}{B_{i,j}} \tag{4.2}$$

The next two equations (4.3) and (4.4) are used to determine the budget multipliers for the divisions during the given time period.

$$b_{i,j} = \frac{r_{i,j\ Plan}}{r_{i,j\ Act}} + x_j \frac{\left(r_{i,j\ Act} - r_{i,j\ Plan}\right)}{r_{i,j\ Act}}, \qquad if\ r_{i,j\ Act} > r_{i,j\ Plan} \tag{4.3}$$

$$b_{i,j} = \frac{r_{i,j\ Plan}}{r_{i,j\ Act}} - y_j \frac{\left(r_{i,j\ Plan} - r_{i,j\ Act}\right)}{r_{i,j\ Act}}, \qquad if r_{i,j\ Act} \leq r_{i,j\ Plan} \tag{4.4}$$

$$B_{i,j+1} = b_{i,j} * B_{i,j} \tag{4.5}$$

It assumed that $x_j < 1$, and that $y_j > 1$. Note the lower the value of $x_j$ the greater the penalty for underpredicting performance (lower value of $r_{i,j\ Plan}$). Likewise, the greater the value of $y_j$, the greater the penalty for over-predicting performance (higher value of $r_{i,j\ Plan}$). It is assumed that the budgets for each division will not change outside of the penalties applied between time periods. Thus, the budget for the following time period is determined based on (4.5)

If symmetrical penalties are desired, $x_j$ and $y_j$ should be chosen so that the average is 1 as shown in (4.6).

$$\frac{(x_j + y_j)}{2} = 1 \tag{4.6}$$

Lastly, if tolerances($t_j$) or bonuses ($u_j$) are employed, then the budget multipliers that are calculated in (4.3) and (4.4) are adjusted using (4.7).

$$If\ 1 - b_{i,j} \leq t_j \quad instead\ , \quad b_{i,j} = 1 + u_j \tag{4.7}$$

## D. GAME SEQUENCE

### 1. The Current Year

During the game sequence, it is assumed that the time period is a year. It is also assumed that the department has been operating for several years and is going to start using this game to determine the budget allocation for the current year and the divisions have not been informed yet.

#### *a. Proposal Stage*

The beginning of this first year starts the proposal stage. This stage will only occur during this first year and not in any future years. During this stage, each division will determine their proposed number of completed projects for the year ($n_{i,1\ Prop}$) and the proposed budget for the year ($B_{i,1\ Prop}$). These numbers are then submitted to the department. The proposed efficiency ratio ($r_{i,1\ Prop}$) can also be calculated at this stage using equation (4.2) for reference in the planning stage.

Once the department, has received the numbers from the divisions, the department needs to meet its budget ($\beta_0$). The most common case is that the sum of the proposed budgets from the divisions will exceed the department's budget. Thus, the department will need to determine how to make budget cuts to meet equation (4.1). In the case that the sum is less than the departmental budget already, the department can either decide to reallocate the money to some or all of the divisions or use the money for something else (thus removed from the game for the model).

#### *b. Planning Stage*

Once the department has decided on how to allocate the budget at the end of the proposal stage, they provide these numbers back to the divisions. This new budget number is now the planned budget for the year ($B_{i,1\ Plan}$).

Also, after the planned budget numbers are returned to the divisions, the department will make an announcement regarding the use of a new model with penalties for future budgets based on the accuracy of the predictions made during this planning stage.

The department will then meet with each division to explain the new model in detail and answer any questions. The emphasis will be placed on the efficiency ratio ($r_{i,j}$) during the planning stage and actual stage and how they determine the budget multipliers ($b_{i,j}$) based on equations (4.3), (4.4), and (4.6). The department will also explain the penalties ($x_j$) and ($y_j$) as well as the tolerances($t_j$) and bonuses ($u_j$) but not the exact values they plan to use for each.

Once each division understands the new model and how it will work, they will determine an updated number of completed projects for the planning stage ($n_{i,1\ Plan}$). It is highly recommended, that the divisions consider the efficiency ratio for the planning stage ($r_{i,1\ Plan}$) as well as from the proposal stage ($r_{i,1\ Prop}$). Divisions that previously strived to be truthful and accurate in their predictions should have efficiency ratios relatively close to one another. However, if the divisions had previously not been as truthful in the past or specifically in the proposal stage, this will serve as a wake-up call, and they may need to adjust their numbers quite a bit in this planning stage to avoid receiving a larger penalty in the future.

Once each division has determined their final numbers for the planning stage, they are submitted to the department and are locked in for the first year.

#### *c. Actual Stage*

At the end of the year, the actual number of projects completed for the year ($n_{i,1\ Act}$), and the actual budget consumed for the year ($B_{i,1\ Act}$) will be determined and then the actual efficiency ratio for the year ($r_{i,1\ Act}$) will be computed for each division.

The department will also determine the final numbers for each penalty ($x_1$) and ($y_1$) as well as the tolerance($t_1$) and bonus ($u_1$) if they are used. Next, the budget multipliers ($b_{i,1}$) for each division are determined based on equations (4.3), (4.4), and (4.6). Then finally the planned budget for year 2 ($B_{i,2\ Plan}$) is determined for each division using equation (4.7).

### 2. The Second Year

#### *a. Planning Stage*

The departments provide the divisions with the new planned budget values from the end of the actual stage ($B_{i,2\ Plan}$) as well as the final numbers for each penalty ($x_1$) and ($y_1$) as well as the tolerance ($t_1$) and bonus ($u_1$) if they were used.

With this information, each division will determine their planned number of completed projects for year 2 ($n_{i,2\ Plan}$) and submit it to the department.

#### *b. Actual Stage*

The actual stage for year 2 follows the same process as the year 1 actual stage.

### 3. Third Year and Beyond

The process for all subsequent years follows the same process as the second year.

## E. CLOSER LOOK AT PENALTIES, TOLERANCES, AND BONUSES

If the department feels they are experiencing more problems in a certain direction either underperformance or over-performance, departments can employ a greater penalty in that direction to discourage the problematic behavior of the divisions more heavily.

Figure 2 is an example with a symmetrical penalty ($x_j = 0.7$ and $y_j = 1.3$ ) as a baseline, Figure 3 presents an asymmetrical case with a greater penalty for underperformance ($x_j = 0.4$ and $y_j = 1.3$) and finally, Figure 4 presents an asymmetrical case with a greater penalty for overperformance ($x_j = 0.7$ and $y_j = 1.6$).

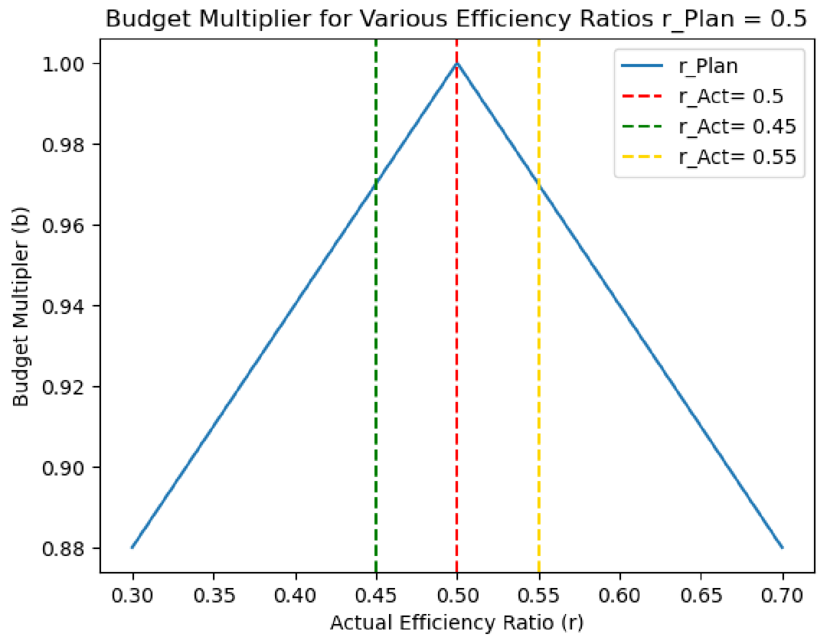


Figure 2. Efficiency Ratios with Symmetrical Penalties, ($x_j = 0.7$ and $y_j = 1.3$ )

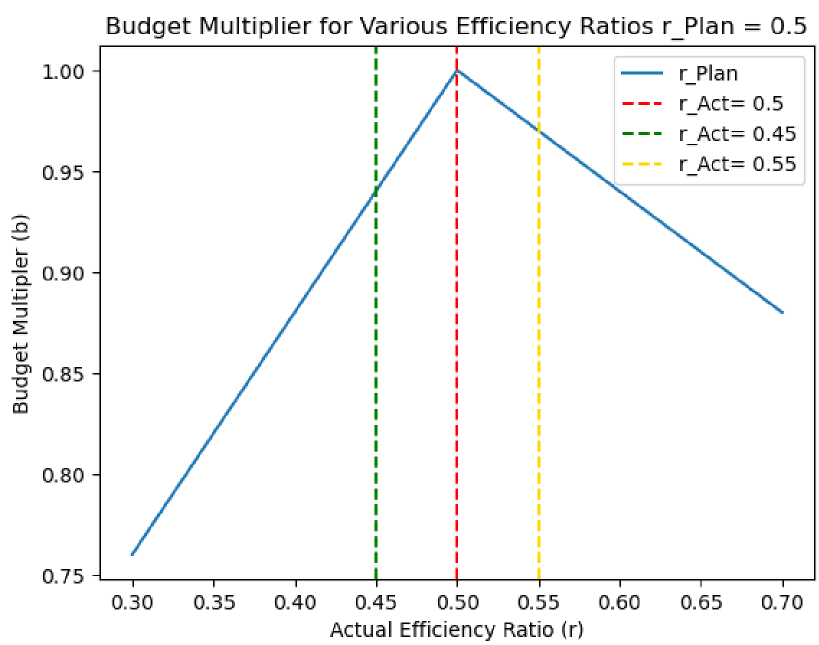


Figure 3. Efficiency Ratios with Greater Penalty for Underperformance, ($x_j = 0.4$ and $y_j = 1.3$)

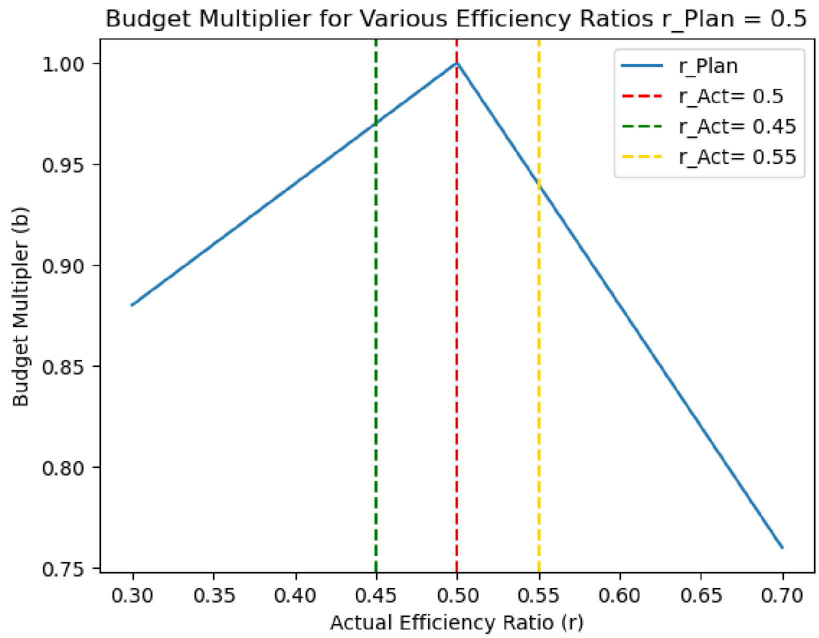


Figure 4. Efficiency Ratios with Greater Penalty for Overperformance, ($x_j =$ 0.7 and $y_j = 1.6$)

For each graph, the x-axis is the actual efficiency ratio ($r_{i,j\ Act}$), and the y-axis is the budget multiplier ($b_{i,j}$). The graph represents a division that had a planned efficiency ratio ($r_{i,j\ Plan}$) of 0.5. The dashed lines represent three different scenarios with three different values for the actual efficiency ratio. The red scenario is one where the division planned accurately, and the actual ratio was the same as planned ratio at 0.5. The green scenario is one where the division underperformed by 10 percent and the actual ratio was 0.45. Lastly, the gold scenario is where the division overperformed by 10 percent with an actual ratio of 0.55.

In Figure 2, the budget multiplier in both the green and gold scenarios is 0.97 representing the baseline for the next two figures. In Figure 3, the multiplier in the green scenario is 0.94 while the multiplier remains 0.97 for the gold scenario. This visually confirms that the lower $x_j$ value has penalized the underperformance in the green scenario more harshly than the baseline.

Likewise in Figure 4, the green scenario's multiplier is 0.97 while gold scenario's multiplier is now 0.94 since in this example overperformance was punished more harshly.

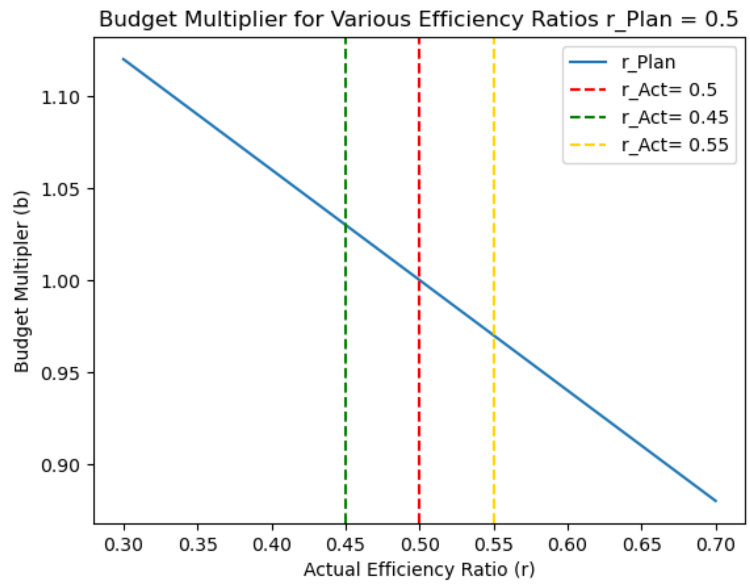


Figure 5. Efficiency Ratios when $x_j > 1$, ($x_j = 1.3$ and $y_j = 1.3$)

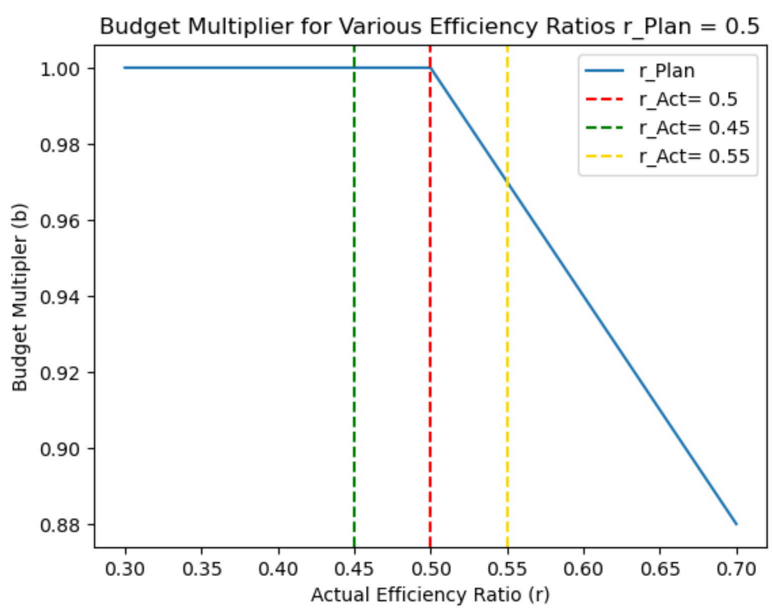


Figure 6. Efficiency Ratios when $x_j = 1$, ($y_j = 1.3$)

Additionally, what happens if the restriction of $x_j < 1$ and is relaxed. Figure 5 demonstrates what happens if $x_j > 1$ specifically, $x_j = y_j = 1.3$ and Figure 6 demonstrates the case where $x_j = 1$ and $y_j = 1.3$.

Notice in Figure 5 that the graph is constantly decreasing and that the underperforming green scenario has multiplier of 1.03 which is higher than the baseline red scenario of 1.0. Thus, instead of penalizing underperforming divisions, $x_j > 1$, instead has the opposite effect and instead becomes a bonus. Thus $x_j > 1$ is not a viable option.

In Figure 6, where $x_j = 1$, the graph begins horizontal and then begins to decrease after $r_{i,j\ Act} = 0.5$. Thus, the multiplier for the underperforming green scenario has the same multiplier as the baseline red scenario of 1.0. Therefore, divisions are indifferent to being accurate or underperforming in this case. So $x_j < 1$ is confirmed to be the only viable option.

Conversely, Figure 7 demonstrates what happens if $y_j < 1$ specifically, $x_j = y_j = 0.7$ and Figure 8 demonstrates the case where $x_j = 0.7$ and $y_j = 1$.

In Figure 7, the opposite of the case in Figure 5 happens. The overperforming gold scenario now receives a bonus instead of a penalty and similarly confirms that $y_j < 1$ is not a viable option

A similar situation applies to Figure 8 with Figure 6; however, the graph instead increases until $r_{i,j\ Act} = 0.5$ and then levels off. A similar conclusion is made that the division is indifferent in the gold scenario to being accurate or overperforming. Thus, $y_j > 1$ is confirmed to be the only viable option.

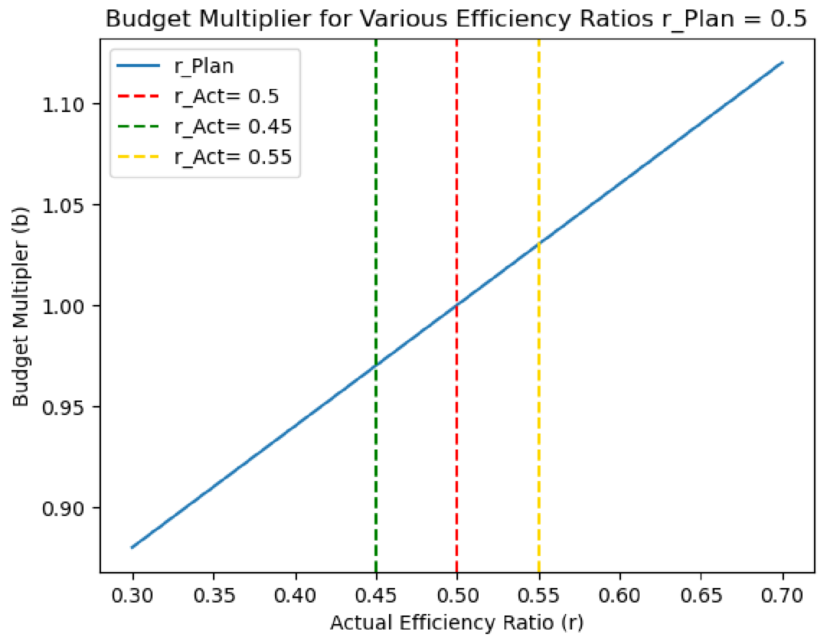


Figure 7. Efficiency Ratios when $y_j < 1$, ($x_j = 1.3$ and $y_j = 1.3$)

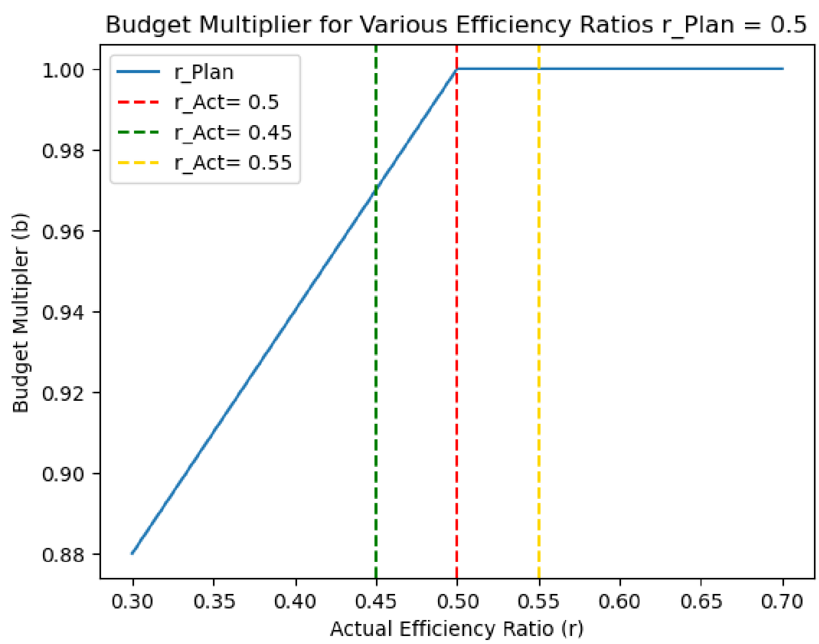


Figure 8. Efficiency Ratios when $y_j = 1$, ($x_j = 1.3$)

To summarize, it has been demonstrated that the only viable options available are $x_j < 1$, and $y_j > 1$ while keeping all other conditions the same.

Additionally, more flexibility can be introduced by introducing the tolerance ($t_j$) and bonus ($u_j$).

Divisions may have more information than the department about the number of projects ($n_{i,j}$) they can complete, however, there is always some level of uncertainty that cannot be eliminated, no matter how much the division wants to be truthful. Thus, it is reasonable to employ a tolerance value to account for this uncertainty. Additionally, a bonus can be introduced to provide an extra incentive for divisions to provide accurate numbers within the tolerance.

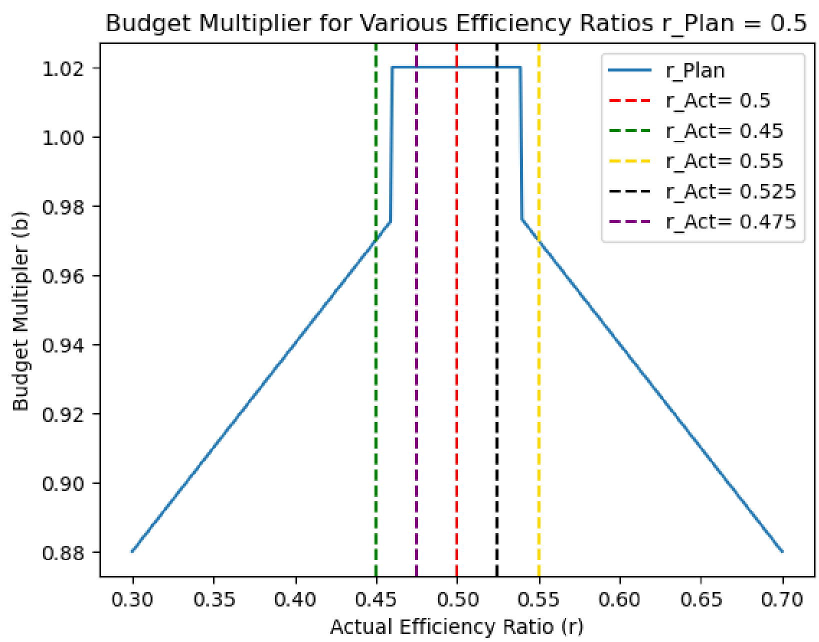


Figure 9. Addition of Tolerance and Bonus ($x_j = 0.7$, $y_j = 1.3$, $t_j = 0.04$, $u_j = 0.02$ )

In Figure 9, two additional scenarios are added, the purple scenario where the division underperformed by 2.5% and the black scenario where the division overperformed by 2.5%. A tolerance of 4% and bonus of 2%, as well as the same penalties as Figure 2, are employed ($x_j = 0.7$ and $y_j = 1.3$ ).

Notice in the purple and black scenarios, the division was not entirely accurate but was within the tolerance. Thus, the division receives the bonus adjusted multiplier of 1.02 in these scenarios. However, when the division performed outside of the tolerance in the green and gold scenarios, the division receives the same penalized multiplier as the baseline scenario of 0.97.

## F. ADDITIONAL MODEL INSIGHTS

### 1. Game Sequence Insights

During the proposal stage of the first year, divisions can anticipate budget cuts and are aware that they will be competing for limited funds. To avoid receiving a larger budget cut than other divisions, they are incentivized to report higher project realization ($n_{i,1\ Prop}$) and request higher budgets ($B_{i,1\ Prop}$). Although the department may not have as much information as divisions, they can still cut budgets for divisions with lower project achievements. If divisions have pre-assessments for their projects, such as NPV or social welfare value, the pressure to perform well is even greater.

At the beginning of the planning stage of the first year, divisions are made aware that $x_j$< 1 and $y_j$ > 1 will be used, but the exact values of $x_j$ and $y_j$ are not disclosed. Divisions understand that the best option is to make the most accurate prediction and report the truth about what can be accomplished with the available budget. However, the uncertainty about the exact size of the punishment for underperformance creates an additional incentive for divisions to report what they truly believe can be achieved with the budget.

If program managers proposed a high efficiency ratio ($r_{i,1\ Prop}$) in the proposal stage and then promise a significantly different efficiency ratio ($r_{i,1\ Plan}$) in the planning stage, the department may become suspicious and adjust the values of $x_j$ and $y_j$ accordingly. For instance, if a program manager reports a much lower planned efficiency ratio ($r_{i,1\ Plan}$) than proposed efficiency ratio ( $r_{i,1\ Prop}$) , the department may use a low value of $x_j$ to punish the division for achieving more than the low projected value of the planned efficiency ratio. However, the uncertainty about the exact punishment size prevents divisions from reporting a very low planned efficiency ratio to avoid a larger budget cut in the next year. Overall, the tendency is for divisions to make the most accurate forecast for the available budget and report the truth to avoid any potential penalties.

What if program managers report a very high number of projects ($n_{i,1\ Plan}$) in the planning stage? For instance, they reported a high proposed efficiency ratio in the proposal stage. They may be tempted to remain somewhat consistent by reporting a higher planned efficiency ratio than is reasonable. However, if they are unable to deliver on their planned number of projects ($n_{i,1\ Plan}$), they can still meet the planned efficiency ratio by returning their remaining budget. Thus, the actual numbers ($n_{i,1\ Act}$ and $B_{i,1\ Act}$) will both be lower than the planned values, and thus actual efficiency ratio will be much closer to the planned efficiency ratio than if they had not returned the remaining budget. This allows divisions to promise high project numbers in the planning stage, as they have the option of doing fewer projects than predicted while still maintaining an acceptable efficiency ratio for the year and avoiding cuts in the next year since a smaller budget will be used and money will be saved.

What if program managers report a very low number of projects ($n_{i,1\ Plan}$) in the planning stage? For instance, if a program manager mistakenly reports a low efficiency ratio value in the proposal stage and is reluctant to indicate a significantly different ratio in the planning stage , they may end up reporting a low planned number of completed projects ($n_{i,1\ Plan}$). In this scenario, the program manager likely has already been given a low number in the planning stage. If the program manager submits a planned efficiency ratio, the only option to fulfill it would be to spend money on unnecessary items, but this could be detected by the department and result in the program manager of the division being deemed underperforming and possibly losing their position, particularly if they already have low project completion numbers. To prevent this from happening, headquarters could adjust the penalties by setting the from the second or third year onward to encourage program managers in this situation to exceed the planned efficiency ratio that they project, thus by setting $x_j$ closer to 1.

### 2. Dynamic Effects

The model promotes transparency among divisions regarding their project execution capabilities. According to game theory, it's considered advantageous for a player to disclose their "type" when the mechanism aligns with the Revelation Principle. In the context here, the "type" pertains to whether a program manager is highly efficient and diligent, or less productive and unmotivated.

Over time, the department might consider utilizing the efficiency ratio from the previous year ($r_{i,j-1\,Act}$) rather than permitting divisions to submit a planned efficiency ratio ($r_{i,j\,Plan}$) that is lower for the current year. This adjustment would result in a modification of the proposed game rule (whereby $n_{i,1\,Plan}$ would be determined by the divisions rather than the department). Although altering the rule might be enticing, it carries the potential for complications. It's worth noting that the penalties $x_j$ and $y_j$ can be adjusted, perhaps allowing the department to potentially apply distinct penalties for different divisions (thus changing a different rule).

As an illustration, consider a program manager leading a division with high productivity, who might naturally report a high planned efficiency ratio. However, if the program manager anticipates that this efficiency ratio value could serve as a minimum benchmark in upcoming years, they would likely adopt a more cautious approach and initially set a lower planned efficiency ratio value. This caution arises from the concern that the initially set level could become the minimum expected performance standard. This dynamic phenomenon is recognized as the Ratchet Principle (Weitzman 1980). Program managers are faced with the task of balancing an uncut budget to achieve strong performance in the present year with the possibility of being assigned more ambitious objectives in subsequent years.

To avoid this ratchet effect, the department should always make the number of completed projects ($n_{i,j\,Plan}$), the choice of division program managers, given the assigned budget ($B_{i,j\ Plan}$). In other words, it will not be used as a minimum performance level in subsequent years.

## G. CONCLUSIONS

The presented model obeys many desirable properties:

Theory: The model conforms to current economic theory and harmonizes with contemporary Mechanism Design Theory and the Revelation Principle. These theories are at the forefront of endeavors aimed at mitigating challenges associated with Information Asymmetry. The game described in this section begins with the desired Nash equilibrium of an accurate and truthful prediction for budget and number of completed projects for all divisions in all phases of the game sequence. The examples provided show that the divisions cannot choose an alternative strategy that is better than an accurate budget. In every case, the divisions are penalized and receive a reduced budget if they are not accurate within the allowable tolerance.

Practice: For practicality, a model should possess simplicity and clarity. This specific model encompasses only a handful of parameters, each of which holds straightforward and applicable meanings. Once the initial-year budget negotiation is complete, program managers are solely tasked with accurately estimating one parameter ($n_{i,j\ Plan}$) .Models designed to encourage precision are both intuitive and attractive.

Application Flexibility: The model should possess the capability to adjust according to the preferences and unique business circumstances of the headquarters. This model offers versatility in defining the extent of penalties, whether they are symmetrical or asymmetrical, through the parameters $x_j$ and $y_j$. Additionally, it accommodates tolerance for forecasting errors and the potential incorporation of a bonus for precise reporting.

Dynamic Effects: Over time, the model's dynamic effect will reveal which program managers are high-productivity and which are low-productivity.

To prevent the ratchet effect for high-productivity program managers, it is important to always communicate that the decision on the future $m_{i,j}$ values will be made by them, and not by headquarters.

# IV. MONTE CARLO SIMULATION AND STOCHASTIC PORTFOLIO OPTIMIZATION

## A. OPERATIONAL AND LOGISTICS DECISION CRITERIA

Besides monetary values, various operational, logistical, and other military values of interest can be developed and utilized in the proposed model, as explained in this report. Below are a few examples of alternative value metrics and are taken directly from *Risk-Based ROI, Capital Budgeting, and Portfolio Optimization in the Department of Defense* by Dr. Jonathan Mun (2020, pp. 84–85). These metrics can be employed in future studies with real data gathered. They can be utilized independently or combined into a composite weighted metric that can be used as $n_{i,j}$ .

- **Inherent Availability** (IA). Measures operational percentage in an ideal support environment per design specifications. Mean Time Between Failure (MTBF) and Mean Time to Repair (MTTR) are used in this calculation.

$$IA = \frac{MTBF}{MTBF + MTTR}$$

- **Effective Availability** (EA). Probability a ship's system is available at any instant during the maximum operational period, accounting for all critical failures, reparable and nonrepairable at sea, and preventive maintenance. Additional metrics here are Mean Down Time (MDT), Maintenance Time (MT) and Mean Time to Failure (MTTF).

$$EA = 1 - \frac{MTTR}{MTBF + MTTR} - \frac{MDT}{MT} - 0.5\frac{MT}{MTTF}$$

- **Mission Reliability** (MR). **Operational Ready Rate** (ORR) at the start of a mission compared to its **Inherent Reliability** (IR).

$$MR = ORR * IR$$

- **Operational Dependability** (OD). Probability a system can be used to perform a specified mission when desired.

$$OD = \frac{MTTF}{MTBF}$$

- Mean Down Time (MDT), Mean Maintenance Time (MMT), Logistics Delay Time (LDT), and their combinations.
- Achieved Availability (AA), Operational Availability (OA), and Mission Availability (MA)

## B. ALTERNATIVE FINANCIAL AND ECONOMIC VALUES

Also outlined in the same 2020 Mun article are alternative financial and economic values and are provided below (p. 84-85).

- **Cost Deterrence and Avoidance**. Soft or shadow revenue (cost savings) over the economic and operational life of the program or system. Milestone A, B, C.
- Traditional Financial Metrics. **Net Present Value** (NPV), **Internal Rate of Return** (IRR), **Return on Investment** (ROI), and other metrics, as long as there are financial and monetary values.
- **Budget Constraint**. FY Budget limitations and probabilities of budgetary overruns.
- **Total Ownership Cost** (TOC) and **Total Life cycle Cost** (TLC). Accounting for the cost of developing, producing, deploying, maintaining, operating, and disposing of a system over its entire lifespan. Uses **Work Breakout Structures** (WBS), **Cost Estimating Categories** (CEC), and **Cost Element Structures** (CES).

- Knowledge Value Added (KVA). Monetizing Learning Time, Number of Times Executed, Automation, Training Time, and Knowledge Content.

- **Strategic and Capability.** Multiple value metrics can be determined by Subject Matter Experts (SME) including:

    - **Expected Military Value**
    - **Strategic Value**
    - **Future Weapon Strategy**

- **Capability Measures** (CM). Difficult to quantify and needs SME judgment:

    - Innovation Index, Conversion Capability, Ability to Meet Future Threats
    - Force Structure (size/units), Modernization (technical sophistication), Combat Readiness, Sustainability
    - Future Readiness (ability to meet evolving threats, ability to integrate future weapons systems)

- **Domain Capabilities** (DC)

    - Portfolios are divided into different domains, and each domain is optimized separately and then combined at the enterprise level and re-optimized; example domains include Coastal Defense, Anti-Air Surface Warfare, Anti-Surface Warfare, Anti-Submarine Warfare, Naval Strike, Multi-Mission Air Control, Sea Control, Deep Strike, Missile Defense, and so on.

## C. MONTE CARLO SIMULATION OF GAME THEORY MODEL

During this Monte Carlo simulation the proposed stage is removed and only year 1 is being simulated. Also, each division is simulated independently of the other divisions, however, the distributions for penalty parameters ($x_j$ and $y_j$), tolerance ($t_j$) and bonus ($u_j$) remain the same.

Ideally, having historical data is preferred to have the most accurate simulation. However, in the absence of historical data, sample stylized data can be used as an alternative. The simulation described in this section uses the latter.

For $B_{i,j\ Plan}$ and $B_{i,j\ Act}$, each is given one of three distributions all with a mean of \$100 Million. The first is the triangular distribution with a minimum of \$90 Million, maximum of \$110 Million, and mode of \$100 Million. The second distribution is the normal distribution with a mean of \$100 Million and standard deviation of \$10 Million. The third and final distribution is the uniform distribution with a minimum of \$90 Million and maximum of \$110 Million. The value of $B_{i,j\ Plan}$ and $B_{i,j\ Act}$ is assigned one of these three distributions for each division.

For $n_{i,j\ Plan}$ and $n_{i,j\ Act}$, each is given one of two distributions each with a mean of 50 projects. Since the number of projects must be a whole number, each is discrete distributions. The first distribution is the discrete uniform distribution with a minimum of 45 projects and a maximum of 55 projects. The other distribution used is the Poisson distribution with a mean of 50 projects. The value of $n_{i,j\ Plan}$ and $n_{i,j\ Act}$ is assigned one of these two distributions for each division.

The final four parameters are typically set by the department. For the simulation a distribution is assigned to cover the range of values that the department could assign to them. Regarding the penalty parameters $x_j$ and $y_j$, the normal distribution is used for both. For $x_j$, the mean is 0.9 and standard deviation is 0.01, while for $y_j$, the mean is 1.1 and standard deviation is 0.01. Additionally, an upper bound of 1.0 is imposed on $x_j$ and a lower bound of 1.0 is imposed on $y_j$ to ensure they stay penalties and do not turn into bonuses as previously discussed. Based on the distributions currently assigned, it is highly unlikely that the bonds will come into play. However, it is important to highlight this point if the distributions are changed, potentially increasing the likelihood the bounds come into play for the tolerance $t_j$ and bonus $u_j$, the exponential distribution with a mean of 0.02 is used for both. Table 2 provides a summary of the distributions used.

Table 2. Distributions of Input Parameters

**Budget Parameters ($B_{i,j\,Plan}$, $B_{i,j\,Act}$) in $Million** : **Mean= 100**

**1**: *Triangular (Min = 90, Max = 110, Mode = 100)*

**2:** *Normal (Mean = 100, SD = 10)*

**3:** *Uniform (Min = 90, Max = 110)*

**Number of Projects ($n_{i,j\,Plan}$, $n_{i,j\,Act}$): Mean = 50**

**1:** *Discrete Uniform (Min = 45, Max = 55)*

**2:** *Poisson (Mean = 50)*

**Penalty Parameters**

$x_j$: *Normal(Mean = 0.9, SD = 0.01) Upper Bound: 1*

$y_j$: *Normal(Mean= 1.1, SD = 0.01) Lower Bound: 1*

**Tolerance and Bonus**

$t_j$: *Exponential (Mean = 0.2)*

$u_j$: *Exponential (Mean = 0.2)*

Table 3. Divisional Distribution Assignments

| | B_Plan | n_Plan | B_Act | n_Act |
|---|---|---|---|---|
| Division 1 | 1 | 1 | 1 | 1 |
| Division 2 | 2 | 1 | 2 | 1 |
| Division 3 | 3 | 1 | 3 | 1 |
| Division 4 | 1 | 2 | 1 | 2 |
| Division 5 | 2 | 2 | 2 | 2 |
| Division 6 | 3 | 2 | 3 | 2 |
| Division 7 | 1 | 1 | 2 | 1 |
| Division 8 | 2 | 1 | 3 | 1 |
| Division 9 | 1 | 2 | 1 | 1 |
| Division 10 | 2 | 1 | 2 | 2 |

Table 3 describes the distributions assigned to the division specific parameters. The rows represent 10 divisions which are given generic names of Division 1–10. The columns represent the different division specific parameters of $B_{i,j\ Plan}$, $B_{i,j\ Act}$, $n_{i,j\ Plan}$, and $n_{i,j\ Act}$. The values represent the numbered distribution from Table 10 corresponding to that parameter's column. For example, Division 8 has $B_{i,j\ Plan} = 2$, $n_{i,j\ Plan} = 1$, $B_{i,j\ Act} = 3$, and $n_{i,j\ Act} = 1$. This means $B_{i,j\ Plan}$ has the normal distribution, $n_{i,j\ Plan}$ has the discrete uniform, $B_{i,j\ Act}$ has the uniform distribution, and $n_{i,j\ Act}$ has the discrete uniform distribution. Notice that budget and/or number of projects parameters do not necessarily have the same distributions between the planned and actual stages.

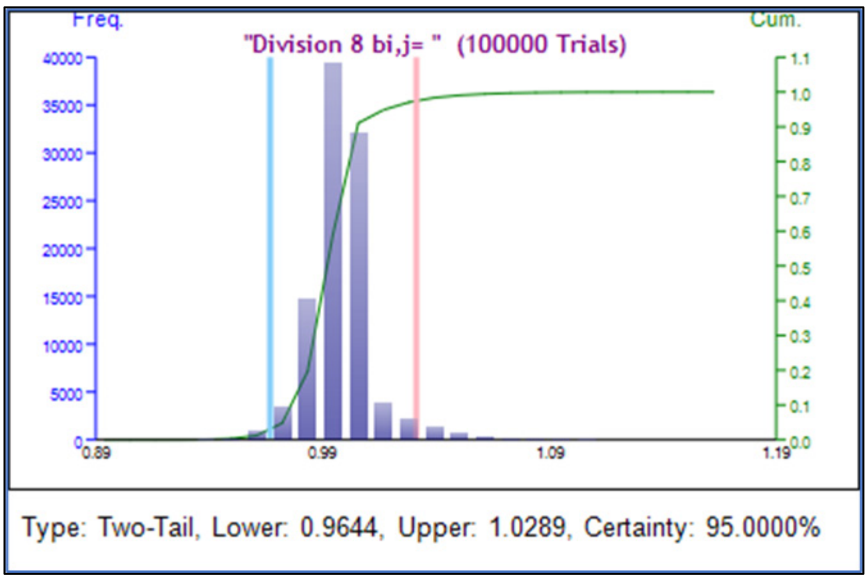


Figure 10. Example Simulation (Division 8)

Figure 10 shows an example of the Monte Carlo simulation for Division 8. The simulation ran for 100,000 trials using the previously described distributions for the parameters. The output of simulation is a range of values for the budget multiplier ($b_{i,j}$). A two-tail 95% confidence interval is applied to the results. The upper and lower bounds of this confidence interval are what will be used for the stochastic portfolio optimization. In this case the lower bound is 0.9644 or 96.44% and upper bound is 1.0289 or 102.89%.

Table 4. Simulation Results

| | Lower Bound of 95% Confidence Interval | Upper Bound of 95% Confidence Interval |
|---|---|---|
| **Division 1** | 97.60% | 103.39% |
| **Division 2** | 95.96% | 102.63% |
| **Division 3** | 97.24% | 103.22% |
| **Division 4** | 94.77% | 102.19% |
| **Division 5** | 93.62% | 101.90% |
| **Division 6** | 94.55% | 102.12% |
| **Division 7** | 96.78% | 102.94% |
| **Division 8** | 96.44% | 102.89% |
| **Division 9** | 96.17% | 102.63% |
| **Division 10** | 94.49% | 102.18% |

Table 4 displays the results of the Monte Carlo simulations that were run for each division in a similar manner to the example for Division 8 with the lower and upper bounds of the confidence intervals displayed. These values will be used in a stochastic optimization for budget allocation.

## D. BUDGET ALLOCATION STOCHASTIC OPTIMIZATION

To understand how the game theory Monte Carlo simulation can be applied to budget allocation stochastic optimization, an example of a more traditional approach of budget allocation stochastic optimization should be explored.

Figure 11 displays an example of an asset allocation optimization model. Column B, *asset class description* is the name of the asset class and is Divisions 1–10 in this case. Column C, *annualized returns* represent the relative return geometric average of each division represented as a percentage. This assumed to be known given information. Column D, *volatility risks* follow the logarithmic relative stock returns approach to calculate the risk associated with each division. This is also known given information. Column E, *allocation weights* are the decision variables in the optimization denoted by the blue column. The next two columns, Column F and G, *required minimum allocation*, $p_i$ and *required maximum allocation,* $q_i$, represent the bounds for the allocation weights.

Column H, the return to risk ratio is the ratio $\frac{R_i}{\sigma_i}$, where $R_i$ is the *annualized return* for division $i$ and $\sigma_i$ is the *volatility risk* for division $i$. The last four columns, Columns I-L, *Returns Ranking, Risk Ranking, Return to Risk Ranking,* and *Allocation Ranking* are the rankings based on the criteria in their title (Mun 2022, p. 671–672).

**ASSET ALLOCATION OPTIMIZATION MODEL**

| Asset Class Description | Annualized Returns | Volatility Risk | Allocation Weights | Required Minimum Allocation | Required Maximum Allocation | Return to Risk Ratio | Returns Ranking (Hi-Lo) | Risk Ranking (Lo-Hi) | Return to Risk Ranking (Hi-Lo) | Allocation Ranking (Hi-Lo) |
|---|---|---|---|---|---|---|---|---|---|---|
| Division 1 | 10.54% | 12.36% | 10.00% | 5.00% | 35.00% | 0.8524 | 9 | 2 | 7 | 1 |
| Division 2 | 11.25% | 16.23% | 10.00% | 5.00% | 35.00% | 0.6929 | 7 | 8 | 10 | 1 |
| Division 3 | 11.84% | 15.64% | 10.00% | 5.00% | 35.00% | 0.7570 | 6 | 7 | 9 | 1 |
| Division 4 | 10.64% | 12.35% | 10.00% | 5.00% | 35.00% | 0.8615 | 8 | 1 | 5 | 1 |
| Division 5 | 13.25% | 13.28% | 10.00% | 5.00% | 35.00% | 0.9977 | 5 | 4 | 2 | 1 |
| Division 6 | 14.21% | 14.39% | 10.00% | 5.00% | 35.00% | 0.9875 | 3 | 6 | 3 | 1 |
| Division 7 | 15.53% | 14.25% | 10.00% | 5.00% | 35.00% | 1.0898 | 1 | 5 | 1 | 1 |
| Division 8 | 14.95% | 16.44% | 10.00% | 5.00% | 35.00% | 0.9094 | 2 | 9 | 4 | 1 |
| Division 9 | 14.16% | 16.50% | 10.00% | 5.00% | 35.00% | 0.8584 | 4 | 10 | 6 | 1 |
| Division 10 | 10.06% | 12.50% | 10.00% | 5.00% | 35.00% | 0.8045 | 10 | 3 | 8 | 1 |
| *Portfolio Total* | *12.6419%* | *4.58%* | *100.00%* | | | | | | | |
| *Return to Risk Ratio* | *2.7596* | | | | | | | | | |

Specifications of the optimization model:

**Objective:** *Maximize Return to Risk Ratio (C18)*
**Decision Variables:** *Allocation Weights (E6:E15)*
**Restrictions on Decision Variables:** *Minimum and Maximum Required (F6:G15)*
**Constraints:** *Portfolio Total Allocation Weights 100% (E17 is set to 100%)*

Additional specifications:

1. One can always maximize portfolio total returns or minimize the portfolio total risk.
2. Incorporate Monte Carlo simulation in the model by simulating the returns and volatility of each asset class and apply Simulation-Optimization techniques.
3. The portfolio can be optimized as is without simulation using Static Optimization techniques.

Figure 11. Example of Asset Allocation Stochastic Optimization. Adapted from Mun (2022, p. 672).

Row 17 represents the *portfolio totals*. Cell C17 represents the overall return for the portfolio. It is computed with the formula $R_p = \sum_{i=1}^{n} R_i\omega_i$, where $R_p$ is the *total portfolio return*, $R_i$ is the *annualized return* for division $i$, $\omega_i$ is the *allocation weight*, and $n$ represents the total number of divisions. Cell D17 represents the overall risk for the portfolio. It is computed with the formula $\sigma_P = \sqrt{\sum_{i=1}^{n} \omega_i^2\sigma_i^2}$, where $\sigma_P$ is the *total portfolio volatility risk*, and $\sigma_i$ is the *volatility risk* for division $i$. This formula assumes that there are no correlations between the divisions. If there are correlations between the divisions the formula becomes $\sigma_P = \sqrt{\sum_{i=1}^{n} \omega_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{m} 2\,\omega_i\omega_j\rho_{i,j}\sigma_i\sigma_j}$ , where $\rho_{i,j}$ is the cross correlation between division $i$ and division $j$. Cell E17 is the total portfolio allocation weight which will always be 100%. Lastly, cell C18 represents the return to risk ratio, which as the name suggests, is the ratio between the overall return for the portfolio (C17) and the overall risk for the portfolio (D17). This is also known as the Sharpe ratio and follows the equation $S_p = \frac{R_p}{\sigma_p}$ where $S_p$ is the Sharpe ratio for the portfolio (Mun 2022, p. 671–672).

The objective of this optimization is to maximize the Sharpe's ratio, $S_p$. The first constraint is the sum of the allocation weights, $\omega_i$ must be 1 or 100%. The final set of constraints is that the allocation weight for each division must be between the minimum, $p_i$, and maximum, $q_i$, required allocation for each division. Table 13 summarizes the formulation of this optimization problem.

Everything expressed up until this point describes static portfolio optimization. In order for this to become stochastic portfolio optimization, some uncertainty needs to be introduced and a few more parameters needed to be added.

It is unlikely that the annualized return, $R_i$, and volatility risk, $\sigma_i$, are perfectly predictable as is implied by the static optimization formulation. Instead, it is likely that there is some range of values that each of these values could. Perhaps with historical data, subject matter expert consensus or some other method, a probability distribution can be assigned to each of these values for each division.

With the addition of uncertainty to the model, Monte Carlo simulation is now introduced as well as two new parameters. The first parameter, N, is the number of simulations, and M is the number of trials conducted. The first step is to simulate the parameters with an uncertainty ($R_i$ and $\sigma_i$), N times. Then the mean value from this round of simulation is used as the actual value in the static optimization. A distribution based on historical data or other means will be assigned to each of these values. The second step is to re-run the previous steps for M trials. That is, run N Monte Carlo simulations to get the mean values to be used for $R_i$ and $\sigma_i$ for the trial and then used in the static optimization model. This process is then repeated for each trial. The solution that produces the best performing objective value, in this case the maximum Sharpe's ratio, is the output of this process. Table 5 depicts the formulation of this optimization problem.

Table 5. Formulation for Budget Allocation Optimization. Adapted from Mun (2022, pp. 671–672).

**Objective:**

$$\max S_p$$

**Subject To:**

$$\sum_{i=1}^{n} \omega_i = 1 \; or \; 100\%$$

$$p_i \leq \omega_i \leq q_i, \quad \forall i$$

**Where:**

$$S_p = \frac{R_p}{\sigma_p}$$

$$R_p = \sum_{i=1}^{n} R_i \omega_i$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2}$$

**Indices:**

| | |
|---|---|
| $i$ | Represents each division in the portfolio |

**Variables:**

| | |
|---|---|
| $\omega_i$ | Allocation weight for division $i$ in percentage |

**Parameters:**

| | |
|---|---|
| $S_p$ | Sharpe's ratio or return to risk ratio |
| $R_i$ | Annualized return for division $i$ |
| $R_p$ | Total portfolio return |
| $\sigma_i$ | Volatility risk for division $i$ |
| $\sigma_p$ | Total volatility risk for the portfolio |
| $p_i$ | Minimum required allocation |
| $q_i$ | Maximum required allocation |

The final output of the stochastic optimization model is the best-case solution to the static optimization model among the M trials, as well as the distribution of values the decision variables took on throughout entirety of the run.

**ASSET ALLOCATION OPTIMIZATION MODEL**

| Asset Class Description | Annualized Returns | Volatility Risk | Allocation Weights | Required Minimum Allocation | Required Maximum Allocation | Return to Risk Ratio | Returns Ranking (Hi-Lo) | Risk Ranking (Lo-Hi) | Return to Risk Ranking (Hi-Lo) | Allocation Ranking (Hi-Lo) |
|---|---|---|---|---|---|---|---|---|---|---|
| Division 1 | 10.49% | 12.55% | 10.71% | 5.00% | 35.00% | 0.8355 | 9 | 3 | 7 | 5 |
| Division 2 | 11.28% | 16.52% | 6.65% | 5.00% | 35.00% | 0.6827 | 7 | 8 | 10 | 10 |
| Division 3 | 11.98% | 15.42% | 8.11% | 5.00% | 35.00% | 0.7771 | 6 | 7 | 9 | 9 |
| Division 4 | 10.75% | 12.42% | 11.22% | 5.00% | 35.00% | 0.8658 | 8 | 1 | 5 | 4 |
| Division 5 | 13.27% | 13.31% | 12.06% | 5.00% | 35.00% | 0.9972 | 5 | 4 | 2 | 2 |
| Division 6 | 14.23% | 14.28% | 11.23% | 5.00% | 35.00% | 0.9962 | 3 | 6 | 3 | 3 |
| Division 7 | 15.34% | 13.98% | 12.64% | 5.00% | 35.00% | 1.0976 | 1 | 5 | 1 | 1 |
| Division 8 | 14.96% | 16.58% | 8.76% | 5.00% | 35.00% | 0.9027 | 2 | 10 | 4 | 7 |
| Division 9 | 14.15% | 16.55% | 8.31% | 5.00% | 35.00% | 0.8551 | 4 | 9 | 6 | 8 |
| Division 10 | 9.98% | 12.48% | 10.31% | 5.00% | 35.00% | 0.7999 | 10 | 2 | 8 | 6 |
| *Portfolio Total* | *12.7041%* | *4.52%* | *100.00%* | | | | | | | |
| *Return to Risk Ratio* | *2.8098* | | | | | | | | | |

Figure 12. Budget Allocation Stochastic Optimization Results (N=1000, M=20). Adapted from Mun (2022, pp. 671–672).

Figure 12 shows the results of the stochastic optimization from Figure 11. Both annualized returns and volatility risk were given normal distributions with a mean of the initial values shown with a standard deviation of 10 percent of the mean. Values of N=1000 and M=20 is used for the optimization. The maximum Sharpe's ratio under these conditions was 2.8098 with the allocation weights in column E as the optimal solution.

## E. STOCHASTIC OPTIMIZATION OF BUDGET GAME THEORY MODEL

In this section, the values that were computed at the end of Section C will be applied to a stochastic optimization model described in Section D. The data used in this example is stylized data not historical data. If historical data is available, distribution analysis can be conducted and used in the place of the distributions used for budgets, number of projects, military value, and military downsides.

Figure 13 displays the results of the stochastic optimization with the game theory model. Note there are some differences between the example model in Section D, and the application to the game theory model.

**BUDGET ALLOCATION STOCHASTIC OPTIMIZATION MODEL**

| Description | WA Military Value | WA Downsides | Allocation Weights | Allocation ($1MIL) | Requested | Minimum Allocation | Maximum Allocation |
|---|---|---|---|---|---|---|---|
| Division 1 | 11.25% | 12.40% | 12.69% | $126.88 | $130.00 | $126.88 | $134.41 |
| Division 2 | 11.35% | 16.25% | 6.72% | $67.17 | $70.00 | $67.17 | $71.84 |
| Division 3 | 11.75% | 15.35% | 7.78% | $77.79 | $80.00 | $77.79 | $82.58 |
| Division 4 | 10.75% | 12.35% | 10.40% | $104.00 | $105.00 | $99.51 | $107.30 |
| Division 5 | 13.25% | 13.10% | 13.11% | $131.07 | $140.00 | $131.07 | $142.66 |
| Division 6 | 14.10% | 14.35% | 10.44% | $104.35 | $110.00 | $104.01 | $112.33 |
| Division 7 | 15.50% | 14.15% | 12.10% | $120.98 | $125.00 | $120.98 | $128.68 |
| Division 8 | 15.15% | 16.40% | 8.68% | $86.81 | $90.00 | $86.80 | $92.60 |
| Division 9 | 14.40% | 16.65% | 8.17% | $81.74 | $85.00 | $81.74 | $87.24 |
| Division 10 | 9.75% | 12.40% | 9.92% | $99.21 | $105.00 | $99.21 | $107.29 |
| ***Portfolio Total*** | ***12.76%*** | ***4.50%*** | ***100.00%*** | ***$1,000.00*** | **$1,040.00** | **$995.16** | **$1,066.92** |
| ***Value to Risk Ratio*** | ***2.8362*** | | Target Allocation | **$1,000** | | | |

Figure 13. Final Results of Stochastic Optimization of Budget Game Theory Model. Adapted from Mun (2022, pp. 671–672).

The *annualized returns* column is replaced with *WA Military Value*. Instead of using monetary returns, it is replaced with a weighted average of the military value of the projects conducted by the specified division. The military values in this sense are any military value described in Sections A and B that are viewed positively. An exact formula for this value will not be explored, as it may change based on the decision-maker. However, a stylized example of its outputs is used for this example.

Similarly, *volatility risk* is replaced *WA Downsides*. The monetary risk is replaced with a similar weighted average of military values, however with those values that are viewed negatively. Again, the exact formula is not explored, and a stylized example of its outputs are used.

The portfolio totals for these values are computed the same way from Section D. The distributions assigned for these is a uniform distribution with the minimum value and maximum values noted in the final 4 columns of Table 6.

Table 6. Amplifying Information of Model Parameters

| Minimum Allocation | Maximum Allocation | Actual Allocated | WA MV Min | WA MV Max | WA DS Min | WA DS Max |
|---|---|---|---|---|---|---|
| 97.60% | 103.39% | 97.60% | 10.13% | 12.38% | 11.16% | 13.64% |
| 95.96% | 102.63% | 95.96% | 10.22% | 12.49% | 14.63% | 17.88% |
| 97.24% | 103.22% | 97.24% | 10.58% | 12.93% | 13.82% | 16.89% |
| 94.77% | 102.19% | 99.04% | 9.68% | 11.83% | 11.12% | 13.59% |
| 93.62% | 101.90% | 93.62% | 11.93% | 14.58% | 11.79% | 14.41% |
| 94.55% | 102.12% | 94.86% | 12.69% | 15.51% | 12.92% | 15.79% |
| 96.78% | 102.94% | 96.78% | 13.95% | 17.05% | 12.74% | 15.57% |
| 96.44% | 102.89% | 96.45% | 13.64% | 16.67% | 14.76% | 18.04% |
| 96.17% | 102.63% | 96.17% | 12.96% | 15.84% | 14.99% | 18.32% |
| 94.49% | 102.18% | 94.49% | 8.78% | 10.73% | 11.16% | 13.64% |

The *allocation weights* column in Figure 13 is the same as described from Section D. However, it is paired with the additional column for *allocation*. Both columns are synonymous with each other, with *allocation weights* as percentages and all *allocation* expressed in millions of dollars. The total departmental budget is $1 billion and equates to an average budget for each division of $100 million.

The *requested* column in Figure 13 represents the requested amount for each division in millions of dollars. This would closely resemble the proposed budget ($B_{i,j\ Prop}$) from the budget game. The exact amounts used are a stylized example. The following two columns *Minimum Allocation* and *Maximum Allocation* from Section D represent the upper and lower bounds of the confidence interval calculated from the Monte Carlo simulations. These values are computed by using the first two columns from Table 14, which are the same values computed in Table 12 from Section C and are multiplied by the *requested* column amount. These values act as lower and upper bounds for the *allocation* column.

Figure 14 displays the results where the money is allocated by reducing each by budget by the same amount, $4 million. This number is calculated by taking the difference between the total requested budget, $1,040.00 million and the department budget of $1,000 million, which is $40 million, then divided by 10 divisions to get $4 million.

**BUDGET ALLOCATION STOCHASTIC OPTIMIZATION MODEL**

| Description | WA Military Value | WA Downsides | Allocation Weights | Allocation ($1MIL) | Requested | Minimum Allocation | Maximum Allocation | Actual Allocation |
|---|---|---|---|---|---|---|---|---|
| Division 1 | 11.25% | 12.40% | 12.60% | $126.00 | $130.00 | 97.60% | 103.39% | 96.92% |
| Division 2 | 11.35% | 16.25% | 6.60% | $66.00 | $70.00 | 95.96% | 102.63% | 94.29% |
| Division 3 | 11.75% | 15.35% | 7.60% | $76.00 | $80.00 | 97.24% | 103.22% | 95.00% |
| Division 4 | 10.75% | 12.35% | 10.10% | $101.00 | $105.00 | 94.77% | 102.19% | 96.19% |
| Division 5 | 13.25% | 13.10% | 13.60% | $136.00 | $140.00 | 93.62% | 101.90% | 97.14% |
| Division 6 | 14.10% | 14.35% | 10.60% | $106.00 | $110.00 | 94.55% | 102.12% | 96.36% |
| Division 7 | 15.50% | 14.15% | 12.10% | $121.00 | $125.00 | 96.78% | 102.94% | 96.80% |
| Division 8 | 15.15% | 16.40% | 8.60% | $86.00 | $90.00 | 96.44% | 102.89% | 95.56% |
| Division 9 | 14.40% | 16.65% | 8.10% | $81.00 | $85.00 | 96.17% | 102.63% | 95.29% |
| Division 10 | 9.75% | 12.40% | 10.10% | $101.00 | $105.00 | 94.49% | 102.18% | 96.19% |
| ***Portfolio Total*** | ***12.77%*** | ***4.51%*** | ***100.00%*** | ***$1,000.00*** | **$1,040.00** | | | |
| ***Value to Risk Ratio*** | ***2.8343*** | | Target Allocation | **$1,000** | | | | |

Figure 14. Budget Allocation Model by Subtracting $4 Million from Requested Budget. Adapted from Mun (2022 pp. 671–672).

Note that the *value to risk ratio* (Sharpe's ratio) is 2.8343 in this case while in the game theory model it was 2.8362. Also, the five cells highlighted in red in the *actual allocated* column represent divisions whose budget allocation under this alternate method falls below the game theory minimum. Thus, not only is the Sharpe's ratio lower, using this method would cut the budget of these divisions more harshly than the game theory model. Therefore, the game theory model is better than using this alternate method.

# V. CONCLUSION/FINDINGS

## A. FINDINGS

This thesis explored the ins and outs of game theory, explored a model that served as the best game theoretic approach to budget allocation. To review, the research questions were: 1) What is the best game theoretic approach to budget allocation in the U.S. Navy? and 2) What is the best way to implement this model into real-world budget allocation?

Game theory constitutes a valuable tool for budget allocation purposes. The game theory model described in Chapter III stands as applicable across various departments within the Navy, aiding in the facilitation of truth-telling and precision within the budget allocation process. Implementation of the model in the actual budget allocation of the real world is not an obligatory requirement for the department. Rather, the department has the option to execute the game in parallel with the real-world budget allocation, thereby discerning the differences presented by the game theory model.

Additionally, a department can use this model to "wargame" a budget allocation process with their program managers. This can provide useful information to the department and program managers and reinforce the idea of accuracy and truth-telling in the budget allocation process.

Alternatively, if the game theory model as described in Chapter III is deemed to be too difficult to implement in a real-world scenario, a department can use its historical data to run simulations on each of the divisions within their department. This process is like the Monte Carlo simulations conducted in Section C of Chapter IV using historical data instead of assumptions about what the distributions might be. Then these simulations can provide upper and lower bounds for the departmental budgets. The department can then use stochastic optimization with these bounds and additional input distributions for military value and military downsides.

## B. CONCLUSIONS

In conclusion, game theory, Monte Carlo simulation, and stochastic optimization can provide great recommendations to decision-makers. However, the problems decision-makers face when allocating the budget will not be easily solved with these techniques. The ideas described in this thesis instead provide the framework for a game-theoretic approach that can be used as an aid for the decision-makers.

## C. LIMITATIONS OF STUDY

The main limitation to the study was the absence of historical data in which to use for the Monte Carlo simulations and stochastic optimization in Chapter IV. Stylized data was used in its place. However, this greatly reduces the ability to conduct analysis on the outcomes of the data. While Navy budget figures are readily available online, how the budget allocation process played out is not. Only the final numbers at the end of the process are available. Also, information that can be used to determine the number of projects and/or how to determine military value and military downsides is not available.

## D. FUTURE RESEARCH

There are two main ideas for future research. The first is to obtain real-world data and use it to run more in-depth analysis with Monte Carlo simulation and stochastic optimization. This removes the main limitation to the study and more specific recommendations can be made that are tailored to this data.

The second is to conduct a wargame scenario with the model described in Chapter III. This wargame can be done in a real Navy department or with some other subject-matter experts as the project managers of the divisions. Depending on the nature of this wargame scenario, real-world or stylized data can be used in this wargame.

# LIST OF REFERENCES


Allen, F., & Morris, S. (2014). Game theory models in finance. In K. Chatterjee and W. Samuelson (eds.), *Game theory and business applications* (pp. 17–41). Springer, Boston, MA.

Anton, A. V. (2019). *Choice of discount rate and agency cost minimization in capital budgeting: analytical review and modeling approaches* (Doctoral dissertation, Victoria University). https://vuir.vu.edu.au/40447/1/ANTON%20Arun-thesis_nosignature.pdf

Araújo, M. (2015). Economics of telecommunications: Capital budgeting under competitive markets using strategic game theory and real options approach. University of Porto – School of Economics, Portugal. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.306.9046&rep=rep1&type=pdf

Ashton, R. H., & Ashton, A. H. (1995). *Judgment and decision-making research in accounting and auditing*. Cambridge University Press.

Bennion, E. G. (1956). Capital-budgeting and game-theory. *Harvard Business Review*, *34*(6), 115–123.

Boquist, J. A., Milbourn, T. T., & Thakor, A. V. (1998). How do you win the capital allocation game? *MIT Sloan Management Review*, *39*(2), 59. http://apps.olin.wustl.edu/faculty/milbourn/MilbournSloanMgtRev.pdf

Denault (2001, Jan.). Coherent allocation of risk capital. *Journal of Risk, 4*, 1–34. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.72.4317&rep=rep1&type=pdf

Ferreira, N., Kar, J., & Trigeorgis, L. (2009). Option games: The key to competing in capital-intensive industries. *Harvard Business Review, 87*(3), 101–107. https://hbr.org/2009/03/option-games-the-key-to-competing-in-capital-intensive-industries

Gibbons, R. S. (1992). *Game theory for applied economists*. Princeton University Press.

Grenadier, S.R. (2002). Option exercise games: An application to the equilibrium investment strategies of firms, *Review of Financial Studies 15*, 691–721

Huisman, K. J. M., Kort, P. M., Pawlina, G., & Thijssen, J. (2003). Strategic investment under uncertainty: Merging real options with game theory. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.545543

Jiang, Z., & Gong, X. (2019). Research on Issues of Budget Performance Management on the Process of Budgeting by Game Theory. *Journal of Financial Risk Management*, *8*(04), 193. https://www.scirp.org/pdf/jfrm_2019111915105540.pdf

Kengatharan, L. (2016). Capital budgeting theory and practice: a review and agenda for future research. *Research Journal of Finance and Accounting*, *7*(1). https://core.ac.uk/download/pdf/234631219.pdf

Li, Y. (2014). Value of more sophistication: capital investment decision-making with competitive dynamics in the mining industry. [MS thesis, MIT] https://www.semanticscholar.org/paper/Value-of-more-sophistication-%3A-capital-investment-Li/42dd25b0b1b3154baac5bc63c0deee90179ab143

Mun, J. (2020). Risk-based roi, capital budgeting, and portfolio optimization in the Department of Defense. *Defense Acquisition Research Journal*, *27*(91), 60–107. https://doi.org/10.22594/dau.19-829.27.01

Murto, P., & Keppo, J. (2002). A game model of irreversible investment under uncertainty. *International Game Theory Review*, 4(02), 127–140. http://lib.tkk.fi/Diss/2003/isbn9512264455/article3.pdf

Ng, K. S. (1968). *The relevance of game theory in its application to decision making in competitive business situations* (Doctoral dissertation, University of British Columbia). https://open.library.ubc.ca/soa/cIRcle/collections/ubctheses/831/items/1.0102410

Nichols, N. A. (1994). Scientific management at Merck: An interview with CFO Judy Lewent. *Harvard Business Review*, *72*(1), 88–99. https://hbr.org/1994/01/scientific-management-at-merck-an-interview-with-cfo-judy-lewent

Nobel Committee for Economic Sciences. (1996). The Sveriges Riksbank Prize in economic sciences in memory of Alfred Nobel 1996. Nobelprize.org. https://www.nobelprize.org/prizes/economic-sciences/1996/summary/

Nobel Committee for Economic Sciences. (2001). The Sveriges Riksbank Prize in economic sciences in memory of Alfred Nobel 2001. Press release. Nobelprize.org. https://www.nobelprize.org/prizes/economic-sciences/2001/press-release/

Pindyck, R. S., & Rubinfeld, D. L. (1995). *Microeconomics* (3rd ed.). Pearson Education

Royal Swedish Academy of Sciences. (2007). *Mechanism design theory*. https://www.nobelprize.org/uploads/2018/06/advanced-economicsciences2007.pdf

Shapiro, A. (2011). Budget negotiations: A study of game theory. https://www.npr.org/2011/04/06/135186332/budget-negotiations-a-study-of-game-theory

Sikalo, M., Arnaut-Berilo, A., & Zaimovic, A. (2022). Efficient asset allocation: Application of game theory-based model for superior performance. *International Journal of Financial Studies*, *10*(1), 20. https://www.mdpi.com/2227-7072/10/1/20

Siziba, S., & Hall, J. H. (2021). The evolution of the application of capital budgeting techniques in enterprises. https://repository.up.ac.za/bitstream/handle/2263/74586/Siziba_Evolution_2021.pdf?sequence=1&isAllowed=y

Smit, H.T.J. (2003). Winter. Infrastructure investment as a real options game: the case of european airport expansion. Finance. Manage, *32*(4), 27–57.

Thakor, A. V. (1991). Game theory in finance. *Financial Management*, 71–94. https://www.jstor.org/stable/3666098

Thijssen, J. J. J., Huisman, K. J. M., & Kort, P. M. (2004). The effect of information streams on capital budgeting decisions. *European Journal of Operational Research*, *157*(3), 759–774. https://pure.uvt.nl/ws/portalfiles/portal/608157/EJOR.pdf

Tian, G. (2013). Lecture notes on microeconomic theory. Texas A&M University. http://econweb.tamu.edu/tian/micro1.pdf

Verbeeten, F. H. (2006). Do organizations adopt sophisticated capital budgeting practices to deal with uncertainty in the investment decision?: A research note. *Management Accounting Research*, 17(1), 106–120.

Young, S. M. (1985). Participative budgeting: The effects of risk aversion and asymmetric information on Budgetary Slack. *Journal of Accounting Research*, *23*(2), 829–842. https://doi.org/10.2307/2490840

Zhang, P., Peeta, S., & Friesz, T. (2005). Dynamic game theoretic model of multi-layer infrastructure networks. *Networks and Spatial Economics*, *5*(2), 147–178.

Zhu, K., Weyant, J.P. (2003). Strategic decisions of new technology adoption under asymmetric information: A game-theoretic model. *Decision Sci*. *34*(4), 643–675.

THIS PAGE INTENTIONALLY LEFT BLANK

# INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California