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Multichannel Assignment in Resilient Network Topologies

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14. ABSTRACT
We study a wireless multichannel assignment problem across a variety of k-resilient distributed wireless network topologies. We study trade-offs between various network graph coloring schemes used as an approach to assign distributed wireless transmitter channels and we discuss assigning colors based upon additional radio coloring constraints. We also introduce a partial coloring algorithm to manage situations in which channel resources are constrained and more optimal coloring cannot be achieved. Using these new algorithms, we demonstrate both channel assignment stability and channel conflict improvements in a series of simulated mobile networks undergoing resilient topology control. Looking toward future work, we also discuss potential multichannel assignment designs and characteristics using a k-resilient connected dominating set (CDS) approach within the network.

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EXECUTIVE SUMMARY

Maturing wireless multichannel mechanisms for increased group communication effectiveness in mobile ad hoc networks will lead to important tactical network enhancements, including improved resiliency, increased capacity, and reduced delay in future wireless networks. This work presents a number of algorithm designs and simulation studies addressing the multichannel assignment problem in distributed, wireless ad hoc networks.

Since multichannel assignment characteristics and performance are related to topology and dynamic characteristics of a network, this report examines solutions alongside various classes of topological resilience and mobility. A variety of graph coloring problems are introduced as applied theoretical approaches to address the complex channel assignment problem. In this approach, the wireless network is modeled as a communication connectivity graph, but the connectivity graph is transformed to represent a radio conflict graph accounting for various transmission conflicts due to hidden terminals within the wireless network. Using the radio conflict graphs, exhaustive simulations are performed across a variety of network sizes and resiliency levels using a set of heuristic multichannel algorithms and the algorithm that iterates over all the nodes of a connectivity graph in "saturation order" (also known as "DSATUR") performs best in terms of limiting the maximum number of channels need to solve the complete radio graph coloring problem. However, existing heuristic coloring algorithms tend to only address network application cases in which a complete coloring solution is desired. To be more comprehensive, we address a solution when complete coloring cannot be achieved due to network channel resource constraints. A partial coloring algorithm is designed and simulated that constructs an imperfect but effective coloring solution when the distributed pool of total available transmission channels is limited. It is demonstrated that this approach manages network conflicts to a reasonable level when network resiliency or density is moderate.

We solve the additional problem of stabilizing channel reassignment in mobile wireless networks by extending our partial coloring algorithm to include a recoloring strategy to minimize temporal change or channel reassignment. We demonstrate how this significantly outperforms more naive, nonmobile-aware strategies. We conclude the report with additional design documentation and experiments that use a connected network backbone to manage channel assignments based upon connected dominating set concepts. We demonstrate that CDS-based approaches can reduce channel assignment requirements while effectively supporting resiliency and group communication exchanges. We are encouraged by the designs and findings of these early simulation experiments and we plan on refining initial designs further and performing detailed network traffic performance experiments in follow-on work.

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MULTICHANNEL ASSIGNMENT IN RESILIENT NETWORK TOPOLOGIES

1. OVERVIEW

1.1 Introduction

In this paper, we research an important wireless network resource optimization problem involving the assignment of orthogonal channels for transmission within a distributed wireless network undergoing topology control. Our use of the terminology ”channel” is fairly general in this report and can represent a frequency, time, or code-based wireless channel providing orthogonal network reception capabilities at multiple receivers with minimal co-interference. We will be examining the multichannel assignment problem (MAP) and considering its application to a series of wireless network classes of various densities resulting from k-resilient topology control strategies. As a strategy to solve the channel assignment problem in our distributed wireless network models, we framework the MAP assignment as a constraint-based graph coloring problem (GCP). The GCP is a generalization of the previously well-studied map coloring problem, but also includes the cases of coloring nonplanar graphs where edges between vertices may cross over each other due to nondirect edge interference and complex wireless propagation characteristics. To also address additional wireless constraints, such as the well-known hidden terminal problem and distributed wireless channel contention arising from network neighbors using shared medium channels, we primarily study the radio coloring problem that extends GCP assignment constraints beyond direct neighbor connections in the graph model.

We will be presenting simulation results of MAP challenges and performance trends as a wireless network increases its average density using a form of k-resilient topology control. The details of the transmission power control modeling cost constraints and the k-resilient topology control construction algorithm, kXTC, are covered by previous publications [1–3]. In previous work, we examined the performance characteristics of adaptive resilient topology control approaches using dynamic graph-based analytic models and network simulation/emulation using both the network simulator version 3 (ns3) and the Extendable Mobile Ad hoc Network Emulator (EMANE). Characteristics studied included the trade-offs between network traffic delivery effectiveness and different topological resiliency levels, but past work was limited to modeling networks using a single shared transmission channel, such as a carrier-sense multiple access collision avoidance (CSMA/CA) approach. It is known that a multiple-channel architecture (MCA) can further reduce the overall contention and can improve the achievable capacity for traffic services in a wireless network even when only single-but-orthogonal channel assignments are used per transmitter [4]. Performance improvements are partially due to the resulting reduction in shared channel traffic contention and the potential for spatial reuse in channel assignment across the network. Despite these earlier findings, effective distributed channel assignment remains nontrivial in practice and its application is worthy of further study. To our present knowledge, the optimization of channel assignment alongside k-resilient topology control, mobility, and limited channel availability has been minimally studied in past work.

1.2 Motivation

Network topology control designs often seek to minimize the required network connections, but in turn, minimization of connections leads to network fragility. Network topological resilience, achieved by adding additional network connections in a structured manner, aids in maintaining protection against:

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- network failures,
- network control errors, and
- network dynamics.

Achieving topology resilience in the overall network construction and dynamic maintenance is known to be nontrivial and there are associated open network technical challenges, such as:

- assigning a minimum transmission power per node,
- supporting efficient multicast or broadcast networking,
- minimizing traffic capacity degradation, and
- minimizing reassignment and collisions given network dynamics.

Applying multiple orthogonal communication channels within a distributed network using frequency or other dimensions (e.g., time, space) can improve the contention and capacity degradation resulting from performing topology control. With advancements in modern wireless front-end hardware and processing, achieving multiple simultaneous communication channel receive capabilities at a wireless network node is now easier and more affordable than in the past. This motivates us to take advantage of such multiple channel receive capabilities to further improve network resilience, contention reduction, and traffic effectiveness when combined with existing resilient topology control schemes. The envisioned outcome of this area of research includes improving future wireless network design and planning aids, as well as maturing new network adaptation algorithms to help combat performance risks and costs in future wireless network deployments.

1.3 Technical Approach

Overall, we will study a problem of optimally assigning a set of available p orthogonal channels to n transmission nodes within a distributed communication network. In this report, we examine optimal coloring characteristics across a series of analytic models of wireless networks adapted for minimum transmit power levels to achieve k resilience in terms of broadcast communications graph connectivity. In these early initial studies, we make the simplifying assumption that wireless receivers can receive and process multiple transmit channels simultaneously, supporting the ability to process simultaneous neighborhood multicast transmissions on multiple channels, and we do not examine related receiver scheduling constraints at this point in the study. We assume omnidirectional channels and limit the architecture to a single-transmission-channel-assignment-per-node approach that we term a single-transmit, multiple-receive (STMR) channel architecture. Future work is underway to extend this design to assign multiple transmit channels at nodes where possible to realize multiple-transmit, multiple-receive (MTMR) network channel architectures with the use of selective transmission directivity as well to enhance the overall wireless network architecture effectiveness.

1.4 Organization

The paper is organized as follows. First, we briefly review the related and past work on MAP solutions and we review the extensive previous work on GCP techniques. Next, we review the modeling and topology control approach used to generate the k-resilient topologies forming the basis for our MAP studies by providing a wide variety of topology types across different k values. We present and review results from simulations of channel assignment using a number of relevant graph coloring algorithms and extensions. Next, since in practice, network channel resources may be constrained, we develop and evaluate partial coloring solutions for cases in which optimal coloring or resource assignment cannot be achieved. We examine the performance trade-off between resulting average network conflicts and the partial coloring strategy. We take our partial coloring designs and extend the algorithm for increased stability in the dynamic network case. Using a variety of mobile network scenarios and constraint cases, we present simulations to study the resulting channel conflict percentage and stability of the recoloring process. Lastly, we present and examine additional design ideas and approaches to hierarchically layer the MAP solution within a wireless network. One such idea involves the construction of key connected relay network nodes to form a connected dominated set (CDS) within the k-resilient topology. We then perform GCP-based channel assignment on these key relay nodes, as they form a critical communication backbone for network routing, transporting both unicast and multicast data. After our backbone nodes are formulated and optimized, we next attach remaining topological leaf nodes to the communication graph, leading to a more simplified and hierarchical approach to channel scheduling at the edges of the network supporting improved network services for more disadvantaged nodes. In this sense, the key routing backbone nodes of the network are the first control-and-management priority in this design, thereby improving overall network capacity and resilience and potentially reducing the minimum number of overall channels needed. To conclude, we discuss areas of further research and summarize the main results and performance trends observed so far.

2. TECHNICAL BACKGROUND

Significant past work has been done in studying channel assignment problems in wireless networks [5–9]. One of our motivations for examining multichannel architectures is to increase capacity in distributed networks while at the same time controlling resiliency of connections. The potential increase in channel capacity achieved from multichannel architectures has been studied in past works, including [4]. In [4], the potential of multichannel capacity increases even with the use of a single interface was outlined. In earlier work [10], a simplified distributed network simulation demonstrated the potential for traffic capacity increases in a network when a multichannel architecture was used involving multiple simultaneous receive channels per node. Abstract simulations of small networks and traffic capacity in [10] also demonstrated the fact that understanding multichannel network capacity performance was complex and was dependent upon multiple variables, including the connectivity ratio, or density, of the communication network. Empirical data from these basic connectivity simulations demonstrated a decrease in overall MCA capacity gains as the network connectivity ratio decreased, but it also demonstrated that maximal achievable capacity gains could be approached using a smaller total number of distributed channel resources than in the highly connected topology case. Dynamic topology control helps to manage and adjust connectivity characteristics as the network operational conditions change, so in a broader sense, topology control is a key factor driving MAP requirements and achievable network gains. We are interested in exploring new MAP designs and performance characteristics in distributed wireless networks, and as mentioned, we will apply recent resilient topology control results from previous work to effectively manage and control network connectivity densities throughout our studies, including both static and mobile network cases.

Here, we briefly review k-resilient topology concepts before proceeding. At a basic level, k-resilient network topology control is analogous to forming a communication graph with a minimum edge connectivity number of k. The connectivity number $v(G)$ of a graph G is defined as the minimum size of the separating set. A graph is k-connected if $v(G) \leq k$ and k is the minimum number of vertices that must be removed in order to break a connected graph into two or more components. As a basic example, a fully connected network mesh is the maximally resilient network structure for an undirected graph and would have a $k = (n - 1)$, where n is the number of nodes in the network. In contrast, the minimum cost Steiner tree (MCST) that spans all network nodes provides an example of minimum cost and sparsity. Our research interest lies in between these two extremes and to control average connectivity levels within our networks, we will use k-resilient topology graphs resulting from an adaptive resiliency and multicast extension of the X Topology Control (XTC) [11] protocol algorithm described in [1, 3].

In our studies, we examine a range of network sizes and k resiliency values. Denser networks will reduce spatial reuse opportunities and in general will require additional channels to be assigned to achieve reduced network collision in the absence of link access scheduling. On the other hand, as a minimized connectivity topology, an MCST topology should require a smaller number of orthogonal channel assignments but will result in the most fragile network structure suffering from more restricted capacity limitations. Overall upper bound capacity gains achievable in multiple hop network topologies are complex to predict, but as a network topology's average connectivity decreases, or the diameter increases, performance bounds can be approached using a smaller number of global transmit channels in the distributed assignment solution. This trend can be illustrated partially by the fact that the minimum number of colors needed to optimally color sparser complex graphs decreases in general. As we will discuss, using a graph coloring approach as a MAP solution in distributed wireless networks involves additional constraints and is less understood than the simpler, yet still computationally complex, coloring problems in general graphs.

2.1 Basic Channel Assignment Problem for STMR

The single-transmit, multiple-receive (STMR) channel-assignment problem relates to the assignment of p individual minimally conflicting channels to a set of n transmit nodes within a distributed network. Throughout this paper, we make the simplifying assumption that receiver nodes within the network model can process and receive multiple wireless channels simultaneously. While not always the case, given advancements in modern wireless receiver hardware, we feel this is a reasonable assumption for many applications. Our future plans include exploring the hybrid dynamic scheduling problem when receiver channel constraints cannot be met. Given this model assumption, in the trivial case of the fully connected mesh network, we would need to assign $p = n$, one orthogonal channel for each transmitter, to avoid the need for additional channel sharing and scheduling considerations across the network transmit group. The minimum channel-assignment requirement as the network connectivity becomes more distributed is nontrivial and network channel-assignment problems can often be viewed as analogous to a form of a graph-coloring problem. In the next section, we review of general graph coloring and explore how past results relate to wireless network MAP challenges.

2.2 Graph Coloring Complexity and Constraints

In its simplest form, graph coloring is way to assign labels (e.g., colors) to vertices or edges of a graph so that no two adjacent vertices or edges in the graph have the same label; when vertices are used for coloring, this is called vertex coloring. It is known to be NP-complete to decide if a given graph admits a j-coloring for a given j colors except for the trivial cases $j \in \{0, 1, 2\}$. Given a graph G, $\chi(G)$, the graph's chromatic number represents the minimum number of colors required for a complete coloring solution. Computing

$\chi(G)$ is known to be NP-hard for nontrivial graphs such as those we examine in this study [12]. Due to its importance and computational complexity for optimality, there has been a plethora of past work on developing and studying heuristic approaches to graph coloring. Prior to presenting our own algorithm designs and extensions for unique cases, we first conduct an evaluation of existing coloring approaches in our studies across the variety of topology types of interest. As discussed earlier, we will need to address technical challenges that go beyond more typical past general graph coloring studies, such as modeling contention and collision issues in wireless access layers that transcend basic bidirectional local connectivity models. Also, we will need to address partial coloring when the number of available p channels ¡ $\chi(G)$ and also consider the case of graph dynamics and mobility in realistic wireless communication networks. To our present knowledge, these additional challenges have been minimally explored in previous work.

To begin examining the STMR channel assignment problem in different topologies, we model a series of candidate wireless networks constructed with a topology algorithm as a graph model. Each graph G is represented by $G(V,E,w)$ where V represent the set of vertices, or wireless nodes, E represents the potential reliable communication links to direct graph neighbors established by the maximum power each V will transmit, and w represents a set of costs associated with each edge, if desired. Our link-cost models for kXTC construction can include signal-to-noise (SNR) threshold calculations for reliable link establishment given a particular model or waveform. If desired, our topology control cost models can include complex propagation models (e.g., terrain) and external network interference constraints that directly lead to nonplanar graph construction, where connectivity probability is not directly distance correlated as in frequently used unit disk graph models. This is important to note because many past theoretical proofs and performance studies of graph coloring are often limited to planar graph models.

As mentioned, classic graph-coloring heuristic methods previously studied and developed focus on direct-neighbor labeling constraints. In STMR radio network modes, the hidden terminal problem as shown in Fig 1 results from the fact that transmitters, nodes A and B, are out of reliable range of each other, but still collide when transmitting to receiver C, which is a direct neighbor of both transmitters. To address this issue, we extend our studies beyond the classic graph-coloring problem to what is termed a radio-coloring problem.

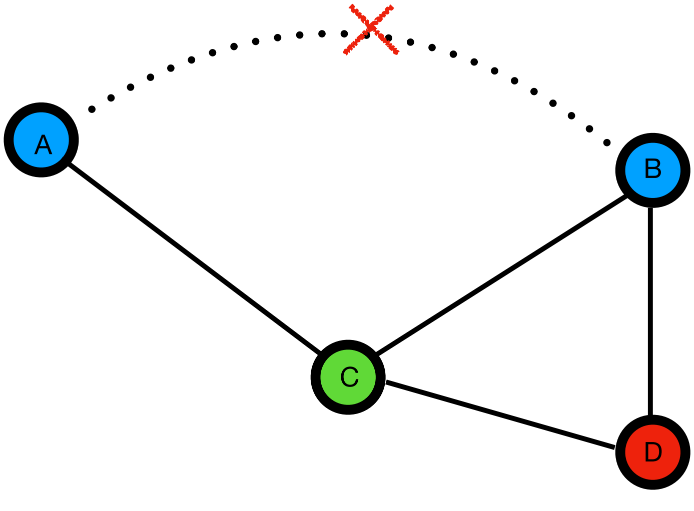
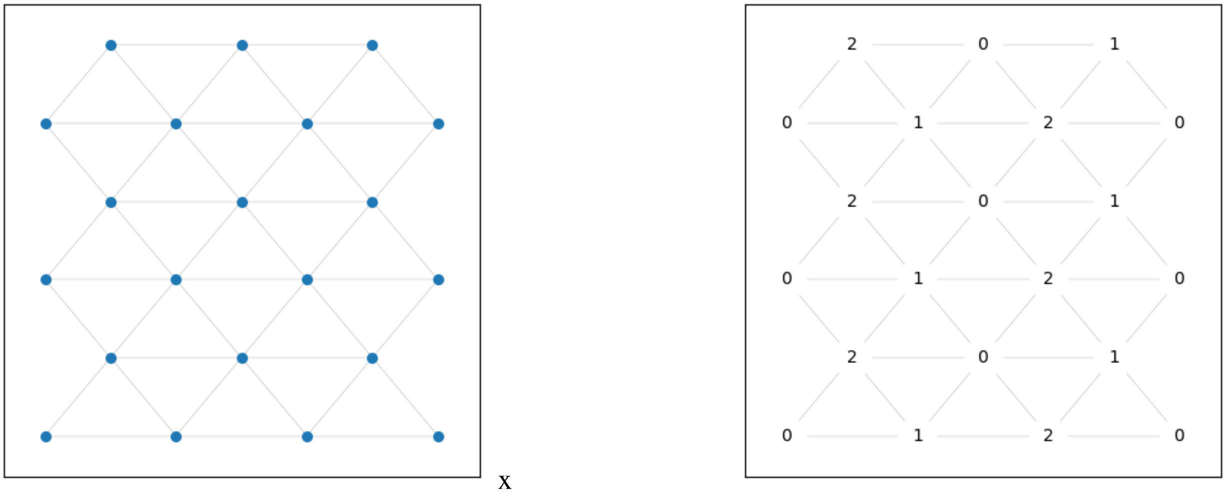


Fig. 1—Radio coloring constraint

To illustrate this point and to present the radio-coloring model, we use the following canonical example of a simple triangular lattice graph.

In Fig. 2(a), we use a canonical $5x5$ triangular lattice graph to illustrate in a basic manner the radio-coloring problem and to demonstrate steps to modify traditional graph-coloring solutions to address a simplified conflict model addresssing radio coloring. The outcome from using a classic coloring algorithm to solve the triangular lattice example is presented in Fig. 2(b), and we can see that we find a solution using only three distinct global colors, but many node assignments have common neighbors with the graph that would result in significant transmission collision and channel contention without additional dynamic traffic scheduling and link management.



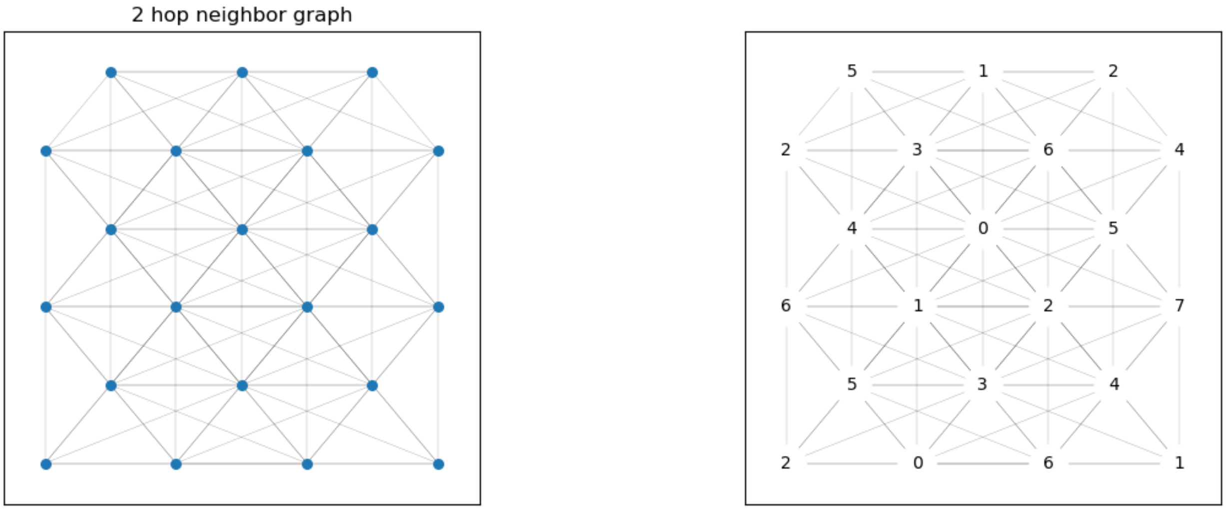
(a) Lattice graph

(b) Basic colored graph

Fig. 2—Basic coloring: lattice graph example

To find a more effective wireless network coloring solution and reduce the likelihood of common but hidden transmit neighbor collisions, we transform the graph in Fig. 2(a) to include two-hop edge conflicts as shown in Fig. 3(a). Reapplying the coloring algorithm to account for two-hop labeling conflicts and then applying those label assignments to the original graph results in Fig. 3(b). We now have a more robust channel assignment to reduce reception collision, but there is the obvious cost of additional channels needed for optimal coloring. In the simple 21-node lattice network case, we now need seven total channels vs. three channels in the original coloring solution.

Prior to presenting simulation results across a variety of algorithms and network types, we demonstrate radio-coloring results for a k=2 resilient topology network using a 25-node radio constraint-based simulation in Fig. 4. We see in this more complex topology that five channels are sufficient for full classical coloring and that eight channels were needed to achieve full radio coloring.



(a) Radio constraint graph

(b) Radio colored graph

Fig. 3—Radio coloring: lattice graph example

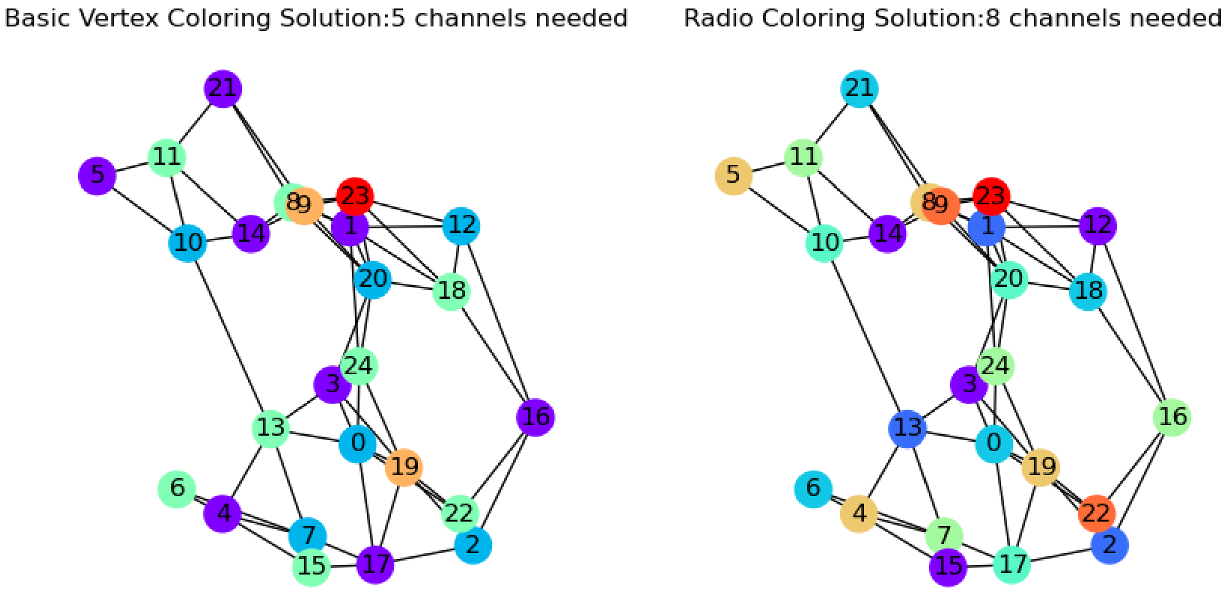


Fig. 4—k=2 resilient topology graph

3. NETWORK MODELS AND INITIAL SIMULATION STUDIES

3.1 Resilient Topology Modeling

As introduced earlier, we will be examining a variety of k-resilient network topologies throughout our simulations. We refer to k-resilient topologies as analogous to the definition of the connectivity number of a network graph. To review, the basic kXTC algorithm used in our simulations derives from the original XTC design ideas presented in [11] but extends this to k-resiliency as described in [3]. In order to evaluate topology coloring across a variety of lower bounded average densities, we examine communication graph models starting from the MCST (providing low connectivity resilience) up through a relatively moderate resilience (i.e., $k = 4$). Also, we reiterate that our variant of kXTC topology control is extended to support efficient multicast reception and group data exchanges within the constructed wireless network. It achieves this by establishing a minimum transmit power per node that reliably reaches the entire subgraph of directly connected topology neighbors for each transmitter node. This is different from topology control that establishes a minimum transmit power per a single receiver and uses a least-cost shortest-path routing model. See [3] for more details on kXTC. Since many collaborative tactical networks may be moderately sized networks, we also chose to examine distributed networks in a node population range of 10–100.

3.2 Radio-Coloring Algorithm Analysis

As mentioned previously, the computational complexity of graph coloring often necessitates the use of heuristic algorithms to compute complex graph solutions. Therefore, we begin our analyses by examining a set of well-known coloring algorithms implemented within the openly available NetworkX Python software library [13, 14] and measure the contrasting statistical behavior in achieving coloring solutions across networks of varying resiliency. Figure 5 presents the results. The summary barplots illustrate the minimum channel assignment averages and distributions required to achieve fully colored topologies. The known heuristic coloring algorithms we examined are broken out along the x-axis of Fig. 5 and are enumerated as follows:

0. Largest First
1. Random Sequential
2. Smallest Last
3. Independent Set
4. Saturation Largest First (DSatur)

The aggregate experimental results presented in Fig. 5 include 50 networks per trial generated from randomized locations within a defined geospatial area. We also generate five different network sizes, $n \in \{10, 20, 30, 40, 100\}$, and for each network experiment trial, we include three levels of k-resilient topologies, $k \in \{1, 2, 4\}$, constructed from the kXTC algorithm. For purposes of initial trend analysis, we use simple free-space loss as the chosen propagation-loss model. We also include additional experimental data for constructed MCST topologies providing a comparison of the likely lower bounds for extremely sparse, minimally resilient network coloring. This results in a total of 1,000 network topologies and 5,000 coloring solution experiments across the entire set. The results here represent the radio-broadcast-conflict coloring scheme throughout as previously discussed. We notice that the Saturation Largest First, or DSatur, algorithm performed well in terms of reducing the required minimum number of channels across the wide variety of topologies tested, and we now limit further experimental comparison results to DSatur-based radio-coloring schemes for the rest of the paper.

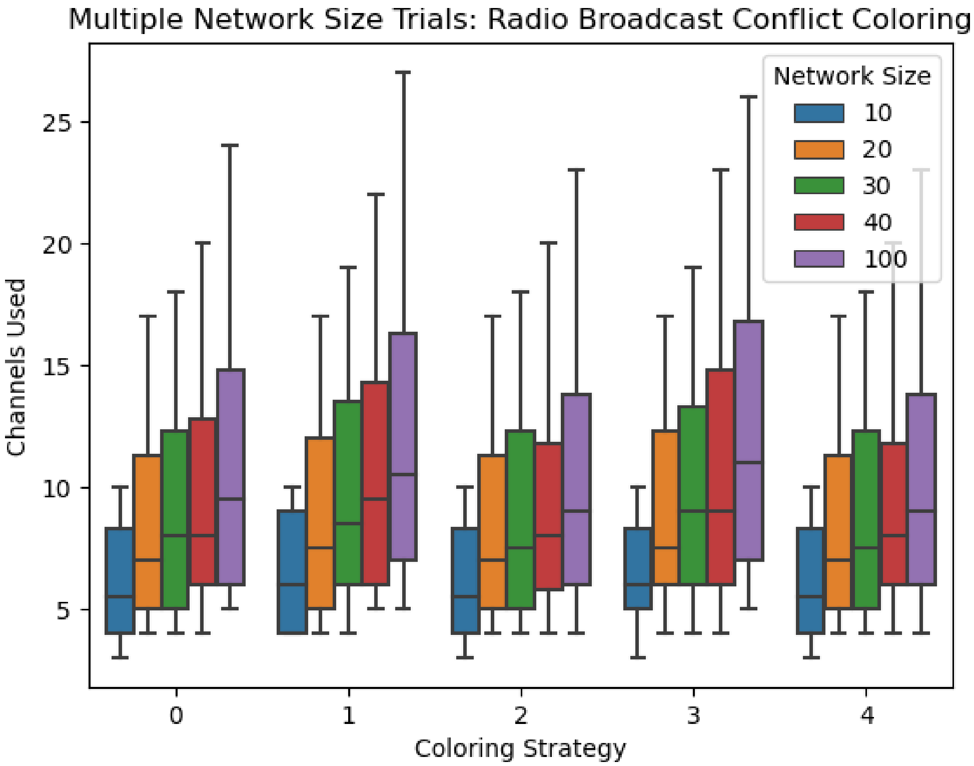


Fig. 5—Size-vs.-channels coloring algorithm comparison

3.3 DSatur Radio Coloring Trend Analysis

While Fig. 5 presents useful insight in pointing out relative radio-coloring algorithm performance metrics across a large sample of network topology classes, we are specifically interested in the coloring performance trends as they relate to specific resilient network topology classes and network sizes. Figure 6 shows our experimental results for DSatur-based radio-coloring solutions across a similar set of 1,000 generated experimental network topologies, but here, we plot the specific minimum colors needed to color each topology class as the networks grow in size. Because of the lower-transmit-power profile and network sparsity resulting in the case of $k = 1$, we can see that the maximum number of colors needed remains relatively low even as the network grows in size. The $k = 2$ results are similar to $k = 1$, with a few more channels generally needed for all network sizes. This provides good insight into potential practical applications, since in previous work, we have shown that $k = 2$ provides significant resiliency and traffic-capacity improvements and it also appears that radio coloring can be achieved with a relatively small number of overall orthogonal channels in this case.

As further discussion of the radio-coloring complexity, we point out that in general, coloring planar topology graphs can use far fewer colors. The well-known proof for the 4-color theorem [15] states that all loopless planar graphs can be colored with four or fewer colors. If more colors are needed, the graph being colored is generally not planar in nature. As mentioned earlier, there are two reasons why coloring distributed wireless networks is additionally complex and may be distinct from the planar graph-coloring problem. The first reason is that our constraint-based kXTC algorithm supports generic communication link-cost constraints

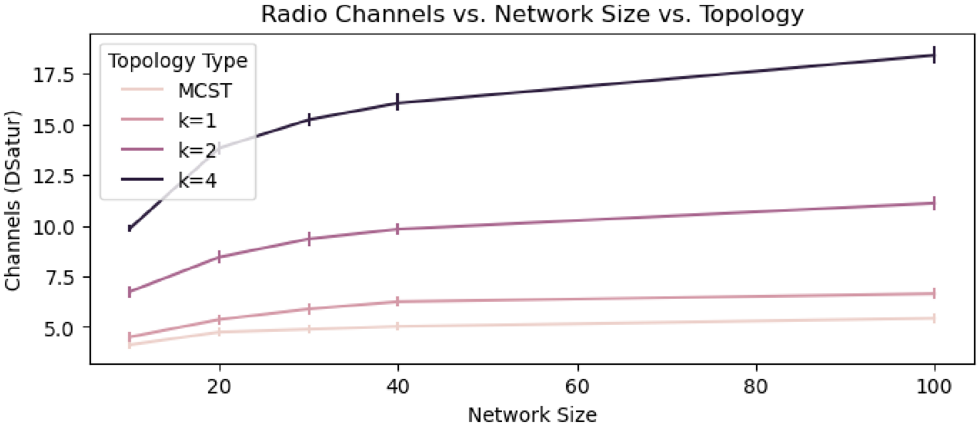


Fig. 6—Simulation comparison of network size and k

that are not necessarily distance correlated, leading to minimum k-resilient topology graphs that are often nonplanar in nature. The second reason is that the radio constraint-based coloring transformation of the original connectivity graph, even if planar, produces nonplanar graph connections between vertices.

4. PARTIAL AND MOBILE NETWORK COLORING SOLUTIONS

4.1 Partial Channel Assignment Problem

The experiments in previous sections explore the channel-assignment characteristics needed to properly satisfy the radio-constraint coloring problem. However, in practice, we are also interested in the problem of coloring when the number of available channel resources is less than the chromatic number, $\chi(G)$, of the graph. Previous work has addressed algorithms to solve distance two edge coloring (D2EC) on a graph G with a k channel constraint (D2EC(G,k)) on disk graphs [15]; however, we attempt to solve this as a vertex-coloring problem on nontrivial graphs. To address this problem, we modify the commonly used greedy coloring technique to develop a heuristic, greedy algorithm. The algorithm makes a greedy choice to select a node after sorting the graph vertices based on their degree. Second-order sorting is based upon node ID, which is naturally occurring due to the stability of the sort. The selected node colors itself with the first instance of least-used channel amongst its neighbors and the next greedy choice is made. What differentiates this algorithm from traditional coloring algorithms is its ability to only use a certain number of channels, passed in by a parameter. As far as we are aware, this is the first work of its kind to look at partial vertex coloring in terms of the radio-conflict coloring problem. However, a caveat to this methodology is that it is not distance discriminative. Both distance one and distance two neighbors are treated equally. While this present design works well for our abstracted transmission conflict model, there are further improvements that can be designed. One such approach is a distance-discriminative algorithm exists that we plan to analyze in future work.

As in our previous studies, we generate a significant number of trial test cases, but we now limit the number of channels available for assignment when coloring the networks. In order to evaluate the performance of this algorithm, we develop a performance metric based upon an average transmitter conflict percentage in the radio graph resulting from the partially colored network solution. We calculate the transmission conflict percentage by counting every edge in the colored radio graph where each of the connected vertices is the same color and then we average this number out by dividing by the total number of edges in the radio graph. Both immediate-neighbor and 2-hop edges are counted when measuring conflict. Figure 7 shows this metric applied over various k-resilient graphs with an example five-channel constraint.

$$Conflict(\%) = \frac{\sum\{(v1, v2) \in E : v1_{color} = v2_{color}\}}{\sum\{(v1, v2) \in E\}} * 100$$

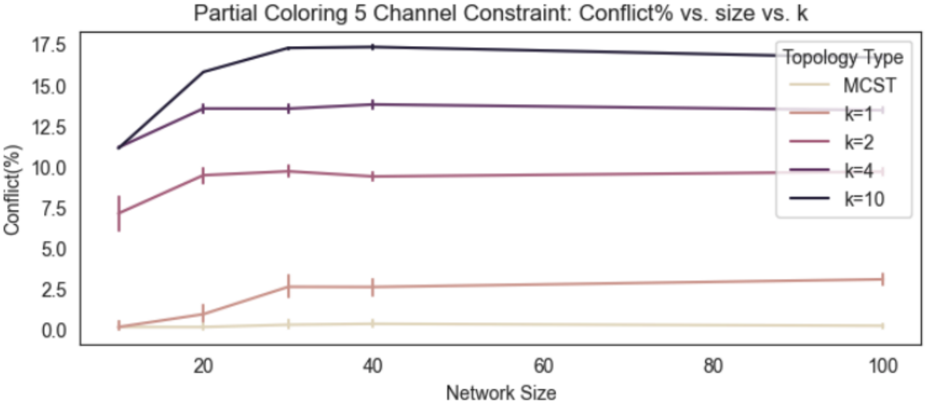


Fig. 7—Partial coloring trials

4.2 Dynamic Partial Coloring Design and Analysis

To add additional application to our work, we analyze and design an approach to partial color wireless networks as they change over time. A main challenge is the ability both to minimize the amount of churn involved in channel reassignment and to reduce the amount of temporal conflict due to partial coloring constraints. Previous related work details agent algorithms [16] designed to color dynamic networks as edges are added and removed. Overall, this previous work demonstrated that known static coloring approaches and ant-based agent algorithms were not effective in solving dynamic coloring problems in terms of the number of colors needed and the required reassignments. We expand upon this area of work by designing and exploring further the partial coloring problem in dynamic topology environments. In our testing model, we generated our dynamic graph using the bonnmotion generator tool and its random waypoint models to establish dynamic movement scenarios and, given these temporal location changes, we periodically update kXTC calculations to generate and maintain dynamic network topologies at a given k value over time [17]. In our presented scenarios, initial position vectors were randomized with a maximum speed of 5 m/sec.

To address the technical challenge of dynamic coloring, which is determined to be NP-hard, we have developed a greedy-based algorithm incorporating knowledge from the previous state of channel assignment to help minimize not just the colors assigned, but to assist in reducing the percentage of channel changes required. This candidate algorithm will be referred to as Least Change (LC) coloring. LC coloring works by comparing the network adjacency matrices between update periods. For each network node, the previous adjacency results are saved and the difference in neighborhood vertices compared with the previous adjacency is sorted in ascending order. The idea behind this heuristic approach is that nodes with the least amount of change in neighbor connectivity are colored first. When it is determined that a node cannot use the current color and must be assigned a new one, it selects a color from available colors that minimally conflicts with its presently assigned neighbors using our previous static partial coloring solution. During testing, we compare this dynamic algorithm to successive iterations of the more dynamically naive static partial coloring algorithm (PC) without temporal adjacency information from the previous section.

The adjacency matrix of the radio graph transformation at time t = 1 is represented as

$$A_{t=1} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix}.$$

The coloring assignment at t = 1 is represented as

$$C_{t=1} = \begin{bmatrix} c_1 & c_2 & \ldots & c_n \end{bmatrix}.$$

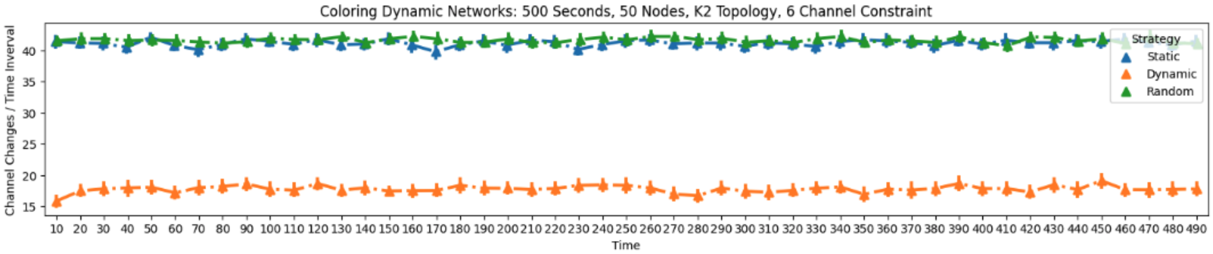
Here, $A_{t=1}$ is the adjacency matrix, and $a_{i,j}$ represents the connection at time t between vertex i and vertex j in the graph. Typically, $a_{i,j}$ is 1 if there is an edge between vertex i and vertex j and 0 otherwise, although weighted edges could be represented as well.

If the adjacency matrix of the graph at time, t = 2, is $A_{t=2}$, we calculate a change, d_i, in the radio graph conflict neighbors for $node_i$ using A_c as follows:

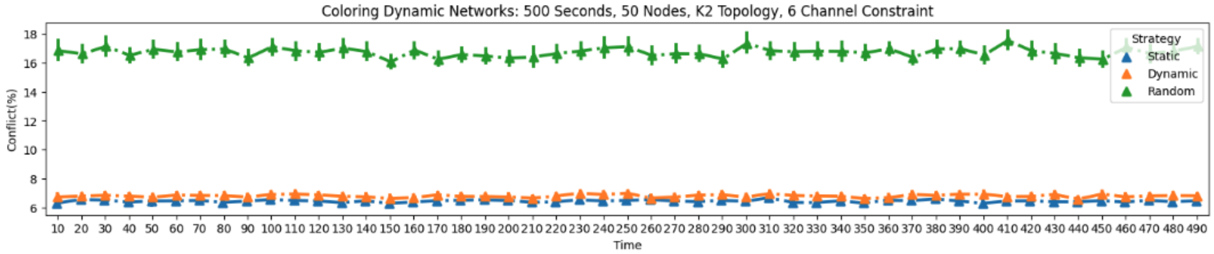
$$A_c = |(A_{t=2}) - (A_{t=1})|$$
$$d_i = \sum_{j=1}^{n} a_{i,j}.$$

The coloring of nodes is sorted in ascending order according to the values of d_i so that nodes with the least amount of change are colored first. If the previous node coloring can be maintained, the assignment remains and the algorithm proceeds to the next node in ascending order.

In the following example of our simulated experiments, we observe a 50-node $k = 2$ graph over a duration of 500 seconds with a maximum channel usage of 6 as shown in Fig. 8. In addition to our two algorithms, we also present a random algorithm that assigns a random transmit channel within the assigned constraint to each node. We use this as an upper bound to prove the effectiveness of our algorithms.



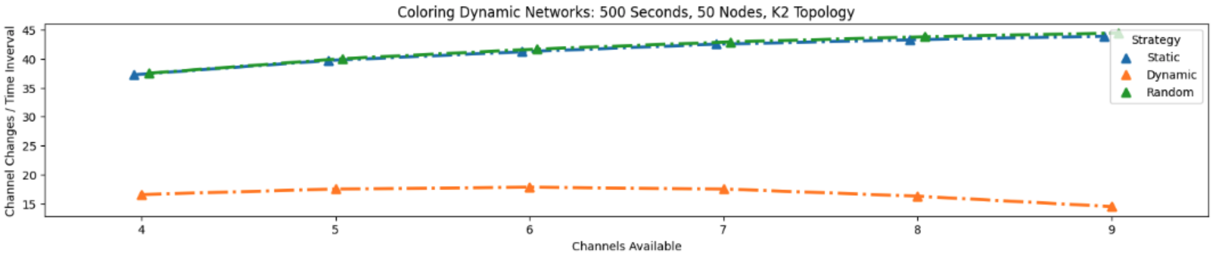
(a) Coloring changes



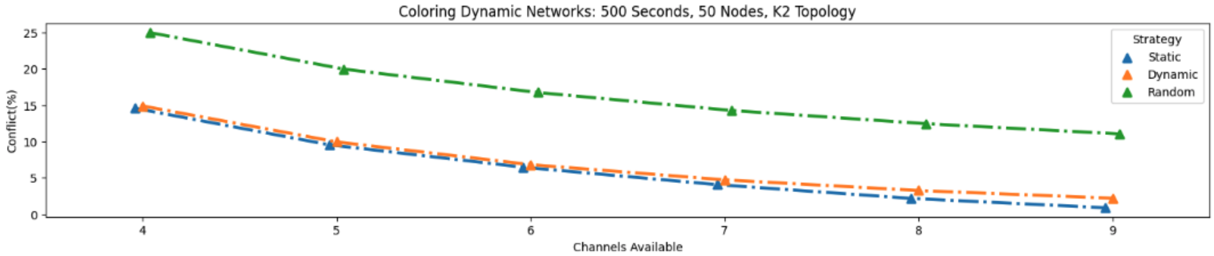
(b) Conflict changes

Fig. 8—Mobile network results 6 channels

We determine that there is little change in the behavior of the algorithms over time, so we display aggregate results of 50 mobility trials for each constraint setting for $k = 2$ in Fig. 9. These results show the algorithmic behavior in correlation with channel constraints in the range of $\{4–9\}$. Across our experiments, we observe that the LC coloring algorithm drastically reduces the total number of channel changes required between successive dynamic time intervals. When channel resources available approach $\chi(G)$, we observe that LC exhibits slightly poorer comparative conflict management, but significant channel change improvements remain against the naive static assignment case.



(a) Coloring changes vs. channels



(b) Conflict changes vs. channels

Fig. 9—Mobile network aggregate results

To examine behavior across topology classes, we further simulated k-resilient topologies with $k \in \{1, 2, 4, 6, 10\}$ across 50 mobility trials with a channel constraint of 5 and we present the aggregate results in Fig. 10. Similarly to our previous experiment, we observe that LC performs more competitively to PC in more challenging channel assignment scenarios. Figure 10 shows LC exhibiting superior conflict management at $k > 2$ but poorer comparative performance at $k \leq 2$. We conclude that basic LC style dynamic coloring demonstrates significant temporal channel reassignment reduction while managing resulting transmission conflict percentages across the network.

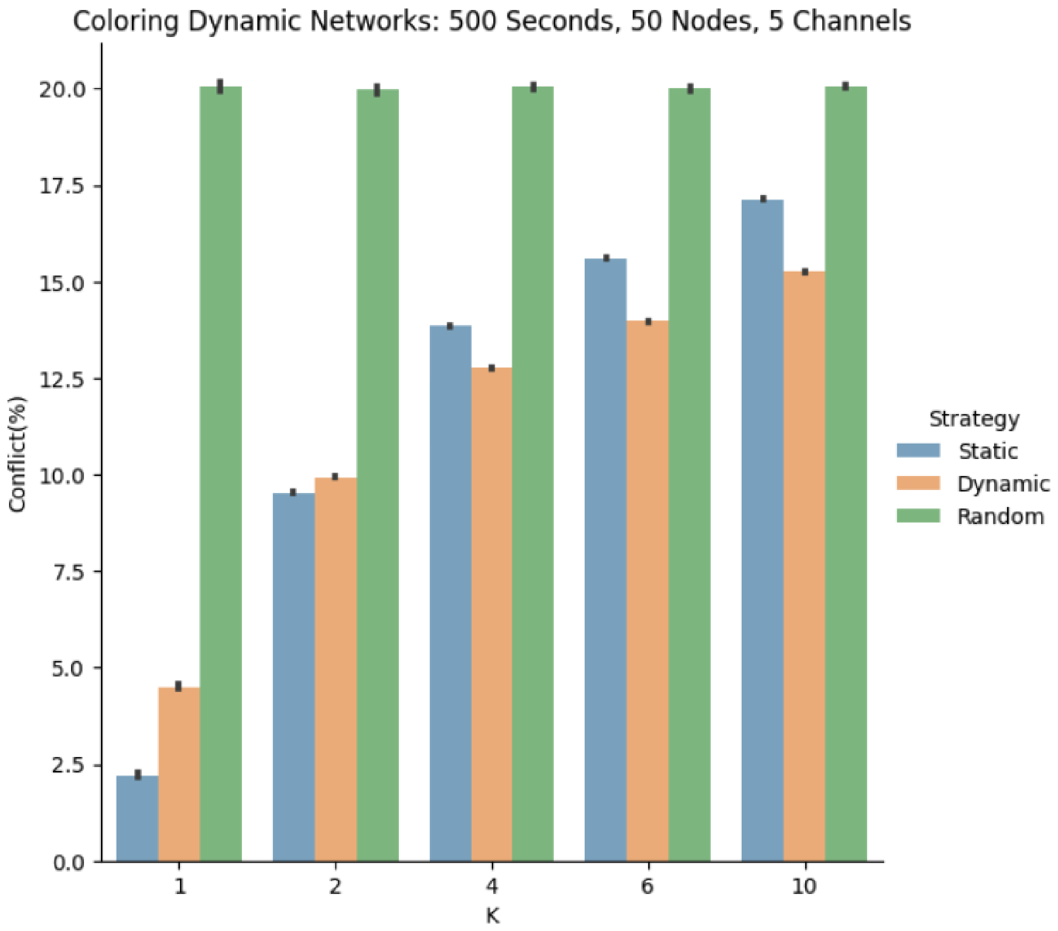


Fig. 10—Mobile network conflict vs. multiple *k*

5. CONNECTED DOMINATING SET (CDS) APPROACH AND EXPERIMENTS

As average network connectivity and size increases, the minimum number of channels required to radio color the network will generally increase. Connected dominating set (CDS) approaches have traditionally been applied in mobile ad hoc network (MANET) networking to reduce topology complexity and to improve network-wide control messages and routing processes [18–20]. We propose that dividing up the MAP or coloring problem to first address a subset of resiliently connected CDS nodes may yield multiple network-performance and design benefits, including fewer required channels, higher network backbone capacity, and supporting disadvantaged tactical edge nodes more effectively by the use of mobile-network-management hierarchy.

To review, A CDS is a set of network nodes forming a connected backbone where every non-CDS network node is a direct neighbor of a CDS node and the set of CDS nodes forms a strongly connected subgraph amongst themselves. The calculation of an optimal CDS within a given graph is a known NP-complete problem, but several heuristic approaches have been adopted in practice, including work by the authors [18, 19]. Many MANET approaches to CDS maintenance support dynamic distributed elections using periodic 2-hop network neighborhood knowledge updates. We hypothesize that as a network's connectivity or density increases, a

CDS-centric coloring approach to channel assignment and management should reduce the overall minimum channels needed and could support a more resilient backbone within heterogeneous node populations.

Figure 11 illustrates an example of topology control and the resulting link cost function to operate the network. In the flat network case, we generate a $k = 2$ resilient topology for the entire network and display the communication edge cost function distribution. To illustrate a CDS-pruned case, we generate a $k = 2$ resilient topology, perform a CDS election process on this topology, and reattach edge leaf nodes to the lowest-cost CDS communication edge available, resulting in a network that has lower overall communication cost but retains a core backbone with some resilient connections between backbone nodes. The extraction of the CDS does not guarantee complete $k = 2$ connectivity resiliency in the resultant CDS subgraph because some of the edge leaf node connections may provide key redundant connectivity that has now been extracted in the CDS creation process. We solve that issue by regenerating any needed k-resilient low-cost edges only between CDS nodes from the original topology cost matrix formulation.

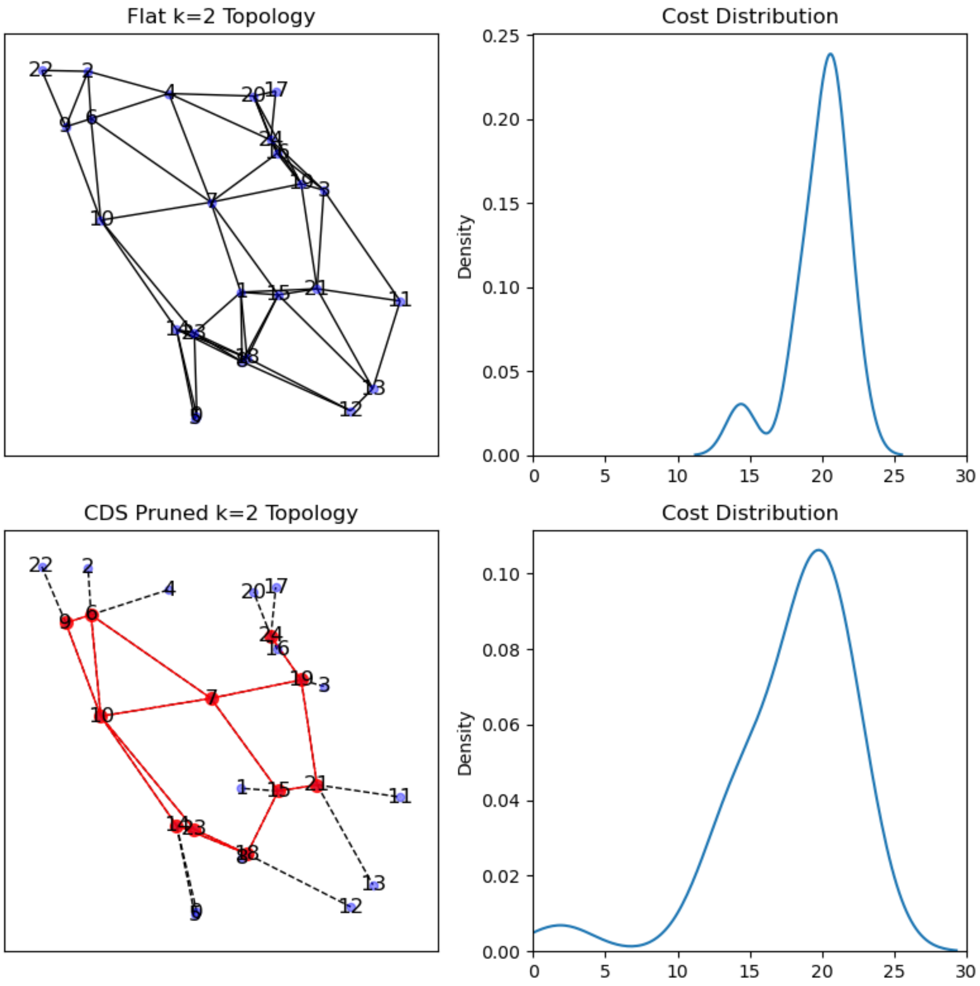


Fig. 11—CDS pruning example

Figure 12 summarizes the results of CDS pruning and coloring experiments for a variety of k-resiliency values and network sizes. Overall, the general coloring trends are similar to those in the non-CDS case. Of more interest is comparing CDS-pruned coloring characteristics to flat coloring. As an example, we take the $k = 2$ topology resilient case from Fig. 12 and additionally plot the flat radio-coloring results as well as CDS subgraph coloring and present the average coloring results in Fig. 13. As expected, we can see even for a relatively sparse $k = 2$ topology, there is a significant reduction in coloring requirement for CDS subgraph coloring.

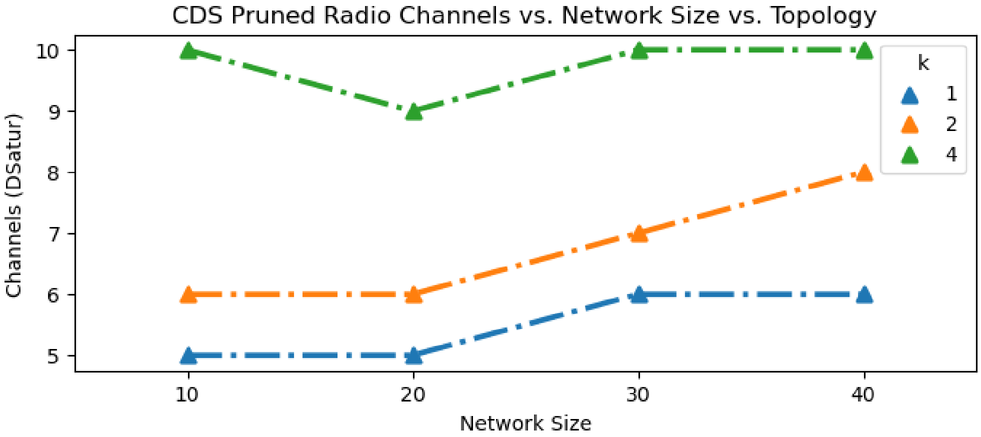


Fig. 12—CDS pruning experiment

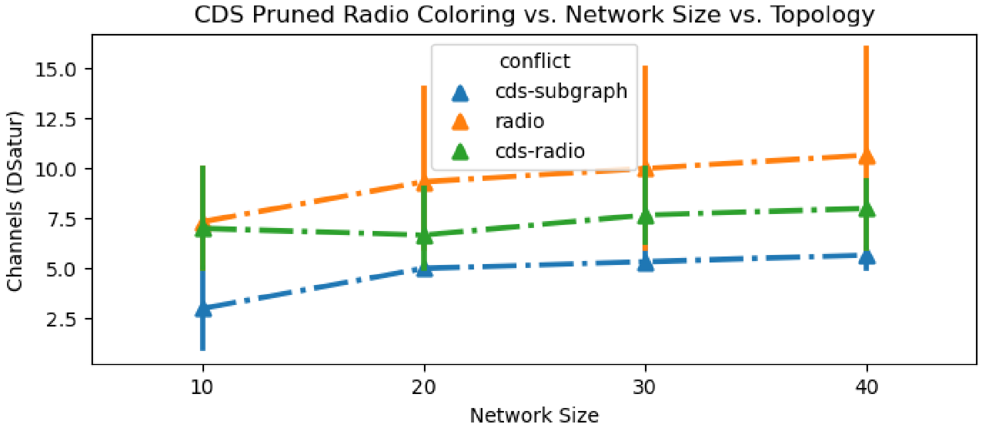


Fig. 13—CDS pruning trials

These CDS-based coloring results and related pruning designs are preliminary, but they are encouraging in the practical sense that channel assignment constraints are reduced when the network is organized hierarchically using a connected backbone. Organizing a resilient, multichannel CDS network backbone to better support single-channel disadvantaged edge nodes may be a practical architecture approach, and we plan to study this approach further and to integrate it with partial and dynamic coloring approaches presented earlier.

6. CONCLUDING REMARKS

6.1 Planned Future Work

Our future planned work includes migrating the models and algorithms developed here to working protocols and testing them within more detailed simulation and emulation environments to measure the channel-contention and traffic-load capacity impacts for both fixed and mobile network scenarios. A key goal of this future work is to gain a better understanding of network-load capacity and traffic-delay impacts of topology control, resiliency, and multichannel usage.

We are encouraged by our initial partial coloring and mobile partial coloring designs and results, but challenges and work remain in achieving channel assignments along with joint scheduling optimization when network conditions are such that $channels < \chi(G)$. Initial partial coloring and stability results under mobile network conditions are encouraging, but there are further research-and-design challenges to be examined in this area.

We also plan to extend work in multichannel optimization and scheduling to include the use of transmission directivity within the network to achieve greater range at lower cost.

6.2 Summary of Results

This report has studied the problem of effectively assigning a set of available p orthogonal channels to n transmission nodes within a distributed communication network with various levels of resilient topology. We first presented related and past work on MAP solutions and reviewed the extensive previous work on graph-coloring techniques. We discussed the additional constraints that must be considered in a wireless network channel assignment solution vs. classical coloring that examine only single-hop vertices. A radio-conflict graph transformation was introduced to improve the coloring solution in handling wireless hidden terminals and distributed contention conditions. Using the radio-conflict graph models, we simulated channel assignment results using a number of relevant graph-coloring algorithms and extensions across a large set of k-resilient topologies to observe trends regarding the number of channels required for fully coloring the representative radio graph. The DSatur algorithm was observed to have good minimization properties across a large set of case scenarios.

In practice, available channels are often less than $\chi(G)$, so we further developed and evaluated partial coloring solutions for cases in which optimal coloring or resource assignment cannot be achieved. We examined the effectiveness of our partial coloring solution by measuring the resulting contention conflicts that arise in the radio graph for a variety of limited channel use cases. We developed and used a conflict percentage metric to examine the performance trade-offs between resulting average network conflicts and partial coloring strategies. We demonstrated that we could manage the conflict percentage to reasonably low levels (e.g., channel max = 5) when the network resiliency is small, thus enabling engineering trade-offs between channel resource constraints and radio contention. Further, we built upon our partial coloring designs and extended the algorithms for increased stability in the dynamic network case. Using a variety of mobile network scenarios and constraint cases, simulation results demonstrated the ability of our partial coloring mobility algorithms to decrease the average channel reassignment rate while managing the resulting average channel conflicts across a large set of mobility trials, various channel constraint settings, and multiple resilient topology classes.

Lastly, we presented and examined additional design ideas and approaches to hierarchically layer the MAP solution using a connected dominated set (CDS) within the k-resilient topology. We performed GCP-based channel assignment on these key relay nodes forming a critical communication backbone for network routing, transporting both unicast and multicast data. After our backbone nodes are formulated and optimized, we attach remaining topological leaf nodes to the communication graph, leading to a more simplified and hierarchical approach to channel scheduling at the edges of the network, supporting improved network services for more disadvantaged nodes. In this sense, the key routing backbone nodes of the network are the first control-and-management priority in this hybrid design, thereby improving network overall capacity and resilience and potentially reducing the minimum number of overall channels needed. To conclude, we discussed areas of further research and summarized the main results and performance trends observed so far.

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