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RPPR Final Report

as of 23-May-2023

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Title: Modeling the Mechanics of Multiaxial Ratcheting

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Major Goals: Ratcheting is the accumulation of plastic strain due to cyclic loading. This phenomenon is known to reduce the life and lead to unexpected failure of piping components in nuclear power and chemical plants, offshore structures, machinery components, buildings, bridges, and any other structure subject to earthquakes, extreme weather, and/or cyclic mechanical and thermal service conditions. Predictions of ratcheting strains are needed to avoid, foresee, and understand structural failure. The largest challenge associated with finding a realistic constitutive model to accurately predict ratcheting is that any acceptable small error in a single cycle, accumulates over many cycles to an unacceptable level, so it is imperative that the material response is described with a high degree of accuracy. Multiaxial ratcheting has proven to be particularly difficult to predict because in addition to the norm, the direction in strain space towards which the plastic strain tensor accumulates may be unknown and yet it must be predicted with high accuracy in order to capture the overall deformation of the structure and assess its integrity. Our research objective is to use the principles of continuum mechanics and thermodynamics to investigate the mechanics of ratcheting and derive a rigorous, simple, and accurate constitutive model for uniaxial and multiaxial ratcheting that is applicable to a wide array of ductile metals. In other words, we aim to answer the scientific questions: Under repetitive plastic loading paths, what are the changes in the material state at the continuum level and how can these changes of the material state be modeled by means of a constitutive model in order to allow successful simulation of ratcheting?

Accomplishments: On the constitutive modeling side, we have been able to considerably improve uniaxial and multiaxial ratcheting simulations, vis-à-vis past efforts, with a new model that uses the Bounding Surface (BS) plasticity framework for metals. BS theory was enhanced by the "BCD" modification (named after Burlet/Cailletaud/Delobelle, the researchers who first proposed this) in a model that we are calling SANIMETAL-BCD, for Simple ANIsotropic METAL with BCD modification. The BS framework allows for a flexible and variable-direction Kinematic Hardening (KH) rule for the Yield Surface (YS) in reference to the BS, without affecting the stress-distance dependent value of the plastic modulus, with the added flexibility of a choice for the KH for the BS independent from the KH of the YS, that is defined relatively to the KH of the BS no matter what the KH of the latter is. This decoupling of modulus and KH direction is a great generic advantage of the BS models that allows successful simulation of Uniaxial Ratcheting (UR) and Multiaxial Ratcheting (MR) with the same model constants. The success for multiaxial ratcheting is mainly due to the proper variation of the flow rule intimately related to choices of the KH for the two surfaces. In the process, it was important to replace the fixed factor of the original BCD modification with a variable factor expressed by an extremely simple function of a measure that reflects the relative "position" of the back stress with respect to its varying saturation/bounding value on the BS, expressed by a proper ratio. This simple change to the BCD modification together with the key choice of a specific KH for the BS allowed the successful simulation of a plethora of UR and MR data under various simple and complex loading

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conditions. These simulations are significantly more accurate than those existing in the literature by other constitutive models that are more complex, in general.

On the experimental side, we have been working with our Czech collaborators to characterize the yield surface using automatic procedures. Thus far, we have been able to characterize the initial and evolved yield surfaces of P250GH N grade boiler steel and yield surfaces after cyclic loading of 304L steel. These characterizations use a fully automatized means of evaluating the elastic modulus and the effective plastic strain so that the offset strain is detected with a high degree of precision and repeatability. In all loading cases studied, we found significant yield surface distortion. Moreover, we characterized the direction of the plastic strain tensor and compared it with the normal to the yield surface direction and found good agreement between the two regardless of prior loading. This suggests that the associative flow rule, which states that the direction of the plastic strain increment is normal to the yield surface and is typically assumed when modeling the plastic behavior of metals, is valid under a wide array of prior loadings.

Again, working with our Czech collaborators, we have implemented into the commercial finite element code ABAQUS one of the previous models best able to replicate ratcheting results at a material point. The model that was implemented is classical plasticity theory (not BS) and uses multi-component backstress to model KH. Specifically, the KH rule used in the finite element implementation was the Multicomponent Armstrong–Frederick with Threshold and r modification (MAFTr) published by Dafalias and Feigenbaum (2011). With this implementation, we were able to simulate ratcheting in straight and elbow shaped pipe components, and for the former compare results with experimental data from the literature. Future work will focus on finite element implementation of the new SANIMETAL-BCD model we recently developed as that clearly shows greater accuracy in one element simulations and, hence, holds a greater promise in predicting ratcheting of whole structures, like piping components, after implementation in finite elements codes.

An analytical description of the constitutive model in full detail and the obtained simulations of various UR and MR data, as well as some of our experimental results and finite element simulations, are included in an associated pdf document uploaded separately.

Finally, it is worth mentioning several accomplishments that were not within the set of initial goals but were motivated by them, and which may have a positive contribution in the future to the already achieved goals. In the corresponding publications, the support by ARO was acknowledged. The first accomplishment is related to the introduction of a Memory Surface (MS) in stress space that hardens both isotropically and kinematically, for the deviatoric cyclic response of sands, in an effort to combine successful simulations under monotonic and cyclic loading, where the MS is active during the latter. The MS can easily be incorporated into the structure of the SANIMETAL-BCD model of the present project. A second accomplishment is quite theoretical and provides a rigorous way to obtain separately in a tensorial formulation with respect to any coordinate system, the part of the stress rate that causes only change of stress principal values and the part that causes only change of principal stress directions. This may be proved very useful in improving multiaxial simulations that introduce different ratcheting characteristics when principal stress values and directions are imposed separately not in combination. In order to realize this idea, a third theoretical achievement was introduced, namely, the framework of “Split stress rate plasticity formulation” which makes use of the previous achievement and can constitute the basis for further improvement of the SANIMETAL-BCD model since split stress rate can be combined with any plasticity model. Last a fourth achievement of instructional character is the writing up of Chapters in a recent book on cyclic plasticity, where the exposition of the basis for BS and DDH are explained at a level and format to be understood and used by graduate students.

Training Opportunities: There was quite a bit of training for a graduate student at NAU and a post-doctoral scholar at UCD during this reporting period. In both cases, one-on-one, mentoring occurred regularly between the PI/Co-PI and the student/post-doc. The student at NAU began in July 2020 and decided not to continue with his studies in June 2022, so that he could earn more money in industry and help support his family. In addition, students and post-docs from the Czech Republic collaborated on this project and were mentored by Dr. Heidi Feigenbaum. Unfortunately, the collaboration and mentorship were remote for a substantial part of the project due to the COVID-19 pandemic. The post-doctoral scholar was supervised and trained by the Co-PI Y.F. Dafalias, and at the end of his training period was appointed Assistant Professor at the Engineering Department of Durham University at U.K.

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Results Dissemination: Results have been communicated through journal papers, conference papers, and book chapters that acknowledge ARO support as follows:

Yannis F. Dafalias, Alexandros Petalas, Heidi Feigenbaum, "Bounding Surface Plasticity Model Modification for Ratcheting of Metals", International Journal of Solids and Structures, under review.

Yannis F. Dafalias: "Split stress rate plasticity formulation", International Journal of Solids and Structures, <https://doi.org/10.1016/j.ijsolstr.2022.111494>, in press, 2022.

Yang, M., Taiebat, M., and Dafalias, Y.F., "SANISAND-MSf: a sand plasticity model with memory surface and semifluidised state", Geotechnique, 72(3), 227-246, 2022. <https://doi.org/10.1680/jgeot.19.P.363>

Klepa? V., Parma S., Feigenbaum H. P., Marek R., Plešek J., Svárovský J.: "Use of advanced kinematic hardening rules for prediction of multiaxial ratcheting" PROCEEDINGS OF COMPUTATIONAL MECHANICS 2021, November 8 - 10, 2021, CZECH REPUBLIC

Svárovský J., Parma S., Stefan J., Ciocanel C., Feigenbaum H. P., Klepa? V., Marek R., ? Plešek J.: "Development of autonomous experimental setup to investigate directional distortional hardening under biaxial loading" PROCEEDINGS OF COMPUTATIONAL MECHANICS 2021, November 8 - 10, 2021, CZECH REPUBLIC

Jan Štefan, Slavomír Parma, René Marek, Jiří Plešek, Constantin Ciocanel, and Heidi Feigenbaum, "Overview of an Experimental Program for Development of Yield Surfaces Tracing Method" Applied Sciences, 11(16), 7606, 2021. <https://doi.org/10.3390/app11167606>

Seyed Behzad Behravesh and Yannis F. Dafalias, 2021. "Two surface cyclic plasticity", Chapter 4, pp. 101-138, Cyclic Plasticity of Metals, edited by Hamid Jahed and Ali Roostaei, Elsevier, 2021.

René Marek, Slavomír Parma, and Heidi Feigenbaum, "Distortional hardening cyclic plasticity: Experiments and Modeling" Cyclic Plasticity of Metals, edited by Hamid Jahed and Ali Roostaei, Elsevier, 2021.

R. Marek, H. Feigenbaum, J. Stefan, S. Parma, J. Plešek, "New geometry-based model of directional distortional hardening for multiaxial ratcheting predictions" XVI International Conference on Computational Plasticity. Fundamentals and Applications, Barcelona, Spain, September 7-9, 2021.

S. Parma, C. Ciocanel, J. Štefan, H. Feigenbaum, R. Marek, R. Halama, J. Plešek, "Experimental Investigation of Yield Surface Evolution and Strain Hardening of Boiler Steel under Biaxial Loading" XVI International Conference on Computational Plasticity. Fundamentals and Applications, Barcelona, Spain, September 7-9, 2021.

V. Klepac, S. Parma, R. Marek, H. Feigenbaum, J. Plešek, "FE Simulation of Ratcheting Using Advanced Kinematic Hardening Rules" XVI International Conference on Computational Plasticity. Fundamentals and Applications, Barcelona, Spain, September 7-9, 2021.

Li X.S. and Dafalias Y.F.: "Non-coaxiality between two tensors with application to stress rate decomposition and fabric anisotropy variable", Journal of Engineering Mechanics, ASCE, Vol. 146 (3), Paper No 04020004, 13 pages, 2020.

J. Štefan, S. Parma, R. Marek, J. Plešek, H. Feigenbaum, and C. Ciocanel, "An Experimental Investigation of Yield Surfaces Anisotropies," in 58th Conference on Experimental Stress Analysis, p. 575, 2020.

Honors and Awards: Nothing to Report

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: PD/PI

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Participant: Heidi Feigenbaum
Person Months Worked: 5.00
Project Contribution:
National Academy Member: N

Funding Support:

Participant Type: Co PD/PI
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Person Months Worked: 7.00
Project Contribution:
National Academy Member: N

Funding Support:

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Project Contribution:
National Academy Member: N

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Project Contribution:
National Academy Member: N

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Project Contribution:
National Academy Member: N

Funding Support:

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Project Contribution:
National Academy Member: N

Funding Support:

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Person Months Worked: 2.00
Project Contribution:
National Academy Member: N

Funding Support:

Participant Type: Faculty
Participant: Constantin Ciocanel
Person Months Worked: 1.00

Funding Support:

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Project Contribution:
National Academy Member: N

Participant Type: Undergraduate Student

Participant: Kate Collette

Person Months Worked: 2.00

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National Academy Member: N

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Article Title: Noncoaxiality between Two Tensors with Application to Stress Rate Decomposition and Fabric Anisotropy Variable

Authors: X. S. Li, Y. F. Dafalias

Keywords: oncoaxiality; Rotational shear; Coaxial and noncoaxial stress rate; Anisotropic critical state theory; Representation theorem

Abstract: The coaxial and totally noncoaxial parts of a tensor in regard to another reference tensor are derived in closed analytical form based on representation theorems of tensor-valued isotropic functions. In the process a new interpretation is obtained for a singular case of representation theorems. As a first application, the coaxial and noncoaxial parts of a stress rate tensor in regard to the stress tensor are analytically expressed, and the findings applied to the following two cases: analytically express the part of a stress rate tensor that induces change of stress principal axes at fixed principal stress values, and change of stress principal values at fixed stress principal axes such that the stress orbit on the stress π -plane is circular. A second application refers to enhancing the definition of the fabric anisotropy variable A so that the orthogonal coaxial and noncoaxial parts of the fabric tensor in regard to the plastic strain rate direction participate in the definition of A .

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Journal: Geotechnique

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Article Title: SANISAND-MSf: a memory surface and semifluidized state enhanced sand plasticity model for undrained cyclic shearing

Authors: Ming Yang, Mahdi Taiebat, Yannis F. Dafalias

Keywords: SANISAND-MSf; undrained cyclic shearing; sand; memory surface; semifluidized state; bounding surface; critical state

Abstract: This paper develops an experimental technique to evaluate the initial yield surfaces of metallic materials, as well as to study their evolution during plastic flow. The experimental tracing of yield surfaces is necessary for deriving and calibrating more robust phenomenological models of directional distortional hardening. Such models can be used to characterize the behavior of structures experiencing complicated and demanding loading modes, such as multiaxial ratcheting. The experimental technique developed in this work uses thin-walled tubular specimens, along with a servo-hydraulic machine, under various modes of tension/compression and torque. Identification of the onset of plastic flow is based on a small proof equivalent plastic strain. A continuous and fully automatized evaluation of elastic moduli and proof plastic strain is assured. The current technique is shown to provide promising results to effectively capture the yield surfaces of conventional metallic materials.

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Article Title: Overview of an Experimental Program for Development of Yield Surfaces Tracing Method

Authors: Jan Štefan, Slavomír Parma, René Marek, Jiří Plešek, Constantin Ciocanel, Heidi Feigenbaum

Keywords: experimental mechanics; phenomenological plasticity theory; yield surface; directional distortional hardening

Abstract: This paper develops an experimental technique to evaluate the initial yield surfaces of metallic materials, as well as to study their evolution during plastic flow. The experimental tracing of yield surfaces is necessary for deriving and calibrating more robust phenomenological models of directional distortional hardening. Such models can be used to characterize the behavior of structures experiencing complicated and demanding loading modes, such as multiaxial ratcheting. The experimental technique developed in this work uses thin-walled tubular specimens, along with a servo-hydraulic machine, under various modes of tension/compression and torque. Identification of the onset of plastic flow is based on a small proof equivalent plastic strain. A continuous and fully automatized evaluation of elastic moduli and proof plastic strain is assured. The current technique is shown to provide promising results to effectively capture the yield surfaces of conventional metallic materials.

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Article Title: Split stress rate plasticity formulation
Authors: Yannis F. Dafalias
Keywords: Plasticity; Constitutive modeling; Stress principal axes rotation; Fabric; Granular mechanics; Dilatancy; Isotropic and anisotropic critical state theory; Stress rate non-coaxiality
Abstract: A novel general constitutive formulation framework of rate independent plasticity is presented where the stress rate tensor is split in two components, and plastic loading is affected separately by each one of these two components in relation to a common loading direction normal to a single smooth yield surface in stress space. This split-stress rate separate effect provides greater flexibility for simulating the material response under combined stress rate components loading. The difficulty arises when loading and unloading events may occur interchangeably between the two stress rate components and special attention is required to avoid irrational results and guarantee continuity of response upon rotation of the total stress rate direction at a loading stress point.
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Acknowledged Federal Support: Y

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Article Title: Bounding Surface Plasticity Model Modification for Ratcheting of Metals
Authors: Yannis F. Dafalias, Alexandros Petalas, Heidi Feigenbaum
Keywords: Plasticity; Constitutive modeling; Bounding Surface; Kinematic Hardening; Ratcheting
Abstract: The Bounding Surface (BS) plasticity model for metals is modified according to the proposition introduced in the works of Burlet and Cailletaud (1986) and Delobelle (1993) for the kinematic hardening of a classical Armstrong/Frederick (AF) model, called the BCD modification from the initials of the foregoing authors. In addition, the BCD modification for the BS has been modified by introducing a properly varying modification factor. This simple feature of the novel BCD modification provides a dramatically improved capability to simulate multiaxial ratcheting (MR), because it affects directly the changing flow rule direction during complex multiaxial loading, without sacrificing accurate simulations under uniaxial ratcheting (UR) since the plastic modulus is not affected. The new model, named SANIMETAL-BCD, is shown to yield superior or equal simulations of UR and very complex MR experimental data for three Carbon Steel specimens.
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Authors: S. Parma, C. Ciocanel, J. Štefan, H. Feigenbaum, R. Marek, R. Halama, J. Plešek
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Date Received: 19-Aug-2022 Conference Date: 08-Nov-2021 Date Published:
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Paper Title: Development of autonomous experimental setup to investigate directional distortional hardening under biaxial loading
Authors: J. Svárovský, S. Parma, J. Stefan, C. Ciocanel, H. P. Feigenbaum, V. Klepa?, R. Marek
Acknowledged Federal Support: **Y**

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Paper Title: Use of advanced kinematic hardening rules for prediction of multiaxial ratcheting
Authors: V. Klepac, S. Parma, H.P. Feigenbaum, R. Marek, J. Plešek, J. Svárovský
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Paper Title: An Experimental Investigation of Yield Surfaces Anisotropies
Authors: J. Štefan, S. Parma, R. Marek, J. Plešek, H. Feigenbaum, and C. Ciocanel
Acknowledged Federal Support: **Y**

Partners

Institute of Thermomechanics
Prague, CZE

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The Czech team is working closely with both Drs. Feigenbaum and Dafalias on the current project. The Czech team is taking the lead on the finite element implementation of the models and is working closely with the NAU team on the experimental characterization of yield surfaces. Prior to the COVID-19 pandemic and again after August 2022, students/researchers from the Czech team were spending extended periods of time at NAU working directly with Dr. Feigenbaum.

I certify that the information in the report is complete and accurate:

Signature: Heidi Feigenbaum

Signature Date: 4/19/23 6:21PM

Project Final Report - Grant # W911NF1910040 (Reporting Period: Feb. 1, 2019 – Jan. 31, 2023)

Modeling the Mechanics of Multiaxial Ratcheting

Heidi Feigenbaum, PI, and Yannis F. Dafalias, Co-PI

Abstract

Ratcheting is the accumulation of plastic strain due to cyclic loading. This phenomenon is known to reduce the life and lead to unexpected failure of piping components in nuclear power and chemical plants, offshore structures, machinery components, buildings, bridges, and any other structure subject to earthquakes, extreme weather, and/or cyclic mechanical and thermal service conditions. Predictions of ratcheting strains are needed to avoid, foresee, and understand structural failure. The largest challenge associated with finding a realistic constitutive model to accurately predict ratcheting is that any acceptable small error in a single cycle accumulates over many cycles to an unacceptable level, so it is imperative that the material response is described with a high degree of accuracy. Multiaxial ratcheting has proven to be particularly difficult to predict because in addition to the norm, the direction in strain space towards which the plastic strain tensor accumulates may be unknown and yet it must be predicted with high accuracy in order to capture the overall deformation of the structure and assess its integrity. Our research objective is to use the principles of continuum mechanics and thermodynamics to investigate the mechanics of ratcheting and derive a rigorous, simple, and accurate constitutive model for uniaxial and multiaxial ratcheting that is applicable to a wide array of ductile metals.

We were able to achieve this goal through a new model that modifies the Bounding Surface (BS) plasticity model for metals according to the proposition introduced in the works of Burlet and Cailletaud [1986] and Delobelle [1993] for the kinematic hardening of a classical Armstrong/Frederick (AF) model, called the BCD modification from the initials of the foregoing authors. The BCD modification was introduced in the relative kinematic hardening between yield surface (YS) and BS, unlike its introduction in the absolute and single kinematic hardening of YS for an AF model, hence, achieving two objectives: first, maintaining the inherent feature of BS for decoupling plastic modulus and direction of kinematic hardening, and, second, allowing a flexibility as to the relative kinematic hardening direction without altering the value of the plastic modulus, a property of BCD modification. In addition, the BCD modification for the BS is significantly modified itself, by introducing a properly varying modification factor instead of the fixed one used in the original works. This simple feature of the novel BCD modification provides a dramatically improved capability to simulate multiaxial ratcheting (MR), because it affects directly the changing flow rule direction during

complex multiaxial loading, without sacrificing accurate simulations under uniaxial ratcheting (UR) since the plastic modulus is not affected. An additional significant contribution to successful UR simulations is provided by the free-to-choose kinematic hardening of the BS, since the BCD modification is applied to the relative kinematic hardening between BS and YS. The new model, named SANIMETAL-BCD, is shown to yield superior or equal simulations of UR and very complex MR experimental data for three Carbon Steel specimens, in comparison with other models, within a much simpler constitutive framework. Shortcomings and future necessary improvements are discussed in details.

1 Objectives

Ratcheting is the accumulation of plastic strain due to cyclic loading. This phenomenon is known to reduce the life and lead to unexpected failure of piping components in nuclear power and chemical plants, offshore structures, machinery components, buildings, bridges, and any other structure subject to earthquakes, extreme weather, and/or cyclic mechanical and thermal service conditions. Predictions of ratcheting strains are needed to avoid, foresee, and understand structural failure. The largest challenge associated with finding a realistic constitutive model to accurately predict ratcheting is that any acceptable small error in a single cycle accumulates over many cycles to an unacceptable level, so it is imperative that the material response is described with a high degree of accuracy. Multiaxial ratcheting has proven to be particularly difficult to predict because in addition to the norm, the direction in strain space towards which the plastic strain tensor accumulates may be unknown and yet it must be predicted with high accuracy in order to capture the overall deformation of the structure and assess its integrity. Our research objective is to use the principles of continuum mechanics and thermodynamics to investigate the mechanics of ratcheting and derive a rigorous, simple, and accurate constitutive model for uniaxial and multiaxial ratcheting that is applicable to a wide array of ductile metals. In other words, we aim to answer the scientific questions: Under repetitive plastic loading paths, what are the changes in the material state at the continuum level and how can these changes of the material state be modeled by means of a constitutive model in order to allow successful simulation of ratcheting?

To reach this goal, we employed several objectives (#1-3) and associated sub-tasks (with years of grant worked on):

1. To develop new constitutive models that improve predictions of ratcheting.
 - (a) Develop of a “segmented” model for yield surface distortion wherein the kinematic hardening (KH) stays the same, but distortion is added so that the effect of distortion on ratcheting predictions can be assessed (Years 1-2, Feb. 2019 – Jan. 2021).
 - (b) Develop of a new bounding surface (BS) model that incorporates the “BCD” modification (named after Burlet/Cailletaud/Delobelle, the researchers who first proposed this) to kinematic hardening in a model that we are calling SANIMETAL-BCD, for Simple ANIsotropic METAL with BCD modification (Years 2-4, Feb.

2021 – Jan. 2023).

2. Experimentally characterize the yield surface to ensure the accuracy of the new models.
 - (a) Develop automated procedures/codes to experimentally measure the yield surface (Years 2-3, Feb. 2020-Jan. 2022).
 - (b) Characterize the yield surface during ratcheting (Year 4, Feb. 2022 – Jan. 2023)
3. Incorporate new models into practice.
 - (a) Develop a systematic way to calibrate plasticity models to predict ratcheting using gradient methods (Year 1, Feb. 2019-Jan. 2020) and machine learning techniques, in particular, neural networks (Year 4, Feb. 2022 – Jan. 2023)
 - (b) Finite element implementation of some existing models for KH that have proven to be more successful at predicting ratcheting than standard KH rules built into commercial finite element software (Years 1-4, Feb. 2019 – Jan. 2023)

2 Findings

2.1 Objective 1 - Develop new constitutive models

Ratcheting is a term referring to the accumulation of plastic strain under the imposition of stress or strain (in multiaxial cases) cyclic loading; in the uniaxial case, the cyclic stress has a non-zero mean value. It is in fact a more difficult response to model than “symmetric” cyclic loading where the cyclic stress has a zero-mean value, but it is also more important in practice because this is the expected way metal components are loaded in all kinds of structures, e.g., piping components in chemical or nuclear plants, beam and column elements as well as their connections in metal frames, etc. Ratcheting is distinguished into Uniaxial and Multiaxial Ratcheting, henceforth abbreviated as UR and MR, respectively.

As per its name, UR is characterized by one non-zero stress component cycling over its own non-zero mean value and, therefore, causing accumulation of the corresponding uniaxial plastic strain component, while because of incompressibility additional predefined strain components appear. In stress-controlled MR, there is at least one cyclically varying stress component of zero or non-zero mean value, imposed on one (at least) pre-existing different non-zero stress component, that usually remains fixed. In strain-controlled MR, a total strain component is cycled instead of the stress. In stress-controlled MR there are more than one plastic strain components that accumulate along the directions of non-zero stress components, notwithstanding the additional strain components needed for the satisfaction of plastic incompressibility for metals. For strain-controlled MR, the cycled strain has fixed values, but there is at least one other strain component that accumulates, while the stress corresponding to the cycled strain component increases (hardening) or decreases (softening). One very common and useful example of MR occurs when a pipe is under fixed or varying internal pressure and is cyclically loaded under axial stress or strain control. Both axial and circumferential strain components accumulate in a stress-controlled experiment, while

if one strain component is prescribed in a strain-controlled experiment, it is the other strain component that accumulates.

The research objective of this project was to address the constitutive modeling of UR and MR of metals, by proper modifications of the plasticity constitutive model introduced in the mid-seventies by Dafalias and Popov [1974, 1975, 1976] and updated by Mahan et al. [2011], that uses the coupling interaction of two surfaces, the Yield Surface (YS) and the Bounding Surface (BS) which encloses the YS. A similar theory was simultaneously introduced by Krieg [1975], but the format was very different and not applicable to the demands for MR simulations, such as the necessity to have the BS obeying Isotropic Hardening (IH) and special kind of Kinematic Hardening (KH) not existing in the Krieg's formulation.

UR and MR have been the objectives of a multitude of research efforts and publications, yet simulation of many available data has not been entirely satisfactory; in fact, it was rather unsuccessful in several cases, particularly for MR, and often required the introduction of complicated additional constitutive features. One can say that the problem of MR simulations, despite its very long history that counts several decades, is not resolved at a satisfactory level from the perspective of both accurate predictions and relatively simple constitutive modeling.

There are two main categories of constitutive models used for metal ratcheting, both based on the concept of back-stress and associated KH, that can also include, if necessary, IH. The first kind is based on the seminal work by Armstrong and Frederick [1966] who proposed the so-called evanescent memory model, named AF model here for abbreviation, that extended the linear KH models of Ishlinskii [1954] and Prager [1956] to a non-linear KH format for the back stress. Due to several inherent problems of this model, mainly for partial unloading/reloading as explained later by Dafalias [1984], Chaboche et al. [1979a] have introduced a Multicomponent Armstrong/Frederick model, named MAF for abbreviation, that additively combines several AF back-stress components to get the total back stress, providing the tools for successful cyclic loading simulations. It was used towards MR with relative success, often requiring modification of the AF basic KH rule by several authors who introduced additional constitutive features such as non-hardening regions or memory surfaces in strain or stress space, etc., a complete list of which is beyond the scope and objectives of this report. MAF-based models are by far the most used and investigated models in cyclic plasticity for metals today. It is worth mentioning that Marquis [1979] has shown that any classical AF model implies the existence of a fixed limit surface in stress space, with which the back stress converges without ever crossing it, such limit surface acting in essence as a fixed BS for the back-stress rather than the stress.

The second category of constitutive models is based on BS plasticity, mentioned earlier. The original two-surface BS version by Dafalias and Popov [1974, 1975, 1976] and Krieg [1975] were motivated by the multi-surface model of Mróz [1967] but were developed on an entirely different platform. The Mróz [1967] model, while very successful for certain kinds of cyclic loading simulations, has not been used for UR and MR simulations because of an inherent early stabilization of the strain accumulation due to its basic conception and formulation that yields closed loops in uniaxial cyclic loading. The successful application of the BS model to random cyclic loading in the uniaxial case and some multiaxial loading paths was established earlier than that of MAF models. However, its application to MR was delayed because, unlike the inventors and followers of MAF models who pursued further

the development of their model, the inventors of BS have switched their attention to its use for soil plasticity constitutive modeling under triaxial and multiaxial cyclic loading, where now BS-based models hold a dominant position under the code names SANICLAY and SANISAND models.

It took more than 15 years after the introduction of BS in 1975, for the research teams of Kyriakides and Hassan (e.g. Hassan and Kyriakides, 1992, 1994b,a, Hassan et al., 1992, Corona et al., 1996, Bari and Hassan, 2001) to discover the importance of two special constitutive features that characterize any BS model, in relation to UR and MR simulations of metals. First, the freedom to choose any kind of KH and IH for the BS, which allows one to obtain a bounding plastic modulus that affects the actual plastic modulus by the stress distance dependence of the two moduli within the general BS theory [Dafalias, 1986]. Second, the ability to define the direction of the KH of the YS independently from the definition of the plastic modulus as described above, which is an inherent property for any BS model. This is often called the “decoupling” or “uncoupling” of these two quantities (e.g., Hassan and Kyriakides, 1992, Bari and Hassan, 2001), but note that the quantities are still connected to satisfy the consistency condition of plasticity. These two features provide great modeling flexibility because one could address the magnitude of plastic strain increment and simulate successfully UR based on the value of plastic modulus, and independently address the direction of KH for the YS that affects the evolution of the flow rule direction, and consequently the relative to each other norms of various plastic strain rate components, a key for successful simulation of MR, without altering the value of plastic modulus due to its independence from the choice of KH for the YS.

These two constitutive features are lacking from all MAF models because the AF KH rule and its variations define simultaneously plastic modulus and KH direction in a coupled and inflexible way. The research team of Kyriakides used the flexibility offered by BS with a small twist, namely, choosing the direction of an AF KH rule for the YS and independently the bounding plastic modulus resulting from a particular KH for the BS introduced in the dissertation of Sayed-Ranjbari [1986] and more recently updated in Mahan et al. [2011], where it was named the Dafalias/Ranjbari (DR) KH. In other words they ended up not using the direction of the DR KH for the BS but only the value of the ensuing bounding plastic modulus, while the direction of KH for the BS was defined by its interaction with the chosen AF KH of the YS. This approach brought the simulation of UR and MR, mainly for Carbon Steel (CS) specimens, in a better state than that of existing MAF models, by simpler constitutive means.

Yet, the final simulation of MR was not very successful in several cases, either by using MAF in all its variants or BS in the format used by the aforementioned researchers. Therefore, as mentioned earlier, the main objective of the present work aims at modifying the BS model in such a way as to achieve simulations of equal or better accuracy for both UR and MR than those obtained by either MAF or previous BS models and achieve this while maintaining the greatest possible simplicity of the formulation. The proposed BS model modification in the sequel is motivated by two modifications introduced quite earlier for Prager and AF KH models. The first was a radial evanescence modification of a Prager linear KH model to a non-linear one, introduced by Burlet and Cailletaud [1986]. The second, and more important modification for the current development, was a KH resulting from combining the aforementioned Prager non-linear and the usual evanescence KH of an AF model,

introduced by Delobelle [1993]. For abbreviation, it will be named the BCD modification from the initials of the aforementioned authors, without repeating the relevant references above. The nice feature of the BCD modification for an AF KH model was that one could modify the direction of KH by means of a new parameter δ , called here the BCD modification parameter, that did not affect the value of the plastic modulus once the consistency condition was imposed, as if δ was never introduced. This provided AF with a flexibility similar to that of BS models (but not quite the same) on the choice of various KH directions with same plastic modulus by varying the value of δ , which could be a function of other quantities such as max and mean values of back stress as well as of a non-radiality parameter [Delobelle, 1993, Delobelle et al., 1995]. In fact, the BCD modification was applied to each component of a MAF model by Bari and Hassan [2001], with a fixed but different for each component BCD modification parameter, which improved MR simulations in many but not all cases.

Dafalias and Feigenbaum [2011] have proven analytically a serious theoretical shortcoming of the BCD modification, namely the choice of $\delta < 0.5$ can cause under specific loading conditions a crossing of the previously mentioned AF implied limit surface [Marquis, 1979], by the corresponding back-stress, i.e., the center of the YS, that can induce unwanted negative plastic modulus value and softening. Many of the successful simulations of prior works using MAF models with the BCD modification were employing such values of $\delta < 0.5$, but the unwanted negative plastic modulus value did not appear simply because the corresponding necessary loading conditions were not encountered in these particular simulations. Subsequently, Dafalias and Feigenbaum [2011] used in lieu of a fixed BCD modification parameter δ , the variable ratio r_i of the norm of the YS center, i.e., the norm of the i^{th} back stress component, to the size of the i^{th} AF limit surface, in their own MAF model with BCD modification that also included a multiplicative version of AF [Dafalias et al., 2008], avoiding the foregoing shortcoming because $r_i = 1$ on the i^{th} limit surface. These changes improved further MR but still with unsatisfactory deviations for several data.

It seems, therefore, that MAF models, even with the BCD modification in various variants, are not capable of addressing satisfactorily MR in all cases, and often need to use additional constitutive features that complicate the formulation. We believe the cause is still the lack of complete decoupling between the KH direction of the YS and the plastic modulus in MAF models, as well as the lack of freedom inherent in BS models to choose specific KH for the BS with the aforementioned beneficiary effects on plastic modulus, features that only BS models can offer together. Hence, it was decided to use two novel constitutive ingredients within the general BS theory. First, the introduction of a BCD modification into the relative KH of the YS with respect to the KH of the BS, such BS acting on the back stress of the YS rather than the stress on YS (as done by the limit surface in any AF model, clearly explained in Marquis, 1979). Second, the use of a variable quantity for the BCD modification parameter, that remains greater than 0.5 when the YS back stress reaches the BS to avoid the aforementioned shortcoming. Such quantity is motivated by, but also is quite different from, the one used in Dafalias and Feigenbaum [2011] for AF models, and its exact definition will be presented in the relevant part of this work. In addition, the BS will obey the DR KH, yielding not only the ensuing bounding plastic modulus used in Kyriakides and collaborators' works but also the implied DR KH direction. This relatively simple and straightforward approach produced successful simulations for both UR and MR that are equal or better than the existing ones by other models of greater complexity. Such

simulations will be reported in the sequel, following first a rigorous analytical formulation as well as a guideline for calibration.

All tensors will be represented in direct notation by boldface symbols, a dot superposed on a quantity will imply its rate, while the symbol “:” between two tensors will imply the summation operation over the adjacent two pairs of indices in reverse order, which become the trace of the product of the two tensors if they are of the same rank. It is also convenient to list below the meaning of the main abbreviations used so far, to be repeated very often in this work:

MR: Multiaxial Ratcheting

UR: Uniaxial Ratcheting

BS: Bounding Surface

YS: Yield Surface

KH: Kinematic Hardening

IH: Isotropic Hardening

AF: Armstrong/Frederick

MAF: Multicomponent Armstrong/Frederick

DR: Dafalias/Ranjbari

BCD: Burlet/Cailletaud/Delobelle

CS: Carbon Steel

Classical plasticity formulation with IH and BCD modification of AF KH

A general classical plasticity formulation with Mises’s type YS obeying IH and KH, consists of the following equations:

$$f = \frac{3}{2} (\mathbf{s} - \boldsymbol{\alpha}) : (\mathbf{s} - \boldsymbol{\alpha}) - k^2 = 0 \quad (1)$$

$$\dot{k} = \langle \lambda \rangle \bar{k} \quad (2)$$

$$\dot{\boldsymbol{\alpha}} = \langle \lambda \rangle \bar{\boldsymbol{\alpha}} \quad (3)$$

$$\dot{\boldsymbol{\epsilon}}^p = \langle \lambda \rangle \mathbf{n} \quad (4)$$

with $f=0$ the analytical expression of YS, $\mathbf{s}$ the deviatoric stress tensor, $\boldsymbol{\alpha}$ the deviatoric back stress tensor, k the “size” of the YS, $\dot{\boldsymbol{\epsilon}}^p$ the plastic strain rate tensor, $\bar{k}$ and $\bar{\boldsymbol{\alpha}}$ appropriate scalar and tensor valued functions, respectively, of the state variables $\mathbf{s}$ (external) and $\boldsymbol{\alpha}$, k (internal), $\mathbf{n} = \sqrt{3/2}(\mathbf{s} - \boldsymbol{\alpha})/k$ the unit-norm tensor normal to the YS such that $\mathbf{n}:\mathbf{n}=1$, λ the loading index (or plastic multiplier), $\langle \rangle$ the Macauley brackets such that $\langle A \rangle = A$ when $A > 0$ and $\langle A \rangle = 0$ when $A \leq 0$. Since $\mathbf{n}$ is a deviatoric tensor it follows that $\dot{\boldsymbol{\epsilon}}^p = \dot{\boldsymbol{\epsilon}}^p$, with $\dot{\boldsymbol{\epsilon}}^p$ the deviatoric plastic strain rate. Clearly the plastic volumetric strain rate $tr\dot{\boldsymbol{\epsilon}}^p=0$, where “tr” signifies the trace of a tensor.

The loading index λ is obtained from the consistency condition $\dot{f} = 0$, which in conjunction with Eqs. (2) and (3), the preferred unit normal formulation, i.e., use of $\mathbf{n}$ instead of $\partial f / \partial \boldsymbol{\sigma}$ for the definition of λ and the relations $\partial f / \partial k = -2k$ and $\partial f / \partial \boldsymbol{\sigma} = 3(\mathbf{s} - \boldsymbol{\alpha}) = -\partial f / \partial \boldsymbol{\alpha} = |\partial f / \partial \boldsymbol{\sigma}| \mathbf{n} = \sqrt{6}k\mathbf{n}$, with $|\mathbf{X}| = \sqrt{\mathbf{X} : \mathbf{X}}$ signifying the norm of a tensor $\mathbf{X}$, yields:

$$\lambda = \frac{1}{K_p} \mathbf{n} : \dot{\mathbf{s}} \quad (5)$$

$$K_p = - \left| \frac{\partial f}{\partial \boldsymbol{\sigma}} \right|^{-1} \left(\frac{\partial f}{\partial k} \bar{k} + \frac{\partial f}{\partial \boldsymbol{\alpha}} : \bar{\boldsymbol{\alpha}} \right) = \sqrt{\frac{2}{3}} \bar{k} + \bar{\boldsymbol{\alpha}} : \mathbf{n} \quad (6)$$

where K_p is the all-important plastic modulus and use of $\mathbf{n} : \bar{\boldsymbol{\alpha}} = \bar{\boldsymbol{\alpha}} : \mathbf{n}$ was made for the last member of Eq. (6). Notice that the plastic strain rate Eq. (4), does not affect the loading index and plastic modulus because the plastic strain does not enter Eq. (1), thus, it does not appear in the consistency condition, $\dot{f} = 0$. Eqs.(1)-(6) provide the complete set of equations for any IH and KH plasticity model with YS given by Eq. (1), and the various forms of such models depend exclusively on the choice of the functions k and $\bar{\boldsymbol{\alpha}}$.

With $\dot{\boldsymbol{\epsilon}}_{eq}^p = \sqrt{(2/3)} \dot{\boldsymbol{\epsilon}}^p : \dot{\boldsymbol{\epsilon}}^p$ the equivalent plastic strain rate, the following most common and effective rate evolution equation is adopted for the IH rule:

$$\dot{k} = c_k (k_s - k) \dot{\boldsymbol{\epsilon}}_{eq}^p = \langle \lambda \rangle \sqrt{\frac{2}{3}} c_k (k_s - k) = \langle \lambda \rangle \bar{k} \quad (7)$$

where use of Eq. (4) with $\dot{\boldsymbol{\epsilon}}^p = \dot{\boldsymbol{\epsilon}}^p$ was made in deriving the third member of Eq. (7), k_s denotes the saturation value (limit value) of k , and c_k is a model constant controlling the pace at which k approaches k_s . Notice that depending on whether $k_s > k$ or $k_s < k$ one has isotropic hardening (increase of k) or softening (decrease of k), respectively. Often k_s is also made function of other quantities, such as max strain amplitude, etc. It follows from Eq. (7) that $\bar{k} = \sqrt{2/3} c_k (k_s - k)$.

Before we introduce the BCD modification, it is expedient to present the format of AF KH used in this work, that offers some elements of convenience for calibration purposes and helps in the subsequent connection with the BCD modification when applied to the BS model. Armstrong and Frederick [1966] in their seminal paper proposed the following rate equation for KH:

$$\begin{aligned} \dot{\boldsymbol{\alpha}} &= \frac{2}{3} h \dot{\boldsymbol{\epsilon}}^p - c \dot{\boldsymbol{\epsilon}}_{eq}^p \boldsymbol{\alpha} = \langle \lambda \rangle \sqrt{\frac{2}{3}} c \left(\sqrt{\frac{2}{3}} \frac{h}{c} \mathbf{n} - \boldsymbol{\alpha} \right) = \langle \lambda \rangle \sqrt{\frac{2}{3}} c \left(\sqrt{\frac{2}{3}} \boldsymbol{\alpha}^s - \boldsymbol{\alpha} \right) \\ &= \langle \lambda \rangle \sqrt{\frac{2}{3}} c (\boldsymbol{\alpha}^s - \boldsymbol{\alpha}) \end{aligned} \quad (8)$$

with h and c being the original AF parameters. Observe the use of $\boldsymbol{\alpha}^s = h/c$ and c in the final expression. The preference to use $\boldsymbol{\alpha}^s = h/c$ instead of h , stems from the fact the $\boldsymbol{\alpha}^s = \sqrt{2/3} \boldsymbol{\alpha}^s \mathbf{n}$ is the saturation or limit tensor towards which the back stress $\boldsymbol{\alpha}$ tends to converge with, at a pace defined by the constant c , such limit providing an easy to measure important material property; to the contrary, h represents the more difficult to measure initial value of uniaxial plastic modulus (initial slope of stress-plastic strain curve).

The BCD modification of Eq. (8) yields:

$$\dot{\boldsymbol{\alpha}} = \langle \lambda \rangle \sqrt{\frac{2}{3}} c (\boldsymbol{\alpha}^s - [\delta \boldsymbol{\alpha} + (1 - \delta)(\mathbf{n} : \boldsymbol{\alpha}) \mathbf{n}]) = \langle \lambda \rangle \bar{\boldsymbol{\alpha}} \quad (9)$$

with the obvious definition of $\bar{\boldsymbol{\alpha}}$ following from the last two members of Eq. (9). δ is the relevant parameter whose role is to create a mixture of AF KH rule occurring for $\delta = 1$, and a Burlet and Cailletaud [1986] non-linear Prager type KH rule along $\mathbf{n}$, occurring for $\delta = 0$ and accounting for the fact $\boldsymbol{\alpha}^s = \sqrt{\frac{2}{3}} \alpha^s \mathbf{n}$. Substitution of $\bar{k}$ and $\bar{\boldsymbol{\alpha}}$ as defined by Eqs. (7) and (9), respectively, in Eq. (6), yields for plastic modulus after some simple algebra the expression:

$$K_p = \frac{2}{3} c_k (k_s - k) + \sqrt{\frac{2}{3}} c (\boldsymbol{\alpha}^s - \mathbf{a}) : \mathbf{n} \quad (10)$$

Notice that δ disappears from the value of the plastic modulus and this is a great advantage because one can calibrate the plastic modulus value for UR or any radial ratcheting loading, and then calibrate the value of δ only for the KH direction that affects MR. Dafalias and Feigenbaum [2011] showed analytically that one must have $\delta > 0.5$ when the back stress $\boldsymbol{\alpha}$ lies on the implied limit or saturation surface with radius $\boldsymbol{\alpha}^s = \sqrt{\frac{2}{3}} \alpha^s \mathbf{n}$, for guaranteeing it will not “cross” it and yield unrealistic negative plastic modulus due to the negative value of the term $(\boldsymbol{\alpha}^s - \mathbf{a}) : \mathbf{n}$ in the foregoing Eq. (10) for K_p , that occurs when $\boldsymbol{\alpha}$ is outside the surface with radius $\boldsymbol{\alpha}^s$. In addition, Dafalias and Feigenbaum [2011] proposed replacing the fixed BCD modification parameter δ with the variable ratio:

$$r = \frac{\sqrt{(3/2) \boldsymbol{\alpha} : \boldsymbol{\alpha}}}{\sqrt{(3/2) \boldsymbol{\alpha}^s : \boldsymbol{\alpha}^s}} = \frac{\alpha}{\alpha^s} \quad (11)$$

with α , α^s the norms of $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^s$, respectively, measuring the proximity of the back stress to its saturation value lying on a saturation surface; naturally $r = 1$ when the back stress is on the saturation surface, hence, being greater than 0.5, it does not induce any negative plastic modulus ever as per the discussion preceding Eq. (11). The fact that r varies, implies a variation of the mixture of the two KH rules, as explained after Eq. (9).

Bounding surface formulation with BCD modification

General formulation The methodology for the formulation follows closely, but not identically, the one in Mahan et al. [2011] where the resulting constitutive model was named SANISTEEL, from the initial letters of Simple ANIsotropic STEEL, addressing mainly steel materials. Since the main theoretical novelty will be the use of the BCD modification while addressing all kinds of different metals, the adopted name for the present model will be

SANIMETAL-BCD, even though the materials simulated in this work will belong to the Carbon Steel (CS) family. Another change from Mahan et al. [2011] is that in this project an evolving BS for the YS back stress, instead of the stress, will be adopted, as already done for soils [Manzari and Dafalias, 1997, Dafalias and Manzari, 2004]. Recall from the Introduction that Marquis [1979] interpreted the limit surface for the back stress implied in any AF model, as a corresponding fixed BS for such back stress.

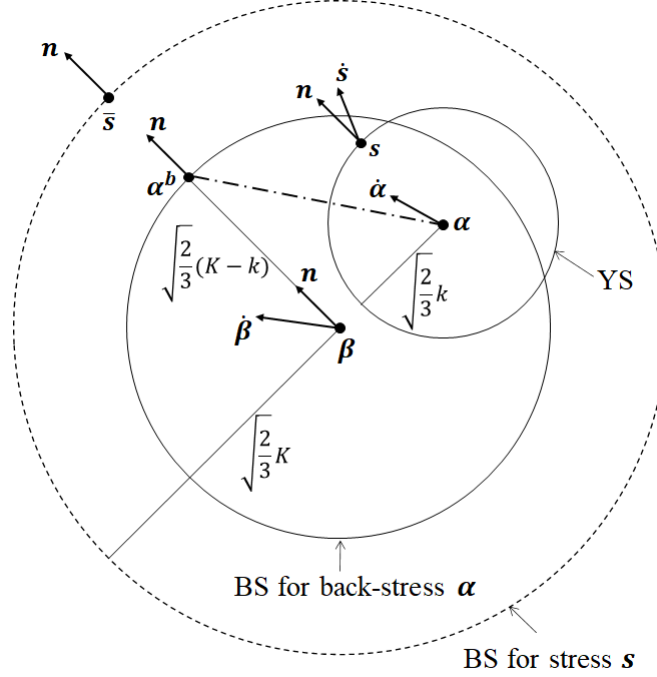


Fig. 1. Illustration of the concepts of YS, BS for the stress $\mathbf{s}$, BS for the back stress $\boldsymbol{\alpha}$, unit norm tensor $\mathbf{n}$ normal to YS at current stress $\mathbf{s}$, stress rate $\dot{\mathbf{s}}$, image back stress $\boldsymbol{\alpha}^b$, image stress $\bar{\mathbf{s}}$, and indication of the KH rates $\dot{\boldsymbol{\alpha}}$ and $\dot{\boldsymbol{\beta}}$.

Figure 1 serves as the key illustration of the analytical description that follows, and one must automatically refer to this figure in the following. The YS, given analytically by Eq. (1), is shown as a circle (sphere in deviatoric multiaxial stress space) of center $\boldsymbol{\alpha}$ and radius $\sqrt{2/3k}$. The current deviatoric stress tensor $\mathbf{s}$ and unit norm tensor $\mathbf{n}$, normal to the YS at $\mathbf{s}$, are also shown, as well as the imposed stress rate $\dot{\mathbf{s}}$. The classical BS for the stress is shown as a dashed circle of center $\boldsymbol{\beta}$ and radius $\sqrt{2/3K}$ on which the image stress $\bar{\mathbf{s}}$ is located along the radius with direction $\mathbf{n}$ emanating from $\boldsymbol{\beta}$. The BS for the back stress is analytically expressed by:

$$F = \frac{3}{2} (\boldsymbol{\alpha}^b - \boldsymbol{\beta}) : (\boldsymbol{\alpha}^b - \boldsymbol{\beta}) - (K - k)^2 = 0 \quad (12)$$

and shown in Figure 1 as a solid line circle with same center $\boldsymbol{\beta}$ and radius the difference

$\sqrt{2/3}(K-k)$ between the radii of BS for stress and YS. The image back-stress $\boldsymbol{\alpha}^b$ of the back-stress $\boldsymbol{\alpha}$, lies on the above BS and is located along the radius with direction $\mathbf{n}$ emanating from $\boldsymbol{\beta}$ as illustrated in Figure 1, and analytically defined by:

$$\boldsymbol{\alpha}^b = \boldsymbol{\beta} + \sqrt{\frac{2}{3}}(K-k)\mathbf{n} \quad (13)$$

This variable $\boldsymbol{\alpha}^b$ is the tensor with which the back stress $\boldsymbol{\alpha}$ of the YS converges. Notice the difference of $\boldsymbol{\alpha}^b$ from its counterpart $\boldsymbol{\alpha}^s = \sqrt{2/3}\alpha^s\mathbf{n}$ entering Eqs. (7) and (9), had no BS been introduced. The isotropic hardening of the BS follows the scheme of IH for the YS as per Eq. (7) and reads:

$$\begin{aligned} \dot{K} - \dot{k} &= [C_K(K_s - K) - c_k(k_s - k)] \dot{\epsilon}_{eq}^p = < \lambda > \sqrt{\frac{2}{3}} [C_K(K_s - K) - c_k(k_s - k)] \\ &= < \lambda > (\bar{K} - \bar{k}) \end{aligned} \quad (14)$$

where K_s denotes the saturation value (limit value) of K , C_k is a model constant controlling the pace at which K approaches K_s , and from the last two members of Eq. (14) clearly follows that $\bar{k} = \sqrt{2/3}c_k(k_s - k)$ and $\bar{K} = \sqrt{2/3}C_k(K_s - K)$.

Postponing for later the definition of the KH of the BS by means of the analytical expression of the rate of its center $\boldsymbol{\beta}$, we focus on the modification of the KH for the back stress, center of the YS, in the presence of the BS in association with the BCD modification, i.e., on the modification of Eq. (9). First and foremost the $\boldsymbol{\alpha}^b$, as given by Eq. (13), must substitute for $\boldsymbol{\alpha}^s$ in both Eqs. (8) and (9). Assuming next radial loading from zero along a fixed direction $\mathbf{n}$, observe first that Eq. (9), the KH rule with the BCD modification, becomes identical to AF KH as per Eq. (8), because $\boldsymbol{\alpha}$ develops along $\mathbf{n}$ and consequently all the terms with δ cancel out. Because $\boldsymbol{\alpha}^b$ lies on a BS, it evolves also along $\mathbf{n}$ due to IH and KH of the BS, as eloquently implied by taking the rate of Eq. (13), providing a moving “target” for the evolving $\boldsymbol{\alpha}$. Because the rate of $\boldsymbol{\alpha}$ depends on the distance $\boldsymbol{\alpha}^b - \boldsymbol{\alpha}$ as per Eq. (8) resulting from Eq. (9) for radial loading along $\mathbf{n}$ (recall substitution of $\boldsymbol{\alpha}^b$ for $\boldsymbol{\alpha}^s$ in the foregoing equations), it will eventually approach the $\boldsymbol{\alpha}^b$ sufficiently close to have a small enough distance from it that necessarily causes the same rate of $\boldsymbol{\alpha}$ with that of $\boldsymbol{\alpha}^b$ along $\mathbf{n}$. Thus, the back stress $\boldsymbol{\alpha}$ may never be able to reach the BS at $\boldsymbol{\alpha}^b$ as it is supposed to do, unless the $\boldsymbol{\alpha}^b$ stops evolving in the case of a fixed BS, as it happens for the fixed limit surface of a classical AF model. The remedy of this unrealistic eventuality, which did not exist for the simple BCD modification of AF as per Eq. (9) where $\boldsymbol{\alpha}^s$ was fixed, is very simple. First, the rate of $\boldsymbol{\alpha}^b$ is obtained by taking the rate of Eq. (13) and using Eq. (14). Second, one can modify Eq. (9) by just adding the rate of $\boldsymbol{\alpha}^b$ to the rate of $\boldsymbol{\alpha}$ while substituting for the fixed δ a variable BCD modification parameter r_α (to be defined in the sequel), and write:

$$\begin{aligned}
\dot{\boldsymbol{\alpha}} &= \dot{\boldsymbol{\alpha}}^b + \langle \lambda \rangle \sqrt{\frac{2}{3}} c \left(\boldsymbol{\alpha}^b - [r_\alpha \boldsymbol{\alpha} + (1 - r_\alpha) (\boldsymbol{\alpha} : \mathbf{n}) \mathbf{n}] \right) \\
&= \dot{\boldsymbol{\beta}} + \sqrt{\frac{2}{3}} \left(\dot{K} - \dot{k} \right) \mathbf{n} + \langle \lambda \rangle \sqrt{\frac{2}{3}} c \left(\boldsymbol{\alpha}^b - [r_\alpha \boldsymbol{\alpha} + (1 - r_\alpha) (\boldsymbol{\alpha} : \mathbf{n}) \mathbf{n}] \right) \\
&= \langle \lambda \rangle \left[\bar{\boldsymbol{\beta}} + \frac{2}{3} [C_K (K_s - K) - c_k (k_s - k)] \mathbf{n} + \sqrt{\frac{2}{3}} c \left(\boldsymbol{\alpha}^b - [r_\alpha \boldsymbol{\alpha} + (1 - r_\alpha) (\boldsymbol{\alpha} : \mathbf{n}) \mathbf{n}] \right) \right] \\
&= \langle \lambda \rangle \bar{\boldsymbol{\alpha}}
\end{aligned} \tag{15}$$

where for the KH of the BS the general expression $\dot{\boldsymbol{\beta}} = \langle \lambda \rangle \bar{\boldsymbol{\beta}}$ was assumed, the $\bar{\boldsymbol{\beta}}$ to be defined in the sequel, and where the $\bar{\boldsymbol{\alpha}}$ is clearly defined by the last two members of Eq. (15) as the quantity in brackets that is multiplied by $\langle \lambda \rangle$. In Fig. 1 $\dot{\boldsymbol{\alpha}}$ is shown schematically along a direction between $\mathbf{n}$ and $\boldsymbol{\alpha}^b - \boldsymbol{\alpha}$, as expected by the mixture of two kinds of KH implied by the BCD modification; the $\boldsymbol{\alpha}^b - \boldsymbol{\alpha}$ is indicated by a dashed-dotted line connecting $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}^b$.

The all-important plastic modulus K_p is obtained by substitution of $\bar{k}$ from Eq. (7) and $\bar{\boldsymbol{\alpha}}$ from Eq. (15) in Eq. (10), yielding:

$$K_p = \frac{2}{3} c_k (k_s - k) + \bar{\boldsymbol{\alpha}} : \mathbf{n} = \bar{\boldsymbol{\beta}} : \mathbf{n} + \frac{2}{3} C_K (K_s - K) + \sqrt{\frac{2}{3}} c (\boldsymbol{\alpha}^b - \boldsymbol{\alpha}) : \mathbf{n} \tag{16}$$

As expected, notice that the BCD modification parameter r_α of Eq. (15) disappears from Eq. (16) for the plastic modulus, indicating that the role of r_α is only to define the KH direction of the YS in reference to the KH of the BS as per Eq. (15). The IH of the YS does not appear in the last member of Eq. (16) for the plastic modulus but has its place in Eq. (15) for the updating of the YS back stress. Finally, compare Eq. (16) with Eq. (10) for the classical AF model with (or without) BCD modification; they are entirely different due to the BS role, such role becoming more prominent when the quantities $\bar{\boldsymbol{\beta}}$ and c are defined as functions of the state in the sequel.

Three important and novel constitutive functions

Eq. (15) constitutes the main novel contribution of the present SANIMETAL-BCD model. It expresses the much sought-after KH of the YS in relation to the KH of the BS using a variant of the BCD modification, such variant depending on the constitutive function that yields the BCD modification parameter r_α . In addition to the usual IH constants c_k , k_s , C_k , K_s for YS and BS entering Eqs. (14) and (15), there are three remaining very important constitutive functions that must be specified for the application of Eq. (15) and the ensuing Eq. (16): the c , $\bar{\boldsymbol{\beta}}$ and r_α . Each one will be addressed separately in the following.

The function c The first two terms of the last member of Eq. (16) express the bounding plastic modulus $\bar{K}_p$ obtained by the consistency condition applied to the BS for the stress [Mahan et al., 2011], when BS obeys KH and IH, respectively, for each term. Thus, Eq. (16) can be written as $K_p = \bar{K}_p + \sqrt{2/3}c(\boldsymbol{\alpha}^b - \boldsymbol{\alpha}) : \mathbf{n}$, which is the quintessential feature of any classical BS plasticity model, namely that the plastic modulus equals the bounding plastic modulus (on BS for stress) plus a projected along $\mathbf{n}$ “distance” $(\boldsymbol{\alpha}^b - \boldsymbol{\alpha}) : \mathbf{n}$ of the current back stress $\boldsymbol{\alpha}$ from its evolving image bounding value $\boldsymbol{\alpha}^b$ on the BS for back stress, multiplied by $\sqrt{2/3}c$. Observe that the aforementioned projection along $\mathbf{n}$ of the “distance” between back stress quantities, substitutes for the exactly equal projection along $\mathbf{n}$ of the “distance” $(\bar{\mathbf{s}} - \mathbf{s}) : \mathbf{n}$ between stress quantities of the classical BS development, as already mentioned (the $\bar{\mathbf{s}} - \mathbf{s}$ tensor can easily be visualized, but not shown, in Figure 1). If the pre-multiplying quantity c is fixed, the distance dependence is linear, resembling the linear dependence on the distance of $\boldsymbol{\alpha}$ from its saturation value $\boldsymbol{\alpha}^s$ in Eq. (10) for a typical AF model. However, a linear dependence is inadequate to simulate the stress-strain curves accurately, hence, a non-linear dependence on the distance will be introduced by means of the function c , along the lines of flexibility provided within the BS constitutive framework.

As early as in Dafalias [1975] and Dafalias and Popov [1976], it was recognized that the value of such distance at the initiation of a new plastic loading process plays a very important role in the determination of c . While in the aforementioned references the distances and BS were referring to stress, in the present development the BS and corresponding distances are related to the back stress and so must the corresponding initial values. Along the lines of models developed for soil plasticity [Manzari and Dafalias, 1997, Dafalias and Manzari, 2004] with BS for the back stress, the following definition of c is proposed:

$$c = \frac{h_0}{\sqrt{3/2} < (\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}) : \mathbf{n} >} \quad (17)$$

where h_0 is a parameter and $\boldsymbol{\alpha}_{in}$ is the value of $\boldsymbol{\alpha}$ at initiation of a new loading process, called simply $\boldsymbol{\alpha}$ initial. The factor $\sqrt{3/2}$ serves to maintain the same value of h_0 under uniaxial loading used often for calibration. The Macauley brackets $< >$ in the denominator are used to signify the initiation of a new loading process any time $(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}) : \mathbf{n} \leq 0$, in which case $\boldsymbol{\alpha}_{in}$ is updated to the current value of $\boldsymbol{\alpha}$. Such definition of initiation of a new loading process makes sense because $(\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}) : \mathbf{n} \leq 0$ means that the current $\mathbf{n}$ forms an angle larger than 90 degrees with $\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}$, the latter quantity being a plausible measure of the overall past direction of a loading process that started with back stress at initial location $\boldsymbol{\alpha}_{in}$ and ends up with the back stress at current location $\boldsymbol{\alpha}$. This eventuality does not happen only during a classical loading/unloading/reloading in reference to the YS along a very different $\mathbf{n}$, but can also address drastic change of $\mathbf{n}$ without ever unloading via an almost neutral loading process tangentially around the YS. The important event for the physics of the situation is the large change of the loading direction, portrayed by $\mathbf{n}$, with respect to $\boldsymbol{\alpha} - \boldsymbol{\alpha}_{in}$, and this is achieved by the above scheme using the Macauley brackets. Notice that at the moment of updating $\boldsymbol{\alpha}_{in}$ to the current value of $\boldsymbol{\alpha}$, the denominator in Eq. (17) becomes zero, yielding an infinite value for c , and consequently an infinite value for the plastic modulus K_p of Eq. (16), that causes a much-desired smooth elastic-plastic transition at the initiation of a new

loading process. Possible discontinuities during the updating of $\boldsymbol{\alpha}_{in}$ or the equivalent initial value of the stress distance, for small unloading-reloading events (the so-called overshooting problem), have been addressed in Dafalias [1984, 1986] and Dafalias and Taiebat [2016], but in general one cannot eliminate entirely a possible discontinuity, a shortcoming of any model that uses discrete memory variables like $\boldsymbol{\alpha}_{in}$.

The value of the parameter h_0 is normally fixed, but if a more sophisticated data set, like random cyclic loading or UR must be simulated, the original suggestion of Dafalias and Popov [1976] to render h_0 function of the initial distance $\delta_{in} = \sqrt{3/2} (\bar{\mathbf{s}}_{in} - \mathbf{s}_{in}) : \mathbf{n}_{in}$ between initial values of deviatoric stress $\mathbf{s}_{in}$ and its image stress $\bar{\mathbf{s}}_{in}$ on the BS in multiaxial space, projected on the initial value $\mathbf{n}_{in}$ of $\mathbf{n}$ is adopted, as:

$$h_0 = \frac{a}{1 + b(\delta_{in}/\sigma_r)^m} \quad (18)$$

with σ_r a reference normalization stress quantity, as for example the double radial “size” $2K$ of the BS for stress, or $2(K-k)$ of the BS for back stress; in this work, the value $2K$ was used. Eq. (18) introduces three constants a , b , m , for which closed-form analytical expressions in terms of specific measurements on the experimental stress-strain curves are obtained in Dafalias and Popov [1976] with additional examples in Behraves and Dafalias [2022] and will not be repeated here. Since we are working in a formulation with BS for back stress instead of stress, δ_{in} should be given in terms of back stress quantities. Thus, accounting for Eq. (13) and the relations $\mathbf{s}_{in} = \boldsymbol{\alpha}_{in} + \sqrt{2/3}k_{in}\mathbf{n}_{in}$ and $\bar{\mathbf{s}}_{in} = \boldsymbol{\beta}_{in} + \sqrt{2/3}K_{in}\mathbf{n}_{in}$, one can easily show that $\delta_{in} = \sqrt{3/2} (\bar{\mathbf{s}}_{in} - \mathbf{s}_{in}) : \mathbf{n}_{in} = \sqrt{3/2} (\boldsymbol{\alpha}_{in}^b - \boldsymbol{\alpha}_{in}) : \mathbf{n}_{in}$, to be used in Eq. (18) for h_0 .

The function $\bar{\boldsymbol{\beta}}$ This function defines the KH of the BS according to $\dot{\boldsymbol{\beta}} = \langle \lambda \rangle \bar{\boldsymbol{\beta}}$. One can use a simple linear Prager-type KH, or an AF KH with or without a BCD modification, or anything else one finds most appropriate. As mentioned in the introduction the special KH called the Dafalias/Ranjbari (DR) KH, was found by Hassan and Kyriakides [1992, 1994b] to be very appropriate for UR, because it induces a parallel translation of the bounds towards the direction of ratcheting in the uniaxial case, a feature that helps to simulate accurately UR, and it is therefore adopted here. The analytical expression for $\dot{\boldsymbol{\beta}}$ can be found in Mahan et al. [2011] and is rewritten below as:

$$\dot{\boldsymbol{\beta}} = \langle \lambda \rangle \left(\frac{2}{3} h_{\beta} \mathbf{n} + \frac{1}{2} \sqrt{\frac{2}{3}} c_{\beta} \left[\sqrt{\boldsymbol{\beta} : \boldsymbol{\beta}} \mathbf{n} - \boldsymbol{\beta} \right] \right) = \langle \lambda \rangle \bar{\boldsymbol{\beta}} \quad (19)$$

introducing two constants h_{β} and c_{β} with an obvious definition of $\bar{\boldsymbol{\beta}}$ as the quantity in parentheses of the second member of Eq. (19) which is identical to Eq. (34a) of Mahan et al. [2011]. Based on this definition the term $\bar{\boldsymbol{\beta}} : \mathbf{n}$ of Eq. (16) becomes $\bar{\boldsymbol{\beta}} : \mathbf{n} = (2/3) h_{\beta} + (1/2) \sqrt{2/3} c_{\beta} [\sqrt{\boldsymbol{\beta} : \boldsymbol{\beta}} - \boldsymbol{\beta} : \mathbf{n}]$, i.e., identical to Eq.(34b) of Mahan et al. (2011). In Fig. 2(c) of the foregoing reference, a graphic illustration of the DR KH response in uniaxial loading is

presented, while Eq. (18a) of that same reference yields the uniaxial expression of the above Eq. (19). Upon monotonic loading it exhibits a linear hardening with slope h_β , while upon unloading and reloading in the opposite direction, the hardening becomes an AF non-linear type controlled by both constants h_β and c_β till again switches smoothly to linear hardening in the opposite direction. It is exactly this smooth interchange between linear and non-linear hardening that induces the parallel translation of the bounds in the direction of ratcheting, discovered and used by Hassan and Kyriakides [1992, 1994b] for better simulation of UR.

The function r_α As already mentioned, the r_α substitutes for the fixed BCD modification factor δ that was replaced in Dafalias and Feigenbaum [2011] by the ratio r which is defined in Eq. (11) for an AF model. In our approach, we will consider r_α a function of r , but now the latter must be redefined because the BS center β moves with KH, unlike the center of the implied limit or saturation surface in an AF model that stays fixed at the origin in stress space. The new r definition reflects the proximity of the back stress α to its bounding value α^b on the BS, such proximity measured relatively to β , thus, it is given by:

$$r = \sqrt{\frac{(\alpha - \beta) : (\alpha - \beta)}{(\alpha^b - \beta) : (\alpha^b - \beta)}} \quad (20)$$

A very simple power function of r is proposed for the BCD modification factor r_α as follows:

$$r_\alpha(r) = c_{r1} r^{c_{r2}} \quad (21)$$

introducing two constants c_{r1} , c_{r2} with the only requirement $c_{r1} > 0.5$ at $r = 1$ so that $r_\alpha(1) = c_{r1} > 0.5$. This is necessary in order to avoid for the BCD modification the possible eventuality of the YS center crossing its BS and inducing softening due to a negative plastic modulus resulting from the last term of Eq. (16), as discussed after Eq. (10). The rigorous analytical proof of the foregoing is given in Dafalias and Feigenbaum [2011].

It is now important to investigate the effect of r_α on the YS back-stress translation direction $\bar{\alpha}$ defined in Eq. (15). It can be seen from Eq. (20) that r takes values from 0 to 1, as α varies from β to α^b , respectively. When $r = 0$, it follows from Eq. (21) that $r_\alpha = 0$ and the relative translation of the YS's center α with respect to its image back stress α^b on the BS, is along $\alpha^b - (\alpha : \mathbf{n})\mathbf{n}$ as per Eq. (15); this is a variation of a non-linear Prager-type KH suggested by Burlet and Cailletaud [1986], because from Eq. (13) the α^b is not along $\mathbf{n}$, as it was the case for α^s in Eq. (9) had it been a BCD modification for a typical AF KH. When $r = 1$ and for $c_{r1} = 1$, one has $r_\alpha = 1$, and the aforementioned relative translation is along $\alpha^b - \alpha$, i.e., a KH of the AF or Mroz type. If the c_{r1} is not equal to 1, then at $r = 1$ one has $r_\alpha = c_{r1}$ which yields a variation of the non-linear AF KH rule for the relative translation of α with respect to α^b . Therefore, the effect of the function r_α on the direction of the relative YS center translation with respect to its image back stress α^b is very significant, and essentially the r_α defines this direction as an interpolation between the aforementioned directions of a variation of a non-linear Prager KH rule when $r = 0$ and

a variation of the non-linear AF KH rule when $r = 1$. The absolute translation $\bar{\alpha}$ is also affected by the KH rule chosen for the BS, namely the tensor $\bar{\beta}$, as well as by the IH of both YS and BS yielding a component along $\mathbf{n}$, as per the first two terms in the brackets of the fourth member of Eq. (15), respectively.

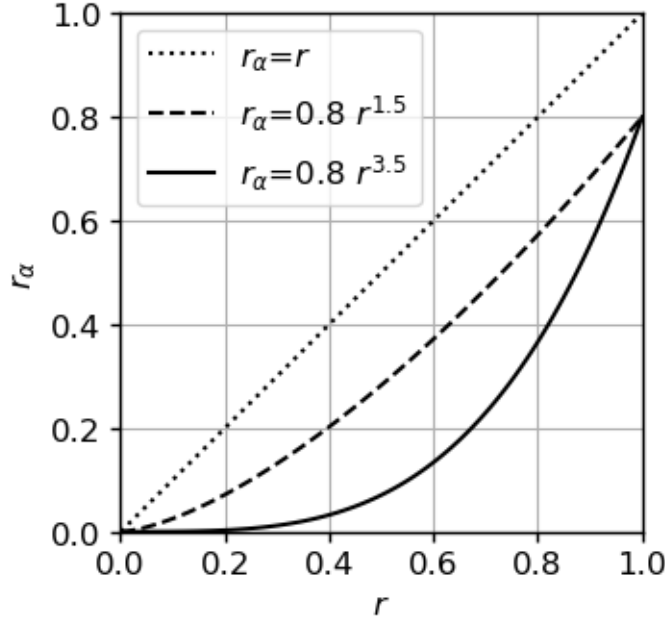


Fig. 2. The power law function for r_α to show how the weighting between AF and Prager KH can vary with the choice of parameters.

It is now clear that the use of r_α rather than r or a fixed δ , allows for a more flexible interpolation of the variations of the nonlinear Prager and AF KH rules as per Eq. (15). In Figure 2 the variation of the “interpolating” BCD modification parameter r_α with r is shown for various choices of the constants c_{r1} , c_{r2} including the case of $c_{r1} = c_{r2} = 1$, namely the case where $r_\alpha(r) = r$. Based on attempted simulations for these various choices it was concluded that ratcheting would be better predicted if the direction of $\bar{\alpha}$ is closer to $\mathbf{n}$ for a larger part of each cycle after the initiation of a new loading process (load reversal). It means that r_α must be close to 0 for a wider initial part of the range in the $[0, 1]$ r domain, after the loading reversal, compared to what could be achieved with the simple $r_\alpha = r$ equation. Accounting for the fact $0 < r < 1$, this can be achieved with the power function $r_\alpha(r) = c_{r1}r^{c_{r2}}$ with $c_{r2} > 1$, and the larger the c_{r2} is, the more pronounced this effect is. This is illustrated in Figure 2 by the dashed line with $c_{r2} = 1.5$ against the solid line with $c_{r2} = 3.5$, where there is a larger range where r_α is near zero for the latter case.

Model calibration

Uniaxial and multiaxial ratcheting simulations and comparisons with published experimental data are presented in the next section. Those experiments were published in Hassan and Kyriakides [1992], Hassan et al. [1992] and Corona et al. [1996] and performed on three different carbon steels, namely CS1018, CS1020 and CS1026, respectively. The input model

constants for each material, together with some notes for their determination, are presented in this section.

Table 1 presents the model constants used for the simulations of both UR and MR for all three materials. In all the experiments, the specimens were cyclically stabilized by strain symmetric cycles in the range of $\pm 1\%$. For details, the reader is referred to the introduction section of Hassan and Kyriakides [1992], Hassan et al. [1992] and Corona et al. [1996]. This cyclic stabilization process was not explicitly simulated in either the above references or here, and the simulations start with cyclic ratcheting from the stabilized state of each specimen. The model constants for UR are derived based on the stabilized strain symmetric cycles results, thus, they reflect the stabilized state of the material. Hassan and Kyriakides [1992], Hassan et al. [1992] and Corona et al. [1996] provided such material constants for a Dafalias/Popov bounding surface plasticity model, with the plastic modulus resulting from the Dafalias/Ranjbari KH for the BS, while an AF KH direction was used independently for the YS (see detailed comments made in Introduction). These model constants constitute a basis for a trial-and-error process of constants determination for the present model because for uniaxial conditions the two formulations are very close to each other, and the constants end up being the same or slightly modified.

Table 1. Model constants for three steel materials, CS1018, CS1020 and CS1026.

Equation	Symbol	CS1018	CS1020	CS1026
-	ν	0.33	0.33	0.33
-	E	26320 ksi	25125 ksi	26320 ksi
1	k_{in}, k_s	23, 23 ksi	25, 25 ksi	19, 20.2 ksi
12	K_{in}^*, K_s	43, 43 ksi	60.15*, 60.15 ksi	38.5, 38.5 ksi
7	c_k	0	0	250
14	C_k	0	0	0
18	a, b, m	91100, 27, 2	146200, 40, 3	71100, 27, 2
19	c_β	30	73.5	15
19	h_β	650	700.35	300
21	c_{r1}, c_{r2}	0.8, 3.5	0.8, 3.5	0.8, 3.5
In few sets of uniaxial experiments, an adjusted K_{in}^* was used. In those cases, the adjusted values are reported explicitly in the relevant locations.				

The elastic Young's modulus E can be easily measured from the slope of the elastic stress-strain curve of the stabilized cycle. A single Poisson's ratio $\nu = 0.33$ is used for all three carbon steels, following the proposition of Hassan and Kyriakides [1992], Hassan et al. [1992]. The initial value k_{in} of the size of the YS that corresponds to the initial yield stress point in uniaxial loading, is measured approximately at the smooth transition from linear elastic to curved elastoplastic stress-strain curve. The initial value K_{in} of the size of the BS can be measured approximately as the image stress point of the initial yield stress point on the bound, such bound representing the significantly "slower" non-linear part of

the stress-strain curve with which the stress merges eventually upon continual loading. No isotropic hardening of the YS and BS are applied for the materials CS1018 and CS1020, thus, the constants c_k , and c_K of Eqs. (7) and (14), respectively, are set equal to zero, and the saturation values k_s and K_s of Eq. (14) are set equal to their initial values, as reported in Table 1, and plotted with dashed lines. For CS1026 a small IH was introduced only for the YS, because it was found that a very small increase in the size of the YS (from $k_{in} = 19$ ksi to $k_s = 20.2$ ksi) improves the simulations, especially for UR; correspondingly a value of $c_k = 250$ was used. The saturation value of K_s is set equal to its initial value as reported in Table 1.

The constants a , b and m of Eq. (18) determine how the parameter h_0 changes, which controls the curvature of the non-linear part of the stress-strain curve, as a function of a measure of the initial distance in stress space between stress and its image on the BS for stress at the initiation of a new loading process, that has been observed to strongly affect the shape of the curve. Instead of using the available closed-form analytical expressions for these constants that require special measurements as mentioned after Eq. (18), it was decided to use a trial-and-error process to define an optimum shape for the highly non-linear part of the stress-strain curve before the stress merges with the image-stress on the bound. This was facilitated by the simple numerical implementation of the model in uniaxial loading and the knowledge of the effect h_0 has on the curve shape.

The constants h_β and c_β for the kinematic hardening of the BS are determined by trying to optimize the simulations of the UR response. The value of h_β that yields a linear KH, reflects the slope of the bound in stress-strain space, thus, the higher its value is the smaller will be the parallel translation of the bound along the direction of ratcheting and, hence, the larger the number of cycles to reach a certain value of accumulated strain. A similar conclusion applies to the constant c_β that in conjunction with h_β controls the AF type non-linear KH for the BS, in reverse loading.

For the power law in Eq. (21), defining the BCD weighting parameter r_α , the values $c_{r1} = 0.8$ and $c_{r2} = 3.5$ led to successful simulations for all the materials tested in this project. As shown in Fig. (2), these values maintain r_α close to zero for a large initial portion of the range $0 < r < 1$, thus, inducing a larger contribution of the variant of Prager non-linear KH type than that of an AF KH, on the relative overall KH of the YS relatively to the BS. Such initial predominance of Prager KH is necessary for the more accurate simulation of the MR response.

Model validation

The following notation is used in all subsequent plots for UR and MR:

- The σ_x signifies the axial stress, σ_{xm} signifies the mean values of σ_x during cyclic loading, and σ_{xa} half the peak-to-peak amplitude of the applied cyclic σ_x .
- The σ_θ signifies the circumferential stress, induced by pressure, $\sigma_{\theta m}$ the mean value of the cyclic σ_θ , and $\sigma_{\theta a}$ half the peak-to-peak amplitude of the cyclic σ_θ .
- The ε_x signifies the axial strain, ε_{xc} half the peak-to-peak amplitude of the cyclic ε_x , while the ε_{xp} signifies the peak ε_x .

- The ε_θ signifies the circumferential strain, and $\varepsilon_{\theta p}$ signifies the peak ε_θ .
- A superposed bar on a stress symbol implies normalization by the yield limit.

Model validation in uniaxial ratcheting (UR) In this section, UR model simulations are compared with data from Hassan and Kyriakides [1992]. The uniaxial ratcheting experiments are stress-controlled with varied amplitude of stress cycles and positive mean stress values. The mean stress and amplitude are normalized by the yield stress which is approximately $\sigma_o = 73$ ksi (based on a 0.2% strain offset yield stress) for CS1020, and $\sigma_o = 40.50$ ksi (based on the “plateau” stress) for CS1026.

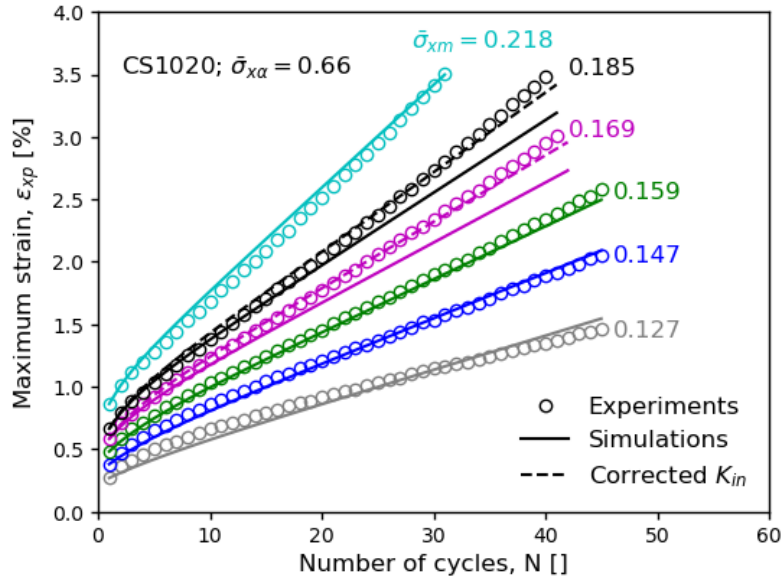


Fig. 3. Uniaxial ratcheting experiments on CS1020. Same applied stress, varying mean stress.

Material CS1020 Two sets of UR experiments on CS1020 specimens are simulated in Figure 3 and Figure 4. In the first set, the cycling amplitude was kept constant, and the mean stress was varied (Figure 3), while in the second set, the mean stress is constant, and the cycling amplitude is varied (Figure 4). In both figures, the experimental data are presented with open circles, and the simulation results with colored dashed and solid lines. Before the discussion of the numerical simulation results and their comparison against the experimental data, two key aspects must be discussed; first, the applied correction of the predicted maximum axial strain, ε_{xp} , at the first cycle, and second, the difference between the dashed and solid lines that are used in some cases for the numerical simulations in Figure 3 and 4.

Regarding the first aspect, a direct specification of the maximum axial strain ε_{xp} , that is generated at the end of the first cycle ($N = 1$) is used in the numerical simulations. Practically, this means that the very first point at $N = 1$ in all the curves of the numerical simulations of Figure 3 and 4 is directly specified to match the corresponding experimental

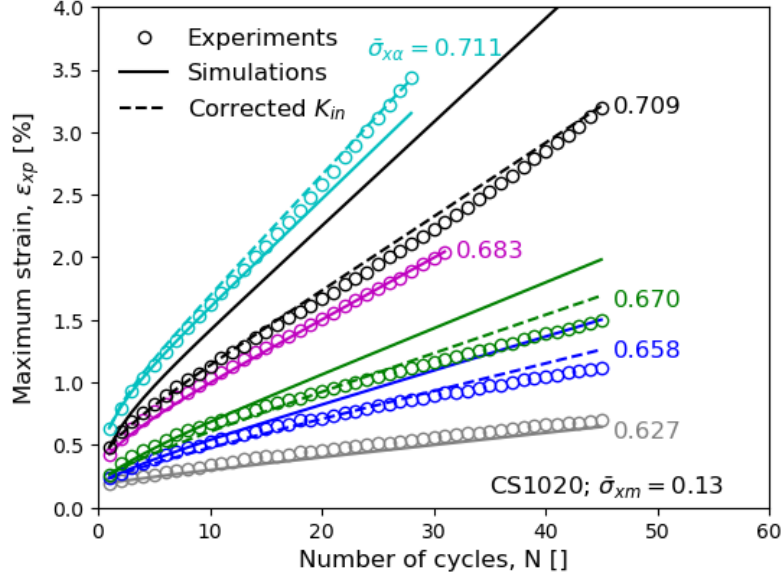


Fig. 4. Uniaxial ratcheting experiments on CS1020. Same mean stress, varying applied stress.

data point exactly. This same process was followed in Hassan and Kyriakides [1992], Hassan et al. [1992] and allowed them to focus on the predicted ratcheting rate after the first cycle ($N > 1$). The reason is that the symmetric cyclic stabilization process of the sample prior to ratcheting is not explicitly simulated, and consequently, the initial state of the material at the start of ratcheting is unknown. Had the stabilization process been simulated, it would have produced slightly different initial points at the end of the first ratcheting cycle than what is shown in the data due to model error in simulating the stabilization and the first cycle.

Regarding the second aspect, notice that there are cases where the numerical simulation results are presented with both solid and dashed lines for the same test (e.g., the purple solid and dashed lines for the case $\bar{\sigma}_{xm} = 0.169$ in Figure 3). When the numerical simulation results are with solid lines, the model constants are those in Table 1. When a dashed line is used for a simulation, the results with those constants did not match well the experimental data and an adjustment of the value of K_{in} only (single parameter adjustment) is applied for higher model accuracy. More specifically, in Figure 3, the tests with $\bar{\sigma}_{xm} = 0.169$ and 0.185 are also simulated with a slightly adjusted $K_{in} = 59.65$ ksi value compared to $K_{in} = 60.15$ ksi in Table 1, and plotted with dashed lines. In Figure 4, the tests with $\bar{\sigma}_{xa} = 0.658$, 0.670 , 0.709 and 0.711 are also simulated with adjusted $K_{in} = 60.912$, 60.912 , 62 and 59.5 ksi values, respectively, and plotted with dashed lines. It is worth noting that Hassan and Kyriakides [1992, 1994b] reported the results of their simulations with only adjusted, when needed, values of K_{in} . In this work, the results with both adjusted and non-adjusted unique K_{in} values, the latter given in Table 1, are presented for completeness.

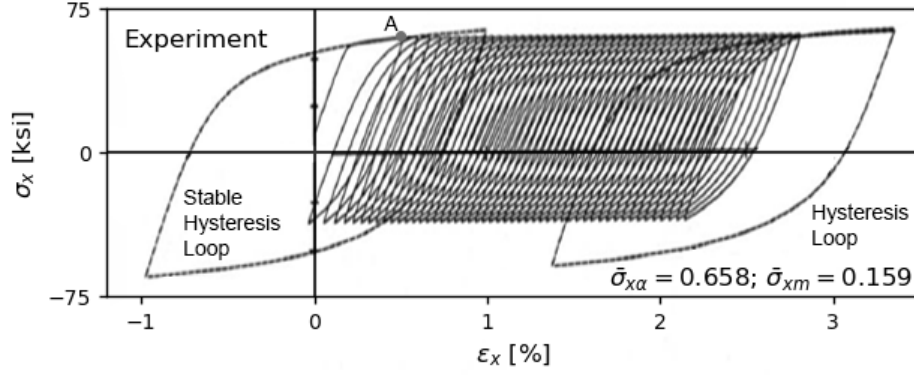
The results in Figure 3 suggest that the model predicts with high accuracy the UR. Moreover, it successfully considers the effect of the applied mean stress, since it predicts with acceptable accuracy the ratcheting rate for the 6 tests of varying mean stress in Figure

3, with a single set of model constants (solid lines). When the adjusted value of K_{in} is applied for the cases of $\bar{\sigma}_{xm}=0.169$ and 0.185 (dashed lines), the accuracy is improved and agreement between the predictions and the experimental data is excellent.

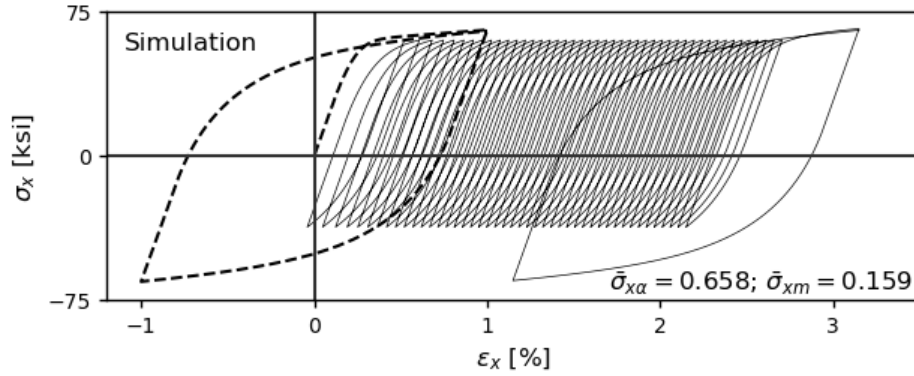
Before we address the results of Figure 4, it is instructive to clarify the three stages of the simulated by the BS model stress-strain relationship, namely, the linear elastic loading (Stage 1, $f < 0$ and $F < 0$); the non-linear elastoplastic loading with axial stress on the YS but within the BS (Stage 2, $f = 0$ and $F < 0$); and the non-linear elastoplastic loading with axial stress on the YS and on the BS (Stage 3 with $f = 0$ and $F = 0$). In the case of the UR experiments of Figure 4, if the maximum axial stress to be applied $\sigma_x = \sigma_{xm} + \sigma_{x\alpha} > k_{in}$ then Stage 2 is initiated, and if $\sigma_x = \sigma_{xm} + \sigma_{x\alpha} > K_{in}$ Stage 3 is initiated because β has a significant smaller value compared to K_{in} and the stress (and thus the YS) reaches the image stress (and thus the BS) when $\sigma_x = K_{in} + \beta$. Stage 1 is always simulated after the initiation of a new loading event. Clearly the same increment of axial stress $d\sigma_x$ acting during each of the three stages would cause a significantly larger increment of axial strain $d\varepsilon_x$ in Stage 3, compared to Stage 2 and 1, in that order. This can be visualized by looking at the initial loading part of the simulated Stable Hysteresis Loop of Figure 5b (black dashed line), where the slope of the stress-strain curve is much flatter just before unloading (Stage 3).

Addressing now the results of Figure 4, let us focus on the cases of $\bar{\sigma}_{xa} = 0.627, 0.658, 0.670$ and 0.683 (grey, blue, green, and purple lines, respectively), where the maximum applied axial stress $\sigma_x = \sigma_{xm} + \sigma_{x\alpha}$ is 55.26, 57.52, 58.4 and 59.34 ksi, respectively, lower than the calibrated $K_{in} = 60.15$ ksi (Table 1), hence, also $\sigma_x = \sigma_{xm} + \sigma_{x\alpha} < K_{in} + \beta$. The simulated stress-strain relationship, in all the ratcheting cycles, is characterized by loading Stages 1 and 2 (linear elastic, and non-linear elastoplastic on the YS but within the BS) and thus the model did not predict as large increments of strains as it would had Stage 3 been activated, as explained before. This simulated response was qualitatively accurate with what was observed in the experiments, and thus those cases were simulated with adequate accuracy when the non-adjusted common K_{in} is used (solid lines). When a small adjustment of K_{in} is applied in the cases of $\bar{\sigma}_{xa} = 0.658$ and 0.670 the accuracy increased (dashed lines), not because the response is qualitatively different by entering for example in the large strain regime of Stage 3, but because the ratcheting slope is slightly improved within each cycle at Stage 2 by making the distance of the current axial stress to the image stress slightly larger in average.

In the last two cases of $\bar{\sigma}_{xa} = 0.709, 0.711$, the maximum axial stress reached in each of the experiments is 61.247 and 61.393 ksi, respectively. When those two experiments are simulated, the maximum applied axial stress per cycle is larger than the calibrated $K_{in} = 60.15$ ksi. Thus, in both cases, the model predicts large strain increments since part of loading is characterized by Stage 3, and thus, significantly larger accumulated strain at the end of each cycle is predicted, as demonstrated by the black and cyan solid lines in Figure 4 for the cases of $\bar{\sigma}_{xa} = 0.709$, and 0.711 , respectively. For the $\bar{\sigma}_{xa} = 0.711$ case (cyan open circles), the simulation results are in good agreement with the experimental results, and the simulation of large strain increments within each cycle (loading Stage 3) explains the steep slope of the cyan experimental and numerical curves. A small adjustment to the value of K_{in} (dashed cyan line) improves the slope of the numerical curve not because the response gets qualitatively different, but by improving the stress-strain curve within each cycle by making the distance of the current axial stress to the image stress slightly larger in average.



(a)



(b)

Fig. 5. Stress-strain response of CS1020 in uniaxial ratcheting. a) Experiment after Hassan and Kyriakides [1992]; b) Simulation.

However, in the case of $\bar{\sigma}_{xa} = 0.709$, the simulation with non-adjusted K_{in} (black solid line) over-predicts the accumulated total axial strain compared to the corresponding experiment, due to the fact that the model predicts large total strains within each cycle (loading Stage 3). However, this is not observed in the experimental results and thus, a significant adjustment (increase) of $K_{in} = 62$ ksi was needed to improve accuracy (dashed black line in Figure 4), by practically forcing the elastoplastic response of the model to remain within Stage 2, i.e. the total applied axial stress 61.247 to remain smaller than $K_{in} = 60.15$ ksi. Note that Hassan and Kyriakides [1992] also applied similar adjustments to K_{in} to increase the accuracy of the Dafalias/Popov BS model in their work.

A full experimental stress-strain curve for uniaxial ratcheting is plotted in Figure 5a, taken from Hassan and Kyriakides [1992], and its simulation is shown in Figure 5b. The strain-symmetric stabilized cycle of 1% axial strain amplitude is plotted with a dashed line, while at the end of the ratcheting cycling another strain-symmetric cycle of the same amplitude is plotted with a solid line (Figure 5a). The model can simulate all the features of this complicated loading history, and the numerical results compare very well with the experimental data.

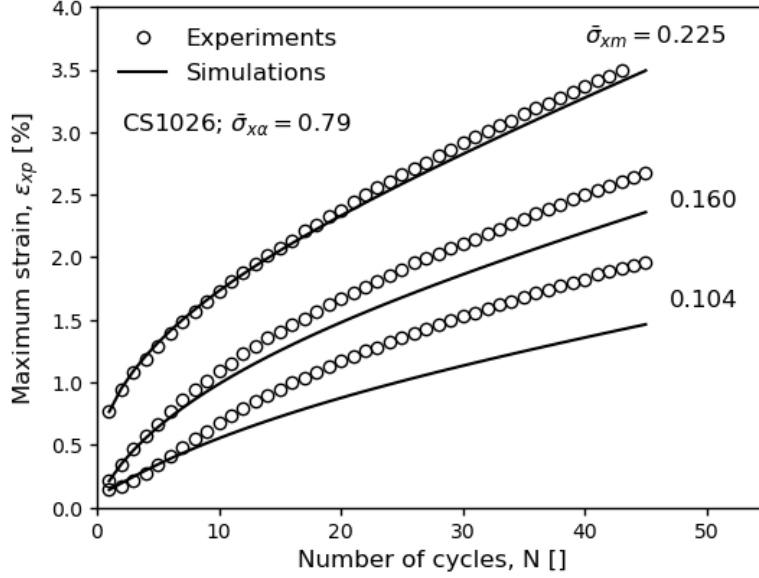


Fig. 6. Uniaxial ratcheting on CS1026. Constant applied stress. Data after Hassan and Kyriakides [1992]

Material CS1026 Nine additional UR experiments are simulated in this section and compared with experimental data on CS1026 presented in Hassan and Kyriakides [1992]. In those experiments, a single set of model constants (including a single K_{in}) is used, as presented in Table 1. Figure 6 presents the simulations and experimental data for three tests with the same cyclic amplitude but varying mean stress. The model shows good accuracy compared to the experimental data, except for the simulation of the experiment with the low $\bar{\sigma}_{xm} = 0.104$, which is significantly less accurate than the other two. Figure 7 presents four tests with constant mean stress and varying cyclic amplitude. With a single set of parameters, the model shows good accuracy for the three tests, except the test when $\bar{\sigma}_{xa} = 0.771$, where it under-predicts significantly the maximum accumulated strain per cycle.

Figure 8 presents the experimental data and numerical simulations of a UR test on CS1026 with $\bar{\sigma}_{xa} = 0.79$ and $\bar{\sigma}_{xm} = 0.193$, and a large number of cycles ($N = 90$). These results suggest that the model can be very accurate even when a high number of cycles are applied. Figure 9 presents the experimental data and numerical simulations of a UR with varying $\bar{\sigma}_{xa}$ during the test and mean normalized applied stress $\bar{\sigma}_{xm} = 0.16$. The agreement between the simulation and experimental data is very good. The full stress-strain relationship of this simulation and the comparison with the experiment is presented in Figure 10.

Model validation in multiaxial ratcheting (MR) In this section, multiaxial ratcheting simulations are compared with experimental data on CS1018 and CS1026 specimens, reported in Hassan et al. [1992] and Corona et al. [1996]. Six simulated loading histories are illustrated in Figure 11, all in the space of the axial strain ε_x , versus circumferential stress σ_θ , which is induced by pressure. The loading types of Figure 11 can be described as follows: (1) axial strain-symmetric cycling at constant pressure, (2) inclined path, positive slope, (3) inclined path, negative slope, (4) bowtie path, (5) reverse bowtie path, and (6)

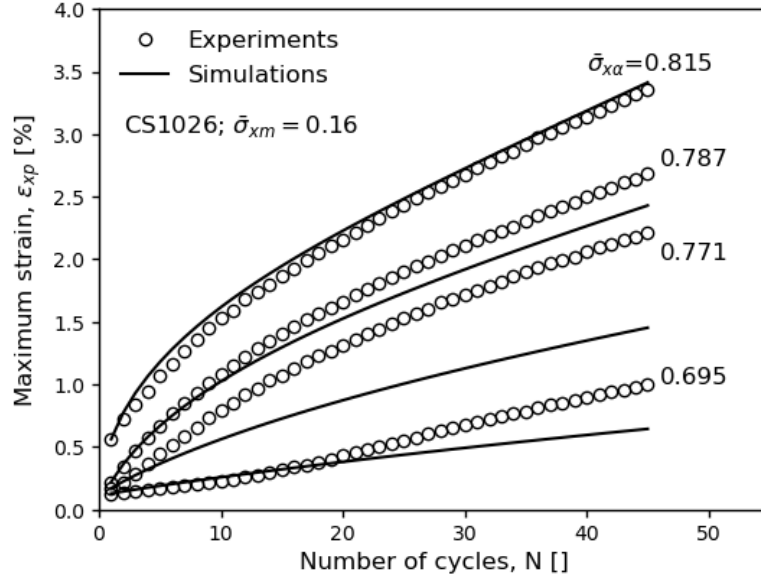


Fig. 7. Uniaxial ratcheting on CS1026. Constant mean stress. Hassan and Kyriakides [1992]

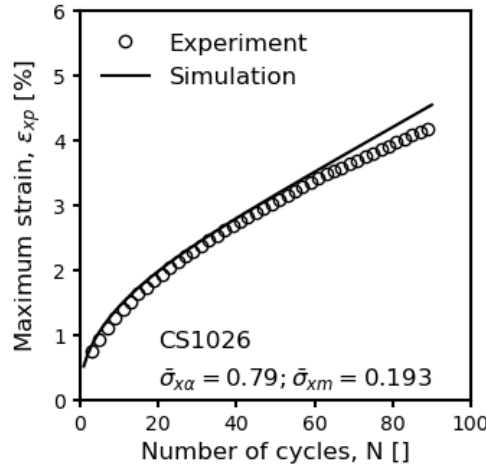


Fig. 8. Uniaxial ratcheting on CS1026 with a large number of cycles. Data after Hassan and Kyriakides [1992]

hourglass path. Both CS1018 and CS1026 specimens are tested after a cyclic stabilization process, as explained in the uniaxial section. In the simulations, only the multiaxial ratcheting stages are simulated, but the stabilized state of the materials is reflected in the chosen model constants and simulations are adjusted such that the initial point matches that seen experimentally, as explained in the uniaxial section, to account for any previous loading.

Material CS1026 The model constants for the multiaxial ratcheting simulations are the same as those used for CS 1026 uniaxial ratcheting simulations and are presented in Table 1. Figure 12 presents the experimental data and numerical simulations for four axial

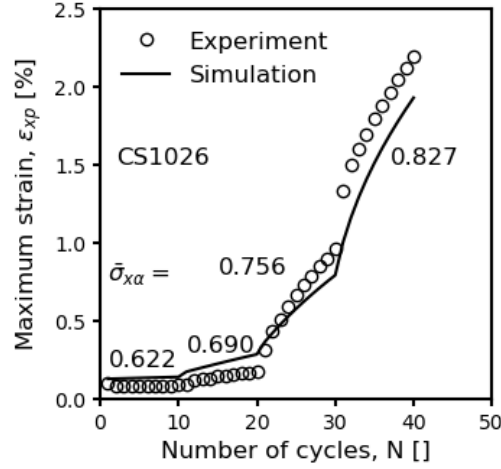
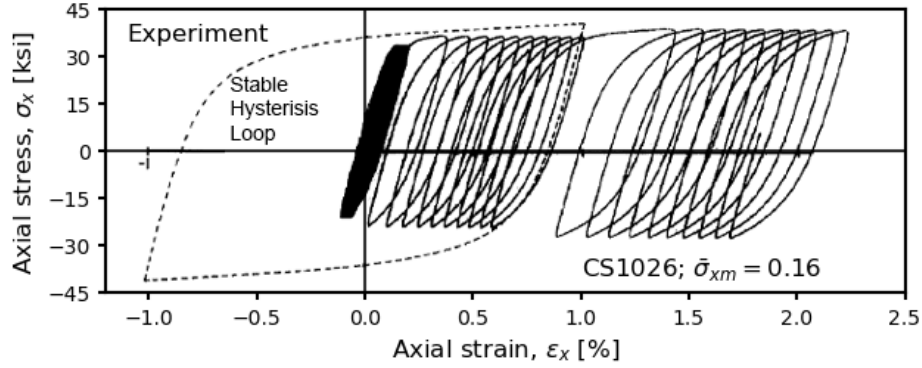
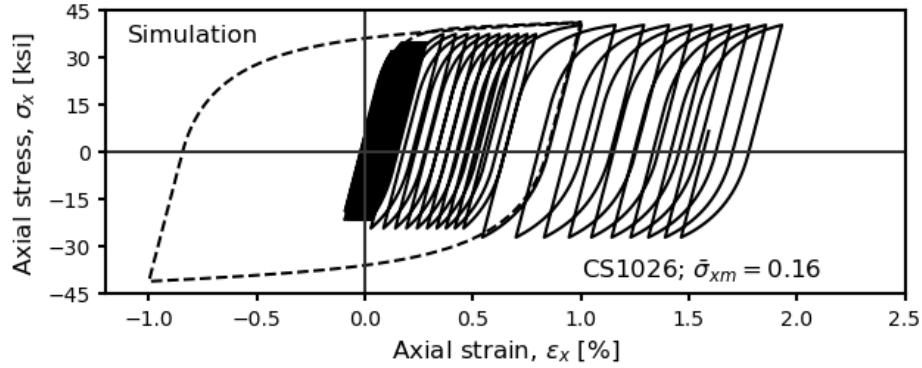


Fig. 9. Uniaxial ratcheting on CS026 with varying axial stress amplitude. Hassan and Kyriakides [1992]



(a)



(b)

Fig. 10. Stress-strain relationship in uniaxial ratcheting with varying applied axial stress for CS1026. Applied stress history of Figure 9. a) Experiment after Hassan and Kyriakides [1992] b) Simulation.

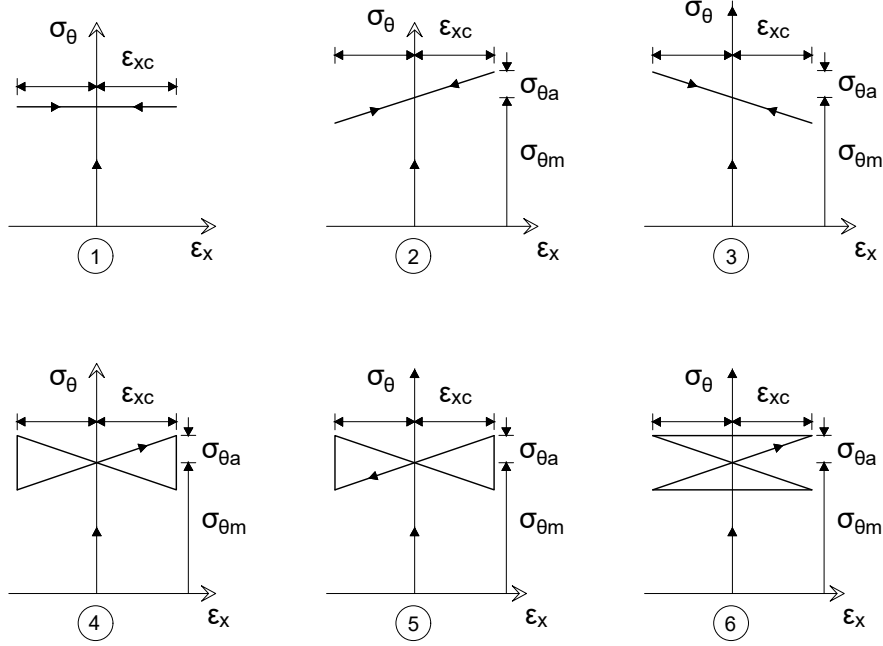


Fig. 11. Cycling loading histories (after Corona et al., 1996).

strain-symmetric cycling tests at constant pressure (Type 1 in Figure 11), for the same applied axial strain amplitude of $\varepsilon_{xc} = 0.5\%$ at various circumferential stresses σ_θ , each one obtained by a corresponding constant internal pressure of the tubular specimen. In Figure 12, as well as in all subsequent figures, the stress quantities are normalized by the yield stress value, hence, the bar over the stress symbol $\bar{\sigma}$. Figure 13 presents the experimental data and numerical simulations for three axial strain-symmetric cycling tests at constant pressure (again Type 1 in Figure 11), but now with the same $\bar{\sigma}_\theta = 0.24$ at various axial strain amplitudes, ε_{xc} . Figure 14 presents the experimental data and simulation of the same Type 1 loading, with $\varepsilon_{xc} = 0.5\%$ and $\bar{\sigma}_\theta = 0.178$, and a high number of cycles ($N = 69$). In all three cases, the model predicts the experimental data with very good accuracy. Finally, Figure 15 presents the simulated strain-path $\varepsilon_x - \varepsilon_\theta$ with $\varepsilon_{xc} = 0.5\%$ and $\bar{\sigma}_\theta = 0.245$ and the comparison against the corresponding experiment. It can be concluded that the model predictions are quite successful.

Two of the more advanced loading histories, namely the inclined path with positive slope (Type 2 in Figure 11) and bowtie path (Type 4 in Figure 11), were also simulated and compared with the experimental data in Corona et al. [1996]. In both simulations, $\varepsilon_{xc} = 0.5\%$, $\bar{\sigma}_{\theta a} = 0.06$ and $\bar{\sigma}_{\theta m} = 0.24$. The experimental data and the simulation results versus the number of cycles N , are plotted in Figure 16. An excellent fit of the numerical simulations with the experimental data was achieved. In addition, for those two loading histories, the simulated strain-paths and the corresponding experimental data are plotted in Figure 17 (Type 2) and Figure 19 (Type 4), while the simulated stress-paths and corresponding experimental

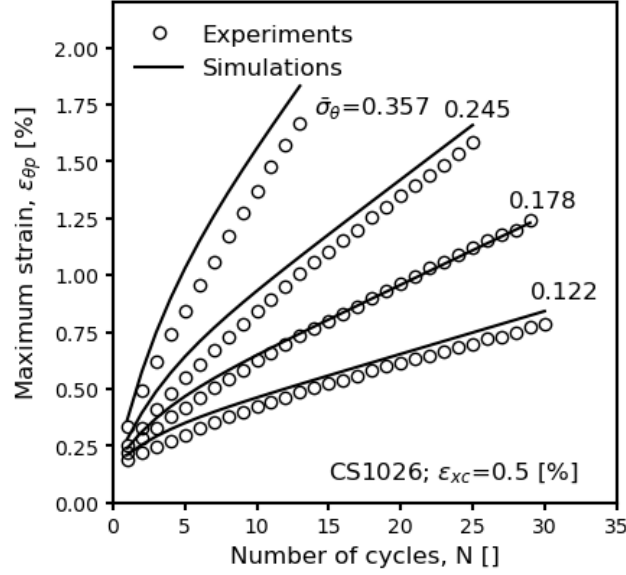


Fig. 12. Axial strain-symmetric cycling at constant pressure. Type 1 of Figure 11. Constant applied axial strain across tests. Material CS1026. Data after Hassan et al. [1992]

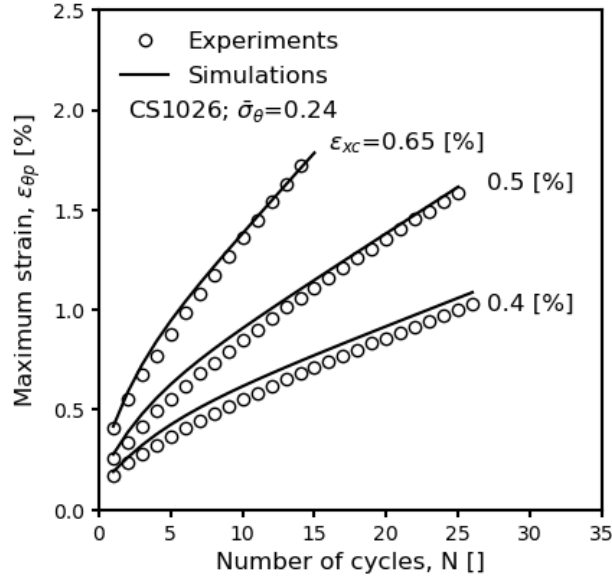


Fig. 13. Axial strain-symmetric cycling at constant pressure. Type 1 of Figure 11. Constant applied circumferential stress across tests. Material CS1026. Data after Hassan et al. [1992].

data are plotted in Figure 18 (Type 2) and Figure 20 (Type 4). The model reproduces stress and strain paths with high accuracy.

Finally, a set of simulations and their comparison with the experimental data for the case of axial cyclic stress-symmetric loading with constant internal pressure are presented in Figure 21. Those tests induce double ratcheting, i.e., the plastic strain accumulates in both the axial and circumferential directions, simultaneously. This is possibly the most difficult

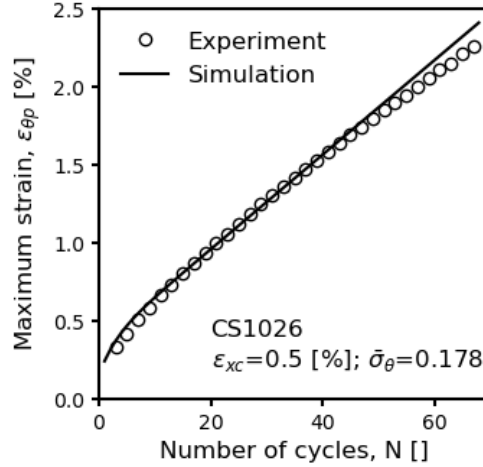


Fig. 14. Axial strain-symmetric cycling at constant pressure. Type 1 of Figure 11. A large number of cycles. Material CS1026. Data after Hassan et al. [1992].

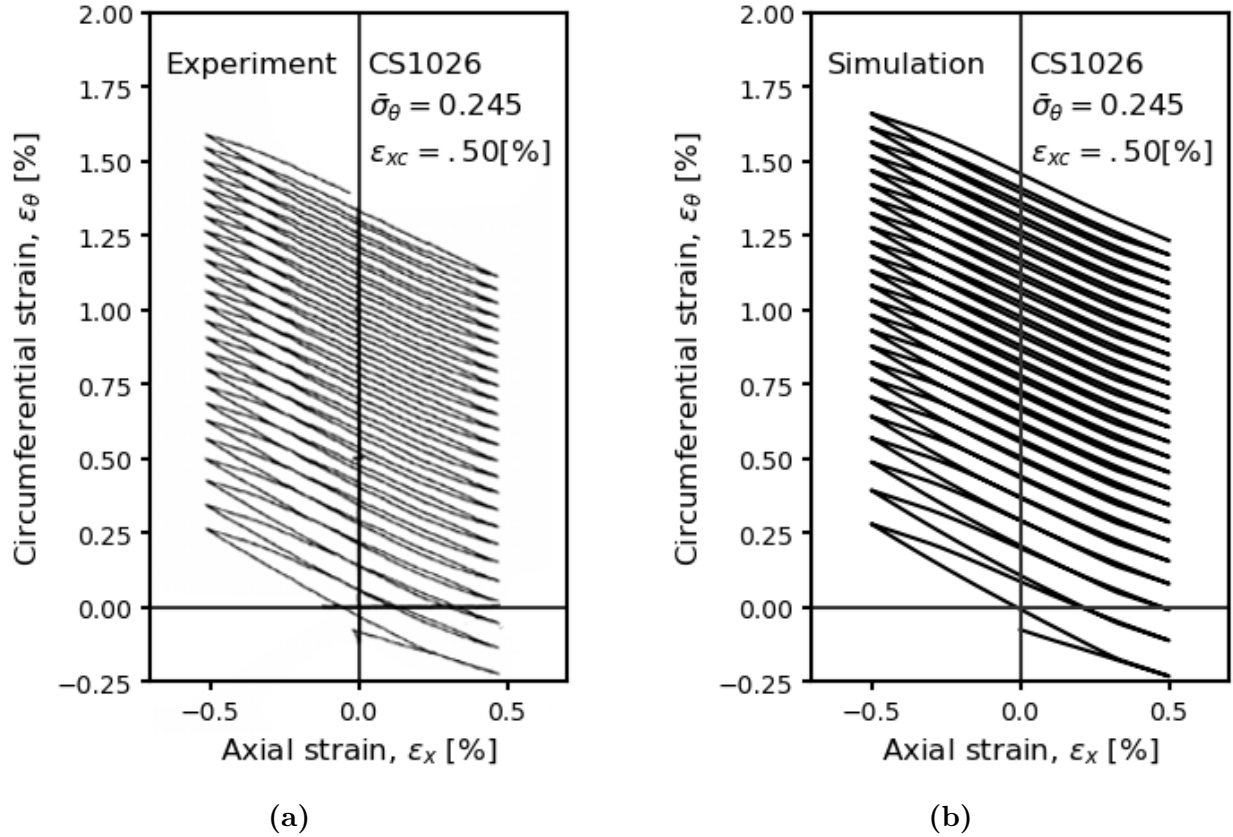


Fig. 15. Strain-path for axial strain-symmetric cycling at constant pressure. Type 1 of Figure 11. Material CS1026. a) Experiment after Hassan et al. [1992]; b) Simulation.

loading to simulate. Some of the simulations are adequately accurate, but several others are not very satisfactory. Overall, though, the model performs better than the original

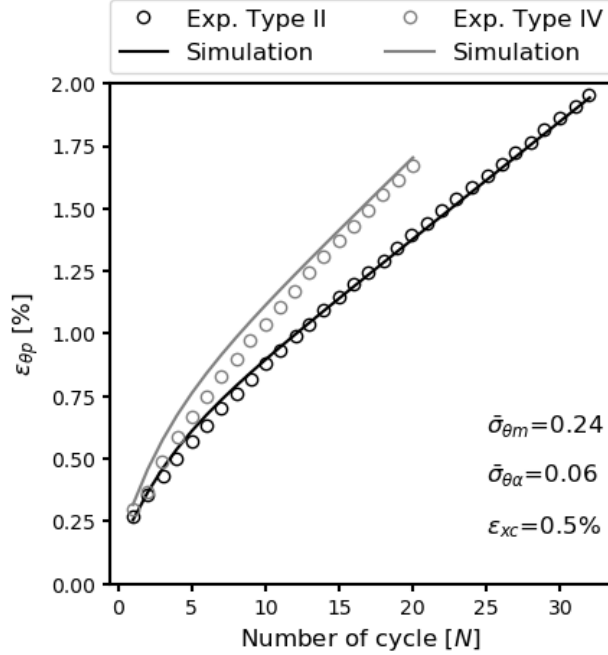


Fig. 16. Inclined path, positive slope, Type 2 in Figure 11, and bowtie, Type 4 in Figure 11, multiaxial ratcheting data and simulations. Material CS1026. Data after Corona et al. [1996].

Dafalias/Popov BS model and other state-of-the-art constitutive models in the literature [Welling et al., 2017, Hassan et al., 1992]. Future research is needed to accurately predict the material behavior under this loading condition.

Material CS1018 Three MR experiments, namely the inclined path, negative slope (3), the reverse bowtie path (5), and hourglass (6) shown in Figure 11, are simulated for CS1018 specimens. Similarly, to the previous cases, the stabilization cycling of the specimens is not simulated, however, the choice of model parameters reflects the stabilized state of the material, and the initial point is adjusted to match the experimental findings. The results are presented in Figure 22. A very good agreement of the simulation results with the experimental data is observed when plotted against the number of cycles N . In addition, the simulated strain paths of those three loading conditions are presented in Figure 23 (hourglass (6), 17 cycles), Figure 25 (inclined path, negative slope (3), 40 cycles) and Figure 27 (reverse bowtie (5), 20 cycles). Similarly, the simulated stress-paths are presented in Figure 24 (hourglass (6)), Figure 26 (inclined path, negative slope (3)), and Figure 28 (reverse bowtie (5)). The corresponding experimental data from Corona et al. [1996] are juxtaposed to the simulations of Figures 23-28 for comparison. The conclusion is that model can predict very accurately these observed stress and strain paths.

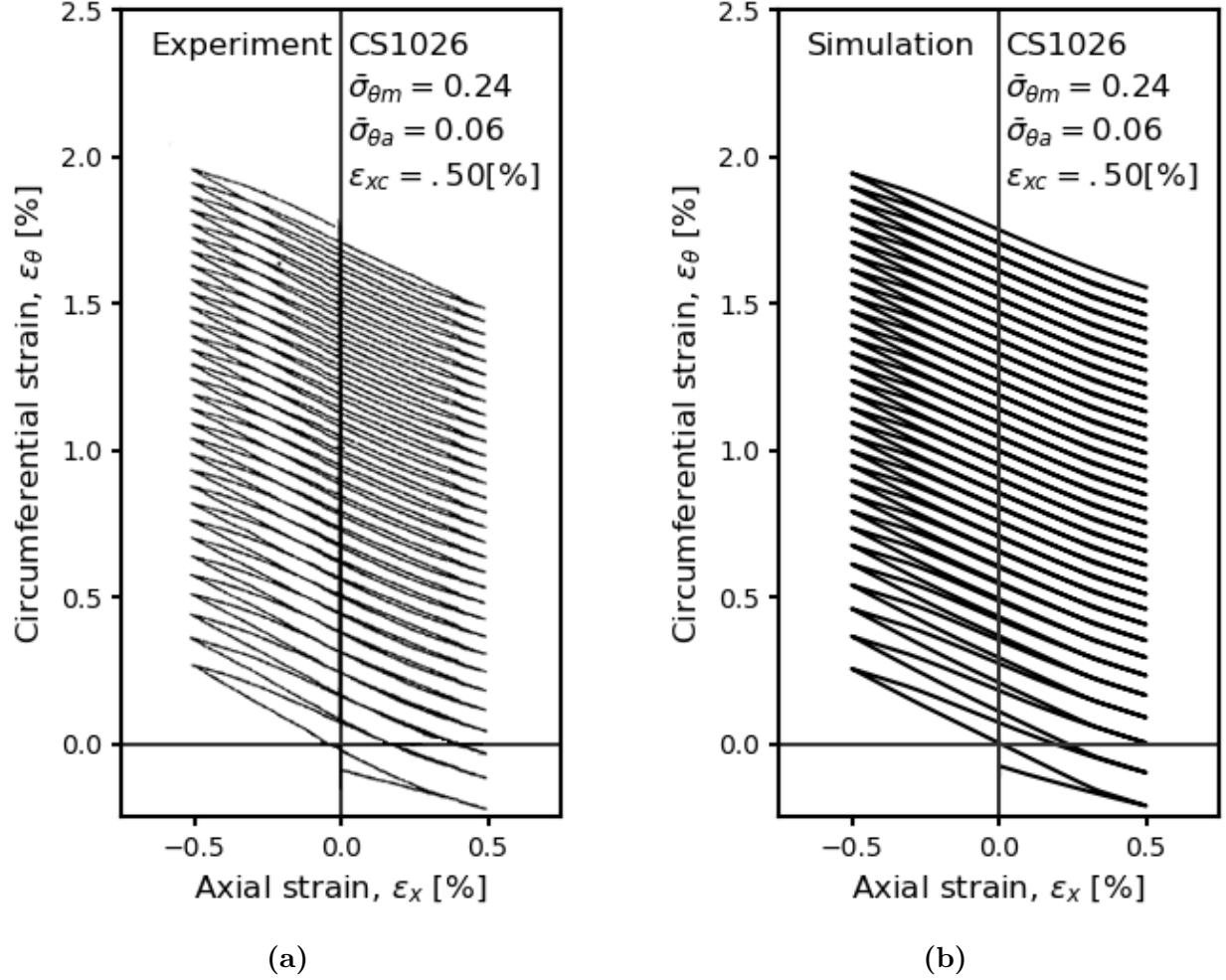


Fig. 17. Strain-path for an inclined path with positive slope. Type 2 in Figure 11. Material CS1026. a) Experiment after Corona et al. [1996]; b) Simulation.

Conclusion

The new SANIMETAL-BCD Bounding Surface (BS) model is based on a simple modification of the existing SANISTEEL BS model [Mahan et al., 2011], without introducing additional complex constitutive features, such as memory surfaces in stress space or non-hardening regions in strain space proposed mainly in conjunction with MAF models. Still, it can provide equally good or better simulations than these other more complex constitutive models for UR and MR cases, without losing the simulative capabilities for the much simpler monotonic or symmetric cyclic loading. This is because all the endemic advantages of BS plasticity are still present in the new model, in particular, the following two:

1. The decoupling of the value of the plastic modulus from the direction of the KH of the YS, and
2. The option to choose any kind of KH for the BS that is deemed necessary for more accurate simulation, without changing the other constitutive features.

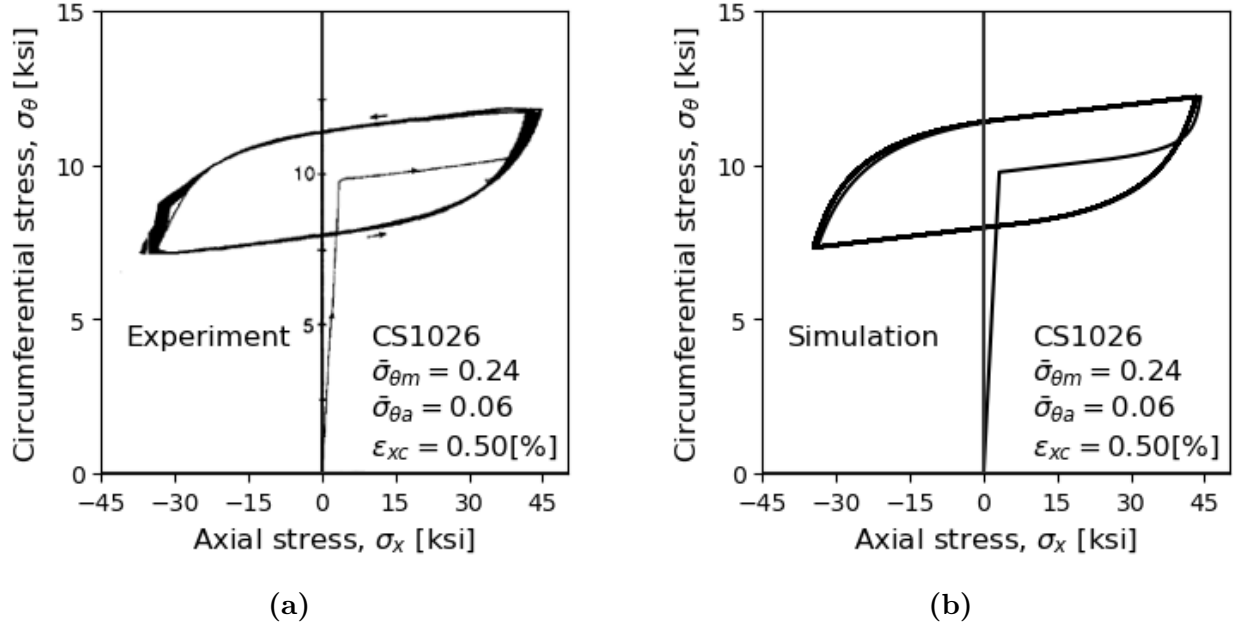


Fig. 18. Stress-path for an inclined path with positive slope. Type 2 in Figure 11. Material CS1026 a) Experiment after Corona et al. [1996]; b) Simulation.

One then can identify the following novel constitutive ingredients of the SANIMETAL-BCD model, in order of appearance. First, the BS acts as a changing bounding locus in stress space for the back stress (center of the YS) rather than the stress itself; this is a feature already introduced in soil models (Manzari and Dafalias, 1997, Dafalias and Manzari, 2004, Taiebat and Dafalias, 2008), but it is the first time it was applied to metals.

Second, and most important, is the introduction of the BCD modification into the relative KH of the YS in regard to the KH of the BS, as eloquently portrayed by the key Eq. (15). So far, the BCD modification was applied to AF type of models, in which case there is no notion of relative KH. This modification was the key to controlling the direction of the plastic strain rate tensor (associative flow rule) during the loading process, hence, it is the basis for the successful simulation of MR.

The third novel ingredient, closely related to the second and motivated by a prior suggestion of Dafalias and Feigenbaum [2011] within the setting of MAF models, was the substitution of the variable r_α for the fixed-value parameter δ in the classical BCD modification of AF and MAF models. The r_α was expressed in Eq. (21) as a function of r , defined in Eq. (20) and measuring the proximity of the back stress to its BS. Despite its simplicity, Eq. (21) plays a role of cardinal importance for the effectiveness of the introduced BCD modification in regard to the successful simulations of MR for complex cyclic loading shown in previous figures for many cases. This has been confirmed by attempting the same simulations with a fixed value of r_α , as is the case of fixed-value parameter δ , which yielded significantly inferior simulations. In addition, the use of r_α easily satisfies the requirement $r_\alpha > 0.5$ at $r = 1$ on the BS, for avoiding crossing of the BS by the back stress of the YS under some loading conditions that can induce unjustified softening, a shortcoming of several MAF models which use $\delta < 0.5$ [Dafalias and Feigenbaum, 2011].

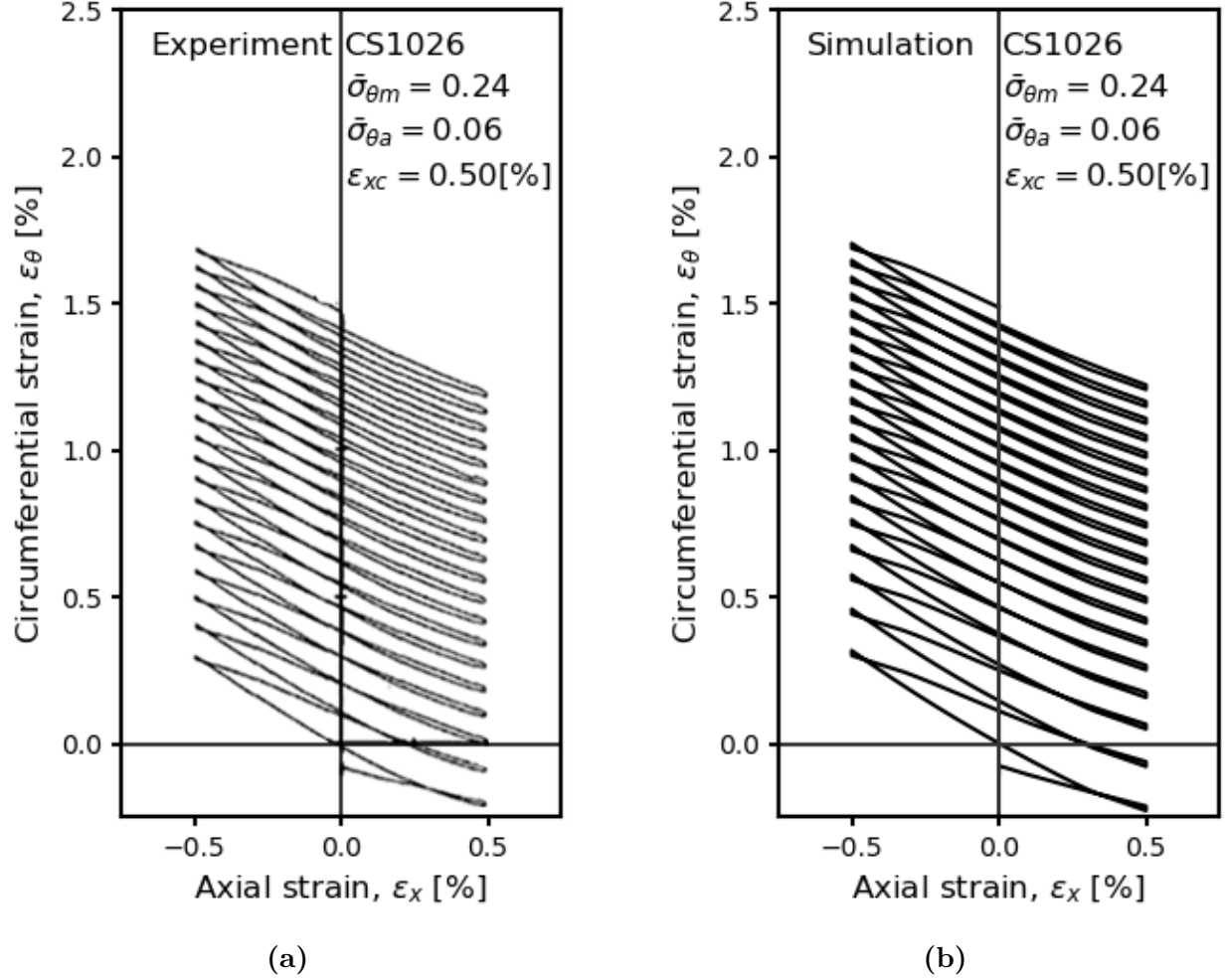


Fig. 19. Strain-path, bowtie. Type 4 in Figure 11. Material CS1026. a) Experiment after Corona et al. [1996]; b) Simulation.

The fourth novel constitutive ingredient is the use of the special Dafalias/Ranzbari (DR) KH (see Mahan et al., 2011) for the BS, which induces a parallel “translation” of the bounds in uniaxial case towards the direction of ratcheting to maintain the continuation of UR, as observed in experiments. As already mentioned in the Introduction, this was a discovery made by Hassan and Kyriakides [1992, 1994b], but in these references the DR KH for the BS was used only for obtaining the ensuing bounding plastic modulus, while the KH direction of the YS was defined by an independent AF KH, followed by the KH for the BS via its interaction with it. In the present work, the approach is simpler and more straightforward: the DR KH was used directly for the BS for obtaining both the important bounding plastic modulus by the consistency condition for the BS, and the direction of KH for the BS, entering Eq. (15) for the relative KH of YS and BS, without the need to specify an independent AF type of KH for the YS. The flexibility of such YS KH was delegated instead to the use of the aforementioned third novel ingredient, namely the BCD variable modification parameter r_α . A word of caution is that the DR KH for prolonged monotonic loading behaves like a linear

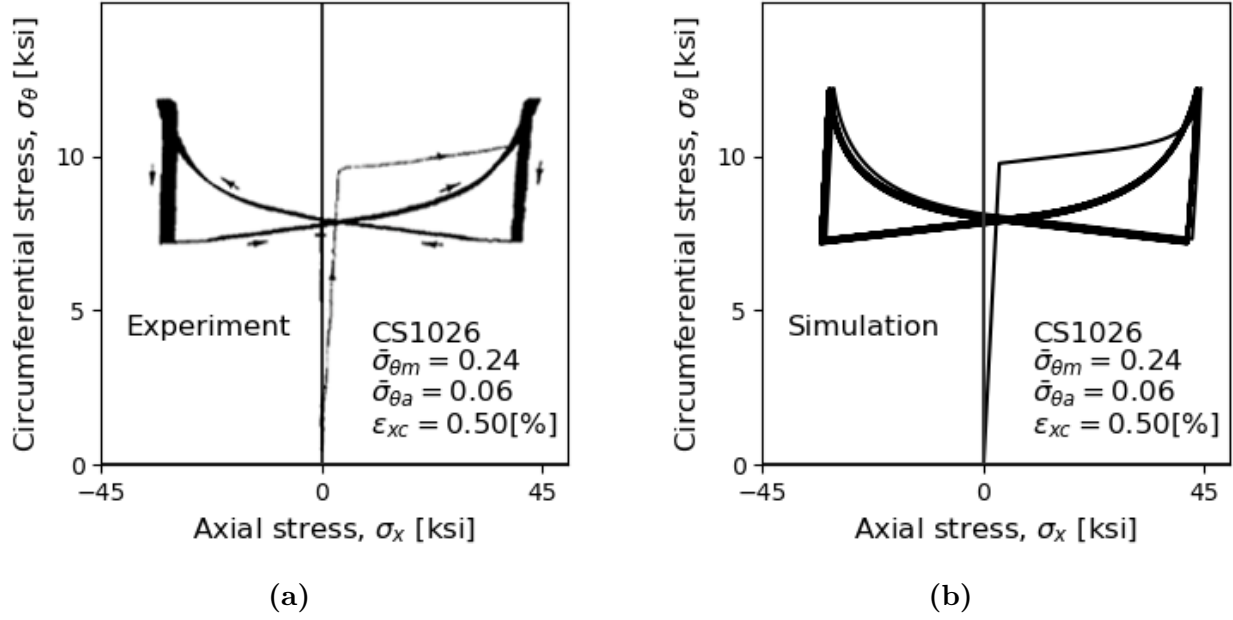


Fig. 20. Stress-path, bowtie (Type 4 in Figure 11). a) Experiment after Corona et al. [1996]; b) Simulation. Material CS1026.

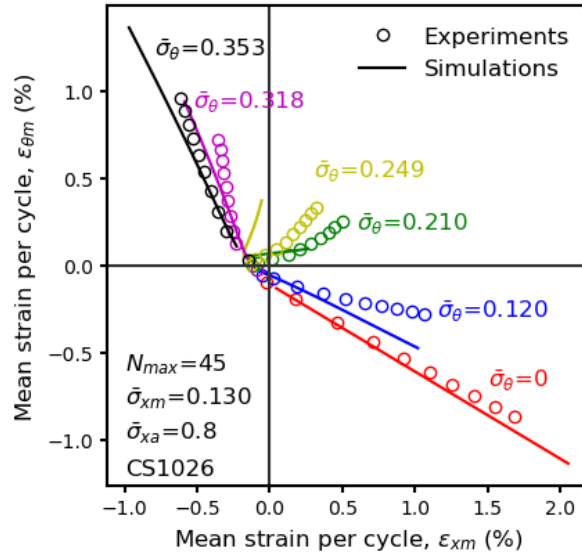


Fig. 21. Mean axial versus mean circumferential strain recorded at each cycle in stress-controlled axial cycling experiments at different values of $\bar{\sigma}_\theta$. Each data point (open circles) equals the values at every fifth cycle. Data after Hassan et al. [1992].

KH, and its unrealistic effect when no reverse loading takes place must be appropriately modified if the need for such prolonged monotonic loading ever arises.

The results show that, in general, the model's simulations agree quite satisfactorily with experimental findings under a wide array of UR and MR loading conditions for all three kinds of carbon steel examined in this work. The lower accuracy of the model in the case

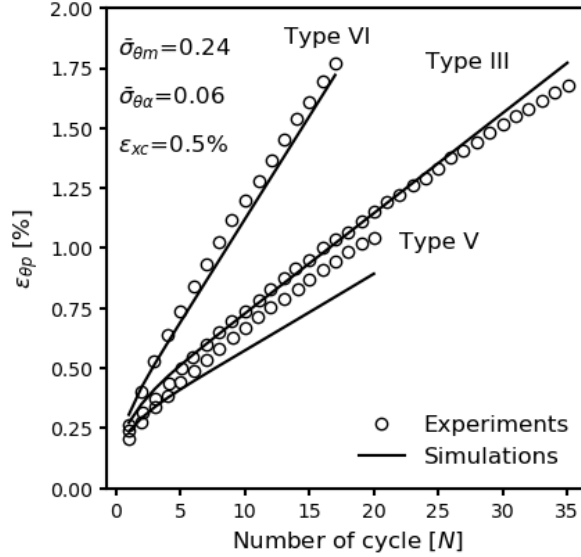


Fig. 22. Multi-axial ratcheting, reverse bowtie path (Type 5), inclined path, negative slope (Type 3), and hourglass (Type 6). Data after Corona et al. [1996]. Material CS1018.

of multi-axial cyclic stress-symmetric loading with constant internal pressure, which induces double ratcheting of plastic strain in the axial and circumferential directions simultaneously, was highlighted in this work and is a shortcoming characterizing all other available state-of-the-art models at present. More research is needed to improve its performance in this most complex case.

Another issue not addressed yet with the present model, is the possible effect of non-proportionality under multi-axial loading on the IH [Hassan and Kyriakides, 1994a]; one of the reasons it was not addressed here is that materials used were cyclically stabilized before being subjected to UR and MR loading paths and after this stabilization, no additional IH is expected. The IH can include also isotropic softening under cyclic loading, which may be seen in this data (see Fig.5), and that can be another objective to be investigated in subsequent research. Finally, it will be important to address non-proportional loading that involves also a change of stress principal axes (non-coaxiality), as it happens when shear stress is cyclically applied; the non-proportional (non-radial) cases presented here, maintained the same stress principal axes directions.

2.2 Objective 2 - Experimentally characterize the yield surface

In this project, we have established an experimental technique to evaluate the yield surfaces of metallic materials, namely a yield surface tracing (YST) method. The concept of a yield surface (YS) is critical to phenomenological plasticity theory, and it is defined as a set of yield points (YPs) within a particular stress space that represents the onset of yield in any combination of mixed-mode loading from a predefined initial point. The yield surface may change size, shape and/or location due to plastic flow. YST is an essential tool for the acquisition of experimental data required for deriving and calibrating robust phenomenological models for plastic deformation. A fine description of yield surfaces is

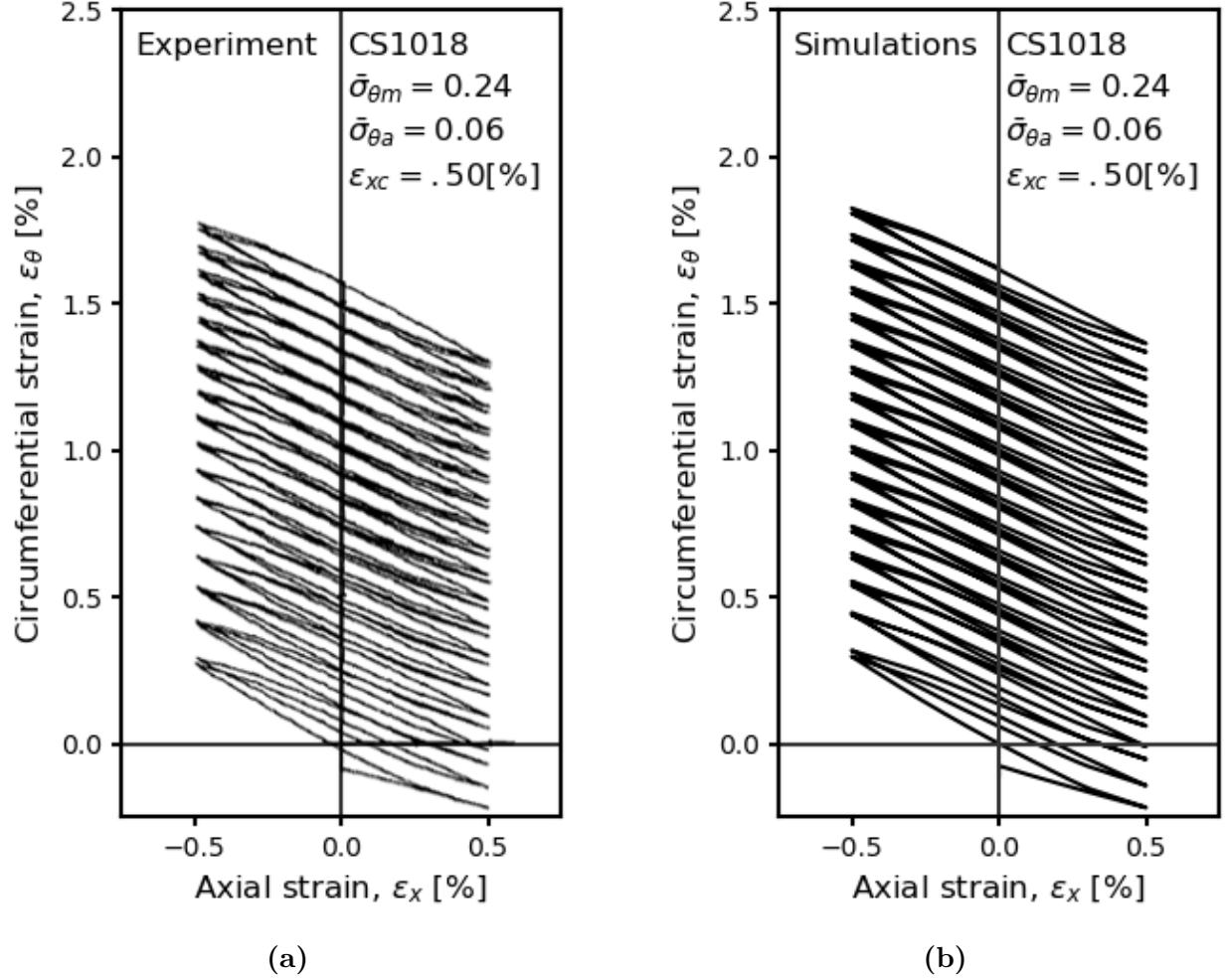


Fig. 23. Strain-path, hourglass (Type 6 in Figure 11). a) Experiment after Corona et al. [1996]; b) Simulation. Material CS1018.

expected to improve the accuracy of phenomenological modeling predictions for structures experiencing complicated and demanding loads.

Methods

The YST method developed in this project identifies a series of representative yield points constituting a YS after some prescribed axial/torsional pre-load. Identification of individual yield points (YPs), “probes”, is achieved using an automatized procedure, and continuously evaluating the prescribed yield point definition. A YP is identified by calculating plastic strain, which is essentially the difference between the measured strain and that predicted by Hooke’s Law. Because the loading is both axial and torsional, a total equivalent plastic strain is calculated. When this equivalent plastic strain reaches a critical value, which is usually quite small ($50\text{-}200\mu$), plastic deformation is said to occur and the YP is identified.

A vital part of the procedure is an algorithm that continuously and automatically performs a least squares linear regression analysis to fit the elastic part of the stress–strain curve

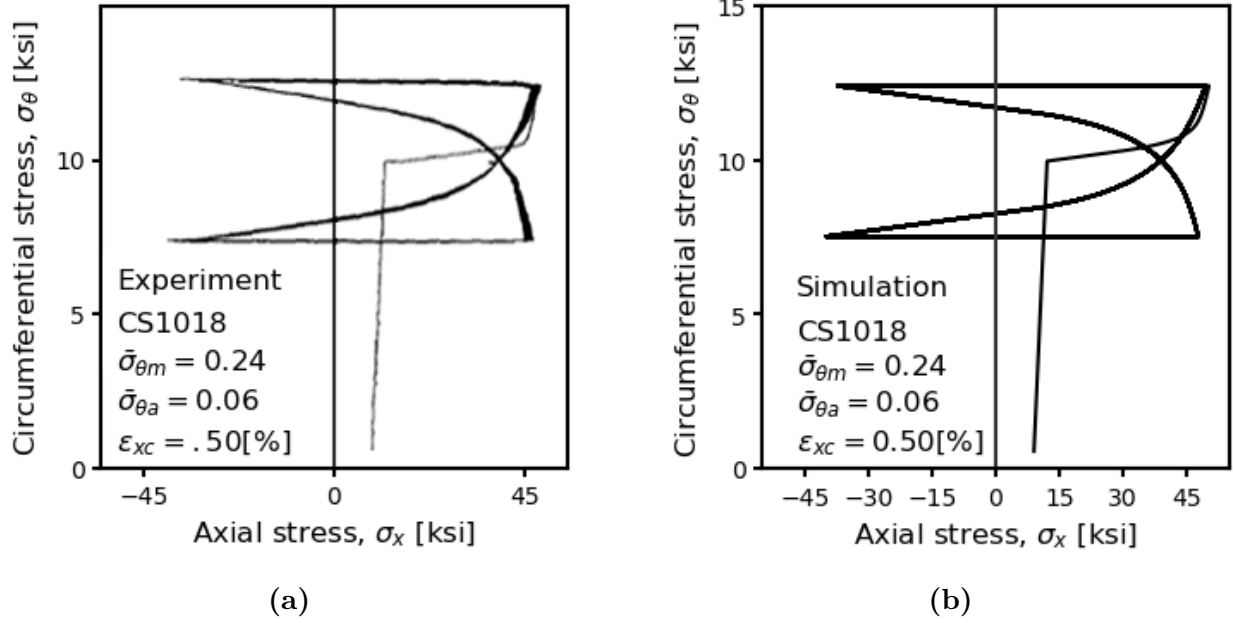


Fig. 24. Stress-path, hourglass (Type 6 in Figure 11). a) Experiment after Corona et al. [1996]; b) Simulation. Material CS1018.

throughout a probe. There are two primary outputs: the elastic moduli (Young's modulus and shear modulus) as slopes, and the residual strains as offsets. Elastic strain components are calculated subsequently and used to evaluate the total equivalent plastic strain accumulated during the actual probe. Calculations are updated with every additional data point evaluated. Hence, the procedure is fully automatized for evaluating unique yield points.

Experimental Set-Up

The axial-torsional testing of the present study was executed utilizing the Instron 8852 system, which comprises of a universal hydraulic testing machine (UTM), a control unit, and a computer. The UTM, capable of applying axial loads of ± 100 kN and torques of ± 1000 Nm, was interfaced with the PC via the two-axis 8800MT digital controller and the Console software, enabling comprehensive system control from the computer. The Console software can also be utilized with other programs, including proprietary software by Instron (e.g., WaveMatrix2, Bluehill Universal) and in-house software, through APIs. Custom software was created using a visual programming environment to control the UTM during the Yield Surface (YS) tracing. The testing setup also includes a biaxial contact extensometer Epsilon Tech 3550, which was utilized to measure axial and shear strains on the test specimens.

Materials

Thin-walled tubular test specimens (TS) were used to find the YS for different materials. First, two conventional non-alloyed ferritic steel grades were used to detect YS after pre-loading in axial and/or shear. These steels are: P250GH (W. Nr. 1.0460, or C22G2), which is often used for applications on weldable parts of pressure vessels and piping exposed to

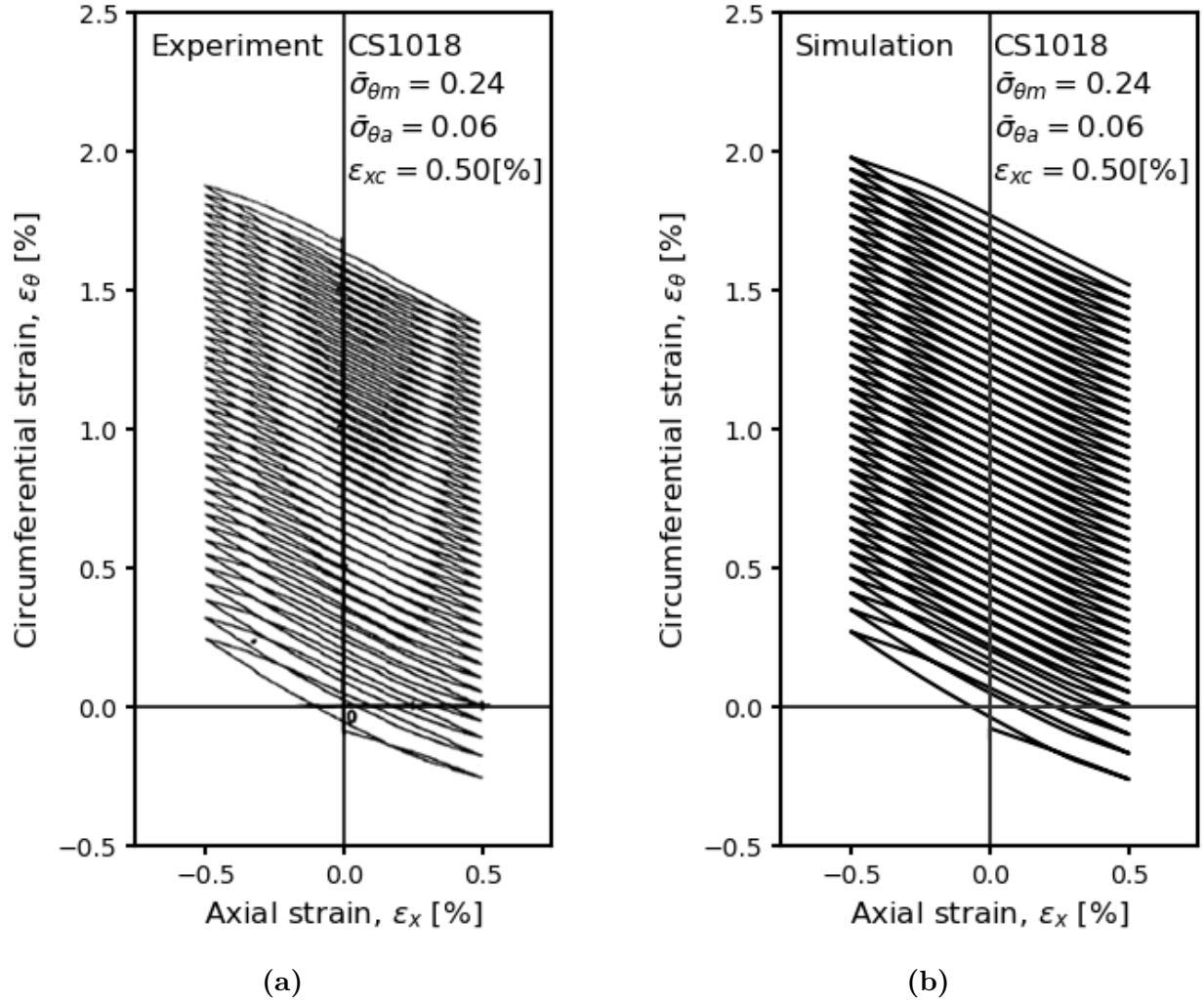


Fig. 25. Strain-path for an inclined path with a negative slope (Type 3 in Figure 11). a) Experiment after Corona et al. [1996]; b) Simulation. Material CS1018.

elevated temperatures. The second grade, and S235JR (W. Nr. 1.0122), which belongs to a group of ordinary weldable steels commonly used for less demanding parts of structures such as halls, bridges, cranes, etc. Subsequently, 304L steel was used to detect the YS after several cycles of loading to determine what the YS may look like after ratcheting.

Results and Discussion

Results for the non-alloyed ferric steels with an off-set strain of 200μ are shown in Figures 29 and 30. In order to emphasize the shapes of experimental YSs and their positions within the stress space, hypothetical von Mises circles were drawn into the figures. As can be seen, the initial YS of S235JR steel conforms to the von Mises criterion better than that of P250GH steel. These “initial YS anisotropies” could be attributed to the raw material production history.

Subsequent YSs depicted in Figures 29(B) and 30(B),(C) provide evidence of pronounced

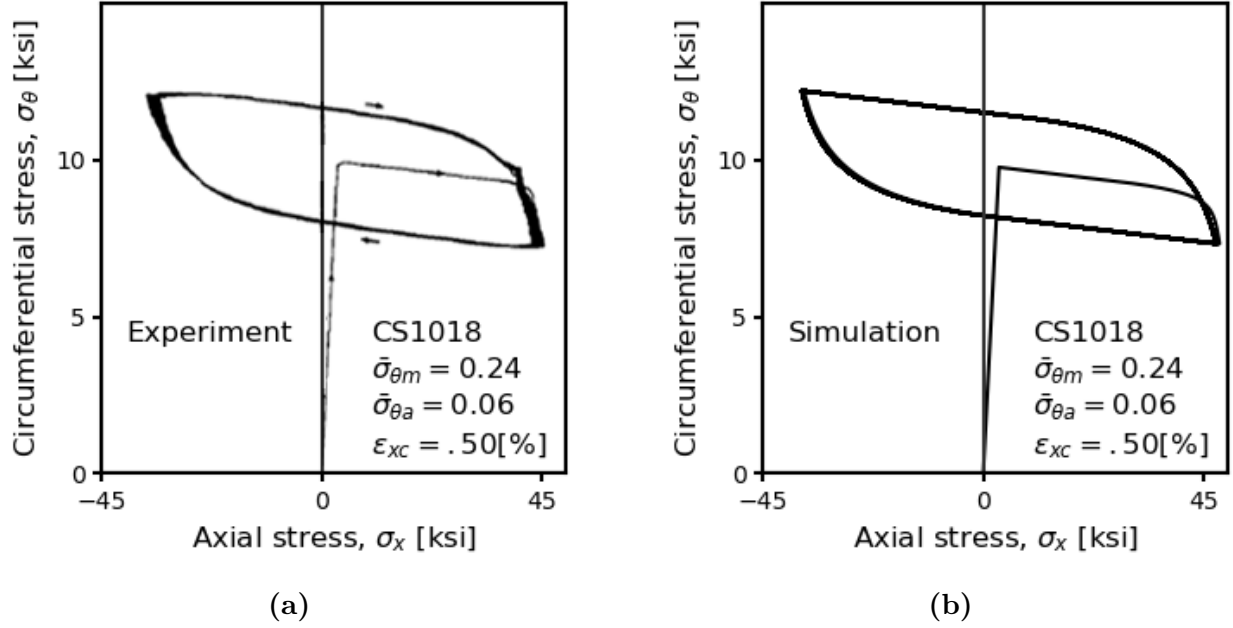


Fig. 26. Stress-path for an inclined path with a negative slope (Type 3 in Figure 11). a) Experiment after Corona et al. [1996]; b) Simulation. Material CS1018.

kinematic hardening. Pure axial pre-strain induced the commonly reported slight sharpening of the forward YS apex, as well as flattening of the rear part. This type of distortion of the YS is called directional distortional hardening (DDH). In the case of P250GH grade, decrease of shear yield points magnitudes indicated a negative cross effect.

Figure 31 shows the results for YS detection of 304L after cyclic loading using an off-set strain of 50μ . The total shear strain was subjected to 5 cycles with the strain amplitude of 0.5% and the mean strain of 0.5%. Following the cycling straining, three yield surfaces were measured. After the multiaxial cycling straining was completed, a 10-minute holding period was purposely implemented in which the test specimen was maintained at a constant strain level. To ensure the accuracy and reliability of results, three repetitions were performed for each yield surface. The results shown in Figure 31 were then taken as an average from the readings.

Also shown in Figure 31, are the flow vectors and normals to the yield surface. To compare flow vectors and yield surface normals, the flow vectors are placed on the yield surface approximation.

One of the key findings from this study is the observation of strong directional distortional hardening (DDH), which emphasizes the material's anisotropy under biaxial loading conditions. In addition to DDH, strong kinematic hardening was observed, in contrast to isotropic hardening, which was of little significance. Furthermore, the results from this study show that the flow vectors are in good agreement with the normal to the yield surface, lending support to the associative flow rule. This finding is particularly important for modeling the behavior of materials under multiaxial loading conditions, as it provides a useful tool for accurately predicting material behavior in a variety of loading paths. Based on these experimental results, future work will focus on adding DDH to the SANIMETAL-BCD model

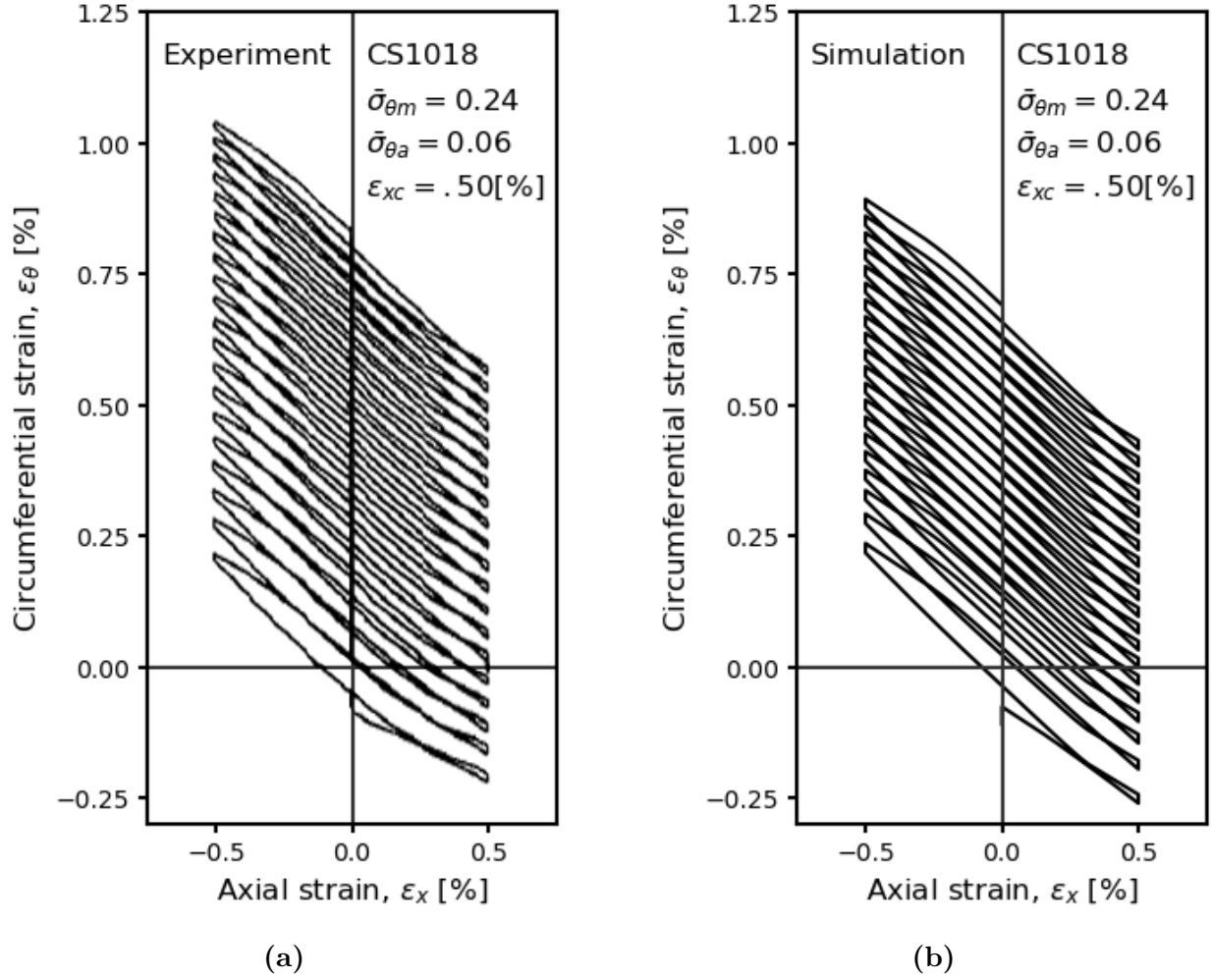


Fig. 27. Strain-path for a reverse bowtie path (Type 5 in Figure 11). a) Experiment after Corona et al. [1996]; b) Simulation. Material CS1018.

presented in Objective 1 as the experiments show that DDH is occurring and it is important to model it in order to accurately capture the plastic strain direction.

2.3 Objective 3 - Incorporate new models into practice

In order to use advanced plasticity models to predict ratcheting in structures, we worked towards a numerical implementation of a rate independent plasticity model designed to improve MR predictions. The model used in this implementation was originally proposed by Dafalias and Feigenbaum [2011] and is referred to as the *Multicomponent Armstrong–Frederick model with Threshold with r modification (MAFTr)*. The numerical implementation uses a commercial finite element (FE) solver Abaqus through the UMAT subroutine coded in FORTRAN language.

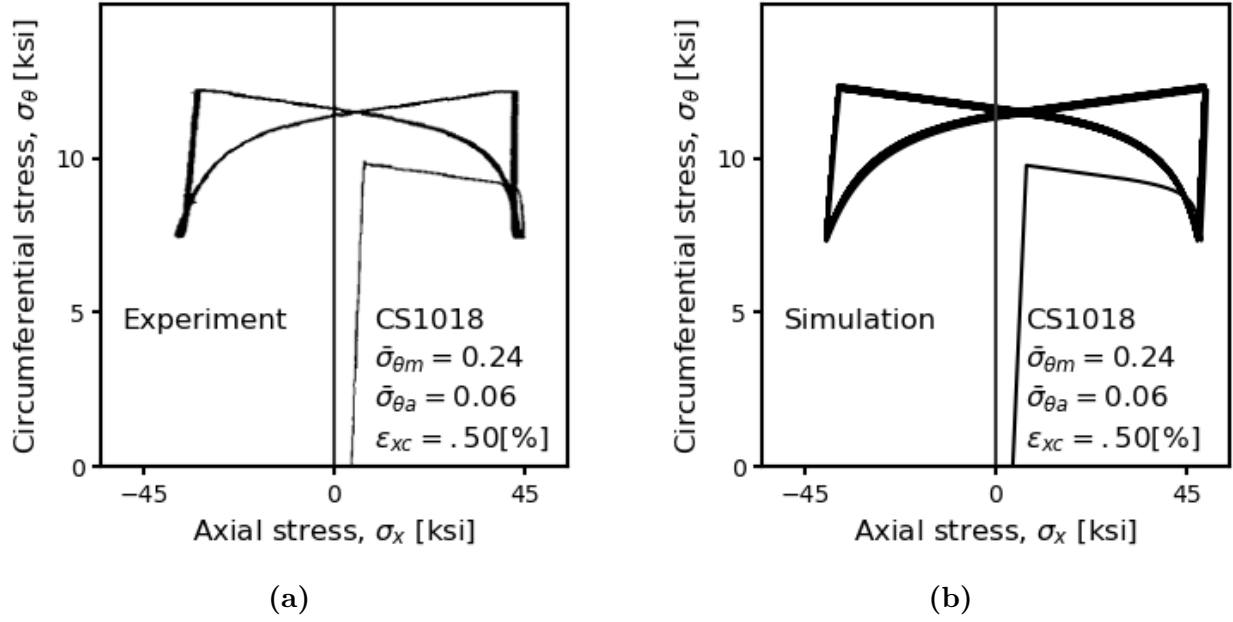


Fig. 28. Stress-path for a reverse bowtie path (Type 5 in Figure 11). a) Experiment after Corona et al. [1996]; b) Simulation. Material CS1018.

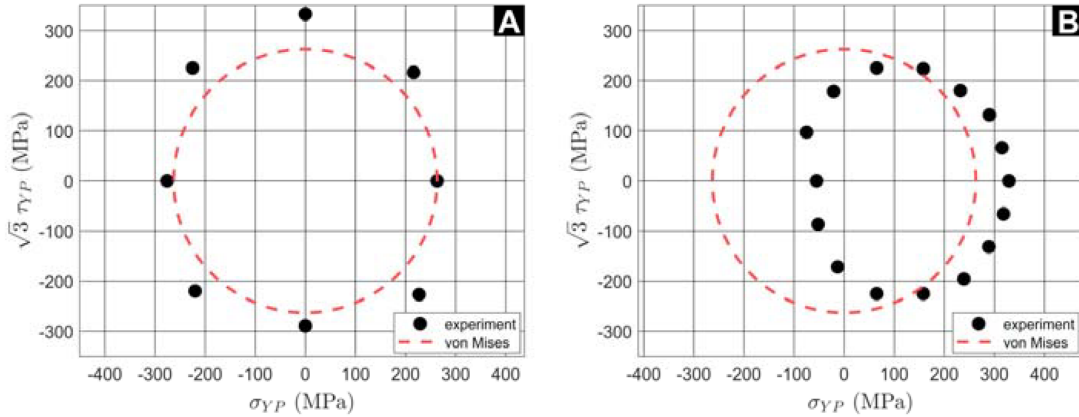


Fig. 29. Initial YS of P250GH steel (A), and subsequent YS after 1% axial plastic pre-strain (B) from Štefan et al. [2021].

2.3.1 *MAFTr* Model

The original paper by Dafalias and Feigenbaum [2011] aims to improve multiaxial ratcheting (MR) predictions via a new KH rule referred to as the *MAFTr* model. Although the authors explicitly employ the von Mises yield condition and associative flow rule, some other constitutive features and equations are not included nor specified, *e.g.*, the elasticity model and isotropic hardening rule. Note that this approach is fully correct as it keeps the model general and highlights the modularity of the plasticity theory. However, as the FE implementation of the *MAFTr* model requires a complete constitutive theory to be established, in this section, the entire set of governing equations to be implemented in FE

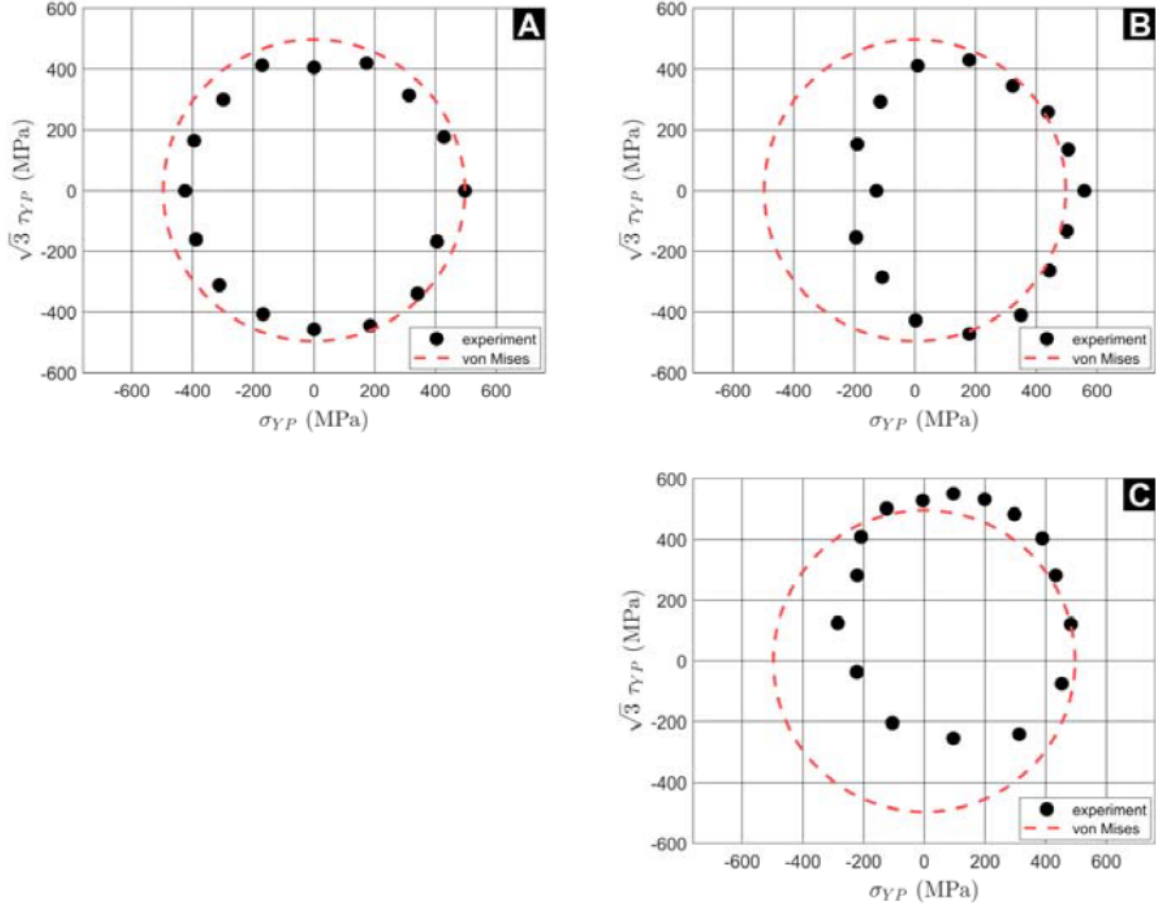


Fig. 30. Initial YS of S235JR steel (A), subsequent YS after 1% axial pre-strain (B), and subsequent YS after further 1% shear pre-strain (C) from Štefan et al. [2021].

code is listed and discussed.

Governing Equations The FE implementation of the *MAFTr* model described in this paper is introduced within the infinitesimal strains framework. Thus, we assume an additive decomposition of the total strain. The elastic behavior of material is assumed to be linear isotropic, in which case the Hooke's law reads

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}^e, \quad (22)$$

where $\mathbf{C}$ is the fourth-order stiffness tensor. Note that the formulation in Eq. (22) with a general stiffness tensor is commonly used among authors who implement elasto-plastic models, and it has no impact on computational complexity or complexity of the model's implementation. Besides, the general formulation benefits from the possibility to model elastically anisotropic materials.

The model makes use of classical isotropic von Mises yield function extended by internal variables of kinematic and isotropic hardening and uses the associative flow rule for the plastic strain increment, as in Eqs. (1) - (4). Note that, unlike the model in Object 1, this

model is not based on a BS formulation, but rather uses classical plasticity theory. Thus this model likely does not predict ratcheting as well, but nonetheless has been proven by Welling et al. [2017] to improve ratcheting predictions over other models in the literature.

The back-stress $\boldsymbol{\alpha}$ inherent to *MAFTr* kinematic hardening rule is given by a superposition of four independent components $\boldsymbol{\alpha}_i$ as in Chaboche et al. [1979b]:

$$\boldsymbol{\alpha} = \sum_{i=1}^4 \boldsymbol{\alpha}_i, \quad (23)$$

where each of the components $\boldsymbol{\alpha}_i$ has its own evolution equation that can be expressed in a similar manner as Eq. (3) by

$$\dot{\boldsymbol{\alpha}}_i = \lambda \bar{\boldsymbol{\alpha}}_i. \quad (24)$$

The first three components hardening rules are given by

$$\bar{\boldsymbol{\alpha}}_i = \sqrt{\frac{2}{3}} c_i \left(\sqrt{\frac{2}{3}} a_i^s \mathbf{n} - (r_i \boldsymbol{\alpha}_i + (1 - r_i) (\boldsymbol{\alpha}_i : \mathbf{n}) \mathbf{n}) \right), \quad (i = 1, 2, 3), \quad (25a)$$

where the weighting factor $r_i = \sqrt{3/2} \|\boldsymbol{\alpha}_i\| / a_i^s$. The fourth component features a threshold $\bar{a}$ which delays the dynamic recovery via a term in Macaulay brackets creating a region of linear hardening as

$$\bar{\boldsymbol{\alpha}}_4 = \sqrt{\frac{2}{3}} c_4 \left(\sqrt{\frac{2}{3}} a_4^s \mathbf{n} - (r_4 \boldsymbol{\alpha}_4 + (1 - r_4) (\boldsymbol{\alpha}_4 : \mathbf{n}) \mathbf{n}) \left\langle 1 - \sqrt{\frac{2}{3}} \frac{\bar{a}}{\|\boldsymbol{\alpha}_4\|} \right\rangle \right) \quad (25b)$$

with $r_4 = \sqrt{3/2} \|\boldsymbol{\alpha}_4\| / (a_4^s + \bar{a})$. The definition of r_4 is not correctly stated in the original paper of Dafalias and Feigenbaum [2011] where it does not take into account the actual asymptotic limit $\|\boldsymbol{\alpha}_4^{\text{lim}}\| = \sqrt{2/3} (a_4^s + \bar{a})$ derived from Eq. (25b) provided

$$\bar{\boldsymbol{\alpha}}_4 = 0 = \sqrt{\frac{2}{3}} a_4^s \mathbf{n} - \|\boldsymbol{\alpha}_4^{\text{lim}}\| \mathbf{n} \left(1 - \sqrt{\frac{2}{3}} \frac{\bar{a}}{\|\boldsymbol{\alpha}_4^{\text{lim}}\|} \right). \quad (26)$$

Since the *MAFTr* model is in fact the kinematic hardening rule, it does not introduce a dedicated internal variable nor evolution equation for the isotropic hardening. Thus, to extend the constitutive capability and usability of the implementation, we have adopted a simple isotropic hardening rule after Feigenbaum and Dafalias [2007] in the form of Eq. (2) where the term $\bar{k}$ is

$$\bar{k} = \kappa_1 (1 - \kappa_2 k), \quad (27)$$

where κ_1 and κ_2 are model's parameters representing hardening rate and inverse asymptotic limit, respectively.

Finally, the full model is specified by loading–unloading conditions that read

$$\lambda \geq 0, \quad f(\boldsymbol{\sigma}, k, \boldsymbol{\alpha}) \leq 0, \quad \text{and} \quad \lambda f(\boldsymbol{\sigma}, k, \boldsymbol{\alpha}) = 0. \quad (28)$$

There are known as a part of *Karash–Kuhn–Tucker conditions* for constrained optimization problem.

2.3.2 Methods

For the sake of this report, the methods to implement the model in FE code will only be briefly outlined.

The model is implemented into FE code Abaqus through UMAT subroutines coded in FORTRAN programming language. The software, compiler, and hardware used for this implementation is shown in Table 2. Abaqus, like most FE solvers, is based on the displacement formulation. In general, the displacement based FE codes discretize a continuous loading in the time domain by a time sequence of loading states. The loading state at time t is characterized by total strain ${}^t\boldsymbol{\epsilon}^{\text{trial}}$ referred to as the trial strain. In the discrete states, FE codes attempt to solve the equilibrium equations, while between the states the constitutive model is integrated.

Table 2. Specification of computational tools.

Software	Abaqus 2021
Compiler	Intel Fortran v. 18.0.2
Hardware	Intel® Xeon® CPU E5-2637 v4 3.50GHz

The incremental problem of plasticity coded using a classic two-step algorithm of predictor–corrector type based on trial strains is introduced, *Cf.* Marek et al. [2015]. An overview of this method is shown in Figure 32. The integration of the model under elasto-plastic loading regime is done by the explicit forward Euler method. Here, a well-known problem occurs that the yield condition is not met inherently at the end of integration steps. *Cf.* Dunne and Petrinic [2006], Marek et al. [2015], and Halilovic et al. [2017]. Due to this, after several integration steps, the yield point may drift off from the yield surface, which corrupts the meaning of the yield condition and, consequently, affects the consistency of the system of equations. In this work, the problem is solved by radial return projection, which is shown conceptually in Figure 33. As these are relatively standard procedures in plasticity model implementation, they are not described in detail here.

2.3.3 Verification and Validation

To verify the model implimentation, components of internail variables computed by the UMAT subroutine were compared with analytical solutions we derived for the *MAFTr* model under proportional loading. We also check the model by changing the step size in the numerical integration. As expected, when the step size decreased, the model converged and we were able to assess the step size needed for accurate results.

2.3.4 Results

To test the performance of subroutines and the implementation, two numerical examples were done. The first one was a planar plate with a hole loaded by pressure in a single direction. The second one was a simulation of an elbow pipe preloaded by pressure and cyclicly loaded by displacement according to the experiment in Hassan et al. [2015]. These examples are depicted in Figure 34. Parameters of *MAFTr* model were calibrated on experimental MR

data published by Hassan et al. [1992]. With the calibrated model, a range of simulations of MR was performed. The results of some of them are depicted in Figure 35. In Figure 35(a), tests differ in the amplitude of prescribed axial strain ε_{xa} . The evolution of ratcheting by FE analysis with implemented *MAFTr* model is underpredicted for $\varepsilon_{xa} = 0.65\%$, and overpredicted if $\varepsilon_{xa} = 0.4\%$. Accurate results were obtained for case of $\varepsilon_{xa} = 0.5\%$. To check the importance of prescribed axial strain, tests with different load and preload stress were performed. Results of these set-ups are shown in Figures 35(b) and 35(c). So far, less promising situation is depicted in 35(d), which shows the evolution of the ratio between circumferential mean strain $\varepsilon_{\theta m}$ and axial mean strain ε_{xm} , when the specimen is preloaded by circumferential stress σ_θ and axial stress σ_{xm} , and cyclicly loaded by axial stress σ_{xa} . Note that these tests were some of the same used to test the model in Object 1.

While these results show discrepancies between the model predictions and experimental findings, that is expected as Welling et al. [2017] also showed that the *MAFTr* could not always predict experimental MR data accurately. Nonetheless, these results show the FE implementation is successful. In the future, we hope to implement the new model from Objective 1 with more accurate predictions of MR in FE software. Such an implementation would use the same methodology as we used in this project for the *MAFTr* model and would allow us to predict and analyze the behavior of an overall structure in cyclic plastic loading with more accuracy.

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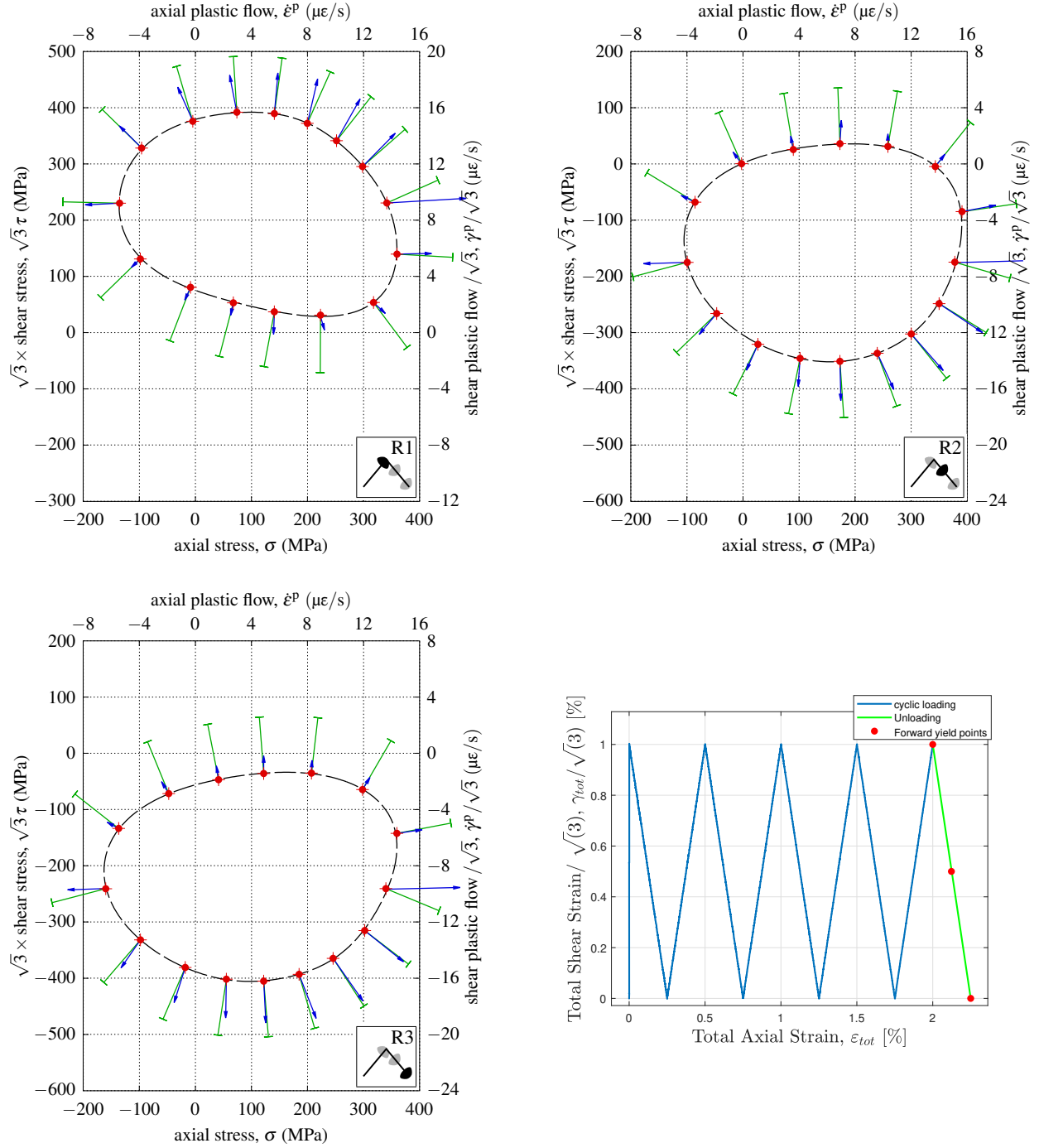


Fig. 31. YS for 304L steel after several cycles of loading. The first YS was measured at 2% axial strain and 1% shear strain (upper left). The second YS was measured at 2.123% axial strain and 0.5% shear strain (upper right). Finally, the third YS was measured at 2.25% axial strain and 0% shear (lower left). To ensure the accuracy and reliability of results, three repetitions were performed for each yield surface, and the results shown are an average from the three readings. The load path is shown in the bottom right where red dots indicate when the YS was detected. Along with the YS are shown the normal to the YS (green lines) and direction of the plastic strain (blue arrows).

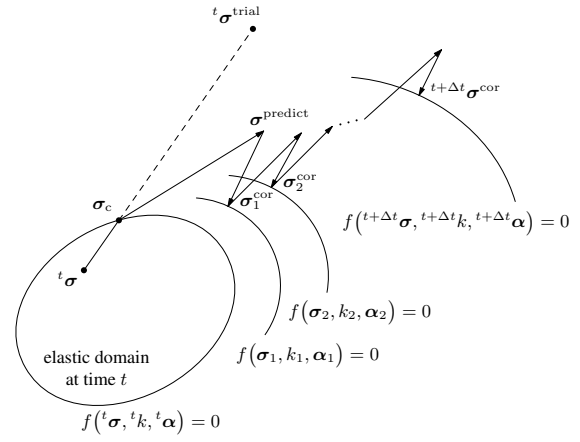
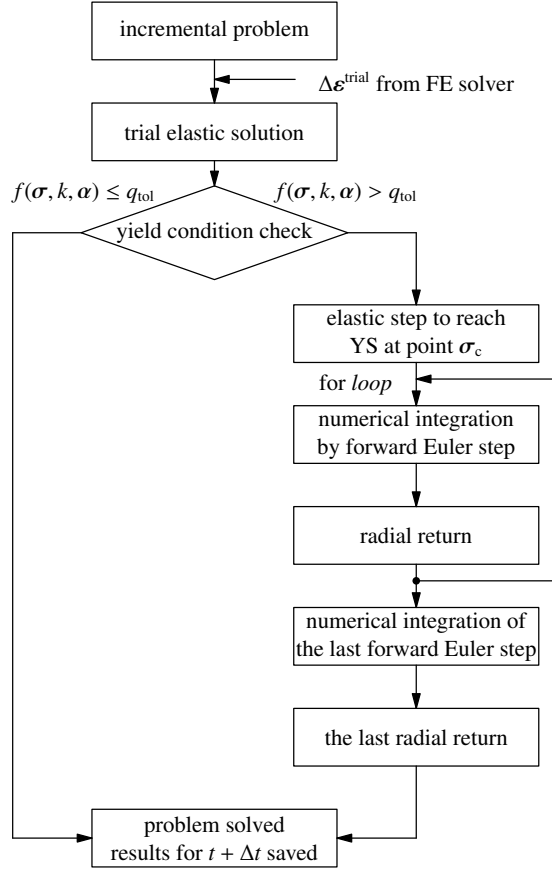


Fig. 32. Flowchart of integration of model. **Fig. 33.** Geometrical interpretation of integration of model in stress space.

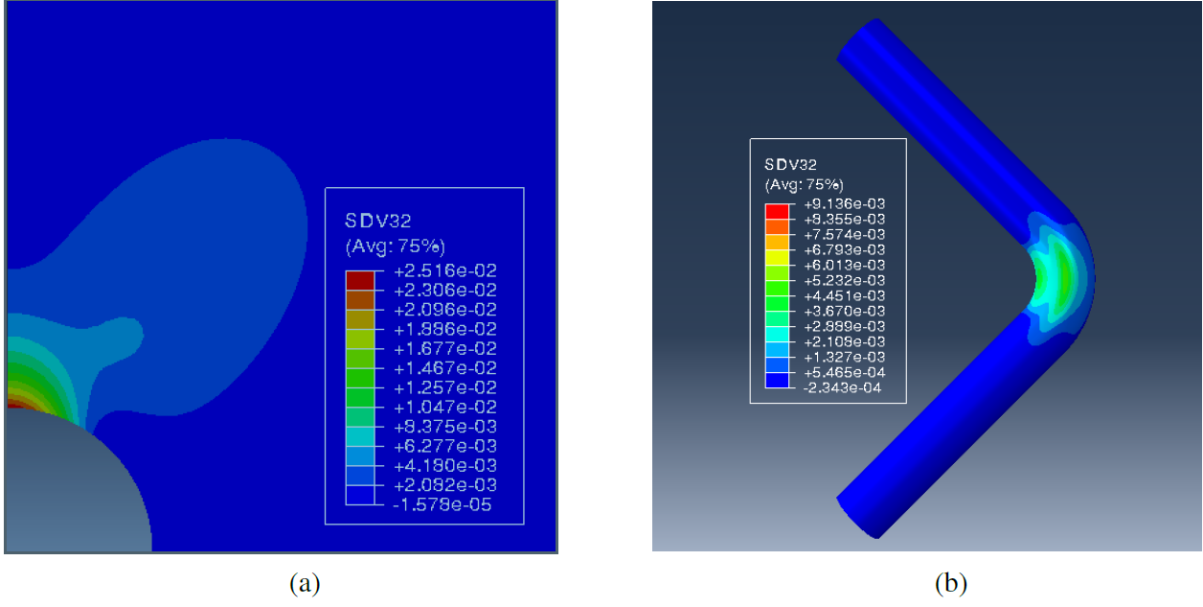


Fig. 34. Numerical examples used to test the performance of UMAT subroutine with implemented *MAFTr* model into Abaqus FE code.

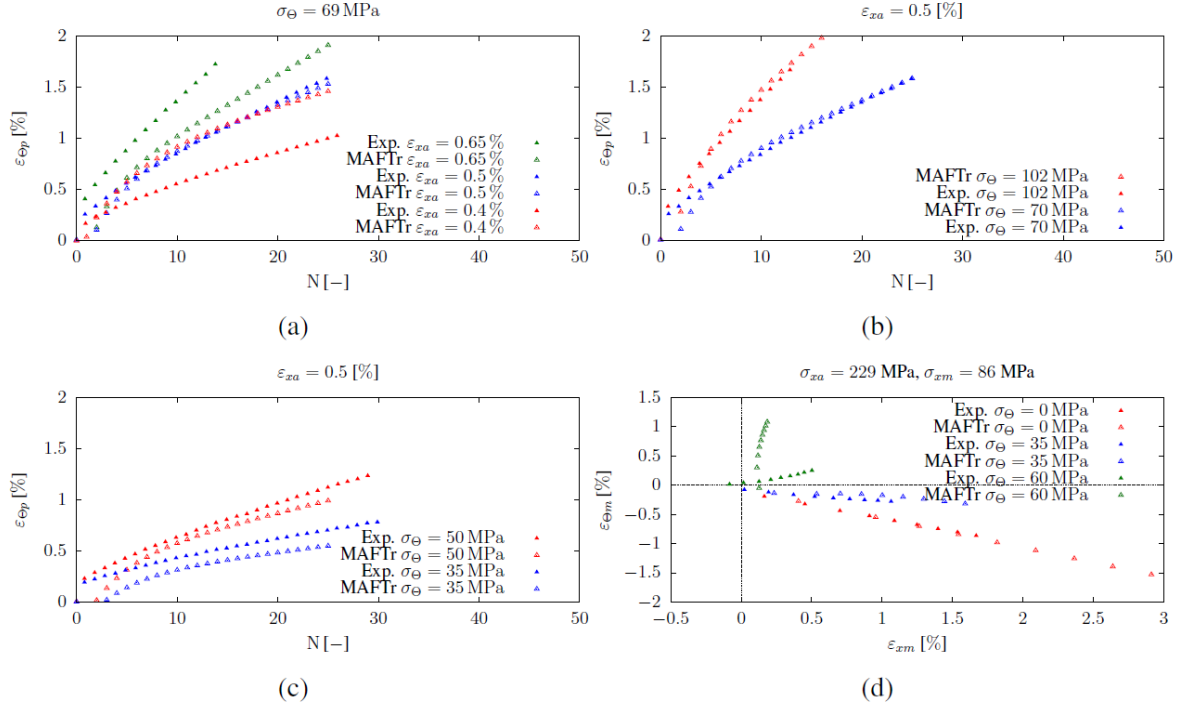


Fig. 35. Biaxial ratcheting predicted by MAFTr model and prediction of ratcheting as a function of the number of cycles. The experimental data was measured on CS 1026 and published by Hassan et al. [1992]. Ratcheting simulations with previously estimated stress increment and their comparison with the experimental data are depicted in subfigures (a) - (d).