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A Stochastic Source-Sink Framework for Orbital Debris

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Abstract

Orbital debris poses a risk to the expanding governmental and commercial use of Earth orbit. In particular, the rising orbital populations increase the risk that debris growth may render parts of the orbital environment unviable for satellite operation. Source-sink evolutionary models are useful for rapidly assessing the impact of various parameters on the orbital population. However, these models, as currently applied, provide insight into only a single evolution of the orbital population. Here, we present a stochastic framework for source-sink models that samples the full probability distribution of population outcomes given a set of rates for relevant processes (e.g., launch, decay, and collisions). We apply our model to investigate the impact of the satellite launch rate, the post-mission disposal (PMD) rate of satellites, and noncompliance with PMD in low Earth orbit (LEO) between 500-2000 km. Our model indicates that parts of this LEO orbital regime can become unviable for satellite operation under the 25-year rule for PMD if the total number of satellites launched grows by roughly a factor of 2-4 over the current total. Using a 5-year PMD rule mitigates this risk in our model. Significant rates of noncompliance with the PMD rule can increase the risk of runaway debris growth, even under the 5-year PMD rule.

1 Introduction

The number of satellites in Earth orbit has increased dramatically in recent years and continues to grow at an unprecedented rate. This expansion of space capabilities offers several benefits, including Internet access, communications, and remote sensing. However, the growing number of satellites may increase the risk of collisions with orbital debris and other satellites. In particular, stakeholders are concerned with the risk of runaway debris growth—the so-called "Kessler syndrome"—that may render portions of the Earth orbital environment unviable for satellite operation. Models of the orbital population are useful for assessing this risk.

Orbital population models fall generally into two categories. Models in one class, including the NASA LEGEND model [1], simulate the propagation of individual objects using high-fidelity orbital mechanics. A second class of models, sometimes referred to as source-sink evolutionary models, simulates populations of objects in orbit changing under processes that either add objects (sources) or remove objects (sinks) [2-4]. While lacking the fidelity of the first category of models, this latter class permits rapid assessment of the impacts of future conditions or policies (e.g., average launch rate or on-orbit lifetime limitations).

These source-sink models describe a single evolution of the population, in which the source and sink processes operate at their average rates. However, a process operating at an average rate will not produce the same number of occurrences within any time period. Rather, the number of any relevant event (e.g., satellite launch and de-orbit, debris decay, collision) will vary stochastically. Consequently, given any set of sources, sinks, and initial conditions, a range of outcomes is possible. This range is not described by standard source-sink models.

In this paper, we present a stochastic source-sink model of the orbital environment. Inspired by techniques used in physical chemistry [5], our model samples the full probability distribution of system evolutions that a given set of processes can generate. This work extends a previous model we developed, which will be submitted for publication in the near future [6]. Our current model consists of populations of satellites and debris occupying altitude shells. We consider active satellites, which can maneuver to avoid collisions, and derelict satellites. We also distinguish between mission-lethal and catastrophically-lethal debris: the former can render an active satellite derelict without breaking it up, while the latter cause catastrophic collisions that generate many debris pieces. The main processes in our model are the launch and post-mission disposal (PMD) of active satellites; the decay of derelict satellites and debris; and collisions between two objects from any of the model populations. We further consider some fraction of the active satellite population that does not comply with PMD. Members of this population become derelict after their mission lifetimes.

We apply this model to study the impact on low Earth orbit (LEO) of the launch rate, PMD rate, and compliance with PMD. Our model suggests that runaway debris growth can render some portions of LEO unviable for satellite operation under increased launch rates and the 25-year rule for PMD within 100 years.[1] We find that imposing a 5-year in our model rule eliminates this risk. However, a high rate of noncompliance with the 5-year rule reintroduces this risk. With noncompliance rates of 40% or greater, some altitude shells in our model become unviable for satellite operation within 100 years.

Our paper is organized as follows. In Section 2, we describe our model's populations and processes, as well as the sampling method used to generate the distribution of population evolutions. In Section 3, we apply our model to investigate the LEO regime. In Section 4, we discuss the results of these simulations. Finally, in Section 5, we summarize our conclusions.

2 Stochastic Source-Sink Model

2.1 Populations

Our model considers populations of satellites and debris in orbital shells around the Earth. As in similar work [2-4], we assume that all objects are in near-circular orbits. The populations of satellites and debris consist of objects with a range of characteristic length scales. We use a characteristic length scale rather than a mass scale because length is the main input to the NASA satellite breakup model (SBM) [7], which we apply in our collision model.

The satellite population consists of active and derelict satellites. Active satellites refer to payloads controlled by an operator. These satellites can maneuver to avoid collisions and can be de-orbited via post-mission disposal (PMD). In contrast, derelict satellites act essentially like large pieces of debris: they can neither maneuver nor be de-orbited. We provide specific details on the collision rates for active and derelict satellites in Sec. 2.2.4 below.

We further divide the active satellites into two categories: compliant and noncompliant. Active compliant satellites will de-orbit via PMD. Active noncompliant satellites, however, are so named because their operators do not comply with PMD. Instead, after a user-defined on-orbit lifetime, these satellites will become derelict.

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[1] By unviable, we mean that there exist system evolutions in which collisions with debris reduce the number of active satellites in an altitude band to zero or near zero.

The debris population also consists of two sub-categories: catastrophically-lethal debris and mission-lethal debris. Catastrophically-lethal debris are large enough to cause catastrophic collisions, i.e., collisions that result in the breakup of the satellite and debris objects [7]. Catastrophic collisions generate a large number of debris pieces. Mission-lethal debris are smaller debris objects that do not break up satellites. Instead, these debris objects can penetrate and "kill" active satellites, rendering them derelict. The two types of debris objects are determined by a length scale, which we refer to as the lethal length. Debris with characteristic lengths above the lethal length are catastrophically lethal; the rest are mission lethal.[2]

2.2 Processes

Figure 1 shows the processes that add and remove objects to each of the populations in our model. We describe these processes in detail in this section. A full description of the processes for a similar model will be provided in a forthcoming publication [6].

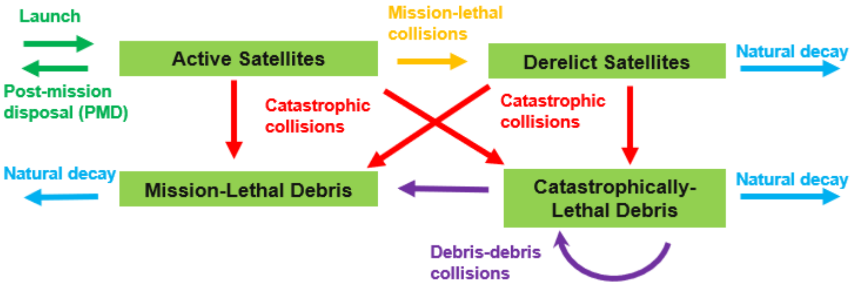

Fig. 1: Object populations and processes in the source-sink model.

2.2.1 Launch

We define the "launch rate" in terms of the number of active satellites added to space per year. This number will exceed the number of actual launch events, as launch vehicles typically transport many satellites. In our model, we specify a launch rate $L(h, l_{sat})$ that varies with altitude h and satellite length scale l_{sat}. This rate defines the average number of satellites at the specified length scale that are launched to the altitude shell with central altitude h within a year. The model is capable of including debris at all length scales generated by launches. In the current implementation, we assume that launches generate no debris pieces. We further assume that no satellites fail immediately after launch.

Launches add active satellites that can be either compliant or noncompliant with the PMD rule. A user-specified fraction of each group of launched satellites will be noncompliant. These satellites do not de-orbit via PMD; rather, they become derelict satellites after an average lifetime, which is a tuneable parameter in our model.

2.2.2 Post-mission disposal

Post-mission disposal (PMD) refers to the active de-orbiting of satellites at the end of their on-orbit lifetimes. This process applies to active satellites. In our model, we assume that each satellite has a

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[2] There exists a large number of debris objects that are so small as to be not even mission-lethal. We do not explicitly consider such small debris in our model. Instead, we take the minimum debris length scale to be the minimum length at which a debris object can be mission-lethal.

given probability of being removed via PMD each year. The total PMD rate at a given altitude and satellite length scale is this individual rate multiplied by the total number of active satellites:

$$\omega_{PMD}(h, l_{sat}) = r_{PMD} n_{ACS}(h, l_{sat}) \tag{1}$$

where $n_{ACS}(h, l_{sat})$ is the number of active compliant satellites at altitude h and characteristic length scale l_{sat}.[3] The effect of PMD is to remove active satellites at the specified altitude and length scale from the simulation.

As Eq. 1 shows, the PMD process assumes any active satellite has a given probability r_{PMD} of de-orbiting via PMD each year. In reality, individual satellites remain on orbit for their lifetime and then de-orbit within some time period following the end of mission life. As our model does not track individual satellites, we cannot model this directly. Rather, we assume that the satellite population is in a steady state, so that there are a large number of satellites near the end of their missions. We therefore model a 25-year or 5-year PMD rate by using the parameters $r_{PMD} = 1/25$ or $r_{PMD} = 1/5$ per satellite per year, respectively. Using this approximation, we can assess the potential impact of the recent U.S. Federal Communications Commission (FCC) decision to require the operators of LEO satellites to dispose of these satellites within 5 years of mission completion [8].

2.2.3 Decay

Natural decay due to atmospheric drag removes derelict satellites and debris from orbit. We apply an exponential model of the atmospheric density in calculating the decay rate for each object. The decay rate for the object population of species i is given by [2-4]:

$$\omega_{decay}(h, l_i) = r_D(h, l_i) n_i(h, l_i) \tag{2}$$

where n_i is the number of objects of species i and

$$r_D = \frac{\rho(h) B \sqrt{\mu R}}{d} \tag{3}$$

In Eq. 3, $\rho(h)$ is the atmospheric density at altitude h, *B* is the ballistic coefficient, *R* is the radius of the orbit from the center of the Earth, μ is the Earth gravitational parameter, *d* is the thickness of the orbital shell, and l_i is the characteristic length scale of the object population. We use the exponential atmospheric density model from the pyatmos Python package [9]. Using an exponential density model is standard practice for source-sink models [3,4]. We plan to improve the density model in future work.

Decay removes objects from a given orbital shell and adds objects to the altitude shell immediately below. Decay from the lowest altitude shell removes objects from the simulation. Our model does not consider the influx of objects from the altitude shell above the highest shell in the simulation.

2.2.4 Collisions

We assume that collisions occur only between objects in the same altitude shell. The relative collision rate between two object classes, indexed by *i* and *j*, in the shell with central altitude h is given by

$$\omega_{coll}(i, j) = \alpha_i \alpha_j \frac{\pi\, v_r(h)}{V(h)} n_i n_j \left(l_i + l_j\right)^2 \tag{4}$$

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[3] "Compliant" indicates that this population of satellites follows the PMD rule. In contrast, the population of active noncompliant satellites does not follow the PMD rule.

where $v_r(h)$ is the average relative impact velocity, which is roughly equal to the mean orbital velocity in the altitude shell centered at altitude *h*; *V(h)* is the volume of that altitude shell; n_i is the number of objects of species *i*; l_i is the characteristic length scale for this object species; and α_i is a reduction factor associated with active satellites. The reduction factor α_i takes values from 0-1 and reflects the relative decrease in the "natural" collision rate due to one of the objects being actively controlled.

Our model enables us to normalize the sum of the relative collision rates in Eq. 4 to a total collision rate. In particular, we use a quadratic normalization

$$\Omega_{tot} = \sum_{i,j} \omega_{coll}(i,j) = r_{coll} N_{tot}^2 \tag{5}$$

where N_{tot} is the sum of all objects in all altitude shells and r_{coll} is a parameter that we fit to the available historical collision data.

Catastrophic collisions can occur between satellites, between satellites and catastrophically-lethal debris, or between catastrophically-lethal debris objects. We apply the NASA standard breakup model (SBM) [7] to calculate the number of catastrophically-lethal and mission-lethal debris generated by each collision. Specifically, the SBM gives the number of debris generated by a catastrophic collision above a given length scale l_c as:

$$n_{deb}(l > l_c) = 0.1(M_1 + M_2)^{0.75} {l_c}^{-1.71} \tag{6}$$

where $M_{1,2}$ are the masses of the colliding objects. The SBM relates the mass to the characteristic length scale of each object via the area-to-mass ratio [7]. Mission-lethal collisions occur between mission-lethal debris objects and active satellites. These collisions turn an active satellite into a derelict satellite. While such collisions do not change the overall population, they have an indirect impact on debris creation because derelict satellites are likelier to collide than active satellites; see Eq. 4.

2.3 Stochastic trajectory sampling

Typical source-sink models calculate a single evolution of the orbital population by using the average rates for all processes and solving a relatively small set of coupled ordinary differential equations (ODEs). In contrast, our model calculates the probability of a range of population outcomes at each point in time. The general equation our model solves is given by

$$\frac{dP(\boldsymbol{n},t)}{dt} = W(\boldsymbol{n},t)P(\boldsymbol{n},t) \tag{7}$$

where *P(**n**,t)* is the probability of having object populations ***n*** at time *t*, and *W(**n**, t)* is a matrix that describes the rates and effects of the source and sink processes.

Equation 7 is typically infeasible to solve directly: there are as many ODEs as there are possible states of the system at each time. Borrowing the approach of Gillespie originally developed for chemical rate equations [5], we approximately solve Eq. 7 by sampling a large number of evolutions of the system. Our sampling process is as follows:

- We set the satellite and debris populations to their initial values.
- For each of a set number of time steps, we repeat the following steps:
  - We generate the number of occurrences of each process from a Poisson distribution based on the average process rates [5]. These rates are defined in the sections above.
    - For population-dependent rates, we use the population at the beginning of the time step.
    - The average number of occurrences is given by: $N_{avg} = \omega \Delta t$, where ω is the

average rate and Δt is the duration of the time step. We typically set $\Delta t = 1$ year.
  - We update the populations based on the results of this collection of process occurrences.

By aggregating these system evolutions, we can approximate the full probability distribution of object populations at each time point.

3 Results

3.1 Launch rate vs. post-mission-disposal rate

Here, we examine the impact of changing launch and PMD rates on the orbital environment. For these simulations, we use 25 altitude bins ranging from 500-2000 km, which covers the bulk of the LEO orbital regime. Our model does not consider the impacts from objects outside these altitude bounds. We use a launch profile that reflects the current populations of our altitude bins obtained from space-track.org [10]; see Fig. 2. We consider three total launch rates of 1500, 3000, and 6000 satellites per year, using this profile. A launch rate of 1500 satellites per year is roughly consistent with the launch rates of the past few years [10].[4] We further include two PMD rates of 1/25 per satellite per year and 1/5 per satellite per year. These parameter choices roughly reflect the FCC's previous 25-year and new 5-year de-orbit rules [8].

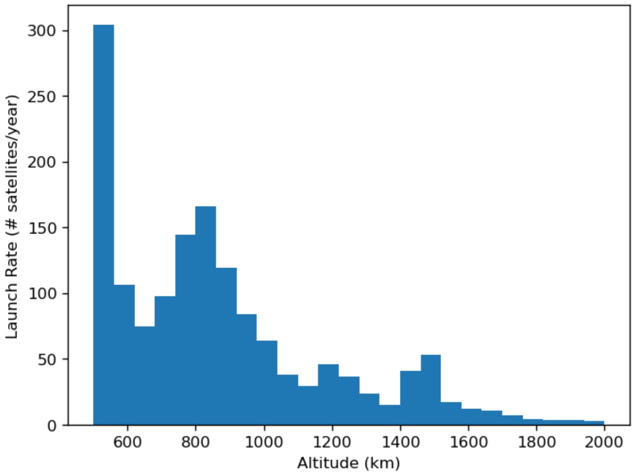

Fig. 2: Satellite launch rate profile. Based on data from space-track.org [10].

For each parameter set, we sample 100 system evolutions, each lasting for 100 years of simulation time. Our time step length is 1 year. We use a single satellite length scale of 2 m and 25 debris lengths between 1 cm and 2 m. We set the lethal length scale at 5 cm and assume that debris pieces larger than this (i.e., between 5 cm and 2 m) cause catastrophic collisions, while all debris pieces smaller than this (between 1-5 cm) are mission-lethal. With this choice, catastrophically-lethal debris sizes correspond roughly to trackable debris, while mission-lethal debris are non-trackable. For collisions, we set the reduction factor $\alpha = 0.2$ if the object is an active satellite and one otherwise [3]. This reduction factor causes a collision involving an active satellite to be 80% less likely than if the satellite were derelict. We use the normalization in Eq. 5 with $r_{coll} = 1.77 \times 10^{-10}$/year. We determined this value by fitting the quadratic relationship in Eq. 5 using the past 20 years of data obtained from space-track.org [10].

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[4] We define the launch rate in terms of satellites per year, which exceeds the number of launch events per year. See Sec. 2.2.1.

Figure 3 compares the active satellite populations obtained with the three total launch rates and two PMD rates at early, mid, and late time points in the 100-year trajectory. The dotted line shows the average population at each point, while the bands around the dotted lines reflect the maximum and minimum populations obtained across the set of 100 system evolutions. The results in Fig. 3 indicate that debris growth can significantly affect the long-term orbital environment for the 25-year PMD rule if the total rate of satellites launched is 3000 or 6000 per year, i.e., roughly 2-4x greater than the current rate. Under these conditions, the minimum active satellite population at 99 years decreases to zero, meaning that there are cases in which all active satellites have been eliminated by collisions.[5] For the 5-year rule, there are no cases in which this occurs.

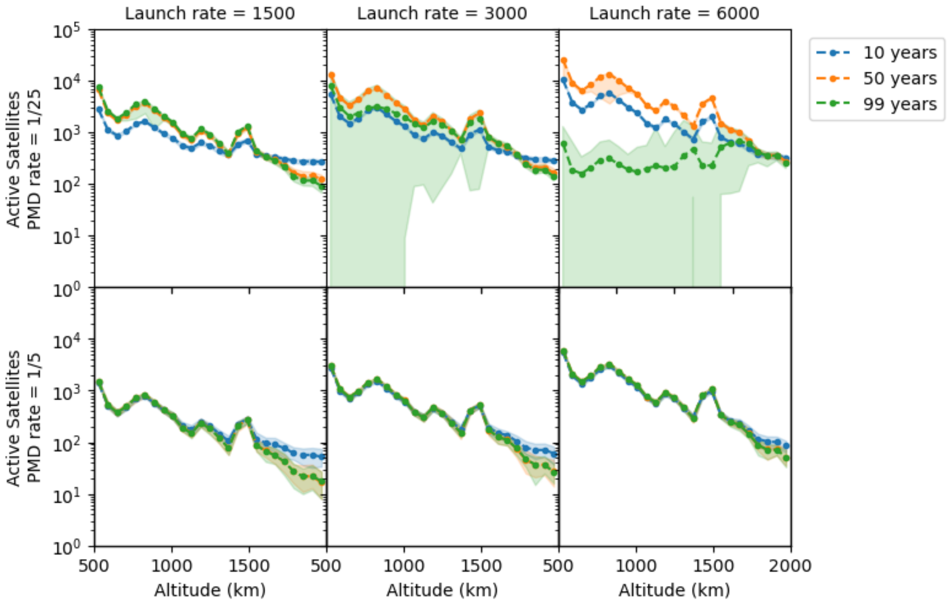

Fig. 3: Active satellite populations by altitude at different time points for a set of launch and PMD rate combinations. The dotted lines show the average population from the 100 system evolutions. The bands show the maximum and minimum populations obtained across the evolutions.

Figure 4 shows similar results to Fig. 3, but for catastrophically-lethal debris objects. The results for 3000 and 6000 launches per year and a PMD rate of 1/25 per satellite per year show significant debris growth at 99 years. This result confirms that runaway debris growth in the model is responsible for the change in the active satellite population at 99 years shown in Fig. 3.

Although we focus here on active satellites and catastrophically-lethal debris, our simulations also generate results for derelict satellites and mission-lethal debris.

3.2 Effect of noncompliance with post-mission-disposal rate

In the simulations described above, we assume that all active satellites comply with PMD. In practice, some fraction of satellites will be left in space at the end of their missions. In this section, we examine the effect of noncompliance with PMD on the orbital environment. To do this, we assume that some fraction of each set of launched satellites will not comply with the PMD rate. In these simulations, we

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[5] In our model, satellites continue to be launched to orbit even in the presence of runaway debris. In reality, we would expect launches to cease under such conditions.

set the lifetime of these active noncompliant satellites conservatively to 25 years and use a PMD rate of 1/5 per satellite per year for active satellites. We use a total launch rate with 1500 satellites per year.

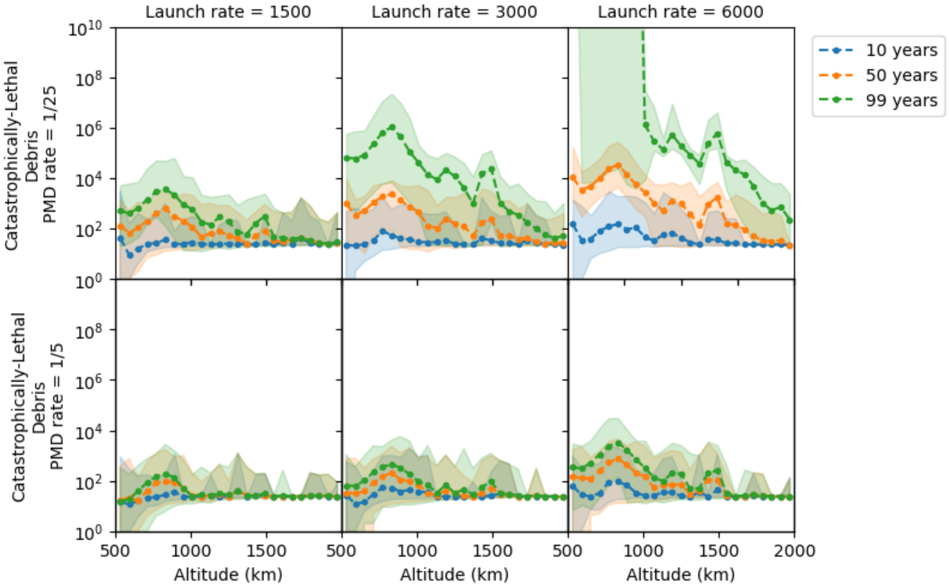

Fig. 4: Similar results to Fig. 3, but for catastrophically-lethal debris.

Figure 5 shows the active satellite populations generated for increasing rates of noncompliance. This active satellite population includes both compliant and noncompliant satellites. The fully compliant case in the top left panel matches the corresponding case in the bottom left panel of Figure 3. If there is 40% noncompliance or higher, however, our model suggests that there is a risk that the active satellite population decreases to zero in most of the altitude bins.

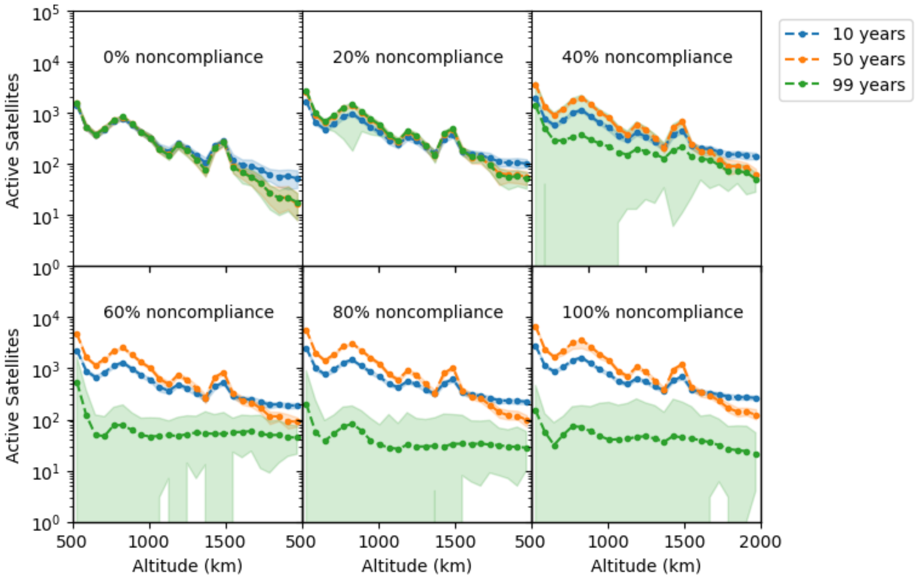

Fig. 5: Active satellites for various compliance rates with a launch rate of 1500 satellites per year.

Figure 6 shows similar results to Figure 5 for the catastrophically-lethal debris population. A noncompliance rate of 40% or greater produces a significant increase in orbital debris at 99 years. In particular, a noncompliance rate of 80% or higher produces a large spike in this population.

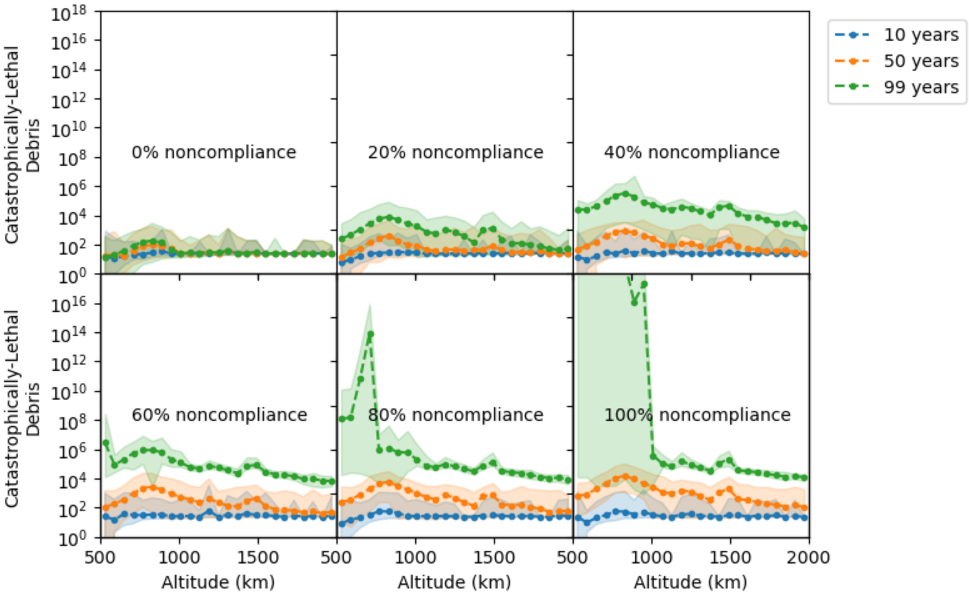

Fig. 6: Same as Fig. 5, but for catastrophically-lethal debris.

4 Discussion

We investigated here the effect of varying launch and PMD rates, as well as compliance with PMD rates, on the LEO orbital regime between 500-2000 km. Our model suggests that debris growth poses a risk to the active satellite population within 100 years if the number of satellites launched increases by roughly 2-4x over current levels and the average time to PMD is 25 years. The 5-year rule for PMD mitigates this risk in our model. However, the model further suggests that a high rate of noncompliance with the PMD rule can decrease the viability of some segments of the orbital regime even under the 5-year rule. With a noncompliance rate of 40% or higher, there is a risk that runaway debris growth drives the active satellite population to zero.

We note that in no case does the average satellite population decrease to zero. Rather, runaway debris growth occurs in some cases, causing there to be a wide range between the maximum and minimum satellite population. This variation arises from the stochastic nature of all the relevant processes: launch, PMD, natural decay, and collisions. Unlike standard source-sink models, our stochastic model enables us to describe the full range of system outcomes and therefore identify the risk of runaway debris growth.

The results presented above depend on several model assumptions. In particular, we normalize the total collision rate based on a quadratic formula. Our current normalization is based on a fit to historical data from the entire Earth environment in the last 20 years [6] and may need to be adjusted to reflect evolving conditions in LEO. Furthermore, we assume that active satellites are 80% less likely than derelict satellites or debris to collide with another object due to their ability to maneuver. Our results will vary with the efficacy of active satellite collision avoidance. Finally, we interpret a 25-year or 5-year PMD rule by imposing PMD rates of 1/25 or 1/5 per satellite per year, respectively. This approximation discounts the average mission life of satellites and therefore underestimates the time an active satellite will spend in orbit.

5 Conclusion

In this work, we applied a stochastic source-sink model to the study of Earth's orbital environment. Our model consists of populations of active satellites, derelict satellites, and orbital debris, together with processes that add or remove objects from these populations. We sample a large number of system evolutions that span the distribution of possible outcomes produced by the underlying processes. Using this stochastic approach, we assessed the stability of the environment to launch rate, PMD rate, and compliance with PMD. We intend to extend these sensitivity studies in future work.

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