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ONLINE OPTIMIZATION FOR ROUTING IN DYNAMIC CONTESTED ENVIRONMENTS

by

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December 2023

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The People’s Republic of China (PRC) seeks control over the Republic of China (ROC) and has increased military exercises in the Indo-Pacific, posing threats to ROC’s territory. The PRC’s ambition escalates political conflict and emphasizes the strategic importance of the region. To address challenges faced by the ROC in navigating risky areas, the ROC aim to establish transportation lines and optimize routes to minimize costs and avoid dangers.

Our approach tackles this issue as an online optimization challenge on an established network. It considers the changing environment, costs, and risks associated with paths within the network. Through an iterative process, we aim to find the most optimal route that reduces costs and bypasses contested areas. We will evaluate our algorithm by comparing its results with fixed and dynamic scenarios under various conditions. We can determine its efficacy by assessing its effectiveness in avoiding contested areas and minimizing costs.

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ONLINE OPTIMIZATION FOR ROUTING IN DYNAMIC CONTESTED ENVIRONMENTS

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ABSTRACT

The People's Republic of China (PRC) seeks control over the Republic of China (ROC) and has increased military exercises in the Indo-Pacific, posing threats to ROC's territory. The PRC's ambition escalates political conflict and emphasizes the strategic importance of the region. To address challenges faced by the ROC in navigating risky areas, the ROC aim to establish transportation lines and optimize routes to minimize costs and avoid dangers.

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List of Acronyms and Abbreviations

AO	area of operation
ALMM	algebraic-logical meta-model
CCP	casualty collection point
CVRP	capacitated vehicle routing problem
DTIC	Defense Technical Information Center
DVRP	dynamic vehicle routing problem
EPSG	European Petroleum Survey Group
FE	first-echelon
IP	integer programing
JIHI	Jurnal Ilmiah Hubungan Internasional
LP	linear programming
MILP	mixed-integer linear programming
MND	Ministry of Defense
MTZ	Miller-Tucker-Zemlin
NPS	Naval Postgraduate School
OR	Operations Research
PGMs	probabilistic graphical models
PLA	People’s Liberation Army
PLAN	People’s Liberation Army Navy

PLAAF	People's Liberation Army Air Force
POI	points of injury
PRC	People's Republic of China
PTSP	Prize-Collecting Traveling Salesman Problem
ROC	Republic of China
SE	second-echelon
SPP	shortest path problems
TSP	traveling salesman problem
UAVs	unmanned aerial vehicles
VRP	vehicle routing problem
2E-VRP	two-echelon vehicle routing problem
U.S.	United States
USAF	United States Air Force

Executive Summary

The present thesis offers an examination of the intricate domain of maritime security. It consists of five chapters that thoroughly investigate tactics, frameworks, and decision-making instruments essential for strengthening security in volatile and disputed maritime settings. At the core of this investigation lies a hypothetical conflict situation involving China, Taiwan, and various other nations.

The thesis begins by highlighting the utmost significance of maritime security in the modern era characterized by geopolitical tensions, territorial conflicts, and evolving dangers. It emphasizes the urgent requirement for sophisticated decision support systems to efficiently navigate the intricacies of naval conflict situations, protect crucial interests, and guarantee the security of maritime activities.

Chapter 2 provides a thorough examination of previous studies and methodologies in the field of maritime security. It encompasses various subjects, including analysis and modeling of threats, decision-making procedures, and the significance of technology. This chapter acts as an essential resource for gaining a comprehensive understanding of the present state of the domain.

Chapter 3 delves into the various approaches utilized, offering detailed explanations of techniques for optimizing online performance, modeling the shortest path, utilizing damage functions, and solving the Capacitated Vehicle Routing Problem (CVRP). These methodologies establish a solid foundation for constructing scenarios that accurately simulate complex real-world maritime conflict situations.

Chapter 4 presents the results that arise from implementing these methodologies to a hypothetical scenario of a maritime conflict. The chapter emphasizes the advantages and disadvantages of the created models and strategies. Noteworthy benefits include thorough coverage and pertinence, while also recognizing certain restrictions like simulating ship movement and enemy behavior.

Ultimately, the final remarks encapsulate the accomplishments of this investigation and outline potential areas for further study in the field of maritime security. A notable achievement lies in the creation of decision-making instruments that enhance efficiency in navigating ever-changing maritime settings. Additionally, it is emphasized that authenticating these systems through real-life implementations is crucial for future exploration and advancement.

The main objective of this experiment is to provide the ROC Navy with an effective tool for making decisions that will improve logistics transportation in complex maritime situations. By simulating dynamic and competitive environments, this model allows for the identification of the most favorable routes for friendly ships, taking into account potential threats. The results highlight the effectiveness of using Linear Programming (LP) as a method to optimize logistics operations. Ultimately, this research greatly enhances decision-making capabilities in challenging maritime scenarios, resulting in increased efficiency in logistics transportation for the ROC Navy.

Acknowledgments

I would like to begin by extending my deepest gratitude to my beloved nation, Taiwan, as well as the military division that has chosen me to represent them at the esteemed Naval Postgraduate School. I am truly blessed to be part of this extraordinary institution that not only boasts exceptional professors but also provides an enriching environment where I am surrounded by exceptional peers. This unique learning space offers an extensive range of expertise and wisdom, ultimately making it an invaluable sanctuary for intellectual development and personal maturation.

I would like to express my appreciation to Professor Jefferson Huang, who has served as my thesis advisor, and Ruriko Yoshida, my co-advisor. These two individuals have not only shown great commitment to their professions but also possess a deep enthusiasm for the subjects they impart knowledge on. Their unwavering support and mentoring have played a pivotal role in shaping my educational path. Moreover, I want to recognize Anthony Tai from the United States (U.S.) Naval Surface Warfare Center as my second reader. His invaluable perspectives have continuously sparked inspiration within me and significantly influenced the trajectory of my thesis.

I am incredibly grateful for the immense support and unwavering dedication shown by my friends and family, and I feel compelled to acknowledge their invaluable presence throughout every step of my academic journey. Their constant presence has not only offered me the necessary strength to overcome challenges, but also provided me with endless encouragement to pursue my educational goals. Their unwavering faith in my decisions has been a constant source of motivation, fueling my determination to succeed. My loved ones have truly been pillars of support, offering guidance during critical moments and standing by me through thick and thin. I owe a great debt of gratitude to them for contributing significantly to my personal growth and academic accomplishments.

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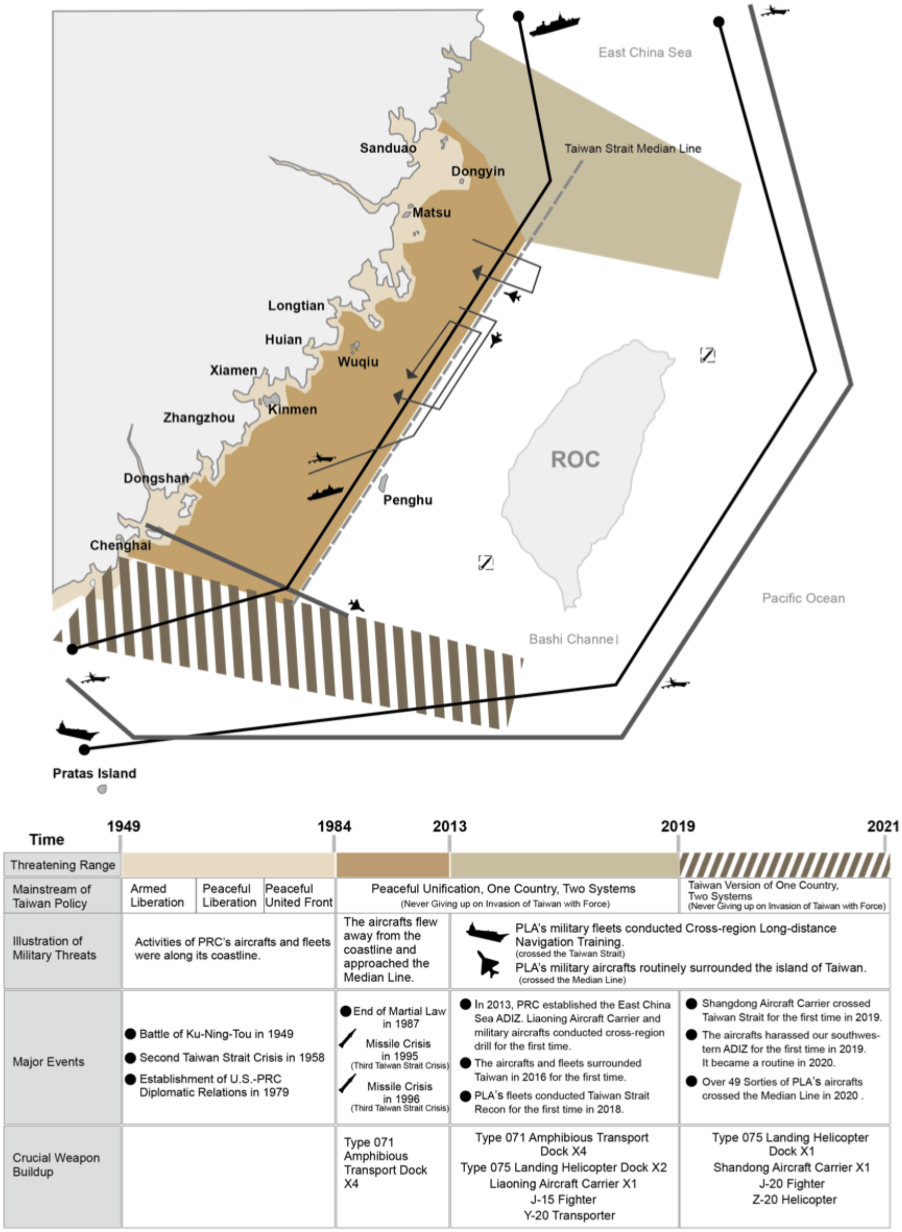
CHAPTER 1:
Introduction

This chapter aims to establish a foundational comprehension of the intricate relationship between the Republic of China (ROC), commonly known as Taiwan, and the People's Republic of China (PRC), commonly referred to as China. It also offers an overview of essential concepts and terminology pertinent to this study.

This chapter commences with examining the author's background and motivation, and then the chapter proceeds to address the central concern surrounding Taiwan. Subsequently, it outlines the specific objectives to be attained and delves into a comprehensive review of relevant literature aligned with the research objective.

1.1 Background and Motivation

The ROC, Taiwan, and the PRC, China, have been embroiled in a tense political conflict for over half a century. China has consistently sought to assert control over Taiwan and has not abandoned its claims to the island. According to the Ministry of National Defense (2021), China enacted the Anti-Secession Law in 2005, which was designed to discourage pro-independence sentiments in Taiwan and legitimize the potential use of force against the island. It is also important to note that China considers the "consolidated" state situation to be an internal conflict. Given these circumstances, China has not wavered in its determination to deter Taiwan through military means (Ministry of National Defense 2021). Furthermore, given the heightened dangers present in the global maritime setting, it becomes imperative to identify a mechanism capable of safely traversing regions that may pose hazards for the ROC (Eckhardt 2022). Figure 1.1 shows Diagram of Evolution of PRC's Policy toward Taiwan and Analysis of Its Military Threats from 1949 to 2021



Time	1949			1984	2013	2019 2021
Threatening Range						▨
Mainstream of Taiwan Policy	Armed Liberation	Peaceful Liberation	Peaceful United Front	Peaceful Unification, One Country, Two Systems (Never Giving up on Invasion of Taiwan with Force)		Taiwan Version of One Country, Two Systems (Never Giving up on Invasion of Taiwan with Force)
Illustration of Military Threats	Activities of PRC's aircrafts and fleets were along its coastline.			The aircrafts flew away from the coastline and approached the Median Line.	PLA's military fleets conducted Cross-region Long-distance Navigation Training. (crossed the Taiwan Strait) PLA's military aircrafts routinely surrounded the island of Taiwan. (crossed the Median Line)	
Major Events	● Battle of Ku-Ning-Tou in 1949 ● Second Taiwan Strait Crisis in 1958 ● Establishment of U.S.-PRC Diplomatic Relations in 1979			● End of Martial Law in 1987 Missile Crisis in 1995 (Third Taiwan Strait Crisis) Missile Crisis in 1996 (Third Taiwan Strait Crisis)	● In 2013, PRC established the East China Sea ADIZ. Liaoning Aircraft Carrier and military aircrafts conducted cross-region drill for the first time. ● The aircrafts and fleets surrounded Taiwan in 2016 for the first time. ● PLA's fleets conducted Taiwan Strait Recon for the first time in 2018.	● Shangdong Aircraft Carrier crossed Taiwan Strait for the first time in 2019. ● The aircrafts harassed our southwestern ADIZ for the first time in 2019. It became a routine in 2020. ● Over 49 Sorties of PLA's aircrafts crossed the Median Line in 2020 .
Crucial Weapon Buildup				Type 071 Amphibious Transport Dock X4	Type 071 Amphibious Transport Dock X4 Type 075 Landing Helicopter Dock X2 Liaoning Aircraft Carrier X1 J-15 Fighter Y-20 Transporter	Type 075 Landing Helicopter Dock X1 Shandong Aircraft Carrier X1 J-20 Fighter Z-20 Helicopter

Figure 1.1. Diagram of Evolution of PRC's Policy toward Taiwan and Analysis of Its Military Threats from 1949 to 2021. Adapted from Ministry of National Defense (2021)

1.2 Problem Statement

This research seeks to support the ROC to maintain transportation lines throughout dynamic contested maritime situations and optimize routing paths to reach desired operation objectives while minimizing costs and avoiding potential risks. According to the ROC Ministry of Defense (MND) report in 2021, in recent years, frequent military exercises carried out by the PRC in the Indo-Pacific region have expanded, presenting potential risks to the territory and sovereignty of the ROC (Ministry of National Defense 2021). These frequent military exercises have intensified the already tense political conflict between the two countries and underscored the strategic significance of the Indo-Pacific region. In a maritime environment operation, the ROC may encounter risky areas to navigate through. Therefore, the aim of this study is to support the ROC in maintaining transportation lines during dynamic contested maritime situations and optimize routing paths to achieve operation objectives with minimal costs and risks.

1.3 Research Objectives

The main objective of this research is to provide assistance to Taiwan in effectively managing transportation routes in challenging and volatile maritime environments. In the case of a potential conflict, Taiwan would be required to rapidly deploy a command line comprised of fast combat support ships, destroyers, frigates and logistics ships to its main harbors. Our suggested approach presents the issue as an online optimization problem across a fixed network. This approach takes into consideration the ever-changing and contested environment, factoring in the varying costs associated with each route based on its risk level determined by proximity to enemy ships. Through a series of optimized path problems, we aim to discover the most efficient path that minimizes costs and avoids areas of contention. By strategically determining routing paths that minimize costs and risks, the ROC can effectively carry out its operations and achieve desired outcomes. To assess the algorithm's effectiveness, we can compare its results with paths followed in both static and dynamic scenarios under different conditions and constraints. This evaluation will allow us to determine how well the algorithm performs in steering clear of contested areas while optimizing costs.

The current research presents significant potential benefits as it seeks to empower individual ships with the ability to independently assess and select their most advantageous course of action. This model would not only result in cost minimization but also ensure avoidance of conflict zones by adapting to the changing positioning of enemy forces over a period. However, an important limitation of this study is its dependence on a centralized approach for calculating routes, which may prove challenging when transitioning towards a decentralized control framework that governs the ships' operations.

CHAPTER 2:
Literature Review

In this chapter, an extensive analysis of the literature pertaining to the research conducted in the subsequent chapters of this thesis is presented. The intention behind conducting a thorough literature review is to establish a comprehensive understanding of the current state of knowledge within the realm of maritime security, threat prediction, routing problems and decision-making processes. By delving into existing scholarly works and relevant sources, this review aims to provide a contextual backdrop for the study and highlight the existing body of knowledge in these crucial areas.

2.1 Maritime Security and Threats

Many experts in international relations all agree that maritime security is of great importance in recent years. According to Yi (2023), maritime security pertains to the safeguarding and preservation of actions and resources associated with the Earth's oceans, seas, and water passages. Several risks to maritime security can result in noteworthy economic, environmental, and geopolitical implications. The task of upholding maritime security and anticipating potential future threats has become an increasingly prominent concern in recent times. According to *E-International Relations*, a prominent website in the field of international relations on a global scale (Yi 2023), China has become a global economic powerhouse in the past two decades. China's notable endeavor to enhance its military capabilities, particularly with regards to extending its naval presence in distant waters, is undeniably remarkable. This significant expansion of China's naval power projection gives rise to inquiries regarding the underlying motivations and potential consequences on the overall state of global maritime security (Yi 2023). Moreover, *Jurnal Ilmiah Hubungan Internasional (JIHI)*, a biannual academic journal that specializes in the study of global affairs, also illustrates a similar topic within its publications. Another article entitled "Maritime Security in the Indo-Pacific: Issues, Challenges, and Prospects," explores the contemporary importance of maritime security within the Indo-Pacific region. According to Tertia and Perwita (2018), in the 21st century, this region has captured global attention due to various factors including China's impressive economic growth, India's steady ascent, the growing assertiveness of regional countries, and an increase in oil exports to the area. As a result of its importance, prominent nations such as China, Japan, India, Australia and Southeast Asian countries that hold significant influence, and even the United States (U.S.) have been vying for influence in this contested region. The article emphasizes the perception of certain regional countries regarding China's expanding presence in the military realm in the Indo-Pacific as a demonstration of its authority (Tertia and Perwita 2018). Tertia and Perwita further note that since implementing its Rebalance policy in 2012, the U.S. has also become a more notable participant in this region. Their paper thoroughly examines the issues faced by maritime security in the Indo-Pacific while also addressing potential challenges and prospects. Last, *Geopolitics, History, and International Relations*, a scholarly publication that is dedicated to contemporary research on geopolitics, demonstrates comparable subject matter in terms of maritime security. The article, titled "Maritime Disputes in the Western Pacific," examines the fact that the Asia-Pacific area is a region that holds significant maritime importance both in terms of geography and economy. According to Till (1996), consequently, it faces various threats such as piracy and disruptions to ensure unhindered passage of ships. The risk associated with the peaceful use of the sea is heightened by criminal activities that take place in its waters, as pointed out by Till. Conflicts stemming from territorial disputes over resource-rich islands such as the Spratlys, Senkaku islands, and South Kuriles can lead to instability within specific regions. This instability becomes especially concerning when major strategic interests are at play (Till 1996). The increasing significance of the sea has caused naval power to flourish within this region, which can have both destabilizing effects and potential crisis management or cooperative efforts. Recent conflicts revolving around island disputes have vividly demonstrated that the sea possesses the capacity to both unite and divide nations (Till 1996).

This thesis and other maritime security experts from academia and the private sector focus on the topic of maritime security and the geopolitical dynamics in the Indo-Pacific region, with a specific emphasis on China and Taiwan. The aforementioned publications provide a comprehensive examination of China's expeditious economic development and the consequential expansion of its naval capabilities, prompting concerns regarding the implications for global maritime security. The maritime security experts focus on the significance of maintaining maritime security in the Indo-Pacific region, which has emerged as a crucial area of interest due to several factors, including China's rapid growth, India's ascent, and heightened assertiveness among neighboring countries. While the body of literature underscores regional countries' perceptions of China's increasing military influence in the Indo-Pacific and delves into the obstacles and opportunities within this contested region, this thesis focuses on the enduring political dispute between the PRC and the ROC and China's use of military tactics to discourage Taiwan. It also recognizes the increased risks in the international maritime domain and the necessity of establishing methods to ensure secure navigation in potentially perilous areas.

Overall, this thesis and body of lecture explore the significance of maritime security and the strategic importance of the Indo-Pacific region. Additionally, we both acknowledge China's involvement in these dynamics. The body of literature, however, offers a more extensive outlook on the geopolitical influences that are molding maritime security in the Indo-Pacific (Till 1996). It explores diverse regional actors and their respective concerns, such as China, India, and the U.S. Additionally, their papers address the consequences of territorial conflicts on maritime security and how naval strength can have both unifying and divisive effects on nations. The thesis, nonetheless, specifically centers on the confrontation between the ROC and the PRC, as well as the imperative for the ROC to maneuver through perilous maritime regions. It underscores the security hurdles confronting the ROC and underscores the significance of establishing a framework to tackle these difficulties. The thesis narrows its scope by focusing on ROC's maritime security and the obstacles it encounters within the framework of PRC's actions.

The evaluation of current academic sources presents significant background information for comprehending the intricate forces at work in the Indo-Pacific area and the significance of safeguarding maritime interests. Many of these sources, like Tertia and Till, provide valuable perspectives on the intentions and actions of prominent participants, such as China and the U.S. For example, Yi (2023) emphasizes the significance of maritime security in relation to China's expanding naval presence and its potential consequences on global maritime security. Tertia and Perwita concentrate on the importance of maritime security in the Indo-Pacific region, highlighting regional perceptions of China's military expansion and the involvement of major nations like the United States. Till explores the geopolitical and security aspects of maritime disputes in the Asia-Pacific region, particularly concerning territorial conflicts over resource-rich islands and their implications for regional stability and international relations. However, these sources do not directly tackle the precise issues and goals stated in the thesis, which focus on the ROC's maritime security under disputed circumstances. Therefore, the relevance and imminence of addressing the specific maritime security requirements of the ROC, considering the ongoing political conflict, are effectively argued in this thesis. Our proposal suggests an optimization approach to aid the ROC in efficiently and securely navigating contested maritime regions.

To summarize, the body of literature provides a more comprehensive perspective on maritime security in the Indo-Pacific, whereas our thesis focuses specifically on the challenges faced by the ROC. The thesis is an important addition as it proposes a possible remedy to address the distinctive concerns of the ROC. Building upon the insights from the articles above with a focus on enhancing national defense measures within Taiwan, the author constructs an experimental scenario set within the waters of the Western Pacific. This scenario centers around exploring interactions between Taiwan, China, and the U.S. Figure 2.1 shows the Defense Policy Trends of Major Countries in the Indo-Pacific:

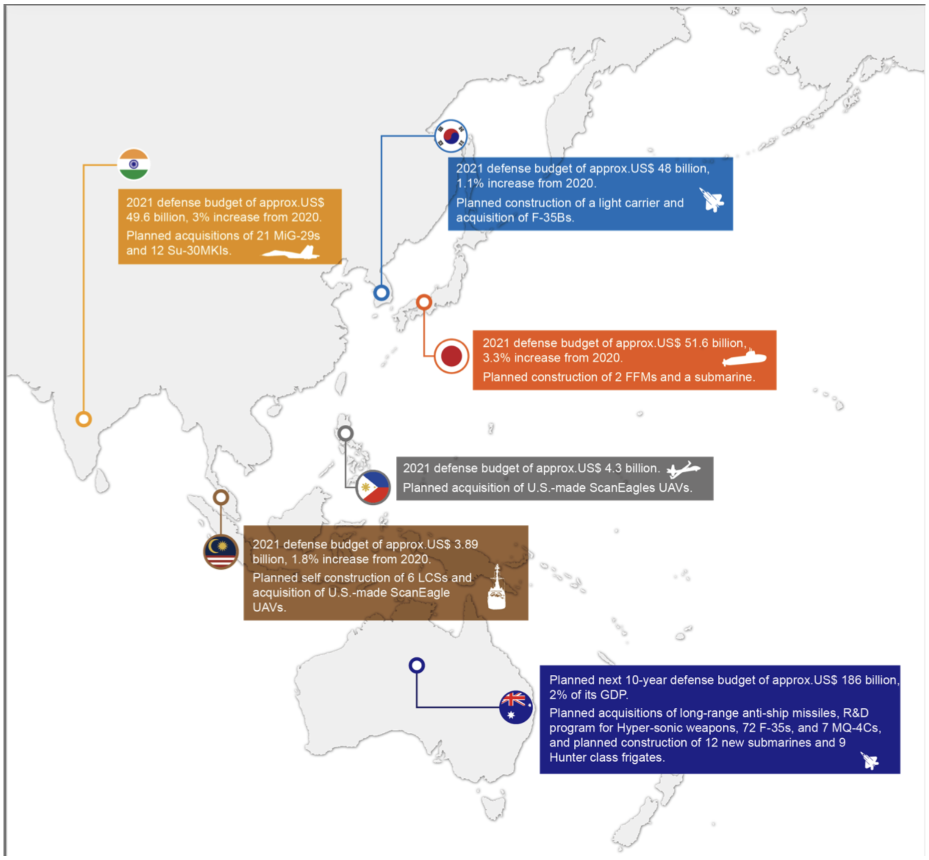


Figure 2.1. Defense Policy Trends of Major Countries in the Indo-Pacific. Adapted from Ministry of National Defense (2021)

2.2 Vehicle Routing Problems

2.2.1 Two-echelons of Vehicles

In recent years, the meandering journey of the two-echelon vehicle routing problem (2E-VRP) has emerged from the shadows of obscurity into the spotlight of logistics and transportation research. In a 2023 journal article titled "Two-Echelon Vehicle Routing Problems: A literature Review," authored by Sluijk et al. (2023), it explains that the traditional vehicle routing problem involves vehicles starting at a central location known as the depot, making deliveries to customers, and then returning to the depot. However, certain constraints such as physical obstacles or legal regulations near customer locations may necessitate the implementation of a two-echelon distribution network. This network consists of urban vehicles and city freighters referred to as first-echelon (FE) and second-echelon (SE) vehicles, respectively, which carry out pick-up and delivery operations. Intermediate facilities called satellites are utilized for consolidating goods between these echelons. This complex arrangement is commonly referred to as the 2E-VRP. Within the 2E-VRP framework, FE vehicles are responsible for transporting goods from depots to satellites where they are then passed on to SE vehicles for final delivery to customers. The concept of a two-echelon distribution network is widely employed in city logistics with the aim of optimizing transportation efficiency while also taking into account considerations such as safety and environmental impact. Moreover, this setup finds applications in various domains including express delivery services, grocery distribution networks, and e-commerce operations, among others. As of late, research has been focused on exploring different aspects of the 2E-VRP along with related problems like location-routing and inventory-routing problems. This comprehensive review seeks to provide an overview of existing studies on 2E-VRPs and their practical implementations within real-world scenarios by examining adaptations inspired by actual industry practices in addressing this problem variant. (Sluijk et al. 2023). Moreover, there exist two tiers of vehicles - primary carriers and secondary shuttle services, according to Yu's literature review from 2016 that has been expansively pondered owing to its worthwhile uses in industries such as e-commerce and nourishment delivery assists . The primary vehicles transport provisions from a central depot, akin to a bustling marketplace, to diverse waystations, while the secondary vehicles carry the goods from these wayposts to single customers. This double-decker strategy allows for a more proficient utilization of means and reduces the all-embracing distance traversed (Yu 2016).

In summary, our model employs two identical vessels, without any specific hierarchy or ranking, to serve the needs of six ports located in Taiwan. The vessels work collaboratively to achieve the common goal of fulfilling the delivery requirements of these ports. We focus on optimizing vehicle routes for the delivery of goods. In the traditional single-depot vehicle routing problem (VRP), goods are transported from a central warehouse to customers directly, often resulting in uneven delivery times and higher transport costs. In contrast, the 2E-VRP offers a more streamlined approach. It involves an intermediate step where goods are initially delivered to local depots, which then further distribute to customers. This approach can lead to more efficient delivery schedules and reduced transportation expenses.

As time progresses and technology advances, there is potential for our model to evolve into a more sophisticated 2E-VRP. This evolution would enable us to assign more complex tasks to the vessels, allowing them to autonomously determine the most optimal routes for deliveries.

2.2.2 Dynamic Vehicle Routing Problem

The dynamic vehicle routing problem (DVRP) addresses optimizing deliveries amid shifting schedules (Hanshar and Ombuki-Berman 2007). This enigma implicates transport traversing terrains to timely tend tasks. By discerning drivers' directions with developing determinations, we navigate the notorious navigation. This conundrum involves significant situations like spanning snacks and packets to rides and emergencies. The DVRP is responsive to the diverse needs of individuals, ranging from providing nourishment to hungry individuals with quick meals delivered over long distances, to efficiently transporting packages to people's residences. This intricate system adapts to the ever-changing demands and requirements of its users. We can determine the most optimal routes for vehicles in this complex logistical language by using algorithms similar to ants exploring boldly or genes slowly acquiring strategies. Like sailors surveying seas for sunny shores or safest routes through rising rapids, this issue involves intelligently routing rides to fulfill fetchings amidst factors' fluidities. In this section, we will wade through discussing the depths and dimensions of this dilemma and its potential impacts across industries. As delivery drivers dash to distribute dozens of daily deliveries, deftly dodging traffic's tides and tangles en route, the evolving efficiency established through examining this exception promises profits across plains.

The DVRP holds a significant position in the field of enterprise logistics (Konstantakopoulos et al. 2022). DVRP issues encompass various dynamics, such as the emergence of customers, travel and service times, and vehicle accessibility. An important aspect of DVRP is the consideration of customer availability, where customers may be revealed during the planning or execution of routes. The article "Dynamic Vehicle Routing Problem—Predictive and Unexpected Customer Availability" written by Kucharska (2019) presented a classification of DVRP problems based on different elements that contribute to dynamism . Its objective is to distinguish DVRP that accounts for the dynamic appearance of customers during route planning or execution. Specifically, it examines the difference between predictable and unexpected changes in customer availability. The predictive variant involves modeling customer availability according to a suitable general rule using the algebraic-logical metamodel (ALMM). According to Kucharska,

> ALMM offers a methodology for making collective decisions across successive process stages rather than individually for each vehicle. The proposed algebraic-logical model addresses the dynamic vehicle routing problem with predicted consumer availability. The paper showcases how ALMM can be applied to both predicted and unexpected customer availability scenarios in dynamic problems. (Kucharska 2019)

According to Liu et al. (2022), their paper examines the use of machine learning techniques for solving complex problems with unmanned aerial vehicles (UAVs). It compares two approaches, Online Optimization and Q-Learning, in solving reward-collecting tasks in different environments. Their study focuses on the problem of Autonomous Casualty Evacuation and highlights the significance of using UAVs for such tasks in military and civilian scenarios. It emphasizes the need for efficient algorithms to enable UAVs to navigate dynamic environments, collect rewards, and avoid adversaries while achieving mission objectives. Their study evaluates two main approaches: First, online optimization uses optimization techniques to make real-time decisions for the UAV's actions. It assumes the UAV has access to environment and adversary information and optimizes its actions for reward collection. The study evaluates the effectiveness of this approach under varying conditions and adversary behaviors. Second, Q-Learning is a reinforcement learning technique where an agent (UAV) learns a policy by interacting with the environment. The agent takes actions and receives rewards, gradually learning the best actions to maximize cumulative reward over time. This thesis explores Q-Learning's performance in different grid environments, including scenarios with multiple rewards and adversaries. Their research experiments on 5x5 and 9x9 grids, testing various adversary movements like clockwise, counterclockwise, and random. It measures key metrics: successful missions, average reward collected, and computational time for both approaches. According to their summary, online optimization is effective when the UAV has information about the environment and adversaries' behavior. It achieves optimal rewards and success in specific scenarios. On the other hand, Q-Learning struggles in larger and more complex scenarios, specifically when faced with capturing multiple rewards and avoiding adversaries in a large state space. Both approaches' computational time increases with bigger grid environments, especially for online optimization. Moreover, their thesis acknowledges limitations and suggests future research directions. It emphasizes the importance of efficient algorithms for UAVs in dynamic environments with uncertainties and adversarial elements. Overall, their thesis contributes to autonomous UAVs by studying online optimization and Q-Learning for solving reward-collecting problems. It provides insights into the trade-offs between computational efficiency and task complexity, guiding future research in this field.

According to Marler (2022), his thesis titled “A Decision Process for Surface Medical Evacuation Routing Under Adversary Threat and Uncertain Demand Using Online Optimization” builds upon the work of Liu et al. (2022) presents a comprehensive examination of the development of a decision process for optimizing surface medical evacuation routing in a challenging context characterized by adversary threats and uncertain demand. The research focuses on online optimization techniques, particularly the application of online convex optimization. The model is set within a military medical evacuation context where transporting critically injured patients is crucial. To test and validate the proposed decision process, a simulation model is utilized with an emphasis on exploring how the presence of adversaries affects routing decisions. The model evaluates various parameters such as distance traveled, fuel constraints, mission accomplishment, and simulation run times to derive meaningful insights. Additionally, the thesis introduces the concept of a distance multiplier that approximates adversary presence in existing simulation models. The conclusion discusses the broader implications of this decision process for autonomous systems, logistics support, and communication strategies when dealing with uncertain and contested environments. Overall, this thesis offers an innovative approach to addressing critical challenges in medical evacuation routing under adverse conditions with potential applications extending beyond the military domain.

According to Cone (2022), his paper titled “Casualty Evacuation Optimization in a Conflicted Environment” investigates how to improve casualty evacuation operations in complex and conflicted settings. The study centers on creating a model that can effectively plan the routes for ground and air vehicles to transport patients from points of injury (POI) to hospitals, taking into account different limitations and factors. The research in Cone’s study uses a mix of Operations Research (OR) and mathematical modeling methods. His thesis presents a model that incorporates ideas from both the traveling salesman problem (TSP) and the Prize-Collecting Traveling Salesman Problem (PTSP), which are well-known problems in OR. The goal is to optimize the number of patients transported while considering limitations such as vehicle capacities, time constraints, and coordination needs. His model's configuration consists of a network portrayal of the contentious surroundings, where nodes symbolize significant places such as POIs, casualty collection point (CCP)s, and hospitals. Ground and air vehicles are assigned with the duty of transporting patients from the POIs to hospitals via CCPs, and the model streamlines their paths. The study examines different situations, including varying quantities of patients, the repercussions of adversary actions, and the influence of vehicle capacities. Cone's research shows that, although the problem is complex, the model developed successfully finds workable solutions for smaller networks. It also examines limitations and assumptions, specifically regarding vehicle capacities and time constraints, which may need more investigation. Suggestions for future research include expanding the network, considering more realistic adversary actions, and exploring different optimization methods. Overall, Cone's thesis presents an important addition to casualty evacuation planning in conflicted environments. It introduces a unique mathematical model that efficiently routes vehicles to save lives. By incorporating operational research techniques and considering real-world limitations, this study offers valuable insights into optimizing casualty evacuation operations. These findings have significant implications for both military and humanitarian missions.

2.3 Conclusion

Marler adopts an online optimization model draws from Liu et al. (2022), which integrates randomized adversaries and patients. However, a significant contrast between our model and Marler's lies in the use of only one vehicle by the latter. On the other hand, the utilization of multiple VRP by Cone serves to coordinate numerous vehicles. A key distinction between Cone's model and our thesis presented here is that the former does not employ online optimization, rendering it unable to accommodate random adversaries. Consequently, our model essentially merges elements from both approaches.

The literature review contained in this chapter serves as the foundation, providing crucial ideas and approaches that are pertinent to the primary goal of this thesis, which is to tackle intricate maritime conflict scenarios. Although existing literature offers valuable perspectives on optimization, route planning, and damage modeling, this thesis stands out by employing these concepts in dynamic maritime conflict situations around Taiwan. Nonetheless, additional research and validation are necessary to evaluate the practical usefulness and effectiveness of the models presented in this thesis for real-world applications. This thorough investigation examines the theoretical and conceptual framework of maritime security, providing insight into the constantly evolving nature of threats. It highlights the urgent requirement for sophisticated predictive models and adaptable decision-making processes to effectively respond to these changing challenges. Furthermore, the literature stresses the increasing significance of technology in strengthening measures for maritime security and emphasizes the importance of conducting comprehensive risk assessments to improve our awareness of potential vulnerabilities and ultimately strengthen our defenses against possible threats. In the forthcoming sections, this thesis will elaborate on existing knowledge by introducing an original experimental model designed to enhance maritime security. The primary objective of this model will be to enhance the ability to evaluate threats and provide decision-making support systems within the maritime industry. By conducting a deeper exploration into these domains, this thesis aims to offer a more exhaustive comprehension of effectively safeguarding maritime environments from possible dangers for Taiwan.

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CHAPTER 3:
Methodology

In this chapter, we describe the methodologies that we utilize for building the scenario and experimental design. This integrated system consists of the following components: online optimization, shortest path model, damage function, TSP model, and capacitated vehicle routing problem (CVRP) model.

3.1 Shortest Path Problem Model

Finding a path typically involves locating the most direct route between two points. This task can also be categorized as a network or transportation issue. The main goal is to discover the shortest possible route for traveling from the starting point to the destination, taking into account that each pair of points may or may not be connected by a directed arc and are assigned their own weight (Zafar et al. 2018). Two well-known approaches for determining the shortest routes between nodes are Dijkstra's algorithm (2023) and linear programming (LP) (Encyclopedia Britannica 2023).

In this section, we use graph theory to visualize a network flow problem by using LP Formulation (Rardin 1998). It is also known as a transportation problem. Given a directed Graph: $G = (N, A)$, where N is a set of nodes; A is a set of arcs where (i, j) denotes arc connecting from node i to node j, where node $i \in N$ and node $j \in N$. Suppose each arc (i, j) is assigned a distance denotes as d_{ij}, assuming initially that $d_{ii} = 0$ and $d_{ij} \geq 0$. If there is no arc directed from node i to node j, then $d_{ij} = \infty$; or, for purpose of digital computation, d_{ij} is taken large (Dreyfus 1969). Numbered arbitrarily a start node s, and a terminal node t. Let x_{ij} be the decision variable that whether arc (i, j) is taken or not. If arc (i, j) is taken, x_{ij} equals to 1; and 0, otherwise, for all $(i, j) \in A$.

We want to find a forward path connecting from start node s to terminal node t with the shortest possible distance. The restriction as follows must be satisfied by the network flow: the sum of x_{ij} minus the sum of x_{ji} equals to 1 if node i equals to node s, -1 if node i equals to node t, and 0 if otherwise, for all i and $j \in N$. It means that the amount of flow into a node equals the amount of flow out of it, unless it is the start node s , which has only outgoing flow, or terminal point t , which has only incoming flow (Ahuja et al. 1988).

The shortest path problems (SPP) Model is as follows:

$$\min_x \sum_{i,j} d_{ij} x_{ij} \quad i,j \in A,$$

Subject to:

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1, & \text{if} \quad i = s \\ 0, & \text{if} \quad i \in N - \{\text{s,t}\} \qquad \forall i \in N \\ -1, & \text{if} \quad i = t \end{cases} \tag{3.1}$$

$$x_{ij} \in \{1,0\} \qquad \forall i,j \in A.$$

Figure 3.1 shows an example of a network flow and its optimal shortest path.

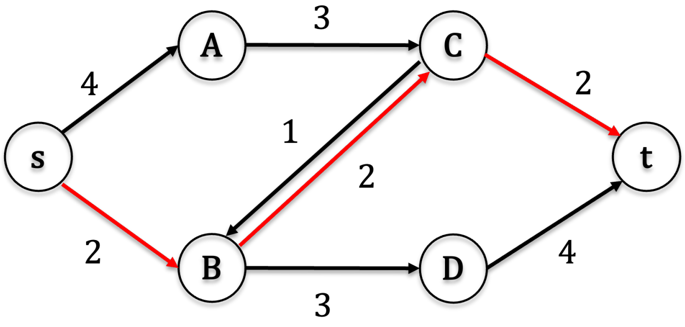

Figure 3.1. Shortest Path Finding in a Network Flow.

3.2 Online Optimization

Online optimization is a significant aspect of computer science and operation research, with applications in real-life decision-making situations. It is employed to find the best possible solution when future knowledge is incomplete or uncertain. This process involves making sequential decisions based on unfolding circumstances, where each decision made at a specific point cannot be altered later. Furthermore, each decision made has consequences that affect the subsequent alternatives available for selection in the future (Online optimization 2023) (Krumke 2002).

The starting point is indicated by the yellow node, while the terminal point is represented by the red node. The nodes in between, highlighted in blue, serve as intermediate points. The numbers assigned to each node symbolize the cost associated with that particular path. The shortest route is denoted by a red arrow line. In the first graph, we observe the initial identification of the shortest path. Moving on to the second graph, we witness a shift in the new starting point as node B takes precedence based on previous findings of the shortest path. Consequently, new costs are randomly generated at this juncture. As we proceed to examine the third graph, we discover a fresh identification of a shorter route. This iterative process continues until we eventually arrive at our desired destination – the terminal point.

Figure 3.2 illustrates the progression of online optimization.

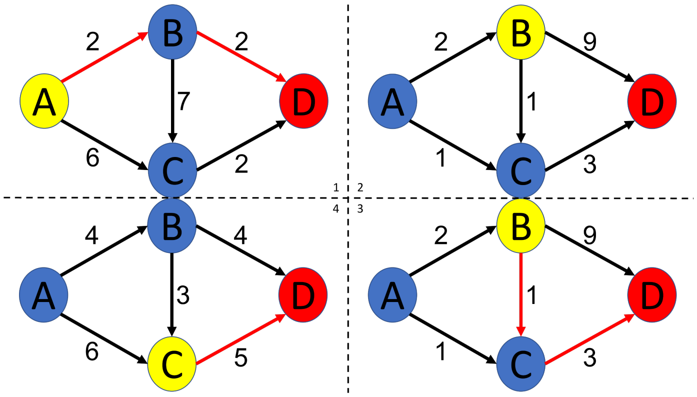

Figure 3.2. Overview of online optimization

3.3 Damage Function

The subsequent four functions are sourced from Alan Washburn's 2002 book, *Search and Detection*.

3.3.1 The Rayleigh Distribution

According to Garczyk and Stach (2021), “the Rayleigh distribution is a continuous probability distribution whose components are independent, positive random variables with a normal distribution.” The Rayleigh distribution is useful in any field where a vector component’s magnitude or a random variable’s amplitude follows a specific distribution. This distribution is versatile and commonly used in probability and statistics.

The probability density and cumulative distribution functions are associated with a random variable that follows a Rayleigh distribution, which is characterized by the parameter b:

$$f_R(r) = \frac{r}{b^2} \cdot e^{-\frac{r^2}{2b^2}}, \tag{3.2}$$

$$F_R(r) = P(R \leq r) = 1 - e^{-\frac{r^2}{2b^2}}. \tag{3.3}$$

3.3.2 The Diffuse Gaussian Weapon

According to Washburn (2003),

> The optimization problem is in general difficult because even calculating the hit probability for a given pattern usually requires numerical integrals. One exception is when errors are normally distributed and the damage function takes on a compatible “diffuse Gaussian” form. In that case, the hit probability can be expressed as an analytic function of the pattern’s aim points, and conventional methods used to optimize it.

The damage caused by the Diffuse Gaussian weapon, which is determined by the value of b, can be described using the Rayleigh distribution with parameter b. Therefore, according to Equation 3.3, we can express the damage function as follows:

$$D(r) = P(R \geq r) = 1 - F_R(r) = e^{-\frac{r^2}{2b^2}}. \tag{3.4}$$

The affected region caused by the use of these weapons is known as the damage area. Therefore, it can be defined as:

$$\begin{aligned} \alpha &= 2\pi \int_0^\infty rD(r)dr \\ &= 2\pi \int_0^\infty re^{-\frac{r^2}{2b^2}}dr \\ d &= 2\pi \int_0^\infty b^2 e^{-u} du \quad \text{(with change of variable} \quad u = \frac{r^2}{2b^2}. \end{aligned} \tag{3.5}$$

Diffuse Gaussian weapons have a wider and less predictable spread of damage. This spread makes them more appropriate when the damage to the target is caused by shell fragmentation, which means pieces of the weapon breaking apart and scattering. Consequently, this spread is a more suitable way to create a realistic scenario.

Figure 3.3 demonstrates the Damage Function of the Diffuse Gaussian.

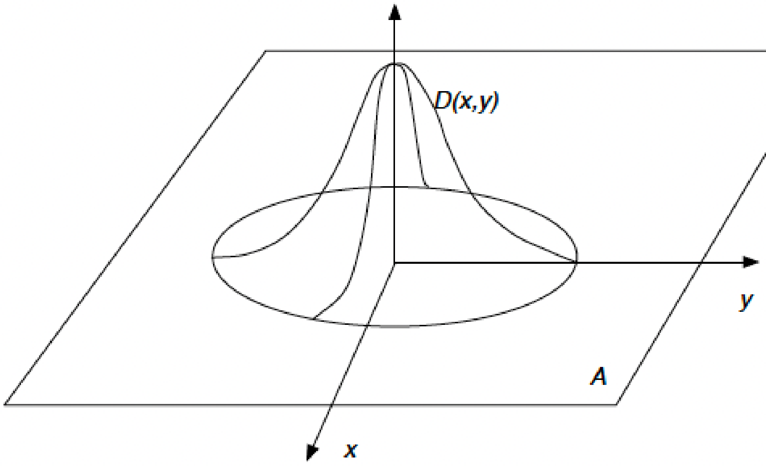

Figure 3.3. Diffuse Gaussian 2-D Damage Function. Adapted from Washburn (2002).

3.4 Capacitated Vehicle Routing Problem (CVRP) Model

The CVRP deals with optimizing the choice of routes to effectively use a group of vehicles in order to serve a particular group of customers (AIMMS 2023). All recipients are associated with shipments, and the requests are fixed, predetermined, and cannot be divided. The vehicles are identical and stationed at a solitary central depot (Toth and Vigo 2002). Every vehicle possesses a designated maximum cargo capacity, and the combined cargo capacity across all vehicles must equal or exceed the total demand. This issue is highly important within the field of optimization problems. The goal is to efficiently assign routes to multiple vehicles, with each vehicle taking the shortest available path (AIMMS 2023).

The CVRP can be formulated as a mixed-integer linear programming (MILP) model with the primary objective of minimizing the total distance while ensuring the fulfillment of customer demands (AIMMS 2023). This optimization problem involves a set of cities, denoted by indices $1, 2, \ldots, n$, and a fleet of vehicles, denoted by indices $1, 2, \ldots, p$. In this context, the variable d_{ij} signifies the distance between city i and city j, with $d_{ij} > 0$ for all distinct pairs i and j in the range $1, 2, \ldots, n$. Additionally, we introduce the binary variable x_{ijk}, which takes on the value 1 if the route from node i to node j is part of the optimal solution and is serviced by vehicle k. Otherwise, x_{ijk} is set to 0. Furthermore, it is assumed that all vehicles share the same capacity, denoted as Q, and each city j has a corresponding demand represented as q_j. We use the Miller-Tucker-Zemlin (MTZ) formulation for eliminating subtours (AIMMS 2023). It incorporates an extra variable called u_i that is assigned to each node, except for the depot. As the vehicle moves from node i to node j, the value of u_j must exceed the value of u_i. Consequently, as new nodes are visited, the u_i value increases to prevent backtracking and circular routes. The set V consists of all the nodes, and node n=1 serves as the depot. Importantly, the depot does not have a corresponding u_i value, allowing for the possibility of starting and ending at the depot without violating this condition.

The CVRP Model is adapted from AIMMS website, an international software company (AIMMS 2023):

$$
\begin{aligned}
&\min_{x} \sum_{k=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ijk} \\
&\text{Subject to:} \\
&x_{ijk} \in \{1, 0\} \qquad \forall k \in \{1, 2, \ldots, p\} \quad \forall i, j \in \{1, 2, \ldots, n\} \\
&x_{ijk} = 0 \qquad \forall k \in \{1, 2, \ldots, p\} \quad \forall i \in \{1, 2, \ldots, n\} \\
&\sum_{i=1}^{n} x_{ijk} = \sum_{i=1}^{n} x_{jik} \qquad \forall k \in \{1, 2, \ldots, p\} \quad \forall j \in \{1, 2, \ldots, n\} \\
&\sum_{k=1}^{p} \sum_{i=1}^{n} x_{ijk} = 1 \qquad \forall j \in \{2, \ldots, n\} \\
&\sum_{j=2}^{n} x_{1jk} = 1 \qquad \forall k \in \{1, 2, \ldots, p\} \\
&\sum_{i=1}^{n} \sum_{j=2}^{n} q_j x_{ijk} \leq Q \qquad \forall k \in \{1, 2, \ldots, p\}. \\
&u_j - u_i \geq q_j - Q(1 - x_{ijk}) \qquad \forall i, j \in V \backslash \{1\} \quad i \neq j \\
&q_i \leq u_i \leq Q \qquad \forall i \in V \backslash \{1\}.
\end{aligned}
\tag{3.6}
$$

The initial restriction guarantees that the quantity of vehicles entering a node matches the number of vehicles exiting from that same node. Together with the first restriction, the second constraint ensures that each node is both entered and exited by the same vehicle. In addition to the first constraint, the third limitation permits every vehicle to depart from and return to its original location. The fourth restriction ensures that the total demands of all customers served by a vehicle are not greater than its cargo capacity. The fifth constraint safeguards against the development of subtours by guaranteeing that the value of u_j is greater than or equal to q_j plus u_i. These values are then utilized to effectively eliminate any subtours. The last constraint establishes the capacity limits of the vehicles.

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CHAPTER 4:
Experimental Design and Analysis

The purpose of this experiment is to develop a decision-making tool that will facilitate prompt actions in intricate and competitive maritime situations between China and Taiwan. This tool has been specifically customized to aid the ROC Navy in minimizing routing expenses and enhancing the overall efficiency of logistics transportation lines. The first section outlines the hypothetical scenario from an instructor at the Naval Postgraduate School (NPS). The subsequent section elaborates on the specific aspects of the model's experimental design. Finally, the concluding section addresses the outcomes of the model as well as its performance and analysis.

4.1 Scenario

The information provided in this section is derived from Kline (2023), an unpublished document that describes the NPS 2023–2024 unclassified conflict scenario. We use this hypothetical scenario for applying the model.

In this scenario, the armed forces of the Philippines and Indonesia have readily accepted and smoothly incorporated the presence of the U.S. Marines, as well as air defense and ground-based anti-ship missile systems from the U.S. Army. However, after a significant frigate, PRESIDENT, belonging to the U.S. Navy was sunk, the U.S. declared all ships, submarines, and aircraft belonging to the People's Liberation Army (PLA) as hostile. As a result of this declaration, there has been an escalation of tensions in the Philippine Sea. Submarines from China's People's Liberation Army Navy (PLAN), along with H-20 bombers from their People's Liberation Army Air Force (PLAAF), have launched attacks on U.S. Navy warships and logistics vessels in order to disrupt their efforts in supplying Taiwan and allied forces within the first island chain. In response to these actions, U.S. submarines and bombers from the United States Air Force (USAF) have initiated strikes against Chinese warships and logistics vessels that are responsible for providing supplies to Chinese forces stationed on Natuna Besar island in the South China Sea. This situation in the Philippine Sea serves as an example of China's planned invasion of Natuna Besar island in Indonesia by 2044.

This annexation is part of a larger coordinated campaign led by China, along with its vassal states Russia and North Korea. Their objectives include claiming land, securing valuable resources, controlling critical maritime chokepoints, while also asserting their authority in establishing a new global order. During China's military occupation of Natuna Besar island for control over its oil fields, they deployed ten warships east of Taiwan and dispatched fifteen submarines to enforce a blockade on all military and energy supplies going into Taiwan - except those provided by China itself. The Chinese leadership framed this action as a means to bolster the economic and political integration between China and Taiwan (Kline 2023).

4.2 Experimental Design

In this particular section, we will explore how we employ the methodologies detailed in Chapter 3 to construct our bespoke experimental model.

4.2.1 Mapping and Geographic Representation

Effective map visualization is essential for conveying complex geographical data and analytical results. Our framework is designed to generate informative and visually appealing maps. It incorporates grid cells, nodes, and arcs, providing a comprehensive view of the spatial relationships within the data (Slocum et al. 2022). A Python-oriented geographical representation and mapping framework is presented in this section, which holds great importance in our research. This framework consists of a collection of functions and utilities enclosed within the "geopandas" and "contextily" packages, among others. It enables the exploration of different aspects of spatial analysis and geospatial visualization, thereby enriching the comprehensiveness and lucidity of our research outcomes. In order to generate these maps, the framework makes use of libraries such as "matplotlib" for overall plotting and "contextily" (Contextily 2023) for the base-map. This amalgamation of tools guarantees that the produced maps not only effectively convey the research findings but also adhere to established principles in cartographic representation.

Our geographic representation and mapping Framework provides a comprehensive range of tools for spatial analysis and visualization (Madhugiri 2023). The grid generation element serves as the foundation for multiple spatial analyses, while the map visualization element guarantees that the findings are presented in an easily understandable way.

4.2.2 Grid Generation

The grid generation aspect of the framework is a crucial element in the process of creating a grid in spatial analysis as it establishes the basis for various subsequent analyses, including spatial interpolation, density estimation, and hotspot detection (Anselin 1988). It facilitates the establishment of a grid across a designated geographic area, enabling detailed spatial analysis. This procedure partitions the region into smaller grid cells, resulting in the formation of a “GeoDataFrame” that not only streamlines spatial calculations but also improves the efficacy of spatial operations (GeoPandas 2023).

The “contextily” package in Python is used to offer features for manipulating web map tiles, enabling the inclusion of background maps in geospatial visualizations and analyses. This process involves utilizing the European Petroleum Survey Group (EPSG):4326 coordinate system, which is a widely accepted standard for defining coordinate reference systems and transformations in geospatial data (Contextily 2023). We are able to accurately represent our grid with the integration of this system.

The arrangement of the grid is created by connecting the central points of both the longitude and latitude lines. Each cell measures 0.2×0.2 degrees in latitude. As a result, a total of 2,121 grids are produced. These grids cover our area of operation (AO), spanning from 19.5 to 30.0 degrees latitude and 119.0 to 129.0 degrees longitude, encompassing the entire range. The map is particularly fascinating due to its extensive coverage, which includes the coastlines of China, Taiwan, Okinawa, and Guam, and it provides a thorough perspective of these noteworthy geographical characteristics. It is important to take note of one particular aspect: for the purpose of this study, we place Guam to the bottom right of the AO, to expedite the simulation computation time for the model. The grids present on the lands are eliminated to prevent direct passage of the ship through them.

4.2.3 Contested Environment Model

In a competitive and ever-changing environment, it is imperative to adjust the decision-making procedures to align with the present or expected circumstances, including the positioning of enemy vessels and evaluations of damage inflicted.

Enemy Deployment

This thesis utilizes two PLAN Type 052D Luyang III-class guided-missile destroyers, specifically Guiyang (pennant number 119), which are equipped with the YJ-18 SSM that has a minimum operational range of 220 km (YJ-18 2023). One destroyer follows a stationary movement pattern, while the other exhibits a random movement pattern. Additionally, we incorporate one Type 054A Jiangkai II-class frigate named Zaozhuang (pennant number 542), armed with the YJ-83 anti-ship missiles capable of reaching distances up to 180 km (YJ-83 2023). This frigate also demonstrates a random movement pattern. According to Chinese media sources in early 2020 and an article from The Diplomat journal in the same year (Gady 2020), these vessels were officially declared combat-ready by the PLAN. Thus, we have selected them as representative enemy ships deployed near Taiwan's eastern sea for our analysis.

Damage Calculation

In this study, we make the assumption that all of these armaments conform to the damage function of a diffuse Gaussian weapon, where the parameter b is determined by dividing the missile range by the square root of 2 (Washburn 2002) that has mentioned in 3.4.

This model accumulates damage values for grid cells that intersect with multiple points by updating their costs. It uses a clipping function to ensure that costs remain within the range of 1 to 10. The resulting cost dictionary contains the costs for each grid cell, reflecting the cumulative impact of the damage points.

4.3 Model Overview

Our model is centered around the AO, which encompasses a specific region characterized by latitudes ranging from 19.5 to 30.0 degrees and longitudes ranging from 119.0 to 129.0 degrees. This region plays a vital role in our approach, as it consists of two key phases that we will explore further. In the first phase, we establish a grid system where each arc within this grid is assigned a uniform cost of 1. To fulfill the demands of the six ports which are Zuoying, Maging, Taipei, Keelung, Su-ao and Hualien in Taiwan, our two friendly ships are each equipped with 4 units of supplies while each port requires only 1 unit. Friendly ships follow the four cardinal directions (east, west, south, and north) to streamline the calculation of travel time steps. Each friendly ship maintains a speed of 12 nautical miles per hour, and in the grid system, 0.2 degrees of latitude roughly translates to 12 nautical miles. Consequently, when a friendly ship advances by one grid during each time step within the model, it signifies an actual travel time of 1 hour, covering a distance of 12 nautical miles. To determine the initial optimal route for our two ships departing from Guam, we employ the CVRP methodology. This methodology allows us to efficiently plan and allocate resources to meet the demands of the six ports located in Taiwan. Moving on to the second phase, we introduce a dynamic environment into consideration. This environment introduces three enemy ships that two exhibit random movement patterns, one exhibits stationary and all of them possess damage levels ranging from 1 to 10. These damage levels are then utilized as new costs for arcs within their range. Our objective in this phase becomes utilizing SPP techniques to identify optimal paths for our friendly ships at each time step, ensuring they navigate through potential threats while guiding them towards their designated ports identified in the previous CVRP phase. By expanding on these two phases and emphasizing how they contribute to our overall approach, we enhance both clarity and depth regarding our model's functioning within the defined AO.

4.3.1 Notation

1. N denotes a set of ports $\{1,2,3\ldots,\text{n}\}$
2. V denotes a set of ships $\{1,2,3\ldots,\text{v}\}$
3. M denotes a set of grids $\{1,2,3\ldots,\text{m}\}$
4. t denotes the time steps
5. s denotes the start point
6. d denotes the destination point
7. g denotes the current grid
8. h_k denotes the target port of vehicle k which is yielded by Phase I CVRP
9. c_{ij} denotes the cost of the path from the start to destination point
10. c'_{lwt} denotes the path cost that is caused by the random movement patterns of the enemy ships from grid l to grid w at time t
11. u denotes the value of the node as the vehicle moves except for the depot
12. q denotes the same demand of each port
13. Q denotes the same capacity of each ship
14. x_{ijk} denotes the route from port i to port j is served by vehicle k.

15. y_{lwkt} denotes the route from grid l to grid w is served by vehicle k at time t
16. r denotes the distance between the destination grid w and the nearest enemy ships
17. R denotes the effective missile range of the enemy ships
18. b denotes a parameter using the Rayleigh distribution equals to $\frac{R}{\sqrt{2}}$

4.3.2 Phase I: CVRP

$$
\begin{aligned}
&\min_{x} \sum_{k=1}^{v} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ijk} \\
&\text{Subject to:} \\
&x_{ijk} \in \{1,0\} \qquad \forall k \in \{1,2,\ldots,v\} \quad \forall i,j \in \{1,2,\ldots,n\} \\
&x_{ijk} = 0 \qquad \forall k \in \{1,2,\ldots,v\} \quad \forall i \in \{1,2,\ldots,n\} \\
&\sum_{i=1}^{n} x_{ijk} = \sum_{i=1}^{n} x_{jik} \qquad \forall k \in \{1,2,\ldots,v\} \quad \forall j \in \{1,2,\ldots,n\} \\
&\sum_{k=1}^{v} \sum_{i=1}^{n} x_{ijk} = 1 \qquad \forall j \in \{2,\ldots,n\} \\
&\sum_{j=2}^{n} x_{1jk} = 1 \qquad \forall k \in \{1,2,\ldots,v\} \\
&\sum_{i=1}^{n} \sum_{j=2}^{n} q_j x_{ijk} \leq Q \qquad \forall k \in \{1,2,\ldots,v\}. \\
&u_j - u_i \geq q_j - Q(1 - x_{ijk}) \qquad \forall i,j \in N \backslash \{s\} \quad i \neq j \\
&q_i \leq u_i \leq Q \qquad \forall i \in N \backslash \{s\}
\end{aligned}
\tag{4.1}
$$

$$
\begin{aligned}
&\text{where} \quad c_{ij} = \min_{y} \sum_{l,w} y_{lw} \\
&\text{Subject to:} \\
&\sum_{w} y_{lw} - \sum_{w} y_{wl} = \begin{cases} 1, & \text{if } l = s \\ 0, & \text{if } l \in M - \{s,d\} \text{ for all } l \in M \\ -1, & \text{if } l = d \end{cases} \\
&x_{lw} \in \{1,0\} \quad \forall l,w \in M.
\end{aligned}
\tag{4.2}
$$

Figure 4.1 provides a diagram that illustrates the results obtained from 4.3.2 Phase I: CVRP. The routes of the two amicable ships are represented by black and blue lines, while the blue nodes indicate the locations of ports on land. The red nodes, on the other hand, represent the nearest grid nodes to each port.

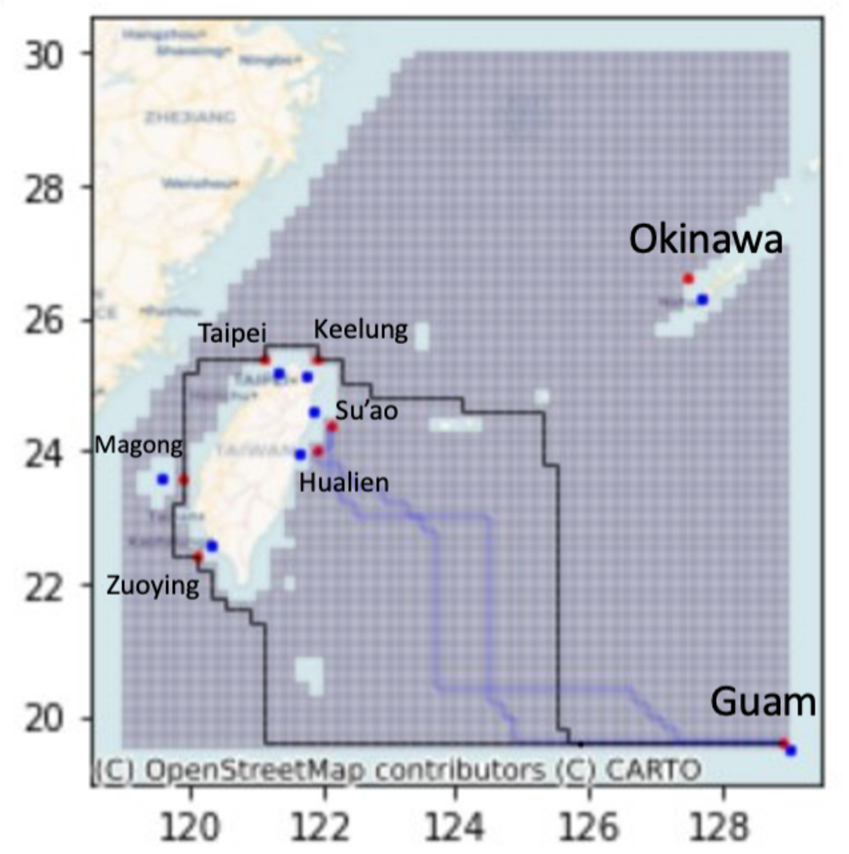


Figure 4.1. Phase I: CVRP

4.3.3 Phase II: SPP

$$
\begin{aligned}
&\min_{y} \sum_{l=1}^{m} \sum_{w=1}^{m} c'_{lwt} y_{lwkt} \\
&\text{where} \quad c'_{lwt} = e^{-\frac{r^2}{2b}} \cdot 10 \quad \text{bounded by 1 and 10} \\
&\text{Subject to:} \\
&\sum_{w} y_{lwk} - \sum_{w} y_{wlk} = \begin{cases} 1, & \text{if } l = g \\ 0, & \text{if } l \in N - \{g, h\} \text{ for all } l \in M \\ -1, & \text{if } l = h_k \end{cases} \\
&y_{lw} \in \{1, 0\} \quad \forall l, w \in M.
\end{aligned}
\tag{4.3}
$$

Figure 4.2 presents the outcomes acquired from 4.3.3 Phase II: SPP integer programing (IP). In this diagram, the optimal routes of the two friendly ships are depicted by black and blue lines at each time step. On the other hand, the random routes of the enemies are represented by red lines at each time step. Additionally, the damage levels are indicated by a gradient circle ranging from 1 to 10.

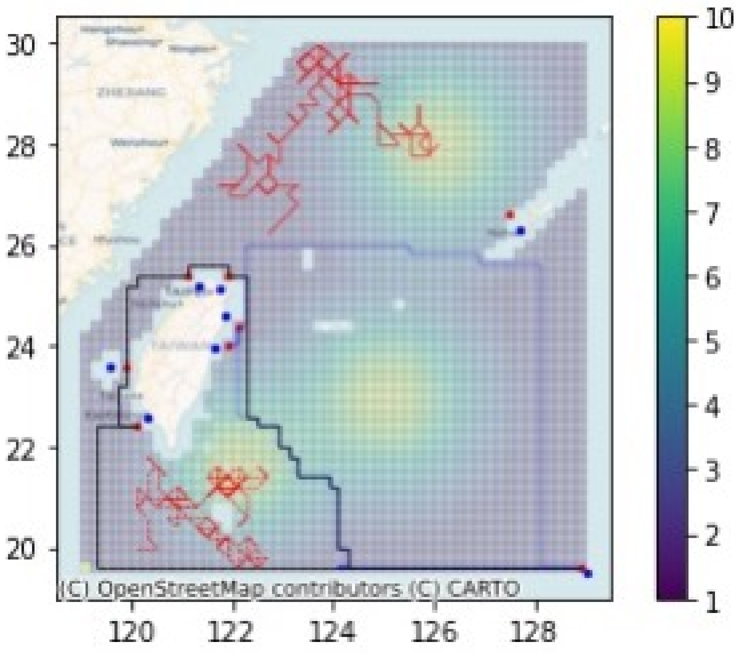


Figure 4.2. Phase II: SPP IP

Figure 4.3 presents the outcomes acquired from 4.3.3 Phase II: SPP LP. In this diagram, the optimal routes of the two friendly ships are depicted by black and blue lines at each time step. On the other hand, the random routes of the enemies are represented by red lines at each time step. Additionally, the damage levels are indicated by a gradient circle ranging from 1 to 10.

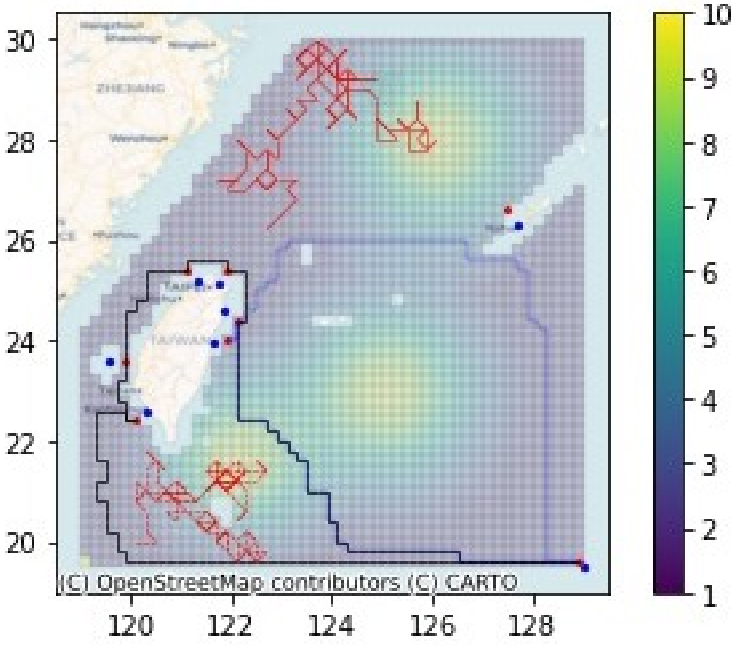
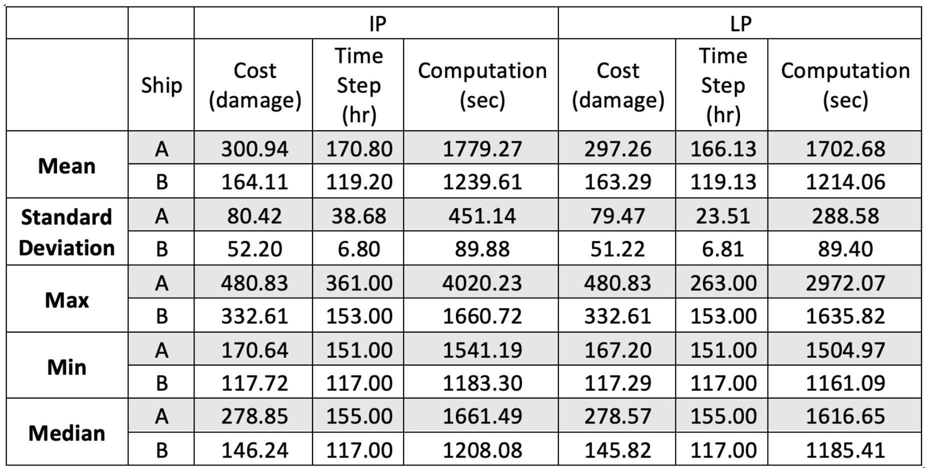


Figure 4.3. Phase II: SPP LP

4.4 Result and Analysis

The outcome and the valuable insights obtained from analyzing the cost, time step taken in hours, and computational duration in seconds for different methodologies such as VRP, IP, and LP will be presented in this section. During 4.3.2, the computational runtime of the VRP is approximately 305 seconds for both friendly ships. The final cost for friendly ship A is 152, with a time step of 152 hours. Similarly, for ship B, the final cost is also 152 with a time step of 152 hours. The initial plan generated during 4.3.2 involves ship A supplying Zuoying, Maging, Taipei, and Keelung, while ship B supplies Hualien followed by Su-ao. Building upon on this initial travel plan, we then add online optimization elements that we mentioned in 4.3.3 to generate the result for IP and LP. The tabulation 4.4 shows a overview of the result using IP and LP in phase II: SSP.



		IP			LP		
	Ship	Cost (damage)	Time Step (hr)	Computation (sec)	Cost (damage)	Time Step (hr)	Computation (sec)
Mean	A	300.94	170.80	1779.27	297.26	166.13	1702.68
	B	164.11	119.20	1239.61	163.29	119.13	1214.06
Standard Deviation	A	80.42	38.68	451.14	79.47	23.51	288.58
	B	52.20	6.80	89.88	51.22	6.81	89.40
Max	A	480.83	361.00	4020.23	480.83	263.00	2972.07
	B	332.61	153.00	1660.72	332.61	153.00	1635.82
Min	A	170.64	151.00	1541.19	167.20	151.00	1504.97
	B	117.72	117.00	1183.30	117.29	117.00	1161.09
Median	A	278.85	155.00	1661.49	278.57	155.00	1616.65
	B	146.24	117.00	1208.08	145.82	117.00	1185.41

Figure 4.4. Overview of the result using IP and LP

4.4.1 Result of the Final Cost

The trend and ultimate cost of ship A for each replication are depicted in Figure 4.5. We can also tell from the Figure 4.6 that overall the final cost of ship A using LP is less than or equal to using IP. Furthermore, the trend and ultimate cost of ship B for each replication are depicted in Figure 4.7. We can also tell from the Figure 4.8 that overall the final cost of ship B using LP is less than or equal to using IP.

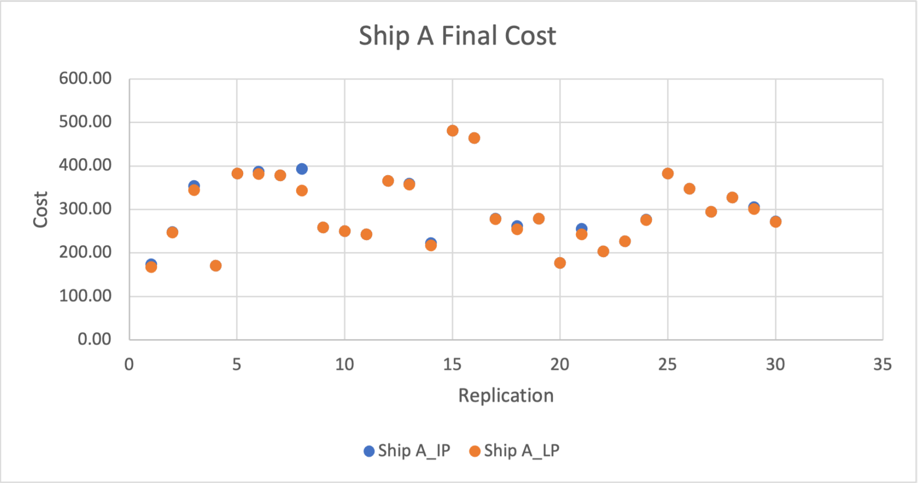


Figure 4.5. Ship A Final Cost Comparison between IP and LP

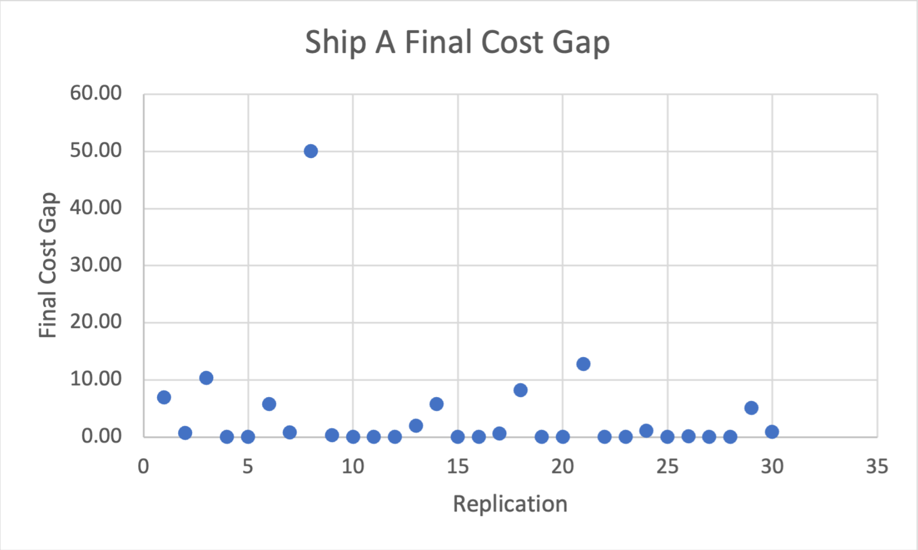


Figure 4.6. Ship A Final Cost Gap Comparison between IP and LP

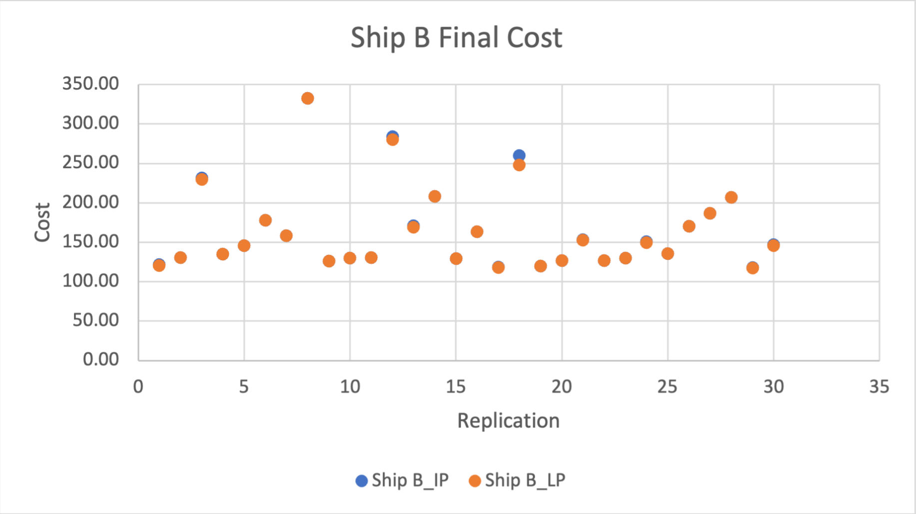


Figure 4.7. Ship B Final Cost Comparison between IP and LP

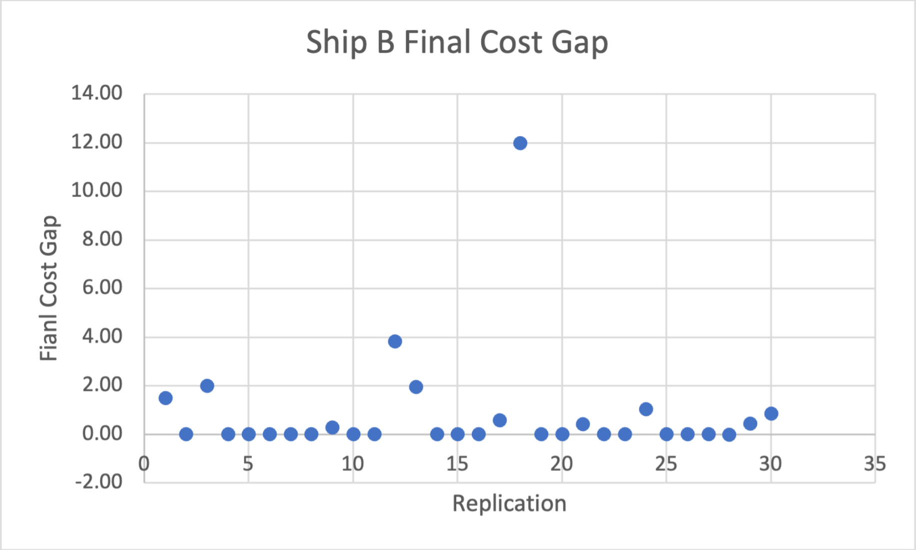


Figure 4.8. Ship B Final Cost Gap Comparison between IP and LP

4.4.2 Result of the Time Step

The trend and ultimate time step of ship A for each replication are depicted in Figure 4.9. We can also tell from the Figure 4.10 that overall the time step of ship A using LP is less than or equal to using IP. Furthermore, the trend and ultimate time step of ship B for each replication are depicted in Figure 4.11. We can also tell from the Figure 4.12 that overall the time step of ship B using LP is less than or equal to using IP.

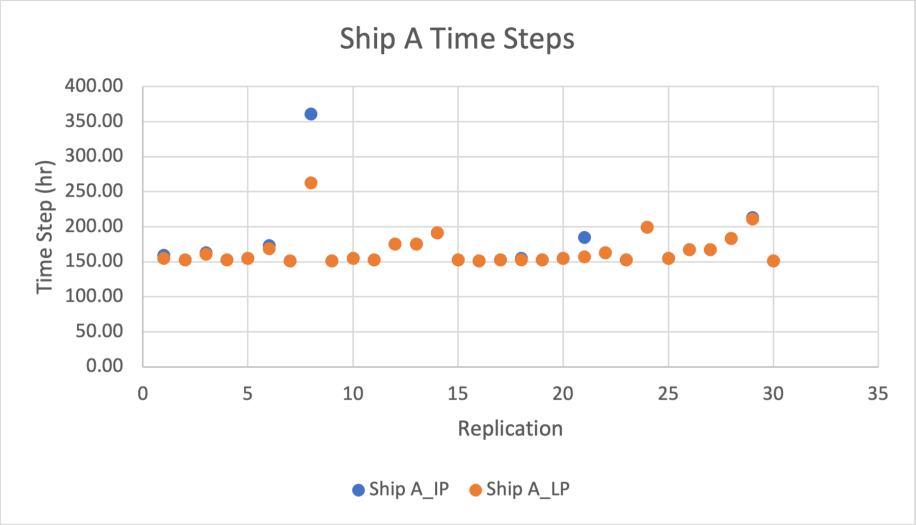


Figure 4.9. Ship A Time Step Comparison between IP and LP

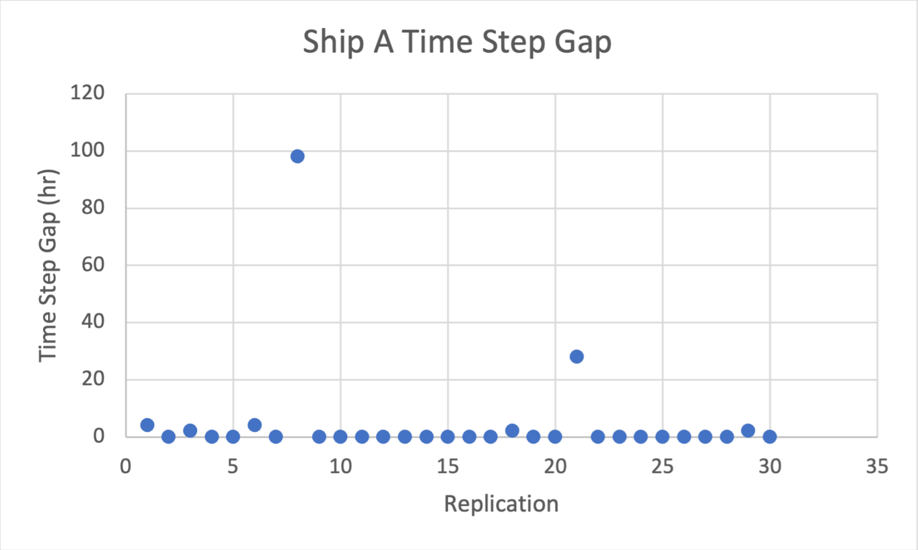


Figure 4.10. Ship A Time Step Gap Comparison between IP and LP

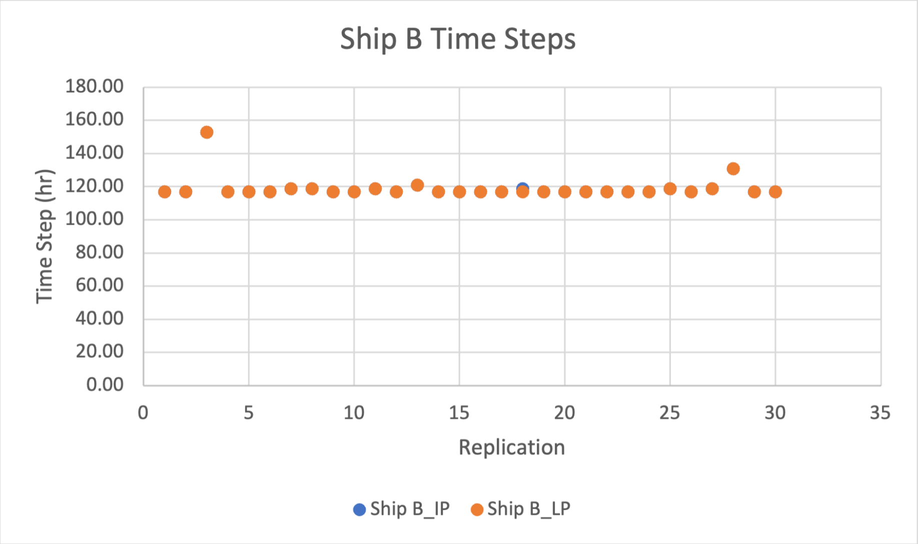


Figure 4.11. Ship B Time Step Comparison between IP and LP

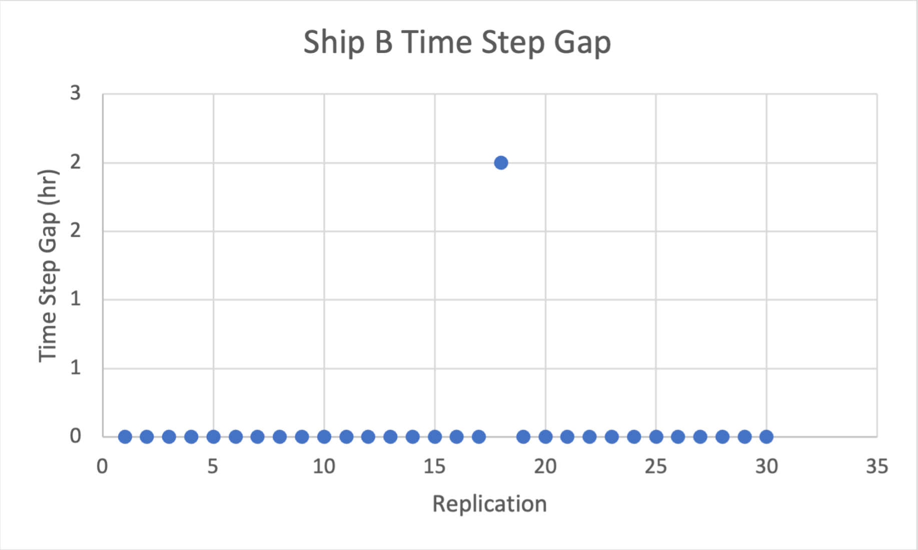


Figure 4.12. Ship B Time Step Gap Comparison between IP and LP

4.4.3 Result of the Computation Run Time

The trend and ultimate computation run time of ship A for each replication are depicted in Figure 4.13. We can also tell from the Figure 4.14 that overall the computation run time of ship A using LP is less than or equal to using IP. Furthermore, the trend and ultimate computation run time of ship B for each replication are depicted in Figure 4.15. We can also tell from the Figure 4.16 that overall the computation run time of ship B using LP is less than or equal to using IP.

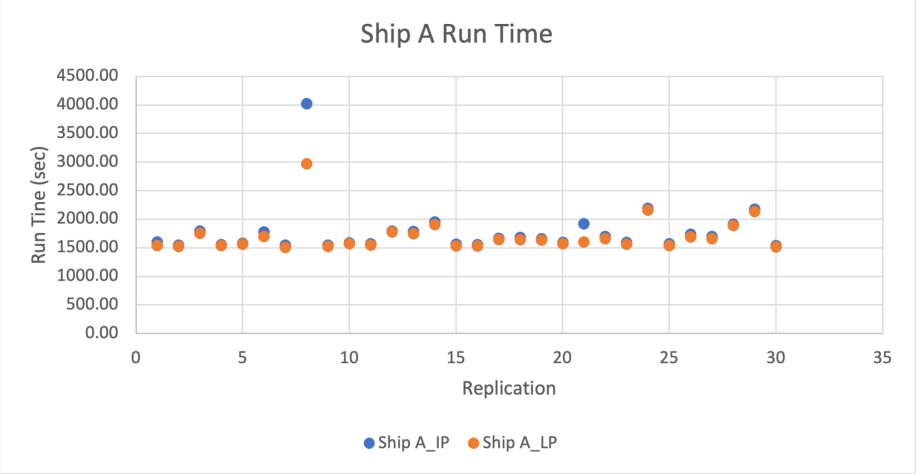


Figure 4.13. Ship A Run Time Comparison between IP and LP

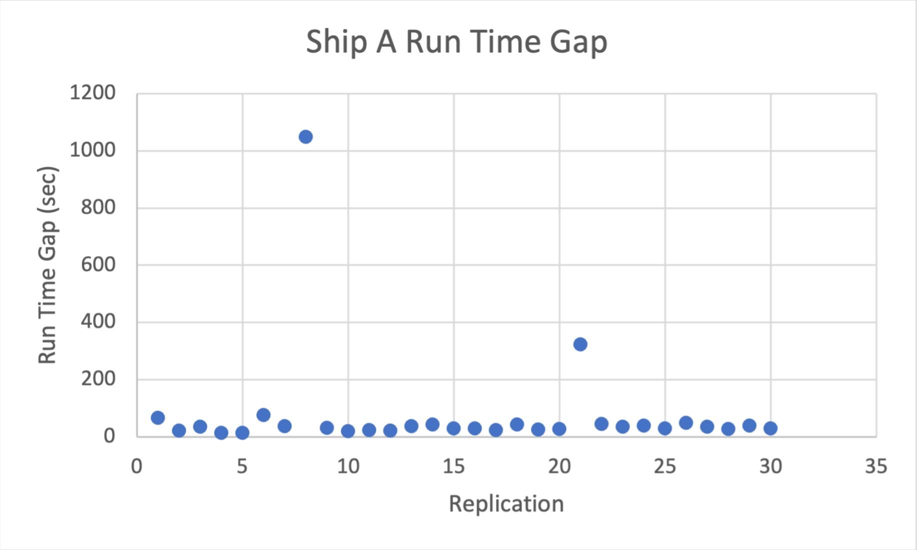


Figure 4.14. Ship A Run Time Gap Comparison between IP and LP

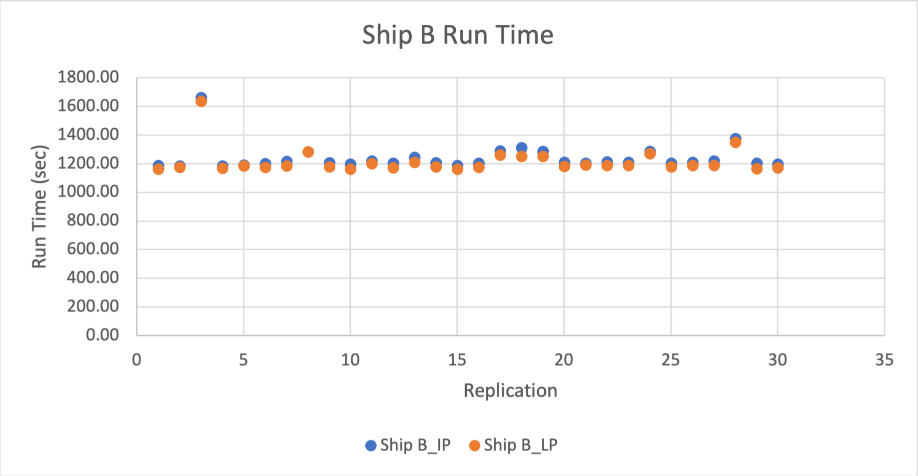


Figure 4.15. Ship B Run Time Comparison between IP and LP

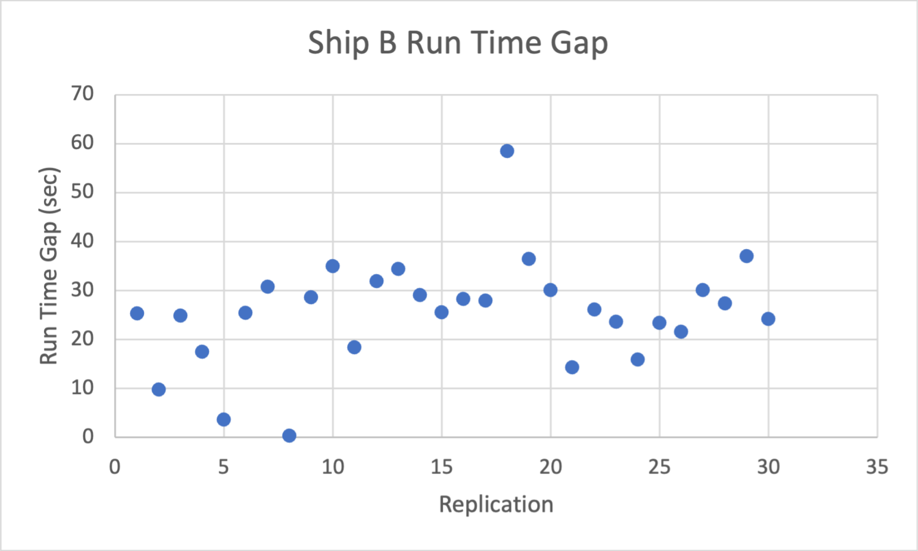


Figure 4.16. Ship B Run Time Gap Comparison between IP and LP

CHAPTER 5:
Conclusion and Future Work

The findings and analysis of this study are transformed into a practical instrument for the MND in this chapter. The initial part of this chapter provides a summary of the study's contribution on using online optimization to find an optimal path in a dynamic contested maritime environment. The subsequent section explores potential enhancements to threat prediction and modeling. Furthermore, there is an emphasis on the significance of incorporating uncertainty and risk into decision-making. Lastly, the concluding section discusses the potential future deployment of this model in actual scenarios through further modifications.

5.1 Summary of Contributions

The thesis centers on strategies for constructing a scenario and experimental design for a particular maritime conflict involving China, Taiwan, and other nations. It incorporates various elements such as online optimization, shortest path modeling, damage functions, TSP modeling, and the CVRP modeling. The scenarios involve military actions and intricate naval operations in a disputed environment. The experimental design includes a geospatial framework for mapping and visualization, grid generation, and map visualization to represent the geographical aspects of the scenario. It also takes into account enemy deployment, damage calculations, and a dynamic approach to account for changes in the environment and movement of targets. These scenarios and models are intricate and necessitate thorough analysis. However, there is an evaluative assessment of the strengths and weaknesses of the thesis as outlined:

Strengths: First, it is important to note that this thesis offers extensive coverage that encompasses a wide assortment of methodologies including optimization techniques, geospatial analysis methods, and mathematical modeling approaches to present an all-encompassing perspective on the scenario. Second, the presented scenario in the thesis accurately mirrors a multifaceted real-world situation that renders it highly pertinent for military strategists conducting analyses. Last, the description of methodologies employed within this research is comprehensive and clearly elucidated which enhances its accessibility to readers.

Weaknesses: The thesis's drawback lies in the limited movement of the ships in the model, as they can only travel in four specific directions. This characteristic does not align with a realistic scenario. Additionally, the enemy movement lacks strategic planning and is instead random, further deviating from realism.

This thesis centers on the resolution of major challenges in the domain of finding an optimal path in a dynamic contested environment with an unexpected random threats, evaluating risks, and making knowledgeable choices to enhance the protection and well-being of maritime activities. The investigation carried out in this study has yielded numerous consequential achievements that are succinctly outlined as the following sections.

5.2 Future Directions

As we progress through the intricate and ever-changing realm of maritime security in the vicinity of Taiwan, there are numerous areas that present enticing prospects for additional exploration and advancement.

5.2.1 Improved Threat Prediction and Modeling

Our study employed a selected Diffuse Gaussian weapon model to analyze threats, which demonstrates unpredictable movement patterns. This decision was made to facilitate the assessment of the various possible routes can arise from such a threat in its environment. Furthermore, the inclusion of random movement in our model permits us to simulate the erratic behaviors exhibited by adversaries. Ultimately, through the implementation of this all-encompassing methodology for threat analysis, we aspire to acquire a better understanding of the dynamics associated with this particular form of threat. However, it is crucial to establish cutting-edge advanced threat prediction models that can effectively provide accurate and efficient threat assessments in maritime environments by utilizing historical data, real-time sensor information, and the power of machine learning techniques. These sophisticated models have been specifically engineered to seamlessly adapt to the ever-changing landscape of evolving threat scenarios, consequentially bolstering overall maritime security measures (Auslander et al. 2012).

The identification of dangers in maritime environments presents a significant obstacle due to the intricate combination of vessel activities occurring simultaneously. Among these activities, only a small portion may display anomalies, raise suspicion, or present a potential threat. Previous investigations on this topic have been limited to examining individual vessels using simple rule-based models that alert watchstanders when a proximity threshold is exceeded. The Defense Technical Information Center (DTIC)-published report titled "Maritime Threat Detection Using Probabilistic Graphical Models" argues that probabilistic graphical models (PGMs) provide a more effective approach to modeling complex maritime scenarios. In this examination, Auslander et al. (2012) evaluates the performance of PGMs in detecting attacks on small boats at sea and describes three distinct types of PGMs that differ in their ability to accurately represent situations and assess their effectiveness in identifying threats using track data from force protection naval exercises involving unmanned sea surface vehicles. The results indicate that the most successful PGMs can outperform the currently employed rule-based strategy for these tasks; however, certain PGMs necessitate significant engineering efforts and are computationally demanding.

By implementing a more effective threat detection system and precise damage function, there is room for improvement in our model when it comes to locating the best route in a dynamic contested maritime setting.

5.2.2 Human Factors and Decision Making

In the field of maritime security, there is a growing interest in understanding the impact of human factors on overall safety and protection. This human factors encompass a range of aspects including crew training, decision-making processes, and how individuals respond to security threats. The aim is to delve deeper into these areas by examining various facets that contribute to effective maritime security measures. By focusing on crew training, we can explore ways to enhance skillsets and knowledge among personnel, equipping them with the necessary tools to mitigate potential risks and threats. Decision-making plays a crucial role in maritime security as well; analyzing the cognitive processes involved in making accurate judgments can lead to improved strategies for handling challenging situations. Additionally, investigating human responses to security threats offers insights into behavioral patterns and can inform future protocols and procedures aimed at ensuring quick and effective countermeasures. Altogether, exploring human factors within maritime security broadens our understanding of the multifaceted nature of this field, ultimately paving the way for more comprehensive approaches towards creating safer environments at sea.

5.2.3 Real-world Deployment and Testing

In order to establish the efficacy and feasibility of our systems in real-world maritime surroundings, we are dedicated to performing thorough testing and verification. Our strategy entails working closely with various industry participants, governmental bodies, and marine operators to ensure a thorough evaluation of our models and decision support systems. Through the cooperative endeavor, our objective is to demonstrate the genuine capabilities of our systems in practical maritime scenarios. In order to promote extensive approval and effective implementation, it is imperative to uphold a consistent practice of testing and improvement in practical situations to guarantee the continual efficiency and significance of our systems in addressing developing dangers and challenges in maritime domains.

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