ARL-TR-9900 ● APR 2024

Biomedical Continuous-Time Digital Signal Processing Applications

by W Michael Crowe, Patrick Jungwirth, and Sina Najmaei

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REPORT DOCUMENTATION PAGE

1. REPORT DATE	**2. REPORT TYPE**	**3. DATES COVERED**	
		START DATE	**END DATE**
April 2024	Technical Report	1 November 2023	29 February 2024

4. TITLE AND SUBTITLE
Biomedical Continuous-Time Digital Signal Processing Applications

5a. CONTRACT NUMBER	**5b. GRANT NUMBER**	**5c. PROGRAM ELEMENT NUMBER**
5d. PROJECT NUMBER	**5e. TASK NUMBER**	**5f. WORK UNIT NUMBER**

6. AUTHOR(S)
W Michael Crowe, Patrick Jungwirth, and Sina Najmaei

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	**8. PERFORMING ORGANIZATION REPORT NUMBER**
DEVCOM Army Research Laboratory ATTN: FCDD-RLA-NC Aberdeen Proving Ground, MD 21005	ARL-TR-9900

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**	**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**

12. DISTRIBUTION/AVAILABILITY STATEMENT
DISTRIBUTION STATEMENT A. Approved for public release: distribution unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Continuous-time digital signal processing (CT-DSP) is an emerging subfield of signal processing. CT-DSP offers three advantages for low power medical electronics and biomedical sensing applications: fewer sample points with higher information content (form of lossless compressive sensing), lower power, and better signal-to-noise ratio compared to conventional digital signal processing.

For continuous time sampling, a 3 times reduction in the number of electrocardiogram sample points has been demonstrated while obtaining a 97% accuracy rate in identifying heart arrythmias. A continuous-time control system for a switching DC-to-DC converter has been developed with 3 times improvement in control system signal overshoot. For a 4-bit equivalent analog-to-digital converter (ADC), an offline reconstruction of a continuous-time signal has achieved greater than 100-dB signal-to-noise-and-distortion (SINAD) ratio. A continuous-time pipeline ADC has been developed. This ADC overcomes limitations present in conventional pipeline ADCs.

A significant issue with continuous-time systems is signal reconstruction. Discrete time systems are linear time-invariant (LTI), and signal reconstruction is conveniently delayed until the final step in the signal processing chain. Continuous-time systems are not LTI, and signal reconstruction is not time-invariant. Research has shown that the simplest CT-DSP reconstruction technique provides modest improvement in SINAD compared to conventional DSP. Initial results from ongoing real-time reconstruction research indicate that it is possible for a 30-dB SINAD improvement.

This report presents an introduction to CT-DSP for medical and biomedical sensing applications. The potential improvements covering low power, better signal-to-noise ratio, and fewer data points offer significant capability improvements for low power, battery operated, biomedical applications.

15. SUBJECT TERMS
continuous-time digital signal processing; digital signal processing; DSP; low power; biomedical applications; Biological and Biotechnology Sciences; Network, Cyber, and Computational Sciences

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**
a. REPORT	**b. ABSTRACT**	**C. THIS PAGE**		
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UU	18

19a. NAME OF RESPONSIBLE PERSON	**19b. PHONE NUMBER** *(Include area code)*
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STANDARD FORM 298 (REV. 5/2020)
Prescribed by ANSI Std. Z39.18

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Acknowledgments

The researchers are grateful to the US Army Combat Capabilities Development Command (DEVCOM) Army Research Laboratory and DEVCOM Aviation & Missile Center for the opportunity to research and develop continuous-time systems. The authors are also grateful for the foundational research work done by Yannis Tsividis and colleagues at Columbia University.

1. Introduction

Digital signal processing cannot offer concurrent low-power, lossless compressive sensing, and higher signal-to-noise ratio (SNR); each must be traded against the others. Continuous-time digital signal processing (CT-DSP) is an emerging subfield of signal processing. CT-DSP offers three advantages for low-power medical electronics and biomedical sensing applications: fewer sample points with higher information content (form of lossless compressive sensing), lower power, and better SNR compared to conventional digital signal processing. Shannon sampling has inherent limitations,[1,2] which cannot be completely overcome with currently available architectures.

Continuous-time (CT) systems were first applied to control systems back in the 1950s. Inose developed the CT ($\Delta\Sigma$) analog-to-digital converter (ADC) in 1962.[3] In 2003, Tsividis[4] published the foundational paper on CT systems and started the modern digital signal processing subfield of CT-DSP.[3,5–20] Kurchuk published fundamental CT research in her 2010 dissertation.[7] Kurchuk et al.[8] developed a GHz-speed CT-level crossing ADC in 2012. Jungwirth and Crowe[20] developed a CT pipeline ADC in 2021. Kim et al.[16] developed a CT $\Delta\Sigma$ low-power ADC for wearable electrocardiogram (ECG) applications in 2022.

2. Review of Shannon Sampling Theorem

Figure 1 block diagram shows ideal Shannon sampling.[1,2] Shannon sampling theory is an ideal mathematical model that proves exact reconstruction is possible and establishes the means for discrete time sampling to accurately capture the details of bandlimited continuous time signals. However, in digital signal processing the ideal low-pass reconstruction filter cannot be built from real-world components. Practical implementations are an approximation to ideal Shannon sampling.

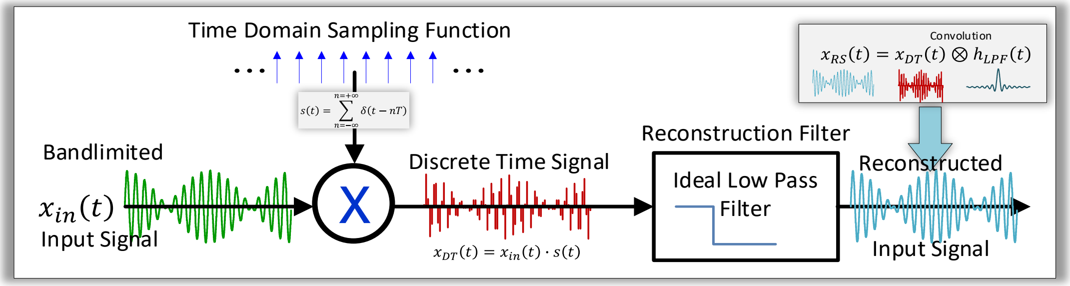


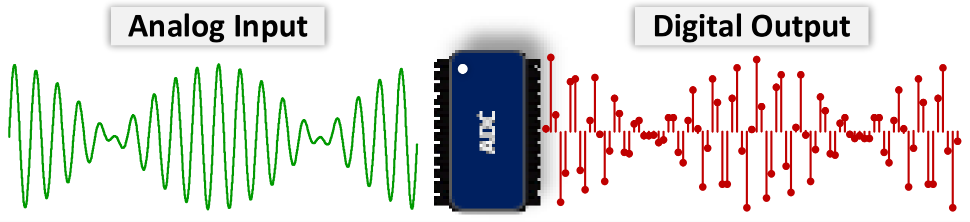
Fig. 1 Shannon sampling block diagram. Ideal low-pass filter can exactly reconstruct the input signal.

In Fig. 1, a bandlimited input signal is sampled, f_s times per second, by the sampling function, $s(t)$, an array of Dirac delta functions. Sampling function, $s(t)$, consists of an infinite number of frequency tones at multiples of the sampling frequency, $f_{c_n} = nf_s$. Discrete time signal, $x_{DT}(t)$, is the product of the bandlimited input signal, $x_{in}(t)$, and the sampling function, $s(t)$. Discrete time signal, $x_{DT}(t)$, contains an infinite number of amplitude modulation (AM) signals at the carrier frequencies, $f_{c_n} = nf_s$. If the sampling frequency is $f_s > 2f_{max}$, where f_{max} is the highest frequency present in the input signal, $x_{in}(t)$, then exact reconstruction is possible. If the sampling frequency is $f_s < 2f_{max}$, then frequency aliasing occurs because the infinite number of AM signals with carrier frequencies at $f_{c_n} = nf_s$ will overlap. Frequency aliasing cannot be removed.

$f_s > 2f_{max}$ Shannon's sampling theorem sampling frequency requirement (1)

3. Review of Quantization

Conventional discrete time ADCs are limited by quantization error. When an input signal is digitized, it is quantized in amplitude and time. The difference between the actual input signal and the quantized amplitude value is called quantization error. Bennett[21] developed and Widrow[22] refined the fundamental SNR equation for both ADCs and digital-to-analog converters (DACs). As illustrated in Fig. 2, the signal-to-quantization-noise ratio (SQNR) is a function of the number of bits n in the ADC or DAC, and the oversampling ratio (OSR). The OSR is defined as $f_{s,OSR} = OSR \cdot f_{s,min}$ where $f_{s,min}$ is the minimum Shannon sampling frequency, $f_{s,min} = 2f_{max}$, and $f_{s,OSR}$ is OSR times the minimum Shannon sampling frequency. For example, if the OSR = 256, the SNR is improved by $10log256 = 24$ dB.



$$\boxed{\mathrm{SQNR}(n, OSR) = 6.02n + 1.76 + 10\log(OSR)\ \mathrm{dB}}$$

Fig. 2 Fundamental ADC and DAC equation

CT systems[3–6] sample a signal when the input signal exactly equals a discrete voltage level. Since the difference between the sampled signal and the discrete voltage level is 0.0, there is no inherent quantization error. This is a significant advantage over conventional DSP. For a 16-voltage level (4-bit equivalent ADC), Vezyrtzis and Tsividis[4] demonstrated offline reconstruction of a CT signal with greater than 100-dB signal-to-noise-and-distortion (SINAD) ratio. An "equivalent" conventional ADC only has a $SQNR = 26$ dB (see Fig. 2 with $n = 4$ and $OSR = 1$). This simple example shows the potential of CT-DSP for DSP applications.

4. CT-DSP Introduction

CT control systems applications were first developed back in the 1950s. In 1962, Inose et al.[3] developed the first CT (or asynchronous) delta sigma ($\Delta\Sigma$) ADC. The $\Delta\Sigma$ ADC was patented in 1960 by Cutler.[9]

CT-DSP[14–20] is asynchronous like analog signal processing with discrete voltage levels like conventional DSP. Kurchuk's dissertation[7] provides a comprehensive characterization of these CT-DSP signals. They have no inherent quantization error, and they self-adapt to the slope of the input signal. The time between sample points is proportional to the slope of the input signal. For input signals with small slopes, only a few sample points are generated. Many data points are captured for high-slope waveforms. This is a form of lossless compressive sensing. Qaisar and Hussain[12] demonstrated this by reporting a 97% accuracy rate in identifying heart arrythmias, along with a 3 times reduction in the number of ECG sample points.

Zero quantization error and lossless compressive sensing are two significant advantages of CT-DSP over conventional DSP. The benefits of CT-DSP offer significant design advantages for low-power, wearable medical devices.[11–16] In the following sections, CT-DSP waveforms and CT-DSP low power applications are described.

4.1 Level Crossing Sampling and Level Crossing Delta

There are two representations of CT, level-crossing signals. In Fig. 3, level crossing sampling (LCS) creates a sample point when the input signal exactly equals a voltage level, resulting in a sequence of scaled Dirac delta functions. LCS signals are often represented as a sequence of time-amplitude vectors $(t_k, n_k\Delta)$.

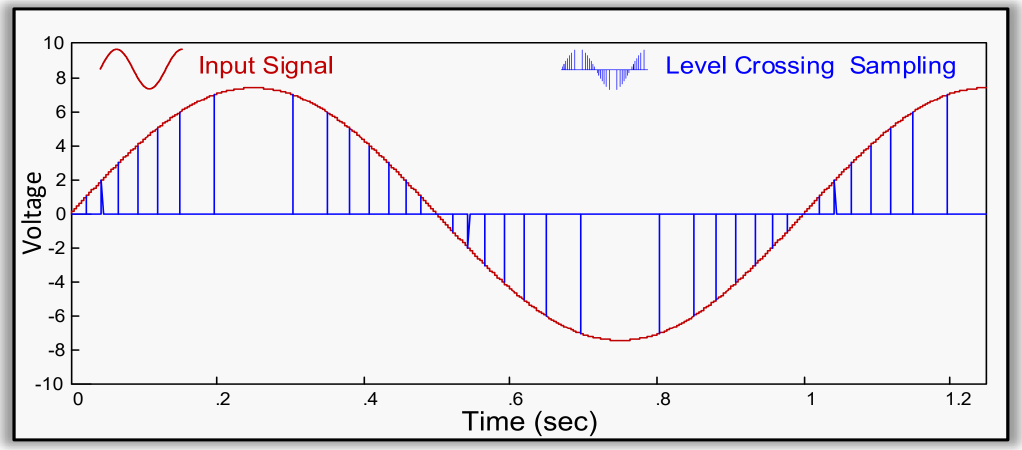


Fig. 3 CT LCS

Figure 4 illustrates level crossing delta (LCD) with half level offset. The LCD uses the same level crossing samples as LCS, but it applies an asynchronous zero-order-hold (AZOH) and half voltage level offset to create step functions. The half-step offset improves waveform symmetry and reduces error, just as it does for clocked ADCs.

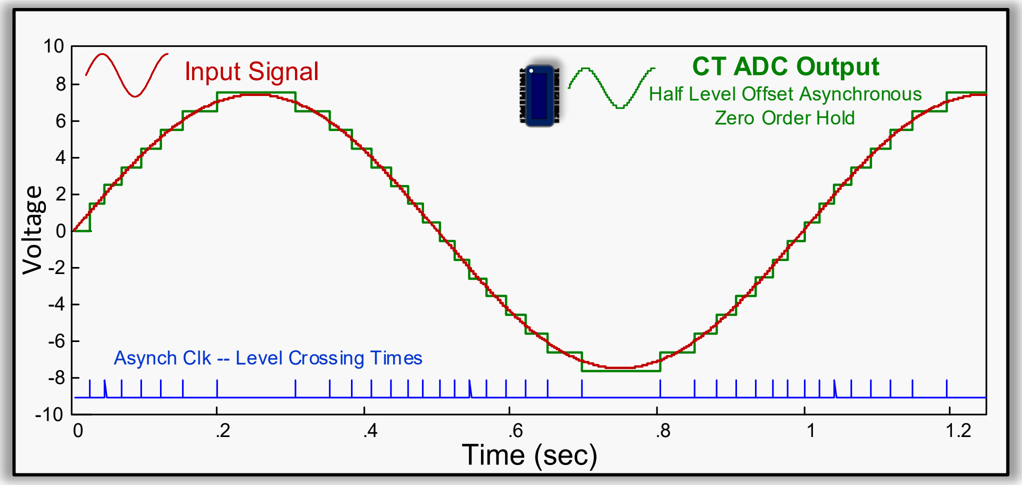


Fig. 4 Half-step offset level crossing (also called LCD)

4.2 CT-DSP Is Not Linear Time-Invariant

Conventional DSP is linear time-invariant (LTI). Since DSP is linear, signal reconstruction can easily be placed as the final signal processing step. In Fig. 5, Jungwirth and Crowe[18] proved LCS is not LTI. LCS requires some form of signal reconstruction prior to any signal processing steps. Tsividis[4] and Kurchuk[7] used the simplest reconstruction for LCS, the AZOH. AZOH is an asynchronous version of the zero-order-hold used in conventional digital signal processing.

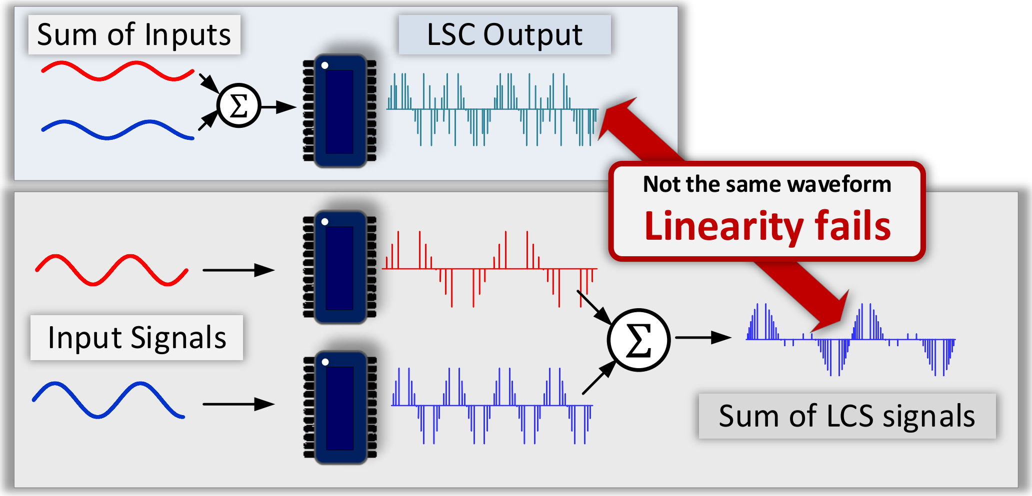


Fig. 5 **Linearity (superposition) fails for LCS signals. LCS signals must be reconstructed before any signal processing.**

4.3 Time Stamp Processing

LCS of a chirp waveform results in a constant number of sample points per cycle. Figure 6 shows eight sample points for every sine wave cycle in the chirp waveform. Conventional discrete time sampling requires a very large oversampling factor to capture a chirp waveform. For conventional discrete time sampling, there are many data points. For CT LCS, the sampling points provide both amplitude and timing information. For discrete time, a waveform must be scanned to find a specific pattern. As illustrated in Fig. 7, the LCS sample points contain time and amplitude information. This makes looking for time domain patterns simpler. The pattern analysis capabilities for CT systems could be very useful for medical sensing and diagnostics. For example, pulse positions may indicate specific heart conditions.

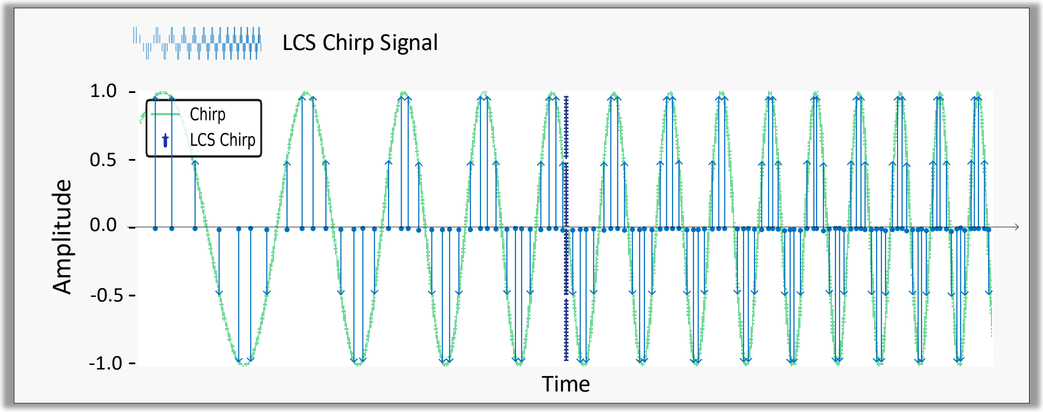


Fig. 6 LCS chirp waveform

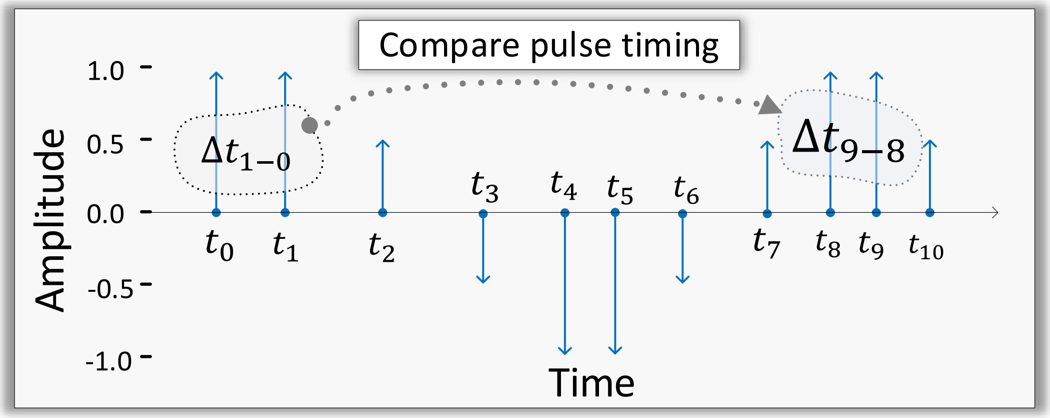


Fig. 7 Pulse timing comparison measurements

4.4 CT-DSP Application Review

Several low-power medical electronics applications have demonstrated the benefits of low-power CT systems: lossless compressive sensing, fewer data points, and power savings. CT-DSP offers significant design advantages for low-power, wearable medical devices such as ECG sensor systems.[12–17] The potential uses of CT pulse timing information, shown in Fig. 7, have not been fully explored. This has the potential of providing another data analysis method for ECG and brain wave use cases and may also benefit from techniques such as wavelet analysis.

Zhao and Prodic[17] developed a CT control system for a DC-DC converter with a 3 times improvement in signal overshoot. Energy efficiency for CT-DSP has been reported with up to 10 times improvement.[10] Figures 6 and 7 show the potential of continuous time for radar signal applications.

Vezyrtzis and Tsividis[5] show the potential for high-resolution CT ADCs. Using offline signal reconstruction, Vezyrtzis and Tsividis show a 16-level, CT ADC (comparable to a conventional 4-bit flash ADC) that achieves better than 100 dB SINAD. Jungwirth and Crowe[19,20] have developed a CT pipeline ADC that solves several performance limitations present in conventional high-speed, high-resolution ADCs. CT ADCs could potentially achieve effective number of bits (ENOB) = 10 bits (equivalent number of bits) at speeds of greater than 10 GHz.

5. CT-DSP ECG Signal Processing

CT systems is an active research area[11–16] for low-power, portable ECG monitoring equipment. As shown in Fig. 8, LCS results in fewer samples that provide very accurate timing information. The time-amplitude, $(t_k, n_k\,\Delta)$, vectors provided by LCS offer new ways to implement time domain signal processing and time domain pattern matching. As illustrated in Fig. 9, the timing information for ECG waveforms can be used for pattern searches as described in Fig. 7.

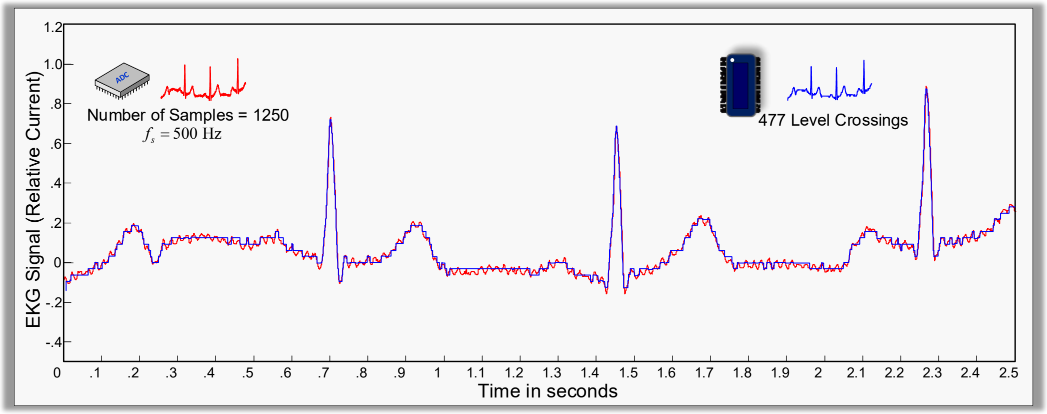


Fig. 8 Conventional DSP and CT-DSP electrocardiogram comparison. CT 64-level ADC only generates 477 samples compared to 1250 samples for conventional DSP. Due to the self-adaptive nature of CT sampling, the sharp ECG pulse is well defined.

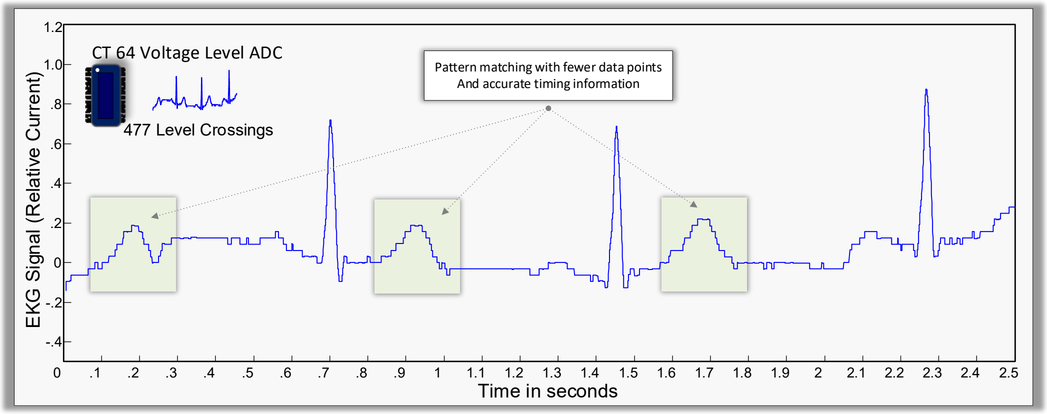


Fig. 9 CT-DSP electrocardiogram graph. CT LCS provides accurate timing information (see Fig. 8) with fewer data points. The combination of fewer data points with accurate timing information can help with pattern matching searches. Using CT for autocorrelations, and pattern matching is an open research area.

6. Conclusion

CT-DSP has already demonstrated its performance benefits for low-power, portable ECG sensing[12–16] and data processing. CT-DSP control systems[17] have demonstrated lower control system lag and much improved control system overshoot.

Medical electronics for insulin pumps and heart pacemakers require the conflicting requirements of low power, high accuracy, and stable control systems. CT techniques can better satisfy some of these signal processing and control system challenges. Analysis of CT pulse timing, which has not been fully explored, has the potential of providing another data analysis method for ECG and brain wave use cases.

For CT-DSP to enter mainstream engineering, three challenges must be overcome: availability of commercial ADCs, lack of mature engineering tools, and the need for high-performance, real-time reconstruction techniques. To address these challenges, the authors have designed a level-crossing ADC, which takes advantage of the nature of these signals and should provide a robust solution to CT signal capture. Engineering tools are currently being developed under an Army Small Business Innovation Research topic, “Continuous-Time Digital Signal Processing (DSP) Using Reconfigurable Devices.” Finally, the authors are developing a reconstruction technique for real-time systems that promises approximately 30-dB improvement in SNR over conventional DSP.

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List of Symbols, Abbreviations, and Acronyms

ADC	analog-to-digital converter
AM	amplitude modulation
AZOH	asynchronous zero-order-hold
CT	continuous-time
CT-DSP	continuous-time digital signal processing
DAC	digital-to-analog converter
DC	direct current
ECG	electrocardiogram
ENOB	effective number of bits
LCD	level crossing delta
LCS	level crossing sampling
LTI	linear time-invariant
OSR	oversampling ratio
SINAD	signal-to-noise-and-distortion
SNR	signal-to-noise ratio
SQNR	signal-to-quantization-noise ratio