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A HIGHLY ACCURATE SQUARE-LAW RECORDER

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ABSTRACT

The square-law recorder is an instrument designed to take signals from zero to one volt rms in a narrow frequency band centered at 50 kc as in input and record in strip-chart form the square of this input times some fixed constant. The squaring process is performed by keeping the power to the filament of a temperature-limited thermionic diode constant by reducing the direct current to the filament as an ac signal is applied. Since the maximum power supplied by the ac signal is only a small fraction of the total filament power, the decrease in direct current is proportional to the square of the alternating current. The system has negligible zero drift, and the power scale is inherently linear without the use of large correction factors. The accuracy of the chart readings is better than one percent for values above ten percent full-scale deflection, and the zero drift is less than one-half percent full scale over a six-hour period.

PROBLEM STATUS

This is a final report on one phase of this problem; work is being continued on other phases.

AUTHORIZATION

NRL Problem R02-13
Project RF 001-02-41-4004

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A HIGHLY ACCURATE SQUARE-LAW RECORDER

INTRODUCTION

The square-law recorder (Fig. 1) was designed at NRL for the Navy Hydrographic Office in connection with a sea-state-analysis problem. Sea-state analysis involves the study of ocean-surface conditions by means of statistical analysis, using the amplitude distribution and power spectrum of sea waves.* A sea-state analyzer which incorporates the square-law recorder was designed for the automatic processing of sea-state data. The sea-state analyzer (Fig. 2), except for the square-law recorder, is similar to an instrument used at NRL in the recording and analysis of audio and subaudio noise.† Samples of sea-wave data at subaudio frequencies amplitude-modulate a carrier that is at a higher frequency in the subaudio or low audio range. The modulated carrier is recorded on magnetic tape that travels at a constant speed of from 3 to 12.5 inches per minute. The samples of wave amplitude can be obtained directly from a wave-staff or other wave-amplitude-sensing devices, or the data can be transcribed onto the tape from samples that have been recorded on chart paper. The tape is cut into lengths that average about five feet, formed into loops, and played back at a speed of either 7.5, 15, or 30 inches per second. The signal from the loop being played back is fed through a linear detector to a narrow-band frequency analyzer in which frequency is scanned linearly with respect to time. The output of the frequency analyzer is recorded as a voltage on the Sanborn strip-chart and as a voltage squared times some fixed constant on the strip-chart of the square-law recorder.

The sea-state analyzer was originally designed to record only the voltage on the Sanborn instrument. From this record, points along the voltage plot had to be taken individually, squared, and then replotted. This was not only time consuming, it was also a source of errors, since the voltage curves had to be smoothed before the squaring process.‡ The square-law recorder eliminates these errors by squaring before smoothing. The steps of squaring individual points from the voltage curves and then replotting are also eliminated.

*W. J. Pierson, Jr., G. Neumann, and R. W. James, "Practical Methods for Observing and Forecasting Ocean Waves by Means of Wave Spectra and Statistics," U. S. Navy Hydrographic Office, Pub. No. 603, 1955.

†D. D. Howard, "Instrumentation for Recording and Analysis of Audio and Sub-Audio Noise," 1958 IRE National Convention Record, Part 5, pp. 176-182, 1958.

‡Smoothing was obtained through electrical damping in circuits preceding the Sanborn recorder and by mechanical damping of the recording stylus assembly. Also, the operator usually attempted to draw smoother curves before taking points. The errors occurred because in smoothing, an attempt was made by the operator to equalize the areas above and below the smoothed curves. However, the power contributed by the voltage represented by the areas above the smoothed line was more than the power not contributed by the voltages represented by equal areas below the line. Because of this, at times the replotted curves had values that were lower than those obtained when the tape data were run through a power meter.

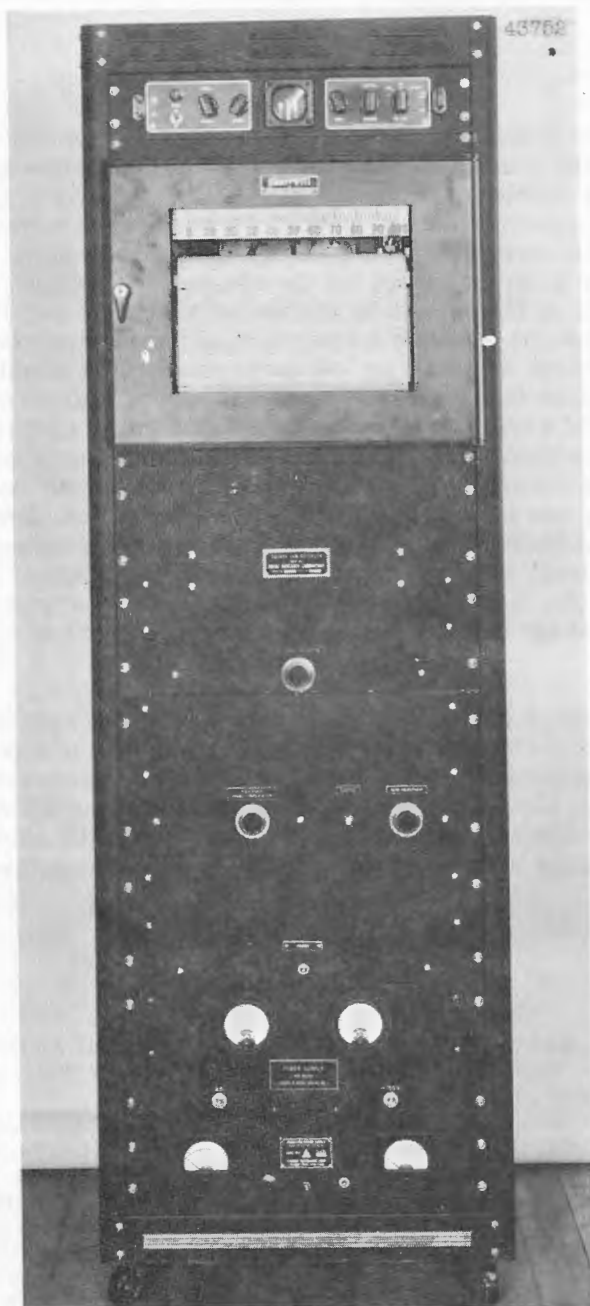


Fig. 1 - The square-law recorder

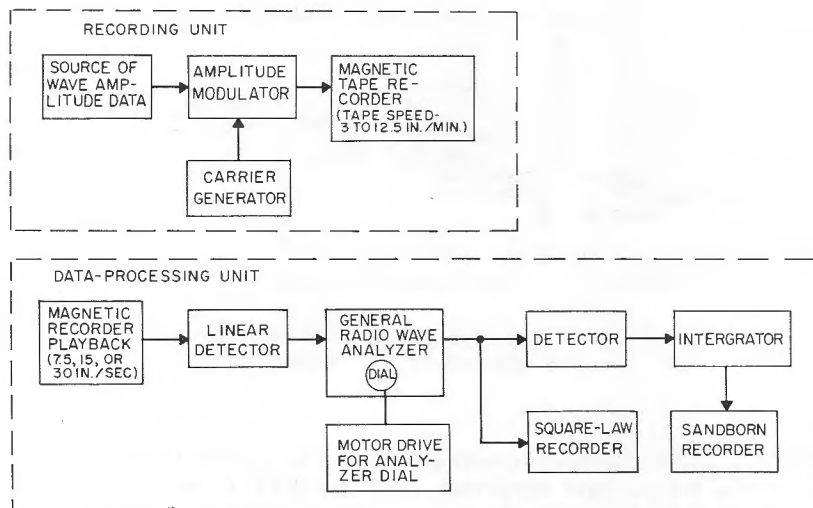


Fig. 2 - Block diagram of the sea-state analyzer

The square-law recorder uses a thermionic diode which is operated in a condition of temperature-limited emission. The filament is maintained at a given operating temperature by a direct current. An ac input signal supplies an additional amount of power to the filament; this is compensated for by removing a small amount of the direct current to keep the filament power constant.¹ The power supplied by the ac signal is small compared with that supplied by the direct current. The direct current therefore decreases as the square of the alternating current, thus providing a measure of the alternating signal power. This is an adaptation of a constant-power circuit developed by Reed Research Inc.*

A mechanical chopper eliminates a first-order drift error in the diode and allows the use of ac instead of dc amplifiers, which insures a high degree of stability. The other first-order instrument errors, which in themselves are small, tend to cancel each other.

THEORY OF OPERATION

Basically the sea-state-analysis system consists of a magnetic tape deck, a General Radio wave analyzer, a Sanborn recorder, and the square-law recorder. The wave analyzer is a commercial unit which has been modified so that the frequency dial can be driven mechanically. The wave analyzer is a heterodyne frequency meter which sweeps slowly through a band of frequencies in the low audio-frequency range. The amplitudes of the corresponding frequency components on the tape, which have been multiplied in frequency from their recorded frequencies to frequencies in the low audio range by the ratio of the playback tape speed to the recording tape speed, determine the amplitude of the wave-analyzer output signal. This signal is obtained from the i-f section of the wave analyzer and is at the i-f frequency of 50 kc. This signal is fed directly to the square-law recorder, and to the Sanborn recorder through a detector and integrating network.

The main components of the square-law recorder are the temperature-limited thermionic diode, a 60-cycle mechanical chopper, an ac amplifier with assorted filters, a servomotor, and a linear potentiometer (Fig. 3). The linear potentiometer, servomotor, and

*R. D. Campbell, "A Constant Power Circuit and Its Application to Instrumentation and Computation," Reed Research Inc., RR-503, Oct. 1949.

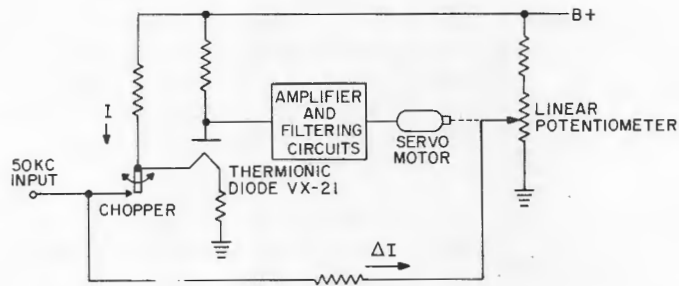


Fig. 3 - Simplified schematic diagram of the square-law recorder

part of the amplifier were obtained commercially as a single unit, the Minneapolis-Honeywell ElectroniK strip-chart recorder, model Y153X18, with modified pen-drive motor. The 50-kc signal is periodically superimposed upon the direct current of the diode filament by the mechanical chopper. The power dissipated by the filament is kept constant when the 50-kc signal is applied by shunting off a small amount of the direct current through the chopper. This can be expressed algebraically as

$$I_{ac}^2 R_f + (I - \Delta I)^2 R_f = c, \quad (1)$$

where I_{ac} is the 50-kc signal current, R_f is the hot-filament resistance, and c is a constant. Actually there are two direct currents involved, I and ΔI . The main or reference filament current, I , remains constant and is much larger than the other, ΔI , which can be considered a decrement of I and is subtracted from the filament current only when I_{ac} is applied. The amount of power dissipated in the filament by I_{ac} is only a small fraction of that dissipated by I . It can be shown through Eq. (1) that

$$I_{ac}^2 = K\Delta I - \Delta I^2, \quad (2)$$

where K , a constant, is in this case equal to $2I$, and ΔI^2 is the term that contributes nonlinearity (Appendix A). The ΔI^2 term can be neglected when ΔI is very small compared with K .

The decremental current ΔI is determined by the position of the linear potentiometer slider. The slider of the potentiometer is connected to the recording pen of the strip-chart recorder. The servomotor, which drives the pen and slider assembly, obtains its driving signal from the plate of the diode. The driving signal is developed when the power supplied to the diode filament with the chopper contact open differs from that supplied with the contact closed. If there is more power supplied to the filament when the chopper contact is closed, a signal is applied to the filament which results in a square wave of power being superimposed on the idling power of the diode filament. The square wave is shaped into a sine wave, which is applied to the motor with a phase that drives the slider of the potentiometer in a direction which increases the amount of current shunted from the filament. If less power is supplied when the contact is closed than when it is open, the phase of the servo driving signal is reversed and the slider moves in a direction to decrease ΔI . When the power to the filament with the contact open is the same as the power with the contact closed, there is no driving signal and the slider remains stationary.

The shaping of the square wave of power on the diode filament into a sine wave to drive the servomotor begins in the diode and is completed in the amplifier and filtering circuits. The square wave is integrated at the filament into a thermal sawtooth wave

which lags the square wave by 90 degrees. The integration is caused by the thermal time lag of the filament. Since the plate current of the diode is temperature limited, a corresponding sawtooth voltage appears across the plate-load resistor. This sawtooth voltage is applied to a frequency-selective amplifier. The frequency-selective amplifier converts the sawtooth wave into a sine wave by passing only the fundamental frequency of the sawtooth wave. The frequency-selective amplifier is also used to reject some of the noise that originates in the diode.

Noise sets a limit on how small ΔI can be made compared with K in Eq. (2). The parameters of the diode circuit were chosen so that the full-scale value of ΔI , defined as ΔI_{max} , was 0.22 ma and the filament reference current I was 5.84 ma (Appendix B). Using these values, the ratio $-\Delta I^2$ to $K\Delta I_{max}$ would be -0.019. If no correction were applied, then the instrument would have a maximum negative error of 1.9 percent occurring at full scale. This is shown in Fig. 4, where $K\Delta I$ is the ideal square-law or true power curve, and $K\Delta I - \Delta I^2$ is the actual diode curve. The figure also shows a curve, $K_1\Delta I - \Delta I^2$, in which the error has been reduced by choosing the zero and full-scale points on the chart scale as the points representing true power (Appendix C). This causes the maximum error to fall at midscale, and here the error is 0.95 percent. In the instrument this error is nearly cancelled by the linear potentiometer loading error (Appendix D). This was brought about by the proper choice of the linear-potentiometer circuit parameters. The potentiometer itself has a linearity error of approximately 0.1 percent. Assuming an ideal potentiometer with zero linearity error, the percent error (full scale) of I_{ac}^2 with respect to the position of the potentiometer slider was computed using the potentiometer circuit parameters and then plotted in Fig. 5. From the figure it can be seen that the linearity error introduced by a combination of the diode nonlinearity and the potentiometer loading effect has been reduced to less than the 0.1-percent linearity error of the potentiometer resistance.

Fig. 4 - Errors in diode square-law curve

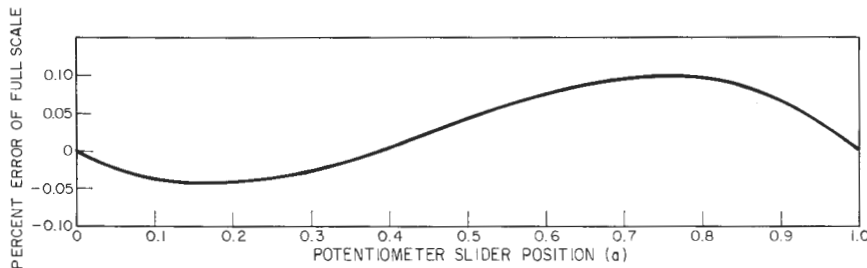
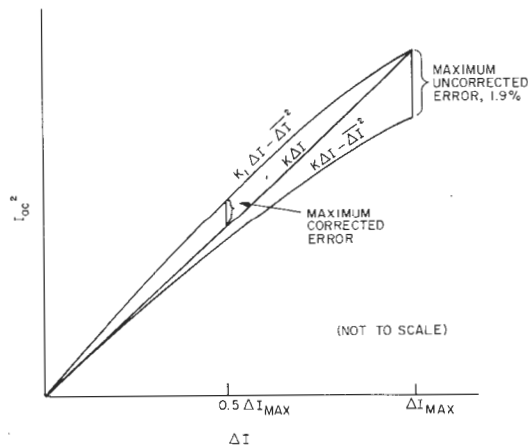


Fig. 5 - Error of I_{ac}^2 as a percentage of full-scale vs position of an ideal linear potentiometer slider

An attempt was made to measure the overall accuracy of the instrument experimentally, but a 50-kc signal source with the desired amplitude stability was not available. An oscillator with a power-output amplitude stability of about one-half percent over short periods of time was used in an experimental setup, and it was verified that the accuracy of the square-law recorder was at least as good as the amplitude stability of the oscillator.

There is a small, long-term drift at the upper end of the instrument scale. This is a secondary effect due to the drift in the diode filament electron emission with age. The diode has a finite plate resistance which changes as the filament electron emission changes. A small portion of the voltage square wave on the diode filament is coupled to the plate by the plate resistance. The square wave on the plate is compensated for in the following stage by an out-of-phase signal obtained from one of the diode filament terminals. The compensation signal is passed through a variable resistor which could remain at a fixed value, if it were not for changes in the plate resistance. The square wave on the diode plate requires compensation at the upper end of the instrument scale because the effects of filament-emission drift are apparent only in this region of the scale.

A first-order effect of the drift in the diode filament emission was eliminated by using the mechanical chopper and an ac servo system instead of a dc system. Besides the change in plate resistance that caused the drift at the upper end of the instrument scale, the drift in the filament emission also caused a change in the plate current of the diode. The drift in plate current causes a drift in the diode plate voltage, which in a dc servo system would not be distinguishable from variations of the plate voltage due to the input signal. The drift in plate voltage would show up as a zero drift in the instrument. With the chopper and ac amplifiers this zero drift is eliminated, since only the diode plate voltages that vary at a 60-cycle rate have any effect on the instrument. The problem of zero drift in the amplifiers themselves was also eliminated by using the ac servo system. This advantage over the dc system was somewhat offset by the requirement of bandpass filters for equalization of the ac system as compared with the simpler and less critical low-pass filters used in dc systems.

DISCUSSION OF CIRCUITS

50-KC Amplifier

Figure 6 is a block diagram of the square-law recorder. A very linear 50-kc amplifier was required between the output of the wave analyzer and the input of the chopper diode. An amplifier with two RC-coupled stages and a cathode-follower output stage was built that would supply an output of 10 volts at 8.3 ma. The open-loop gain was reduced with negative feedback from approximately 200 to 10 in order to improve linearity and stability.

Chopper-Diode and Preamplifier

The chopper-diode circuit was discussed previously. The 50-kc signal current I_{ac} and the incremental direct current ΔI are respectively applied to and removed from the chopper by separate high-value isolation resistors. The chopper interrupts the flow of these currents through the diode filament at a 60-cycle rate and causes a 60-cycle square wave of power to appear on the filament. The square wave of power is integrated into a thermal sawtooth wave which lags the square wave by 90 degrees. Since an increase in the filament temperature produces an increase in the emission of electrons, and since the diode is temperature limited, a voltage waveform corresponding to the thermal sawtooth wave on the filament appears on the diode plate.

There is a 510- $\mu\mu\text{f}$ capacitor across the diode output to filter out high-frequency signals, such as the 50-kc signal feedthrough, that are not needed in the following circuits.

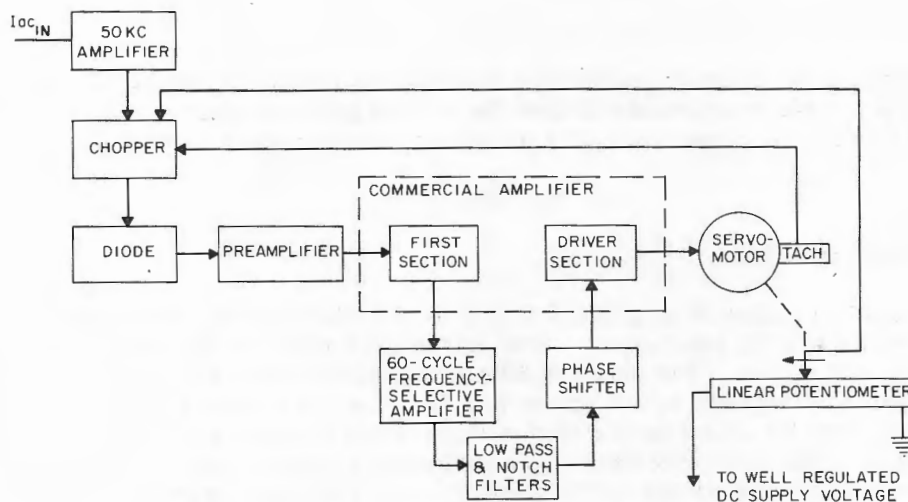


Fig. 6 - Block diagram of square-law recorder

The sawtooth wave from the diode plate is amplified by a triode preamplifier, which is RC coupled to the diode. The main filament power of the diode is obtained from a 150-volt regulated plate supply through a dropping resistor. The triode preamplifier also obtains its filament power in this manner. A 500-ohm resistor between the diode filament and ground was one of the parameters used in adjusting the accuracy of the instrument. If it were desired that the value of ΔI_{\max} be changed, a change in the value of this resistor and in the zero adjustment should be made.

The diode and triode tubes are microphonic, and for this reason they were sealed with wax into brass weights, which were in turn clamped between foam rubber blocks mounted to the chassis. This was done to prevent high-frequency mechanical vibrations occurring in the chassis from being transmitted to the tubes. Also, the chopper had to be set on shock mounts to prevent a 60-cycle mechanical vibration from being transmitted to the tubes. The entire chopper-diode and preamplifier assembly is enclosed to shield against stray electrical fields.

60-Cycle Frequency-Selective Amplifier

Besides the 60-cycle sawtooth wave, the output of the preamplifier also contains noise, most of which originates as shot noise in the diode. The sawtooth wave of the diode is not much above the noise level, because the linearity requirement necessitates that I_{ac} and ΔI_{\max} be much less than I_{dc} . To reduce this noise, the output of the preamplifier is fed into a frequency-selective amplifier which has a bandwidth of about 0.7 cycle. The frequency-selective amplifier also converts the sawtooth wave into a sine wave by attenuating the harmonics and amplifying only the fundamental.

The frequency-selective amplifier consists of a main amplifier section and a feedback section. The main amplifier is a triode section directly coupled to a cathode follower. The feedback section consists of a 60-cycle parallel-T rejection filter and an amplifier similar to the main amplifier section.

Filtering

The output of the frequency-selective amplifier is passed through a 120-cycle and a 180-cycle notch filter to attenuate further the second and third harmonics of 60 cycles. A low-pass filter with a corner at 120 cycles was used to reduce further the high-frequency noise.

Servomotor and Damping

There are two stages of amplification and an adjustable phase shifter between the filtering circuits and the servomotor. The servomotor receives its reference voltage from the 115-volt ac line. The phase shifter was adjusted so that the servo driving voltage would be 90 degrees out of phase with the reference voltage. The servomotor is damped electrically with a tachometer voltage which is applied to the chopper along with I_{ac} and ΔI . The main functions of the tachometer damping are to reduce the tracking-pen overshoot and to smooth the curve being traced. The tachometer is connected directly to an extension of the servo armature shaft. There is a 60-to-1 gear reduction from the servo armature to the pen and potentiometer slider pulley. The gear-train reduction ratio, which was originally 10 to 1, and the tachometer shaft input, which was originally taken from the reduction-gear output, were both changed to increase the smoothing effect. The system appeared to record the power of the signals from the tapes faithfully before the changes were made, but large excursions produced by the raw data from the tapes made it difficult to draw smooth curves through the curves traced by the instrument pen. It was therefore decided to limit the distance the pen could travel during one cycle of tape-loop travel by making the changes above. This distance can be varied by adjusting the damping control, which regulates the amount of tachometer voltage that is fed to the chopper.

The reason for using the tachometer to obtain damping instead of narrowing the bandpass of the carrier by filtering or by using a narrower 60-cycle frequency-selective amplifier was that to obtain the desired degree of damping, the bandwidth of the system would have to be narrower than the band over which variations in the line frequency occurred. Another reason of nearly equal importance was the difficulty of obtaining a stable system with the degree of narrow-banding required.

Figure 7 shows the response of the instrument to pulses of 30-second duration with just enough damping to eliminate overshoot, while Fig. 8 shows the instrument response with the damping control set at the position desired for typical sea-swell data. The chart moves one inch in 15 seconds. When the instrument is damped just enough to eliminate overshoot, the pen will move from zero to full scale in about six seconds. Before the gear-train modification, the pen would move from zero to full scale in about one second. Figure 9 is a plot of power spectral density made from a wave-staff signal.

Other Circuit Details Worthy of Note

There is a 60-cycle resonant-filter circuit in the output of the tachometer. This is because the reference voltage of the servomotor sets up a 60-cycle vibration in the servo armature when the drive signal is near zero. Without the filter, the tachometer, which is connected directly to the servomotor armature, injects a 60-cycle voltage into the servo drive-signal circuit. The injected signal is 90 degrees out of phase with the drive signal. This reduces the torque of the servo around the balance point and causes a sluggish response to small input signals. The amount of rejection required, plus the fact that a large inductance would cause the system to break into oscillation, necessitated the use of a relatively low inductance with a high Q in the resonant filter to obtain the required impedance level.

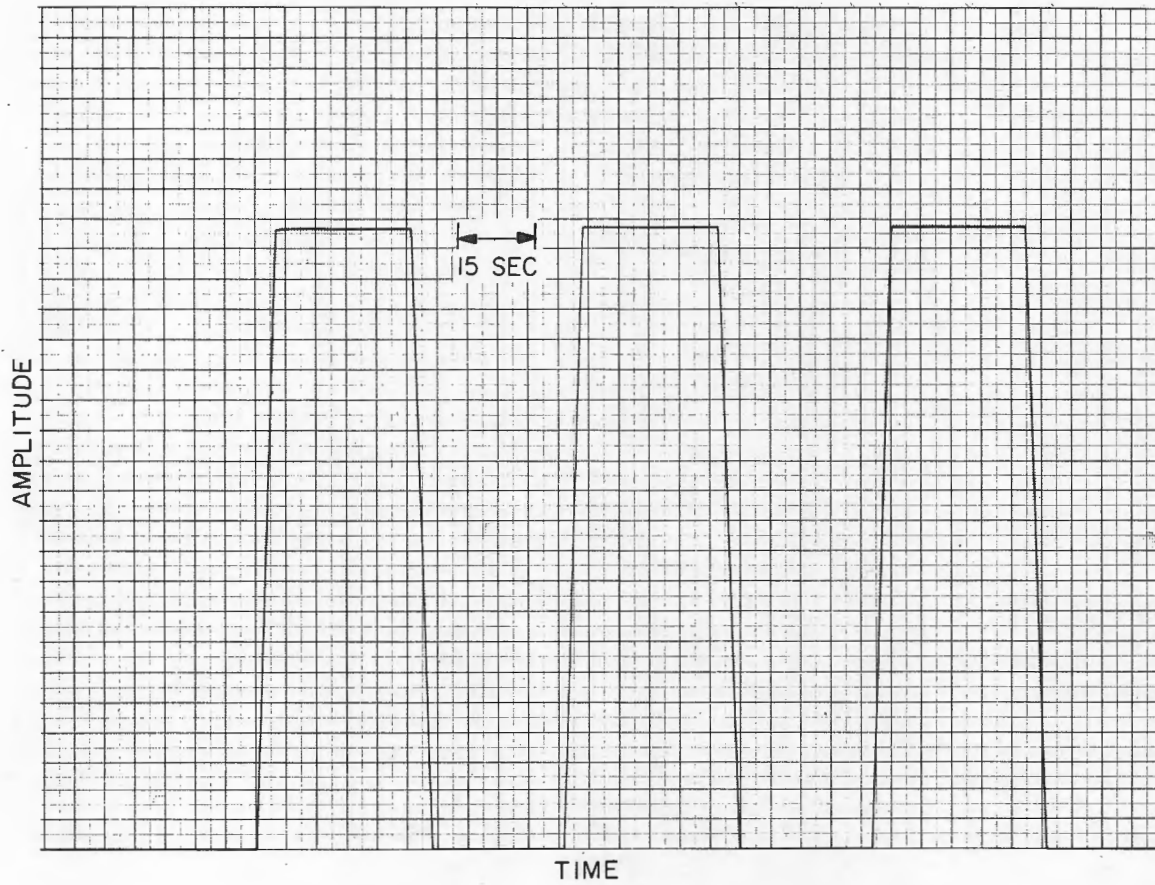


Fig. 7 - Response of instrument to rectangular pulses of 30-seconds duration with just enough damping to eliminate overshoot

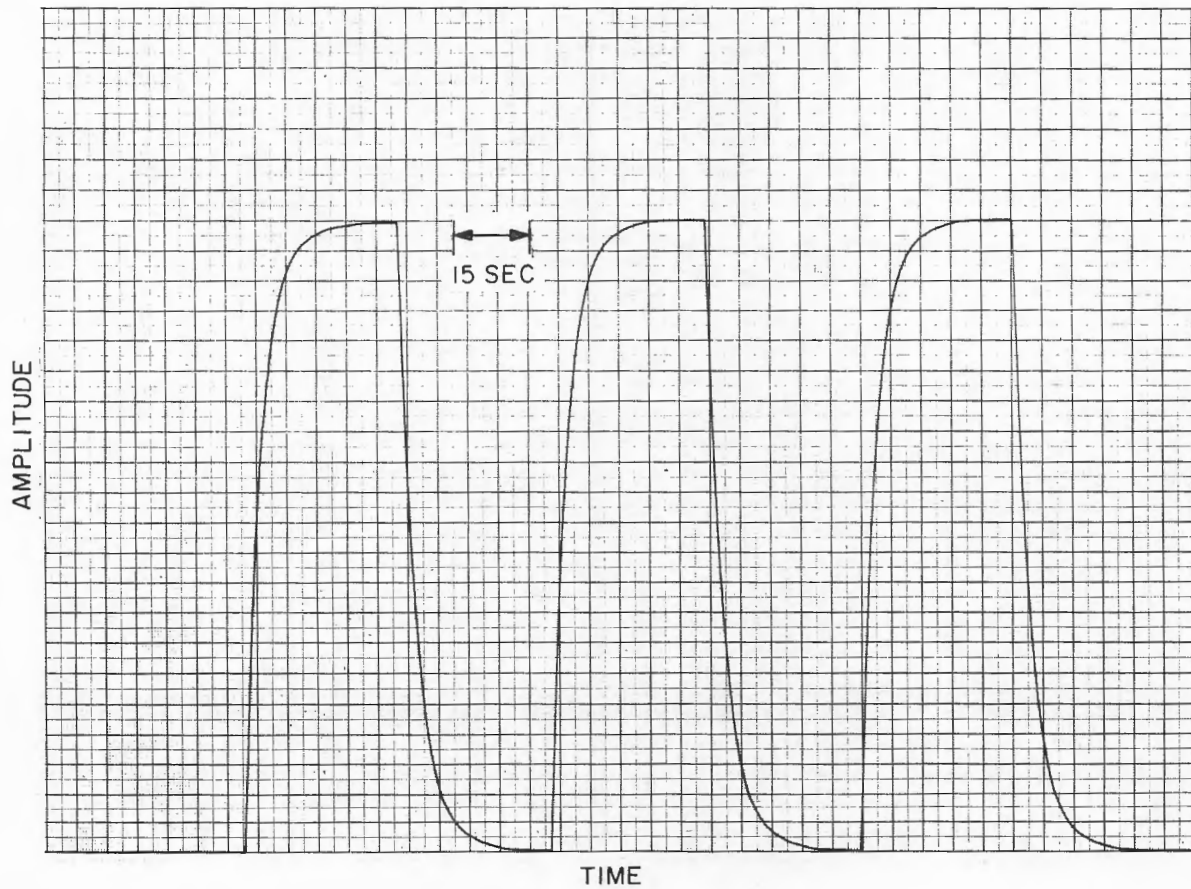


Fig. 8 - Response of instrument to rectangular pulses of 30-seconds duration with damping control set at a position desired for typical sea-swell data

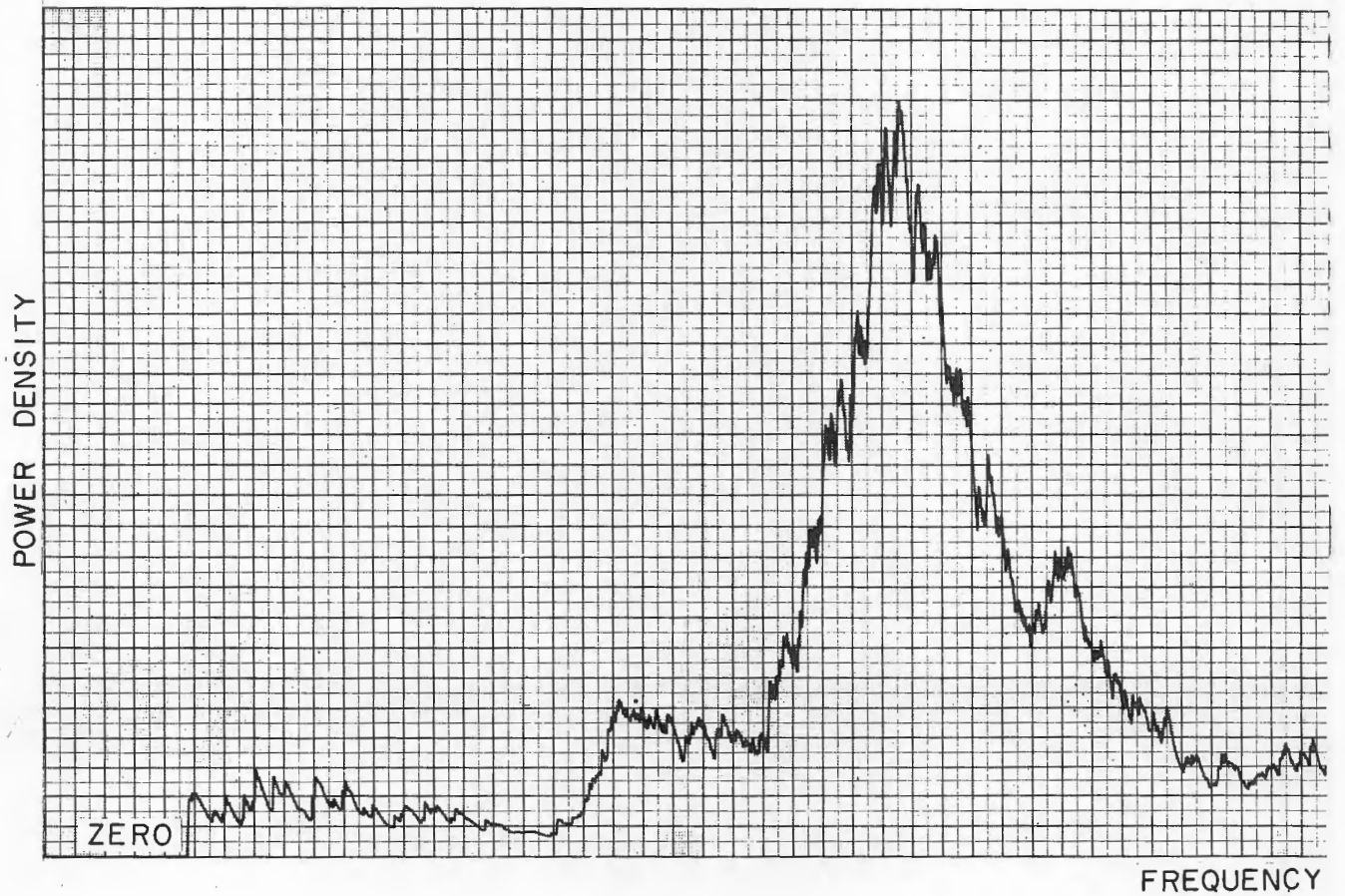
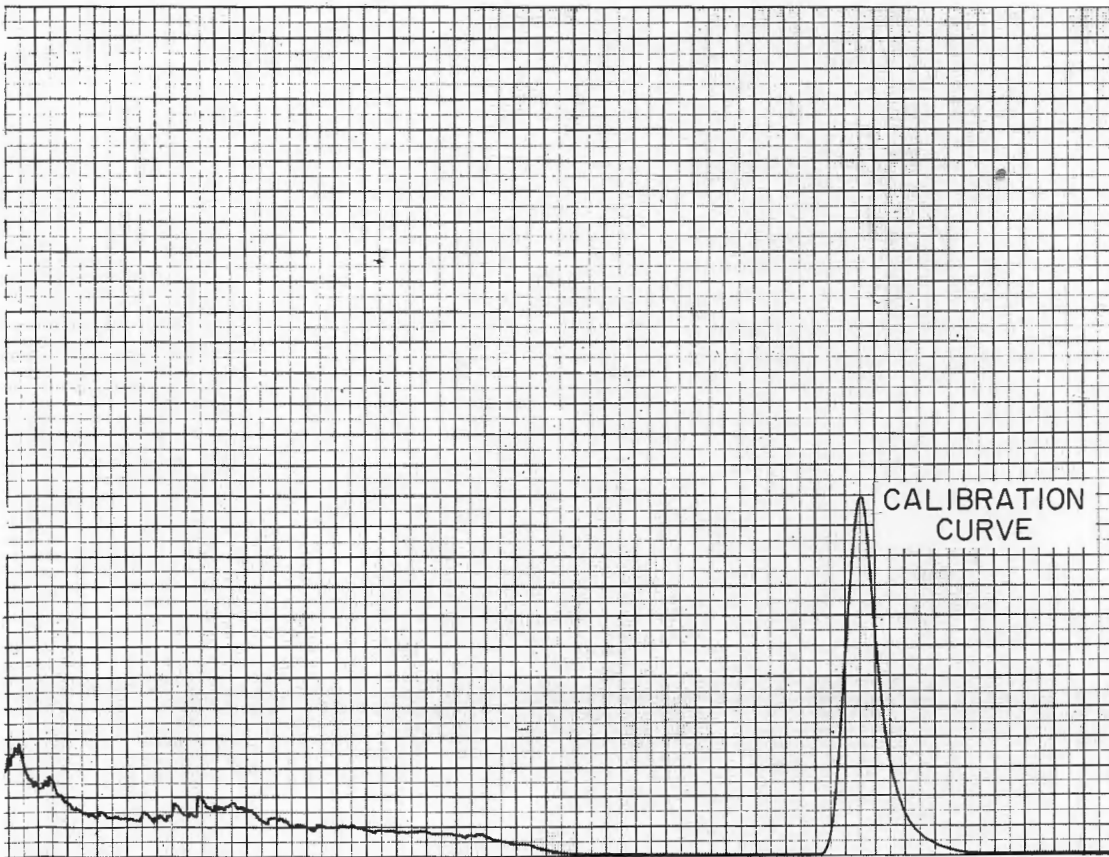


Fig. 9 - Power-spectral-density plot



of a wave-staff record

OTHER POSSIBLE APPLICATIONS OF THE INSTRUMENT

The square-law recorder may be adapted to record ac power from a few hundred cycles to well into the megacycle range by bypassing the 50-kc amplifier. The instrument would then require a 10-volt input from a low-impedance source for full-scale deflection. The lower frequency range of the instrument would then be determined by the thermal time constant of the diode and the chopper frequency. The frequency of the input signal must be high enough above chopper frequency so that any shift between the phase of the two frequencies will not cause a shift in power indication. The upper frequency range of the instrument would be limited by the series inductance and shunt capacitance of the diode filament and filament leads.

The rate at which the signal can be varied or modulated depends upon the maximum speed at which the pen will travel across the chart paper and upon the damping. When the modulating frequency is limited only by the maximum speed of pen travel, there is no definite corner frequency as with electrical damping, but the maximum frequency at which an input signal will be faithfully reproduced is proportional to the reciprocal of the amplitude of the input signal. Assuming the input signal to be a sine wave of power, the frequency limit is approximately one-third the reciprocal of the product of the maximum pen-drive speed times the height of the waveform on the chart. If the frequency of the input signal is increased above this point while its amplitude remains constant, the waveform on the chart will begin to distort, while the amplitude of the waveform will remain constant until the frequency has increased one and one-half times. As the frequency increases beyond this point, the amplitude of the waveform decreases. The pen of the square-law recorder requires six seconds to travel the width of the chart. Pen drives may be obtained which require from one-fourth to 24 seconds to travel the width of the chart. As the pen excursions become small, a point is reached at which the pen-drive speed is no longer a limiting factor on the frequency response of the instrument. At this point a transition takes place in which the mechanical damping of the moving parts and the electrical damping set in by the damping control become the sole factors determining the frequency response of the instrument, and a definite corner frequency is established.

ACKNOWLEDGMENTS

The author would like to acknowledge the many contributions of Mr. Arno M. King, who originally suggested the combination of the chopper and thermionic diode as the squaring device and who did much of the preliminary design work. The author also wishes to thank Mr. Christopher M. Morrow and Mr. Chauncey G. Myers for their advice on circuit design, and Mr. Calvin L. Moody for his assistance in construction of the equipment.

APPENDIX A

DERIVATION OF THE DIODE SQUARE-LAW EQUATION

The total power to the filament of the diode is

$$P = I_{dc}^2 R_f + I_{ac}^2 R_f, \quad (A1)$$

where R_f is the resistance of the hot filament, I_{dc} is the filament direct current, and I_{ac} is the current in the filament supplied by the 50-kc signal. When the direct current to the filament decreases as the current due to the 50-kc signal increases, the power to the filament remains constant. If the direct current that flows through the filament when the 50-kc current is zero is chosen as a constant reference current I , and if the symbol ΔI is used to represent the amount of direct current subtracted from I as I_{ac} increases, then

$$P = c = I^2 R_f \quad \text{when } I_{ac} = 0 \quad (A2)$$

and

$$I_{dc} = (I - \Delta I). \quad (A3)$$

Substituting Eq. (A3) into Eq. (A1),

$$P = (I - \Delta I)^2 R_f + I_{ac}^2 R_f \quad (A4)$$

or

$$P = (I^2 - 2I\Delta I + \Delta I^2) R_f + I_{ac}^2 R_f. \quad (A5)$$

Since the mode of operation requires that the power remain constant at all times regardless of the value of I_{ac} , Eqs. (A2) and (A5) can be set equal to each other.

$$(I^2 - 2I\Delta I + \Delta I^2 + I_{ac}^2) R_f = I^2 R_f. \quad (A6)$$

Dividing Eq. (A6) by R_f and then subtracting I^2 from both sides of the equation gives

$$I_{ac}^2 = 2I\Delta I - \Delta I^2, \quad (A7)$$

or letting $2I$ equal a constant K ,

$$I_{ac}^2 = K\Delta I - \Delta I^2. \quad (A8)$$

If there were no ΔI^2 term, then the $K\Delta I$ term would be proportional to the square of I_{ac} .

APPENDIX B

COMPUTATION OF DIODE FILAMENT CURRENTS

The values of I , the dc filament current when the ac power is zero, and ΔI_{\max} , the maximum incremental direct current, were determined in the following manner. Figure B1 is a diagram of a portion of the diode and linear potentiometer sections. All of the resistors are 1/10-percent precision except the linear potentiometer, which has a precision of about 1 percent, and the 10-K resistor, which is composed of a 7.5-K 1/10-percent precision resistor in series with a 5-K Helipot. The diode filament resistance was found by measuring the voltage drops across the 500-ohm resistance and the filament, and then solving the voltage-to-resistance ratios for the filament resistance.

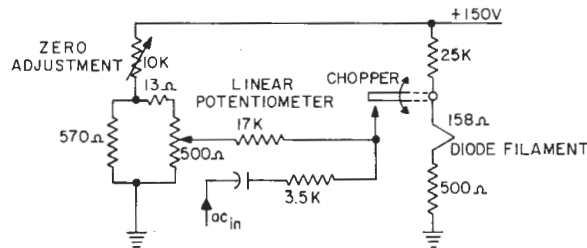


Fig. B1 - Diode and linear potentiometer circuits

Since no current flows through the chopper contact when the 50-kc current and ΔI are zero, the value of the filament reference current I was found by dividing the 150-volt supply voltage by the sum of the 25-K, the 500-ohm, and the filament resistances. The value of ΔI_{\max} was found by determining the current through the diode filament and 500-ohm resistor when they are shunted by a 17-K resistor and then subtracting this current from I . The value of I was found to be 5.84 ma, and that of ΔI_{\max} was 0.22 ma..

APPENDIX C

INCREASING ACCURACY OF INSTRUMENT BY ADJUSTMENT OF FULL-SCALE ERROR

If no corrections are made, and if the instrument is allowed to record on the exact diode curve, $K\Delta I - \Delta I^2$, there will be a maximum error at full scale. Using the diode filament currents that were found for the system in Appendix B, where $I = 5.84$ ma and $\Delta I_{\max} = 0.22$ ma, the maximum error δ is

$$\delta = \frac{-\Delta I_{\max}^2}{2I\Delta I_{\max}} = -0.019. \quad (C1)$$

This error is obtained when the slope of the line $K\Delta I - \Delta I^2$ equals the slope of line $K\Delta I$, at zero. If the instrument is set up so that the function curve of the instrument crosses the line $K\Delta I$ at the zero and full-scale points, the instrument will operate along a new curve,

$$I_{ac}^2 = K_1\Delta I - \Delta I^2, \quad (C2)$$

where

$$K_1 = K + b, \quad (C3)$$

K being equal to $2I$ and b being a constant. From Eqs. (C2) and (C3),

$$I_{ac}^2 = (2I + b)\Delta I - \Delta I^2. \quad (C4)$$

At full scale

$$I_{ac_{\max}}^2 = (2I + b)\Delta I_{\max} - \Delta I_{\max}^2 \quad (C5)$$

and also since the two curves intersect at full scale,

$$I_{ac_{\max}}^2 = 2I\Delta I_{\max} \quad (C6)$$

so that

$$2I\Delta I_{\max} = 2I\Delta I_{\max} + b\Delta I_{\max} - \Delta I_{\max}^2. \quad (C7)$$

Subtracting $2I\Delta I_{\max}$ from both sides of Eq. (C7) and then bringing the $b\Delta I_{\max}$ term to the left side of the equation,

$$b\Delta I_{\max} = \Delta I_{\max}^2. \quad (C8)$$

Dividing Eq. (C8) by ΔI_{\max} ,

$$b = \Delta I_{\max},$$

so that the new instrument function curve is

$$I^2_{ac} = (2I + \Delta I_{\max}) \Delta I - \Delta I^2. \quad (C9)$$

The error is now:

$$e = (2I + \Delta I_{\max}) \Delta I - \Delta I^2 - 2I\Delta I \quad (C10)$$

or

$$e = \Delta I_{\max} \Delta I - \Delta I^2. \quad (C11)$$

Differentiating Eq. (C11) with respect to ΔI and setting $de/d\Delta I$ equal to zero gives the point of maximum error at

$$\Delta I|_{e_{\max}} = \frac{\Delta I_{\max}}{2}. \quad (C12)$$

Substituting Eq. (C12) into Eq. (C11) and using the value for ΔI_{\max} found in Appendix B, the maximum error value is found to be 0.0095. This is one-half the value obtained with no corrections applied. By choice of points other than zero and full scale for the intersections of the two curves, $K\Delta I - \Delta I^2$ and $2I\Delta I$, the error could be reduced further. Full scale was chosen as a point of intersection because of its convenience as a check point when the operator is calibrating the instrument.

The diode nonlinearity is further reduced by the linear potentiometer loading effect. The combined effect of the linear potentiometer loading error and the diode nonlinearity on the instrument accuracy is determined in Appendix D.

APPENDIX D

EFFECT OF DIODE NONLINEARITY AND LINEAR POTENTIOMETER LOADING ON ACCURACY OF INSTRUMENT

The nonlinearity produced by potentiometer loading can be adjusted so that the nonlinearity of the diode may be nearly canceled by adjusting the values of the 10-K, 17-K, and 500-ohm resistors of the diode and linear potentiometer sections shown in Fig. B1. The dc equivalent circuit of the circuit of Fig. B1 with the chopper contact closed is shown in Fig. D1(a). Figure D1(b) is a Thevenin's equivalent circuit of Fig. D1(a). In Fig. D1(c) the position of the slider of the linear potentiometer is indicated as a linear function of a , where a is zero when the slider is furthest from ground and one when the slider is at ground. Figure D1(d) is a Thevenin's equivalent circuit of Fig. D1(c). This circuit was solved for the mesh currents, I_1 and I_2 . Current I_{dc} was found by subtracting I_2 from I_1 .

$$I_{dc} = \frac{5.84 - 0.214a - 0.0748a^2}{1.00 + 0.00136a - 0.0128a^2} \quad (D1)$$

$$I = I_{dc}|_{a=0} = 5.84 \text{ ma} \quad (D2)$$

$$\Delta I = I - I_{dc} = 5.84 - \frac{5.84 - 0.214a - 0.0748a^2}{1.00 - 0.00136a - 0.0128a^2} \quad (D3)$$

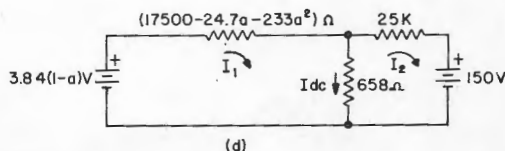
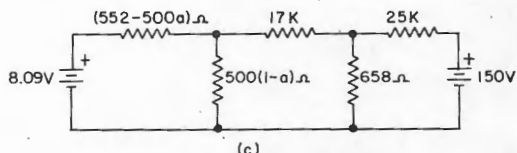
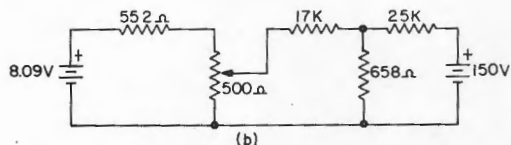
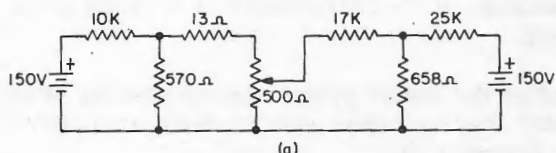


Fig. D1 - DC equivalent circuits of the diode and linear potentiometer sections

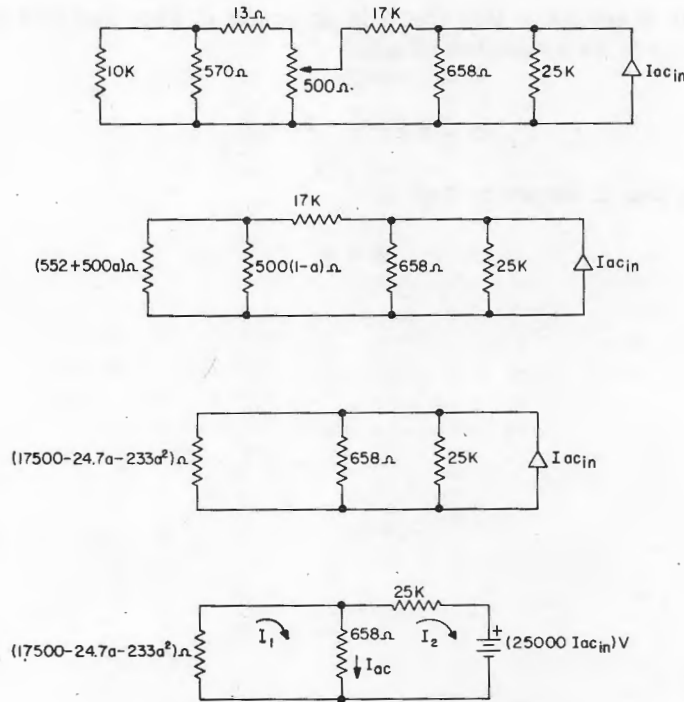


Fig. D2 - AC equivalent circuits of the diode and linear potentiometer sections

The ratio of the 50-kc current at the input of the chopper-diode circuit $I_{ac_{in}}$ to the 50-kc current in the diode filament I_{ac} also varies with a . Figure D2(a) is the equivalent ac circuit with chopper contact closed of the portion of the diode and linear potentiometer sections shown in Fig. B1. In Fig. D2(b), the resistances on the left in Fig. D2(a) have been combined, and in Fig. D2(c) the resistors on the left in Fig. D2(b) are combined. Figure D2(d) is a Norton's equivalent circuit of Fig. D2(c). By subtracting the mesh current I_2 from I_1 , I_{ac} is found in terms of $I_{ac_{in}}$.

$$I_{ac} = \left(\frac{0.939 - 0.001316a - 0.0125a^2}{1 - 0.00136a - 0.01233a^2} \right) I_{ac_{in}} \quad (D4)$$

From Eq. (A7),

$$I_{ac}^2 = 2I\Delta I - \Delta I^2 \quad (D5)$$

Substituting the functions of $I_{ac_{in}}$ and a of Eqs. (D2), (D3), and (D4) into Eq. (D5),

$$I_{ac_{in}}^2 = \left[2(5.84) \left(5.84 - \frac{0.0748a^2 + 0.214a - 5.84}{0.0128a^2 - 0.00136a - 1.000} \right) - \left(5.84 - \frac{0.0748a^2 + 0.214a - 5.84}{0.0128a^2 - 0.00136a - 1.000} \right)^2 \right] \left(\frac{0.01233a^2 + 0.00136a - 1.000}{0.0125a^2 + 0.001316a - 0.939} \right)^2 \quad (D6)$$

With the instrument is set up so that there is no error at zero and full scale ($a = 0$ and 1 respectively), the error as a function of a is

$$e_1 = 2.924a - Iac_{in}^2. \quad (D7)$$

A graph of this function is shown in Fig. 5.

* * *